Accounting for agronomic rotations in crop production: A theoretical investigation and an empirical modeling framework

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Abstract

As far as crop acreage choices are concerned, a consensus seems to exist among agricultural scientists and extension agents: crop rotation effects and the related constraints are major determinants of farmers' crop choices. Crop rotation effects are inherently dynamic. They are generally ignored in multicrop models with land as an allocable input found in the literature since most of these models are developed within a static framework.
The aim of this paper is twofold (*i*) to propose a new approach and tools for investigating dynamic crop acreage choices accounting for crop rotation benefits and constraints and (*ii*) to illustrate the impacts of crop rotation effects and constraints on farmers' acreage choices through simulation examples. The models proposed in this paper are sufficiently simple for being empirically tractable either in simulation studies or in econometric and mathematical programming analyses.
Our simulation results tend to show responses of the optimal dynamic acreages to simple price shocks which are much more complex than those implied by static models. They also demonstrate that farmers' perceptions of the future economic context are crucial determinants of their acreage choices. In fact current acreage choices may appear suboptimal in a static sense but are fully consistent when dynamic effects of crop rotations are specified.

1. Introduction

In their critical review of the literature on the theory and measurement related to farmers' choices, Just and Pope (2001) forcefully and convincingly argue that "potentially large gains may come from understanding more of the structure that underlies the production technology for investigating and modeling farmers' choices". In particular these authors ask: "What elements of technology should economists consider essential?" (Just and Pope, 2001, p.722). As far as crop acreage choices are concerned, a consensus seems to exist among agricultural scientists and extension agents: crop rotation effects and the related constraints are major determinants of farmers' crop choices.

Crop rotation effects are inherently dynamic. They are generally ignored in multicrop models with land as an allocable input found in the literature since most of these models are developed within a static framework. The models developed by Ozarem and Miranovski (1994) (see also the references therein), Thomas (2003) or Livingston *et alii* (2008) are exceptions in this respect. But these analyses focus on the effects of specific rotation sequences and/or on stock management issues. Oude Lansink and Stefanou (2001) consider dynamic acreage choice models but impose exogenous acreage adjustment costs and ignore the effects of crop rotation. In fact crop rotation effects are generally not explicitly considered in multicrop models but they are often used for arguing assumptions underlying these models, either in the multicrop econometric literature (ME) (see, *e.g.*, Chambers and Just, 1989) or in the (positive) mathematical programming (P)MP (see, *e.g.*, Howitt, 1995). These assumptions motivate crop diversification (see, *e.g.*, Heckeleï and Wolff (2003) and the references therein). Crop rotation effects partly underlie the crop acreage bounds used in MP models, the implicit cost function used in the PMP literature or the standard decreasing marginal return to crop acreage (DMR) assumption used in both the PMP and ME literatures.

The aim of this paper is twofold (*i*) to propose a new approach and tools for investigating dynamic crop acreage choices accounting for crop rotation benefits and constraints (sections 2 and 3) and (*ii*) to illustrate the impacts of crop rotation effects and constraints on farmers' acreage choices through simulation examples (sections 4 and 5). Our dynamic approach build on the agronomic result that present technological possibilities are influenced by past crop productions. In fact our approach generalizes available frameworks and offers several advantages. First we show that farmers' perceptions of the future economic context are crucial determinants of their acreage choices, even in the short run due to the production dynamics implied by the crop rotation effects and constraints. Second the dynamic responses to anticipated and/or non anticipated shocks to economic incentives are quite complex. For example, we can observe first positive and then negative (own and cross) price effects because farmers optimally manage their land allocation in a dynamic way. These evolving responses to economic incentives certainly partly explain the negative viewpoint expressed by Just and Pope regarding our inability to correctly explain farmer behaviors. Third our analysis also shows that to explicit account for crop rotations does not necessarily provide arguments for DMR assumption on all crops. This assumption is widely used in our profession but not validated by agronomic science. Fourth the implementation of our approach raises new challenges but we prove that it can be applied with data usually available to economists.

More precisely, accounting for crop rotations in acreage choice models is especially challenging, for at least three main reasons. (*i*) Crop rotation effects in the crop production technologies must be represented in addition to the other determinants of acreage choices (such as the management of labor and capital at peak-times). (*ii*) A dynamic programming framework must be employed and, finally, (*iii*) dynamic crop acreage choices involve inter-temporal trade-offs implying highly constrained choices with potentially many corner solutions due to the discrete feature of the crop rotation choices. These corner solutions imply the examination of numerous qualification constraints and generate severe discontinuities in the solution functions. The combination of these challenges may explain the relative scarcity of studies on acreage choices involving crop rotation effects and constraints. Our modeling framework is specifically developed for coping with them. The models proposed in this paper are sufficiently simple for being empirically tractable either in simulation studies or in econometric and (P)MP analyses.

2. Modeling framework

2.1. Assumptions

The proposed modeling framework builds on four main assumptions related to the multicrop technology. First, we assume production dynamics of order 1 even if the proposed framework can be extended to dynamics of higher order. This first order dynamic already shows interesting results. The previous crop of a crop grown on a given plot can generate three types of effects: (*i*) it partially determines the level of pest and disease, as well as weed, infestation levels, (*ii*) it partially determines the nutrient levels available at the beginning of the cropping process and (*iii*) it partially determines the soil structure of the plot and, as a consequence, its properties with respect to the development of the plant root system, with respect to the soil flora and fauna or with respect to water drainage or holding capacity. These effects can directly affect the yield levels as well as the input use levels.

Second, it is assumed that the expected return of crop *k* ($k \in \mathcal{K}$ with $\mathcal{K} \equiv \{1,...,K\}$) on plots with previous crop *m* is a function of input uses for crop *k*, of the input and crop *k* prices, and of the form of the production technology specific to the (m,k) sequence. More specifically, the random variable $\tilde{\pi}_{mk}$ represents the crop *k* profit per unit of land at the end of the crop sequence (m,k). It is to be interpreted a short-run indirect profit function in a risky context, *i.e.* $\tilde{\pi}_{mk} \equiv \pi_{mk}(\tilde{v}_k, \mathbf{w}_k, \tilde{\boldsymbol{\varepsilon}}_{mk})$ where $\tilde{v}_k$ is crop *k* (random) price, $\mathbf{w}_k$ is the (non random) price vector of crop *k* variable inputs and $\tilde{\boldsymbol{\varepsilon}}_{mk}$ is a vector of random events potentially affecting the production of crop *k* on previous crop *m*. Farmer's are assumed to be risk neutral.[1] Crop rotation effects are economically measured by the differences $\tilde{\pi}_{mk} - \tilde{\pi}_{nk}$. It is

[1] This assumption does not simplify the analysis of the acreage choice dynamic aspects, *i.e.* the impacts of accounting for crop rotations. But it greatly simplifies the specification of the empirical modelling framework.

generally possible to order the different elements of $\tilde{\boldsymbol{\pi}}_k \equiv (\tilde{\pi}_{mk}, m \in \mathcal{K})$, at least according to their means. The present framework ignores the quantitative role of previous input uses and yield levels, *e.g.* it ignores the variations in nutrient stocks due to fertilizer uses and crop intake. It focuses on "average" nutrient stocks. Hennessy (2007) considers similar approximations in his analysis of the structure of crop rotations at the plot level. Crop rotation constraint for previous crop m states that $\boldsymbol{\iota}'\mathbf{s}_{(m)} \leq p_m$ where $\mathbf{s}_{(m)} \equiv (s_{mk}, k \in \mathcal{K})$ with s_{mk} being the current acreage share of the crop sequence (m,k), $\boldsymbol{\iota}$ denotes the dimension K unitary column vector and p_m denotes the previous acreage share of crop m. The differences in the elements of the $\tilde{\boldsymbol{\pi}} \equiv (\tilde{\boldsymbol{\pi}}_k, k \in \mathcal{K})$ vector induce trade-offs for choosing the plots on which to plant the different crops given the previous crops on these plots and taking into account that the current crop choices will be next year previous crops.

Third, it is considered that the limited quantities of quasi-fixed factors generate implicit costs depending on the current acreage choices but not on current variable input uses. In this respect, the quasi-fixed factor management costs are a motive for crop diversification as the increase in the acreage of a given crop generate costly peak loads for labor and machinery. This assumption is consistent with the observations of agricultural scientists and of extension agents. Farmers are more reluctant to change their cropping practices than to change their acreage (within the ranges implied by their quasi-fixed factor endowment) in the medium run. In other words, farmers manage the allocation of their quasi-fixed factor services by adapting their acreage in order to not constrain their variable input uses at the plot level.[2] This stylized fact also motivates our using of an implicit management cost function denoted by $C(\mathbf{a})$ where $\mathbf{a} \equiv (a_k, k \in \mathcal{K})$ with $a_k \equiv \boldsymbol{\iota}'\mathbf{s}_k$ and $\mathbf{s}_k \equiv (s_{mk}, m \in \mathcal{K})$. This implicit cost function can be interpreted as a smooth reduced form approximation of the implicit costs generated by the limited

[2] Cropping practice "standards" are designed by agricultural scientists, extension agents and farmers as coherent sets of input use rules of thumb which are (approximately) economically optimal within identified price ranges. A drastic modification of, say, a given input use level may destroy the coherency of the cropping practice and may have a significant negative impact on yields.

quantities of quasi-fixed factors. It is related to the one used in the PMP framework but does not include crop rotation effects and constraints as this seems to be implicit in the PMP framework.[3]

Our fourth assumption is that the (m,k) production technology of any crop k with any previous crop m exhibits constant return to acreage. This assumption rules out scale (as well as scope) effects.

According to this assumption set farmers' acreage choices do not affect their variable input choices at the plot level. This implies that the crop variable input level choices and the acreage choices can be separately analyzed. This paper focuses on the later choices.

2.2. General modeling framework

According to this assumption set, we will consider optimization problems involving profit functions of the form:

$$(1)\qquad \Pi(\mathbf{s};\boldsymbol{\pi}) \equiv \mathbf{s}'\boldsymbol{\pi} - C(\mathbf{a}) = \mathbf{s}'\boldsymbol{\pi} - C(\mathbf{As})$$

where $\mathbf{s} \equiv (\mathbf{s}_k, k \in \mathcal{K})$. In what follows we will use the matrices $\mathbf{A} \equiv \mathbf{I} \otimes \boldsymbol{\iota}'$ and $\mathbf{D} \equiv \boldsymbol{\iota}' \otimes \mathbf{I}$ where $\mathbf{I}$ is the dimension K identity matrix in order to define the vector of the total acreage of the different crops "supplies" induced by $\mathbf{s}$ as $\mathbf{a} = \mathbf{As}$ and the vector of the total acreage of the previous crop "demands" induced by $\mathbf{s}$ as $\mathbf{p} \equiv \mathbf{Ds}$. The implicit cost function is a function of $\mathbf{a}$ because it is assumed that the acreage of crop k generates similar management costs whatever the previous crop is for this acreage. Of course, the form of the implicit management cost function $C(.)$ depends on the farmer's quasi-fixed factor endowment. According to the form of the profit function $\Pi(\mathbf{s};\boldsymbol{\pi})$, the choice of the acreage $\mathbf{s}$ basically involves a trade-off between the short-run profits of the chosen acreage, $\mathbf{s}'\boldsymbol{\pi}$, and its management costs $C(\mathbf{As})$.

The considered farmer being risk neutral, his dynamic optimization problem is described by:

$$(2)\qquad \max_{\mathbf{s}_t, t=1,\ldots,T} \left\{ E_1 \sum\nolimits_{t=1}^{T} \delta^{t-1} \left[\mathbf{s}_t'\tilde{\boldsymbol{\pi}}_t - C(\mathbf{As}_t) \right] \text{ s.t. } \mathbf{s}_t \geq \mathbf{0} \text{ and } \mathbf{Ds}_t \leq \mathbf{p}_t \equiv \mathbf{As}_{t-1} \text{ for } t=1,\ldots,T \right\}$$

[3] As well as in Oude Lansink and Stefanou's (2001) adjustment cost function

with $T \le +\infty$ and where $\delta \in \left]0,1\right[$ is a discount factor. It is assumed that that the farmer faces price and/or output risks when choosing his acreage, *i.e.* that $\tilde{\boldsymbol{\pi}}_\tau$ is random for $\tau \ge t$. The rotation constraints $\mathbf{Ds}_t \le \mathbf{p}_t \equiv \mathbf{As}_{t-1}$ state that the total acreages "demanded" with the different previous crops, $\mathbf{Ds}_t$, cannot exceeds the corresponding acreage of the corresponding previous crop, $\mathbf{p}_t \equiv \mathbf{As}_{t-1}$. Operators E_t denote the expectation operator conditional on the information set available at *t* (as it is perceived by the farmer in period 1). Considering the problem dynamics there is vector of (endogenous) state variables at time *t*, $\mathbf{p}_t \equiv \mathbf{As}_{t-1}$.[4] The term $\mathbf{s}_0$ is a parameter of the considered problem since it directly determines the initial value of the state variable, $\mathbf{p}_1 \equiv \mathbf{As}_0$.

We now impose a technical assumption set. It is assumed that the cost function $C(\mathbf{As})$ is strictly increasing and strictly convex in $\mathbf{s} \ge \mathbf{0}$, implying that the maximization problems considered throughout this paper have a unique solution. It is also assumed throughout the paper that the crop rotation constraints always bind at the optimum acreage choices, meaning that the farmer never chooses to set land aside, whatever $\mathbf{p}_t$ and the $\tilde{\boldsymbol{\pi}}_\tau$ for $\tau \ge t$ are. This reasonable assumption can be checked afterwards.

The Bellman equations corresponding to this problem are provided by:

$$(3) \qquad V_t(\mathbf{p}_t) = \max_{\mathbf{s}_t} \left\{ E_t \tilde{\boldsymbol{\pi}}_t' \mathbf{s}_t - C(\mathbf{As}_t) + E_t V_{t+1}(\mathbf{As}_t) \text{ s.t. } \mathbf{s}_t \ge \mathbf{0} \text{ and } \mathbf{Ds}_t \le \mathbf{p}_t \equiv \mathbf{As}_{t-1} \right\}$$

for $t = 1,...,T$ with the convention that $V_{T+1}(\mathbf{As}_T) = 0$. $V_t(\mathbf{p}_t)$ is the value function of the considered optimization problem in period *t*, *i.e.* the discounted sum of the expected (optimized) crop profits of the considered farmer.

[4] Thanks to farmer's risk neutrality $E_t \tilde{\boldsymbol{\pi}}_t$ can also be considered as another vector of (exogenous with respect to $\mathbf{s}_t$) state variables.

At this point it appears useful to compare our dynamic modeling framework with the standard, static, one. Most of the multicrop models with land as an allocable input are special cases (See, *e.g.*, Heckeleï and Wolff, 2003) of profit functions of the form:

$$\Pi(\mathbf{a}_t;\tilde{\mathbf{v}}_t,\mathbf{w}_t,\tilde{\boldsymbol{\varepsilon}}_t)\equiv\sum\nolimits_{k\in\mathcal{K}}a_{k,t}\pi_k(a_{k,t};v_{k,t},\mathbf{w}_{k,t},\tilde{\boldsymbol{\varepsilon}}_{k,t})-C(\mathbf{a}_t)$$

The average crop profit functions $\pi_k(a_{k,t};v_{k,t},\mathbf{w}_{k,t},\tilde{\boldsymbol{\varepsilon}}_{k,t})$ are assumed to decrease in a_k for $k\in\mathcal{K}$, the so-called DMR assumption. One way to link the dynamic framework presented here and the standard static framework is to interpret the $\Pi(\mathbf{a}_t;\tilde{\mathbf{v}}_t,\mathbf{w}_t,\tilde{\boldsymbol{\varepsilon}}_t)$ profit function as a, necessarily crude, approximation of the farmer's period *t* objective function, *i.e.* of $\tilde{\boldsymbol{\pi}}_t'\mathbf{s}_t-C(\mathbf{a}_t)+E_tV_{t+1}(\mathbf{A}\mathbf{s}_t)$ plus the crop rotation and nonnegativity constraints. This interpretation is only meaningful if the economic context is "stationary" enough, *i.e.* in particular if the $E_tV_{t+1}(\mathbf{A}\mathbf{s}_t)$ functions do not vary too much across periods.

But even in this favorable case, three points are worth noting with respect to the $\Pi(\mathbf{a}_t;\tilde{\mathbf{v}}_t,\mathbf{w}_t,\tilde{\boldsymbol{\varepsilon}}_t)$ function. (*i*) This modeling framework can only be used for investigating short run responses to "small" economic shocks. (*ii*) The crop rotation effects and constraints partly determine the form of the $\pi_k(.)$ functions as well as the form of the implicit cost function $C(.)$. These functions can only be considered as approximately constant across time if farmers use almost constant rotation schemes. (*iii*) Optimal (and stable) crop rotation choices do not guarantee the validity of the DMR assumption. For instance, if the farmer expects a price increase for a "marginal" crop only, he may extend the acreage of this crop on more land with beneficial crop rotation effects, leading to an increase of both acreage and average return for this crop.

3. Empirical modeling framework: dealing with corner solutions and programming issues

3.1. The corner solution issue

The nonnegativity constraints $\mathbf{s} \geq \mathbf{0}$ imply the occurrence of many corner solutions in **s**, implying the existence of many different cases to be considered as well as important discontinuities in the solution functions. This corner solution problem is overcome in what follows thanks to a specific computational device. This device is fairly simple. Instead of directly using the cost function $C(\mathbf{As})$ we use a "working" cost function $C(\mathbf{s};\rho)$ parameterized by a "working" parameter ρ. This "working" cost function is chosen so as to possess three properties. (*i*) $C(\mathbf{s};\rho)$ is strictly increasing and strictly convex in **s** with $\lim_{\rho\to+\infty} C(\mathbf{s};\rho) = C(\mathbf{As})$, *i.e.* $C(\mathbf{s};\rho)$ is a cost function which can be interpreted as a "perturbed" version of the cost function $C(\mathbf{As})$. The fact that $C(\mathbf{s};\rho)$ can be considered as a good approximation of $C(\mathbf{As})$ if ρ is sufficiently large can be empirically checked. (*ii*) $C(\mathbf{As})$ is only defined for strictly positive acreage levels, *i.e.* for $\mathbf{s} > \mathbf{0}$, implying that the solution, $\mathbf{s}^*$, to any optimization problem in **s** defined with $C(\mathbf{s};\rho)$ satisfies $\mathbf{s}^* > \mathbf{0}$. (*iii*) The solutions of the (approximating) problems defined with $C(\mathbf{s};\rho)$ converge to the solutions of the (original) problems defined with $C(\mathbf{As})$ as $\rho \to +\infty$. The use of the function $C(\mathbf{s};\rho)$ in place of that of $C(\mathbf{As})$ allows us to focus on the crop rotation effects and constraints and to ignore the nonnegativity constraints $\mathbf{s} \geq \mathbf{0}$.

As a result we now consider the dynamic problem (2) and the Bellman equations given in equation (3) without the nonnegativity constraints $\mathbf{s}_t \geq \mathbf{0}$ for $t = 1,...,T$ and with $C(\mathbf{s};\rho)$ replacing $C(\mathbf{As})$.

3.2. Dynamic programming issues

We now show that standard duality results allow reformulating this dynamic acreage choice model with crop rotation effects and constraints as a fairly simple "Euler type"[5] dynamic optimization problem. The dual version of the original primal problem allows reducing the resolution of the constrained acreage choice to the search of the root of a simple equation system.

[5] According to Rust's (1996) terminology.

The basic idea underlying the dual approach is fairly simple. The farmer's past acreage choices supply him with quantities of land with different crop rotation effects, *i.e.* with different characteristics. The acreage the farmer would like to currently choose as a response to the crop and input market prices induce his demand of land with specific characteristics. The Lagrange multipliers related to the crop rotation constraints are the virtual prices leading to the equilibrium, at the farm level, of the farmer's supplies and demands of land of different characteristics. To be pedagogic, we first start with a myopic problem where the farmer neglects the effects of present decisions on future state variables. We then explain the resolution of the fully dynamic case.

The myopic problem. We consider first the myopic optimization problem, *i.e.* the problem that ignores the future effects of the current choices, defined by:

(4) $\max_{\mathbf{s}} \{\mathbf{s}'\boldsymbol{\pi} - C(\mathbf{s};\rho) \text{ s.t. } \mathbf{Ds} \leq \mathbf{p}\}$.

A Lagrangian function corresponding to the myopic problem under crop rotation constraints is defined as $L(\mathbf{s},\boldsymbol{\mu},\lambda;\boldsymbol{\pi},\mathbf{p};\rho) \equiv \boldsymbol{\pi}\mathbf{s} - C(\mathbf{s};\rho) + \boldsymbol{\mu}'(\mathbf{p}-\mathbf{Ds}) + \gamma(1-\mathbf{j}'\mathbf{s})$ where $\mu_K \equiv 0$ and $\mathbf{j}$ is the dimension K^2 unitary column vector. The γ term is the total land use constraint Lagrange multiplier whereas $\boldsymbol{\mu}$ is the crop rotation constraint (for the first $K-1$ crops) Lagrange multiplier vector. This formulation of the Lagrangian function is chosen so as to characterize the solution to the myopic problem with the solution function, $\mathbf{s}^*(.)$, of the standard land allocation problem under total land use constraint:

(5) $\mathbf{s}^*(\boldsymbol{\pi};\rho) \equiv \arg\max_{\mathbf{s}} \{\mathbf{s}'\boldsymbol{\pi} - C(\mathbf{s};\rho) \text{ s.t. } \mathbf{j}'\mathbf{s} \leq 1\}$.

We will consider that $\mathbf{s}^*(.;\rho)$ has an analytical closed form. Of course we have $\mathbf{j}'\mathbf{s}^*(\boldsymbol{\pi};\rho) = 1$ and $\mathbf{s}^*(\boldsymbol{\pi};\rho) > \mathbf{0}$ due to required properties of $C(\mathbf{s};\rho)$. Rearranging the terms in $L(\mathbf{s},\boldsymbol{\mu},\lambda;\boldsymbol{\pi},\mathbf{p};\rho)$ we obtain $L(\mathbf{s},\boldsymbol{\mu},\lambda;\boldsymbol{\pi},\mathbf{p};\rho) \equiv \mathbf{s}'(\boldsymbol{\pi} - \boldsymbol{\iota}\otimes\boldsymbol{\mu}) - C(\mathbf{s};\rho) + \boldsymbol{\mu}'\mathbf{p} + \gamma(1-\mathbf{j}'\mathbf{s})$ and we easily recognize a Lagrangian function corresponding to the maximization problem defined in equation (5) where $\boldsymbol{\pi} - \boldsymbol{\iota}\otimes\boldsymbol{\mu}$ replaces

$\boldsymbol{\pi}$. The myopic maximization problem being strongly dual and provided that we assume that the crop rotation constraints always bind, it is easily shown that its solution, $\mathbf{s}^*$, is uniquely characterized by:

(6a) $\mathbf{s}^* = \mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}^*; \rho)$

where:

(6b) $\boldsymbol{\mu}^*$ is the root in $\boldsymbol{\mu} \in \mathcal{M}$ of $\mathbf{D}\mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}; \rho) = \mathbf{p}$

with $\mathcal{M} \equiv \mathbb{R}^{K-1} \times \{0\}$. The term $\boldsymbol{\mu}^*$ is a virtual price vector of the previous acreage choice of the farmer. The $\mathbf{D}\mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}; \rho)$ term is the farmer's demand for land quantities with the different previous crops if these land quantities were to be rented at price $\boldsymbol{\mu}$. As a result the equation characterizing $\boldsymbol{\mu}^*$ states the farmer's demand for lands with the different previous crops must equal its supply $\mathbf{p}$. The optimal Lagrange multiplier vector $\boldsymbol{\mu}^*$ can be interpreted as the "market clearing price vector", at the farm level, of the land quantities with different previous crops.

Provided that $\mathbf{s}^* = \mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}^*; \rho)$ has an analytical closed form, to find $\mathbf{s}^*$ just consists in computing the unique root in $\boldsymbol{\mu}$ of $\mathbf{D}\mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}^*; \rho) = \mathbf{p}$, *i.e.* in computing $\boldsymbol{\mu}^*$.

The dynamic problem with uncertainty. The dual approach described above can easily be adapted for characterizing the solutions to the dynamic optimization problems. Two features of these problems make them relatively easy to solve. First the considered problem is of the "Euler type" [6] in the "choice" variables $(\mathbf{s}_t, \boldsymbol{\mu}_t, \gamma_t)$. Second, the dynamic programming optimality principle implies that the marginal effect of $\mathbf{p}_t$ on $V_t(\mathbf{p}_t)$ is given by $\tilde{\boldsymbol{\mu}}_t^* + \tilde{\gamma}_t^* \boldsymbol{\iota}$ for $t = 1, ..., T$.

Using the dual approach and these remarks, it is relatively easy to show that the solutions to the Bellman equation in period *t* are characterized by:

(7a) $\tilde{\mathbf{s}}_t^* = \mathbf{s}^*(E_t \tilde{\boldsymbol{\pi}}_t - \boldsymbol{\iota} \otimes \tilde{\boldsymbol{\mu}}_t^* + \delta E_t \tilde{\boldsymbol{\mu}}_{t+1}^* \otimes \boldsymbol{\iota}; \rho)$

[6] According to Rust's (1996) terminology.

where $\tilde{\boldsymbol{\mu}}_t^*$ is the root in $\tilde{\boldsymbol{\mu}} \in \mathcal{M}$ of:

(7b) $\mathbf{Ds}^*(E_t\tilde{\boldsymbol{\pi}}_t - \boldsymbol{\iota} \otimes \tilde{\boldsymbol{\mu}} + \delta E_t \tilde{\boldsymbol{\mu}}_{t+1}^* \otimes \boldsymbol{\iota}; \rho) = \mathbf{As}^*(E_{t-1}\tilde{\boldsymbol{\pi}}_{t-1} - \boldsymbol{\iota} \otimes \tilde{\boldsymbol{\mu}}_{t-1}^* + \delta E_{t-1}\tilde{\boldsymbol{\mu}} \otimes \boldsymbol{\iota}; \rho)$ for $t = 2,...,T$

with the conventions that $\mathbf{s}^*(E_0\tilde{\boldsymbol{\pi}}_0^* - \boldsymbol{\iota} \otimes \tilde{\boldsymbol{\mu}}_0^* + \delta E_0 \tilde{\boldsymbol{\mu}}_1 \otimes \boldsymbol{\iota}; \rho) \equiv \mathbf{s}_0$ and that $\tilde{\boldsymbol{\mu}}_{T+1}^* \equiv \mathbf{0}$. The form of the static profit function and of the rotation constraints implies that accounting for the effects of the past choices on the current choices and accounting the future consequences of the current choices is fairly simple. In period *t*, the farmer needs to "rent" from himself his land with previous crop *m* at price $\tilde{\mu}_{m,t}^*$ and he can expect to earn in the future $E_t\tilde{\mu}_{k,t+1}^*$ from "lending" to himself in period $t+1$ one unit of land with crop *k* in period *t*.

Despite the simplicity of the obtained characterization of the optimal decision rule, solving for $\mathbf{s}_1$ remains a difficult task due to the curse of dimensionality affecting any dynamic optimization problem with uncertainty.

Dynamic problems without uncertainty. Of course significant simplifications occur in the cases without uncertainty, *i.e.* when $\tilde{\boldsymbol{\pi}}_t = \boldsymbol{\pi}_t$ is known in period 1 for for $t = 1,...,T$. The expressions derived in the case with uncertainty hold with the notable exception that the $\tilde{\boldsymbol{\mu}}_t^*$ Lagrange multipliers are now fixed terms for $t = 2,...,T$ whereas they were random in the dynamic problem with uncertainty ($\boldsymbol{\mu}_1^*$ was fixed). In particular we have $\tilde{\boldsymbol{\mu}}_t^* = E_s\tilde{\boldsymbol{\mu}}_t^* = \boldsymbol{\mu}_t^*$ for $s = t-1, t, t+1$. To solve the considered dynamic problem just consists in finding a root, $(\boldsymbol{\mu}_1^*,...,\boldsymbol{\mu}_T^*)$, in $(\boldsymbol{\mu}_1,...,\boldsymbol{\mu}_T) \in \mathcal{M}^T$ of the equation systems given in equation (7a) and (10), and then in using equation (7a) to compute the optimal acreage sequence, *i.e.* the $\mathbf{s}_t^*$ terms for $t = 1,...,T$.

In the infinite horizon stationary case, the objective is to compute the root, $\boldsymbol{\mu}^*$, in $\boldsymbol{\mu} \in \mathcal{M}$ of the equation $\mathbf{Ds}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu} + \delta\boldsymbol{\mu} \otimes \boldsymbol{\iota}; \rho) = \mathbf{As}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu} + \delta\boldsymbol{\mu} \otimes \boldsymbol{\iota}; \rho)$ characterizing the unique steady state in **s**, $\mathbf{s}^*(\boldsymbol{\pi} - \boldsymbol{\iota} \otimes \boldsymbol{\mu}^* + \delta\boldsymbol{\mu}^* \otimes \boldsymbol{\iota}; \rho)$.

4. Empirical models and computational issues

The above discussions take for granted the availability of an analytical closed form for $\mathbf{s}^*(.;\rho)$ ensuring that $\mathbf{j}'\mathbf{s}^*(\boldsymbol{\pi};\rho) = 1$ and $\mathbf{s}^*(\boldsymbol{\pi};\rho) > \mathbf{0}$ for any $\boldsymbol{\pi}$. The Multinomial Logit (MNL) framework developed by Carpentier and Letort (2008) (CL hereafter) provides such analytical closed forms. These authors show that if the implicit cost function in the maximization problem (5) defining $\mathbf{s}^*(.;\rho)$ has the "Nested MNL" functional form:

(8a) $$C(\mathbf{s};\mathbf{r},\alpha,\rho) \equiv \alpha^{-1}\sum\nolimits_{k\in\mathcal{K}} a_k(\ln a_k - \ln r_k) + \rho^{-1}\sum\nolimits_{k\in\mathcal{K}}(\mathbf{s}'_k \ln \mathbf{s}_k - a_k \ln a_k)$$

with $0 < \alpha \le \rho$ and $\mathbf{r} \equiv (r_k, k \in \mathcal{K})$ with $\mathbf{r} > 0$ and $\mathbf{r}'\boldsymbol{\iota} = 1$, then:

(8b) $$s^*_{mk}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho) = \frac{\exp(\rho\pi_{mk})}{\sum_{n\in\mathcal{K}}\exp(\rho\pi_{nk})}\frac{\exp\left[\alpha\rho^{-1}\ln\sum_{n\in\mathcal{K}}\exp\left[\rho(\pi_{nk}+\alpha^{-1}\ln r_k)\right]\right]}{\sum_{\ell\in\mathcal{K}}\exp\left[\alpha\rho^{-1}\ln\sum_{n\in\mathcal{K}}\exp\left[\rho(\pi_{n\ell}+\alpha^{-1}\ln r_\ell)\right]\right]}.$$

The term $\rho^{-1}\sum_{k\in\mathcal{K}}(\mathbf{s}'_k \ln \mathbf{s}_k - a_k \ln a_k)$ in the "Nested MNL" implicit cost function $C(\mathbf{s};\mathbf{r},\alpha,\rho)$ is a "perturbation" term. It vanishes as $\rho \to +\infty$ and $\lim_{\rho\to+\infty} C(\mathbf{s};\mathbf{r},\alpha,\rho) = C(\mathbf{a};\mathbf{r},\alpha) \equiv \alpha^{-1}\mathbf{a}'(\ln \mathbf{a} - \ln \mathbf{r})$ is a "generalized entropic" implicit cost function depending on the total crop acreage vector **a**. The $C(\mathbf{a};\mathbf{r},\alpha)$ function defines the "Standard MNL" implicit cost function in CL.[7]

The advantage of the "Nested MNL" implicit cost function $C(\mathbf{s};\mathbf{r},\alpha,\rho)$ is threefold. (*i*) It leads to optimal acreage $s^*_{mk}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)$ strictly lying between 0 and 1 and summing to 1. (*ii*) It is continuously

[7] The results are presented for problems involving an implicit cost function $C(\mathbf{a};\mathbf{r},\alpha) \equiv \alpha^{-1}\mathbf{a}'(\ln \mathbf{a} - \ln \mathbf{r})$ of the "Standard MNL" form. These results hold for more general implicit cost functions defined within the "MNL" framework, *e.g.* for the "Nested MNL" implicit cost functions defined in CL.

differentiable in $(\boldsymbol{\pi},\alpha,\rho)$. This property is crucial if the considered models are to be employed in econometric analyses as it allows using standard statistical frameworks. (*iii*) The $s_{mk}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)$ functions provide reliable approximations of the solutions to the problem for large values of ρ, since we have: $\lim_{\rho\to+\infty}\mathbf{s}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)=\arg\max_{\mathbf{s}}\left\{\mathbf{s}'\boldsymbol{\pi}-C(\mathbf{As};\mathbf{r},\alpha)\text{ s.t. }\mathbf{s}\geq\mathbf{0}\text{ and }\mathbf{s}'\mathbf{j}\leq 1\right\}$.

In other words, $\mathbf{s}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)$ is a smooth function which can be used to approximate the solution to a maximization problem with nonnegativity constraints without explicitly considering these constraints. Of course, $\mathbf{s}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)$ positively biases the corner solutions in $\mathbf{s}$ (implying globally negative biases for the interior solutions) but these biases vanish as $\rho\to+\infty$.

In the implicit cost function $C(\mathbf{a};\mathbf{r},\alpha)$ the $\mathbf{r}$ term can be interpreted as a reference acreage vector. It is the unique minimand of $C(\mathbf{a};\mathbf{r},\alpha)$. This reference acreage is the one for which the farm's quasi-fixed factor endowment is best suited. The more $\mathbf{a}$ is distant from $\mathbf{r}$ according to the metric defined by the generalized entropic criteria, the larger is the implicit cost function $C(\mathbf{a};\mathbf{r},\alpha)$. The interpretation of the acreage cost function $C(\mathbf{a};\mathbf{r},\alpha)$ is similar to that of the usual "adjustement cost function" used in dynamic optimization models with "exogenous" adjustment costs (See, *e.g.*, Oude Lansink and Stefanou, 2001). In these models, the "reference" acreage is the previous acreage and is not fixed throughout the optimization problem as it is the case here. The α parameter can be interpreted as a relative "weight parameter". If α is large, the considered farmer basically choose to grow the most profitable crop in the most profitable sequence. In particular, if (and only if) π_{mk} is the unique maximum element of $\boldsymbol{\pi}$ then $\lim_{(\alpha,\rho)\to+\infty}s_{mk}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)=1$. If α is small, the $s_{mk}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)$ terms are mostly determined by $C(\mathbf{s};\mathbf{r},\alpha,\rho)$. We have $\lim_{\alpha\to 0}\boldsymbol{\iota}'\mathbf{s}_{k}^{*}(\boldsymbol{\pi};\mathbf{r},\alpha,\rho)=r_{k}$ for $k\in\mathcal{K}$.

We theoretically proved the convergence of simple numerical procedures for computing the optimal rotation constraint Lagrange multipliers in the myopic problem as well as in the dynamic problems without uncertainty. These procedures are based on the ideas underlying the contraction mapping principle and exploit the specific structure of the considered problems. They are fairly easy to code and they perform well in practice (as far as our applications prove this). Of course the risk issue is important but also much more challenging. This problem is not specific to the optimization problems we consider. Further research is required on this issue.

5. Illustrative simulations: impacts price shocks on the acreage choice dynamics

The objective of the following simulation exercise is threefold. First, we aim at illustrating the tractability of our modeling framework. All presented results are obtained with the empirical framework presented in the preceding sections and were computed using the numerical procedures we defined. Second, we aim at demonstrating that accounting for crop rotations is crucial for understanding, and thus modeling, farmer's acreage choices. Third, we demonstrate that farmers' perceptions with respect to the future evolution of the economic context plays a major role in their acreage choices, even in the short run, due to the dynamic effects implied by the crop rotation effects and constraints. We also show very complex dynamic results that may explain, *e.g.* why econometric estimation of the price elasticity of farm production is difficult.

We consider a typical farm in French regions with medium productivity levels, around the large Paris basin. The usual crop rotation choices are determined with standard wheat-barley-rapeseed sequence as the reference rotation scheme. The crop profit levels given past crop production are presented in the left hand side of Table 1. They are calibrated by using the results of CL together with "expert data" on the rotation effects gathered from interviews with agricultural scientists and extension agents. Profits per crop sequences are reported in Table 1. The right hand side gives the initial acreage allocation which corresponds to the economic context observed from the mid 1990's to the mid 2000's with low grain crop prices. In order to simulate our model, we need to determine the parameters of the cost function. We assume that alpha equals 3 so as to simulate non negligible effects while the reference acreages are all equal to one third. Finally we find that the $\rho = 35$ "working" parameter level is sufficient for providing satisfactory approximations of the acreage corner solutions: the approximate acreage shares corresponding to corner solutions in the "true" model range from 10^{-9} to 10^{-4} in the "approximate" model.

Table 1. Simulation parameters and steady state acreages

		Crop profit levels, stationary baseline				Acreages (steady state), stationary baseline		
		Crop				Crop		
		Wheat	Barley	Rapeseed		Wheat	Barley	Rapeseed
	Wheat	3.5	3.5	2.8	Wheat	0.00	0.16	0.34
Prev. crop	Barley	5	3.5	2.8	Barley	0.16	0.00	0.00
	Rapeseed	6	5	0.5	Rapeseed	0.34	0.00	0.00
					Total	0.50	0.16	0.34
		Crop profit levels, doubled cereal prices / baseline				**Acreages (steady state), doubled cereal prices / baseline**		
		Crop				**Crop**		
		Wheat	Barley	Rapeseed		Wheat	Barley	Rapeseed
	Wheat	10	9.5	2.8		0.00	0.50	0.00
Prev. crop	Barley	12.5	10	2.8		0.50	0.00	0.00
	Rapeseed	14	12	0.5		0.00	0.00	0.00
					Total	0.50	0.50	0.00
$\alpha = 3$, $\rho = 35$, $r_1 = r_2 = r_3 = 1/3$, $\delta = 0.95$								

The results presented in Table 1 for the baseline situation show that the rapeseed acreage is consequent, 34 per cent, despite the fact that this crop provides short run profits systematically lower than those of wheat and barley. This is mainly[8] due to the beneficial effects of growing cereals after rapeseed. The rapeseed-cereals rotation reduces the required pesticide use for cereals and increases the cereal yields because it provides a mean to "agronomically" control for pests which are otherwise difficult to control with pesticides. Note that rapeseed monoculture is strongly unwarranted, mainly due to uncontrolled infestations of bugs. Wheat is mainly grown after rapeseed but also after barley: wheat allows a high valuation of the benefits due to the rapeseed-cereals rotation and appears to be more profitable than barley. Note that an increase in the barley acreage can increase the average net returns of barley if this

[8] This is also partly due to the diversification effect generated by the acreage management cost function.

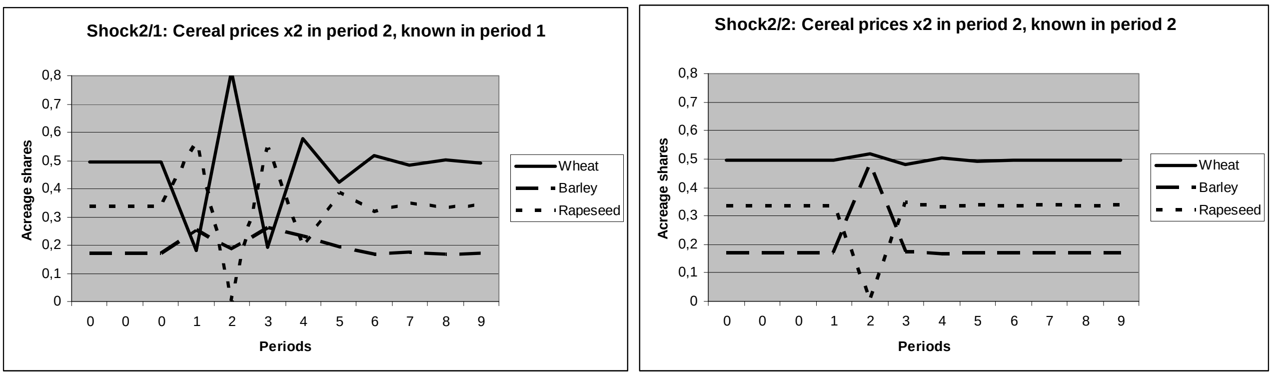
increase in acreage implies that barley is grown after rapeseed. This would follow, *e.g.*, a sharp decrease in the price of wheat.

We now conduct simulations considering first a 100% increase in the price of wheat and barley. This increase in the cereal prices induces a much higher increase in profit because variable costs increase much less than gross returns (see the left hand side of Table 1). Note that rapeseed production would disappear if this increase in the price of cereals would be permanent (see the right hand side of Table 1). In this case, farmers' would basically alternate the production of wheat with that of barley on their plots. The opportunity cost of growing rapeseed for its rotation benefits for cereals is too high.

The impacts of an anticipated one-shot 100% increase in the price of cereals, *i.e*. an anticipated price shock, on acreage choices are very interesting. They are depicted in the left hand side of Figure 1 (Shock 2/1). In this second simulation, in period 1 the farmer is aware (*e.g*. thanks to the information conveyed by future markets) that the price shock will occur in period 2, and in period 2 only. It appears that the year before the price increase, the producer decreases his wheat acreage and increase the land allocated to rapeseed. By such he is building a "capital" such as to optimally ripe the positive effects of the subsequent price increase. A large part of the "baseline" wheat acreage is substituted for rapeseed acreage in period 1 in order to increase the profit of producing wheat in period 2, and especially wheat after rapeseed. Note also that after the price shock the wheat and rapeseed acreages oscillate around their steady state levels. They only progressively get to their steady state levels demonstrating that crop rotation effects provide incentives for crop diversification across time, not necessarily across the farm plots in a given year. The "stabilization process" of the crop acreages at their steady state levels would be slower with larger values of α. The year following the price shock the farmers chooses a large rapeseed acreage because he meanly grew wheat the year before and also because he intends to have again a large cereal acreage after rapeseed.

In order to analyze the impact of anticipations, we conduct a third simulation where the period 2 price shock is unanticipated by the farmer in period 1. The farmer's acreage choices are shown in the left hand side of Figure 1 (Shock 2/2). In this case, the farmer mainly substitutes rapeseed for barley in his acreage for benefiting from the increase in the cereal prices. Whereas the barley acreage is mainly unaffected by the anticipated price shock, it fully “captures” the impact of the unanticipated price shock. By contrast, the impact on wheat acreages is much more muted.

Figure 1. Simulation results: Anticipated and unanticipated price shocks on cereals ($\times 2$) in period 2 and dynamic acreage choices



Of course uncertainties on the price shock occurrences and levels would certainly attenuate the effects shown in these simulations. It still remains that it appears difficult to investigate the effects of the economic determinants of farmers’ acreage choices without considering both the impacts of crop rotation benefits and constraints, and farmers’ perceptions of the future production price distributions, especially in a static modeling framework. This may also explain why it appears to be so difficult to statistically measure the price elasticities of the agricultural production supply.

6. Concluding remarks

This paper presents an empirically tractable framework for modeling dynamic acreage choices, *i.e.* acreage choices accounting for crop rotation effects and constraints. This modeling framework combines (*i*) original results stemmed from a theoretical analysis of farmers’ dynamic programming problems using duality, (*ii*) a simple device for coping with corner solutions in acreage choices implied by the discrete feature of the crop rotation choices, (*iii*) the use of well-behaved static acreage share choice models and (*iv*) simple numerical procedures for solving the involved static and dynamic optimization problems.

Simulation results illustrate two main points. First, the effects of crop rotations on crop returns and the constraints imposed on acreage choices by the crop rotations are crucial determinants of farmers' acreage choices. Second, these results also demonstrate the key role played by farmers' perceptions of the future economic context on their current production choices. Although this result is well-known for investment choices, our analysis demonstrates it also holds for short run production choices due to the production dynamics implied by crop rotations.

Similar simulation results and comparisons with our dynamic modeling framework can also be used for assessing the limits and merits of the standard static multicrop models found the agricultural production literature.

Our main results are for problems without uncertainty. Of course, risk issues are essential in dynamic analyses. But they are also challenging for empirical applications. Our first investigations show that our modeling framework can be adapted for investigating small horizon dynamic problems.

Our empirical modeling framework is designed for simulation purposes but also for econometric analyzes. Our first investigation demonstrates that the presented modeling framework can be employed for specifying empirically tractable econometric models. The implementation of the statistical inference remains an issue due to the complexity of the acreage choice dynamics. Ongoing researches investigate both topics.

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