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in a Two-sector Endogenous Growth Model*

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Abstract

This paper develops a new mechanism for local indeterminacy in a constant-return, two-sector, human capital enhanced growth model, with productive public spending financed by the income taxation in the goods sector. The use of productive public goods services is subject to an external congestion effect in association with the quantity of aggregate physical as well as human capital used in the economy. We establish local indeterminate equilibrium paths driven by the congestion effect. The possibility of local indeterminacy emerges because under constant returns, the congestion effect reduces the marginal contribution of public goods services and increases the marginal contribution of physical as well as human capital, thereby making the social marginal products to deviate from those of the firm's perspective.

Keywords: two-sector model, indeterminacy, and congestion.

JEL Classification: D90, O40, O41.

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1. Introduction

Can negative externalities generate multiple and indeterminate equilibria in an endogenous growth model? The answer appears to be negative in existing literatures. However, this paper shows that a negative externality in association with public goods services (the congestion effect) can lead to local indeterminacy in a constant-return, two-sector, human capital enhanced endogenous growth model. Local indeterminacy emerges because under constant returns in production, a sufficiently strong congestion effect makes the marginal products of physical and human capital, while decreasing from a firm's perspective, to increase in a social perspective.

Several economic growth models have recently established the existence of indeterminate equilibrium paths. The indeterminacy of equilibrium paths is significant as it lays the groundwork for endogenous growth fluctuations that provide for an explanation for large, persistent variations in growth experiences, where different countries follow different equilibrium trajectories toward a balanced growth path. Conventional wisdom generates endogenous growth fluctuations based on increasing returns in production, including those analyzed in a one-sector model (e.g., Benhabib and Farmer, 1994; Boldrin and Rustichini, 1994) and those explored in a two-sector model (e.g., Benhabib and Perli, 1994; Xie, 1994). These two-sector paradigms were extrapolated in the Lucas (1988)-Uzawa (1965) type of human capital enhanced models. While in these two-sector models increasing returns generate local indeterminacy, there are other two-sector, human capital enhanced models with constant returns where local indeterminacy depends on other devices.[1]

Two mechanisms are employed to generate local indeterminacy in these constant-return, two-sector, human capital enhanced growth models. One of the mechanisms is to introduce sector-specific externalities. Benhabib, Meng and Nishimura (2000) and Mino (2001) belong to this line of thought and establish the existence of indeterminacy in a two-sector, constant-return model with sector-specific externalities. The other mechanism is distortionary factor taxation. Bond, Wang and Yip (1996) limit the government activity to the collection of taxes and payment of transfers, and find that the balanced growth path is locally indeterminate if the factor tax rates are very different both across sectors and across factors. Raurich (2001) extends the model and shows that with the consideration of unproductive public spending, indeterminacy is empirically plausible. Finally, Ben-Gad (2003) combines both the distortionary factor taxation and the

[1] Indeterminacy also arises from other mechanisms in two-sector overlapping generation models (Reichlin, 1992, Michel and Venditti, 1997), and in two-sided search models (Laing, Palivos and Wang, 1995; Chen, Mo and Wang, 2002). See an extensive survey in Benhabib and Farmer (1999).

sector-specific externalities in an endogenous growth model with unproductive government spending, and finds that indeterminacy is not possible when either one of the two mechanisms is added to the model in isolation. With elastic labor supply and physical capital employed in both sectors, indeterminacy emerges for varying combinations of factor taxation and sector-specific externalities.

The purpose of this paper is to introduce another mechanism for local indeterminacy in a constant-return, two-sector, human capital enhanced model, with productive public spending in the fashion à la Barro (1990), financed by the income taxation in the goods sector. The use of a productive public goods service is subject to an external congestion effect in association with the quantity of aggregate physical as well as human capital used in the economy. We establish local indeterminate equilibria by calibrating the model based on the U.S. economy. Under different values of the congestion parameter, we simulate the value of the region of the external effect of public goods services upon the goods sector and the education sector. We find the conditions for local indeterminacy can occur under realistic parameter values.

The possibility of local indeterminacy materializes because the congestion effect makes the marginal product in a social perspective to deviate from that of the firm's perspective. The marginal product of human/physical capital in the goods sector is decreasing in the fraction of human/physical capital employed in the sector from a private perspective, but it is increasing from a social perspective because of the presence of the congestion effect.

Under the requirement of constant returns to scale in a technology, the public service congestion reduces the contribution of public capital services and increases the contribution of the other two inputs. As a result, the public service congestion mitigates the direct negative effect of larger human (resp. physical) capital upon the marginal product of human (resp. physical) capital. When the degree of public service congestion is sufficiently large, the indirect positive effect from the resulting larger physical (resp. human) capital because of the Pareto complements between physical and human capital, dominates the direct negative effect, thereby making the net marginal product of human (resp. physical) capital in the goods sector to increase in the amount of human (resp. physical) capital employed in this sector.

When marginal product is decreasing at the firm level, larger physical capital stock reduces the rewards to physical capital relative to the rewards to human capital. However, at a social perspective, such larger physical capital stock increases the ratio of the marginal products of human capital in the goods sector to that in the education sector. In order to equalize the marginal product of human capital in both sectors, human capital is optimally reallocated from the goods to the education sector, and with the Pareto complements in human and physical capital more physical capital is employed in the education sector. As a result, output in the goods sector decreases while output in the education sector increases, in so doing resulting in a higher shadow price of the goods relative to the education.

The above result indicates that the Rybczynski theorem fails. As a duality, the Stolper-Samuelson theorem also stops working, and thereby a rise in the reward of physical capital relative to human capital results in a fall in the price of the goods. When the representative agent expects a higher price of the goods relative to the education, it would increase physical capital accumulation, as the goods sector is more physical capital intensive from its own perspectives. Such an increase in physical capital accumulation leads to reallocation of human capital, and the resulting reallocation of physical capital, from the goods sector to the education sector. As a consequence, output in the goods sector decreases relative to output in the education sector, thereby resulting in a higher price of the goods relative to the education. Therefore, the expectations are self-fulfilling in equilibrium.

The rest of the paper is organized as follows. While Section 2 sets up the basic model and analyzes the balanced growth path, Section 3 analyzes and quantitatively assesses the possibility of equilibrium indeterminacy. In Section 4 the reasons underlying local indeterminacy are explored. Finally, some concluding remarks are provided in Section 5.

2. The Model

2.1 The Environment

The model is based upon Barro (1990), Rebelo (1991) and Bond, Wang and Yip (1996). The economy is comprised of a representative agent and two production sectors. In the two sectors, referred to as the goods sector, *Y*, and the education sector, *X*, both human and physical capital are necessary inputs, with their productivity enhanced, but with a congestion effect, by public spending in each sector.

More specifically, the technology in both sectors is of constant returns, and to simplify the analysis a Cobb-Douglas form is employed as follows:

$$y = A(vk)^{\alpha}(uh)^{\beta} G_y^{\gamma}, k(0) > 0 \; and \;\; h(0) > 0 \;\; given, \tag{2.1}$$

$$x = B[(1-v)k]^{\eta}[(1-u)h]^{\theta} G_x^{\delta}, \tag{2.2}$$

in which $0<v<1$ and $0<u<1$ are the share of physical, *k*, and human capital, *h*, respectively, allocated to the goods sector. Parameters $A>0$ and $B>0$ are productivity coefficients; $0<\alpha<1$ $(0<\eta<1)$ and $0<\beta<1$ $(0<\theta<1)$ are the shares of physical and human capital in the goods (education) sector. In addition, G_y (G_x) is the perceived amount of public goods services received by the representative agent in the goods (education) sector, with parameter γ (δ) representing the degree to which the public spending affects the productivity in the goods (education) sector, respectively.

The perceived amount of public goods services received by the representative agent is assumed to be the same in the two sectors and is given in the following form,[2]

$$G_y = G_x = G \cdot (\frac{1}{KH})^{\psi} \tag{2.3}$$

where G is aggregate public spending, and K and H are aggregate private physical and human capital, respectively. Parameter ψ indicates the degree of congestion, and if $\psi=0$, the public service is non-rival and non-excludable and is therefore a pure public good.

The following parametric restrictions are imposed.

Assumption 1 (*i*) $\alpha - \gamma\psi > 0$, $\beta - \gamma\psi > 0$, $\eta - \delta\psi > 0$, and $\theta - \delta\psi > 0$;

(*ii*) $\alpha + \beta - 2\gamma\psi = 1 - \gamma$ and $\eta + \theta - 2\delta\psi = 1 - \delta$

(*iii*) $0 < \psi \leq 0.5$.

While Assumption (*i*) assures that physical capital and human capital are productive in both sectors in an aggregate economy, Assumption (*ii*) imposes constant returns to scale in all growing inputs, i.e., K, H and G, in order to ensure the existence of a balanced growth path. Finally, in Assumption (*iii*) while restriction $\psi>0$ makes certain a congestion effect, $\psi\leq 0.5$ ensures the dominance of the positive effects of public spending service over the congestion effect.

Public spending is financed by income taxes in the goods sector, with a flat income tax rate, $\tau>0$, as in Barro (1990) and many others. For simplicity, we also assume there is no depreciation for the stocks of physical and human capital. As a result, the motions of the two kinds of capital stock for the representative agent are given by:

$$\dot{k} = (1-\tau)y - c \tag{2.4a}$$

$$\dot{h} = x \tag{2.4b}$$

The preference of the representative agent is assumed to possess a discounted lifetime utility,

[2] The parametric form follows from Turnovsky (1996) and Glomm and Ravikumar (1997), which indicates that an individual's usage of the public services is congested by the aggregate usage making the congestion effect to increase in the absolute size of the economy.

with a felicity prevailing a constant, intertemporal elasticity of substitution as follows:

$$U = \int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt, \tag{2.5}$$

in which parameter ρ>0 is the instantaneous time-preference rate, and σ is the reciprocal of the intertemporal elasticity of substitution.

Finally, to complete the model, the government budget constraints need to balance:

$$G = \tau Y. \tag{2.6}$$

2.2 The Optimization and Equilibrium

Given the tax rates, public spending and initial $k(0)$ and $h(0)$, the representative agent's problem is to choose c, v, u, k and h, in order to maximize its discounted lifetime utility in (2.5), subject to constraints in (2.1), (2.2), (2.3), and (2.4a.b). Denote λ and μ as the co-state variables associated with k and h, respectively. Then, the necessary conditions are:

$$c^{-\sigma} = \lambda, \tag{2.7a}$$

$$\lambda(1-\tau)\alpha \frac{y}{v} = \mu\eta \frac{x}{1-v} \tag{2.7b}$$

$$\lambda(1-\tau)\beta \frac{y}{u} = \mu\theta \frac{x}{1-u} \tag{2.7c}$$

$$\lambda(1-\tau)\alpha \frac{y}{k} + \mu\eta \frac{x}{k} = \rho\lambda - \dot{\lambda}, \tag{2.7d}$$

$$\lambda(1-\tau)\beta \frac{y}{h} + \mu\theta \frac{x}{h} = \rho\mu - \dot{\mu}, \tag{2.7e}$$

together with transversality conditions $\lim_{t\to\infty} e^{-\rho t} \lambda k = 0$, and $\lim_{t\to\infty} e^{-\rho t} \mu h = 0$.

Condition (2.7a) equates the marginal utility of consumption to the marginal cost, the shadow price of physical capital, while (2.7b) and (2.7c) equate the marginal product of physical capital and human capital between the goods and the education sector. Finally, (2.7d) and (2.7e) are the Euler equations governing the optimal accumulation for physical and human capital, respectively.

Proposition 1 *The share of human capital and the share of physical capital employed in the final goods sector are positively related.*

Proof: If we divide (2.7b) by (2.7c), we get

$$v = \frac{\alpha\theta u}{\alpha\theta u + \eta\beta(1-u)} \equiv v(u) < 1 \text{ if } u < 1, \tag{2.8}$$

which leads to $v'(u)>0$.

The positive relationship between v and u results from the Pareto complements in physical and human capital in the technology. We are now ready to define the market equilibrium.

Definition: *Given an income tax rate* τ *and initial physical and human capital* $K(0)$ *and* $H(0)$*, a perfect foresight equilibrium* (*PFE*) *is a tuple* $\{Y/H, X/H, G/H, G_y/H, G_x/H, v, u, \dot{K}/K, \dot{H}/H, \dot{C}/C, \lambda, \mu\}$ *that satisfies:*

(i) *production technologies,* (2.1), (2.2), (2.3);

(ii) *household budget constraint and law of motions,* (2.4a-b);

(iii) *household optimization,* (2.7a)-(2.7e), *together with the two transversality conditions;*

(iv) *government budget constraints,* (2.6).

An economic system with non-stationary variables is difficult to analyze. Therefore, it is necessary to transform the system into a tractable one with stationary variables. Following Bond, Wang and Yip (1996) we transform the economic system into the structure with variables $\{p, s, z\}$, where $p \equiv \mu/\lambda$, $s \equiv C/H$, and $z \equiv K/H$. In the following we briefly describe the transformation, with detailed mathematical derivation found in the Appendix.

Next, if we utilize (2.7c-2.7e) and (2.6), we rewrite (2.7b) as

$$\Phi_1 \left(\frac{u}{1-u} \right)^{-\delta} (v(u)u)^{\frac{\psi\gamma(1-\delta)}{1-\gamma}} [(1-v(u))(1-u)]^{-\delta\psi} \left(\frac{v(u)}{u} \right)^{\frac{\Lambda_1}{1-\gamma}} = z^{\frac{-\Lambda_1}{1-\gamma}} p, \qquad (2.9a)$$

where $\Phi_1 = \frac{(1-\tau)\beta A}{\theta B} \left(\frac{\eta\beta}{\alpha\theta} \right)^{-(\eta-\delta\psi)} (\tau A)^{\frac{\gamma-\delta}{1-\gamma}}$, and $\Lambda_1 = \alpha\theta - \eta\beta - \psi[\gamma(\theta-\eta) - \delta(\beta-\alpha)]$.

Using relationship in (2.8), (2.9a) can be rewritten as

$$u = u(p, z), \qquad (2.9b)$$

with $\frac{\partial u}{\partial p} = \frac{-(1-\gamma)\alpha\theta u(1-u)}{p\Gamma_1} \gtrless 0 \text{ if } \Gamma_1 \lessgtr 0$, $\frac{\partial u}{\partial z} = \frac{\Lambda_1 \alpha\theta u(1-u)}{z\Gamma_1} \gtrless 0 \text{ if } \Gamma_1 \gtrless 0$, and

$\Gamma_1 = [\alpha\theta - \eta\beta]\Lambda_1 v(1-u) + \alpha\theta\left[\delta(1-\gamma)(1-2\psi) - \psi(2-v-u)(\gamma-\delta)\right]$.

Finally, while (2.7b) and (2.7d) lead to an expression for $\frac{\dot{\lambda}}{\lambda}$, (2.7c) and (2.7e) yield an expression for $\frac{\dot{\mu}}{\mu}$, and (2.7a) and (2.7d) lead the following relationship

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}[(1-\tau)r(u(p,z),p)-\rho], \tag{2.10}$$

where $r(u(p,z),p)=\Phi_2\left[\frac{[1-u(p,z)]^{\delta(1-\psi)}}{pu(p,z)^{\delta}[1-v(u(p,z))]^{\delta\psi}}\right]^{\frac{\beta-\gamma\psi}{\Lambda_1}}[v(u(p,z))u(p,z)]^{\frac{\gamma\psi(\theta-\delta\psi)}{\Lambda_1}}\geq 0,$ and $\Phi_2=\alpha A(\tau A)^{\frac{\gamma}{1-\gamma}}\Phi_1^{\frac{\beta-\gamma\psi}{\Lambda_1}}$.

Moreover, (2.4a), with (2.7b, d) and (2.6), yields an expression for $\frac{\dot{K}}{K}$, whereas (2.4b), along with (2.7c, e) and (2.6), leads to an expression for $\frac{\dot{H}}{H}$. Therefore, we transform the equilibrium conditions into a stationary system as follows,

$$\frac{\dot{p}}{p}=\frac{\dot{\mu}}{\mu}-\frac{\dot{\lambda}}{\lambda}=(1-\tau)r(u(p,z),p)-w(u(p,z),p), \tag{2.11a}$$

$$\frac{\dot{s}}{s}=\frac{\dot{C}}{C}-\frac{\dot{H}}{H}=\frac{1}{\sigma}[(1-\tau)r(u(p,z),p)-\rho]-\frac{[1-u(p,z)]}{\theta}w(u(p,z),p), \tag{2.11b}$$

$$\frac{\dot{z}}{z}=\frac{\dot{K}}{K}-\frac{\dot{H}}{H}=\frac{(1-\tau)v(u(p,z))}{\alpha}r(u(p,z),p)-\frac{s}{z}-\frac{[1-u(p,z)]}{\theta}w(u(p,z),p), \tag{2.11c}$$

where $w(u(p,z),p)=\Phi_3 p^{\frac{\eta(1-\gamma)+\delta(\alpha-\psi)}{\Lambda_1}}\left[\frac{u(p,z)^{\delta}[1-v(u(p,z))]^{\delta\psi}}{[1-u(p,z)]^{\delta(1-\psi)}}\right]^{\frac{\alpha-\gamma\psi}{\Lambda_1}}[v(u(p,z))u(p,z)]^{\frac{-\gamma\psi(\eta-\delta\psi)}{\Lambda_1}}\geq 0,$

$$\Phi_3=\theta B\left(\frac{\eta\beta}{\alpha\theta}\right)^{\eta-\delta\psi}(\tau A)^{\frac{\delta}{1-\gamma}}\Phi_1^{\frac{-\eta(1-\gamma)-\delta(\alpha-\psi)}{\Lambda_1}}.$$

System (2.11a)-(2.11c) determines p, s and z. Using the results, we determine u and v using (2.9b) and (2.8), respectively. Ratios, $\frac{Y}{H}$, $\frac{X}{H}$, $\frac{G}{H}$ and $\frac{G_y}{H}$, thus $\frac{G_x}{H}$, are determined using (2.1), (2.2), (2.6) and (2.3). Finally, growth rates, $\frac{\dot{C}}{C}$, $\frac{\dot{K}}{K}$, $\frac{\dot{H}}{H}$, $\frac{\dot{\lambda}}{\lambda}$, and $\frac{\dot{\mu}}{\mu}$, are determined from (2.7a), (2.4a), (2.4b), (2.7d) and (2.7e).

2.3 Balanced Growth Path

We now analyze the equilibrium in a steady state. A steady-state equilibrium is a PFE with a balanced growth path (BGP) under which great ratios p, s and z and fractions u and v are constant, and the quantity variables grow at a constant rate. As a result, $\dot{p}/p=\dot{s}/s=\dot{z}/z=0$ in (2.11a)-(2.11c) along a BGP. Therefore, $\dot{C}/C, \dot{K}/K, \dot{H}/H, \dot{Y}/Y, \dot{X}/X$ and $\dot{G}/G$ are constant and equal in a BGP. To determine a BGP, we substitute (2.11a) into (2.11b) to obtain:

$$\Phi_4\left[\left(\frac{u}{(1-u)^{1-\psi}}\right)[1-v(u)]^{\psi}\right]^{\frac{(\beta-\gamma\psi)\delta}{\Lambda_2}}[v(u)u]^{\frac{\gamma\psi[\eta+\delta(1-\psi)]}{\Lambda_2}}\left[\frac{1}{\sigma}-\frac{(1-u)}{\theta}\right]=\frac{\rho}{\sigma}, \qquad (2.12)$$

where $\Phi_4=(1-\tau)\alpha A\left[\frac{\theta B}{(1-\tau)\alpha A}\right]^{\frac{\beta-r\psi}{\Lambda_2}}(\tau A)^{\frac{\gamma\Lambda_2+(\delta-\gamma)(\beta-r\psi)}{\Lambda_2(1-\gamma)}}\left(\frac{\eta\beta}{\alpha\theta}\right)^{\frac{(\eta-\delta\psi)(\beta-\gamma\psi)}{\Lambda_2}}$, and

$$\Lambda_2=(1+\eta)(1-\gamma)-\alpha(1-\delta)+\psi(\gamma-\delta)>0.$$

Consider:[3]

Condition S. $\sigma>\frac{1}{2}\left(\alpha+\theta+\sqrt{(\alpha-\theta)^2+4\eta\beta}\right).$

We are now ready to analyze the existence and uniqueness of a non-degenerate BGP. We establish the result in three steps.

First, examining (2.12), the right-hand side is $0<\rho/\sigma<\infty$, while the left-hand side is a function of u, whose value is zero at $u=(\sigma-\theta)/\sigma$, a positive value under Condition S, is increasing in u for $u\geq(\sigma-\theta)/\sigma$,[4] and is approaching ∞ at $u=1$. Therefore, there exists a unique interior u^* in a BGP with $0<\frac{\sigma-\theta}{\sigma}<u^*<1$, which using (2.8) implies a unique interior v^*, with $0<\frac{\alpha(\sigma-\theta)}{\alpha(\sigma-\theta)+\eta\beta}<v^*<1$. The result that $u^*<1$ and $v^*<1$ indicates $r^*>0$ and $w^*>0$.

Next, substituting $0<u^*<1$ into $\dot{p}=0$ in (2.11a) yields $(1-\tau)r^*=w^*>0$, which determines a unique p^*. Substituting p^* into $\dot{s}=0$ in (2.11b) leads to $\frac{1}{\sigma}[(1-\tau)r(u(p^*,z),p^*)-\rho]=[1-u(p^*,z)]w(u(p^*,z),p^*)>0$, and determines z^*. Finally, $(1-\tau)r(u(p^*,z^*),p^*)=w(u(p^*,z^*),p^*)>0$ and $\dot{z}=0$ in (2.11c) lead to $s=z^*[v/\alpha-(1-u)/\theta]w^*>0$ under Condition S.

Finally, using (2.11a) and (2.11b) along the BGP, we obtain $(1-\tau)r^*=\frac{\theta}{\theta-\sigma(1-u^*)}\rho>\rho$, as $\frac{\sigma-\theta}{\sigma}<u^*<1$. As a consequence, the economic growth rate in (2.10) is positive along the BGP.

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[3] Condition S is easily met as $\sigma>1$, $\alpha+\theta<2$, $(\alpha-\theta)^2<1$ and $4\eta\beta<4$. For example, suppose $\gamma=\delta=.08$, $\psi=.25$, $\alpha=.3$, $\beta=.64$, $\eta=.24$, and $\theta=.7$ so production functions in both sectors satisfy constant returns to scale. Then the right hand side of Condition S is less than 0.96.

[4] Differentiating the left-hand side of (2.12) with respect to u yields
$$\frac{\rho}{\sigma}\left\{\frac{1}{\Lambda_2 u(1-u)}\left[\delta(\beta-\gamma\psi)[1-(v+u)\psi]+\gamma\psi[\eta+\delta(1-\psi)](2-v-u)\right]+\frac{\sigma}{\theta-\sigma(1-u)}\right\}>0.$$

We summarize the above results in:

Proposition 2 *Under Assumption 1 and Condition S, there exists a unique balanced growth path with a positive economic growth rate.*

3 Transitional Dynamics

3.1 Local Dynamics

To investigate the local dynamics of the economic system, we take a linear Taylor's expansion of (2.11a)-(2.11c) in the neighborhood of the unique BGP. The expansion leads to:

$$\begin{pmatrix} \dot{p} \\ \dot{s} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} J_{11} & 0 & J_{13} \\ J_{21} & 0 & J_{23} \\ J_{31} & -1 & J_{33} \end{pmatrix} \begin{pmatrix} p - p^* \\ s - s^* \\ z - z^* \end{pmatrix} \tag{3.1}$$

where $J_{11} = \{\Sigma_1 - \Sigma_2\} p^*$,

$$J_{13} = \frac{\alpha\theta\Lambda_1}{\Gamma_1}(\Sigma_3 - \Sigma_4) p^* \frac{u^*(1-u^*)}{z^*},$$

$$J_{21} = \left\{\frac{1}{\sigma}\Sigma_1 - \frac{1}{\theta}\Sigma_2(1-u^*) - \frac{(1-\gamma)\alpha}{\Gamma_1}\frac{w^* u^*(1-u^*)}{p^*}\right\} s^*,$$

$$J_{23} = \frac{\alpha\theta\Lambda_1}{\Gamma_1}\left\{\frac{1}{\sigma}\Sigma_3 - \frac{1}{\theta}\Sigma_4(1-u^*) + \frac{w^*}{\theta}\right\} s^* \frac{u^*(1-u^*)}{z^*},$$

$$J_{31} = \left\{\frac{1}{\alpha}\Sigma_1 v^* - \frac{1}{\theta}\Sigma_2(1-u^*) - \frac{(1-\gamma)\theta}{\Gamma_1}\left[u^*(1-u^*) + \alpha v^*(1-v^*)\right]\frac{w^*}{p^*}\right\} z^*,$$

$$J_{33} = \frac{\alpha\theta\Lambda_1}{\Gamma_1}\left\{\frac{1}{\alpha}\Sigma_3 v^* - \frac{1}{\theta}\Sigma_4(1-u^*) + \left[\frac{1}{\alpha}\frac{v^*(1-v^*)}{u^*(1-u^*)} + \frac{1}{\theta}\right] w^*\right\} u^*(1-u^*) + \frac{s^*}{z^*},$$

with

$$\Sigma_1 = \frac{-1}{\Lambda_1\Gamma_1}\left\{(\beta - \gamma\psi)\left[\Gamma_1 - \alpha\theta\delta(1-\gamma)[1-\psi(v^*+u^*)]\right] + \alpha\theta\gamma\psi(1-\gamma)(\theta-\delta\psi)(2-v^*-u^*)\right\}\frac{w^*}{p^*},$$

$$\Sigma_2 = \frac{-1}{\Gamma_1\Lambda_1}\left\{(\eta-\delta\psi)\left[\Gamma_1 + \alpha\theta\gamma\psi(1-\gamma)(2-v^*-u^*)\right] + \alpha\theta\delta(1-\gamma)(\alpha-\gamma\psi)[1-\psi(v^*+u^*)] + (\alpha\delta-\eta\gamma)\Gamma_1\right\}\frac{w^*}{p^*},$$

$$\Sigma_3 = \frac{1}{\Lambda_1}\left\{\gamma\psi(\theta-\delta\psi)(2-v^*-u^*) - \delta(\beta-\gamma\psi)[1-\psi(v^*+u^*)]\right\}\frac{w^*}{u^*(1-u^*)},$$

$$\Sigma_4 = \frac{1}{\Lambda_1}\left\{-\gamma\psi(\eta-\delta\psi)(2-v^*-u^*) + \delta(\alpha-\gamma\psi)[1-\psi(v^*+u^*)]\right\}\frac{w^*}{u^*(1-u^*)}.$$

The Jacobean matrix in (3.1), denoted as *J*, determines the local dynamic properties of the economic system. The 3x3 system includes a state-like variable, whose initial value *z*(0) is predetermined, and two control-like variables, *p* and *s*, which can adjust instantaneously. Therefore, the equilibrium path in the neighborhood of the unique BGP is locally determinate if the Jacobean matrix has only one eigenvalue with negative real parts (stable roots). In contrast, if the Jacobean matrix has two, or a larger number of, eigenvalues with negative real parts, then the equilibrium path near the unique BGP is locally indeterminate.

According to the Routh-Hurwitz theorem, the number of characteristic roots with positive real parts equals the number of variations of signs in $\{-1, TrJ, -BJ + DetJ/TrJ, DetJ\}$, in which *TrJ*, *BJ* and *DetJ* denote the trace, the determinant of the Bordered Hessian, and the determinant of matrix *J,* respectively. It follows that there is a total of eight possible types of variations in sign. In Table 1, the first four rows exhibit cases in which the number of changes in signs are less than or equal to one and thus the number of eigenvalues with negative real parts is larger than or equal to two. If any of the cases in the first four rows in Table 1 emerges in equilibrium, then the BGP is locally indeterminate.

[Insert Table 1 here]

The determinant of the Jacobean (the product of the eigenvalues) is:

$$DetJ = \frac{-\alpha\theta u(1-u)sw^2}{z\Gamma_1}\left\{\left(\frac{1}{\sigma}-\frac{1-u}{\theta}\right)\frac{1}{u(1-u)}\left[\delta(\beta-\gamma\psi)[1-(v+u)\psi]+\gamma\psi(2-v-u)[(\eta+\delta(1-\psi)]\right]+\frac{\Lambda_2}{\theta}\right\}$$

The terms in the large braces in *DetJ* are positive under Condition S. Therefore, the sign of *DetJ* is opposite to the sign of Γ_1 defined in (2.9b), which substituting into Λ_1 in (2.9a) becomes

$$\begin{aligned}\Gamma_1 \equiv &\left\{(\alpha\theta-\eta\beta)^2 v(1-u)\right\} + \left\{\delta(1-\gamma)\alpha\theta\right\} \\ &-\psi\left\{[(\gamma(\theta-\eta)-\delta(\beta-\alpha))(\alpha\theta-\eta\beta)v(1-u)] + [2\delta(1-\gamma)\alpha\theta] + [\psi(\gamma-\delta)(2-v-u)\alpha\theta]\right\}\end{aligned} \tag{3.2}$$

The sign of *DetJ* is important in determining the local dynamics of the model. If the number of negative roots is smaller than 3,[5] local indeterminacy is generally associated with a positive value of *DetJ* and thereby, a negative value of Γ_1.[6] To study the possibility of local indeterminacy, we investigate the elements in Γ_1.

The first large braces in Γ_1 involve only those of standard effects, whereas the second large braces contain those of public spending effects and the third large braces include those in relation

[5] As our simulation results below show, it is impossible to obtain three negative roots in this dynamic system in the range of appropriate parameter space. As a result, local indeterminacy arises in this model only in the situation with a positive determinant for the Jacobean matrix, i.e. Γ_1<0.

[6] Even though a source is also associated with a positive determinant, it also requires with other conditions.

to congestion effects.[7] If public spending is not productive, then $\gamma=\delta=0$ and this model is reduced an otherwise standard model (e.g., Rebelo, 1991) with $\Gamma_1=\Gamma_1^1\equiv(\alpha\theta-\eta\beta)^2 v(1-u)>0$ entailing only a standard effect. If the public services are productive, pure public good, then $\gamma>0$, $\delta>0$, $\psi=0$, and $\Gamma_1=\Gamma_1^2\equiv\Gamma_1^1+\delta(1-\gamma)\alpha\theta>0$ and is even larger than Γ_1^1.

In contrast, when a congestion effect emerges in the use of the public services, Γ_1 is the complicated form in (3.2), which is smaller than Γ_1^2 through the third braces. Envisaging the third large braces, the first brackets arise from the interaction of the congestion effect with the standard effect, while the second brackets come from the crowding out of the public services due to the congestion effect and the third brackets appear in relation to an asymmetry in the congestion effects in the use of public services between the two sectors. It is easier to envisage the signs of Γ_1 if we rewrite (3.2) to obtain:

$$\Gamma_1=\{(\alpha\theta-\eta\beta)\Lambda_1 v(1-u)\}+\{\delta(1-\gamma)(1-2\psi)\alpha\theta\}+\{\psi(2-v-u)(\delta-\gamma)\alpha\theta\} \qquad (3.3)$$

In the above expression, the first large braces are positive as Λ_1 is generally positive, and the second large braces are also positive under $\psi\leq 0.5$. In order to obtain $\Gamma_1<0$, the third large braces must be negative, which is possible only if $\gamma>\delta$ and ψ is sufficiently large. While $\gamma\psi$ presents the congestion effect in Sector Y, $\delta\psi$ is the congestion effect in Sector X. It is necessary that the congestion effect in Sector Y dominates the congestion effect in Sector X, in order to obtain $\Gamma_1<0$.

However, the signs of *TrJ* and *–BJ+DetJ/TrJ* are too complicated to conduct an algebraic analysis. In the following we calibrate the model economy and quantitatively envisage the possibility of local indeterminacy.

3.2 Calibrating the Model Economy

We now calibrate the model economy. We start by choosing parameter values, followed by solving the endogenous variables in a BGP. Finally, we establish the values of *TrJ*, *BJ* and *DetJ* of the Jacobean matrix *J*, and determine the signs of eigenvalues.

We normalize the parameter values for the productivity coefficient in Sector Y by *A*=1. We calibrate the economy based upon the following parameter values representative of the economy in

[7] In conventional works, elements in Γ_1 include only those in the first large braces, so we refer to them as a standard effect. With the consideration of productive public spending, the effect appears in the second large braces in Γ_1, thus referred to as a public spending effect. Finally, when the congestion effects emerge in the use of the public services, the third large braces appear in Γ_1, and thereby are referred to as a congestion effect.

the U.S. and consistent with a 2% long-run, real economic growth rate.

The total tax revenues in the US, on average, account for 20% of its GDP after 1980, and hence τ=20% is chosen. Following Turnovsky (2000) we choose the degree of externality of public spending upon the goods sector at γ=0.08; in a similar and symmetric fashion, we set the degree of externality of public spending upon the education sector at δ=0.08. Finally, there are no estimates for the degree of congestion. We choose the median value, ψ=0.25, to represent the degree of congestion. We must point out that different sets of values for ψ will not change the quantitative results, as the resulting calibrated value for B below is insensitive to a different value of ψ.

For the time preference rate, ρ, we set it at 0.025 in accordance with Benhabib and Perli (1994). As to the intertemporal elasticity of substitution, $1/\sigma$, we set σ=1.5 and consistence with Jones, Manuelli and Rossi (1993) and Ben-Gad (2003). Finally, following Ben-Gad (2003), Benhabib and Perli (1994), and Raurich (2001), α=0.3 and η=0.2 are chosen.[8] With the degree of public spending externality and under a constant return to scale technology, the share of human capital in the goods and the education sector, β and θ, is implied. Using the above parameter values, we calibrate the productivity coefficient in the educational sector, B, in consistence with a 2% long-term, real economic growth rate in the U.S. We summarize these parameters in Table 2.

[Insert Table 2 here]

3.3 Conditions for Indeterminacy

We now simulate the model to see whether the BGP is a source, a sink or a saddle. In the simulation, we maintain all the parameter values in Tables 2 and 3 except for parameter values concerning externality, γ and δ, and congestion, ψ. Under different values for $\psi \le 0.5$, we then simulate the region in the (δ, γ) plane separating equilibrium paths in the neighborhood of the unique BGP that is a source, a sink and a saddle. We have found that for the region of δ<0.4 and γ<0.4, a sink appears for $\psi \in$[0,15, 0.27] and $\psi \in$[0.47, 0.5]. We illustrate the simulation results in a figure for a value of ψ in [0,15, 0.27] and [0.47, 0.5], respectively.

First, we demonstrate the set of (δ, γ) separating a saddle, a sink and a source under ψ=0.2 in

[8] While Ben-Gad (2003) chose α=0.285 and η=0.2, Raurich (2001) chose α=0.42 and η=0.1 in his decentralized economy and α=0.35 and η=0.2. in his centralized economy. Benhabib and Perli (1994) used α=0.25. Thus, our value of α and η lies in the range of the parameter values in these works. Moreover, we will increase and decrease the value of α and η to assure the robustness of our quantitative results.

Figure 1, where the shaded area presents the set of (δ, γ) that exhibits indeterminate local dynamic paths in the neighborhood of the unique BGP, indicating that the BGP is a sink. Several observations are in order. First, the unique BGP is a sink under reasonable degrees of public investment externality, at δ=0.5% and γ=14.6%, and the region for a sink increases in (δ, γ). Second, it is easy to see that under a degree of public spending externality in Sector Y, γ, the unique BGP is a saddle when the degree of public spending externality in Sector X, δ, is large, and the BGP becomes a sink as δ decreases to a certain value. Finally, for a given δ, the BGP is a saddle when γ is small and as γ increases, the BGP becomes a sink.

[Insert Figure 1 here]

Next, we report in Figure 2 the simulation results of the set of (δ, γ) separating a saddle, a sink and a source under ψ=0.5. In this situation a sink is obtained under reasonable degrees of public spending externality, with a higher δ at 9% and a lower γ at 12.2%. For other features, they are similar to those in Figure 1.

[Insert Figure 2 here]

Finally, in order to examine the sensitivity of the above quantitative results, we conduct some robustness analysis. In the analysis, we alter the value of one parameter from the benchmark case while keeping the values of all other parameters unchanged, and calibrate the value for *B* in consistence with the 2% economic growth rate. We have experimented using a higher and lower time preference rate, a 50% increase and a 50% decrease in the reciprocal of the intertemporal elasticity of substitution, an increase and a decrease in the share of physical capital in Sector Y and in Sector X. The results are reported in Table 3, illustrating the range of ψ and the shape of the region of (δ, γ) that gives rise a sink. In all cases there are two separating interval in the range of ψ for a sink, except for two situations. The first case emerges when the contribution of physical capital in the Y sector is large, at α=0.42, under which the two intervals of the range of ψ for a sink are connected into an interval. When ψ is small, the shape of the region of (δ, γ) that gives rise to a sink is similar to Figures 1, and when ψ increases, the shape in Figure 1 disappears and only the shape in Figure 2 emerges. The second case emerges when the contribution of physical capital in the X sector is equal to or larger than its counterpart in the Y sector in the benchmark case, namely η≥α=0.3. In this case, the two intervals of the range of ψ for a sink are collapsed into an interval, and a smaller interval as η increases further. Moreover, only the shape of the region of (δ, γ) that gives rise to a sink in Figure 1 appears.

[Insert Table 3 here]

The above results document the combination of (δ, γ) for a BGP to be locally indeterminate. The required values of δ and γ for a sink seem to be reasonable. When the congestion effect is taken into account, the simulated values of δ and γ net of the congestion effect imply a small required net degree of the external effect in public spending for local indeterminacy.[9]

It should be noted that the region of (δ, γ) for a sink in all Figures is located between the region for a source and the region for a saddle. This is so because *DetJ* is positive for a source and is generally positive for a sink (cf. Table 1). For a source, it needs a sufficiently large γ and also requires *TrJ*>0 and –*BJ*+*DetJ/TrJ*<0 (Row 5 in Table 1). Under a given γ, we find that *TrJ*>0 becomes quantitatively smaller as δ increases, and thus *DetJ/TrJ*>0 gets larger as δ increases. As δ increases to a critical value, *DetJ/TrJ*>0 dominates *BJ*>0 so –*BJ*+*DetJ/TrJ* changes from a negative to a positive value, and therefore the BGP changes from a source to a sink (Row 2 in Table 1). When δ is increased again, *TrJ* changes from a positive to a negative value, so both *TrJ* and *DetJ/TrJ* become negative, resulting in the condition in Row 3 in Table 1. Finally, Γ_1 becomes positive as δ is increased further, making *DetJ* to change from a positive to a negative value. At the same time, *TrJ* and *BJ* also change signs and become positive and negative, respectively. Thus the BGP changes from a sink to a saddle (Row 6 in Table 1).

To summarize the results, we obtain:

Proposition 3 *In a two-sector, human capital enhanced growth model with a congestion effect large enough, the equilibrium path in the neighborhood of the unique BGP is locally indeterminate under reasonable values for the degree of externality of the public services.*

4 The Intuition

The possibility of locally indeterminate equilibrium paths in the neighborhood of a BGP emerges when the marginal product from a firm's perspective deviates from that from a social perspective. Specifically, in this paper the marginal product of human/physical capital in Sector Y is decreasing in the fraction of human/physical capital employed in Sector Y from a private

[9] For example, in a study using the panel data in U.S. and Japan, Shioji (2001) finds that the degree of externality for public capital is between 0.10 and 0.15.

perspective, but is increasing from a social perspective. The reasons are as follows.

Under a decreasing marginal product from a private perspective, a higher physical capital stock reduces the rewards to physical capital relative to the rewards to human capital. However, such a higher physical capital stock also increases the ratio of the social marginal product of human capital in Sector Y, that is more physical capital intensive, to that in Sector X, that is less physical capital intensive. At optimum, human capital is reallocated from Sector Y to Sector X, in order to reduce the marginal product of human capital in Sector Y to equalize the marginal product of human capital in both sectors. Moreover, the fraction of physical capital employed in Sector X increases due to Pareto complements in the technology. As the result of capital reallocation, output in Sector Y reduces while output in Sector X increases, resulting in a higher shadow price of goods Y to goods X.

Given the above result that a higher capital stock reduces output in Sector Y, that is more physical capital intensive from a private perspective, the Rybczynski theorem fails. As a duality, the Stolper-Samuelson theorem also stops working, and thereby a rise in the reward of physical capital relative to human capital results in a fall in the price of goods Y. In this situation, when the representative agent expects a higher price of goods Y relative to goods X, it would increase physical capital accumulation, as goods Y is more physical capital intensive from its own perspectives. Such an increase in physical capital accumulation leads to reallocation of human capital, and the resulting reallocation of physical capital, from Sector Y to Sector X. As a result, output Y decreases relative to output X, resulting in a higher price of goods Y relative to goods X. Therefore, the expectations are self-fulfilling in equilibrium.

Formally, this possibility is better understood if we derive the marginal product of human capital in Sectors Y and X from a private perspective by differentiating (2.1)-(2.2), together with (2.3), to obtain:

$$MPH_y = \frac{\partial y}{\partial (uh)} = [1-\alpha-\gamma(1-2\psi)]\frac{y}{uh}, \tag{4.1a}$$

$$MPH_x = \frac{\partial x}{\partial [(1-u)h]} = [1-\eta-\delta(1-2\psi)]\frac{x}{(1-u)h}, \tag{4.1b}$$

where a substitution is made of the restrictions of constant returns on parameters, that is, $\beta = 1-\alpha-\gamma(1-2\psi)$ and $\theta = 1-\eta-\delta(1-2\psi)$.

Output y and x in (4.1a)-(4.1b) is evaluated at a social perspective, which is obtained if we substitute into (2.1) and (2.2) the government balance of budget in (2.6):

$$y=(A\tau^{\gamma})^{\frac{1}{1-\gamma}}(vk)^{\frac{\alpha}{1-\gamma}}(uh)^{\frac{1-\alpha-\gamma(1-2\psi)}{1-\gamma}}\left(\frac{1}{KH}\right)^{\frac{\gamma\psi}{1-\gamma}}, \tag{4.2a}$$

$$x=B(\tau A)^{\frac{\delta}{1-\gamma}}[(1-v)k]^{\eta}[(1-u)h]^{1-\eta-\delta(1-2\psi)}(vk)^{\frac{\delta\alpha}{1-\gamma}}(uh)^{\frac{\delta[1-\alpha-\gamma(1-2\psi)]}{1-\gamma}}\left(\frac{1}{KH}\right)^{\frac{\delta\psi}{1-\gamma}}. \tag{4.2b}$$

In order to see how the marginal products of human capital in Sectors Y and X change as the fraction of human capital employed in Sector Y, u, increases, we differentiate the marginal products of human capital at a social perspective in Sectors Y and X with respect to u to obtain

$$\begin{aligned}\frac{1}{MPH_y}\frac{\partial MPH_y}{\partial u}&=\frac{\alpha}{1-\gamma}\frac{1-v}{u(1-u)}-\frac{(\alpha-2\gamma\psi)}{1-\gamma}\frac{1}{u}\\&=\frac{2\gamma\psi(1-u)}{(1-\gamma)u(1-u)}-\frac{\alpha(v-u)}{(1-\gamma)u(1-u)},\end{aligned} \tag{4.3a}$$

$$\begin{aligned}\frac{1}{MPH_x}\frac{\partial MPH_x}{\partial u}&=\frac{-\eta v}{u(1-u)}+\frac{[\eta+\delta(1-2\psi)]}{1-u}+\frac{\delta\alpha}{1-\gamma}\frac{1-v}{u(1-u)}+\frac{\delta[1-\alpha-\gamma(1-2\psi)]}{1-\gamma}\frac{1}{u}\\&=\frac{2\delta\psi(1-u)}{(1-\gamma)u(1-u)}+\frac{\delta(1-2\psi)(1-\gamma)-[\delta\alpha+\eta(1-\gamma)](v-u)}{(1-\gamma)u(1-u)}.\end{aligned} \tag{4.3b}$$

From (4.3a), the effect comes from two sources. First, other things being equal, a larger fraction of human capital employed in Sector Y directly reduces the marginal product of human capital in Sector Y, represented by the second term in the first equality. Moreover, a higher u increases the fraction of physical capital employed in Sector Y as well,[10] which indirectly increases the marginal product of human capital in Sector Y through Pareto complements, governed by the first term in the first equality. In the situation without public service congestion, $\psi=0$, and the direct negative effect always dominates the indirect positive effect. Indeed, the net effect depends upon the sign of $-(v-u)$ in the second equality, which is negative under the construction that Sector Y is more physical capital intensive. However, in the case when there is public service congestion, $\psi>0$. Under the requirement of a technology with a constant return in K, H, and G, the public service congestion reduces the marginal contribution of public services and increases the marginal contribution of the other two inputs. As a result, the public service congestion mitigates the direct negative effect on the marginal product of human capital. In the condition when ψ is sufficiently large, the indirect positive effect dominates the direct negative effect, making the net marginal product of human capital in Sector Y to increase in the fraction of human capital employed in this Sector.

[10] See (2.8), where the optimal u and v are positively related, because of the Pareto complements in K and H in the technology and the marginal rate of technical substitution between sectors at optimum.

From (4.3b), the effect originates from three channels. First, other things being equal, a larger fraction of human capital employed in Sector Y and thus, a smaller fraction of human capital employed in Sector X, directly increases the marginal product of human capital in Sector X, represented by the second term in the first equality. Second, a larger fraction of human capital employed in Sector Y also leads to a larger fraction of physical capital employed in Sector Y and thus, a smaller fraction of physical capital employed in Sector X, which indirectly reduces the marginal product of human capital in Sector X through Pareto complements, whose effect is represented by the first term in the first equality. Finally, a larger fraction of human capital employed in Sector Y, other things being equal, increases the output in Sector Y that increases public goods provision, which indirectly enhances the marginal product of human capital in Sector X, represented by the third and fourth terms in the first equality. When there is no public service congestion, ψ=0, and the net effect is generally positive as long as δ is not too small. When ψ>0, the public service congestion reduces the direct positive effect while increases the indirect positive through public goods provision, but the net effect is still positive as long as δ is not too small. Therefore, the net effect upon the marginal product of human capital in Sector X is generally increasing in the fraction of human capital employed in Sector Y, regardless of the magnitude of the congestion parameter.

Based on the above analysis, it is obvious that the marginal product of human capital in Sector Y relative to that in Sector X is decreasing in the fraction of human capital employed in Sector Y, *u*, when there is no public service congestion. When there is public service congestion, the relative marginal product of human capital between Sectors Y and X may be increasing in the fraction of human capital employed in Sector Y if the value of ψ is sufficiently large. When ψ>0, it is required that γ be larger than δ in order for the relative marginal product of human capital between Sectors Y and X to increase in the fraction of human capital employed in Sector Y.

In the case when γ=δ, there is a symmetric public service congestion effect in Sectors Y and X. In this case, comparing the effect of a larger the fraction of human capital employed in Sector Y upon the marginal product of human capital between Sector Y and X,[11] the first terms in the second equality in (4.3a) and (4.3b) totally offset each other, so the effect is determined by $-\frac{\alpha(v-u)}{(1-\gamma)u(1-u)}-\frac{\delta(1-2\psi)(1-\gamma)-[\delta\alpha+\eta(1-\gamma)](v-u)}{(1-\gamma)u(1-u)}$, which is decreasing in the fraction of human capital

[11] The effect of *u* on the relative marginal product of human capital is $\frac{MPH_y}{MPH_x}\left(\frac{1}{MPH_y}\frac{\partial MPH_y}{\partial u}-\frac{1}{MPH_x}\frac{\partial MPH_x}{\partial u}\right)$.

employed in Sector Y. Alternatively, when $\gamma>\delta$, the first term in the second equality in (4.3a) is greater than that in (4.3b). When $\gamma-\delta>0$ is sufficiently large and the resulting positive effect through ψ dominates the negative effects formed by other terms, the relative marginal product of human capital between Sectors Y and X is increasing in the fraction of human capital employed in Sector Y.[12] With these two cases, we are ready to illustrate the key results in relation to the Rybczynski theorem.

First, consider the case when $\psi>0$ and $\gamma=\delta$ and when $\gamma>\delta$ and ψ is small. In this case, $\Gamma_1>0$ and the relative marginal product of human capital between Sectors Y and X is decreasing in the fraction of human capital employed in Sector Y. This case is illustrated in Figure 3 with the demand for human capital in Sector Y relative to Sector X being:

$$\frac{w_y}{w_x}=\frac{(1-\tau)MPH_y}{pMPH_x}=\frac{\beta(1-\tau)}{\theta p}\frac{1-u}{u}\frac{Y}{X}=\Phi_1\frac{1}{p}(vu)^{\frac{\psi(\gamma-\delta)}{1-\gamma}}(\frac{u}{1-u})^{-\delta}(\frac{vu}{(1-v)(1-u)})^{\psi\delta}[(\frac{v}{u})z]^{\frac{\Lambda_1}{1-\gamma}} \tag{4.4}$$

where Φ_1 and Λ_1 are as defined in (2.9a). From (2.8c), given p, the rates of return to human capital between sectors must be equal at optimum, i.e. $w_y/w_x=1$ at optimum.

[Insert Figure 3 here]

Now, suppose that there is an increase in physical capital and thus, a higher physical to human capital ratio. For a given relative shadow price of commodity, p, a higher z shifts the relative factor demand curve upwards (c.f. (4.4)), reasons being that it increases the marginal product of human capital in Sector Y relative to Sector X, since Sector Y is more physical capital intensive (Figure 3). As the marginal product of human capital in Sector Y is decreasing in the fraction of human capital employed in Sector Y while the marginal product of human capital in Sector X is increasing in the fraction of human capital employed in Sector Y, a reallocation of human capital is made from Sector X to Sector Y to reduce the marginal product of human capital in Sector Y relative Sector X so an optimum is recovered. Through the Pareto complements in the technology, a fraction of physical capital is reallocated from Sector X to Sector Y.[13] As a result, a higher physical capital increases the output of the more physical capital intensive sector, Y, and decreases the output of the less physical capital intensive sector, X. Such an outcome is consistent with the Rybczynski theorem. As the duality of the Rybczynski theorem, the Stolper-Samuelson theorem

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[12] Specifically, slope $\frac{\partial[MPH_y/MPH_x]}{\partial u}=\frac{-MPH_y}{MPH_x}\frac{\Gamma_1}{u(1-u)\alpha\theta}>0$ if $(\gamma-\delta)\psi>0$ is so large that $\Gamma_1=(\alpha\theta-\eta\beta)\Lambda_1 v(1-u)+[\delta(1-\gamma)(1-2\psi)+\psi(2-v-u)(\delta-\gamma)]\alpha\theta<0$.

[13] This also can be seen from the positive relationship between u and v at optimum in (2.9a).

must hold. Therefore, a higher physical capital stock decreases the return to physical capital relative to that to human capital, and decreases the shadow price of goods Y that is more physical capital intensive relative to the shadow price of goods X that is less physical capital intensive, leading to a positive relationship between relative factor rewards and relative commodity prices (Batra, 1973, pp27-29).

Finally, consider the case when $\gamma-\delta>0$ and ψ is sufficiently large so $\Gamma_1<0$. In this case, the relative marginal product of human capital between Sectors Y and X is increasing in the fraction of human capital employed in Sector Y, resulting in a relative factor demand curve for human capital between Sectors Y and X to increase in the fraction of human capital employed in Sector Y. See Figure 4.

[Insert Figure 4 here]

Now, suppose that there is an increase in physical capital and thus, a higher physical to human capital. For a given relative shadow price of commodity, a higher z increases the marginal product of human capital in Sector Y relative to Sector X shifting the relative factor demand curve upwards (Figure 4). At the original optimal human capital allocation, the marginal product of human capital in Sector Y is larger than that in Sector X (cf. point A′). As now the marginal products of human capital in both Sectors Y and X are increasing in the fraction of human capital employed in Sector Y, it is necessary for human capital to reallocate from Sector Y to Sector X, in order to reduce (increase) the marginal product of human capital in Sector Y (X). The reallocation is carried on until the marginal product of human capital equalizes in both sectors. Through the Pareto complements in the technology, a fraction of physical capital is reallocated from Sector Y to Sector X. As a result, output in Sector Y decreases while output in Sector X increases, which is not consistent with what is implied by the Rybczynski theorem. As a duality, the Stolper-Samuelson theorem must fail to satisfy. Therefore, when physical capital stock increases, the returns to physical capital relative to that to human capital decrease, but output in Sector Y relative to output in Sector X decreases, thereby increasing the shadow price of goods Y relative to that to good X. As a consequence, the relationship between the relative factor rewards for physical capital to that for human capital and the shadow price of goods Y, that is more physical capital intensive, to the shadow price of goods X, that is less physical capital intensive, is negative, rather than positive (Batra, 1973, pp30-32).

5. Concluding Remarks

This paper establishes local indeterminacy in a constant-return, two-sector, human capital enhanced growth model with productive public spending. The novelty of this paper has been to uncover the negative external effect in association with public goods service as a mechanism for establishing local indeterminate equilibrium. Local indeterminacy in the paper emerges because a sufficient large congestion effect makes the marginal product, which is otherwise decreasing in a firm's perspective, to increase in a social perspective.

There are other extensions of this model, and we mention just one. Two mechanisms, factor taxation and sector-specific externality, are employed to establish local indeterminacy in a constant, two-sector, human capital enhanced growth model. We may introduce into these models the productive public services to examine the robustness of their results concerning local indeterminacy. Moreover, we may add the congestion effect in the use of public spending services, and investigate whether the congestion effect helps to establish local indeterminacy in the model.

Appendix

This appendix derives in detail how the transformed system (2.11a)-(2.11c) is obtained. First, when we substitute (2.7b) into (2.7d), and substitute (2.7c) into (2.7e), together with (2.1), (2.2) and (2.6), we obtain:

$$(1-\tau)\alpha\frac{y}{vk} = (1-\tau)\alpha\Phi_5 (vu)^{\frac{\gamma\psi}{1-\gamma}}\left(\frac{vk}{uh}\right)^{\frac{-(\beta-\gamma\psi)}{1-\gamma}} \equiv (1-\tau)r = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{A.1}$$

$$\theta\frac{x}{(1-u)h} = \theta\Phi_6\left(\frac{u}{1-u}\right)^{\delta}(vu)^{\frac{\delta\gamma\psi}{1-\gamma}}[(1-v)(1-u)]^{\delta\psi}\left(\frac{vk}{uh}\right)^{\frac{\eta(1-\gamma)+\delta(\alpha-\psi)}{1-\gamma}} \equiv w = \rho - \frac{\dot{\mu}}{\mu}, \tag{A.2}$$

where $\Phi_5 = A(\tau A)^{\frac{\gamma}{1-\gamma}}$, $\Phi_6 = B(\tau A)^{\frac{\delta}{1-\gamma}}\left(\frac{\eta\beta}{\alpha\theta}\right)^{\eta-\delta\psi}$.

Next, combining (2.7a) and (A.1), we get:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left[(1-\tau)r - \rho\right], \tag{A.3}$$

Third, we use (A.1) and (2.4a), and (A.2) and (2.4b) to derive:

$$\frac{\dot{K}}{K} = \frac{v(1-\tau)r}{\alpha} - \frac{s}{z}, \tag{A.4}$$

$$\frac{\dot{H}}{H} = \frac{(1-u)w}{\theta}, \tag{A.5}$$

where *r* and *w* are as defined in (2.11a)-(2.11c), with $s \equiv \frac{C}{H}$ and $z \equiv \frac{K}{H}$.

Fourth, with the help of (A.1) and (A.2), we rearrange (2.7b) to obtain:

$$\frac{\mu}{\lambda} \equiv p = \Phi_1 \left(\frac{u}{1-u} \right)^{-\delta} (vu)^{\frac{\psi\gamma(1-\delta)}{1-\gamma}} [(1-v)(1-u)]^{-\delta\psi} \left(\frac{vk}{uh} \right)^{\frac{\Lambda_1}{1-\gamma}}, \quad \text{(A.6)}$$

where Φ_1 and Λ_1 are as defined in (2.9a). This equation is just (2.9a) and also indicate the *vk*/*uh* is a function of *u* and *p*.

Finally, the system (2.11a)-(2.11c) is derived as follows. We derive (2.11a) using (A.1), (A.2) and (A.6), obtain (2.11b) using (A.3) and (A.5), and get (2.11c) using (A.4) and (A.5).

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Table 1

-1	TrJ^*	$-BJ^* + DetJ^* / TrJ^*$	$Det\ J^*$	Number of negative roots	BGP
−	−	−	−	3	sink
−	+	+	+	2	sink
−	−	−	+	2	sink
−	−	+	+	2	sink
−	+	−	+	0	source
−	+	+	−	1	saddle
−	+	−	−	1	saddle
−	−	+	−	1	saddle

Table 2

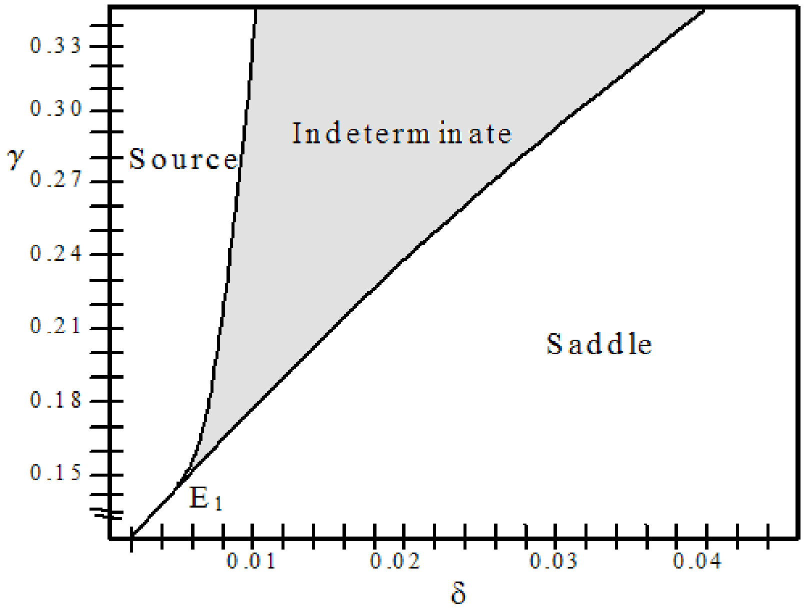
A	Growth rate	τ	γ	δ	ψ	ρ	σ	α	η	B
1	0.02	0.2	0.08	0.08	0.25	0.025	1.5	0.3	0.2	0.07872

Table 3: Robustness

Variations	*B*	Range of ψ for a sink	Shape of region (δ,γ) for a sink
Benchmark[1]	0.07987	[0.15, 0.27] ; [0.47, 0.50]	Figure 1 and Figure 2
ρ=0.01	0.04852	[0.11, 0.21] ; [0.48, 0.50]	Similar
ρ=0.05	0.13802	[0.16, 0.25] ; [0.48, 0.50]	Similar
σ=1	0.04802	[0.15, 0.27] ; [0.48, 0.50]	Similar
σ=3	0.19037	[0.17, 0.26] ; [0.48, 0.50]	Similar
α=0.25	0.08224	[0.10, 0.16] ; [0.35, 0.50]	Similar
α=0.42	0.07091	[0.28, 0.50]	Similar[2]
η=0.1(<α)	0.08567	[0.24, 0.35] ; [0.46, 0.50]	Similar
η=0.25(<α)	0.07515	[0.12, 0.20] ; [0.48, 0.50]	Similar
η=0.3(=α)	0.06974	[0.06, 0.14]	Similar to Figure 1
η=0.35(>α)	0.06394	[0.06, 0.12]	Similar to Figure 1

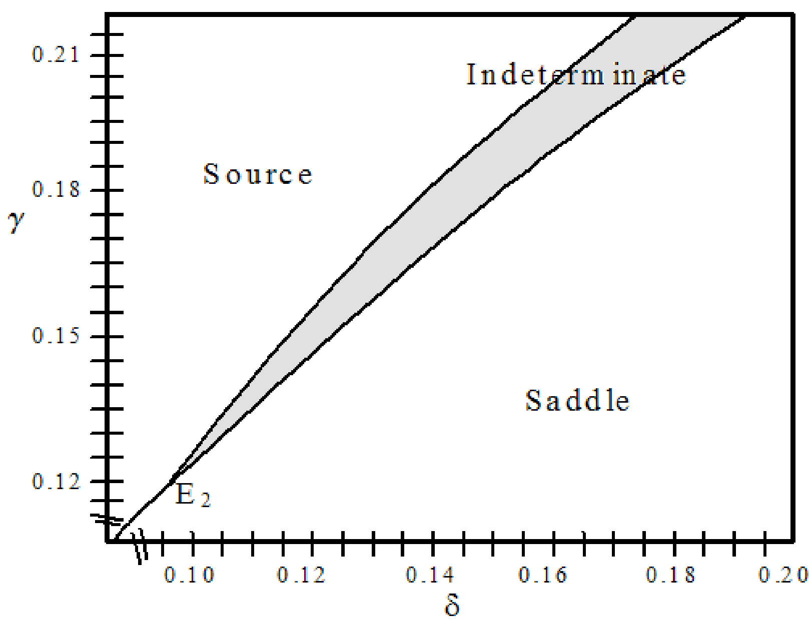
1. Benchmark parameters: *A*=1, *B*=0.07987, τ=0.2, ρ=0.025, σ=1.5, α=0.3, η=0.2, γ∈[0.01, 0.40], δ∈[0.01, 0.40].
2. As ψ increases from 0.28, the shape in Figure 1 disappears and only the shape in Figure 2 emerges.

Figure 1: ψ=0.2



Note: A=1, B=0.07987, τ=0.2, ρ=0.025, σ=1.5, α=0.3, η=0.2. The shaded area presents the region of indeterminacy and E_1 is the lowest point at (δ, γ)=(0.005, 0.147).

Figure 2: ψ=0.5



Note: A=1, B=0.07987, τ=0.2, ρ=0.025, σ=1.5, α=0.3, η=0.2. The shaded area presents the region of indeterminacy and E_2 is the lowest point at (δ, γ)=(0.09, 0.122).

Figure 3: Human Capital Allocation between Sectors,
Case with $\Gamma_1>0$

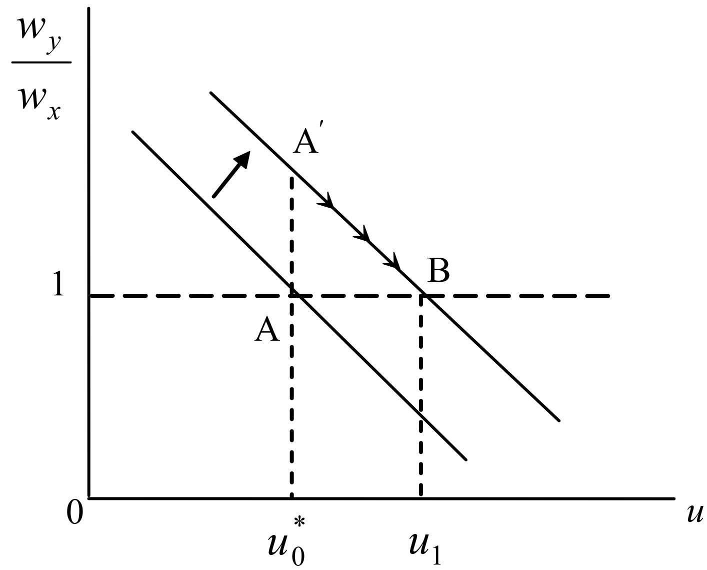
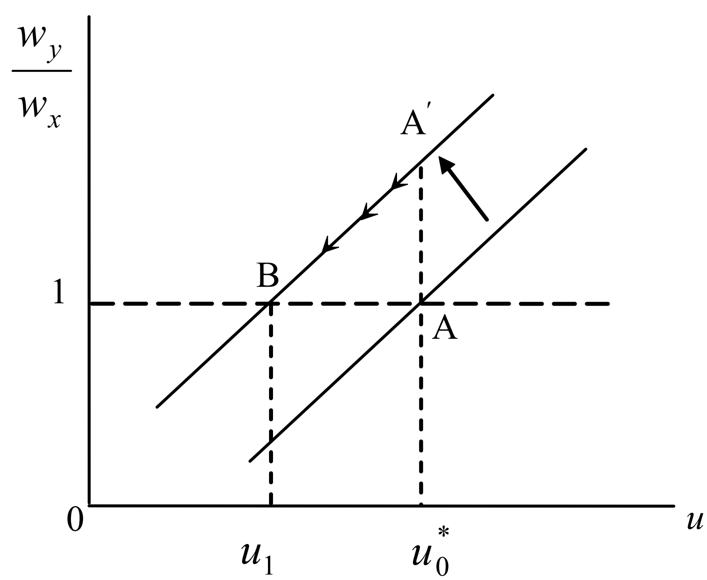


Figure 4: Human Capital Allocation between Sectors,
Case with $\Gamma_1<0$



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