MAGYAR NEMZETI BANK

MNB WORKING PAPERS

2010/10

YULIYA LOVCHA–ALEJANDRO PEREZ-LABORDA

Is exchange rate – customer order flow relationship linear? Evidence from the Hungarian FX market

Is exchange rate – customer order flow relationship linear? Evidence from the Hungarian FX market

October 2010

MNB Working Papers 2010/10

Is exchange rate – customer order flow relationship linear? Evidence from the Hungarian FX market*

(Lineáris-e az árfolyam és az order flow kapcsolata? Vizsgálatok a magyar devizapiacon)

Written by: Yuliya Lovcha–Alejandro Perez-Laborda

Published by the Magyar Nemzeti Bank

Szabadság tér 8–9, H–1850 Budapest

http://www.mnb.hu

ISSN 1585 5600 (online)

* University of Alicante. This paper was partly written while Lovcha was visiting researcher of the Department of Financial Analysis of the Magyar Nemzeti Bank (Central Bank of Hungary, MNB). The authors are grateful to the MNB for providing the data and financial support. We also thank Ian Marsh, Gabriel Perez-Quiros, Maximo Camacho, Aron Gereben and all seminar participants of the MNB and 14th ICMAIF conference for their comments and suggestions. The usual disclaimer applies. For correspondence: Departamento de Fundamentos del Analisis Economico, Universidad de Alicante, Spain. E–mail: yuliya@merlin.fae.ua.es, alex@merlin.fae.ua.es.

Contents

Abstract 5

1 Introduction 6

2 Data description and Empirical Preliminaries 8

Data description 8

Preliminary Estimations and Linearity Analysis 9

3 Non-linear Estimation and Model Fitting 12

Modeling Non-linearity 12

Estimation Results: MS and TR 14

In-Sample Fitting 16

Meese-Rogoff Test 17

4 Conclusions 18

Appendices 19

A Tables and figures 19

B STR estimation 27

References 29

Abstract

Over the last decade, the microstructure approach to exchange rates has become very popular. The underlying idea of this approach is that the order flows at different levels of aggregation contain valuable information to explain exchange rate movements. The bulk of empirical literature has focused on evaluating this hypothesis in a linear framework. This paper analyzes nonlinearities in the relation between exchange rates and customer order flows. We show that the relationship evolves over time and that it is different under different market conditions defined by exchange rate volatility. Further, we found that the nonlinearity can be captured successfully by the Threshold Regression and Markov Switching models, which provide substantial explanatory power beyond the constant coefficients approach.

JEL: C22, F31, G15.
Keywords: customer order flows, nonlinear models, microstructure, exchange rate.

Összefoglalás

A devizaárfolyamok alakulását a piaci mikrostruktúra alapján leíró modellek ez elmúlt évtizedben egyre nagyobb népszerűségnek örvendenek. A mikrostruktúrán alapuló megközelítés alapfeltevése az, hogy a piaci szereplők order flow-ja az árfolyamok alakulására vonatkozó információt tartalmaz. Az empirikus irodalom nagy része ezt a feltevést lineáris modellek felhasználásával tesztelte. A jelen írás az ügyfél order flow és az árfolyam kapcsolatát nemlineáris modellkeretben vizsgálja. Megmutatjuk, hogy ez a kapcsolat időben változik, és függ attól, hogy az adott időszakban mekkora az árfolyam volatilitása. Ezen túl azt találtuk, hogy a nemlinearitás sikeresen megragadható *Threshold Regression* és *Markov Switching* modellek alkalmazásával. Az állandó együtthatós esethez képest a paraméterek időbeli változása jelentősen növeli a modellek magyarázó erejét.

1 Introduction

In standard macroeconomic models exchange rate is determined by fundamental factors, which are observed by all agents in the economy and constitute public knowledge. In these models, there is no private information and price determination is straightforward and immediate. Unfortunately, their empirical performance is very poor. Meese & Rogoff (1983a) and Meese & Rogoff (1983b) show that the structural macro models almost do not have power to explain exchange rate movements and cannot out-perform a naïve random walk in out-of-sample fitting (see Isard (1995) or Taylor (1995) for a survey).

Inspired by Lyons (1995), the market microstructure approach has recently become popular. According to this approach, the information on the market is asymmetric, i.e., some agents have private information. When the market is not fully efficient, the informed agents can exploit this information to get profit. In the classical framework (Evans & Lyons (2002)), market-makers get informed about local demand by observing orders from their own customers (1st-round trading), and receive information about global demand by trading with other dealers in a 2nd-round. Consequently, market-makers can infer the private information from the order flows, and adjust the quotes accordingly trying to close with zero net open positions. In this way, the information is embedded into the market through the order flows.

A growing amount of empirical literature focuses on examining the relation between exchange rates and order flows, mainly by estimating a linear regression of the price changes on the net order flows. The overall conclusion is that order flows contain relevant information for exchange rate determination (see e.g. Evans & Lyons (2002) for inter-dealer order flow or Bjønnes *et al.* (2005) and Marsh & O'Rourke (2005) for customer order flows).

Although linearity has been a maintained assumption in the empirical literature, a time varying relation between exchange rates and order flows is, a priory, more realistic: why should the relation be the same under dramatically different market situations as crises or periods of growth? According to theoretical models (Admati & Pfleiderer (1988), Subrahmanyan (1991) or Easley & O'Hara (1992)), exchange rate volatility is one of the key variable identifying the portion of information embedded to the market. Since the order flows are means of information transmission, the relationship between them and exchange rate is thought to be different in periods of high and low volatility.

In this paper, we first conduct an intensive analysis of the linear relation between exchange rate and customer order flow. Results of this analysis reveal that the relationship evolves over time and that it is different under distinct market conditions defined by volatility. We provide further evidence of this result through the estimation of two types of nonlinear models: Markov Switching (MS) and Threshold Models (TR). We use data on the Hungarian forint (HUF) - euro (EUR) spot exchange rate and different types of spot customer order flows: foreign participants and domestic non-banks. The data set was provided by the Magyar Nemzeti Bank (Central Bank of Hungary). In this sense, our work is related to Gereben *et al.* (2006), who use a similar data set to study the same relationship in a standard linear setting.

Although most of the empirical analysis has focused on the major currencies, the analysis of a CEEC currency is of major interest since the better understanding of the information transmission mechanism to the foreign exchange market has strong policy implications for countries that (as Hungary) plan to join to European Monetary Union, and that, eventually, have to make use of foreign exchange intervention to keep the currency into the prescribed bands.

Our main results include the following. First, we confirm that the customer order flows considered in this paper contain valuable information to explain contemporaneous HUF/EUR exchange rate movements. Second, the relationship between order flows and exchange rate is clearly nonlinear. Order flows have higher impact to exchange rates and are more informative in periods of high volatility. This result is in line with the information uncertainty model developed by Easley & O'Hara (1992), the conclusions obtained by Subrahmanyan (1991). Third, the nonlinear relation between exchange rate and order flows can be successfully captured by nonlinear models. In particular, the nonlinear models, and specially the MS, provide substantial explanatory power beyond the constant coefficient (OLS) approach.

Despite the amount of literature devoted to nonlinear exchange rate modeling over the last decade, the question of the nonlinearity in the exchange rate – order flow relationship has still not received much attention. Up to our knowledge, only Lyons (1996), and Berger *et al.* (2008) touch this issue slightly [1], although none of them analyzes the nonlinearity intensively. A noticeable exception can be found in Luo (2001)[2]. The main differences of this work with the existent literature and, in particular, with Luo (2001) are the following. First, this is the first work which studies nonlinearity in the exchange rate - customer order flow relation. The role of the customer order flows is central in the microstructure literature. They are the prime source of the private information in the market and a catalyst for the inter-dealer market activity in all canonical models (Kyle (1985), Glosten & Milgrom (1985) or Evans & Lyons (2006)), and therefore more important in the determination of exchange returns than inter-dealer order flow.

Second, there are differences in the set of models employed. Initially, we remain a priory agnostic about the sources driving the nonlinearities in the exchange rate - order flow relationship by estimating a MS model. Since the seminal work of Boothe & Glassman (1987), there is increasingly strong evidence that the conditional distribution of the nominal exchange rates is well described by a mixture of normal distributions, and that MS models fit exchange rate data very well. However, in spite of the sizable attention that MS models have currently received in the exchange rate literature, they still have never been employed for the exchange rate - order flow analysis. Given that we find very high correlation between the estimated regime changes and exchange rate volatility, we after consider several TR specifications using volatility as a threshold variable. In this way, we evaluate if volatility by itself can direct regime changes. Since contemporaneous daily volatility is potentially endogenous (that could be one of the main critics to Luo (2001)), we evaluate the severity of this problem through the estimation of a Threshold Regression with Endogenous Threshold (THRET) model recently proposed by Kourtellos *et al.* (2009) that controls for endogeneity of the threshold variable.

Finally, given that the statistical significance of the nonlinearities is a necessary but not sufficient condition for model adequacy, we evaluate the ability of the nonlinear order flow specifications to explain in- and out-of-sample exchange rate movements. Together with the estimation of a MS model, the capacity of the nonlinear models to explain the data remained, up to our knowledge, unexplored.

The paper is organized as follows. Section 2 contains a description of the data set, describes the preliminary estimations and the nonlinearity testing. Section 3 makes a short description of the nonlinear models employed in this work, presents their estimation results and discuss their fitting performance, both in- and out-of-sample. Section 4 concludes.

[1] Lyons (1996) tests weather the information content of the order flow increases with market intensity. He concludes that the order flow is more informative when trading intensity is high. Berger *et al.* (2008) find evidence of nonlinearities by comparing the estimates of the OLS regression of exchange rates on order flows with the ones obtained from a nonparametric regression.

[2] Luo (2001) tests the equally information hypothesis of the exchange rate inter-dealer order flow relationship under different market conditions. The author finds that the information contained in the order flow tends to increase with spread and volatility, and decrease with the volume of trade. He also finds that an interaction model and a Logistic Smooth Threshold Regressive (LSTR), are able to capture this nonlinearities. However, the author does not control for the potential endogeneity of contemporaneous volatility nor evaluates the ability of any of the nonlinear specifications to explain exchange rate movements.

2 Data description and Empirical Preliminaries

DATA DESCRIPTION

The data set consists from the HUF/EUR exchange rate and corresponding customer order flows data at daily frequency covering the period from the 2nd January 2003 to the 15th July 2009. The exchange rate contains quotes from the Reuters D2000-2 system. We use the midpoint of the best bid and ask quotes at 5:00 PM each day. We apply the logarithmic transformation to the exchange rate series to circumvent "Siegel paradox". After, we take first difference and multiply it by 100 to create a series of daily returns. The data on customer order flows is provided by the Central Bank of Hungary. The source of the data is the Daily Foreign Exchange Report of the Bank, which contains all foreign exchange transactions of significant size carried out by commercial banks residing in Hungary. In this sense, our order flows data set is not complete because it does not cover transactions produced by the financial institutions located offshore. According to the Central Bank of Hungary information, the offshore turnover is significant, although the most important market-makers are said to be the locally based ones[3]. Additionally, we do not include the central bank order flow into analysis because of confidentiality. Central bank order flow was relatively stable in our data span with the only exception of the speculative attack on January 15 and 16, 2003, when the Bank was carrying out large-scale Hungarian forint sales to defend the exchange rate band[4]. Consequently, we exclude first 11 observations from the sample.

It is important to remark that compared with the data sets used so far, this data set has important advantages. First, our study focuses on customer (end-of-user) order flow data. Although the bulk of empirical literature studies the intra-dealer order flow, customer order flows are consistently more important in exchange rate determination than inter-dealer order flow according to the market microstructure literature (see e.g. Lyons (1995), Lyons (1995), Evans & Lyons (2005) or Sager & Taylor (2008)). Moreover, the customer order flow data is broken down according to the nature of customer: foreign participants and domestic non-banks, allowing us to test different information content of each of these order flows. Different types of customers can have different sources of private information and different aims, making the analysis particularly interesting. Concentration by the consumer allows us to use highly aggregated data with information content[5]. Second, the coverage of the data set is relatively high. Even taking into account that it does not contain information on transactions produced by the institutions located offshore, it provides a more complete picture of the market than the one offered by studies that use data from a single market-maker (Froot & Ramadoari (2002), Carpenter & Wang (2003) and others). Finally, as opposite to commercial data sets, the data from the MNB is not revealed to the market[6], and is comparatively long for the standard microstructure literature[7], so it is particularly well suited to study of nonlinearities, since sample size is crucial to detect regime-change dynamics in the data.

Figure 1 plots the log-HUF/EUR exchange rate returns (left axe) and two cumulated customer spot order flows: foreign participants and domestic non-banks (right axe). Table 1 summarizes the descriptive statistics of the series.

---

[3] For more detailed data description look Gereben et al. (2006), Appendix 1.

[4] The Hungarian forint's exchange rate is allowed to fluctuate within +/- 15 per cent band relative to euro.

[5] Completely aggregated over a trading day, total signed order flow of a bank is equal to zero, at least as an approximation. Therefore, aggregated over all banks total order flow is only randomly different from zero and uncorrelated with exchange rate changes.

[6] The power of commercial datasets to explain exchange returns has been recently questioned by Sager & Taylor (2008) .

[7] Similar spans can only be found in Bjønnes *et al.* (2005) and Gereben *et al.* (2006) for the Swedish krona/euro and Hungarian forint/euro respectively. Comparatively with last, our data set is more aggregated and contains 4 years (approximately) more of observations.

PRELIMINARY ESTIMATIONS AND LINEARITY ANALYSIS

In this section we analyze the linearity assumption in the relationship between exchange rate and consumer order flows. To do so, we start by computing the sample cross-correlations between the HUF/EUR returns and the order flows. Results are presented in the Figure 2. The contemporaneous cross-correlations between returns and foreign participants, domestic non-banks order flows, are in both cases statistically different from zero at low lags, negative for the first series and positive for the second.

As a benchmark, we estimate a generic linear model for the exchange rate – order flow relationship. As in Bjønnes *et al.* (2005), we perform separate regressions for each type of customers (foreign participants and domestic non-banks):

$$y_t = \alpha_i + \beta_i X_{i,t} + \epsilon_{i,t} \tag{1}$$

where y_t is the exchange rate returns and $X_{i,t}$ is the respective net customer order flow (in trillions forints) at t. The results of this estimation are presented in the Table 2[8]. The estimated slope coefficients associated to the order flow is in both cases strongly significant and, as expected by the cross-correlation analysis, negative for the foreign participants and positive for the domestic non-banks order flow. In this respect, our estimations satisfy the notion of "push" and "pull" customers (see e.g. Bjønnes *et al.* (2005)) and can be interpreted accordingly.

Push customers are though to provide information to the market that is not yet common knowledge, initiating the orders and causing price changes (they are 1st-round traders according to the Evans & Lyons (2002) setting). As a result, their trading will be positively correlated with price movements. Instead, pull customers are assumed to provide liquidity to the market, absorbing the open positions of market-makers generated when trading with push customers. Therefore, their buying order is likely to be negatively correlated with currency appreciations[9]. Pull customers are attracted into the market by prices which suit them because they wish to trade on a certain side of the market and decide to act now rather than postpone the trade in the hope of achieving a better price. Motivated by the cross-correlation analysis and the results of the estimation of the linear model (1), we identify the foreign participants push customers and the domestic non-banks as pull. These was also the roll assigned to foreign participants and domestic non-banks in Gereben *et al.* (2006), who study the standard linear model using a very similar data set [10].

As can also be seen in the table, the foreign participants appear to be the most informative order flow in the linear setting, explaining a 37% of the variability in the returns, while domestic non-banks explains only a 12%, which is consistent with the idea that foreign participants are the main drivers of the market. We further test the estimated residuals for the presence of heteroscedasticity for all customers' order flows with the Breusch-Pagan and Cook-Weisberg tests. The null hypothesis of constant variance is rejected at 5% significance level in all estimations. We therefore impose a GARCH(2,1) structure for the conditional variance, but it does not increase the fitting significantly. In general, the results obtained with the linear model are in accordance with the ones obtained by Gereben *et al.* (2006).

Next, we proceed to question the linearity assumption in the relation given by (1). As a preliminary stage, we apply a standard BDS test to detect lack of independence in the estimated residuals of the linear model.

[8] We also performed the analysis plugging both order flows in the same equation $y_t = \beta_0 + \sum \beta_i X_{i_t} + e_t$. However, as in Gereben *et al.* (2006), in this case severe multicollinearity is detected. Although this problem does not affect the predictive power of the model, it affects the coefficient estimates. Multicollinearity is easily explained when one takes into account that cumulated daily customer order flow should be equal to 0. It means that two disaggregated flows should contain very similar information.

[9] Note that the usual quotation of the currency pair is HUF/EUR, therefore an increase in the exchange rate corresponds to a depreciation of the forint. Negative coefficient for the order flow in the regression (1) indicates buying orders that cause Forint appreciations, while positive coefficients indicate a depreciating impact.

[10] As the authors note, this definition is not completely in line with previous findings where the distinction that matters for determining the sign of the order flows' impact on the exchange rate is between financial and non-financial customers (Bjønnes *et al.* (2005) and Marsh & O'Rourke (2005)). A potential explanation that the authors give to this results "stems from the fact that Hungary is an emerging market economy, relying heavily on foreign capital flows. A large share of the economic fundamentals governing the forint's exchange rate are dependent on external factors. As a result, it is likely that foreign customers are more likely to convey non-public information about future fundamentals through their trades than domestic customers."

The BDS is a fairly general linearity test that has power against most nonlinear dynamics, although the type of nonlinearity can not be exactly determined. The null of independent and identically distributed errors is rejected at 5% significance level for all estimations.

In order to inspect the linear relationship further, we implement a moving window regression to the equation (1). This allows us to check how the coefficients associated with the order flows evolve over time. We choose a window length equal to one hundred observations (five months approximately). The window regression slope coefficients (β_i^t) and their confidence intervals are plotted in the Figure 3, together with the series of returns. As can be seen in the figure, the slope coefficient for all the order flows changes significantly with time, increasing (in magnitude) in periods when returns are more volatile.

To confirm this result, we proceed to test the stability of the estimated parameters under different volatility states. We start by constructing a simple daily exchange rate volatility series z_t as:

$$z_t = \frac{\left|\log\left(p_t\right) - \log\left(p_{t-1}\right)\right|}{\sqrt{2/\pi}} \tag{2}$$

where p_t is the exchange rate at t. After, we divide the sample into two sub-samples according to the volatility regime: high volatility periods, when daily volatility at t is higher than its mean, and low volatility periods otherwise. After estimating equation (1) by separately in the two sub-samples, we test for equality of the estimates across sub-samples with Chow test. The results are presented in the Table 2. As can be seen in the table, the equality hypothesis is strongly rejected for both order flows. The slope coefficients are much larger (in absolute value) in high volatility periods. For example, an increase in the foreign participants order flow by one trillion forints leads to a 17% decrease in the HUF/EUR exchange rate in high volatility periods, whereas in low volatility periods the decrease is only of a 3.5%. Hence the price impact (sensitivity) of the order flows increases with volatility. It is also interesting to note that the informativeness of both order flows also increase with volatility. The foreign participants order flow explains a 47% of the returns variation in periods of high volatility but only a 10% in periods of low volatility. For the domestic non-banks this increase is even more dramatic. In low volatility periods this order flow explains less than 1% of the variation of returns increasing up to a 20% in high volatility periods.

As stated in the introduction, the result that the impact and the informativeness of order flows vary with volatility is not new in the economic literature. According to theoretical models (Admati & Pfleiderer (1988), Subrahmanyan (1991) or Easley & O'Hara (1992)) exchange rate volatility is a key variable identifying the portion of information embedded to the market. However, there is no clear consensus in the theoretical models about when the order flow should be more informative, during high or during low volatility periods.

In the Easley & O'Hara (1992) model, abnormal high volume (over the expected liquidity trading) is interpreted by the market maker as a signal of the presence of informed traders on the market. Since price movements (in the sense of price converging to the new values) require the existence of new information and abnormal volume, during the period when volume is high, volatility will also be high and order flow will be more informative. The prediction that periods of high volume of trade are also periods of high volatility has been documented in many studies (see e.g. Lamoreaux & Lastrapes (1990)), and tested by Diz & Finucane (1993) who found that approximately two-third of volatility interventions can be explained by abnormal volume. In the Admati & Pfleiderer (1988) model, liquidity traders (uninformed customers) prefer to trade at the same time, making trading concentrated. Informed traders also choose to trade during concentration in order to camouflage their trading and minimize the price impact. Consequently, during concentration the volume is higher because both types of trade occur at the same time and volatility is also higher because of the more informed trade. Opposite to Easley & O'Hara (1992), the authors discuss the possibility that the order flow is less informative during concentration because of the clump of liquidity trading. However, Subrahmanyan (1991) argues that conclusions of Admati & Pfleiderer (1988) rely on the assumption of risk neutrality of informed traders. Under risk aversion, the order flow become more informative during concentration because number of informed traders trading on the market is higher. Despite their different implications concerning the informativeness of the order flow, all models state a connection between the episodes of high volatility (which is a known characteristic of exchange rate data) and the new information's assimilation by the market.

New information before its incorporation into prices constitutes private knowledge. The microstructure literature identifies two types of private information. First, payoff related information, i.e. information on fundamentals still not released by the central bank or government agencies (as in the model of Admati & Pfleiderer (1988))[11]. The second type is payoff unrelated information obtained by market makers by observing the interim states of the market only. In both cases the new information is incorporated to prices when the market maker, observing his order flows, changes quotes accordingly to match opened positions.

Our inspection of the linear relationship between exchange returns and customer order flow given by (1), provides substantial evidence that there exist important nonlinearities in the data not captured by the generic model. Further, we find that volatility is strongly related to these nonlinearities, increasing both the price impact of the order flows to returns and their informativeness, which is consistent with the theoretical implications of Subrahmanyan (1991) and Easley & O'Hara (1992).

[11] It is found that surprising realizations of fundamentals do have impact to exchange rate and thus contribute to exchange rate volatility (see e.g. Frömmel *et al.* (2008)). However, if the released information on fundamentals is not different from expected, it is already incorporated into prices and does not influence exchange rate.

3 Non-linear Estimation and Model Fitting

MODELING NON-LINEARITY

In the following section we provide further evidence of the results obtained through the estimation of several nonlinear specifications. To start we describe briefly two types of nonlinear models that are used in this paper: Markov switching and Threshold Regression.

Markov Switching Model

Markov Switching Models (MS) became popular for exchange rate modeling since seminal work of Engel & Hamilton (1990) and Engel (1994). Several studies relate exchange rates and macroeconomic fundamentals in a MS context (see e.g. Marsh (2000), Bessec (2003), Sarno *et al.* (2004) or Frömmel *et al.* (2005)) or analyze spot and forward exchange rates comovements (see e.g. Clarida *et al.* (2003)). Up to our knowledge, this is the first attempt to model the relation between exchange rates and order flows in a MS framework.

Allowing for two regimes in the coefficients, the relationship between y_t and X_t can be written in a MS as :

$$y_t = \alpha_{s_t} + \beta_{s_t} X_t + e_t, \quad t = 1 \ldots T, \tag{3}$$

where y_t denotes exchange rate returns, X_t is a net order flow at t and e_t is a disturbance with zero mean and variance σ^2.

The unobserved state variable $s_t = \{1,2\}$ follows a two-state, first order Markov process with the following transition probability matrix:

$$P = \begin{pmatrix} \Pr\left(s_t = 1 | s_{t-1} = 1\right) & \Pr\left(s_t = 1 | s_{t-1} = 2\right) \\ \Pr\left(s_t = 2 | s_{t-1} = 1\right) & \Pr\left(s_t = 2 | s_{t-1} = 2\right) \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix}$$

where the transition probabilities p_{hh} give the probability that state h will be followed by another state $h = \{1,2\}$. These transition probabilities are assumed to remain constant between successive periods. With the additional assumption that e_t in (3) are normally distributed (conditional to the information available at time t, Ω_t), the conditional density of y_t is normal.

The model can be estimated applying the Expectation Maximization algorithm (see e.g. Hamilton (1994) for further details). We will refer to the probability to be in state h based on information of the whole sample Ω_T as smoothed probability ($\Pr\left(s_t = h | \Omega_T\right)$).

Threshold Model

Threshold regressive (TR) models were first proposed by Tong (1978), Tong & Lim (1980) and Tong (1983), and a comprehensive statistical analysis was made by Hansen (2000). The main idea is that the evolution of the process, governing the dependent variable at any point of time, depends on the value of an observed *threshold variable* z_{t-d} relative to a *threshold value* c.

Formally, the Threshold Model can be written as:

$$y_t = b_1' X_t + b_2' X_t F\left(z_{t-d}, \gamma, c\right) + e_t \tag{4}$$

where X_t is a vector of explicative variables, which may contain lags of the endogenous variable, e_t is a martingale difference sequence with constant variance σ^2 and $F\left(z_{t-d}, \gamma, c\right)$ is known up to parameters vector γ and scalar c. In standard autoregressive specifications (Threshold Autoregressive or TAR) , the threshold variable z_{t-d} is usually chosen to be a lagged value of the dependent variable (y_{t-d}), although it could be any other exogenous variable. The transaction function $F\left(z_t,\gamma,c\right)$ can be either continuous logistic

$$F\left(z_{t-d},\gamma,c\right)=\left[1+\exp\left\{-\gamma\left(z_{t-d}-c\right)\right\}\right]^{-1}$$

or exponential

$$F\left(z_{t-d},\gamma,c\right)=1-\exp\left\{-\gamma\left(z_{t-d}-c\right)^2\right\}$$

or discontinuous with $\gamma\to\infty$, giving back 1 if $z_{t-d}\leq c$ and 0 otherwise. In the first case the model is logistic or exponential smooth TR (LSTR or ESTR), in the second, a standard TR.

Since we are interested in analyzing the contemporaneous relationship between exchange rate and customer order flow, as before we define y_t to be the exchange rate returns and X_t - a particular customer net order flow. According to theoretical models, regime changes depend on the contemporaneous ($d=0$) market conditions defined by the threshold variable z_t.

One of the crucial assumptions of the TR model is that the threshold variable is exogenous. If correlation between the threshold variable with the contemporaneous error e_t is suspected, the exogeneity assumption cannot be insured. To overcome this problem a recent model developed by Kourtellos *et al.* (2009) can be employed: the Threshold Regression with Endogenous Threshold variable (THRET)[12]. The model and the estimation method proposed by the authors (THRET-C2SLS) are detailed below.

The model is the following:

$$y_t=\beta_1'X_t+e_{1t},\quad \text{if}\quad z_t\leq c \tag{5}$$

$$y_t=\beta_2'X_t+e_{2t},\quad \text{if}\quad z_t> c \tag{6}$$

$$z_t=q_t\pi+v_t \tag{7}$$

Two first equations describe the relationship between the variables of interest in each of the two regimes, z_t is the threshold variable with c being the sample threshold value. The third equation is the selection equation that determines the regime that applies. The matrix q_t is $T\times l$, $q_t=\left[q_{1t},q_{2t}\right]$, where q_{1t} is $T\times l-1$ matrix of instruments for the threshold variable and the vector q_{2t} contains X_t. The variance covariance matrix of the errors $\left(e_{1t},e_{2t},v_t\right)$ has the following properties: $E(e_{1t},e_{2t})=0$, $E(e_{it},v_t)=\sigma_{ve_i}>0$, $E(e_{it}^2)=\sigma_i^2>0$, $i=1,2$, and $E(v_t^2)=\sigma_v^2=1$ due to a normalization. Notice that if $\sigma_{vei}^2=0$, $i=1,2$, the Threshold variable is exogenous.

We can also rewrite the THRET model (5), (6) and (7) in a single equation:

$$y_t=\beta_2'X_t+(\beta_1-\beta_2)'X_{1t}(c)+\varkappa_2\lambda_t\left(c-q_t\pi\right)+(\varkappa_1-\varkappa_2)\widetilde{\lambda}_{1t}\left(c-q_t\pi\right)+\epsilon_t, \tag{8}$$

where $X_{1t}(c)=X_tI\left(z_t\leq c\right)$, $\lambda_t\left(c-q_t\pi\right)=I\left(z_t\leq c\right)\lambda_{1t}+I\left(z_t>c\right)\lambda_{2t}$. The terms $\lambda_{1t}=-\frac{\phi(c-q_t\pi)}{\Phi(c-q_t\pi)}$ and $\lambda_{2t}=\frac{\phi(c-q_t\pi)}{1-\Phi(c-q_t\pi)}$ are inverse Mills bias correction terms, where ϕ and Φ represent the pdf and cdf of a standard Normal. In the above equation we have also defined $\widetilde{\lambda}_{1t}\left(c-q_t\pi\right)=I\left(z_t\leq c\right)\lambda_{1t}$, and re-named the covariance between error v_t and e_{it} as $\varkappa_i=\sigma_{ve_i}$ for $i=1,2$. The error term in (8) is given by $\epsilon_t=I\left(z_t\leq c\right)\left(e_{1t}-\varkappa_1v_t\right)+I\left(z_t>c\right)\left(e_{2t}-\varkappa_2v_t\right)$. It can be easily seen that if the threshold variable is exogenous, or $\varkappa_1=\varkappa_2=0$, the expression (4) is equivalent to (8) with $b_1=\beta_2$, $b_2=\beta_1-\beta_2$, $F\left(z_t,\gamma,c\right)=I\left(z_t\leq c,\gamma\to\infty\right)$ and $e_t=\epsilon_t$.

[12] Caner & Hansen (2004) developed a generalized method of the model estimation for Threshold Regression models with endogenous regressors. Kourtellos *et al.* (2009) extended the model of Caner & Hansen (2004) to allow for endogeneity of the threshold variable.

The estimation procedure has three steps. First, we estimate the parameter vector π in the threshold equation (7) by Least Squares (LS). Second, we estimate the threshold parameter by minimizing a concentrated two stage least squares (THRET-C2SLS) criterion using the estimates of $\widehat{\pi}$ from the first stage:

$$\widehat{c} = \arg\min_{c} S_n(c)$$

where

$$S_n(c) = \left(y_t - X_t\beta_2 + X_{1t}(c)(\beta_1 - \beta_2) + \varkappa_2\lambda_t\left(c - q_t\widehat{\pi}\right) + (\varkappa_1 - \varkappa_2)\widetilde{\lambda}_{1t}\left(c - q_t\widehat{\pi}\right)\right)^2$$

Third, we estimate the parameters in (5) and (6) based on the split samples implied by $\widehat{c}$ by LS. For detailed description of the model, estimated procedure and its asymptotic properties see Kourtellos *et al.* (2009). The advantage of the THRET model is that it controls for the endogeneity of the threshold, but from another side this model restricts the transaction function to be zero or one. In this sense STR models are more flexible allowing the transaction function to be a function of the difference between threshold variable and the threshold parameter. Unfortunately, the IV estimation has still not been extended to STR models with endogenous threshold variable (but is a promising field of potential econometric research).

ESTIMATION RESULTS: MS AND TR

The previous analysis of the linear regression (1) provided us evidence that, although the intercepts α appear to be not significant, the coefficients associated with the order flows evolve with time $\beta = \beta_t$. We remain a priory agnostic about the sources driving this nonlinearities by the estimation of a fairly standard MS model. Testing linearity against MS-type nonlinearity is not straightforward, since transition probabilities are not identified under the null, and conventional statistics does not follow asymptotic χ^2 distribution. Formal tests have been proposed by Hansen (1992), Hamilton (1996), Garcia (1998), and Sanzo (2009) and Carrasco *et al.* (2009). Last authors derive an optimal test against the alternative of Markov switching that requires only the estimation of the model under the null. Motivated by our previous findings, we follow here the approach of Carrasco *et al.* (2009) for the special case where the alternative is a model with MS in the coefficient associated to the net order flow (see e.g. Hamilton (2005) for more details about this test):

$$y_t = \alpha + \beta_{s_t} X_t + e_t, \quad t = 1...T.$$

Results of this test can be found in the Table 3. Empirical critical values are computed by parametric bootstrap from 1000 iterations for a sample size equal to the size of the original data. As can be seen in the table, the test rejects strongly the null of a linear model versus a Markov-switching alternative, suggesting that, at least, a model with two regimes in the coefficients associated to the order flows should be used to fit the data.

We then proceed to estimate a more general MS model allowing for additional regime shifts in the intercepts, as in (3). Results of the MS estimation are presented in the Table 4. In line with the results obtained in the previous section, the intercepts are small and usually not significant. Opposite, the coefficients associated with the order flows (β_{s_t}), are always significant and very different from one state to another. For both order flows, the slope coefficients have always the same sign as in the OLS estimation (negative for foreign participants and positive for domestic non banks) but the magnitudes are larger in the state 2 (high sensitivity state) and smaller in the state 1 (low sensitivity state): $|\beta_1| \leq \left|\beta^{OLS}\right| \leq |\beta_2|$.

We then test the hypothesis of equality of the intercepts in both states using a likelihood ratio (LR) in addition to constructing another LR for the null of equal slope coefficients (see Krolzig (1997)). As can be seen in the Table 4, the null of no regime dependence in the intercept can not be rejected for the foreign participants order flow at usual significance levels. However, the test strongly rejects the null of equal slope coefficient across states for both order flows. In general, the results of this testing procedure indicate that regime switches in the exchange rate - order flow relationship are characterized by different impact of the order flow on returns.

From the estimated transition probabilities p_{11} and p_{22}, we can calculate the duration of being in each regime[13]. Since $p_{22} < p_{11}$ for both order flows, the high sensitivity periods have shorter duration than low sensitivity periods. For instance, in the case of the foreign participants order flow, the transition probabilities are estimated 94.2% and 75% (Table 4); this indicates that the average expected duration of being in the low sensibility regime (state l) is about 17 days compared to 4 days in the high sensitivity (regime 2).

Our analysis of the linear relationship pointed out that the impact of the order flows to the exchange returns was larger in high volatility periods. Since the results of the MS estimation indicate a higher impact of order flow during the regime 2 (high sensitivity), a natural question arises: how much is related the probability of being in the high sensitive state to volatility? Figure 4 (right column) depicts the smoothed probabilities of being in the high sensitive state (right axe) together with the exchange returns (left axe). Visual inspection of the two series shows that higher values of the smoothed probabilities correspond to periods of higher volatility. In fact, the correlation among the volatility and the smoothed probability series is positive and very high (0.64 and 0.69 for the foreign participants and domestic non-banks order flows, respectively). Note that, this result is not a trivial implication of the MS model. The regime changes are not driven by observed volatility, but by an unobserved random variable S_t, which it is only assumed to follow an ergodic Markov Chain [14].

We then proceed to check if exchange rate volatility by itself can direct regime changes through the estimation of a threshold model (TR) using contemporaneous daily volatility defined as in (2) as a threshold variable z_t[15].

When the threshold variable is lagged (as in standard autoregressive specifications), it is relatively easy to assume that it is uncorrelated with contemporaneous noises as long as the model is dynamically complete. This assumption is a priory more difficult to hold, if the relationships appear between contemporaneous variables. Note, however, that exchange rate returns are well-known to be unconditionally symmetric and highly leptokurtic. If the distribution of exchange returns y_t is symmetric, the dependent variable y_t is completely uncorrelated with contemporaneous volatility z_t [16]. In fact, the empirical correlation coefficient is very small ($\simeq 0.1$) making us suspect that endogeneity cannot be strong. In order to evaluate the severity of endogeneity, we estimate a THRET model as in (8) using lagged values of volatility as instruments, together with a standard TR model (without instrument the threshold variable). Results of these estimations are reported in the Table 4. For all the order flows, all the estimated parameters (including estimated threshold and variance) are statistically identical[17] in both TR and THRET models. This result suggests that, if endogeneity exists, it is very small and does not cause parameters to be biased[18]. As can be seen in the table, the estimated intercepts are again often not significant while the estimated slope parameters are statistically significant and different across regimes. The sign of the slope coefficients are the same in OLS and MS estimation and, as expected, their magnitudes are larger during the high volatility regime.

After optimal thresholds have been identified, a conventional Chow test can be conducted to test the null of linearity against the Threshold Regression specification. Since threshold parameters are not identified under the null, the test statistic has nonstandard distribution. Following Hansen (1997), we employ 200

[13] The average duration of each state can be calculated as (see e.g. Hamilton 1994): $\sum_{i=1}^{\infty} i\, p_{hh}^{i-1}\left(1-p_{hh}\right)=\left(1-p_{hh}\right)^{-1}$

[14] In addition, the empirical correlation between the smoothed probabilities of the two order flows (FP, DNB) regressions is also quite high (approx. 0.7), indicating a common underlying process. This last result also suggests that a joint estimation of the two equations (system MS) may be helpful to improve regime detection. We however proceed in a different way testing directly weather volatility is precisely the common process driving the regime changes in both equations.

[15] We also find that lagged values of volatility are not able to correctly detect regime changes. The simple volatility estimator also performs better than other measures of intraday volatility as the Parkinson's High-Low or the bid-ask spread.

[16] Let $y_t \sim \mathcal{F}_t\left(0,\sigma_t^2\right)$ where $\mathcal{F}$ denotes the cumulative distribution function. If the distribution is symmetric: $Cov\left(y_t, |y_t|\right) = E\left(y_t |y_t|\right) = -\int_{-\infty}^{0} y_t^2 \frac{f(x)}{1-F(0)} dx + \int_{-\infty}^{0} y_t^2 \frac{f(x)}{F(0)} dx = 0$, leading to the result.

[17] The results of the TR model belong to the 95% confidence of the estimated THRET.

[18] Additionally, we test $\varkappa_1 = \varkappa_2 = 0$ in (8) and we cannot reject H_0 at any standard significance level. This result suggests exogeneity of the threshold variable.

parametric bootstrap replicas, and a modified grid search to find critical values. For both order flows the linearity hypothesis is strongly rejected (Table 3).

As commented before, the TR (and THRET) models impose strong restrictions to the shape of the transaction function that may reduce the ability of the model to track the data. Motivated first by the small correlation between volatility and exchange rate returns, later by the similar estimation results of the TR and THRET and the results of the exogeneity testing of the threshold, we proceed to estimate a standard STR model, which allows for flexibility in the transaction function. The ability of this last model to explain in-and-out exchange rate movements is presented together with the results of the MS and THRET models in the next section. In this way, we have a better picture of how regime changes directed by volatility can explain exchange rate returns, which is one of the main purposes of this work. The results of the estimation and testing of the standard STR models can be found in the Appendix II.

As an overall, the results of the estimation of nonlinear models confirm that the conclusions obtained in the last section are robust. The relationship between order flows and exchange rate is not linear, and the price impact of the order flows increases with volatility, which is consistent with the implications of the theoretical models of Subrahmanyan (1991) and Easley & O'Hara (1992).

IN-SAMPLE FITTING

To asses the ability of the nonlinear models to explain in-sample exchange rate movements, we compare their fitting performance with the usual linear (OLS) model. As a benchmark, we also include a Random Walk (RW). In order to evaluate fitting performance, we compute two standard measures: the Mean Absolute Errors (MAE) and the Root Mean Squared Errors (RMSE). Results are presented in the Table 5. The values in the first colum are the MAE (up) and RMSE (down) of the RW. The following four columns (from two to five) report the same error measures for the competing order flow model relative to the ones of the RW. Last three columns present the relative performance of nonlinear specifications with respect to the OLS. In both cases, a number smaller than one indicates better fitting of the competing model. Percentage gain can be obtained by subtracting those numbers from one.

In general, the OLS fits the data better than the RW, especially for the foreign participants order flow. For the domestic non-banks order flow, however the gain is rather small. When considering the nonlinear specifications, the fitting increases substantially for both order flows. For the MS, which is the model that performs better, the fitting gain ranges between a $18-35\%$ better than the RW and a $15-20\%$ better than the OLS, depending on the order flow considered and the measure employed. The threshold models perform in general a bit worse than the MS, although the results are very similar once we allow the transaction function to have enough flexibility (STR).

To asses how the fitting changes with volatility, we also check the performance of all models relative to the RW in a particular period inside the sample characterized by a relatively high volatility. The period runs from 04/02/09 to 15/07/09 and corresponds to the last five months in our sample. It is worth to note that it lies in the middle of the current crisis and the average volatility in this period is about two times larger than in the whole sample. Results are reported in second part of the Table 5. The fitting performance relative to the RW in this period is higher than in whole sample for all the order flow specifications, both linear and nonlinear. This result is very intuitive taking into account that the RW predicts "no changes" and in the high volatility period the changes are huge. Thus, whatever model that can predict at least direction of the change will bit the RW easily. Even though, the increase of the relative fitting performance for the OLS specification is rather modest, reflecting the fact that the linear specification is clearly better than the RW, but still not flexible enough to catch fast and strong exchange rate movements in high volatility period. The relative performance of the nonlinear specifications with respect to the RW is 10-15% higher in the period of high volatility than in whole sample, depending on measure employed and specification considered. The relative performance of the nonlinear specifications with respect to the OLS is a 5-10% higher in period of high volatility than in the whole sample and the resulting percentage gain of fitting nonlinear specifications in the high volatility period is around 15-25%.

Figure 5 plots the fitted exchange rate returns by the MS and the two threshold models considered (THRET and STR) respectively (in green) together with the OLS results (in red) in this particular period. As can be observed in the figures, the nonlinear versions are able to track the data better than OLS capturing the peaks and dips presented in exchange rate returns.

When evaluating the order flow models by the type of customer, the foreign participants is the order flow that fits the data better. This result holds for all the models and is robust to evaluating the fitting performance in the high volatility period only. However, the explanation power of the domestic non-banks order flow increases substantially when considering the nonlinear specifications. In particular note that, according to the OLS, the domestic non-banks order flow has almost no additional power to explain exchange rate returns movements than consider "no changes" at all (RW). However, with the nonlinear specifications, the same order flow outperforms the RW by a 15% according to the same measure.

MEESE-ROGOFF TEST

In order to evaluate out-of-sample fitting, we employ a Meese & Rogoff (1983b) type exercise [19]. The Meese-Rogoff test has become the standard tool of model evaluation in the FX microstructure literature. In fact, microstructure oriented models are often able to beat the RW benchmark in out-of-sample fitting, even at short horizons. This result contrasts with the poor out-of-sample fitting of the structural macroeconomic models that are usually unable to outperform a RW at horizons shorter than a year.

Our evaluation period runs from 04/12/07 to 15/07/09 (last four hundreds observations). We estimate the models recursively by adding one observation each time, and we compute the predicted exchange rate returns. Table 6 gives detailed results for one day and one week ahead horizons using the MAE and RMSE as measures of accuracy. As before, the results of the RW can be found in the first column, and the following report the prediction errors of the competing specifications relative to the RW (from two to five) and the OLS (last three). Thus, again, numbers smaller than one indicate better performance of the competing specification. For model comparisons, we make use of the Diebold & Mariano (1995) test statistic. We employ a data dependent truncation lag to estimate the spectral density at zero frequency as described by Andrews (1991). The results are in line with the ones obtained in the in-sample fitting and indicate a clear superiority of the nonlinear specifications.

For one day ahead forecasting, all the order flows models (linear and nonlinear) perform much better than the RW according to both measures, although the gains of the OLS with the domestic non-bank order flow are rather modest. For one week ahead predictions, the relative performance of all the order flows models to the RW increases a lot. This last result is not surprising since, by construction, the Meese-Rogoff exercise penalizes the RW as the horizon increases. The nonlinear versions of the models substantially outperform the linear OLS at both time horizons. The improvements range between $20-25\%$ for the MS, which again is the model that shows better overall performance. As before, the Threshold Specifications almost catch up the MS once enough flexibility in the transaction function (STR) is provided.

We also perform the Meese-Rogoff test in the relatively high volatility sub-sample described in the previous section (Table 6). The relative prediction accuracy measures for all order flow models are substantially smaller in periods of high volatility. As before, the performance of the nonlinear versus the linear specification also increases in the high volatility period. Finally, when evaluating the different order flow specifications by group of customers, the foreign participants is again the order flow that performs better for all specifications, as in the in-sample fitting analysis.

[19] The Meese-Rogoff test relies on ex-ante data to estimate parameters but makes use of contemporaneous independent variables to produce the exchange rate 'forecasts'. Note that the Meese-Rogoff test cannot be consider as true out-of-sample forecasting exercise, since future information (except the estimated parameters) is used to produce forecasts. Rather, it should be considered as an out-of-sample fitting test, a test of stability of the estimated parameters or as a "weak" forecasting.

4 Conclusions

In this work we have questioned the linearity of the relationship between exchange rates and order flows that has been a maintained assumption in the empirical literature. We have employed a long database on customer order flows, which are the cornerstone of the microstructure literature. A first examination of the linearity hypothesis has revealed that the relationship evolves over time and that it is different under different market conditions defined by volatility. We have provided further evidence of this result through the estimation of a Markov Switching and two Threshold Models with volatility as a threshold variable (THRET and STR). The Markov Switching has received a lot of attention in the recent exchange rate literature, but has never been employed for the order flow analysis. Our main findings are, first, that the price impact and the information transmitted by the order flows increase with volatility, which is in line with the uncertainty model developed by Easley & O'Hara (1992) and the conclusions of Subrahmanyan (1991). The second finding is that the nonlinearity can be captured successfully by the Threshold Models and, specially, Markov Switching, which provide substantial power to explain exchange rate movements beyond the constant coefficient approach. In particular, explicitly modeling nonlinearities is crucial to understand the information content of the order flow from determinate groups of customers.

In this research we have been primarily concerned with providing evidence that the relation between exchange rates and order flows is not linear, and that nonlinear models can explain exchange rate movements better than linear specifications. Future research might, consequently, analyze the source of these nonlinearities further and involve the nonlinear models into real exchange rate forecasting. In fact, order flows have already demonstrated their potential to explain future exchange rate movements (Evans & Lyons (2005)). In the light of our results, the increase in efficiency provided by the nonlinear specifications may lead to an improvement of forecast accuracy. However, one might be a priory cautious about the obtained results given that, although the models perform quite well conditional to be in a determined regime, the forecast of the regime changes may be wrought with difficulty. A reliable forecast of exchange rate volatility may help to this purpose. An appealing line of future research could include the use of cointegration between exchange rates and order flows (as in Bjønnes *et al.* (2005)) together with nonlinear specifications.

Appendices

A Tables and figures

Table 1

Summary statistics

Variable	T	Mean	Std	Min	Max
100Δlog(Exchange rate)	1635	0.0091	0.6521	−3.3205	5.9372
Foreign participants (FP)	1635	−0.0024	0.0363	−0.2144	0.9023
Domestic non-banks (DNB)	1635	−0.0008	0.0141	−0.1020	0.1087
Volatility	1635	0.4120	0.47	0	5.9372

Notes: a) T - number of observations available, Mean and Std - mean and standard deviation of a series, Min and Max - minimum and maximum values. b) Volatility is defined as absolute value of the first difference of natural logarithms of the HUF/EUR exchange rate. c) Order flows are NET order flows = Δ Cumulated order flows, in trillion forints.

Table 2

Results of estimation of the generic model under different volatility conditions

		α	β	R^2	F stat
FP	OLS	-0.0330^{**}	-13.30383^{***}	0.3661	–
	OLS by Volatility:				
	Volatility(t)=L	-0.0163^{**}	-3.5481^{***}	0.102	–
	Volatility(t)=H	-0.0511^{*}	-17.31^{***}	0.4694	157.31
DNB	OLS	0.0197	15.1743^{***}	0.1184	–
	OLS by Volatility:				
	Volatility(t)=L	−0.0099	1.2252^{**}	0.0045	–
	Volatility(t)=H	0.0517	24.93^{***}	0.2024	65.15

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow respectively. b) Volatility is defined as the absolute value of the first difference of natural logarithms of the HUF/EUR exchange rate. c) F stat is F statistics for the $H_0 : \alpha(L) = \alpha(H)$ and $\beta(L) = \beta(H)$. d) *, **, *** indicate significance at 10%, 5%, and 1% respectively.

Table 3

Carrasco *et al.* (2009) test statistics (MS) and Hansen (1997) Chow-type test (THRET) for the null of linearity against the respective nonlinear model

Order flow	Carrasco *et al.* (2009), MS	Hansen (1997), THRET
FP	16.410***	158.866***
DNB	9.993***	214.444***

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) The numbers in the columns are F-statistics. c) *, **, *** indicate significance at 10%, 5%, and 1% respectively.

Table 4

Estimation Results, MS, TR and THRET

(a) MS

Order flow	α_1	β_1	α_2	β_2	σ^2	p_{11}	p_{12}
FP	−0.042*	−9.090***	0.019	−31.427***	0.184***	0.942**	0.750***
DNB	−0.023	6.381***	0.307*	64.700***	0.249***	0.901***	0.301***

(b) TR

Order flow	α_1	β_1	α_2	β_2	σ^2	c
FP	−0.024*	−6.752***	−0.028	−20.932***	0.208***	0.768***
DNB	−0.005	5.6316***	0.024	52.925***	0.257***	1.003***

(c) THREAT

Order flow	α_1	β_1	α_2	β_2	$\sigma^{2(A)}$	$c^{(A)}$
FP	−0.014**	−5.234***	−0.013	−19.618***	0.212	0.626
DNB	−0.004	5.635***	0.011	53.134***	0.272	0.998

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) *, **, *** indicate significance at 10%, 5%, and 1% respectively. $^{(A)}$ The THRET-C2SLS procedure does not allow to test the significance of the estimated σ^2 and c.

Table 5

In-sample Fitting Results: MAE, RMSE

(a) Whole period

		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	**mae**		0.812	0.689	0.747	0.709	0.848	0.919	0.873
	rmse	0.414	0.796	0.650	0.727	0.684	0.816	0.913	0.859
DNB	**mae**	0.625	0.971	0.823	0.878	0.847	0.847	0.903	0.872
	rmse		0.939	0.755	0.835	0.811	0.804	0.889	0.864

(b) High volatility period

		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	**mae**		0.800	0.630	0.678	0.625	0.788	0.844	0.781
	rmse	0.818	0.763	0.602	0.684	0.636	0.789	0.896	0.834
DNB	**mae**	1.0349	0.902	0.698	0.740	0.694	0.773	0.810	0.770
	rmse		0.884	0.666	0.740	0.698	0.753	0.828	0.790

Notes: a) The numbers in the cells indicate the MAE and RMSE of the competing model relative to RW (first 3 columns) or to OLS (last 2). A number smaller than 1 indicate better performance of the competing model. % Fitting gain can be obtained by subtracting the number in the cell from 1. b) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. c) Whole period: 01/02/2003 – 15/07/2009, High volatility period: 04/02/2009 – 15/07/09. d) STR is Logistic-STR.

Table 6

Out-Sample Fitting Results (Meese-Rogoff): MAE, RMSE

(a) Whole period, 1 day

		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	mae		0.767***	0.625***	0.6961***	0.633***	0.815***	0.904***	0.825***
	rmse	0.687	0.766***	0.617***	0.6908***	0.631***	0.806***	0.900***	0.824***
DNB	mae	0.955	0.938***	0.720***	0.7981***	0.735***	0.767***	0.848***	0.784***
	rmse		0.920***	0.745***	0.7819***	0.771***	0.810***	0.848***	0.838***

(b) Whole period, 1 week

		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	mae		0.322***	0.267***	0.2923***	0.271***	0.828***	0.904***	0.842***
	rmse	1.622	0.340***	0.288***	0.3075***	0.300***	0.848***	0.900***	0.882***
DNB	mae	2.145	0.394***	0.307***	0.3355***	0.314***	0.778***	0.848***	0.797***
	rmse		0.408***	0.340***	0.3482***	0.348***	0.832***	0.849***	0.853***

(c) High volatility period, 1 day

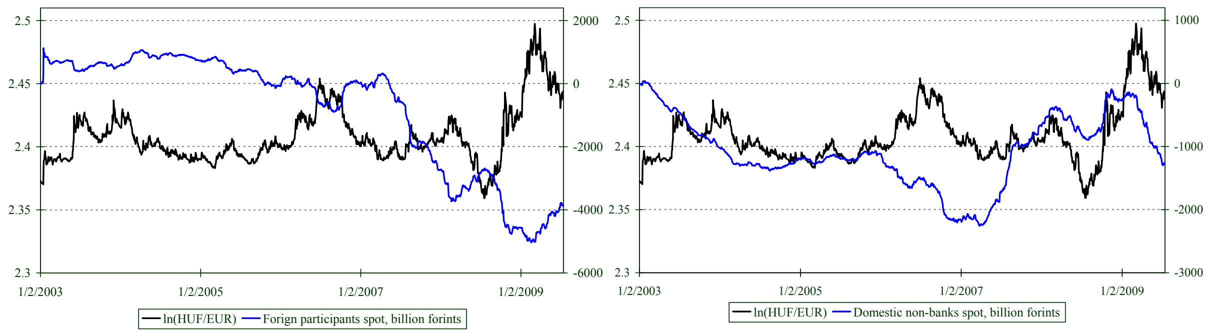
		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	mae		0.810**	0.631***	0.7046***	0.631***	0.779***	0.846***	0.779***
	rmse	0.818	0.771***	0.604***	0.6962***	0.644***	0.783***	0.898***	0.835 ∗∗
DNB	mae	1.0349	0.915***	0.696***	0.7548***	0.706***	0.761***	0.806***	0.772***
	rmse		0.894***	0.666***	0.7359***	0.711***	0.745***	0.820***	0.796***

(d) High volatility period, 1 week

		RW	Model/RW				Model/OLS		
			OLS	MS	THRET	STR	MS	THRET	STR
FP	mae		0.317***	0.248***	0.2674***	0.249***	0.781***	0.842***	0.786***
	rmse	1.986	0.327***	0.256***	0.2950***	0.277***	0.784***	0.902***	0.847**
DNB	mae	2.370	0.356***	0.265***	0.2875***	0.275***	0.746***	0.808***	0.774***
	rmse		0.375***	0.277***	0.3122***	0.303***	0.738***	0.831***	0.806***

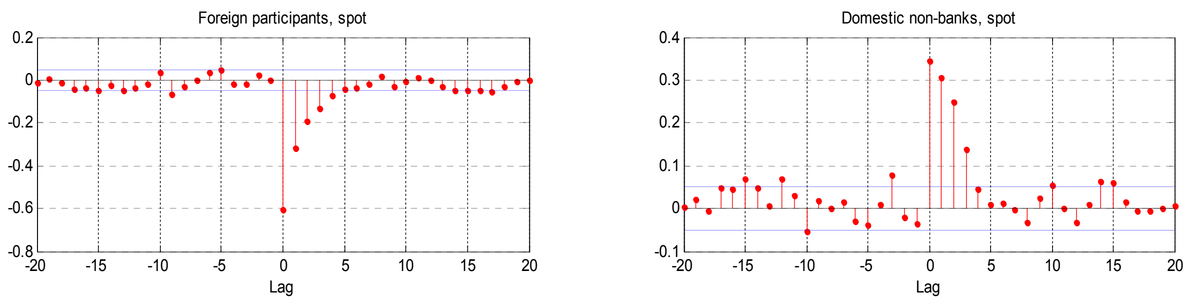
Notes: a) The numbers in the cells indicate the MAE and RMSE of the competing model relative to RW (first 3 columns) or to OLS (last 2). A number smaller than 1 indicate better performance of the competing model. % Fitting gain can be obtained by subtracting the number in the cell from 1. b) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. c) Whole period: 01/02/2003 – 15/07/2009, High volatility period: 04/02/2009 – 15/07/09. d) STR is Logistic-STR. e) *, **, *** indicate statistical significance at 10%, 5%, 1% resp., according to the pDM statistic.

Figure 1
The data: log-exchange rate and cumulated order flow



Notes: exchange rate on the left axis, cumulated order flow on the right axis.

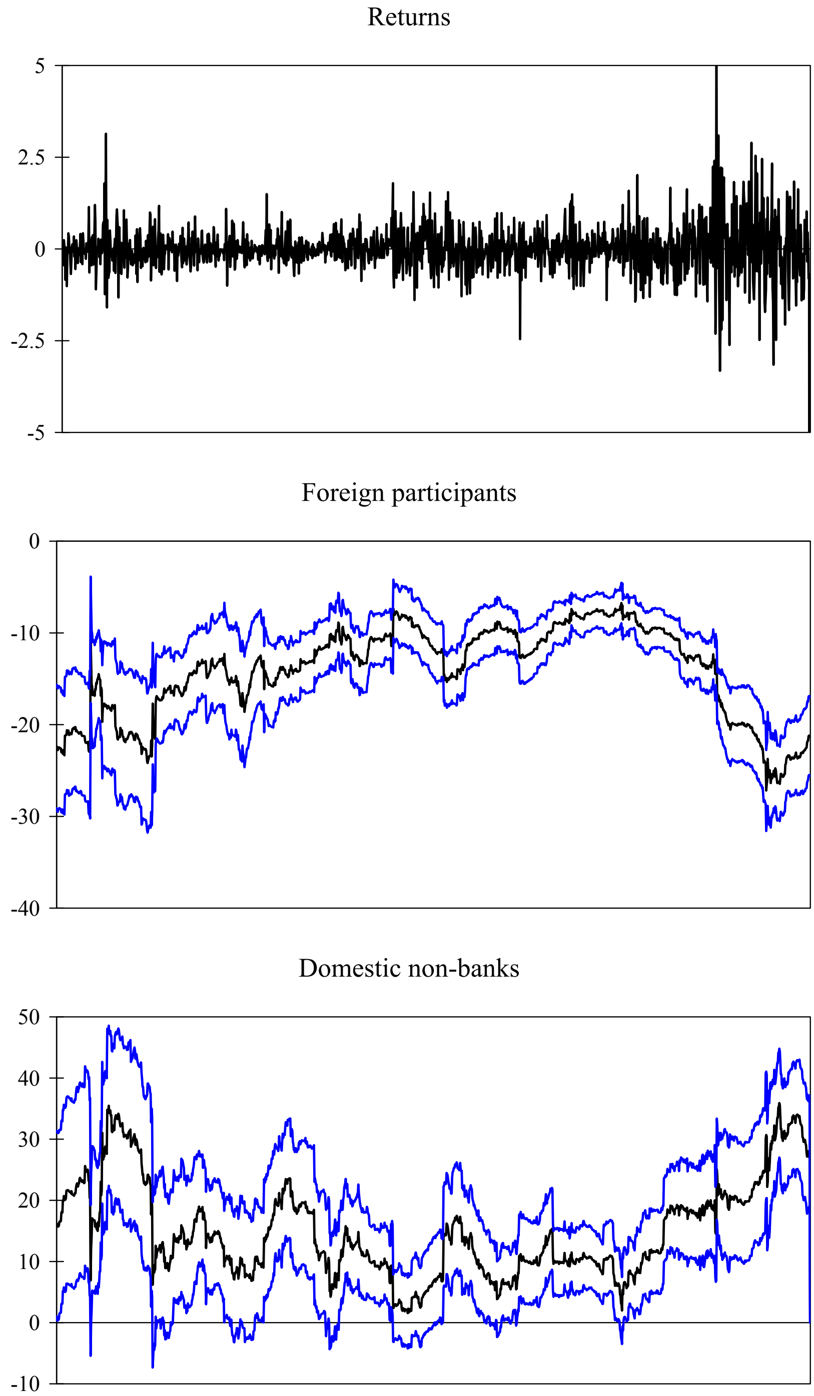
Figure 2
Sample Cross Correlation: Returns, net Order flows



Notes: a) Returns: 100*dlog(HUF/EUR)) (% change in exchange rates).
b) Confidence intervals in the graph correspond to 5% significance level.

Figure 3

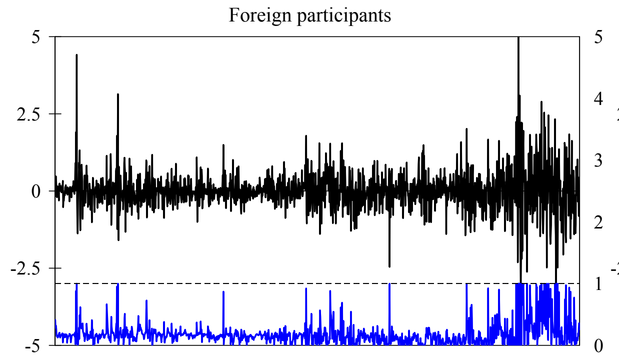
Results of the Moving Window regression



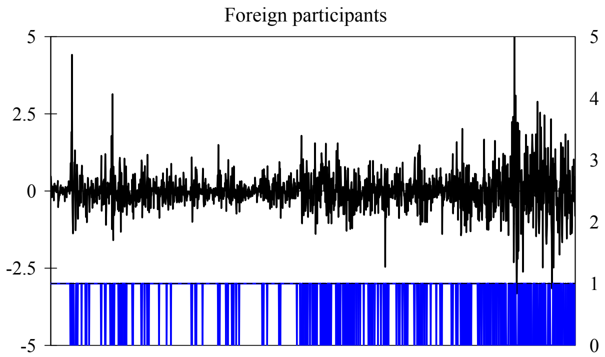
Notes: Upper and lower bounds are 5% confidence intervals.

Figure 4

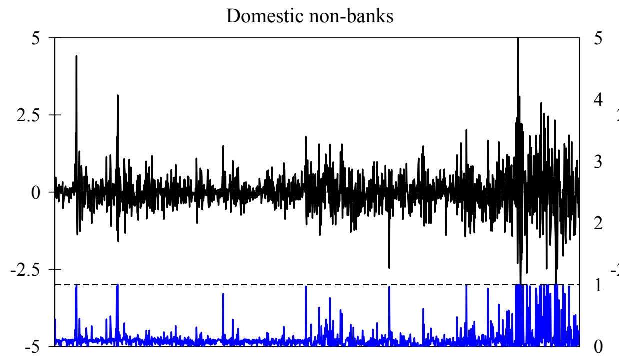
Returns, Values of transaction function (THRET) and Smoothed probabilities (MS)



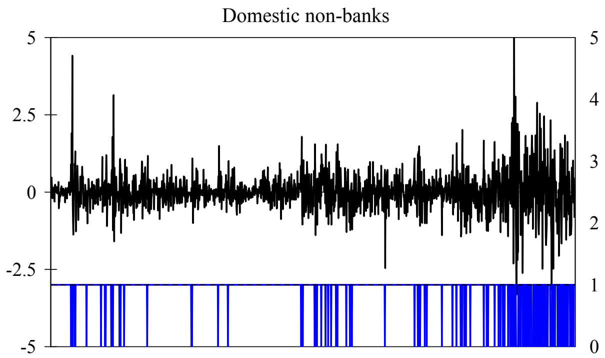
(a) Returns and Smoothed Probability. MS



(b) Returns and Transaction function. THRET



(c) Returns and Smoothed Probability. MS

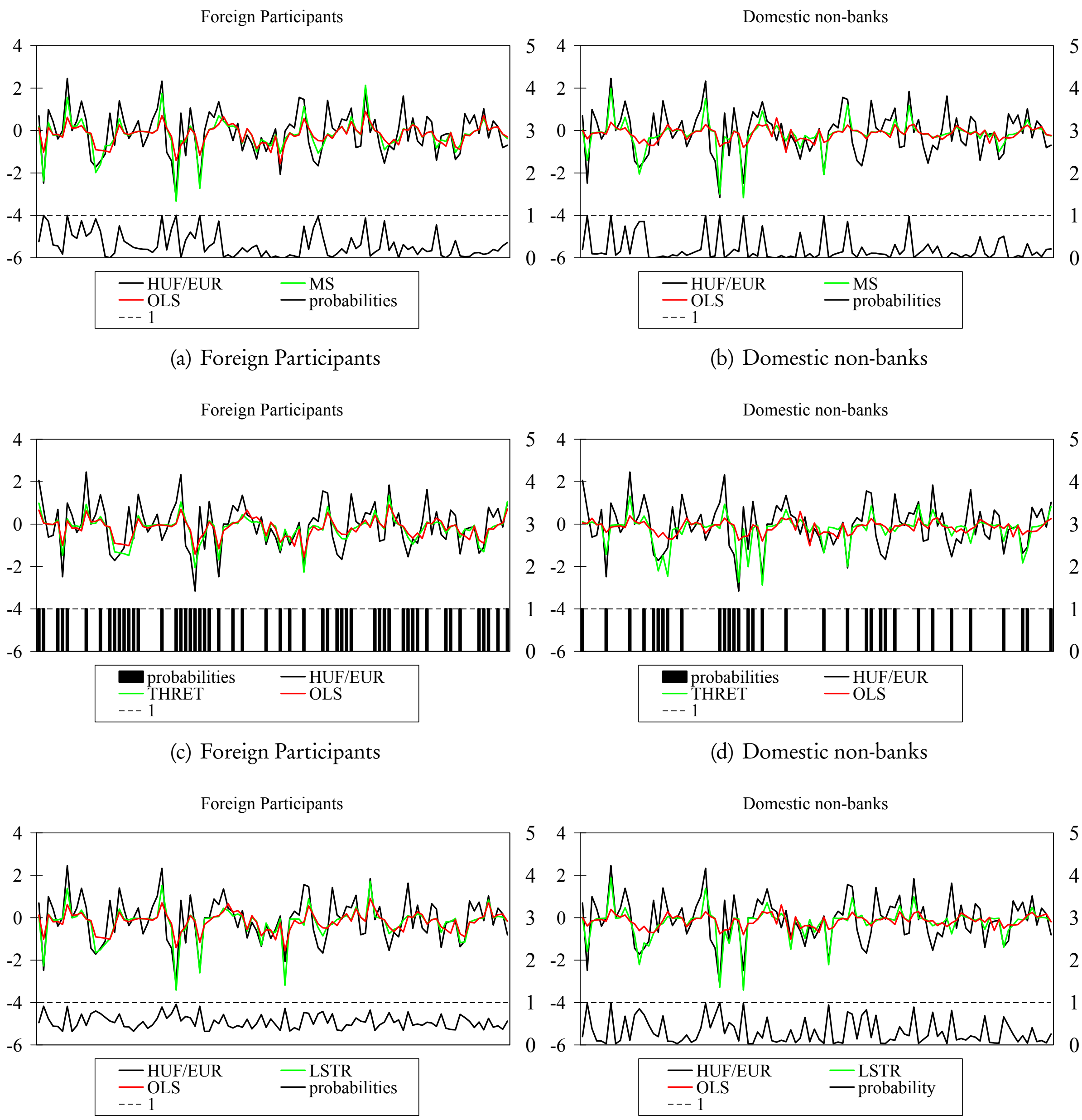


(d) Returns and Transaction function. THRET

Notes: a) Left Scale: Returns = dlog(HUF/EUR) x 100 (% change in exchange rates).
b) Left Column - Smoothed probability of high sensitivity state (MS) Right Scale: Right column -Values of transaction function (THRET).

Figure 5

In-sample Fitting: High volatility sub-sample MS, THRET, LSTR and OLS



(a) Foreign Participants (b) Domestic non-banks

(c) Foreign Participants (d) Domestic non-banks

(e) Foreign Participants (f) Domestic non-banks

B STR estimation

Testing linearity against STR-type nonlinearity implies testing the joint null hypothesis $H_0: \alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ in (2). However, under the null, parameters γ and c are not identified. To overcome the difficulties with testing, we use the method based on an auxiliary regression proposed by Escribano & Jorda (2001) developed on the basis of the testing procedure proposed by Saikkonen & Luukkonen (1988), Terasvirta (1994), Terasvirta *et al.* (1994). Additionally, this procedure allows us to test model specification for the STR: logistic (LSTR) against exponential (ESTR). The method involves the following steps:

First, we run an auxiliary regression of the type:

$$y_t = \alpha + \beta X_t + \delta_1 X_t z_t + \delta_2 X_t z_t^2 + \delta_3 X_t z_t^3 + \delta_4 X_t z_t^4 + e_t, \tag{9}$$

where as before, y_t is the returns, X_t is the consumer order flow, and z_t is the observed threshold variable (volatility). The linearity hypothesis has as null: $H_0: \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$. Second, if linearity is rejected, we can proceed to select the specification of the model by computing usual F-statistics for the following null hypothesis: $H_0: \delta_2 = \delta_4 = 0$ and $H_0: \delta_1 = \delta_3 = 0$ in (9). If the F-statistic associated to the first hypothesis is higher than the one associated to the second the resulting specification is Exponential (ESTR), and if opposite, the resulting specification is logarithmic (LSTR).

Table 7

Escribano-Jorda Procedure

Escribano-Jorda procedure	p-values (F-statistics)			Resulting specification
H_0	$\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$	$\delta_2 = \delta_4 = 0$	$\delta_1 = \delta_3 = 0$	
FP	157.37***	10.40	33.22	LSTR
DNB	126.95***	2.11	19.20	LSTR

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) The numbers in the columns are F-statistics. c) FL and FE are the values of the F-statistic for each of the hypothesis. d) *, **, *** indicate significance at 10%, 5%, and 1%, respectively.

For all order flows the hypothesis of linearity is rejected at all reasonable significance levels. The testing procedure suggests the Logistic STR for all the specification. To estimate the model, we maximize the likelihood function, assuming that the errors are normally distributed. To get the starting values for the estimation we apply the methods described in Frances & van Dijk (2000), p.91. The smoothness parameter γ is restricted to be positive ($\gamma > 0$) and the threshold value c is estimated internally. The results of the estimation of the standard TR and LSTR models are presented in Table 8.

Table 8

Estimation results, LSTR

	LSTR						
Order flow	α_1	β_1	α_2	β_2	σ^2	γ	c
FP	0.0132	14.7672*	−0.0943	−48.958***	0.1832**	1.2489***	0.6081**
DNB	−0.0320*	−2.5431*	0.2662***	70.8212***	0.2575***	3.0752***	1.1485***

The estimated threshold coefficients are positive and statistically significant. Estimates of smoothness parameters are significant and quite high. It means that the coefficients change fast when the latent variable is around the threshold. The estimated intercepts are often not statistically different from zero. Opposite, the slope coefficients β are always strongly significant and statistically different in each regime. The signs of the resulting estimated slope coefficients for period t, $\beta_t = \beta_1 + \beta_2 F(z_t, \gamma, c)$ are always as in the OLS, MS and THRET/TR estimation (negative for spot foreign participants, and positive for domestic non banks), and increasing, in magnitude, with volatility.

The ability of the Logistic-STR model to explain in and out of sample exchange rate movements is presented together with the one of MS and THRET models in Tables 5, 6.

References

Admati, A. R. & P. Pfleiderer (1988). "A theory of intraday patterns: Volume and price variability." *Review of Financial Studies*, 1, 3–40.

Andrews, D. W. K. (1991). "Heteroskedasticity and autocorrelation consistent covariance matrix estimation." *Econometrica*, 59, 817.

Berger, D. W., A. P. Chaboud, S. V. Chernenko, E. Howorka & J. H. Wright (2008). "Order flow and exchange rate dynamics in electronic brokerage system data." *Journal of International Economics*, 75, 93–109.

Bessec, M. (2003). "Mean reversion vs. adjustment to PPP: The two regimes of exchange rate dynamics under the EMS 1979-1998." *Economic Modelling*, 20, 141–164.

Bjønnes, G. H., D. Rime & H. O. A. Solheim (2005). "Liquidity provision in the overnight foreign exchange market." *Journal of International Money and Finance*, 24, 175–196.

Boothe, P. M. & D. A. Glassman (1987). "The statistical distribution of exchange rates: empirical evidence and economic implications." *Journal of International Economics*, 22, 297–319.

Caner, M. & B. E. Hansen (2004). "Instrumental variable estimation of a threshold model." *Econometric Theory*, 20, 813–843.

Carpenter, A. & J. Wang (2003). "Sources of private information in FX trading." Unpublished.

Carrasco, M., L. Hu & W. Ploberger (2009). "Optimal test for markov switching parameters." Unpublished.

Clarida, R. H., L. Sarno, M. P. Taylor & G. Valente (2003). "The out-of-sample success of term structure models as exchange rate predictors: a step beyond." *Journal of International Economics*, 60, 61–83.

Diebold, F. X. & R. S. Mariano (1995). "Comparing predictive accuracy." *Journal of Business and Economics Statistics*, 13, 253–263.

Diz, F. & T. J. Finucane (1993). "The time-series properties of implied volatility of S&P 100 index options." *Journal of Financial Engineering*, 2, 127–154.

Easley, D. & M. O'Hara (1992). "Time and process of security price adjustment." *Journal of Finance*, 47, 57–605.

Engel, C. (1994). "Can the Markov Switching Model Forecast Exchange Rates?" *Journal of International Economics*, 36, 151–165.

Engel, C. & J. D. Hamilton (1990). "Long swings in the dollar. Are they in the data and do markets know it?" *American Economic Review*, 80, 683–783.

Escribano, A. & O. Jorda (2001). "Testing nonlinearity: Decision rules for selecting between logistic and exponential STAR models." *Spanish Economic Review*, 3, 193–209.

Evans, M. D. D. & R. K. Lyons (2002). "Order flow and exchange rate dynamics." *Journal of Political Economy*, 102, 170–180.

Evans, M. D. D. & R. K. Lyons (2005). "Meese-Rogoff Redux: Micro-based exchange rate forecasting." *American Economic Review*, 95, 405–414.

EVANS, M. D. D. & R. K. LYONS (2006). "Understanding order flow." *International Journal of Finance and Economics*, 11, 3–23.

FRANCES, P. H. & D. VAN DIJK (2000). *Non-Linear Time Series Models in Empirical Finance*. Cambridge University Press, Cambridge.

FRÖMMEL, M., R. MCDONALD & L. MENKHOFF (2005). "Markov switching regimes in a monetary exchange rate model." *Economic Modelling*, 22, 485–502.

FRÖMMEL, M., A. MENDE & L. MENKHOFF (2008). "Order flows, news, and exchange rate volatility." *Journal of International Money and Finance*, 27, 994–1012.

FROOT, K. & T. RAMADOARI (2002). "Currency returns, Institutional investor flows, and exchange rate fundamentals." *Technical Report 9101*, NBER.

GARCIA, R. (1998). "Asymptotic null distribution of the likelihood ratio test in Markov switching models." *International Economic Review*, 39, 763–788.

GEREBEN, A., GY. GYOMAI & N. M. KISS (2006). "Customer order flow, information and liquidity on the Hungarian foreign exchange market." *Working Paper 2006/8*, MNB.

GLOSTEN, L. & P. MILGROM (1985). "Bid, ask and transaction prices in a specialist market with heterogeneous informed agents." *Journal of Financial Economics*, 14, 71–100.

HAMILTON, J. D. (1996). "Specification and Testing in Markov-Switching Time-Series Models." *Journal of Econometrics*, 70, 127–157.

HAMILTON, J. D. (2005). "What's real about the business cycle?" *Working Paper 11161*, NBER.

HANSEN, B. E. (1992). "The Likelihood Ratio Test under Non-Standard Conditions." *Journal of Applied Econometrics*, 11, 195–198.

HANSEN, B. E. (1997). "Inference in TAR models." *Studies in Nonlinear Dynamics and Econometrics*, 2, 1–14.

HANSEN, B. E. (2000). "Sample splitting and threshold estimation." *Econometrica*, 68, 575–604.

ISARD, P. (1995). *Exchange Rate Economics*. Cambridge University Press, Cambridge, UK.

KOURTELLOS, A., T. STENGOS & C. M. TAN (2009). "Structural threshold regression." *Working Paper 22-09*, The Rimini Centre for Economic Analysis.

KROLZIG, H. M. (1997). *Markov-switching Vector Autoregressions*. Springer, New Work.

KYLE, A. (1985). "Continuous auctions and insider trading." *Econometrica*, 53, 1315–1335.

LAMOREAUX, C. & W. LASTRAPES (1990). "Heteroscedasticity in stock return data: volume versus GARCH effects." *Journal of Finance*, 45, 221–229.

LUO, J. (2001). "Market conditions, order flow and exchange rate determination." Unpublished.

LYONS, R. K. (1995). "Tests of microstructural hypotheses in the foreign exchange market." *Journal of Financial Economics*, 39, 321–351.

LYONS, R. K. (1996). "Foreign exchange volume: sounds of furry signifying nothing?" In J. A. Frankel, G. Galli & A. Giovannini, editors, "The Microstructure of Foreign Exchange Markets," 183–201. University of Chicago Press, Chicago, IL, USA.

MARSH, I. W. (2000). “High frequency Markov switching models for exchange rate forecasting.” *Journal of Forecasting*, 19, 123–134.

MARSH, I. W. & C. O’ROURKE (2005). “Customer order flow and exchange rate movements: is there really information content?” Cass Business School, London.

MEESE, R. & K. ROGOFF (1983a). “Empirical exchange rate models of the seventies.” *Journal of International Economics*, 14, 3–24.

MEESE, R. & K. ROGOFF (1983b). “The out-of-sample failure of empirical exchange rate models.” In J. Frenkel, editor, “Exchange rate and International Macroeconomics,” University of Chicago Press, Chicago, IL, USA.

SAGER, M. & M. TAYLOR (2008). “Commercially Available Order Flow Data and Exchange Rate Movements: Caveat Emptor.” *Journal of Money, Credit and Banking*, 40, 583–625.

SAIKKONEN, P. & R. LUUKKONEN (1988). “Lagrange multiplier test for testing nonlinearities in time series models.” *Scandinavian Journal of Statistics*, 15, 55–68.

SANZO, S. D. (2009). “Testing for linearity in Markov switching models: a bootstrap approach.” *Statistical Methods and Applications*, 18, 153–168.

SARNO, L., G. VALENTE & M. E. WOHAR (2004). “Monetary fundamentals and exchange rate dynamics under different nominal regimes.” *Economic Inquiry*, 42, 179–193.

SUBRAHMANYAN, A. (1991). “Risk aversion, market liquidity, and price efficiency.” *Review of Financial Studies*, 4, 417–441.

TAYLOR, M. P. (1995). “The economics of exchange rates.” *Journal of Economic Literature*, 33, 13–47.

TERASVIRTA, T. (1994). “Specification, estimation and evaluation of smooth transition autoregressive models.” *Journal of the American Statistical Association*, 89, 208–218.

TERASVIRTA, T., D. TJOSHEIM & C. W. J. GRANGER (1994). “Aspects of modelling nonlinear time series.” In R. F. Engle & D. McFadden, editors, “Handbook of Econometrics,” , volume 4Elsevier, Amsterdam.

TONG, H. (1978). “On a threshold model.” In C. H. Chen, editor, “Pattern Recognition and Signal Processing,” 101–141. Sijthoff and Noordhoff, Amsterdam.

TONG, H. (1983). *Threshold Models in Non-Linear Time Series Analysis*. Springer, Berlin Heidelberg New York.

TONG, H. & K. S. LIM (1980). “Threshold autoregressions, limit cycles, and data.” *Journal of the Royal Statistical Society B*, 42, 245–292.

MNB Working Papers 2010/10
Is exchange rate – customer order flow relationship linear? Evidence from the Hungarian FX market

Print: D-Plus
H–1037 Budapest, Csillaghegyi út 19–21.