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# *Catastrophic Health Expenditure and Household Well-Being*

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# Catastrophic Health Expenditure and Household Well-Being

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## Abstract

According to the catastrophic health expenditure methodology a household is in catastrophe if its health out-of-pocket budget share exceeds a critical threshold. We develop a conceptual framework for addressing three questions in relation to this methodology, namely: 1. Can a budget share be informative about the sign of a change in welfare? 2. Is there a positive association between a household's poverty shortfall and its health out-of-pocket budget share? 3. Does an increase in coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures?

*Keywords:* Catastrophic health expenditure, welfare change, poverty, performance of health insurance schemes.

*JEL codes:* I<sub>1</sub>, I<sub>3</sub>.

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## 1. Introduction

Risk averse individuals will appeal to insurance mechanisms as a means of diversifying their risks. This diversification of risk is important and takes on many forms, institutional and informal. In developing countries such as India, Townsend (1995) finds that via informal mechanisms individuals are able to absorb some health related risks. However, for more serious and chronic illnesses, Gertler and Gruber (2002) find that health shocks can have a major impact on consumption and can severely disrupt household welfare. There is similar evidence about the effect of health shocks in relation to developed countries such as the United States where health insurance until recently was not mandatory. There, it has been documented (cf. Feenberg and Skinner, 1994; Waters et al., 2004) that illness can cause households to reallocate substantial shares of their spending to out-of-pocket (OOP) health expenditures.

Thus it has been proposed to ascribe to a situation where health OOP expenditures exceed a critical share of the household's total outlay the state of catastrophic health expenditure (Xu et al 2003; Wagstaff and van Doorslaer 2003). There is no exact consensus about the critical threshold level. Some studies choose values of 5% (Berki, 1986), 10% (Waters et al., 2004) and up to 40% of non-subsistence spending (Xu et al., 2003).

In this growing literature, the measurement of catastrophic health care payments appears to serve three main objectives surveyed below: (i) to identify changes in levels of well-being, (ii) to assess the extent of poverty / low levels of living at the household level and (iii) to assess the performance of existing health insurance schemes. There is empirical evidence regarding each of these issues, though an economic conceptual framework appears to be missing. Our aim is to try to fill this gap in the literature, by attempting to provide satisfactory answers to the following three questions:

1. Can a budget share be informative about the sign of a change in welfare?
2. Is there a positive association between a household's poverty shortfall (the difference between the household's income and the poverty line) and its health out-of-pocket budget share?
3. Does an increase in coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures?

The plan of the paper is the following. Section 2 surveys the literature in relation to the three questions stated above. The following three sections deal with each of questions 1 to 3. The final section contains a summary and concluding

comments.

## 2. What does catastrophic expenditure aim to measure?

To date, we can distinguish three major purposes in relation to the measurement of catastrophic health expenditure. First, interest in the measurement of catastrophic health payments stems from the fact that in the absence of health insurance, high expenditures on health care can severely disrupt household living standards. For instance Berki (1986) states that "An expenditure for medical care becomes financially catastrophic when it endangers the family's ability to maintain its customary standard of living". Ideally, this change in welfare would be assessed with longitudinal data through examination of how health shocks disrupt consumption paths (Gertler and Gruber, 2002). In the absence of longitudinal data, OOP health payments in excess of a threshold budget share have been used as a proxy for severe disruptions to household living standards. Regarding this point, Van Doorslaer et al. (2007) write "We focus on payments that are catastrophic in the sense of severely disrupting household living standards, and approximate such payments by those absorbing a large fraction of household resources". Thus it may be argued that a catastrophic situation may be used to capture a change in household welfare.

Second, our reading of the literature suggests an implicit association between the state of poverty and the state of health catastrophic expenditure. In the economic literature on poverty, one distinguishes an ethical approach from a levels of living approach (Atkinson, 1987). In the former, an ethical position is used to argue that every member of society should be entitled to a minimum level of resources. In the levels of living approach, poverty is associated with insufficient consumption resulting in a low level of welfare. We find parallels to these two approaches in the health payments literature. In presenting the methodology on the measurement of catastrophic expenditure, Wagstaff and Van Doorslaer (2003) write "The ethical position is that no one ought to spend more than a given fraction of income on health care"<sup>1</sup>. There are also authors who suggest that catastrophic health expenditures are associated with low levels of living. Referring to the costs of health services, Xu et al (2003) write "However accessing these

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<sup>1</sup>The authors also propose measures of catastrophic expenditure drawing on similar tools as in the poverty literature. This leads to "By analogy with the poverty literature, one could define not just a catastrophic payment headcount but also a measure analogous to the poverty gap, which we call the catastrophic payment gap".

services can lead to individuals having to pay catastrophic proportions of their available income and push many households into poverty. ” More explicitly, in defining the *medical poverty trap*, Whitehead et al. (2001) state that ”Rises in OOP costs for public and private health-care services are driving many families into poverty, and are increasing the poverty of those who are already poor.” Finally, Flores et al. (2008) examine how households finance OOP payments (the problem of coping with health care costs) and the implications of *coping strategies* for the measurement of poverty. Clearly, in order to understand the overall relation between the incidence of poverty and catastrophic expenditure one needs to explore, at the micro-level, the relation between the Engel curve for health OOPs and the poverty shortfall <sup>2</sup>. This is what we set out to do in Section 4 below.

Catastrophic health care payments are also used to measure the performance of prevailing health insurance schemes. The understanding is that a large fraction of individuals experiencing catastrophic health payments is associated with an insufficient coverage in relation to health insurance contracts. According to Waters et al. (2004), ”One rationale for health insurance coverage is to provide financial protection against catastrophic health expenditures.” By *insufficient coverage* researchers often refer to the small percentage of the population in benefit of any health insurance scheme (Scheil-Adlung et al., 2006). But it may equally refer to the lack of generosity of the health insurance scheme, with respect to copayments (Gertler and Gruber, 2002.) A Mexican study (Knaul et al. 2006) concludes that the prevalence of health catastrophic expenditure is reduced by an increased coverage of the population by health insurance schemes. Likewise a joint ILO, WHO and OECD study covering three developing countries (Scheil-Adlung et al., 2006) finds that membership in health insurance schemes contributes to reducing the probability of incurring catastrophic health expenditures. Nonetheless this study shows that the protective effect of being insured is not general: in South Africa it only concerns the richest quintile of the population who is able to afford more comprehensive packages (Lamiraud et al., 2005). The U.S. study of Waters et al. (2004) also reveals that low income and the occurrence of multiple chronic conditions, alongside the lack of health insurance, increase the probability of catastrophic health payments.

In the sections below we develop an economic framework for addressing the questions stated in the Introduction in relation to this literature. We begin with asking to what extent a budget share can be informative about a change in welfare.

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<sup>2</sup>Throughout the paper, we refer to the Engel curve as being the cross-sectional relation between household income and the budget share allocated to a specific good.

### 3. Catastrophic expenditure: a measure of change in welfare?

When panel data on a household's consumption are not available, it has been suggested to proxy disruptions in household welfare via the use of the level of the budget share of health OOP expenditure. Our first question, formulated in general terms therefore is: can a budget share be informative about the direction of a change in household welfare?

We examine this question from two different perspectives. Firstly, at a general level, we assume the consumer's utility function does not explicitly depend on health, but health matters to consumption decisions to the extent that health shocks translate into income shocks. Thus, this analysis examines at the most general level the effect of an income shock on the budget share of some arbitrary good. Then, we turn things around, by considering a specific problem where preferences are function of an exogenous health parameter, and the consumer maximizes utility by choice of a consumption good and a health input. In this second perspective, we assume income stays constant before and after changes in the exogenous health parameter and we examine the effect of the health shock on the demand for the health input.

#### 3.1. Income shocks

We consider the following problem: by choice of quantities  $q_1$  and  $q_2$  of two goods, a household maximizes its utility  $u(q_1, q_2)$ <sup>3</sup> subject to a budget constraint  $p_1q_1 + p_2q_2 = m$ . We let  $\lambda$  denote the Lagrange multiplier,  $p \doteq [p_1, p_2]$ , and we denote the consumer's indirect utility function by  $v(m, p)$ . In this conceptual framework, a change in welfare arises from either a change in household income  $m$ , a change in one or more prices, or finally a change in prices and income. The literature most often focuses on income shocks as a source of welfare disruptions, since these are household specific, whereas changes in prices are often perceived to affect all individuals alike.

Let  $m_0$  denote the household's base period income, and let  $m_1$  denote the household's current period income. Consider then an income change  $\Delta m \doteq m_0 - m_1$  (which need not be a negative quantity). Approximating the resulting change in welfare using a first-order Taylor approximation, we have:

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<sup>3</sup>The results below easily generalize in the context of  $n > 2$  goods.

$$v(m_0, p) \simeq v(m_1, p) + \frac{\partial v}{\partial m}(m_1, p) [m_0 - m_1] \quad (3.1)$$

From the envelope theorem we have that  $\frac{\partial v}{\partial m}(m_1, p) = \lambda > 0$ , since the marginal utility of income is always positive. Letting  $\Delta v \doteq v(m_0, p) - v(m_1, p)$ , we can write (3.1) in a more compact fashion as

$$\Delta v = \lambda \Delta m \quad (3.2)$$

Since the marginal utility of income  $\lambda$  is a positive quantity, we can state the following preliminary result:

**Lemma 1.1** *The change in welfare  $\Delta v$  is always of the same sign as the change in income  $\Delta m$ .*

Let  $w_i \doteq p_i q_i / m_1$  denote the budget share for good  $i$  in the current period. Our next purpose is to write (3.1) in terms of  $w_i$ . We first use Roy's identity to write the marginal utility of income as :

$$\lambda = -\frac{\partial v / \partial p_i}{q_i} \quad (3.3)$$

and since  $q_i = w_i m_1 / p_i$ , we have that

$$\lambda = -p_i \frac{\partial v / \partial p_i}{w_i m_1} \quad (3.4)$$

Replacing (3.4) in (3.2) we have that

$$\Delta v = -p_i \frac{\partial v / \partial p_i}{w_i m_1} \Delta m \quad (3.5)$$

Observe that  $\partial v / \partial p_i < 0$  (the indirect utility function is decreasing in prices), so that, as stated in the Lemma,  $\Delta v > 0$  if and only if  $\Delta m > 0$ . Finally, rearranging terms, we obtain the desired relation between the budget share  $w_i$  and the welfare change  $\Delta v$  :

$$w_i = -p_i \frac{\partial v / \partial p_i}{m_1} \frac{\Delta m}{\Delta v} \quad (3.6)$$

Because  $\Delta v$  has the same sign as  $\Delta m$ , a high level of the budget share  $w_i$  is equally compatible with a scenario where  $\Delta v \leq 0$  (for which  $\Delta m \leq 0$ ) and with

a situation such that  $\Delta v > 0$  (corresponding to a  $\Delta m > 0$ ). Accordingly, the answer to our first question is the following:

**Proposition 1** *Without additional information about the sign of the income change, the level of a budget share cannot be informative about the sign of the change in welfare.*

The scope therefore for identifying households who experience a severe decline in their levels of living using a budget share is limited, unless the data analyst is sure that the household has experienced an income drop. Such information about changes in income is however not always available in cross-section type household surveys. The main problem is that it is hoped to identify a change in a variable (household welfare) by means of another variable (a budget share) measured in levels.

### 3.2. Health shocks

Given the above general result, we may wish to consider a more specific problem that is implicit in much of the health economics literature. Let  $N$  denote health inputs and let  $s$  denote an exogenous health endowment. Health  $H$  is produced from these two variables via a health production function  $H(N, s)$  and the household is assumed to maximize by choice of  $N$  a utility function  $u \left[ \frac{m - p_N N}{p_C}, H(N, s) \right]$  where  $(m - p_N N)/p_C$  is the remaining outlay available for spending on an aggregate consumption good  $C$  (cf. for instance Koc, 2004). Our question in this narrower context is the following: does an exogenous health shock, signifying a reduction in  $s$ , necessarily entail a rise in the demand for health inputs, and thus other things equal, a rise in the relevant budget share?

To answer the above question we first examine the first and second order conditions of the above problem and then turn to comparative statics. Firstly, the first order necessary condition for an optimum choice of  $N$ ,  $\Gamma(N; m, p_N, s) \doteq \partial u / \partial N$ , entails a level  $\hat{N}$  such that  $\Gamma(\hat{N}; m, p_N, s) = 0$ :

$$\Gamma(N; m, p_N, s) = -\frac{p_N}{p_C} u_1 + u_2 H_1 = 0 \quad (3.7)$$

where  $u_i$  and  $H_j$  are first derivatives of  $u(\cdot, \cdot)$  and  $H(\cdot, \cdot)$  with respect to their  $i$ th and  $j$ th arguments. Differentiating (3.7) again, we have

$$\frac{\partial \Gamma}{\partial N} = \left( \frac{p_N}{p_C} \right)^2 u_{11} - 2 \frac{p_N}{p_C} u_{12} H_1 + u_{22} H_1^2 + u_2 H_{11} \quad (3.8)$$



The first order condition (3.7) at a point  $\hat{N}$  is thus sufficient under the usual assumptions that the marginal utility of income and health are both positive and decreasing, that the marginal product of health inputs is positive and decreasing; and additionally that  $u_{12} \geq 0$ . We gather these assumptions about preferences and the health production functions under [A1 – A2] below:

- [A1]  $u_1, u_2 > 0, u_{11}, u_{22} \leq 0$  and  $u_{12} \geq 0$  for all  $C, N > 0$ .
- [A2]  $H_1, H_2 > 0$  and  $H_{11} \leq 0$  for all  $N > 0$

Let  $v(m, p_C, p_N; s)$  denote the household’s indirect utility function. Then the effect of a health shock (an exogenous change in  $s$ ) on household welfare is given by

$$\frac{\partial v}{\partial s} = \left( -\frac{p_N}{p_C} u_1 + u_2 \right) \frac{\partial N}{\partial s} \Big|_{\hat{N}} + u_2 H_2 \Big|_{\hat{N}} \quad (3.9)$$

The first right hand side of (3.9) is zero from the envelope theorem. Accordingly,  $u_2 H_2 \Big|_{\hat{N}} > 0$  entails that welfare increases with health. Thus, a health shock results in a welfare deterioration under the above assumptions [A1 – A2]. Following Koç (2004), we use the implicit function theorem to obtain  $\frac{\partial N}{\partial s} = \frac{-\partial \Gamma / \partial s}{\partial \Gamma / \partial N}$ . Thus, given assumptions [A1 – A2],  $\partial \Gamma / \partial N < 0$  and accordingly  $\partial N / \partial s$  will be of the same sign as  $\partial \Gamma / \partial s$ . The latter is of the form

$$\frac{\partial \Gamma}{\partial s} = -\frac{p_N}{p_C} u_{12} H_2 + u_{22} H_1 H_2 + u_2 H_{12} \quad (3.10)$$

The demand for health inputs, and accordingly the health budget share, rises as a consequence of a health shock ( $\partial N / \partial s < 0$ ) when the above derivative in (3.10) is negative. While the first and second right hand side terms of (3.10) are negative under assumptions [A1 – A2], the overall effect is ambiguous when  $H_{12} > 0$ .

It is most plausible to assume  $H_{12} > 0$ , meaning that the marginal product of health inputs is higher for those in better health. Under  $H_{12} = 0$ , the marginal product of health inputs is not affected by health shocks, while  $H_{12} < 0$  is synonym to assuming that the marginal product of health inputs is lower for individuals enjoying a higher exogenous health endowment  $s$ . Though the additional restriction  $H_{12} < 0$  would entail that  $N$  rises as a consequence of a health shock, this assumption is rarely made, as it runs counter to common sense (Koç, 2004.)

We summarize our discussion with the following corollary to Proposition 1:

**Corollary 1.2** *Let the household maximize by choice of  $N$  a utility function  $u \left[ \frac{m - p_N N}{p_C}, H(N, s) \right]$  which satisfies assumptions [A1-A2]. Then:*

(i) If  $H_{12} \leq 0$ , the budget share for health inputs always rises in response to a welfare deterioration arising from an exogenous health shock.

(ii) If  $H_{12} > 0$ , a welfare deterioration arising from an exogenous health shock has an undetermined effect on the budget share for health inputs.

To conclude then, if it is taken that  $H_{12} > 0$  there is again limited scope for equating catastrophic levels of out of pocket spending with welfare disruptions arising from health shocks. This result arises because it cannot be ruled out that the demand for health inputs may fall as a result of a deterioration in  $s$ .

#### 4. Catastrophic expenditure and poverty

We now turn to our second question, where we investigate the household level relation between the poverty shortfall, the difference between a household's resources and the poverty line, and the budget share for health OOPs. Our purpose here is to inquire as to the existence of a positive association between these two variables. The answer to this question would appear to be straightforward if we were willing to assume the existence of a decreasing Engel curve relation between the health budget share and income. With this assumption we could certainly conclude that catastrophic expenditure rises with poverty and thus we could argue that to the extent that economic development reduces poverty, it would also reduce the incidence of catastrophic expenditure.

However, the assumption of a decreasing Engel curve, or more specifically of a monotonic Engel curve is not as natural as it seems. Modern empirical research as well as economic theory of consumer choice highlights the importance of nonlinearities in the Engel curve relation. Stated differently, economic theory does not rule out that a good could be a necessity at some income intervals and a luxury at others. This is where we begin our investigation of our second question.

Let  $z$  denote the poverty line and define  $\pi(m, z)$  as a household's poverty shortfall from the poverty line<sup>4</sup>. Because poverty measures respect the Pareto principle (see Atkinson, 1987) the function  $\pi(m, z)$  is monotonically decreasing in  $m$ , for  $m < z$  and is further assumed to be zero for  $m \geq z$ . The function  $\pi(m, z)$  is thus invertible and we define

$$m \stackrel{\circ}{=} h(\pi, z) \tag{4.1}$$

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<sup>4</sup>That is, a poverty measure  $P(m, z)$  relates to the sum of individual family level poverty shortfalls via the identity  $P(m, z) \doteq \int_0^z \pi(m, z) dF$ , where  $F(m)$  is the distribution of income.

to be the resulting inverse function. We also write the general Engel curve relation for health OOP as  $w \doteq \psi(m)$ , where we observe once again that this relation need not be monotonic.

Substituting for  $m$  in the Engel curve relation using (4.1), we have

$$w = \psi[h(\pi, z)] \tag{4.2}$$

Accordingly, in the region  $\pi > 0$  we have <sup>5</sup>

$$\frac{dw}{d\pi} = \frac{d\psi}{dh} \frac{\partial h}{\partial \pi} \tag{4.3}$$

Because  $m = h(\pi, z)$  is decreasing in  $\pi$ , the second right-hand-term is non-positive and it follows that in  $(\pi, w)$  space the function (4.2) will indeed have a positive slope for all  $\pi > 0$  provided the Engel curve relation  $\psi(m)$  is monotonically decreasing at all income levels. In such a situation, it is the case that for any budget share threshold defining the state of catastrophic expenditure, increases in income will simultaneously result in a decline in the intensity of catastrophic expenditure and poverty.

Now consider a situation where the Engel curve is non-monotonic. The simple relation (4.3) informs us that the non-linearities in the Engel curve will be depicted by the function (4.2). To illustrate our point, we may borrow the specification of quadratic logarithmic demands from Banks et al. (1997) <sup>6</sup>. In a cross-section environment where prices are taken to be constant, the resulting budget share relation is of the form

$$w(m) = \phi_0 + \phi_1 \ln m + \phi_2 (\ln m)^2 \tag{4.4}$$

where  $\phi_1$  and  $\phi_2$  are coefficients that may be of opposite or identical signs. More specifically, when  $\phi_1$  and  $\phi_2$  are of opposite signs the resulting Engel curve can be increasing over some income interval and decreasing over another range of

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<sup>5</sup>At  $\pi = 0$ , the function  $h(\pi, z)$  rises to infinity, and accordingly the derivative  $\frac{\partial h}{\partial \pi}$  is no longer defined.

<sup>6</sup>The budget shares underlying quadratic logarithmic demands are of the form

$$\bar{w}(m, p) \doteq \bar{\phi}_0(p) + \bar{\phi}_1(p)[\ln m - \ln a(p)] + \bar{\phi}_2(p)[\ln m - \ln a(p)]^2$$

where each of the functions  $\bar{\phi}_1$  and  $\bar{\phi}_2$  may either be both positive or negative over the price space. If we abstract from price variations across family units, the resulting function  $w(m)$  is of the form (4.4).

incomes. For the individual poverty shortfall function consider the simple specification based on Watts (1968):

$$\begin{aligned}\pi(m, z) &\doteq \log(z/m) & m < z \\ \pi(m, z) &\doteq 0 & m \geq z\end{aligned}\tag{4.5}$$

The resulting relation between  $\pi$  and  $w$  is given by

$$w = \phi_0 + \phi_1 \ln z + \phi_2 (\ln z)^2 - [\phi_1 + 2\phi_2 \ln z]\pi + \phi_2 \pi^2\tag{4.6}$$

Preliminary research by Lamiraud and Abul Naga (2006) using South African data suggests that the Engel curve for health OOP is  $U$ -shaped. Accordingly, in Figure 1 we plot a hypothetical situation where we set  $\phi_0 = 0.40$ ,  $\phi_1 = -0.80$  and  $\phi_2 = 0.50$ . We also set the poverty line at  $z = 4.5$ , and consider a range of income values in the interval  $m \in [1; 4]$  pertaining to individuals experiencing poverty. The NorthEast quadrant plots the Engel curve relation which, given the parameter values, is  $U$ -shaped, and reaches its minimum at  $m^* = \exp(0.80) = 2.23$ . The SouthEast quadrant plots the relation (4.5) between income and the poverty shortfall. To obtain the relation between  $\pi$  and  $w$ , we draw a  $45^\circ$  degree line in the SouthWest quadrant of the diagram. Finally, in the NorthWest quadrant we obtain the desired relation (4.6). The graph illustrates in a simple fashion (4.3), that is, the fact that the relation between  $w$  and  $\pi$  will reflect the curvature properties of the Engel curve.

Algebraically, the relation drawn in the NorthWest quadrant, given our choice of functional forms and parameter values, is readily obtained from (4.6) as  $w = 0.33 - 0.7\pi + 0.5\pi^2$  such that the curve reaches its minimum at  $\pi^* = 0.7$ . The example may also illustrate one common policy concern. When society's primary objective is the alleviation of catastrophic expenditure at very low income levels (say  $m < m^*$ ), then, in the example, this is the segment of the distribution of income where there is a positive association between the poverty shortfall and the budget share.

As an answer to our second question, the preceding discussion is summarized by means of the following proposition:

**Proposition 2** (a) *Let the Engel curve for health out-of-pocket expenditure be a non-monotonic function of income below the poverty line. Then the resulting relation between the budget share and the poverty shortfall is also non-monotonic.*

(b) *A sufficient condition to obtain a positive association between the budget share and the poverty shortfall is that the Engel curve for health OOP be a decreasing function of income below the poverty line.*

It is therefore an empirical question to ask as to whether the Engel curve for OOP health expenditures is indeed a declining function of income so as to guarantee a positive association between poverty and catastrophic expenditure at the economy-wide level.

## 5. Catastrophic expenditure: a measure of performance of health insurance systems?

As discussed in Section 2, it has been advocated that the intensity of catastrophic health care payments may be used as an index of the performance (i.e. under-coverage) of prevailing health insurance schemes. In this section therefore we construct a simple model for the demand of health in the developing country context in order to address our third question, namely: Does an increase in coverage of a health insurance scheme always result in a reduction of the prevalence of catastrophic expenditures? As simple as it is, the model will serve to highlight some weaknesses in the pursuit of minimizing catastrophic expenditure as a public policy objective.

Assume the exogenous health endowment  $s$  is a random variable, and individuals can insure themselves against the adverse health states. Our starting point is a situation whereby a hypothetical population is in benefit of a compulsory health insurance, offering a co-insurance  $0 \leq \sigma \leq 1$  to individuals, in exchange for a premium  $\Pi(\sigma)$ . The function  $\Pi(\cdot)$  is decreasing, with  $\Pi(0) = K > 0$  and  $\Pi(1) = 0$ . The parameter  $\sigma$  is chosen by the government, and  $\sigma = 0$  corresponds to a situation of full insurance, while  $\sigma = 1$ , is the opposite case where there is no insurance coverage <sup>7</sup>.

Returning to the framework of Section 3, we define the individual's preferences over two goods; a consumption aggregate denoted  $C$  and a health input denoted  $N$ . We allow the individual to opt for a corner solution  $N = 0$ , as there exists empirical evidence documenting cases where insured individuals in poor health forego health-care at low income levels <sup>8</sup>. We normalize the price of health inputs such that  $p_N = 1$ . Under such circumstances, the consumer's out of pocket expenditure is simply  $\sigma N$ . The household's constraints take the form of a budget

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<sup>7</sup>In the U.S. and many developing countries the consumer has the decision to insure or to opt out of the insurance scheme. Such considerations are examined in Abul Naga and Lamiraud (2008). The results and conclusions presented here are qualitatively similar to those of Abul Naga and Lamiraud (2008) where the consumer endogenously chooses whether or not to insure.

<sup>8</sup>See in particular Lamiraud et al. (2005).

constraint as well as a non-negativity constraint on  $N$  :

$$p_C C + \sigma N = m - \Pi(\sigma) \quad (5.1)$$

$$N \geq 0 \quad (5.2)$$

Within the class of utility functions which comply by assumptions [A1 – A2] we shall choose a quasi-linear specification of the form

$$u[C, H(N, s)] = \alpha \log C + H \quad (5.3)$$

and a health production function

$$H(N, s) = G(s)N + B(s) \quad (5.4)$$

where  $G(s)$  and  $B(s)$  are increasing functions of  $s$ .  $B(s)$  is the individual's health  $H(0, s)$  when  $N = 0$ , whereas  $G(s)$  is the marginal product of the health input. The specification is relevant in the developing country context, since (5.3) entails that individuals initially allocate their spending to the consumption good, and upon reaching a certain threshold, start consuming the health good  $N$ . For (5.3–5.4) to satisfy assumptions [A1 – A2] of Section 3 we further require  $\alpha > 0$ ,  $G'(s), B'(s) > 0$  and  $G''(s), B''(s) < 0$ .

The optimum level of spending on the health input is obtained as the solution of the following constrained optimization problem:

$$\begin{aligned} \max_{C, N} L(C, N, \lambda_1, \lambda_2; s) &\doteq \alpha \log(C) + E_s [G(s)N + B(s)] + \\ &+ \lambda_1 [m - \Pi(\sigma) - p_C C - \sigma N] + \lambda_2 N \end{aligned} \quad (5.5)$$

where  $E_s(\cdot)$  is the expectations operator with respect to the distribution of  $s$ ,  $\lambda_1$  is a Lagrange multiplier and  $\lambda_2$  is a Kuhn-Tucker multiplier. The optimum level of out of pocket expenditures obeys the simple rule:

$$\sigma \widehat{N} = 0 \quad m \leq \tilde{m}(\sigma) \quad (5.6)$$

$$\sigma \widehat{N} = m - \tilde{m}(\sigma) \quad m > \tilde{m}(\sigma) \quad (5.7)$$

where  $\tilde{m}(\sigma)$  is the critical income threshold below which households allocate their entire expenditure to the consumption good:

$$\tilde{m}(\sigma) \doteq \Pi(\sigma) + \alpha \sigma / E_s[G(s)] \quad (5.8)$$

The Engel curve for out of pocket expenditures is obtained as the ratio of (5.6-5.7) to the household's income net of the health insurance premium:

$$w_N(m; \sigma) = 0 \quad m \leq \tilde{m}(\sigma) \quad (5.9)$$

$$w_N(m; \sigma) = \left( \frac{m - \tilde{m}(\sigma)}{m - \Pi(\sigma)} \right) \quad m > \tilde{m}(\sigma) \quad (5.10)$$

Given a critical budget share threshold  $\omega \in [0, 1]$ , households for which  $w_N > \omega$  are defined to be in a state of catastrophic expenditure. The question we turn to below is whether when  $\sigma$  falls from an initial value  $\sigma_0$  to a new value  $\sigma_1 < \sigma_0$  it is always the case that the incidence of catastrophic expenditure declines.

Differentiating (5.8) with respect to  $\sigma$ , we have

$$\frac{d\tilde{m}}{d\sigma} = \Pi'(\sigma) + \alpha/E_s[G(s)] \quad (5.11)$$

The first right hand side term above is negative (the premium falls as the co-payment rate rises)<sup>9</sup>, while the second is positive. The overall effect of a rise in  $\sigma$  on the critical income is therefore ambiguous.

Below we distinguish two cases for the relation between  $\sigma$  and the change in catastrophic expenditure, depending on whether the derivative of (5.11) is positive or negative.

### 5.1. Case 1 model: $\tilde{m}(\sigma_1) < \tilde{m}(\sigma_0)$

Assume first that for  $\sigma_1 < \sigma_0$  the resulting effect is that the critical income at which households begin to incur positive OOPs also declines:  $\tilde{m}(\sigma_1) < \tilde{m}(\sigma_0)$ . Then, at any income  $m \geq \tilde{m}(\sigma_1)$  the new Engel curve  $w_N(m; \sigma_1)$  lies everywhere above the initial Engel curve  $w_N(m; \sigma_0)$ <sup>10</sup>. This is illustrated in Figure 2.

That is, while there is an increase in coverage of the health insurance scheme, in the sense that a larger share of health expenditures is taken on by the insurer, the incidence of catastrophic expenditure is increased. The Case 1 model therefore provides a counter-example to the hypothesis that catastrophic expenditures are associated with an insufficient coverage of a given health insurance scheme.

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<sup>9</sup>While it is always the case that  $\Pi'(\sigma) < 0$ , the curvature (i.e. second derivative) properties of  $\Pi(\sigma)$  depend amongst other things on the distribution of income, and whether the insurance premium is fair. See Cutler and Zeckhauser (2000) for further discussion.

<sup>10</sup>See the first part of Proposition 4 in the appendix for a proof of this statement.

On the positive side, it is to be noted that when the critical income falls, there is an additional share of the population (individuals with incomes in the interval  $\tilde{m}(\sigma_1) \leq m \leq \tilde{m}(\sigma_0)$ ) who are consuming health inputs. If the probability of morbidity decreases with the consumption of health inputs, then in this Case, catastrophic expenditure rises but potential morbidity falls when there is an increase in insurance coverage. As such, one lesson to learn from this Case 1 model is the importance of making explicit the means by which catastrophic expenditure is to be reduced, or yet to keep in mind that population health may actually improve when the incidence of catastrophic expenditure rises.

## 5.2. Case 2 model: $\tilde{m}(\sigma_1) \geq \tilde{m}(\sigma_0)$

Next, consider the case whereby for  $\sigma_1 < \sigma_0$  the critical income at which households begin to incur positive OOPs rises:  $\tilde{m}(\sigma_1) \geq \tilde{m}(\sigma_0)$ . Then, at the new critical income  $\tilde{m}(\sigma_1)$  we have that  $w_N(\tilde{m}(\sigma_1); \sigma_0) > 0$  while  $w_N(\tilde{m}(\sigma_1); \sigma_1) = 0$ . That is, the new Engel curve lies initially below the former curve  $w_N(m; \sigma_0)$ . In Proposition 4 of the Appendix we show that there can be at most one crossing of the two Engel curves over the interval  $m \geq \tilde{m}(\sigma_1)$ .

Let us consider in turn these two possible scenarios. First, consider the situation where the two Engel curves do not cross at any income level  $m > \tilde{m}(\sigma_0)$ . Then, in contrast with the Case 1 model, when the critical income rises, catastrophic expenditure is reduced and there is an additional share of the population (individuals with incomes in the interval  $\tilde{m}(\sigma_0) \leq m \leq \tilde{m}(\sigma_1)$ ) who are *no longer* consuming health inputs.

Next, consider a scenario such that the two Engel curves intersect at some finite income  $t \in [\tilde{m}(\sigma_1), \infty)$ . This is illustrated in Figure 3. Individuals with resources  $m < t$  are allocating smaller shares of their resources to health OOPs whereas those with resources  $m > t$  are allocating larger shares. The overall effect of a change in  $\sigma$  on the incidence of catastrophic expenditure depends on the value at which the critical threshold  $\omega$  is set in relation to  $w_N(t, \sigma_1)$ . Let  $F(m)$  denote again the cumulative distribution of income, and assume first that  $\omega \leq w_N(t, \sigma_1)$ . Then there exist incomes  $m_a$  and  $m_b$ , with  $m_a > m_b$ , such that  $w_N(m_a, \sigma_1) = w_N(m_b, \sigma_0) = \omega$ . Catastrophic expenditure under the new scheme is  $1 - F(m_a)$ , which is less than the former level of catastrophic expenditure  $1 - F(m_b)$ .

Conversely, if  $\omega > w_N(t, \sigma_1)$  the new scheme will now entail a larger share of the population experiencing catastrophic expenditure (see Figure 3). Here the



incomes  $m_a$  and  $m_b$  that satisfy  $w_N(m_a, \sigma_1) = w_N(m_b, \sigma_0) = \omega$  are such that  $m_a < m_b$ ; so that now  $1 - F(m_a) \geq 1 - F(m_b)$ .

Thus, in the Case 2 model, the effect of adopting a more generous insurance package has an undetermined effect on the incidence of catastrophic health expenditure: if the two Engel curves fail to cross, then clearly the incidence of catastrophic expenditure will be reduced. In the alternative case, a crossing of the Engel curves entails that there exist values of the critical threshold  $\omega$  that will produce a rise in the incidence of catastrophic expenditure.

### 5.3. Change in coverage: overall effect

The Case I model, where the critical income falls, entails that the incidence of catastrophic expenditure is on the rise when population coverage of the health insurance scheme increases via a reduction in the copayment rate  $\sigma$ . In the Case II model the overall effect of a change in population coverage on the incidence of catastrophic expenditure is ambiguous. If a crossing of Engel curves occurs, then there is a range of critical thresholds  $\omega$  which entail a reduction in the incidence of catastrophic expenditures, and there is also a range of critical thresholds which entails a rise in the incidence of catastrophic expenditures. We may summarize our above discussion with the following Proposition:

**Proposition 3** (a) *Under the case I model ( $\tilde{m}(\sigma_1) < \tilde{m}(\sigma_0)$ ), a reduction in the copayment rate results in an increase in the incidence of catastrophic expenditure.*

(b) *Under the case II model ( $\tilde{m}(\sigma_1) \geq \tilde{m}(\sigma_0)$ ), a reduction in copayment rate results in: (1) a reduction of the incidence of catastrophic expenditures provided the two Engel curves  $w_N(m; \sigma_0)$  and  $w_N(m; \sigma_1)$  do not cross for all values  $m \geq \tilde{m}(\sigma_0)$ ; (2) has an undetermined effect on the incidence of catastrophic expenditure if the two Engel curves  $w_N(m; \sigma_0)$  and  $w_N(m; \sigma_1)$  cross at some finite income level  $t > \tilde{m}(\sigma_1)$ .*

## 6. Concluding comments

We may summarize our answers to the three questions raised in the paper as follows:

1. An observed budget share level for a given good is compatible with both a drop or a rise in levels of living. Thus, without additional information about

the direction of income change, the level of a budget share cannot be informative about the sign of the change in welfare.

2. In the general case where below the poverty line the Engel curve for OOPs is a non-monotonic function of income, the resulting relation between the budget share and the poverty shortfall is also non-monotonic. However, if below the poverty line the income elasticity for OOPs is smaller than one (i.e. the Engel curve is decreasing), this will indeed entail a positive association between the poverty shortfall and the budget share for health OOPs.

3. An increase in coverage of a health insurance scheme need not always result in a reduction of the prevalence of catastrophic expenditures.

In relation to our first question, we may conclude therefore that the scope for using cross-section data to identify households who experience a severe decline in their levels of living using a budget share is considerably limited, unless the data analyst is sure that the household has experienced an income drop. Such information about changes in income is readily available from panel data, but is rarely encountered in cross-section type household surveys.

We note from our discussion in relation to the second question that we have addressed in the paper, that it does not follow that catastrophic health expenditure increases with poverty. Empirical work is therefore needed in order to further explore the curvature properties of the Engel curve for health care spending.

Finally, with the help of a simple model, we have shown that catastrophic expenditure could well increase when the share of health payments covered by a health insurance scheme is increased. Also, one lesson we retain from this analysis is the importance of making explicit the means by which catastrophic expenditure is to be reduced, since it is quite conceivable that population health may actually improve when the incidence of catastrophic expenditure rises. Thus, more work is needed in order to better understand how the overall performance of a health insurance scheme relates to the incidence of catastrophic expenditures, and more generally, to population health.

## 7. Appendix

In this appendix we study the intersection of the two Engel curves before and after a change in  $\sigma$ , the co-insurance rate.

Define the function  $\Delta w_N : [\tilde{m}(\sigma_1), \infty) \rightarrow [-1, 1]$  as the difference in Engel

curves for out of pockets:

$$\Delta w_N(m) \doteq \left( \frac{m - \tilde{m}(\sigma_1)}{m - \Pi(\sigma_1)} \right) - \left( \frac{m - \tilde{m}(\sigma_0)}{m - \Pi(\sigma_0)} \right) \quad (7.1)$$

Let  $NUM1(m) \doteq m - \tilde{m}(\sigma_1)$ ,  $DENOM1(m) \doteq m - \Pi(\sigma_1)$ , respectively denote the numerator and denominator of the first left-hand term of (a1). Likewise define  $NUM2(m) \doteq m - \tilde{m}(\sigma_0)$ ,  $DENOM2(m) \doteq m - \Pi(\sigma_0)$ . We have the following result:

**Proposition 4 :**

(i) For  $\sigma_1 < \sigma_0$  let  $\tilde{m}(\sigma_1) < \tilde{m}(\sigma_0)$ . Then  $\Delta w_N(m) > 0$  everywhere on its domain  $[\tilde{m}(\sigma_1), \infty)$ .

(ii) For  $\sigma_1 < \sigma_0$  let  $\tilde{m}(\sigma_1) > \tilde{m}(\sigma_0)$ . Then there exists at most one finite income  $t \in [\tilde{m}(\sigma_1), \infty)$  such that  $\Delta w_N(t) = 0$ .

**Proof:** (i) Since  $\Pi(\sigma_1) > \Pi(\sigma_0)$  and from the assumption underlying Case A, namely,  $\tilde{m}(\sigma_1) < \tilde{m}(\sigma_0)$ , we have the joint inequalities  $NUM1(m) > NUM2(m)$  and  $DENOM1(m) < DENOM2(m)$ . Thus, in (a1) the first RHS ratio is always greater than the second RHS ratio, and accordingly  $\Delta w_N(m) > 0$  everywhere over  $[\tilde{m}(\sigma_1), \infty)$ .

(ii) Here we show that there can be at most one crossing of the two Engel curves in the interval  $[\tilde{m}(\sigma_1), \infty)$ . Firstly observe that  $\Delta w_N[\tilde{m}(\sigma_1)] < 0$ . If there exists a point  $t \in [\tilde{m}(\sigma_1), \infty)$  such that  $\Delta w_N(t) = 0$ , it is a solution to

$$((t - \Pi(\sigma_0))((t - \tilde{m}(\sigma_1))) = ((t - \Pi(\sigma_1))((t - \tilde{m}(\sigma_0))) \quad (a2)$$

Upon simplifying, this gives:

$$t = \frac{\Pi(\sigma_1)\tilde{m}(\sigma_0) - \Pi(\sigma_0)\tilde{m}(\sigma_1)}{\Pi(\sigma_1) - \Pi(\sigma_0) - (\tilde{m}(\sigma_1) - \tilde{m}(\sigma_0))} \quad (a3)$$

If the point  $t$  lies outside the interval  $[\tilde{m}(\sigma_1), \infty)$ , then  $\Delta w_N(m)$  is everywhere negative and the two Engel curves do not intersect over the relevant domain of definition <sup>11</sup>.  $\square$

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<sup>11</sup>By using the definition of  $\tilde{m}(\sigma)$  in (5.8), one can verify that the denominator of (a3) is always positive provided  $\sigma_1 < \sigma_0$ .

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**Figure 1: Health OOP budget share and poverty shortfall**

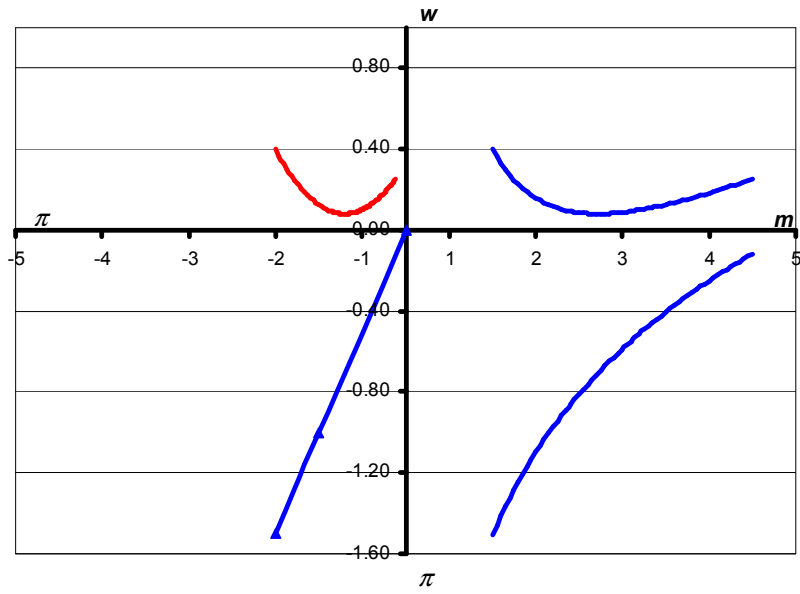


Figure 2: Engel curve for OOP spending (case 1 submodel)

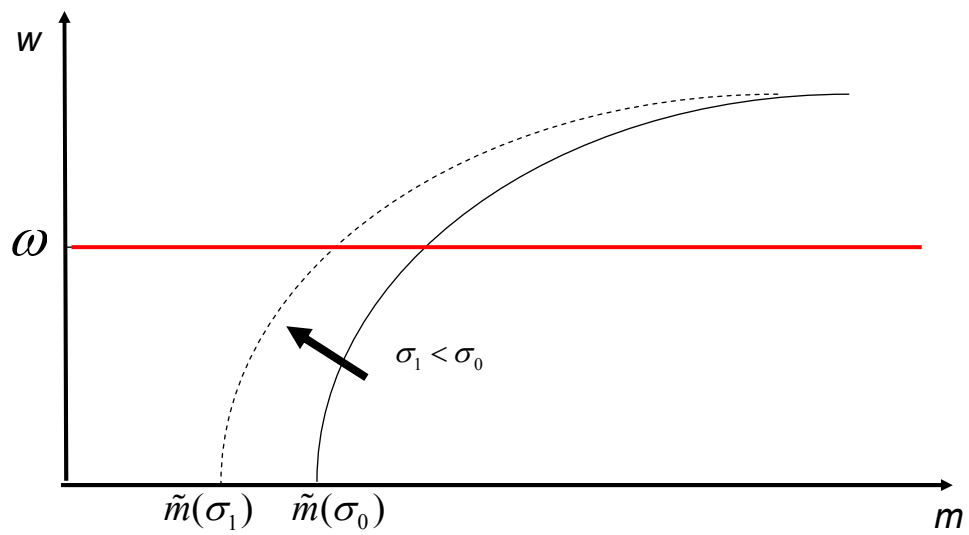
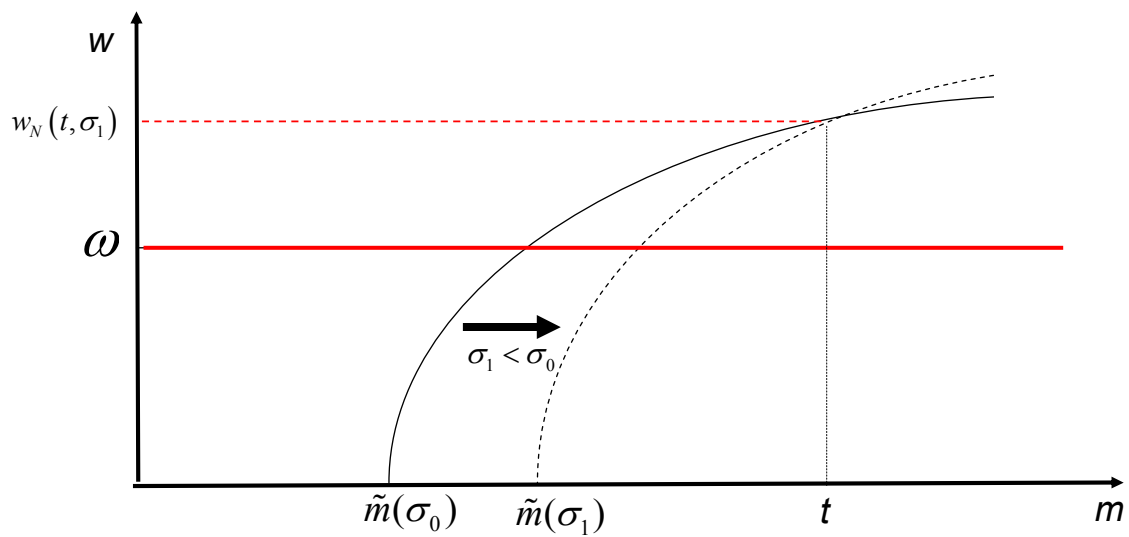


Figure 3: Engel curve for OOP spending (case 2 submodel)





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