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IEAS Working Paper

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IEAS Working Paper No. 04-A012

May, 2004

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May 27, 2004

Acknowledgements: I thank Ron Lee, Ken Wachter, David Steinsaltz, seminar participants at U.C. Berkeley and Academia Sinica for their useful comments and suggestions. Any remaining errors are of course my own.

Abstract

Existing economic theories of the evolution of altruism between kinship members usually emphasize the role that altruism can play in facilitating coordination among kin to achieve an otherwise unachievable efficient (in terms of fitness) equilibrium. In this paper, we explore the background environment against which backward altruism was likely to appear. The instinct of sustaining one's own life drives one to save for one's old age. However, since social mechanisms were not sophisticated in a primitive society, the rate of return on savings was not high. As a consequence, the resources that remain for the children might be limited. Suppose a cultural menchanism or a mutation caused an individual to become backward-altruistic. She would then expect her children to adopt the same attitude as herself, and take care of her in her old age. With this expectation in mind, she would avoid inefficient savings voluntarily so that her children could obtain more resources. Thus, backward altruism in our model does not play a role of coordination, but helps parents to avoid inefficient resource disposition. We analyze the possible appearance of backward altruism as the rate of return on savings changes.

1 Introduction

Biologist W. D. Hamilton made two distinct lines of contribution to the theory of evolution of human behavior.[1] The first (1966) concerns the evolutionary impact of changes in age-specific mortality; in particular, it is proved that the if a mutation decreases the mortality of post-reproduction ages, then it should not have any positive selection effect. Hamilton's second line of contribution (1964 a,b) concerns the evolution of altruism among close relatives; it explains when and why certain altruistic behavior can arise as an outcome of evolutionary dynamics among kinship members.[2]

One subtle tension between these two lines of contribution by Hamilton is related to the existence of "backward altruism," by which we mean the altruism (filial piety) from reproductive children to their post-reproductive parent. Specifically, transferring goods from the young child to the old parent obviously increases the former's mortality while reducing that of the latter. Since the young child is still reproductive and the old parent is not, according to Hamilton (1966), such an altruism-supported backward transfer is unlikely to be selected. This is the subtle tension we just referred to.

There have been several hypotheses in the literature that try to explain the appearance of kinship altruism. Bergstrom (1995) shows that in a prisoners' dilemma game, as long as the average payoff of being cooperative is large (relative to the payoff corresponding to the selfish strategy), then a cooperative mutant can survive or even dominate when these games are repeatedly

[1]The contribution itself is not restricted to the human species, but here we only emphasize its implications for human beings.

[2]Several economists have also contributed to the literature on the evolution of human preferences. See for instance Jack Hirshleifer (1978), Robert Frank (1988), Alan R. Rogers (1994), Ted Bergstrom (1995), and Oded Stark (1995); we shall come back to some of them later. Ingeman Hansson and Charles Stuart (1990) and Arthur Robson (1992) had some discussion concerning the evolution of emotion and risk attitude.

played by siblings. Oded Stark (1995) shows that if a young person's filial attitude toward her old parents can influence her own children's attitude toward herself in the future, then she will have an incentive to provide backward transfers, just to demonstrate to her kids how they are expected to behave when they grow up.[3] The common role of altruism in such literature is to coordinate or facilitate kin agents to adopt a mutually-beneficial strategy combination, which would not have been chosen otherwise.

If the existence of post-reproductive parents can benefit the children directly or indirectly, then it is not at all surprising to observe the appearance of backward altruism, as was shown in Lee (2003). But strictly speaking, being nice to people who can potentially help us is not really altruistic; altruism is better captured by the scenario of being nice to people who are *unable* to help us back. In human history and in this day and age, indeed we observe much backward altruism toward weak and post-reproductive parents, who are unable to assist their children in various aspects, and appear to be "useless" in terms of selection. The purpose of this paper is to investigate the possible appearance of backward altruism when parents are neither productive nor reproductive. It may not be appropriate to dichotomize the old parents into useless and useful types, but it is theoretically important to explore the possible appearance of backward altruism when parents are unable to provide downward service or transfer to their children.

There is a difference between the case of parents who are assumed useful

[3]There are also some economic theories of kinship transfers instead of kinship altruism. Gary Becker (1976) argues that, as long as the parent provides net transfers to her children, the children's backward transfers may actually be a strategic move of showing their superficial obedience, with the expectation that their parent will eventually leave them more net bequests. This is known as "the rotten kid hypothesis". Hillard Kaplan (1994) and Ronald Lee (2003) show that if old parents, although at their post-reproductive ages, can provide familial support such as helping take care of the grandchildren, then backward transfers may still be consistent with a positive selection.

and the case of those who are assumed useless. When parents are useful in terms of selection, their survival is a desirable status. The only reason that their children may not support them is some kind of coordination failure, either due to obstacles imbedded in a prisoners' dilemma (Bergstrom 1995), or due to the lack of commitment by future generations (Stark 1995). In either case, backward altruism often serves the purpose of enhancing the (subjective) reward of adopting the cooperative strategy. But when parents are useless, supporting the old no longer seems to be a desirable thing worth coordinating. Then the question is: might backward altruism arise for other reasons? Analyzing this scenario helps us derive the objective environment against which backward altruism is likely to appear. These conditions then may help historians or anthropologists infer the transfer structure of a given ancient society.

To make our discussion compatible with the conceptual scenario of Hamilton's, we consider an age-specific life structure with a condensed two-age life periods: young and old. Suppose people can produce goods and reproduce offspring only in their young age. If a young person has the instinct to survive to old age, then she has to save for her old age. In a primitive society, however, there were no efficient ways of savings. These ineffective savings crowded out the consumption of children, which in turn diminished their survival probability. If, however, an individual expected her children to support her old age's necessary consumption, then she did not have to save as much, and as a result the the resources that would otherwise have gone into savings could be used instead to support more of the children's survival, which obviously would improve selection. In short, out theory predicts that the appearance of backward altruism may be related to the rate of return or efficiency of the saving mechanism. Evidently, the coordination between kinship members, be it softwired culturally or hardwired genetically, was still important for the appearance of backward altruism in a primitive society. Thus, the theory proposed here is complementary to that in Bergstrom (1995) or Stark (1995).

The remainder of this paper is arranged as follows. In section 2 we introduce the formal model, and derive the dynamic decision specifications of individuals and the corresponding selection criterion. Sections 3 interprets the literature of backward transfers using our model, and explains the distinction embodied in our approach. Sections 4 and 5 present the main propositions and explain under what conditions backward altruism is likely to appear. Section 6 uses some numerical evidence to provide us with a more specific understanding of the advantage of backward altruism. The seventh section relaxes our assumptions and generalizes the analysis along some directions. The final section concludes.

2 The Model

Let us consider a one-sex overlapping generation model, in which each person lives either 1 or 2 periods. The first period is called young, and the second is coined old.[4] To facilitate our presentation, we always write "she" and "her" as the subject and object.

2.1 A Simplified Life Structure

At the beginning of a person's young age, she has a natural fertility of bearing $\bar{n}$ children. For a typical individual, the number of her children that can survive to the beginning of their own young life period may be any non-negative integer ($\tilde{n}$) less than or equal to $\bar{n}$. For technical convenience, we assume that there are only two possible realizations of $\tilde{n}$, low and high,

[4]Note that since there is only one overlapping life period, the grandparent, parent and child never coexist in the same period, and hence in our model the preference-shaping mechanism of Stark (1995) cannot be operative by assumption.

respectively denoted as $\tilde{n} = n_l$ and $\tilde{n} = n_h$. To avoid notational complexity, we simply assume $n_l = 0$ and $n_h \equiv n$.

The probability of having n (instead of zero) children surviving to their young age, denoted $p(.)$, depends on their nutrition condition as well as their parent's possible support to the young. We assume that

$$p = \begin{cases} p(c), & \text{if the old parent is dead;} \\ \mu p(c), & \text{if the old parent is alive,} \end{cases} \tag{1}$$

where $\mu \geq 1$ characterizes the possible help of old parents toward the survival of children, and $p(c)$ is an increasing and concave function of family consumption c, satisfying

Assumption 1: $p(0) = 0$, $p(c) \leq 1$ *for all* c, $p'(c) > 0$, *and* $p''(c) < 0$.

Clearly, if $\mu = 1$ in (1), then the post-reproductive old parent is "useless". We shall come back to its discussion later.

The parent may keep some savings for the consumption of her old age. If c_o is available for her old-age consumption, then the probability that she survives her old (second) period of life is $p_2(c_o)$. Because all concavity needed for an interior maximization has been provided by $p(c)$ according to Assumption 1, to simplify our analysis we shall assume that p_2 is linear in the relevant range:

Assumption 2: $p_2(c_o) = 0$ *if* $c_o \leq 0$; $p_2(c_o) = ac_o$ *if* $0 < c_o \leq b/a$; *and* $p_2(c_o) = b$ *if* $b/a < c_o$.

2.2 Two Kinds of Dynastic Utility Functions

As with all species, people are assumed to have forward altruism toward their offspring, in the sense that they care about the survival of their offspring. We therefore have a lineage "dynasty" similar to the one described in Laitner (1979) and Chu (1991). Suppose every person attaches a constant utility k to all current and future surviving offspring in the lineage. Because we have an overlapping generation structure, in each period there are family members of two different ages. We assume that the old are relatively weak to make any resource allocation decisions; hence all allocations are dictated by the young.

For a young individual with forward altruism, her dynasty utility in period 0 is assumed to have the following form:

$$U = k + \delta p_2(c_o)k + \sum_{t=1}^{\infty} \delta^t N_t Pr(N_t)[k + \delta p_2(c_t)k], \tag{2}$$

where k is the utility of surviving, $\delta < 1$ is the discount rate, c_o is the consumption she prepares for her own old age, $p_2(c_t)$ is the probability that the individual can survive her own old age given the period-t consumption c_t, and $Pr(N_t)$ is the probability that the individual has N_t young-age surviving descendants in period t.[5] In the above expression, the first two terms are the utility of the decision maker herself, and the summation term captures the utility from the survival of all future descendants.[6]

An alternative preference structure is that, other than the natural forward altruism specified in (2), each young individual also has "backward altruism",

[5]Since our purpose is to "explain" the origin of backward altruism, it is better that we do not "assume" other unreasonable preferences to start with. It seems then necessary for us to explain why in (2) there is a time preference $\delta < 1$. Fortunately, the literature has done some of the work. Rogers (1994) argues that if preferences have been shaped by natural selection, then individuals should be indifferent to choices that have the same fitness index. He shows that a time preference factor consistent with biological theories is about 2 percent per year. See also Hansson and Stuart (1990).

[6]Here we do not assume any utility of consumption for the following three reasons: First, it increases notation complexity without providing additional insight. Second, for most periods in the primitive environment, as the Malthusian theory describes, human beings were indeed fighting for their survival. Third, if we allow variable consumptions, then there will be variable savings, which in turn imply variable initial incomes for children. In that case we will have *income* as a continuous state variable. The corresponding Euler-Lotka equation will then be integrated, in addition to age, over variable incomes, which is unnecessarily complicated for our purpose.

in the sense that she cares about the survival of her old parent. In this case, the utility structure specified in (2) should be revised as

$$U = \beta p_2(m)k + k + \delta p_2(c_o)k + \sum_{t=1}^{\infty} \delta^t N_t Pr(N_t)[\beta p_2(m_t)k + k + \delta p_2(c_t)k], \quad (3)$$

where the first term indicates her utility from seeing her parent surviving her second period of life, m is the consumption of the currently old parent, m_t is the consumption of the period-t decision maker's parent, and β is the backward-altruism parameter assigned to the parent. Other terms in (3) are the same as those in (2). Evidently, if $\beta = 0$, then (3) degenerates to (2).[7]

2.3 Budget Constraint and Income Dynamics

As mentioned above, the young individual is the one who makes the resource allocation decision. If she has y amount of income to start with, then she has to divide it into consumption by family members (c), savings for her own future (s), and filial support (m) due to possible backward altruism. In general, the more family consumption there is, the better nutrition children will have, and hence the larger the survival probability the children face. This is why we propose Assumption 1 and write $p(.)$ as $p(c)$.

Suppose that for every unit of food saved, there will be η units for next-period consumption. For a young decision maker, her budget constraint is

$$c + s + m \leq y. \quad (4)$$

The young individual should maximize (2) or (3) subject to the constraint (4).[8]

[7]Suppose a child and a parent are mutually altruistic toward each other. Let the former's utility be $U = u + a_1 V$ and the latter's utility be $V = v + a_2 U$. The reduced form looks like $U = (u + a_1 v)/(1 - a_1 a_2)$. In our equation (3), the parameter of backward altruism, β, can be treated as $a_1/(1 - a_1 a_2)$. See Bergstrom (1997) for more details of the discussion.

[8]In reality, food consumption for the young and the old may be chosen differently, and

We now follow Lee (2003) and specify how the income dynamics are determined. Suppose the land size is A and the young population in the society during period t is P_t. The total output in the primitive society is $f(P_t, A)$. Assume that all young workers share the output equally, then each of them gets $w_t = f(P_t, A)/P_t$ of return. In a stationary state the population size is fixed,[9] so that w_t will be a constant w. We shall first concentrate on this stationary state, and then later come back to the more general case with variable w_t.

2.4 The Euler-Lotka Selection Parameter

Since there are only two life periods in our model, we have a simple discrete version of birth identity:

$$B_t = \sum_{a=1}^{2} B_{t-a} l_a m_a,$$

where l_a is the probability an individual can survive to age a, and m_a is the average number of births for a person of age a. Because we assume that people are reproductive only in their first period of life and that the number of surviving children is n with probability $p(c)$, the above equation can be further simplified as

$$B_t = B_{t-1} np(c).$$

Substituting $B_t = B_0 e^{rt}$ into the expression above presents the following Euler-Lotka equation:

$$e^r = np(c). \tag{5}$$

According to Hamilton (1966) and Rose (1991), a behavior or a mutation is said to be positively selected if it corresponds to a larger parameter r.

hence the young may have a wider decision domain. However, this concern complicates the analytical structure without warranting any further insight of inter-dependent utilities.

[9]There was nearly no population growth in the primitive society over a long period of human history. See Robson and Kaplan (2003).

Equation (5) clearly says that this positive selection can happen only if the corresponding c is larger.

3 An Interpretation of the Literature

As we mentioned in the previous section, if μ is significantly larger than one in (1), then the old parent's survival may help the selection. But even in this case, the children might not want to support their helpful old parents for various reasons. Below we shall provide a typical cause and relate it to the existing literature.

Suppose the parent needs z amount of resource to survive, which renders each child an extra burden $z/n \equiv \Delta x$. When each child contributes this resource, her optimization problem specified in section 2 changes. Suppose it costs her $c \equiv \Delta r/\Delta x$ in terms of fitness, where r is the Euler-Lotka parameter in (5). The survival of the parent increases the probability (by $\Delta\mu \equiv \mu - 1$) of live births, which is assumed to correspond to a benefit $b \equiv \Delta r/\Delta\mu$ in terms of fitness. If parents are helpful, we know that $b > c$. However, some of the parent's help to her n children may be in the form of wisdom or experience, which is a public good. Any individual child may have an incentive to free-ride on her siblings by not-supporting the parent while "overhearing" the wisdom told. This defection strategy benefits the child herself at the cost of her siblings.

As such, the problem among siblings reduces to the problem of coordination failure in a prisoners' dilemma game played by siblings, just like the one described in Bergstrom (1995). He showed that as long as the average payoff of being coorperative is large (relative to the payoff corresponding to the selfish strategy), a coorperative mutant can survive or even dominate. Intuitively, what backward altruism can do in this case is to raise the subjective payoffs corresponding to the "support-the-parent" (coorperative) strategy, and hence facilitate the realization of the cooperative equilibrium. Since parents' survival is beneficial to selection by assumption ($b > c$), the backward altruism in the context of being cooperative will be selected.

An implicit feature in the above example is the presumption that old parents are "useful" to selection and should be supported, but it may not be the case in general. If the ancient environment was so tough that most species die before the end of their reproductive ages, then they all have to fight for their survival to be selected. A long enough evolution would breed in nearly all species the tendency of fighting for their survival. After the exogenous environment gradually improves, even when the survival of the post-reproductive old may not be helpful to selection, the instinct of fighting for survival does not seem to disappear easily, as borne out by human experience and the experience of other species. This is consistent with the evolutionary pattern described in Charlesworth (1994 p.187).[10] But when "useless" people fight to survive, they crowd out resources of other individuals. We shall argue that, other than facilitating the coordination between siblings or across generations, backward altruism may also help avoid the adoption of inefficient strategies by the old. To highlight our point, in what follows we shall consider the extreme case where old parents are completely useless in the sense that they are not only post-reproductive but also no longer provide any downward transfers to their children. As such, we force ourselves to find out the possible objective causes that may push the children to support their old parents.

[10]For economists, it is not surprising to have a preference structure not entirely consistent with genetic fitness. Indeed, when people are assumed to maximize their "utility" function, almost by definition the utility index does not have to be related to genetic fitness. When demographic economists talk about "quantity-quality tradeoffs" of children, there seems to be little reason to believe that the quality improvement in children can compensate, in terms of the Euler-Lotka parameter, the drop in the quantity of children.

4 Should One Save for One's Old Age?

From now on we shall consider the case with $\mu = 1$ in (1), meaning that post-reproductive old parents are useless to their children. In this section we shall separate our discussion into two cases: without backward altruism and with backward altruism. Our strategy is to derive conditions under which parents' optimal savings are zero in the latter case and positive in the former case. Under these conditions, the role of children as "refrigerators" (efficient means of preserving resources other than savings) in the case with backward altruism is most evident, and the potential advantage of backward altruism is intuitively clear. Later we shall derive the exact parameter range that supports the selection of backward altruism.

4.1 Without Backward Altruism: Shall Save

For the preference structure with only forward altruism, as described in (2), it can be easily seen that the young individual's problem is characterized by the following Bellman equation:[11]

$$\begin{aligned} v(w) &= \max_{c+s\leq w} k + \delta p_2(\eta s)k + \delta\{n \cdot p(c) + 0 \cdot [1 - p(c)]\}v(w) \\ &= \max_{0\leq s\leq w} k + \delta p_2(\eta s)k + \delta np(w-s)v(w) \equiv \max_{0\leq s\leq w} \phi(s), \end{aligned} \tag{6}$$

where ϕ is the maximand on the right-hand side of (6). The meaning of the above expression is clear: the decision maker takes whatever her children can achieve $[v(w)]$ as given and then maximizes the sum of her own utility and the expected utility from all her surviving children. For (6) to be well defined, economists often make the following assumption, of which the interpretation is given below.

Assumption 3: $\delta n < 1$.

[11]Concerning the logic behind the Bellman principle, see for instance Sheldon Ross (1992) for details.

Because the random variable n is discrete (countable) and the $v(.)$ function is independent of n, the measurability of the product set spanned by c, s, and n is trivially true. Furthermore, $k + \beta p_2(\eta s)k$ is evidently positive, bounded, and continuous in s, and Assumption 3 warrants the boundedness of the value function. Applying Theorem 9.2 of Stokey and Lucas (1989 pp. 246-7), we know that expression (6) does characterize the optimal solution of the agent under Assumptions 1-3. Note that although v is not affected by the decision variables c and s, it is indeed a function of parameters such as η and δ. We shall use this property in our proof in the Appendix.

Now we can establish our first Proposition:

Proposition 1: *Suppose Assumptions 1-3 hold. As long as c_o has not reached the upper bound b/a in Assumption 1, the young individual's optimal savings are positive in the scenario without backward altruism if and only if*

$$\Big(a\eta - \frac{np'(w)}{1-\delta np(w)}\Big) > 0.$$

The proof is given in Appendix 1. Intuitively, the larger $a\eta$ is, the more effective it will be for the individual to save for her own old-age support. The larger $np'/[1-\delta p(w)]$ is, the more valuable will be the expenditure on family consumption. Proposition 1 simply spells out the exact relationship of the above tradeoff.

4.2 With Backward Altruism: Shall Not Save

We next come to the case where the preference structure has forward as well as backward altruism, as described in (3). The young individual's problem, given that her old parent has s_0 savings, is characterized by the Bellman equation below:

$$v(w, s_0) \quad = \quad \max_{c+s+m\leq w} k + \beta p_2(\eta s_0 + nm)k + \delta np(c)v(w, s) + \delta[1-p(c)]p_2(\eta s)k$$

$$
\begin{aligned}
= \quad & \max_{c,s} k + \beta p_2(\eta s_0 + n(w-c-s))k \\
& +\delta np(c)v(w,s) + \delta[1-p(c)]p_2(\eta s)k \equiv \max_{c,s} \zeta(s,c). \qquad (7)
\end{aligned}
$$

In the above expression, the first term is the utility of the young decision maker for her being able to survive, and the second term is her utility of seeing her old parent survive, of which the probability is $p_2(\eta s+nm)$. The argument in $p_2(.)$ has a term nm other than the old parent's remaining savings ηs, if each of the n now-surviving children donates m to the parent due to their backward altruism. The third term is the utility from future offspring; the v function has a second argument s because the state variable now includes the savings kept by the surviving parent.[12] If the decision maker has n surviving children in the next period, which has probability $p(c)$, then she will enjoy indirect utility $\delta nv(w,s)$.[13] The final term will be realized only if the decision maker does not have any surviving children, which has probability $[1-p(c)]$. In that case, she will count on her own savings in her old age and have $p_2(\eta s)$ probability of survival. From (7), we can establish:

Proposition 2: *Suppose Assumptions 1-3 hold. As long as c_o has not reached the upper bound b/a in Assumption 1, s and m cannot be positive at the same time. Furthermore, if $\delta\eta < \min\{n\beta, 1\}$, then the optimal saving (filial support) of an agent with backward altruism is equal to (larger than) zero. If $\delta\eta > \max\{n\beta, 1\}$, then the optimal saving (filial support) is larger than (equal to) zero.*

The proof of proposition 2 is given in Appendix 2. Because we have assumed a linear function for $p_2(.)$, it is not surprising that m and s, which are both means of preparing old-age consumption, cannot coexist, for one of

[12]Term s does not enter the indirect utility v of (6) in the case without backward altruism because children do not care how much the surviving parents save.

[13]Note that $m \geq 0$ warrants that the parent will do better by relying on her children (than living alone), hence (7) indeed characterizes her optimal solution in terms of residing status.

the two must have a larger marginal return. The second part of Proposition 2 tells us that if backward altruism (β) is strong enough, or if the return rate on savings (η) is small enough, then individuals would rather replace savings by other spendings. Intuitively, with strong backward altruism, people's old-age support is expected to be supplied by children, and hence it is unnecessary to use the means of ineffective savings to secure their old age. The only case of being insecure is when the person has no surviving children, which is an event with probability $p(c)$. And that is why we vary the values of $p(c)$ in our calculation in Appendix 2 to obtain our final parameter condition.

4.3 The Critical Range of η

Combining Propositions 1 and 2, we obtain a range of parameters in which an individual *will* save without backward altruism and *will not* save with backward altruism. The range $[\omega_1, \omega_2]$ is denoted R:

$$\omega_1 \equiv \frac{np'(w)}{a[1-\delta np(w)]} < \eta < \frac{\min\{1, n\beta\}}{\delta} \equiv \omega_2. \quad \text{[R]} \tag{8}$$

Expression (8) is important for our later analytical discussion, because when η is in range R, the unsaved resources under backward altruism can be used to feed the young children in order to increase the number of future descendants. In this sense, children become the *refrigerator* for their parents, which helps the latter "preserve" the food.

In (8), if η is very small, then the rate of return to savings is so low that the individual will not save anyway, even if there is no backward altruism. In this case, children will have no incentive to support the parents who would *not* dispose their resources inefficiently. Thus, backward altruism cannot be supported in this scenario. If η is very large, then savings are so rewarding that the individual will never want to give up this channel of resource disposition, even if there is backward altruism. In this case, the strategy of using children as refrigerators is no longer efficient, the optimal filial feedback (m) is zero, and backward altruism cannot be supported either. Expression (8) says that when the rate of return on savings is somewhere in between the two extremes, backward altruism is more likely to appear.

5 Comparing the Two Regimes in [R]

We showed in the previous discussion that expression (8) characterizes a *possible* region for backward altruism to be sustained. Now we propose to study when backward altruism will really arise.

Suppose expression (8) holds. As we showed in section 4, the optimal savings in (7) are always zero, so that the variable s and s_0 in the value function can be dropped. As such, since $p_2(0) = 0$ by Assumption 2, we can rewrite (7) as

$$v(w) = \max_{c+m \leq w} k + \beta p_2(nm)k + \delta np(c)v(w). \tag{9}$$

Comparing (9) with (6), we find that they are extremely similar: if we redefine the control variable m in (9) as s, and let parameter β equal δ and n in $p_2(.)$ equal η, then (6) and (9) are exactly the same. The similarity between (6) and (9) allows us to compare the size of optimal consumptions derived.

Specifically, we write an auxiliary equation which is a variation of (6) and (9):

$$v(w) = \max_{m \leq w} \Big\{ k + \alpha p_2(qm)k + \delta np(w-m)v(w) \Big\} \equiv \max_{m \leq w} \psi(m). \tag{10}$$

It is easy to see that (9) can be approached from (10) by letting α go to β and q go to n. Alternatively, if $\alpha = \delta$ and $q = \eta$, then we have the original state of (6). As α and q increase, we want to establish the comparative statics with respect to the optimal consumption c.

The first-order condition of (10) is

$$\frac{\partial \psi}{\partial m} = \alpha a q k - \delta n p'^{*} v(w) \equiv \Delta_m = 0, \tag{11}$$

where p'^{*} is the short-hand writing of $p'(w - m^{*})$. Assumption 1 guarantees that the second-order condition of maximizing ψ is satisfied. We prove in Appendix 3 the following two Propositions.

Proposition 3: *Suppose Assumptions 1-3 hold. If expression (8) is satisfied and (10) has an interior solution, then the sign of* $\frac{\partial m}{\partial \alpha}$ *is positive, meaning that the optimal consumption* $(w - m^{*})$ *decreases as* α *increases.*

Proposition 4: *Suppose Assumptions 1-3 hold. If expression (8) is satisfied and (10) has an interior solution, then the sign of* $\frac{\partial m}{\partial q}$ *is positive, meaning that the optimal consumption* $(w - m^{*})$ *decreases as* q *increases.*

Proposition 3 is intuitively clear: an increase in the backward altruism parameter increases a person's intention to leave more to the old parent, and hence the remaining consumption must be reduced. The interpretation of proposition 4 is similar, and is therefore omitted. Propositions 3 and 4 combined tell us that the optimal c^{*} decreases when α or q increases. As such, we can draw an iso-consumption line, as shown by l_1 in Figure 1, which must be negatively-sloped according to Propositions 3 and 4. To the northeast (southwest) of l_1, the consumption will decrease (increase).

Suppose the original $(\alpha_0, q_0) = (\delta, \eta)$ combination is at point A in Figure 1. If we consider a new scenario with $(\alpha_1, q_1) = (\beta, n)$, we want to know where the location of (β, n) is that may allow itself to be selected. Let the hyperbola passing through A be line l_2. From (8) we know that $\delta\eta < n\beta$ must hold in range R, so that the (β, n) point should be on the northeast direction of the hyperbola l_2. Expression (8) also implies $\omega_1 < \eta$, which defines an upper bound for n, denoted n^{*}, in Figure 1.

There are two possible cases for the shape of l_1, depending on whether or not line l_1 is steeper than the tangent line of l_2 at A. If l_1 is steeper (flatter), then case a (b) of Figure 1 applies. Combining the above, we know that for (β, n) to be selected, it can only be located in the lined area in either case. Note that the actual region for (β, n) to be selected is larger than the shaded area because $\eta < \omega_2$ characterizes the *sufficient* condition of zero-saving in the scenario with backward altruism; it is not a necessary condition.

Suppose, for instance, we are in a primitive society, so that η may be less that 1. Since $n > 1 > \eta$, we know that backward altruism can be selected only in case a of Figure 1. In this case, the parameter of backward altruism should be less than the discount rate δ. Another interesting contemporary case is when there is a high interest rate across periods so that η may be larger than n. In this situation, (β, n) may be selected only in case b of Figure 1, in which β is larger than δ.

6 Numerical Analysis

In section 5 we discussed extensively the change of c^* with respect to changes in (α, q). In this section we use a simulation to demonstrate the advantage of backward altruism when the rate of return on savings η and other parameters change.

We consider the functional form $p(c) = [c/(w+z)]^\theta$, and specify the following parameter values: $n = 2,\ \beta = .4,\ w = 1,\ a = b = .7,\ \delta = .45,\ k = 1, z = .001,$ and $\theta = .01$. It turns out that the critical values of η are

$$\frac{np'(w)}{a[1 - \delta np(w)]} = .2857, \quad \frac{1}{\delta} = 2.2222, \quad \frac{n\beta}{\delta} = 1.7778.$$

According to Propositions 1 and 2, we know that in the case without backward altruism the optimal savings amount is zero (so that the optimal consumption is 1) if and only if η is smaller than .2857. In the case with backward altruism, the optimal savings is positive if η is larger than 2.2222. Both these are confirmed in our simulation, as shown in Figure 2.

We now compare the consumption paths derived under different cases. For the case without backward altruism, our Proposition 4 says that the optimal consumption is a decreasing function of η, as long as a corner solution in $p_2(.)$ is not reached. When η reaches roughly 1.25, as one can see from Figure 2, this corner solution is reached, and savings (consumption) begin to decrease (increase). For the case with backward altruism, when η is less than 2.2222, the optimal savings are zero, so that the optimal consumption does not change with η, the rate of return on savings. When η is larger than 2.2222, the agent begins to take advantage of this good return and save. This helps to make her optimal consumption increase. In this case, savings become the main means of old-age support, and the role of children is not important. Thus, the two optimal consumption lines with or without backward altruism merge in this range. This is what we see in Figure 2.

Under the numerical values chosen, we see that the two optimal consumption lines cross each other. This means that there is a critical range of η^* (roughly $[.6, 2.0]$) such that only for η in this range, the optimal consumption corresponding to the case with backward altruism is larger, and hence backward altruism will be selected. Note that backward altruism will not be selected if the rate of return on savings is too low or too high, when the advantage of children acting as refrigerators is not revealed. Thus, in a gatherer-hunter society when most resources can only sustain for several days, or in our times where savings are likely to generate a rate of return much larger than rearing children, our theory as well as simulation suggest that such environment is not suitable to develop or to sustain backward altruism.

Now we consider the case of varying β. We choose $\eta = 1.25$ and reset $a = .5$; all other parameters are the same as before. As one can see from

Figure 3, the optimal consumption path with backward altruism is higher only when β is not very large. When β is larger than 0.28 in Figure 3, we see that backward altruism is no longer selected. This is consistent with what we described in previous sections: since old parents in our setup is not "useful" by assumption, a strong backward altruism cannot be selected, for it pays too much attention to the useless group. Useless parents by definition cannot provide much support to children; but even people who cannot provide help may still be less "harmful", if they can avoid disposing resources in an inefficient way.

7 Relaxing Some Assumptions

We have made some simplifying assumptions in order to derive the analytical results in previous sections. In this section we shall see how the structure should be modified if these assumptions are weakened. Here, we only point out the directions of possible extensions; analytical or simulation details are not specified here.

7.1 Useful Parents

In the previous few sections we have assumed, mainly for the purpose of argument, that $\mu = 1$ in (1), so that post-reproductive parents are not "useful" to their children. But as Lee (2003) pointed out, there are various forms of downward transfers, such as cooking and other household work that post-reproductive old folks can help their adult children. It can be seen that in this case, adult children will have some incentives to support their parents.

Specifically, with $\mu > 1$, equation (6) should be rewritten as

$$\begin{aligned} v(w, s_o) &= \max_{m,s,c}\Big[k + \delta\tilde{p}_2 k + \delta n p(c)\cdot[p_{2o}\mu v(w,s) + (1-p_{2o})v(w,s)]\Big] \qquad (12) \\ \tilde{m} &= \arg_m \max_{m,s,c}\Big[k + \delta\tilde{p}_2 k + \delta n p(c)\cdot[p_{2o}\mu v(w,s) + (1-p_{2o})v(w,s)]\Big] \end{aligned}$$

where $\tilde{p}_2 = p_2(\eta s + n\tilde{m})$, $p_{2o} = p_2(\eta s_o + nm)$, and the second argument in v characterizes the initial savings of the old parent. Individuals now know that their parents' survival is helpful to increasing p_1, and may have incentives to choose $m > 0$, as one can easily see from (12). In a steady state equilibrium, the optimal m chosen by a child must be the same as the one chosen by her mother, as shown by the second equation of (12). For the case with backward altruism, a similar formulation can be written. It is clear that attaching a positive term of backward altruism to the objective function helps increase the individual's incentive of backward transfers. When μ is large, backward altruism is certainly more likely to be selected.

7.2 More General Behavior Rules

Implicit in equation (7) was the assumption that once there is a change in period t that makes individual A carry the altruistic parameter β, the same attitude will be held for all future offspring of A. However, we know that this is not true, especially when the society is comprised of a variety of people and the reproduction of children involves parents from different origins. How should the analysis be revised under this situation?

Let $v(w, b)$ be the value function of an individual when the parameter of backward altruism she has is b. Suppose that $b = 0$ originally, and a change makes $b = \beta > 0$. Suppose there is a probability u that her children will have $b = \beta$ and $(1 - u)$ probability that her children will have $b = 0$. In this case, the individual should solve the following system of equations:

$$\begin{aligned}
v(w, s_0; \beta) \quad &= \max_{c+s+m \leq w} k + \beta p_2(\eta s_0 + nm)k + \\
&\delta np(c)[u \cdot v(w, s; \beta) + (1 - u) \cdot v(w, s; 0)] + \delta[1 - p(c)]p_2(\eta s)k \\
v(w, s_0; 0) \quad &= \max_{c+s \leq w} k + u \cdot \Big[\delta np(c)v(w, s; \beta) + \delta[1 - p(c)]p_2(\eta s)k\Big] + \\
&(1 - u) \cdot [\delta np(c)v(w, s; 0) + \delta p_2(\eta s)]. \qquad (13)
\end{aligned}$$

We shall skip the interpretation of the above equations.

An analysis of (13) will be difficult in general. However, it is not hard to see that if $u \to 1$, then $v(w, s_0; 0)$ is not relevant, the equation system in (13) actually degenerates to (7), and our original result remains true. The more complicated case is when u is variable according to the population composition in the society. The analytics along this line will have to rely on some simplified assumptions.[14]

7.3 Variable Number of Children

In our set-up in sections 3 and 4, there are only two possible realizations of the number of children: $n_l = 0$ and $n_h = n$. If, as it is in general, any positive integer $n < \bar{n}$ may be realized, then each decision maker may face a different number of siblings, which in turn involves a change in our setting. For an altruistic individual, the existing number of her siblings affects how the filial-support expenses are to be shared. The more surviving siblings there are, the less burden an individual will have from supporting her parent. Thus, other than the wage, the number of siblings will also become a state variable.

When there are many possible realizations of n, the Bellman equation in (7) should be rewritten as

$$\begin{aligned} v(w, s_0, n_0) \quad &= \max_{c+s+m \leq w} k + \beta p_2(\eta s_0 + n_0 m)k + \\ &\sum_{n=1}^{\bar{n}} \Big[\delta n p(n|c) v(w, s, n)\Big] + \delta[1 - \sum_{n=1}^{\bar{n}} p(n|c)] p_2(\eta s)k. \end{aligned} \tag{14}$$

In the above expression, $p(n|c)$ is the probability of having n surviving children when the family consumption is c, n is the number of her children, and n_0 is the size of surviving siblings of the decision maker. The meaning of (14) should be clear and hence we skip its interpretation.

[14]See for instance Juang (2001) and the references therein for more details.

As the reader can see, when we relax our assumption of fixed number of children, the space of state variables will become complicated. Conceptually, it may be the case that individuals with few surviving siblings tend not to support their parents, because the cost-sharing is less efficient. The analytical comparative statics seem to become difficult, but complications like this should not interfere with the simulation analysis.

8 Conclusions

As Hamilton (1966) pointed out, a well-known principle in biology is that a change in behavior which reduces the mortality rate of post-reproductive ages will not be selected. A corollary of the above proposition is that species are unlikely to develop backward altruism, because any resource distributed to an old parent must imply a reduction of resources to the children. Economists have made some contributions in justifying the existence of kinship altruism. It has been argued that if kinship altruism can facilitate the coordination between kins and promote the realization of an efficient outcome, then it will be selected.

In this paper we propose a theory of the evolution of backward altruism complementary to the existing ones. We assume that in a primitive economy, there are few goods or capital stocks that the old parent can save efficiently, although the instinct desire of sustaining one's own life induces the parent to save for her old age. Hence, the resources remaining for the children would be limited. Suppose cultural pressure or a mutation causes an individual become backward-altruistic. She then expects that her children may have the same attitude as hers, and therefore they may take care of her when she is old. With this expectation in mind, this individual can avoid inefficient savings and hence her children may obtain more food. Thus, the backward altruism serves the purpose of avoiding an inefficient way of disposing resources. Parents do not have to be useful to their children in order for the latter to support the former; in fact, backward altruism renders the parents "less harmful".

The altruistic support for the old in our model is somewhat like a pay-as-you-go pension system. These transfers can be rationalized if the young are likely to prepare their old-age support and dispose resources in an inefficient way. This is more likely to happen in ancient times when social mechanisms are primitive. Our expression (8) proposes a range of rate of return to savings in which backward altruism is likely to arise. It may be worthwhile for historians to study and verify whether such an inequality helps explain the origin of backward altruism in a human society.

Appendix 1: Proof of Proposition 1

Differentiating ϕ with respect to s and using Assumption 2 yield

$$\frac{\partial \phi}{\partial s} = \delta ak\eta - \delta np'(c)v(w) \equiv \Omega_s. \quad (A1)$$

We want to see when we will have an interior solution for s, and hence we should evaluate expression $(A1)$ at $s = 0$. If savings are set to be $s = 0$, then c must be $c = w$. Assumption 2 implies that the corresponding value function $v(.)$ from (6) should satisfy

$$v(w)_{|s=0} = k + \delta np(w)v(w)_{|s=0},$$

which gives us the solution $v(w)_{|s=0} = k/[1 - \delta np(w)]$. Evaluating $(A1)$ at $s = 0$ and substituting the above $v(w)_{|s=0}$ result back into $(A1)$, we have

$$\frac{\partial \phi}{\partial s}_{|s=0} = \delta k\Big(a\eta - \frac{np'(w)}{1 - \delta np(w)}\Big). \quad (A2)$$

Since $\partial^2\phi/\partial s^2 < 0$ by Assumption 1, we obtain Proposition 1.

Appendix 2: Proof of Proposition 2

We want to study when the optimal savings s will be zero in the scenario with backward altruism. Differentiating ζ in (7) with respect to s, we have

$$\frac{\partial \zeta}{\partial s} = -\beta akn + \delta np(c)\frac{\partial v(w,s)}{\partial s} + \delta[1 - p(c)]ak\eta. \quad (A3)$$

Using the envelop theorem, we see from (7) that $\partial v/\partial s_0 = \beta a\eta k$ for all s_0 in the relevant range. Substituting this result back into $(A3)$, we have

$$\frac{\partial \zeta}{\partial s} = ak\{\delta\eta[p(c)(n\beta - 1) + 1] - n\beta]. \quad (A4)$$

Note that the right-hand-side of $(A4)$ is independent of s, and only depends on c. This means that for any given s, unless the upper bound of $c_o = b/a$ of $p_2(.)$ is reached, $\partial\zeta/\partial s$ is either positive or negative, and hence m and s cannot be positive at the same time.

Concerning the sign of $(A4)$, we separate our discussion into two cases. If $n\beta \geq 1$, then the right-hand side of $(A4)$ attains its maximum at $p(c) = 1$. In this case we see that $\partial\psi/\partial s < akn\beta(\delta\eta - 1)$, which is negative if $\delta\eta < 1$. If $n\beta < 1$, then the right-hand side of $(A4)$ attains its maximum at $p(c) = 0$. In this case we see that $\partial\psi/\partial s < ak(\delta\eta - n\beta)$, which is negative if $\delta\eta < n\beta$. Thus, the optimal saving is zero if $\delta\eta < \min\{1, n\beta\}$.

Note that the optimal saving will be positive if the right-hand-side of $(A4)$ is positive. We can follow the same logic and see that $\delta\eta > \max\{n\beta, 1\}$ is a sufficient condition for s to be positive. In summary, we have Proposition 2.

Appendix 3: Proof of Propositions 3 and 4

Because the comparative static analysis of the Bellman equation is not conventional, in what follows we shall be more detail in our derivation steps.

Equation (10) tells us that $v(w) = k(1 + \alpha aqm^*)/(1 - \delta np^*)$, where $p^* \equiv p(w - m^*)$. Substituting this result back into the first-order condition (11), we have

$$\frac{\partial\psi}{\partial m} = \frac{\alpha aqk(1 - \delta np^* - \delta np'^* m^*) - \delta np'^* k}{1 - \delta np^*} = 0. \tag{$A5$}$$

By Assumption 3, $1 - \delta np^* > 0$. It is clear from the numerator of $(A5)$ that

$$1 - \delta np^* - \delta np'^* m^* > 0 \tag{$A6$}$$

must hold in order for $(A5)$ to have any interior solution.

Totally differentiating $\Delta_m = 0$ in (11) yields

$$\Delta_{mm} dm + \Delta_{m\alpha} d\alpha = 0.$$

We know that $\Delta_{mm} < 0$ by the second-order condition. As for $\Delta_{m\alpha}$, we have

$$\Delta_{m\alpha} = aqk - \delta np'^* \frac{\partial v}{\partial \alpha}.$$

From (10) we have $\partial v/\partial\alpha = am^*qk + \delta np^*(\partial v/\partial\alpha)$. Thus, $\partial v/\partial\alpha = am^*qk/(1 - \delta np^*)$. Substituting this formula back into $\Delta_{m\alpha}$, we get

$$\Delta_{m\alpha} = aqk\Big[\frac{1 - \delta np^* - \delta np'^* m^*}{1 - \delta np^*}\Big].$$

Inequality ($A6$) tells us that the numerator in the above square brackets is positive. Hence, we have Proposition 3.

Totally differentiating (11) we have $\Delta_{mm}dm + \Delta_{mq}dq = 0$. The sign of Δ_{mm} is negative by the second-order condition. Concerning Δ_{mq}, we have $\Delta_{mq} = \alpha ak - \alpha np'^{*}(\partial v/\partial q)$. Differentiating (10) with respect to q yields

$$\frac{\partial v}{\partial q} = \frac{\alpha akm^{*}}{1 - \delta np^{*}}.$$

Using this result, we see that

$$\Delta_{mq} = \frac{\alpha ak[1 - \delta np^{*} - \delta np'^{*}m^{*}]}{1 - \delta np^{*}}.$$

Similar to what we did to derive ($A6$), we see that $(1 - \delta np^{*} - \delta np'^{*}m^{*})$ must be positive in order to have an interior solution of c^{*}. Thus, $\Delta_{mq} > 0$ and we have Proposition 4.

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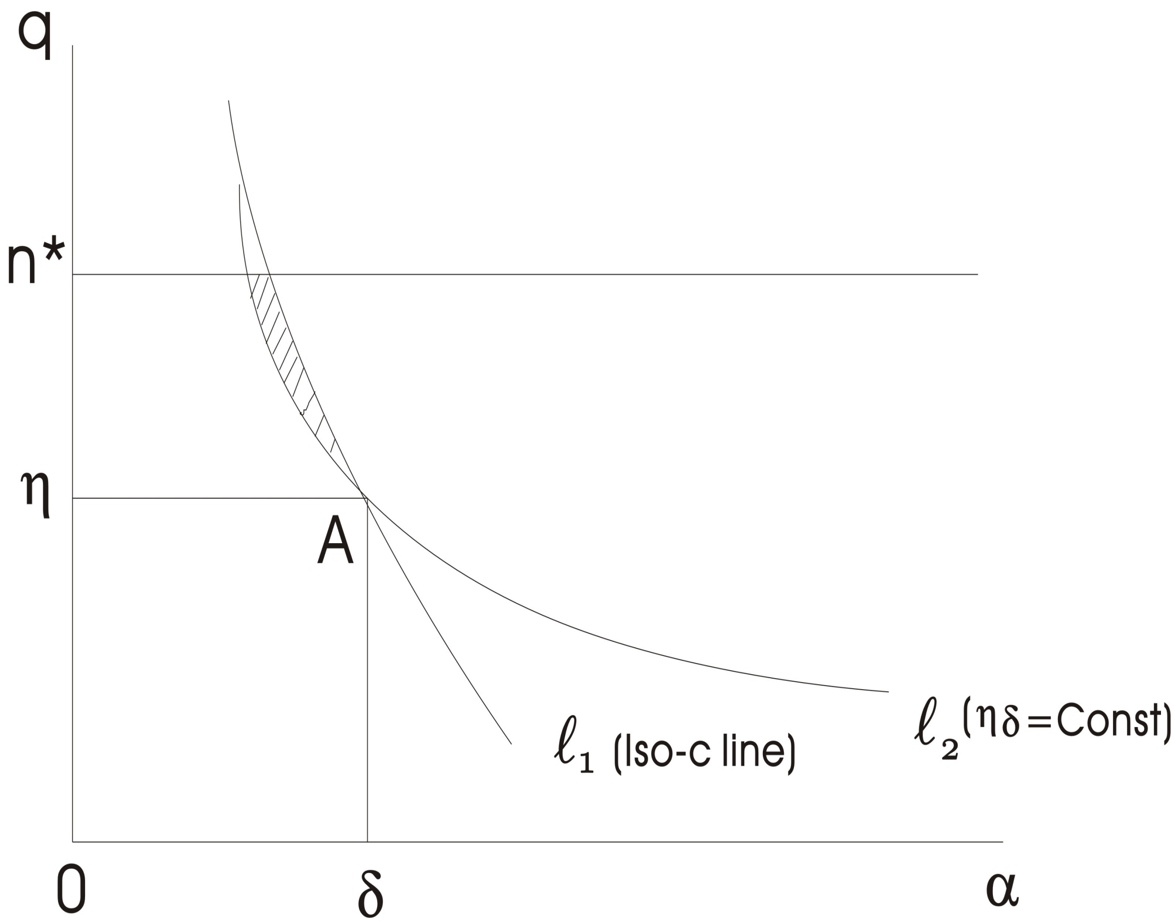

Figure 1a: The Parameter Range of (β, n) for Positive Selection, Case a.

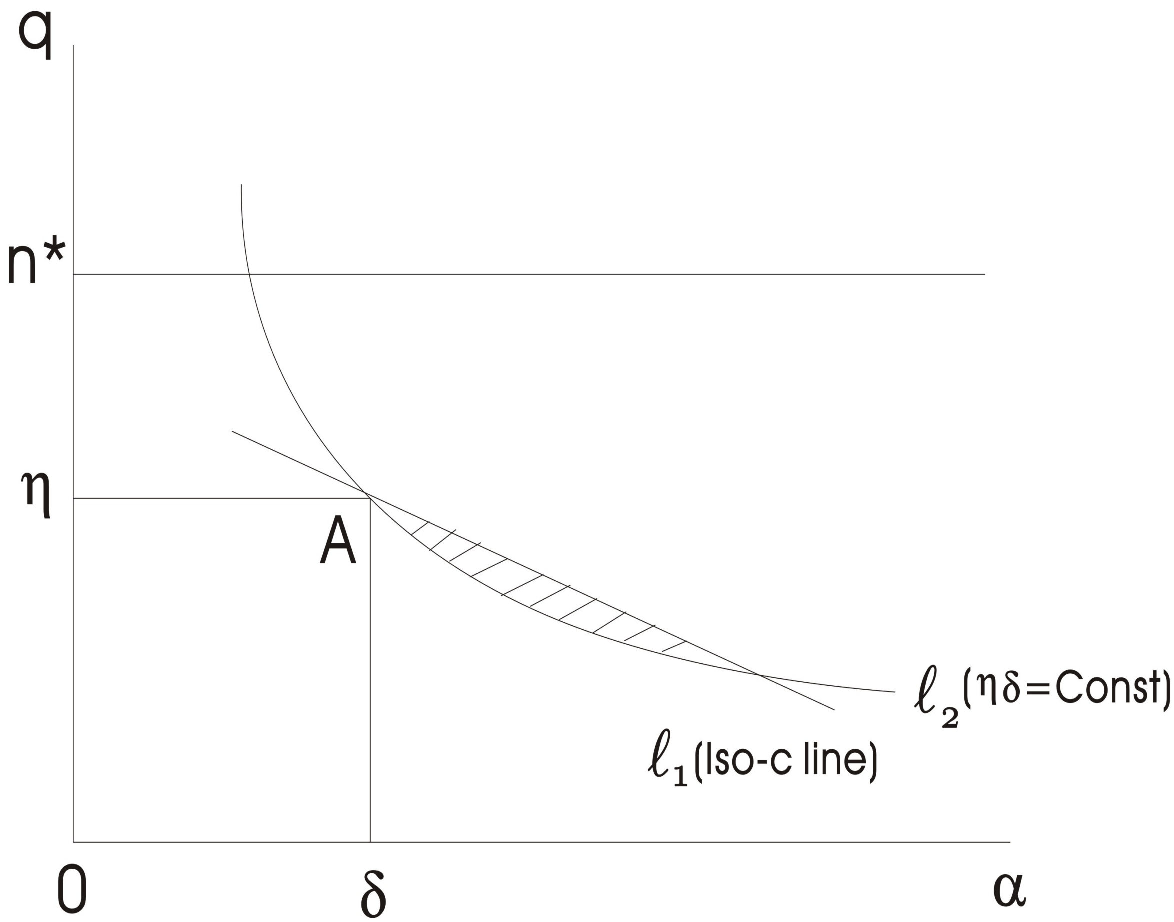

Figure 1b: The Parameter Range of (β, n) for Positive Selection, Case b.

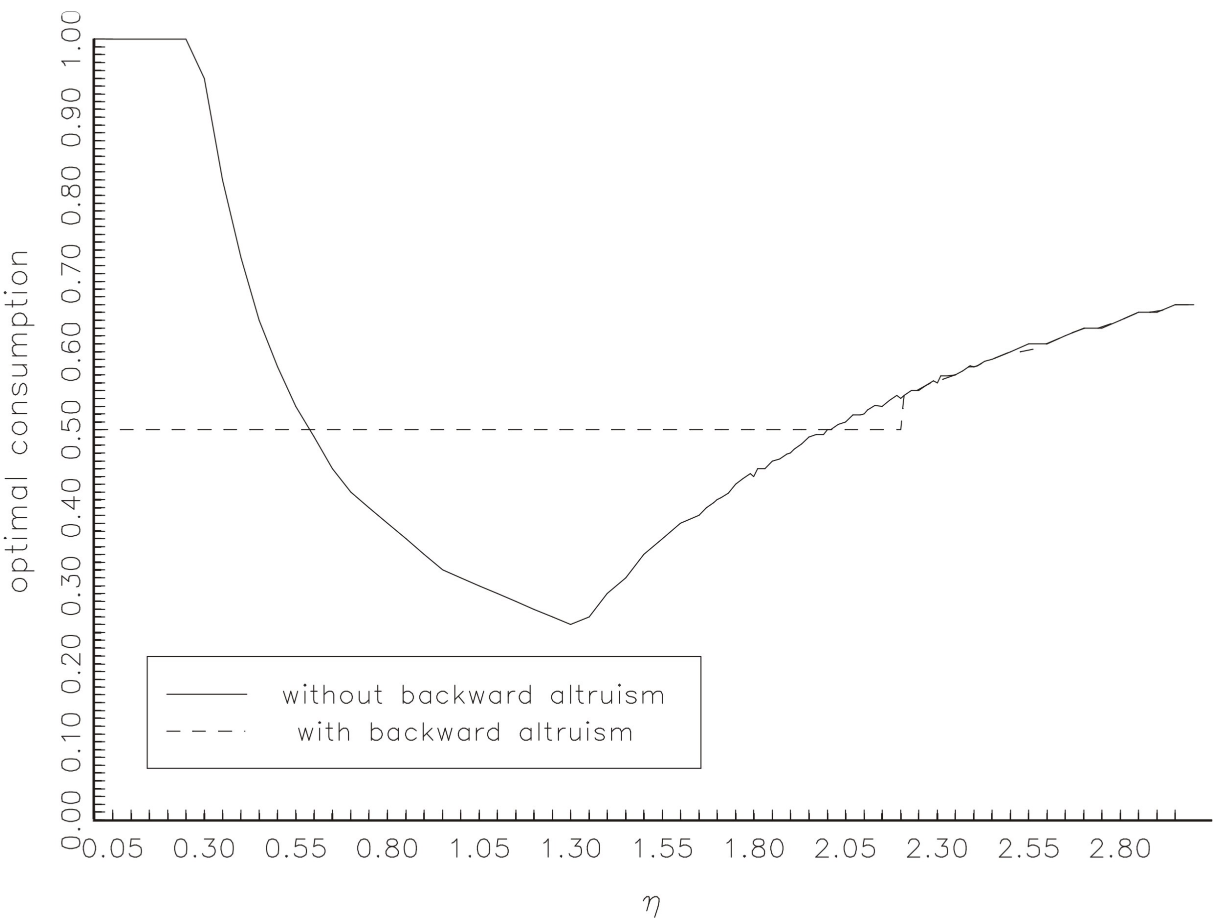

Figure 2: The correspondence between η and the optimal consumption.

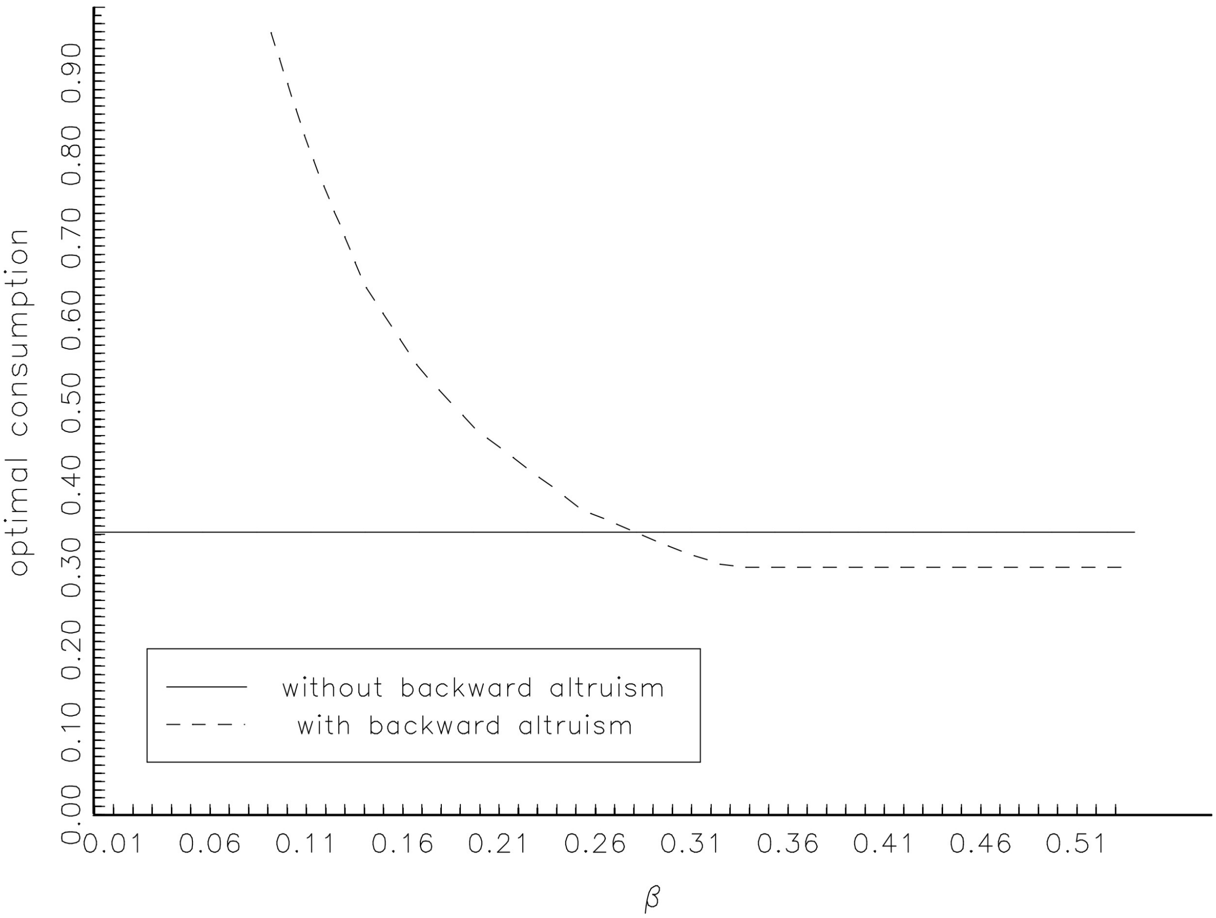

Figure 3: The correspondence between β and the optimal consumption.

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