An Agent-Based Model of Exurban Land Development

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Abstract: In contrast to urban areas that are aptly characterized by a large population base and scarce land supply, exurban regions have limited households and plentiful land. This basic difference has far reaching implications for spatial equilibrium in exurban land markets. Rather than bidding their maximum willingness-to-pay and reaching a spatial equilibrium in which households are indifferent to location, as is the central condition of urban economic models, we argue that exurban households will be able to retain some amount of surplus in moving to an exurban location and therefore will choose the location that maximizes this locational surplus. In this paper, we first review the handful of structural spatial models of exurban land development that have been developed. We then develop a structural spatial model of exurban land development that captures these hypothesized features of exurban land markets using an auction model to represent household bidding and adapting the Capooza and Helsley (1990) model to represent landowners' optimal timing of development. A key innovation of our approach is that, in the absence of full capitalization of land or location differences into land prices, households have preferences for some locations over others and thus it is possible to order household location choices in time and space. This greatly facilitates modeling of land use dynamics by enabling us to model location and land use decisions sequentially in time rather than assuming that all development is instantaneous for given levels of population and income in the region. In addition, the spatial agent-based simulation method that is used to implement the model permits an explicit examination of the implications of exurban land market conditions for the evolution of urban development pattern. Specifically, we ask whether these exurban market conditions explain the emergence and persistence of so-called leapfrog development that is characteristic of exurban regions.

1. Introduction

Urban and urbanizing land, while only a small fraction of the total land area worldwide, generates a disproportionate share of impacts on aquatic and terrestrial ecosystems (Alberti, 2005; Collins et al., 2000; Grimm 2008). These impacts arise not only from the amount and density of urban development, but also the spatial distribution of urban land and its edges relative to ecological features. In the United States, the overwhelming majority of urban land development occurs outside urban and even suburban areas in so-called exurban areas. According to Brown et al. (2005), the amount of land at urban densities (more than one house per acre) increased from less than 1% to nearly 2% between 1950 and 2000 while the amount of exurban land (between 1 and 40 acres per house) increased from about 5% to 25% in this same time period. This rapid expansion of low density exurban land use has had substantial impacts on ecological functioning, including biodiversity and water quality (Hanson et al. 2005, Lohse et al. 2008). Scientists have only recently begun to identify the differential effects of exurban development, in large part due to poor land use data that did not record low density exurban development. The use of parcel data has greatly facilitated the identification of these ecological impacts and shown that spatial variations in urban development at the parcel scale are a critical determinant of impacts (Irwin et al. 2009).

A growing interest in the underlying processes of land use and land cover patterns has led to an explosion in spatial modeling of land use change, much of it coming from outside economics (Irwin 2010). Within economics, progress has been made in estimating econometric models of land development that account for spatial interactions and heterogeneity and in extending urban spatial models to account for multiple sources of spatial heterogeneity. However, despite the preponderance of urbanization in exurban areas, very few structural models of exurban land development have been developed. Of the handful of models that have been developed, most adapt the basic urban economic model of growth by Capozza and Helsley (1989, 1990) to represent exurban household location choice and rural landowner development decisions. However, exurban land markets are not the same as urban land markets and the efficacy of applying this model to exurban regions is questionable. The urban economic model rests on the fundamental assumption of a spatial equilibrium in which land and housing rents adjust to make households indifferent to location. This equilibrium outcome depends on two fundamental assumptions of the model: first, that the number of households demanding location is very large and second, that each land parcel is a unique location (Fujita 1989). These market conditions correspond to excess demand for land at zero rents and generate equilibrium land rents that are equal to households' maximum willingness-to-pay, their so-called bid rent, so that all gains from trade are captured by landowners.

In contrast to a large population base and scarce residential land supply, exurban regions are characterized by just the opposite: plentiful land that is highly substitutable in terms of its locational features and a limited number of households. This basic difference has far reaching

implications for spatial equilibrium. At rents equal to the households' maximum willingness to pay, the land market is characterized by excess supply and thus the standard spatial equilibrium assumption does not hold. Instead, households will bid down rents to something less than this maximum value and in doing so will retain some amount of location-specific surplus. Only in the long run, as increasingly more households move into the exurban region and it transitions into a suburban area, will this locational surplus will be bid away and eventually the standard spatial equilibrium as defined by the urban economic model reached.

We start from this basic premise, that migration to exurban areas is neither costless nor instantaneous and therefore that exurban regions are characterized by excess supply at rents equal households' maximum willingness-to-pay. This invalidates the standard spatial equilibrium model and necessitates an alternative approach to modeling spatial equilibrium in exurban land markets. We develop a structural spatial model in which utility-maximizing households choose a location according to the expected locational surplus that it will generate, landowners make optimal timing decisions regarding land development given current and future expected household bids and market rents are generated from interactions among multiple landowners and a limited number of households. Under such conditions, spatial equilibrium is defined not by spatial indifference among households, but instead by each household having located on the parcel that maximizes their locational surplus given the location choices of all other households and given the supply of residential land as determined by profit-maximizing landowners. This corresponds to a more complicated set of conditions that describe spatial equilibrium that are difficult to fully quantify analytically. For this reason, we use agent-based computational methods to solve the household bidding and landowner reservation rent problems. A two-dimensional grid is used to track the locational demands and choices of households, the locational features of land parcels and the land parcels supplied for residential development in each time period. The grid is a highly stylized representation of an exurban region, in which transportation costs to the boundary of the urban area are the only source of spatial heterogeneity.

Our approach to modeling exurban land markets is advantageous in several ways. First, the model is the first to account for the essential differences in relative land demand and supply that distinguish urban and exurban markets and that have very different implications for spatial equilibrium in exurban land markets. Second, our approach uses agent-based methods to derive the optimal household bids, landowner reservation rents and market land rents. Despite meaningful advances in economic agent-based models of land markets, ours is the first agent-based model land market model to derive bids and reservation rents from a fully structural model of demand and supply of urban land. Third, relaxing what is essentially a static spatial equilibrium assumption has definite advantages to modeling changes in land use dynamics. Rather than instantaneous price adjustments that make households indifferent to location, households have preferences for some locations over others and thus it is possible to order household location choices in time and space. This allows us to model location and land use

decisions sequentially in time rather than assuming that all development is instantaneous for given levels of population and income in the region. Lastly, our spatial simulation approach permits an explicit examination of the implications of these land market conditions for the evolution of urban land use pattern. Specifically, we ask whether these short run exurban market conditions can explain the emergence and persistence of so-called leapfrog development that is characteristic of exurban regions in the U.S. and other countries that do not have strict rural land use controls.

In the remainder of the paper we first review the handful of spatial land use models that have been developed to explain exurban land development. We compare and contrast two modeling approaches that differ in their treatment of price formation: models that rely on the standard urban economic spatial equilibrium assumption to derive market rents versus agent-based models that derive individual prices of land as the result of bilateral trades explicitly modeled among heterogeneous buyers and sellers. We then present our model of exurban land markets, in which the market equilibrium is characterized by each household choosing their best location given the choices of all other households and given the current supply of residential parcels. At the beginning of each period, a new set of potential migrants bids on land parcels and thus the region fills up over time as more people decide to location there. Rebidding by existing residents ensures that the rents paid by residents correspond to market rents and that the long run spatial equilibrium of the urban economic model is eventually reached. We calibrate the model using secondary data and estimates from the literature on transportation costs and apply it to a highly stylized exurban landscape to investigate whether the model can explain persistent versus temporary leapfrog development.

2. Spatial models of exurban land markets

Structural models of exurban land markets that incorporate greater spatial complexity (including spatial dynamics and multiple sources of spatial heterogeneity) constitute a small, but growing body of work. A distinguishing feature of these models is the way in which price formation is modeled. In some cases, the assumption of spatial equilibrium is employed to derive a set of spatial equilibrium prices that evolve over time in response to exogenous changes. In other cases, agent-based models[1] are used to derive individual prices of land or houses that are the result of bilateral trades explicitly modeled among heterogeneous buyers and sellers. The advantage of the latter is their ability to account for so-called "out of equilibrium" dynamics (or what others might call transitional or short run equilibrium dynamics) that can account for endogenous interactions or feedbacks in a recursive manner.

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[1] For a more comprehensive discussion of issues related specifically to agent-based models of land markets, see Parker and Filatova (2008) and Parker et al. (2003).

Caruso et al. (2007) provides an example of the first approach, in which the evolution of land use patterns is modeled over time using a conditional spatial equilibrium approach that adjusts in each time period following the entrant of a new migrant. Resulting changes in land use generate local spillover effects by influencing the desirability of nearby locations and thus create endogenous local feedback effects that generate local spatial dynamics. A spatial simulation model is needed to account for the incremental change in land use pattern that is capitalized into land rents that then influence the next round of decision making. The model is simulated over many periods to study the implications of these multiple sources of spatial heterogeneity for the evolution of residential development patterns. Various patterns of residential land use clustering and scattering emerge depending on the magnitude and spatial scale of the land use spillovers. The approach is innovative because it demonstrates how local spatial dynamics can arise from a microeconomic model of location choice and land use and influence land use patterns at a regional scale. However, it also points to the awkwardness of the spatial equilibrium assumption in a model that seeks to explicitly represent local spatial dynamics. At the beginning of each period, the new migrant is assumed to have monopsony power, which allows him to pay only the reservation price of the farmer and thus is able to choose the location that generates the largest utility gain. Only after the new migrant chooses a location are prices assumed to adjust to a spatial equilibrium, implying that all market power is then transferred to landowners so that each household must pay their maximum willingness-to-pay and is indifferent to location. This awkward set of assumptions is one solution to modeling spatial differences in utility that lead to sequential location choices. Otherwise, if a spatial equilibrium were continuously imposed, the spillover effects of any land use change would be fully and instantaneously capitalized in price and new migrants would always be indifferent to location. If households are always indifferent to location, then it is impossible to sequentially order their location or land use choices. These modeling trade-offs highlight the difficulty of incorporating recursive spatial dynamics into a traditional spatial equilibrium framework.

Because they depart from aggregate market equilibrium assumptions, agent-based models offer a means of explicitly representing recursive interactions, price adjustments and the sequencing of location and land use decisions over time and space. Given the initial specifications of the economic system, the transitional dynamics are driven solely by agent trading that is not typically subject to an aggregate market clearing constraint or other market-level equilibrium conditions. However, the lack of an aggregate market clearing condition opens up difficult questions about how agent bidding, price formation and the possibility of spatial arbitrage should be modeled. The endogeneity of land rents presents a challenge to deriving agents' willingness-to-pay for a particular location from the standard constrained utility maximization framework, since the budget constraint includes the market rent of per-unit housing (or land) at that location, which is of course endogenous to the household's bid. The assumption of a spatial equilibrium solves this problem by ensuring that the household's bid and the market rent for each location are consistent with each other.

Parker and Filatova (2008) discuss this problem and other theoretical and methodological challenges associated with implementing agent-based land market models. They suggest several approaches to modeling agent bidding, ranging from ad hoc specifications of agents' willingness-to-pay (WTP) functions, to an approach that assumes that agents form expectations over the market price of housing (or land) at a given location and then derive their WTP function from a constrained utility maximization problem given this expected price. Agents act given expected prices and update their beliefs over time as they observe the realized market price for a given location. While this is a plausible approach and one that is theoretically grounded in utility maximization, it relies on the researcher having information about how agents form these expectations and how they modify their beliefs about prices given observed realizations of market prices. Given the dearth of empirical data on how households, land owners and developers form expectations,[2] this is a challenging approach to implement and raises the usual concerns about model robustness.

A central question in spatial models of land use change is how locational advantages or disadvantages should be reflected in bids and market prices associated with housing or land at a particular location. The spatial equilibrium assumption solves this question by assuming that any locational difference over which households have preferences is exactly offset by equilibrium prices that instantaneously adjust to these differences. The implicit assumption is that competition among many footloose households for the more desirable land parcels results in equilibrium rents that are equal to households' full WTP, so that in equilibrium households are indifferent to location. Because each land parcel is assumed to be unique, landowner competition is ignored and all gains from trade accrue to landowners. The spatial equilibrium assumption is most appropriate for large urban areas, in which households are mobile, land is scarce and many households compete for unique locations. In contrast, exurban regions are characterized by plentiful land and a limited number of households. This basic difference has far reaching implications: instead of fully capitalizing households' willingness-to-pay for each parcel, the market price for exurban land is determined by a limited number of households that compete and the decisions of multiple landowners who own similar parcels. Under such conditions, the transacted market price does not necessarily correspond to the household's maximum willingness-to-pay and thus an alternative approach to modeling household bids and landowner expectations is needed.

Agent-based models provide a means of modeling transitional or short run dynamics in the absence of an exogenous growth mechanism or constraint, but require an alternative approach to modeling price formation. While some have developed agent-based models of housing markets with aggregate hedonic pricing models (e.g., Miller et al. 2004; Waddell et al. 2003), others have

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[2] In contrast, because data on traders are much more readily available, a good deal of work has been done on how expectations are formed by agents in ABMs of financial markets for financial assets (e.g., see excellent summaries of the literature in the chapters by LeBaron, Hommes, Tesfatsion, and Duffy in Tesfatsion and Judd (2006).

taken advantage of the disaggregation of agent-based models by explicitly modeling price formation as the result of household offer bids, seller ask bids and the interactions between individual buyers and sellers. These models differ in how these market interactions are modeled and in particular, how agents' WTP, willingness-to-accept (WTA) and perceptions of market competitiveness are accounted for in their formulation of optimal offer and ask bids. Filatova et al. (2009a, 2009b) begin with an ad hoc specification of households' WTP function as $WTP = \frac{Y*U^2}{b+U^2}$, where Y is income net of transportation costs and expenditures on a? composite good; U is household utility and b is a parameter that is assumed to represent the price of the composite good. This WTP function mimics standard demand relationships, such as increasing WTP with income. Expenditures on the composite good enter indirectly: given the functional form assumption, households never spend all their income net of transportation costs on housing and thus implicitly, the remaining income is spent on the composite good. Because households never will spend all their net income on housing, this specification of WTP is not comparable to the urban economic spatial equilibrium model of bidding, in which households are always assumed to spend all their residual income (net transportation costs and a fixed amount on the composite good) on housing. Instead of being defined by a set of prices, a spatial equilibrium is defined in a static sense: an equilibrium is reached when no further incentives exist for a household to enter the region or for a household to sell her property, i.e., all gains from trade have been exhausted given current bids and offers, resulting in constant population, prices and land use pattern. A logical next step, and one that the authors are currently pursuing (Parker and Filatova, personal communication), is to introduce household rebidding and relocation, so that existing households could readjust their housing consumption if their location has become suboptimal over time. This is particularly important for considering the dynamic effects of local spatial spillovers, such as the loss of open space or rising congestion levels, that generate feedbacks and will cause additional market and land use adjustments.

To account for agent's responses to market conditions, Filatova et al. (2009a, 2009b) follow a logical two-step approach proposed by Parker and Filatova (2008) in which the individual WTP and WTA bids are first specified as above and then adjusted by a multiplicative factor (1+ε), where $ε = (NB – NS)/(NB + NS)$, NB = number of buyers and NS = number of sellers.[3] This allows bidding to be adjusted based on agent perceptions of market conditions, so that individual offer (ask) bids will increase (decrease) as the number of buyers (sellers) increases. Given positive gains from trade (i.e., offer bid $\geq$ ask bid), then the transaction price is set assuming that the buyer and seller divide these gains equally (i.e., the transaction price is the arithmetic mean of the offer and ask bids). Filatova et al. (2009a) find that the transaction prices of identical locations are not the same over time because of changes in the market conditions (*NB* and *NS*) that are reflected in the bids. The authors call this an emergent property of the model,

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[3] This approach to modeling price adjustments follows standard models used in agent-based computational finance models, e.g., for representing the "market-oriented traders" pricing strategy (LeBaron 2006).

since changes in *NB* and *NS* are endogenously determined in the model. This is a potentially interesting feature of the model. However, the divergence in prices over time can also be explained by the fact that the model does not consider rebidding by households, so that once a location is occupied by a household, no other household may bid for it. As a result, the price is frozen at the time of the initial development. With rebidding and in the absence of endogenous feedbacks or other sources of spatial heterogeneity that would cause the characteristics of two identically located parcels to differ over time, this divergence would disappear as a result of land prices that would subsequently adjust to current market conditions.

Magliocca et al. (2011a, 2011b) follow a somewhat similar strategy as Filatova and coauthors to modeling the household bidding process. First, optimal rents for each possible house that a household could rent are determined by calculating the rent for each house that would provide the utility level of the house with the highest utility. This ensures that the bids reflect the specific features of a housing type and location, analogous to the spatial equilibrium assumption in the traditional model. This amount is then adjusted upwards or downwards depending on the degree of housing market competition, which is determined by the number of bidders relative to the number of available houses, and by the magnitude of the potential surplus that a household could obtain if they did not have to pay above the developer's ask price. In contrast to other agent-based models, the model developed by Magliocca et al. includes both a land and housing market and thus explicitly model the market interactions of farmers and housing developers, in addition to those between developers and households. Farmers and developers employ various strategies to form expectations about future returns from selling and developing rural land respectively and formulate optimal offer and ask bids based on these expectations that seek to maximize their respective profits. Magliocca et al. provide the most serious treatment of price expectations in an agent-based land market model to-date, allowing for a number of different strategies and exploring how various approaches influence market outcomes. For example, each farmer is randomly assigned a set of prediction models that vary in the length of time over which past prices matter, the functional form of the effect of past prices on current prices, and the influence of landowner competition. Farmers adapt their prediction models according to the success of past predictions. In addition, the model allows for household rebidding: each household is randomly assigned a "residence time" when they initial move into a house. When the household's residence time is exceeded, they re-enter the housing market as buyers and the house that they occupied is put back on the market. Current residents and in-migrants are then able to bid on existing houses, which provides a means of updating housing prices based on current conditions.

Magliocca et al. parameterize their model using secondary data from the Census of Agriculture, Bureau of Transportation Statistics, and U.S. Census Bureau, as well as parameter estimates of developer infrastructure and on-site costs and locational demand parameters from the literature. The model is applied to a 10 mile square landscape and run over a 20 year time period. The model predicts sprawl and leapfrog patterns of development as a result of the various

sources of heterogeneity in the model, including in the agricultural productivity of land parcels, consumers' housing preferences, and farmers' and developers' expectations of future prices.

Ettema (2011) takes a different approach to price formation. Rather than deriving explicit WTP or WTA functions and then adjusting according to market conditions, he models households' responses to a stated list price for a given house. These responses are shaped by subjective probabilities that reflect their perception of the market competitiveness of a given list price for a given house, which is determined by the deviation of the list price from its mean price. Based on this, each buyer formulates a probability that she will be offered a house at that price and each seller formulates a probability that she will sell her house at that price within a certain time period. A seller will simply attempt to maximize the list price at which she offers a given house. A buyer's optimal choice is determined by a given list price L of a house that maximizes her expected utility from the house, where her expected utility is the weighted sum of her utility from obtaining the house at the given price L and a slightly higher price $L+\alpha_2$, where the weights are equal to the respective probabilities of obtaining the house at prices L and $L+\alpha_2$. These probabilities are modified over time by individuals using a Bayesian updating rule based on past transactions simulated by the model. An advantage of this approach is that it incorporates the agent's perceptions of market conditions into their optimal choice in a probabilistic manner that is reflected in their optimal bidding behavior. However, it is unclear that modeling these subjective probabilities as a function of deviations from the mean of past prices captures the relevant factors that determine market competitiveness, especially over time. For example, exogenous population migration over time will force household offer bids to become more competitive over time. From this vantage point, an explicit accounting of the relative numbers of buyers and sellers is more sensible. Another problem with this formulation is that it relies on very limited heterogeneity in housing attributes in order to formulate deviations from a mean price. Implementing this approach could be much more problematic in a spatial model, in which every house or lot is potentially distinct from every other one.

In concluding this discussion, it is useful to contrast the approaches taken by agent-based models to modeling price formation with the traditional urban economic spatial equilibrium models. Because agent-based models seek to relax the restrictive assumptions of the traditional model, they must grapple with additional questions of market conditions and specification that are side stepped by the spatial equilibrium assumption. This challenge combined with the complications that arise from adding spatial complexity to the model make deriving a fully structural modeling of price formation a difficult problem to solve. The difficulty arises because the model must simultaneously account on one hand for the heterogeneous and possibly unique set of spatial attributes that distinguishes one location from another and, on the other hand, for market conditions, including the relative number of buyers and seller, the substitutability of locations and heterogeneity among households or landowners. Spatial equilibrium models solve this complex set of issues by imposing an implicit set of assumptions about market conditions, namely a large number of buyers and a lack of competition among landowners, so that

competition among households for more desirable locations forces households to bid all residual income—i.e., income net of transportation costs and optimal expenditures on a composite good for a given utility level or population level—on land or housing. Because desirable land is scarce and landowners do not compete (since each location is assumed to be unique), landowners are able to extract from households the full value of their land parcel. Thus, the resulting transaction price is equal to the household's maximum WTP and all gains from trade accrue to the landowner.

Agent-based models offer a methodological approach that can relax these implicit assumptions about market conditions, but doing so requires an alternative mean of deriving household bids and market prices. The approaches by Filatova, Parker and collaborators and by Maggliocca and collaborators model price formation explicitly as a three-step process:[4] (i) a WTP function is specified to represent the individual-level demand for location (and, in the case of Magliocca et al., housing type); (ii) bid prices are formulated taking account of market conditions (excess demand or supply for a given housing type or land parcel); (iii) given favorably terms of trade, the transaction (i.e., market) price is determined by dividing the gains from trade between the buyer and seller. The approach imposes a set of assumptions about bidding and how market conditions influence the bidding process, following models of agents' pricing strategies developed in agent-based financial economics. By explicitly modeling how market conditions affect bidding and how the subsequent gains from trade are divided, these models allow for consideration of how other types of market conditions (e.g., a buyers market) influence spatial price and land use outcomes. However, because they impose the WTP functions, offer bids and determination of transaction prices rather than deriving these from an underlying model of agent behavior (e.g., utility or profit maximization), the models are not derived from a fully structural model of exurban land markets.

3. An agent-based model of exurban land development

We present preliminary results from a spatial agent-based computational model that is derived from a fully structural approach to modeling household bidding and price formation. This spatial simulation model is derived from a set of models in which farmers optimally choose the timing of land conversion, residents optimally select and bid for land parcels, and land developers optimally choose the parcels to convert. The key innovation of our modeling approach is in the household bidding model. Rather than deriving a WTP function from the household's utility maximization problem, we use an auction model to derive the household's optimal bid accounting for preferences, income, market conditions and uncertainty over future growth. Specifically, households identify their optimal bid by choosing the bid that maximizes their expected surplus, defined as the differences between their maximum WTP and their actual bid for the land. The expectation is taken based on the probability that their bid is the winning

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[4] We are grateful to Tatiana Filatova for pointing this out to us.

bid of all N bidders against whom they are bidding for any given parcel. Any surplus that is achieved with a winning bid that is less than the household's maximum WTP is assumed to be spent on the composite good, thus generating a higher utility for that household. This leads to the principle difference between our modeling framework and that of the traditional spatial equilibrium model: in the small N case, utility is *not* equalized across all locations and thus it is possible to sequentially order household location choices in time and space, something that is critical for modeling transitional spatial dynamics.

To further clarify how our approach to price formation compares to the traditional spatial equilibrium model and the agent-based approaches discussed above, note that the household's maximum bid is equal to their maximum WTP, which corresponds to the spatial equilibrium bid. Under conditions that correspond to a seller's market (i.e., a large number of competing bidders), households will bid their maximum WTP, resulting in the same equilibrium set of prices as the spatial equilibrium model. However, when the number of competing bidders is sufficient small (the "small N" case), households are not forced to bid their maximum WTP and thus the winning bid will be less than their maximum WTP, resulting in transactions prices that are less than the long run spatial equilibrium prices. Land conversion occurs if the winning household's bid for a particular parcel is equal or greater than the landowner's reservation rent (described in further detail below).Gains from trade are determined by two mechanisms: (1) the household's surplus is the difference between their maximum WTP and their winning bid, which is equal to the transaction price and (2) landowner surplus is the difference between their reservation rent and the transaction price. Thus, household WTP and offer bids, landowners ask bids and transaction prices are all derived from a fully structural model of household utility maximization and landowner expected profit maximization.

Household demand for residential location

Let the income of households that are potential migrants into the exurban area be denoted by Y, which is a random variable drawn from distribution with cumulative distribution function $F(Y; \theta)$, where θ is a vector of parameters.[5] Households derive utility based on the consumption of a numeraire good X and land L and must obtain a utility from migrating into the region that is at least equal their reservation utility U_0. Assuming a monotonic utility function, one can invert $U(X_0, L) = U_0$ to obtain the minimum consumption level X_0. At any location z, the income Y of a potential migrant can be no less than $Y_0(z) = X_0 + Tz$ so that the household has some income remaining to bid for a residential location (or parcel).

A household with income Y will bid for all affordable parcels, i.e. parcels within a radius of z to the CBD for which $Y \geq Y_0(z)$. Each household's bid for a parcel at location z is

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[5] In contrast to Capozza and Helsley (1990), income is deterministic, but is distributed randomly across the population of potential migrants.

determined by maximizing the expected residual income, $Y - Y_0(z) - b(Y; \varphi)$, where φ is a vector of parameters that will be discussed in detail later.

Let us first outline the derivation of the bid function $b(Y; \varphi)$ given the income level Y of a household. We assume a first price sealed auction (Krishna 2002) and determine $b(Y; \varphi)$ by performing the following maximization problem. Denote $w = b(Y; \varphi)$. The household agent would like to increase w to increase the chances of winning. At the same time, he would like to reduce the bid w so as to obtain more residual income $Y - Y_0(z) - b(Y; \varphi)$ that can be used on the numeraire good for greater utility. The probability that the household wins the auction given w is given by the probability that the bids of all the other households are less than w. Thus the household agent faces the following maximization problem in formulating his optimal bid for a given residential location:

$$max_w \quad (Y - Y_0(z) - w)F(Y)^{N-1} b^{-1}(w),$$

where F is the cumulative probability distribution of the random variable Y with $w = b(Y; \varphi)$ or $Y = b^{-1}(w)$. It is reasonable to suppose that $b(Y; \varphi) = 0$ for $Y \leq Y_0(z)$, because there is no income left for the auction.

Denote $Q(Y) = F(Y - Y_0)^{N-1}$ as the distribution of the maximum income in an auction of N participants whose income are i.i.d. The maximization problem leads to

$$b(Y; \varphi) = \frac{1}{Q(Y)} \int_{Y_0}^{Y} du \, u \, \frac{dQ}{du} - Y_0 \left[1 - \frac{Q(Y_0)}{Q(Y)}\right],$$

where $\varphi = (Y_0(z), N)$. Integrating the first term by parts, we have:

$$b(Y; \varphi) = \int_{Y_0}^{Y} du \left[1 - \frac{Q(Y_0)}{Q(Y)}\right].$$

For finite values of N, we are unable to evaluate this analytically except for the uniform distribution. However, the integral for a general distribution function $F(Y; \theta)$ can be evaluated numerically and tabulated.

Note that the household's bid increases with N because a higher number of participants implies more competition. If w is the optimal bid with N participants, then, *ceteris paribus*, an increase in N implies that the likelihood that the bid w wins the auction decreases. The bidder tends to increase the bid so as to increase the probability of winning. However, an increase in the bid decreases the residual income and therefore, the household must balance these two opposing forces in formulating the optimal bid. If we allow the income distribution to evolve over time, the income distribution can be formulated as $F(Y; \theta_t)$ and the bid function becomes time-dependent.

Landowner supply of land for residential use

In period t, the landowner of a parcel of agricultural land at location z will receive a series of bids from households that are potential migrants. The landowner compares the highest bid with his land reservation rent, which represents the landowner's opportunity cost of converting the land in the current period. If the highest bid exceeds his reservation value, he will convert the land and receive the residential rent at this original bid level for all periods in the future.[6]

Now let us consider a problem of optimal land conversion time in which the landowner chooses the optimal land conversion time s to maximize the present value of cash flows. The above-mentioned reservation rent is the rent level that generates the same present value of cash flows as the solution of this optimal land conversion time. If the land is kept in agricultural use, it generates a constant agricultural rent R_a for the current period. Also, assume that the landowner takes the number of participants in the auction N as given.[7]

Denote the current period as $t=0$. If the land is converted in a future period s, the expected land value composes of the agricultural rents before the conversion time s and the expected cash flow generated from the auction that will take place in period s. Therefore, the problem of the optimal conversion time can be formulated as:

$$V(z, N) = max_{\{s=1,2,...T\}} \sum_0^{s-1} \frac{1}{(1+r)^t} R_a + \sum_{t=s}^{+\infty} \frac{1}{(1+r)^t} E_0[b(Y_s; \varphi)],$$

subject to

$Y_{t+1} \sim F(Y; \theta_{t+1})$, and $\theta_{t+1} = \Gamma(\theta_t)$.

Because the bid function $b(Y; \varphi)$ depends on the vector of parameters $\varphi = (Y_0(z), N)$ and the income distribution function F, but does not depend on the land conversion decision of the landowner, the conditional expectation at $t = 0$ of the money payoff from the auction at time s, i.e. $E_0[b(Y_s; \varphi)]$, is equivalent to the *ex ante* expectation of the money payoff of the auction: $E[b(Y_s; \varphi)]$. To calculate this expected money payoff, we need to multiply the bid function $b(Y; \varphi)$ by the probability that this bid will win in an auction of N participants, $Q(Y) = F(Y - Y_0; \theta_s)^{N-1}$, times the probability that this values occurs in the range between Y and $Y+dY$, which is $f(Y; \theta)dY$:

$$E[b(Y_s; \varphi)] = \int_{Y_0}^{+\infty} dY b(Y; \varphi) Q(Y) f(Y; \theta_s).$$

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[6] This version of the model does not allow for relocation and therefore, once a household with a given income occupies the space, the household remains in that location. Future versions of the model will relax this restrictive assumption, so that residential rents of developed parcels may be bid up or down over time as more households enter the region.

[7] The number of participants in the auction (N) may change over time. However, this greatly complicate the problem. More importantly, we do not know how to reasonably model the expectation for the time evolution of N. As a result, we assume that landowner's expected N is held as constant when he calculates his reservation value.

Because $Q(Y) = F(Y - Y_0; \theta_s)^{N-1}$, we can rewrite the expected payoff as

$$E[b(Y_s; \varphi)] = \int_{Y_0}^{+\infty} dY \frac{dF(Y; \theta_s)}{dY} \int_{Y_0}^{+\infty} du \, uQ'(u).$$

Integrating the second term by parts, we have:

$$E[b(Y_s; \varphi)] = \int_0^{+\infty} du \, uQ'(u)[1 - F(Y; \theta_s)] + [1 - F(Y_0; \theta_s)] \int_{Y_0}^{+\infty} dY \frac{dF(Y; \theta_s)}{dY},$$

which can be evaluated numerically and tabulated. The expected value will depend on the number of participants in auction N, the minimum required expenditure $Y_0(z)$, and the income distribution $F(Y; \theta_s)$ at time s. Theoretically s can take any value from the set $\{1, 2, \dots + \infty\}$. However, in reality individuals rarely can anticipate the economic conditions decades into the future. It is not unreasonable to constrain the choice set to be finite, i.e. $s \in \{1, 2, \dots T\}$, so that people only form expectations for a limited number of future periods. This greatly simplifies the optimization problem and makes it possible to tabulate the expected payoff $E_0[b(Y_s; \varphi)]$ for $s \in \{1, 2, \dots T\}$, when transition function $\theta_{t+1} = \Gamma(\theta_t)$ is known. The optimal conversion time s^* is then the one that generates the highest present value. Denoting this highest present value as $V(z, N)$ and the residential rent that will generate equivalent cash flow as R_b, we have

$$\sum_0^{+\infty} \frac{1}{(1+r)^t} R_b = V(z, N).$$

The reservation rent is then the maximum of this residential rent R_b and the agricultural rent R_a:

$$R^* = max(R_a, R_b).$$

If the landowner receives a bid that is higher than the reservation rent in the current period, then he will convert the land in that period. Otherwise, he will keep the land in the agricultural use and reconsider the land conversion decision in the next period.

Household-landowner market interactions

We structure the household bidding, landowner expectations formation and exchange processes as an auction. At the beginning of each period, the income distribution $Y_t \sim F(Y; \theta_t)$ and transition function $\theta_{t+1} = \Gamma(\theta_t)$ are known for all future periods up to time $t + T$. Every potential migrant will first determine the set of affordable parcels, i.e. all parcels at location z that satisfies $Y \geq Y_0(z)$. For each parcel, the number of bidders in the current period for the parcel located a distance z, $N(z)$, is the number of potential migrants that can afford the parcel. Because transportation cost increases with distance to the urban center, the number of people that can afford the parcel is expected to decrease with distance z.

Once the number of bidders for each parcel is determined, $N(z)$ becomes public knowledge. Each migrant then formulates bids for each parcel that she can afford and submits

these bids to an auctioneer. The landowner formulates his reservation rent R* for the current period and submits this to the auctioneer as well. The auctioneer first picks out the highest bid for each parcel. If one migrant wins multiple parcels, the parcel that gives the migrant the highest residual, i.e. $Y - Y_0(z) - b(Y; \varphi)$, will be assigned to the migrant. For the rest of the parcels, the second highest offer will be picked out. If there is more than one migrant who wins multiple parcels, the auctioneer will repeat the process until no one wins multiple parcels. Then the auctioneer compares the winning bid with the reservation rent R^* of the landowner. If the winning bid exceeds the reservation rent, the land is developed and occupied by the household with the winning bid. Residential land rents, R_u, are equal to this winning bid. If the winning bid is less than the reservation rent, then the land remains in agriculture for that period and the household does not move into the region in that period.

Model calibration

The parameters of the model are derived using data on residential subdivision development from Carroll County, Maryland and auxiliary data from secondary sources. NetLogo, agent-based modeling software, is used to implement the model and simulate predicted patterns of residential land rents, household income and land development. Here we describe the specification and implementation of the model in NetLogo.

A two-dimensional grid that represents the exurban region into which households migrate is defined within NetLogo. Land parcels are assumed to be equally sized and are represented by the cells of the grid. We assume that development of a rural land parcel occurs as a residential subdivision and calibrate the size of a cell according to the mean subdivision size in Carroll County, which is 36.5 acres. This implies that each parcel has a side with length approximately equal 0.24 mile. Space is defined solely in terms of distance from an urban center. Distance is calculated as the Manhattan distance from the lower left corner of the two-dimensional grid, which represents the closest location within the exurban region to the urban center.

The demand-side parameters of the model include unit travel costs T, reservation consumption level X_0, and the income distribution $F(Y; \theta)$. We assume that fuel costs are an insignificant part of travel cost and focus on the time cost of commuting. Specifically, we assume the time cost of one hour travel time corresponds to 1/8 of the median U.S. household income in 1990, which was around $30,000.[8] If people drive at an average speed of 30 miles per hour, this time cost translates into $30 to traverse the length of a rural land parcel as represented by a single grid cell in the model. In terms of thousands of dollars, we set the per unit travel cost $T = 0.03$. The reservation consumption level X_0 is specified as $15,000, which is based on census data and consumer expenditure report. Specifically, according to the consumer expenditure report, around 50% of household income in the mid 1990s was allocated for

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[8] http://www.census.gov/hhes/www/income/data/historical/metro/msa1.html

transportation and housing and the remaining 50% on all other consumption. The income distribution function $F(Y; \theta)$ is assumed to be a lognormal distribution. The vector θ contains the two parameters of the lognormal distribution, which are determined using maximum likelihood estimation using 1990 household counts by income category from the 1990 U.S. Census Bureau.

The supply-side parameters include agricultural rent R_a, the interest rate r and the transition function $\theta_{t+1} = \Gamma(\theta_t)$. We use USDA data[9] on cropland cash rents to calibrate R_a. Average cropland cash rents in 2000 were \$70 per acre. Multiplying this by the parcel size of 36.5 acres, agricultural rent per parcel R_a is roughly 2.6 thousand dollars. The interest rate is set to be 5%, which is the average annual interest rate for one-year treasury bill (secondary market) between 1990-1999 based on the Federal Reserve data.[10] To calibrate the transition function $\theta_{t+1} = \Gamma(\theta_t)$, we assume $\Gamma(\theta_t)$ takes a simple linear form: $\theta_{t+1} = \theta_t + d\theta$. We then estimate the θ for the 2000 U.S. Census income data and set $d\theta = (\theta_{2000} - \theta_{1990})/10$.

4. Model results

To assess the influence of expected competition levels and heterogeneous income on development patterns and the land rent gradient, we first consider a simplified version of the model in which household incomes are all constrained to the same mean level. Given homogeneous income for all potential migrants, the household's expected level of competition is then varied by varying N, the number of potential migrants in a given period. As discussed above, the lower the expected level of competition, the lower the optimal bid will be by households who will attempt to increase their income residual by bidding lower when competition is less. It is unclear how this will influence the development pattern and land rents and whether variation in N will have any influence on the development pattern when household incomes are homogeneous. Figures 1 and 2 illustrate the cases for N=10 and N=20 respectively. In both cases, we find that the development pattern evolves in a fashion identical to a growing monocentric city in which development is contiguous and concentrated around the urban center and the outer boundary pushes outward over time. However, in contrast to a spatial equilibrium model, in which households instantaneously fill up all locations within the city boundary and the city boundary pushes out over time due to growth, the mechanism underlying the pattern here is quite different. In this case, households bid less than their maximum value or willingness-to-pay, defined as $Y - Y_0(z)$, and retain some residual income that increases their utility. This residual is greater the less the transportation costs are and therefore, there is always an incentive for the household to locate as close to the urban center as possible. In theory, this is true regardless of the value of N: so long as N is finite and does not change over space, the household

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[9] http://usda.mannlib.cornell.edu/usda/nass/AgriLandVa//2000s/2000/AgriLandVa-07-07-2000_Cash_Rents.pdf

[10] http://www.federalreserve.gov/releases/h15/data.htm

will retain some residual income, although this income becomes increasingly smaller as N increases.

Although we do not illustrate this case here, it is worth considering the model prediction assuming an infinite N. In this case, households will bid their maximum value with no income remaining, i.e., $b(Y; \varphi) = Y - Y_0(z)$, and the land market will be in a spatial equilibrium. Households will be indifferent to any location that is affordable, i.e. any parcel within radius of z to the CBD for which $Y \geq Y_0(z)$. Landowners will convert their land parcel if the household's bid is greater than their reservation rent, i.e., if $Y - Y_0(z) > R^*$. Assuming that income is sufficient high so that this holds for some parcels, then the distance z* at which $R^* = Y - Y_0(z)$ determines the outer boundary of development. For $z \leq z^*$, household bids are greater than reservation rents and therefore any bid to a landowner located within this boundary will be accepted and development will occur. This is identical to the standard monocentric city result and, in the long run, as predicted by this model, all parcels within this boundary will be developed. However, in the short run, if the number of migrants is less than the number of parcels within this boundary, then development that occurs within this boundary will be randomly distributed. This results from the spatial equilibrium condition: because households bid their full maximum value and bids exactly offset transportation costs, households are indifferent to all location for which $z \leq z^*$ and thus there is no spatial ordering to development for parcels located within this region. Over time, as new migrants arrive, the area will fill up with development until the long run contiguous development pattern is reached.

Figure 3 illustrates the residential rent gradient for each of the homogeneous cases illustrated in Figures 1-2. In both cases, we see that the residential rents of developed parcels decline from the city center such that transportation costs are exactly offset. At the boundary of the developed area, the rent gradient drops to zero. However, this graph does not illustrate land rents on the remaining portion of undeveloped land, i.e., the landowner's reservation rents. Land rents at this point do not fall to zero since the landowner's reservation rents are greater than zero. Recall that the reservation rent for a given location z is equal to either the expected rents associated with future returns from development R_b, which decline with z, or agricultural rents R_a, which are constant across z. For small z, $R^* = R_b \geq R_a$ and reservation rents will decline with z. At some sufficiently large value of z, $R^* = R_b = R_a$ and for locations beyond this point, $R^* = R_a$ and land rents are constant across space. Therefore, if we plotted the land rents for undeveloped parcels with the residential rents of developed parcels, we should find a negative and continuously declining rent gradient that is analogous to the monocentric model.[11]

Next, we allow income to be heterogeneous by specifying the income distribution over households as described earlier. Figures 4 and 5 illustrate representative model output from two cases that again differ by the expected level of competition. Potential migrants are drawn

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[11] We plan to show this in future versions of the paper.

randomly from the income distribution $F(Y; \theta)$. In some cases, we may draw households for which $Y < Y_0(z)$ for all z, in which case these households will be unable to move into the region and therefore are dropped as potential migrants. For this reason, we cannot directly control N, the number of potential migrants, but we can control this indirectly by varying the number of draws from the population that are allowed in a given period. Therefore, our limited competition case (Figure 4) corresponds to lower number of draws in each time period and the greater competition case (Figure 5) to a higher number of draws. While there are differences in degree, we find that in both cases, patterns of leapfrog development emerge. In the case of limited competition (Figure 4), leapfrog development occurs in the first and second periods, followed by development in the locations closest to the urban center. In the case with greater competition (Figure 5), development initially concentrates around the urban center in the first period, followed by subsequent development that skips to outer areas and leaves intervening land that is closer to the city undeveloped. In both cases, we observe multiple rounds of leapfrog development accompanied by further development that is contiguous to the urban center. In the case of greater expected competition (Figure 5), the geographic extent of leapfrogging is much greater.

The mechanism that generates this leapfrogging arises from the spatial incentives that occur in the heterogeneous income case. Differences in income imply that the set of affordable parcels, i.e., the set of parcels for which $Y \geq Y_0(z)$, will vary across households. Lower income households will have a lower value of z for which $Y = Y_0(z)$ and therefore have a smaller radius that defines the set of locations for which they are able to submit bids. On the other hand, higher income households will have a larger geographical extent that defines their set of affordable locations, which gives them a critical advantage in bidding. Specifically, N is lower for locations that are outside the affordable set of lower income households, which permits the winning bid to be lower (and residual income to be higher) than it would be if N were constant across space. Therefore, higher income households have an incentive to leapfrog to the nearest location at which N is lower. Of course, the incentive to locate close to the city remains and therefore, some higher income households will still locate near the city while others will leapfrog to the nearest location with lower N.

Figure 6 shows the rent and income gradients for these two heterogeneous income cases. We see that in both cases, residential rents are initially declining from the urban center, reflecting the same spatial ordering and competition for land near the urban center as in the homogeneous case. In the heterogeneous case, however, we also find that the initial income gradient is also negative and smoothly declining for locations that are near the urban center. This reflects the higher income households' ability to outbid lower income households for the closest locations. These residential rent and income gradients decline to zero, as a result of the lack of bidding at locations that are somewhat further away. The subsequent spikes in these gradients with increasing distance reflect the leapfrog pattern of development. In comparing the level of income at these leapfrog locations to the income gradient close to the city, we see that income at the

leapfrog locations is lower than the income at the center, but higher than the income along much of the continuous portion of the gradient at locations slightly further from the center. Interestingly, the residential rents associated with the leapfrog locations is lower than most of the rents along the continuous portion of the residential rent gradient. These results support the provided intuition above, that leapfrog development is occurring as a result of additional utility gains that higher income households are able to achieve through the more limited competition for these locations.

Lastly, we hold the level of expected competition constant and reduce the variance of income to examine whether the model's predictions vary with the degree of income heterogeneity. To examine this, we fix mean income at the same level for both cases and vary the variance of income. Figures 7 shows the case for which the income mean and variance are estimated from the 1990 data and Figure 8 shows the case for which the variance is reduced to half this value. We allow the potential number of migrants to be large, which implies that the leapfrog pattern will exhibit a large geographical extent. To account for this, we redefine the region to be long and narrow with the parcel closest to the urban center located at the extreme left side of the region. In both cases, we find that the number of bidders declines with distance, creating an incentive for higher income households to locate farther away. This effect is extreme in the higher heterogeneity case (Figure 7), in which we find that no development occurs at the urban center in the initial period and that all develop occurs at the outer edges of the region. In comparison, when income heterogeneity is lower (Figure 8), we find that the number of bidders declines more rapidly with distance and that development occurs throughout the region, but is somewhat concentrated around the urban center and at the outer boundary. In this case, households of different income levels have an incentive to locate either at the center or in a location that has the lowest N that they can afford. This will vary for households with different income levels and therefore development occurs at different locations throughout the region. Because N is lowest at the outer boundary, some of the richest households will concentrate there. These relationships are evident in the residential rent and income gradients associated with the developed parcels. The income associated with locations that are developed increases with distance.

5. Conclusions

A spatial model of an exurban land market was developed to examine the implications of key features of exurban land markets for land rents and land use pattern outcomes in exurban regions. By starting from a structural model of household optimal location choice and optimal landowner development, we find that spatial equilibrium in exurban markets is very different from the urban economic model. Namely, we allow households to identify their optimal bid by choosing the bid that maximizes their expected surplus associated with a given location. Any surplus that is achieved with a winning bid that is less than the household's maximum WTP is assumed to be spent on the composite good, thus generating a higher utility for that household.

This leads to the principle difference between our modeling framework and that of the traditional spatial equilibrium model: when the number of households is limited, utility is *not* equalized across all locations and thus it is possible to sequentially order household location choices in time and space, something that is critical for modeling the sequence of locational choices and land use changes over time.

The resulting urban development patterns and their evolution over time are found to depend crucially on the degree of heterogeneity among households. Here we consider only heterogeneity of income. We find that, when income is homogeneous across households, the household bidding mechanism generates a spatial ordering of land development over time that are similar to the patterns of a dynamic monocentric model, but that arise in the short run as a result of (more or less) competitive bidding that occurs in the absence of a spatial equilibrium. Variation in utility across space that results from this bidding process gives rise to a spatial ordering of development over time. While changes in the expected level of competition among bidders alters the level of utility, it does not change the spatial ordering of development. In contrast, a model in which an instantaneous spatial equilibrium is imposed generates random development across the set of parcels that are affordable for households and whose reservation rent is below the winning household bid. This set of parcels, defined by $z \leq z^*$, will be developed randomly in the short run as households move into the region. Eventually, this area will be fully developed, the marginal household will no longer have an incentive to move into the region, and the long run development pattern will correspond to a monocentric pattern.

Predictions of leapfrog development emerge when we allow income to be heterogeneous among a finite set of households that bid for location. Given income heterogeneity, leapfrog development emerges as a result of spatial constraints that give an advantage to higher income households that are able to increase their utility by bidding on locations that have more limited competition. In this simple model, limited competition arises from transport costs that constrain lower income households. In a more realistic model, spatial variation in competition for locations may arise from other spatial constraints or heterogeneity in preferences. For example, households whose time costs are lower or that have stronger preferences for rural areas may be able to take advantage of lower competition for parcel farther away from the urban area and as a result, will leapfrog to these areas. In this case, or in other cases that we could hypothesize, the factors that lead to spatial variation in competition may be different, but the economic response is the same: locations that have an advantage over others due to short run constraints that limit competition for these locations, then this creates a spatial ordering of the market in the short run to the timing of development across space. Thus it is feasible to model short run dynamics, something that is not possible when instantaneous adjustment to a spatial equilibrium is assumed.

The model in its current form is highly stylized and useful only to explore the predictions of short run dynamics in land rents and development patterns relative to the traditional monocentric model that is unable to consider short run dynamics. In this context, the model

identifies an alternative explanation for leapfrog development. Rather than being caused by differences in land use returns in a growing monocentric city model (Ohls and Pines 1976; Mills 1981) or from the repulsion of neighboring development (Irwin and Bockstael 2002) or the attraction of open space (e.g., Turner 2005; Wu 2006; Wu and Plantinga 2003), leapfrog arises due to spatial variation in competition that allows households to obtain a higher utility by bidding for parcels located farther away from the urban center. This is a hypothesis that, with the proper controls for other influences on location and land use patterns, should be empirically testable.

Finally, we argue that if agent-based economic models such as the one developed here can be further developed to incorporate more realistic features of the landscape and of household and landowner decision making, they would be useful for policy scenarios in which the short run market and land use dynamics are predicted under baseline and alternative policies. This relies, however, on improved parameterization of these models using data on household location decisions and landowner conversion decisions to empirically specify the model. Here we see promise in adopting similar estimation techniques to those used in empirical structural models. For example, current work in urban and regional economics uses simulation methods to predict certain moments of the data, e.g., residential density associated with a given distance from the urban center, and then either generalized or simulated method of moments to estimate the model parameters by matching the predicted moments to observed moments from the data (e.g., Brinkman 2010; Paciorek 2010). Similarly, an agent-based model could be simulated to generate certain moment conditions in a dynamic context, e.g., the rate of change in land rents or development density at a given location, and the parameters of the model estimated by matching these predicted moments to those observed in the data.

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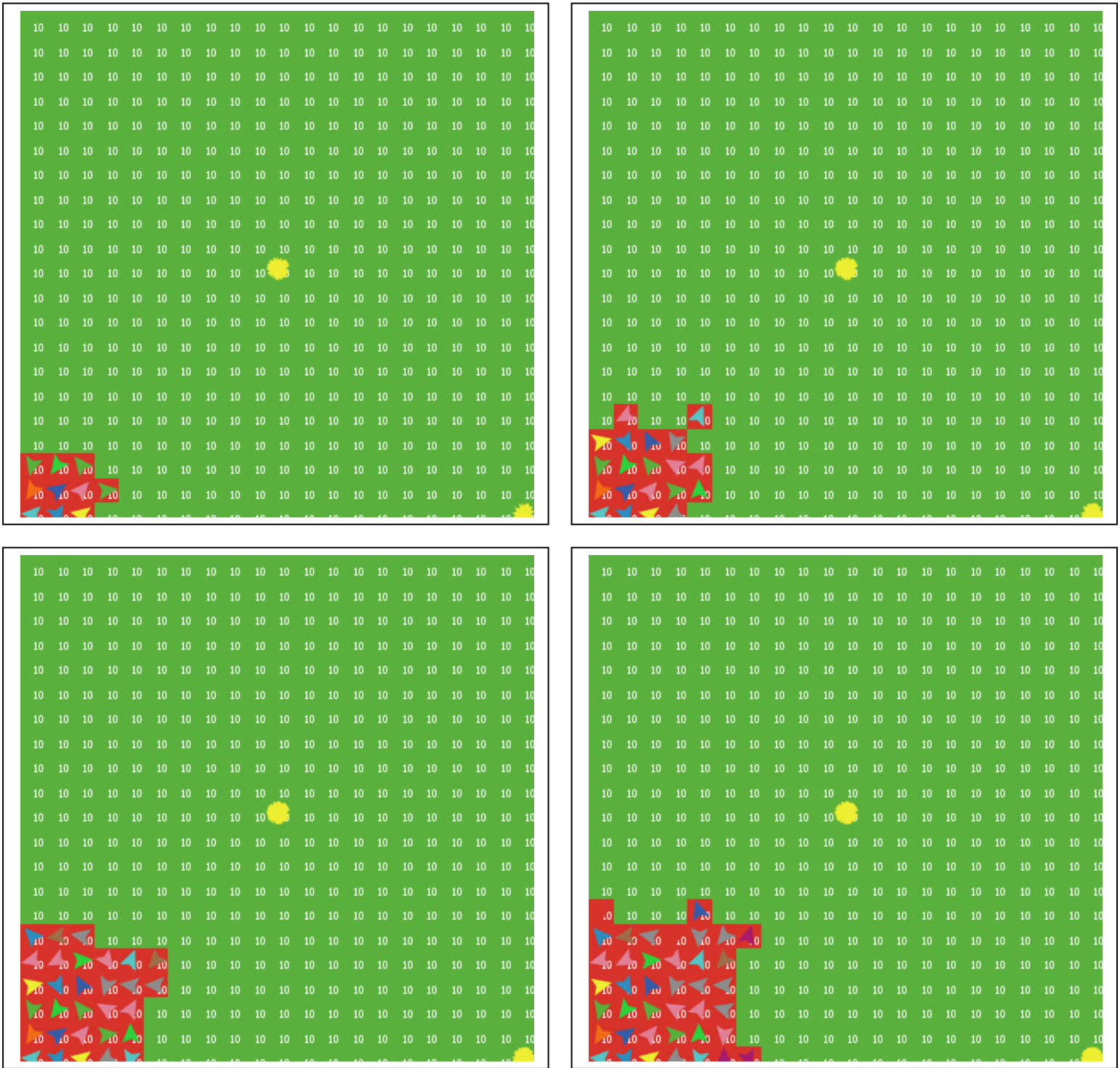
Figure 1: Homogeneous income with limited expected competition (N = 10).
Model evolution over time illustrated from upper left to lower right panels (time periods 1-4 respectively). Distance from urban center increases from lower left corner.

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location
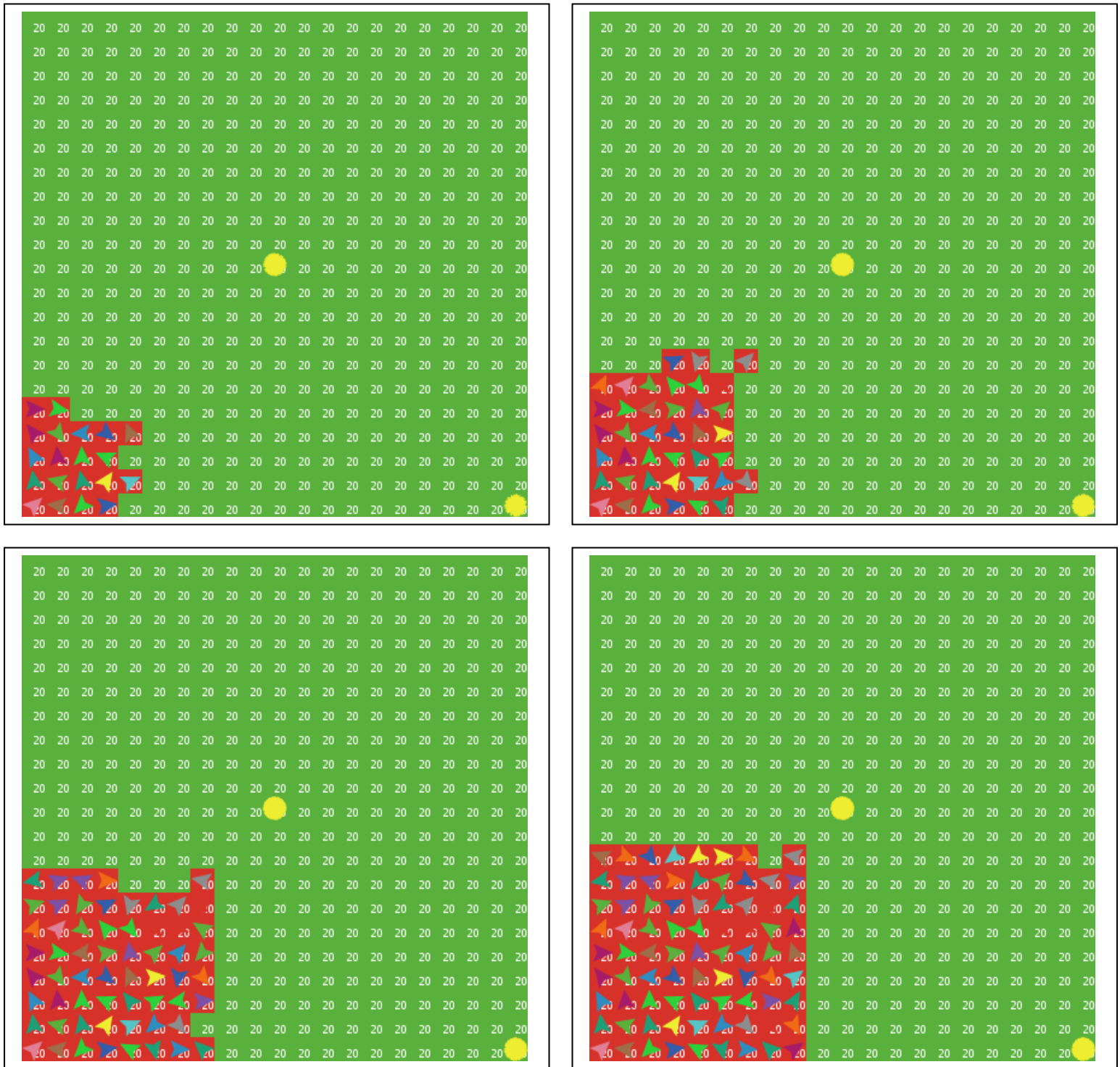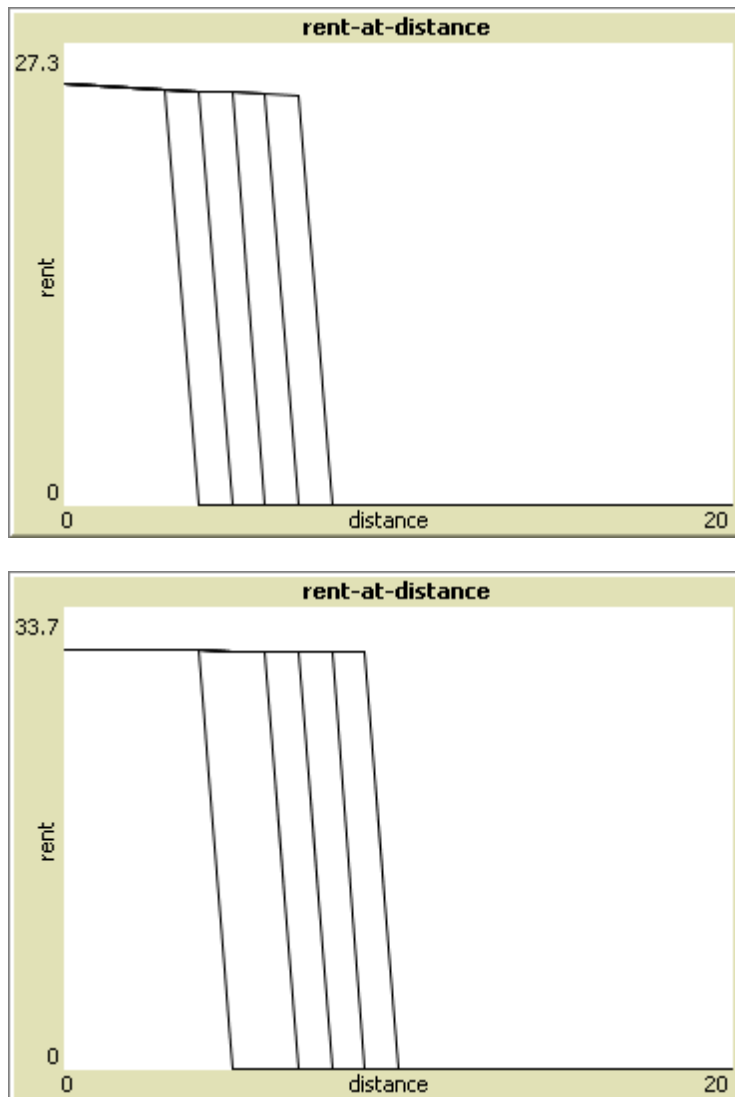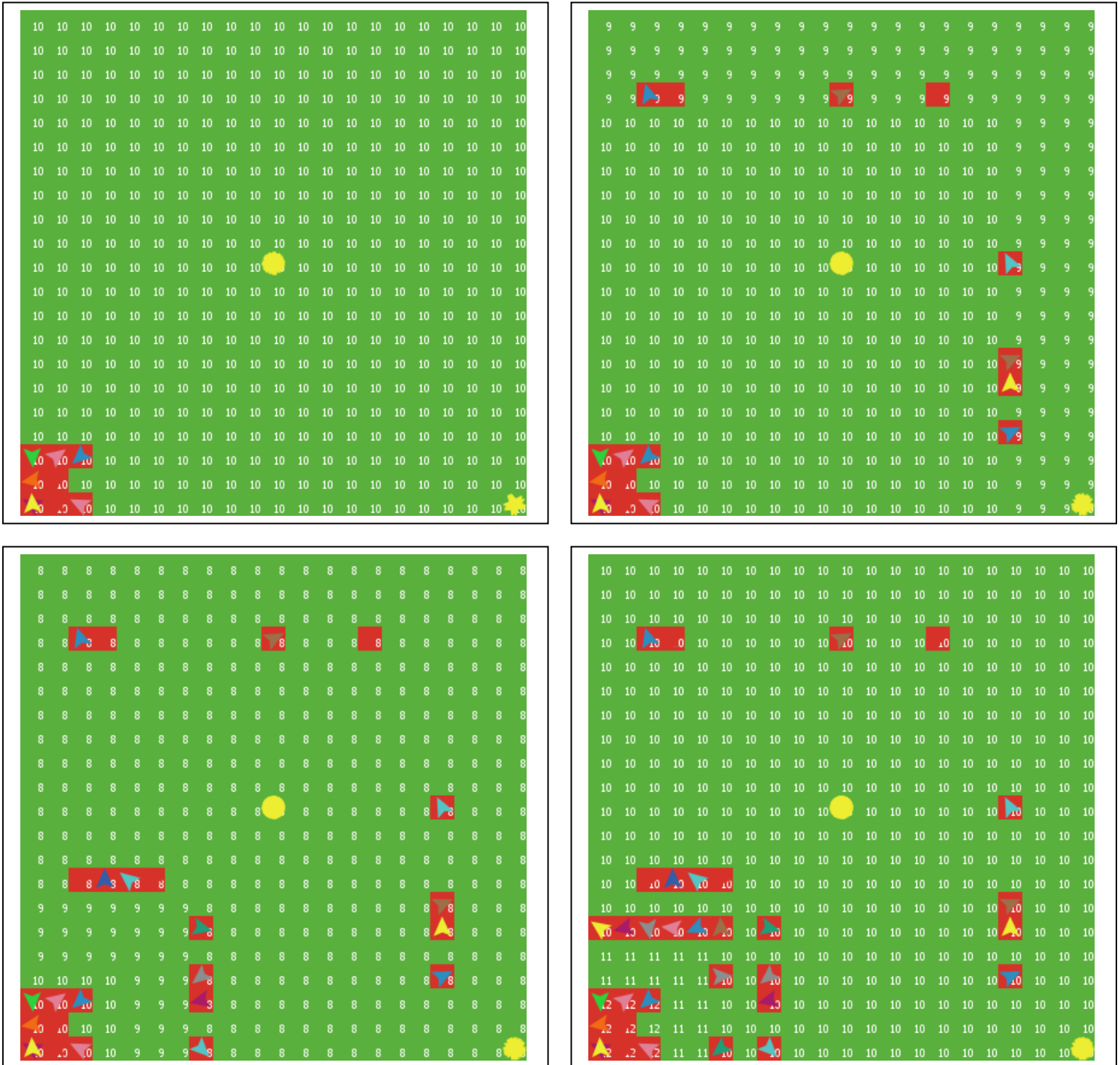
Figure 2: Homogeneous income with greater expected competition (N = 20)
Model evolution over time illustrated from upper left to lower right panels (time periods 1-4 respectively). Distance from urban center increases from lower left corner.

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location
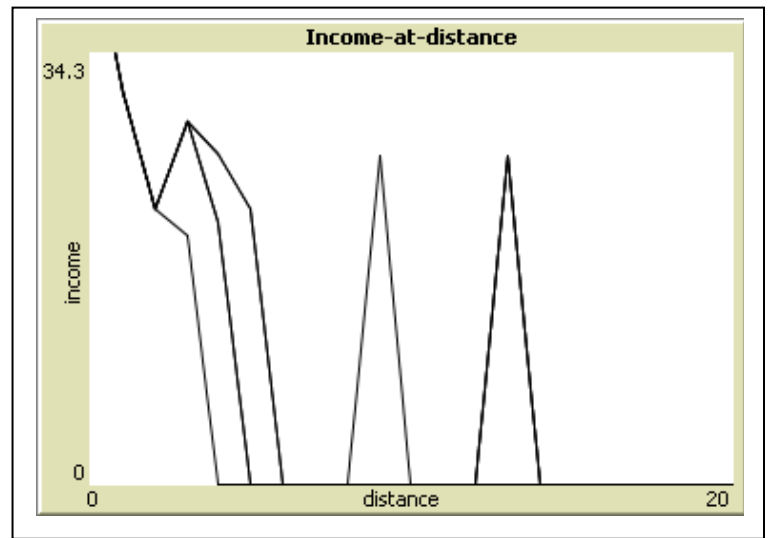
Figure 3: Residential land rent gradient for homogeneous income cases. Top panel: limited competition (N = 10). Bottom panel: greater competition (N = 20)

Figure 4: Heterogeneous income with limited expected competition

Model evolution over time illustrated from upper left to lower right panels (time periods 1,-4 respectively). Distance from urban center increases from lower left corner.

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location
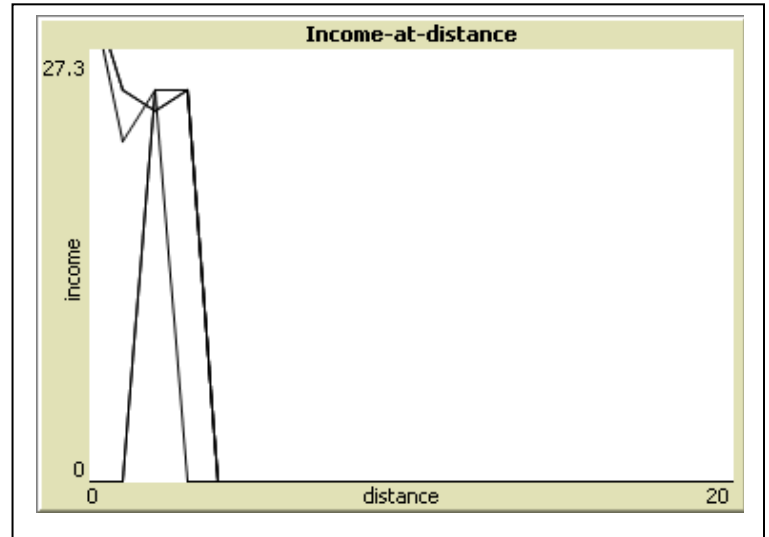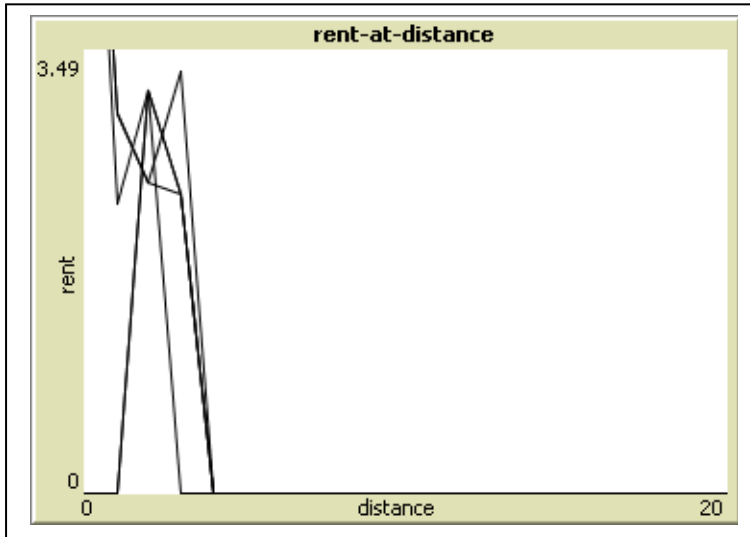
Figure 5: Heterogeneous income with greater expected competition

Model evolution over time illustrated from upper left to lower right panels (time periods 1-4respectively). Distance from urban center increases from lower left corner.

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location

Figure 6: Residential land rent and income gradients for heterogeneous income cases. Top left panel: Residential rent gradient with limited competition. Bottom left panel: Residential rent gradient with greater competition. Top right panel: Income gradient with limited competition. Bottom right panel: Income gradient with greater competition.
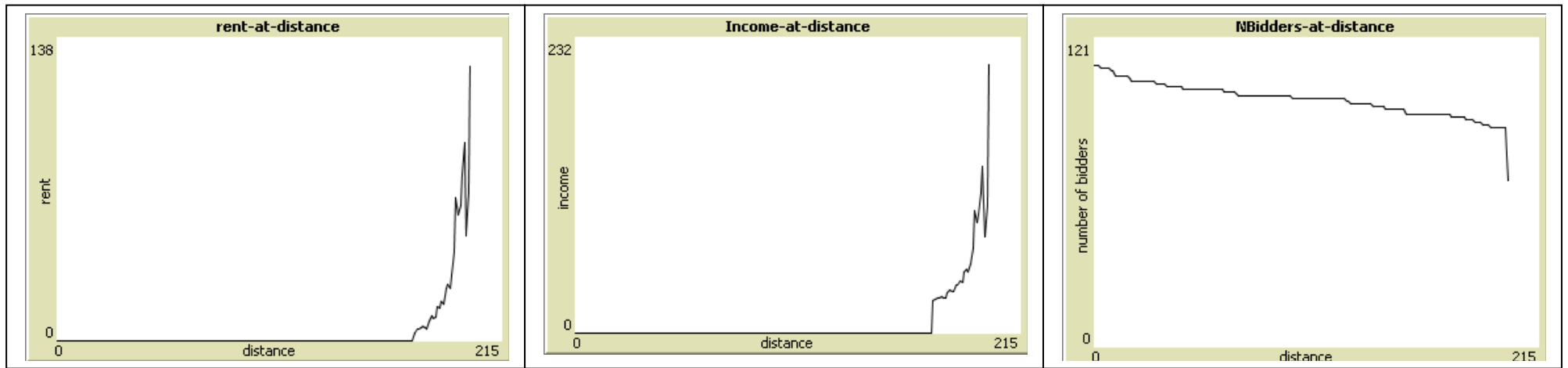
Figure 7: Greater Income heterogeneity with competitive bidding
Left panel: residential rent gradient associated with developed parcels; middle panel: income gradient associated with developed parcels; Right panel: number of bidders as a function of distance

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location
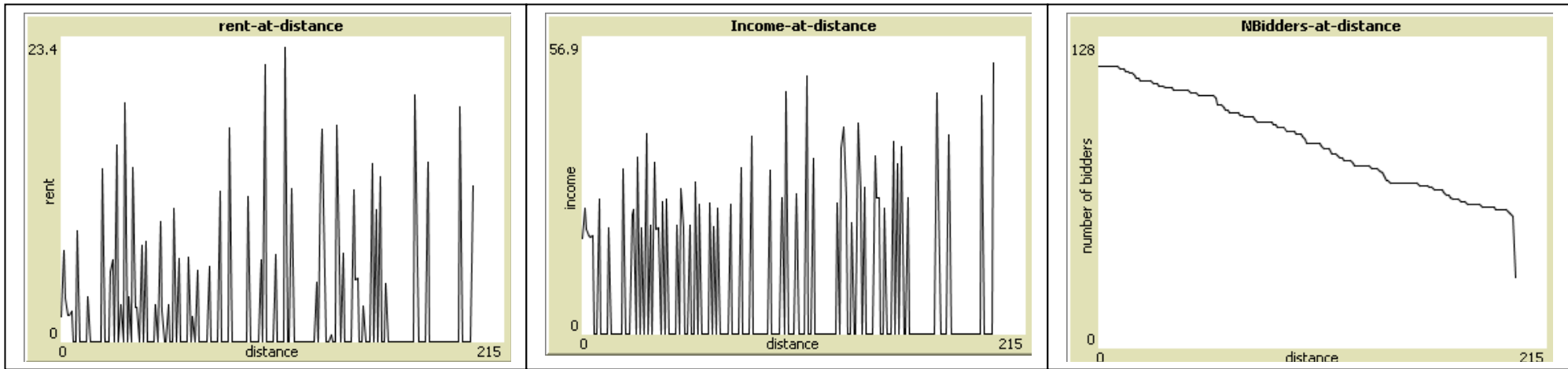
Figure 8: Lower Income heterogeneity with competitive bidding

Left panel: residential rent gradient associated with developed parcels; middle panel: income gradient associated with developed parcels; Right panel: number of bidders as a function of distance

Legend: Green = undeveloped; red = developed; the number in each cell indicates the number of actual bidders for that location