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## Demand Estimation for Agricultural Processing Co-products

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## Table of Contents

	<u>Page</u>
List of Tables .....	ii
Abstract .....	iii
Highlights.....	iv
Introduction .....	1
Demand Estimation .....	2
Methods.....	4
Livestock Classes and Nutritional Requirements .....	4
Ingredient Classifications and Prices .....	7
Linear Programming Model.....	7
Demand Aggregation.....	9
Results and Conclusions .....	10
Demand Estimations.....	10
Co-product Demand.....	11
Sugarbeet Pulp .....	11
Wheat Middlings.....	13
Concluding Comments.....	16
Literature Cited.....	18
Appendix – Demand System Equations .....	20

## List of Tables

<u>Table No.</u>		<u>Page</u>
1	Livestock Classes.....	5
2	Demand for Sugarbeet Pulp by Livestock in the Central Crop Reporting District of North Dakota.....	12
3	Demand for Wheat Middlings by Livestock in the Central Crop Reporting District of North Dakota.....	13
4	Demand for Potato Waste by Livestock in the Central Crop Reporting District of North Dakota.....	15

## Abstract

Co-products of processing agricultural commodities are often marketed through private transaction rather than through public markets or those in which public transaction information is recorded or available. The resulting lack of historical price information prohibits the use of positive time series techniques to estimate demand. Demand estimates for co-products are of value to both livestock producers, who obtain them for use in livestock rations, and processors, who must sell or otherwise dispose of them. Linear programming has long been used, first by researchers and later as a mainstream tool for nutritionists and producers, to formulate least-cost livestock rations. Here it is used as a normative technique to estimate step function demand schedules for co-products by individual livestock classes within a crop-reporting district. Regression is then used to smooth step function demand schedules by fitting demand data to generalized Leontief cost functions. Seemingly unrelated regression is used to estimate factor demand first adjusted for data censoring using probit analysis. Demand by individual livestock classes is aggregated over the number of livestock within a region. Quantities demanded by beef cows for each of the three co-products considered, sugarbeet pulp, wheat middlings, and potato waste, are large relative to other species because of their predominance in the district. At the current price for sugarbeet pulp, quantity demanded by district livestock is low. However quantity demanded is price elastic and becomes much greater at lower prices. Wheat middlings can be an important component of livestock rations, even at higher prices. At a price slightly below the current price, local livestock demand would exhaust the wheat middlings produced at the district's only wheat processing plant. Potato waste is most appropriate for ruminant diets because these animals are able to consume a large quantity of this high moisture feedstuff. Potato waste can be a cost-effective component in beef and dairy rations. Practically, livestock markets for potato waste must be in close proximity to a potato processing plant. Its high moisture content limits the distance it can be economically transported. At current prices, potato waste can be economically included in the ration for beef cows on a farm nearly 100 miles from the processing plant, although storage challenges may restrict use of the feed to closer operations.

**Key words:** co-products, demand estimation, econometrics, linear programming

## Highlights

North Dakota and bordering counties in western Minnesota are home to many firms which process agricultural commodities including sugarbeets, wheat and durum, and potatoes. Large quantities of agricultural co-products result. When their use is limited to livestock rations, identifying their economic value in this role provides an estimate of demand. This information is important for both livestock producers and firms that process agricultural commodities.

When competitively priced with more traditional feed ingredients, sugarbeet pulp, wheat middlings, and potato waste enter least-cost rations of North Dakota livestock. When prices of ingredients, other than co-products available to livestock rations, are at their 20-year average:

- Ingredients in least-cost rations for beef cows include only forages and all three co-products. No concentrates enter the least-cost ration.
- Ingredients in least-cost dairy cow rations are all three co-products and corn, barley, soybean meal, corn silage, and alfalfa. Neither oats nor prairie hay enters the ration.
- Ingredients in least-cost ewe rations include all three co-products, alfalfa, prairie hay, and soybean meal. No cereal grains or corn silage enter least-cost ewe rations.
- High-energy requirements for growing beef, concentrate limits for growing lambs, and high protein requirements for swine constrain their rations to the inclusion of specific feeds and limit the inclusion of co-products.
- Over a wide range of prices, quantities of co-products demanded by beef cows are large relative to other species because of their predominance in the district.

### Sugarbeet Pulp

- Cows with calves are the main consumers of sugarbeet pulp although this co-product is included in the ration for all species considered over a wide price range.
- At current prices, quantity demanded by the district's livestock is low.
- At lower prices, demand by the district's livestock for the primary co-product from the seven processing plants in the adjacent region becomes much greater. Strong demand by local livestock at slightly lower than current prices may prove important should demand drop in other domestic or overseas markets.
- Quantity of sugarbeet pulp demanded by beef cows is price elastic at prices just under current market prices, while that demanded by feedlot beef cattle, dairy cattle, and ewes is much less price elastic. Quantity demanded by feedlot beef cattle and lambs is limited due to the high nutritional requirements of these growing animals.

### Wheat Middlings

- Wheat middlings are abundant throughout North Dakota with approximately five wheat-processing plants in operation.

- Wheat middlings are a good source of protein compared to other concentrates commonly used in North Dakota livestock rations, such as corn and barley, and enter rations as a substitute for these feeds at various prices.
- Quantity demanded is price elastic for inclusion in beef cow rations, although quantity demanded by dairy cows and ewes is much less because of dietary limitations. Quantity demanded is constant over the price range considered for growing beef cattle, lambs, and swine.
- At high prices, dairy cows become the most important consumers of wheat middlings. Quantity demanded by dairy cows will change nearly proportionate with changes in the herd size.

#### Potato Waste

- Potato waste is a high moisture feed. Beef and, to a lesser extent, dairy cows have the ability to consume the large quantity necessary to meet their nutritional requirements.
- Potato waste can be a least-cost ration ingredient in beef cow rations at prices up to \$13 per ton and in dairy cow rations to prices of \$11.80 per ton. Demand by individual dairy cows is similar to that by individual beef cows at lower prices and is always greater at prices higher than \$5.80 per ton.
- Quantity demanded by beef cows is price elastic at prices higher than \$7 per ton. Quantity demanded by dairy cows is much less sensitive to changes in price.
- Sheep demand small amounts of potato waste and the co-product does not enter the ration for feedlot beef cattle, lambs, and swine.
- Current base prices for potato waste are as low as \$7 per ton. At this price, it could be transported up to 95 miles to beef cow operations and up to 80 miles to dairy operations for inclusion in a least-cost ration.
- Practically, livestock markets for potato waste must be in close proximity to a potato processing plant. Its high moisture content limits the distance it can be economically transported. Furthermore, it can be difficult to store, in transit and on the farm. The cold winters in North Dakota require special equipment such as lined delivery trucks to prevent freezing .

# Demand Estimation for Agricultural Processing Co-products

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## INTRODUCTION

Agricultural co-products result from the processing of an agricultural commodity into a consumable or industrial product. Use in livestock feeds represents a major market for co-products. Since co-product markets are very localized and transactions typically are private, little information is available for estimating the nature of these markets using traditional econometric methods. However, information regarding co-product quality and value are important for both livestock producers, who purchase and use co-products, and processing companies, which may generate revenues from co-product sales.

Co-products are an increasingly important source of feed for livestock producers throughout the United States. Using regionally produced co-products in rations can reduce feed costs, which comprise one of the largest expenses in livestock production (Kubic and Stock; and Schroeder). Further fueling interest in the use of co-products in North Dakota livestock rations is an increase in their availability in the region due to the creation of a number of value-added agricultural processing facilities.

The objective of this research is to generate demand schedules for various co-products available in the Northern Great Plains as sources of livestock feed. Factors affecting the demand for co-products include their physical and nutritional characteristics and costs associated with their transportation, handling, and storage. Least-cost ration models are used to derive the demand schedules for co-products under a range of co-product prices as well as prices of alternative feeds. Resulting demands can be used to measure the value of co-products for use in rations of various livestock in specific regions. Co-product demand information can also be used by processing firms to set revenue-maximizing prices as well as to guide longer run decisions, such as plant location. Demands are estimated for sugarbeet pulp, wheat middlings, and potato waste by livestock in the Central Crop Reporting District of North Dakota.

### *Use of Co-products in North Dakota Livestock Rations*

Meeting animal nutritional needs is fundamental to ensuring optimal growth and production. Although high-forage diets are often fed to beef cows and sheep, modern livestock diets for growing and finishing animals and lactating dairy cows are often high in energy and other nutrients. For example, typical feedlot cattle rations in the region are comprised of 80 percent concentrates (*e.g.*, corn) and 20 percent roughages (*e.g.*, corn silage). Rations are supplemented with soybean meal or urea to provide additional protein (Drake *et al.*). However, ration cost may be reduced when high nutrient co-products (*e.g.*, sugarbeet pulp, potato waste, and wheat

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middlings) are used. Substitution of traditional feedstuffs with co-products may retain the nutrient value of the ration but at a lower cost.

North Dakota and bordering counties in western Minnesota are home to many firms that process agricultural commodities including sugarbeets, wheat and durum and potatoes. Processing plants of American Crystal and Minn-Dak Farmers' cooperatives located throughout the Red River Valley process sugarbeets from nearly 600,000 acres. Sugarbeet processing results in a wet pulp co-product (pressed pulp is between 22 to 30 percent dry matter) that may be dried to 90 to 92 percent dry matter. Beet pulp is not high in protein (9 percent crude protein), but offers at least 85 percent of the energy value of corn and 95 percent of the energy value of barley and is an excellent source of calcium (Schroeder).

Wheat middlings result from the milling of flour or semolina from wheat and durum. They typically contain 17 to 18 percent crude protein, above that provided by most feed grains but below that offered by high protein oilseed meal, such as soybean meal (Dhuyvetter, Hoppe, and Anderson). Wheat middlings are also a good source of crude fiber and phosphorus. The increased number of regional wheat and durum processing plants has increased the availability of wheat middlings for livestock feeding (Dhuyvetter, Hoppe, and Anderson).

Potato waste is high in energy [85 percent total digestible nutrients (TDN)], is easily digestible, and contains a moderate amount of protein (8 percent crude protein). It is often used as a substitute for corn or corn silage in livestock rations. When transportation distance is relatively short, potato waste can serve as an inexpensive ration ingredient, especially for beef and dairy diets (Schroeder). Two potato processors are located in North Dakota: AVIKO, located in Jamestown, and Simplot, located in Grand Forks. Potato waste from the plant in Jamestown is high in moisture (75 to 80 percent), resulting in a high transportation cost per mile of nutrient shipped. Lined trucks are necessary to distribute the product, contributing to the transportation cost.

## DEMAND ESTIMATION

Two methods commonly used to estimate demand functions are econometrics, a positive approach, and primal optimization, a normative approach (Konyar and Knapp). If price and quantity data are available, the positive approach allows for estimates based on observed rather than simulated behavior (Acharya-Madnani). However, data are sometimes not available or the use of historical data may mask changes in technology and management practices (Konyar and Knapp). Alternatively, the normative approach provides price and quantity information under explicit assumptions of optimizing behavior and can provide expected demand and supply conditions when transparent markets do not exist (Konyar and Knapp; and Johnson and Varghese).

The normative approach is particularly appropriate for estimating demand for new products or those on which little or no historical data is available. An additional advantage of the normative technique is that it allows demand estimation for individual groups of animals (*e.g.*,

within a particular production stage).<sup>2</sup> Assigning economic values to ingredients in livestock rations based on their ability to meet the nutrient requirements of individual animals allows livestock, to which co-products offer the highest value, to be identified.<sup>3</sup>

Linear programming has long been used by nutritionists and practitioners to formulate least-cost rations, and by researchers to evaluate the effect of ration composition and other management and marketing practices on the profitability of farm enterprises (*e.g.*, see Brennen and Hoffman). Linear programming accommodates complex problems with multiple constraints and results in specific information about the value of individual feed ingredient characteristics (*e.g.*, protein content). Marginal costs of individual nutritional requirements result from the dual formulation of the linear programming model. More refined estimates of feeds as components of least-cost rations and of their contribution to the nutrient requirements of individual animals result than when other normative estimation techniques are used (Peeters and Surry).

Demand schedules for individual livestock classes can be aggregated to forecast potential demands for individual feeds within a region. Johnson and Varghese and Voorhees-Watson estimated demand for feed barley and sunflower meal, respectively, using this method. Johnson and Varghese used this technique to estimate livestock demand for feed barley in California and North Dakota. They used regional price and livestock sector data and explicitly considered the affect of barley characteristics on demand. Voorhees-Watson selected the same method to formulate individual livestock diets. However, linear programming and other normative approaches have not been widely used to estimate the economic value of agricultural co-products.<sup>4</sup>

Normative demand estimation requires specific information about the nutritional characteristics of individual ration components, nutrient requirements for individual animals, number of animals within a region, and current and estimated feed prices. Resulting demand schedules are also not differentiable. Basic variables in linear programming models may be insensitive to price changes within a range. As prices are varied outside of these ranges, discrete

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<sup>2</sup>Surry noted that when econometrics is used to estimate feed demand at an aggregate level, it fails to account for diversity in production systems.

<sup>3</sup>Peeters and Surry provide an extensive review of the use of synthetic, linear programming, and econometric methods to estimate price elasticity of demand for feed ingredients in the European Union (EU). They note that the underlying assumptions of much of the static empirical work on feed ingredients in the EU are that demand for feed ingredients is directly linked to livestock production and that optimal feed quantities are derived from the optimizing behavior of the producer. These assumptions are adopted for the present research.

<sup>4</sup>Drake *et al.* used a simple cost function optimization to determine the value of feather meal and potato-corn waste in Midwestern beef cattle finishing rations. Optimization equations determine an equitable pricing scheme for substituting feather meal for soybean meal under feeding periods of different lengths in a static framework.

jumps in the levels of the basic variables occur. Consequently, elasticities derived from these step-wise demand schedules are meaningless. Since they are based on demand elasticities, optimal pricing strategies for co-product suppliers are thus difficult to devise.

A common approach to derive differentiable demand schedules is to smooth the price-quantity relationships resulting from the least-cost ration model using regression techniques. Use of linear programming ‘pseudo-data’ has been employed to estimate demand for various feeds (*e.g.*, see McKinzie, Dauribert, and Ituerta; Peeters; and Peeters and Surry). Consistent system estimates are derived for feed demands using a two-step procedure recently developed by Shonkwiler and Yen.

## METHODS

A system of demand equations for traditional feeds and three co-products (sugarbeet pulp, potato waste, and wheat middlings) are estimated for the North Dakota Central Crop Reporting District. Co-products are available from processing firms in this or an adjacent region. Regional use in livestock rations is or has the potential to be a major market for the three co-products. Demands are estimated for the major classes of livestock produced in the Central Crop Reporting District (beef cattle, dairy cattle, sheep, and swine).

Least-cost rations are estimated using linear programming for different species of animals in different growth stages with varying levels of animal performance and under varying feed prices. Resulting step function demand schedules are then smoothed. The unknown livestock feeder’s cost function is approximated using a generalized Leontief cost function, a flexible functional form. Use of a generalized Leontief cost function results in demands that are homogeneous of degree zero in feed prices and permit substitutions among feed ingredients. Since the range of prices used in the derivation of least-cost rations resulted in solutions in which many feed ingredients do not enter the ration, numerous cases arise in which the endogenous variable is zero. Failure to account for this censoring would result in biased demand estimates. A two-step procedure developed by Shonkwiler and Yen is used to correct for this bias. Finally, aggregating demand from individual animals within the district provides an estimate of regional demand.

### *Livestock Classes and Nutritional Requirements*

Species are separated into livestock classes according to size or age (*e.g.*, 900 vs. 1,120 lb. steer), production (*e.g.*, dairy cow producing 66 versus 88 lbs. of milk per day), or production stage (*e.g.*, gestating versus lactating sow). Table 1 specifies the 19 classes used to represent livestock in the district. Each class of livestock represented has unique nutrient requirements. Nutrition required by livestock and provided by each feedstuff was obtained from the National Research Council guidelines and modified for use based on advice by specialists in the Animal and Range Science Department at North Dakota State University.<sup>5</sup>

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<sup>5</sup>Specialists include Dr. Greg Lardy and Dr. Marc Bauer (beef and dairy), Dr. Roger Haugen (sheep), and Dr. Robert Harrold (swine). Specific nutrient requirements for each livestock class are available upon request from the authors.

Table 1. Livestock Classes

Species	Classification	Description
Beef Cattle	C1	900 lb. feedlot steer
	C2	1,120 lb. feedlot steer
	C3	1,200 lb. beef cow, 20 lbs. peak milk, <sup>a</sup> 3 months since calving
	C4	1,200 lb. beef cow, 20 lbs. peak milk, 11 months since calving
	C5	1,400 lb. beef cow, 20 lbs. peak milk, 3 months since calving
	C6	1,400 lb. beef cow, 20 lbs. peak milk, 11 months since calving
Dairy Cattle	D1	1,320 lb. dairy cow, 66 lbs. milk per day
	D2	1,320 lb. dairy cow, 88 lbs. milk per day
Sheep	S1	Flushing <sup>b</sup> 150 lb. ewe
	S2	Gestating <sup>c</sup> 150 lb. ewe
	S3	Lactating <sup>d</sup> 150 lb. ewe
	S4	Growing 50 lb. lamb
	S5	Finishing 80 lb. lamb
Swine	H1	22 to 44 lb. growing hog
	H2	44 to 110 lb. growing hog
	H3	110 to 176 lb. growing hog
	H4	176 to 265 lb. growing hog
	H5	Gestating sow, 386 lb., 12 piglet litter
	H6	Lactating sow, 386 lb., 12 piglet litter

<sup>a</sup> Peak milk - maximum milk production per day.

<sup>b</sup> Flushing- feeding for gain of weight before breeding season to increase lambing percentage.

<sup>c</sup> Gestating- animal that is carrying unborn young.

<sup>d</sup> Lactating- animal that is nursing young.

Rations include minimum levels of energy, crude protein, calcium, and phosphorus to meet specified performance criteria. Dietary constraints are specified as percent dry matter, necessitating constraints specifying minimum dry matter intake. Imposing constraints on the level at which bulky feeds can enter the ration specifies a maximum daily intake (gut fill).

Energy requirements and availability from each feed are unique to each species. Total digestible nutrients (TDN) is used for beef and dairy cattle, metabolizable energy (ME) is used for sheep, and digestible energy (DE) is used for swine. Constraints are also imposed for minimum protein, calcium, and phosphorous.

Each species also has unique characteristics requiring additional dietary constraints.<sup>6</sup> Six livestock classes are used to represent beef cattle in the district. Individual classes represent 900 lb. (gaining 4.0 lbs. per day) and 1,120 lb. (gaining 4.5 lbs. per day) feeder steers in feedlots (C1 and C2, respectively). Four classes represent beef cows that have calved (C3 through C6). Beef cow classes are differentiated based on the weight of the cow and the age of her nursing calf.

<sup>6</sup>Unless otherwise indicated, all constraints are specified on a dry matter basis.

Beef cattle rations are restricted in their composition of by-products, concentrates, and roughage. No more than 50 percent of any beef cattle ration can be comprised of co-products. Excessive levels may cause acidosis, which can harm the stomach of the beef animal. Concentrates cannot comprise more than 50 percent of the beef cow diet. Excessive levels may also result in acidosis, reducing a cow's productive life. Alternatively, feedlot cattle should consume a ration that is adequate in concentrates to limit the time necessary for them to reach fed weight. Composition of the feedlot cattle ration is however limited to a maximum of 90 percent concentrates, 20 percent corn silage, and 10 percent hay.

Dairy cattle rations are formulated only for cows that are currently milking. Two classes representing dairy cattle are 1,320 lb. cows producing 66 (D1) and 88 (D2) lbs. of milk per day. In a dairy cow diet, beet pulp and potato waste can comprise no more than 50 percent of the ration and wheat middlings can comprise no more than 24 percent. Greater consumption of wheat middlings may reduce milk production (Acedo, Bush, and Adams). Additional constraints limit roughages and concentrates in the ration to between 40 and 60 percent. A lactating dairy cow should consume 1.5 percent of her live weight in hay equivalent dry matter (NRC, 1989). A minimum level of acid detergent fiber (ADF) is required to ensure maximum dry matter and energy intakes (NRC, 1989). If fiber intake is not adequate, ruminal fermentation and fiber degradation may be impaired, decreasing milk fat percentage.

Five sheep classes are specified. Three classes represent 150 lb. ewes that are flushing (S1), gestating (S2), or lactating (S3).<sup>7</sup> Two classes represent early-weaned finishing lambs gaining 0.6 lbs. (S4) or 0.8 (S5) lbs. per day. In addition to the minimum calcium requirement common to all species, calcium must also be restricted in the sheep diet. Excessive dietary calcium may result in a deficiency of other nutrients such as phosphorus (NRC, 1985). High calcium to phosphorus ratios for long periods of time can result in bone resorption.

Constraints that specifically affect the ewe's diet are restrictions on beet pulp and concentrate consumption. Beet pulp can comprise no more than 15 percent of the ration because it quickly fills the gut and rumen and makes the ewes very laxative. Concentrates should comprise no more than 25 percent of a modern ewe ration. Roughage provides adequate nutrition to meet production goals. Lamb rations are constrained to include between 30 and 40 percent roughage to ensure adequate consumption of concentrate to meet performance criteria. In addition, wheat middlings can comprise no more than 20 percent of the lamb ration and sugarbeet pulp must be less than 10 percent. Higher levels of wheat middlings may reduce feed consumption because of the high digestibility of this co-product. As with ewes, beet pulp quickly fills the gut and rumen and can make lambs very laxative.

Six livestock classes are used to represent swine. Four represent an even gender mix of growing finisher hogs gaining .72 lbs./day between 22 and 44 lbs. (H1), 44 and 110 lbs. (H2), 110

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<sup>7</sup>Flushing is a common practice wherein ewes are fed to gain weight approximately two weeks prior to breeding to increase lambing percent (Robert Haugen, personal communication, August 2000).

and 176 lbs. (H3), and 176 and 265 lbs. (H4). Two represent 386 lb. sows that are in the gestation period (H5) or are lactating (H6) based on an average litter size of 12 piglets. In addition to crude protein, swine diets are balanced to meet specific amino acid requirements. Other nutrient requirements specific to swine rations include a minimum level of ash and minimum and maximum levels of ADF.

### *Ingredient Classifications and Prices*

Feed ingredients commonly used in North Dakota were used in the models. Roughages included alfalfa, prairie hay, and corn silage. Roughages were limited to use by ruminant animals. Swine can consume little or no roughage because they are inefficient at fiber digestion. Alternatively, microbial digestion in the rumen allows ruminants to efficiently digest roughages and extract available nutrients. Concentrates contain a high level of energy. They are added to a ration to increase energy intake or the energy density of the ration (Church and Pond). Concentrates made available to livestock rations include cereal grains (corn, barley, and oats), supplements,<sup>8</sup> and co-products.

Twenty years of historical prices (1980 to 1999) were used to represent the cost of ration ingredients. Weighted average annual prices of barley, corn, alfalfa, prairie hay, and oats were obtained from the North Dakota Agricultural Statistics Service. The per ton price of corn silage is represented as eight times the per bu price of corn (Hendrix). A simple average of weekly soybean meal prices obtained from *Feedstuffs Magazine* represents annual price. Prices of the traditional feed ingredients were represented using a single vector of prices for each year. Prices of supplements including salt, vitamin premix, selenium, trace minerals, dical, and limestone were fixed at recent prices because of the lack of available historic price records and because their price does not influence demand for other feed ingredients since they are used in fixed quantities.

No market information was available about the co-products of sugarbeet pulp, potato waste, and wheat middlings. Least-cost rations were identified for each livestock class using 540 feed ingredient price combinations; 27 possible combinations of co-product prices (three prices of each of three co-products,  $3^3 = 27$ ), each with 20 one-year price vectors representing prices of traditional feeds.

### *Linear Programming Model*

The least-cost ration problem is mathematically stated as:

$$\text{Minimize } \sum_{i=1}^n r_i x_i$$

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<sup>8</sup>A supplement is a form of concentrate that assists in meeting specific nutritional requirements such as for protein, mineral, or vitamin deficient diets.

subject to

$$\sum_{i=1}^n a_{ji} x_i \geq b_j, \quad j = 1, \dots, m$$

$$x_i \geq 0$$

where  $r_i$  and  $x_i$  are the price and amount of feed input  $i$ , respectively. The objective function minimizes the ration cost of producing a specified level of output as defined by the production stage and performance level of the animal represented. The  $m$  constraints are unique to each livestock class, where  $a_{ji}$  is the amount of nutrient  $j$  available from ingredient  $i$ , and  $b_j$  is the nutrient level requirement for the animal. Solving least-cost rations using the described price vectors results in up to 540 points on a demand schedule for each ingredient in the ration.

### *Demand Smoothing*

Demand equations consistent with a generalized Leontief cost function are:

$$x_i(y, r) = y \left( \mathbf{b}_{ij} + \sum_j \mathbf{b}_{ij} \left( \frac{r_j}{r_i} \right)^{1/2} \right)$$

where  $x_i$  is quantity of ingredient  $i$  demanded,  $y$  is output level (e.g., milk production, animal gain),  $r_i$  is the price of feed ingredient  $i$ , and  $\mathbf{b}$ 's are parameters to be estimated.

Least-cost rations frequently did not include one or more feed ingredients. A large number of zero observations in the endogenous variable result in biased parameter estimates if not corrected (Pindyck and Rubinfeld). The two-step estimation procedure for systems of equations with limited dependent variables proposed by Shonkwiler and Yen was used to correct for bias introduced by data censoring. The first step involves a Probit analysis to determine the probability of observing a zero or positive level of the individual feed in the ration given a set of explanatory variables (here we used the feed prices):

$$\begin{aligned} \text{Prob}(Y = 1) &= \int_{-\infty}^{\mathbf{a}'r} \mathbf{f}(t) dt \\ &= \Phi(\mathbf{a}'r) \end{aligned}$$

where  $\phi$  is the probability density function,  $\Phi$  is the cumulative distribution function that a feed will enter the ration at a non-zero level given the observations on prices  $r$ . Estimation identifies values of  $\mathbf{a}$  that best fit observed levels of the feed to be either 'zero' ( $Y=0$ ) or positive ( $Y=1$ ), conditional upon values of the exogenous variables,  $r$ . The results of the probit were used to weight individual demand functions in the system estimation to give consistent parameter estimates:

$$x_i = \Phi(\mathbf{a}'r) x_i(y, r) + \mathbf{d} \mathbf{f}(\mathbf{a}'r)$$

for feeds exhibiting a large number of 'zero' observations, and  $x_i = x_i(r, \mathbf{b})$  for feeds with few or no 'zero' observations. The statistical significance of  $\mathbf{a}$  indicates whether data censoring was necessary to correct for bias originating from the large number of 'zero' observations.

Demand for least-cost rations of 19 livestock classes were solved in GAMS but the smoothing procedure was used for only 9. Co-products demanded for 10 of the livestock classes did not change with price. The nine livestock classes for which demands were smoothed were beef cows (C3 to C6), dairy cows (D1 and D2), and ewes (S1 to S3). Adjustments were made in feeds available to beef cows and lactating ewes prior to demand smoothing. Barley and soybean meal were removed from beef cow diets because these feeds were not, in general, present in the least-cost ration. Forages were combined for beef cows because they tended to enter and exit the ration as blocks without substitution. As a result, the data matrix was singular, preventing solution of the probit model. Alfalfa, prairie hay, and corn silage were combined into a single variable (FORAGES). Sugarbeet pulp and potato waste were eliminated prior to estimating the ration for lactating ewes because neither entered the least-cost ration.

### *Demand Aggregation*

Co-product factor demands were next aggregated into demands by individual species and by all livestock. Co-product demand for each animal unit within a livestock class was first multiplied by the number of animals within the district and the number of days in a year represented by the growth stage of the individual livestock class. Animal inventories within the Central Crop Reporting District were obtained from the North Dakota Agricultural Statistics Service (U.S. Department of Agriculture). There are approximately 121,594 cows and heifers, 14,685 feedlot cattle, 9,953 ewes, 10,671 growing lambs, 1,134 sows, 6,859 growing pigs, and 6,100 dairy animals in the region. Animal numbers within each livestock class were represented as a portion of animal inventory within the species it represents. The portion represents the number of days an individual animal is best represented by the livestock class over the production period (*i.e.*, year or time in the feedlot). For example, a growing beef feeder reaches market weight in 210 days. For 90 days, the animal was characterized as a 900 lb. feeder (represented by the C1 livestock class). The remaining 120 days, the animal was characterized as a 1,120 lb. feeder (represented by the C2 livestock class). Feeder steers were therefore represented by C1 during 43 percent of days on feed and by C2 for the remaining 57 percent. It was assumed that production systems operate on a round turn basis for production periods of less than one year (*e.g.*, a feeder steer marketed at 210 days is replaced). District animal inventory numbers were considered constant over the year (*e.g.*, multiplying beef feedlot inventory by 360 annual feeding days and by .43 to represent the relevant portion of inventory provides the number of annual animal days supported by a daily ration for livestock class C1).

Because of their importance in demand estimation aggregated over species and all livestock, methods employed to estimate the number of animals represented by each livestock class are detailed. Inventory of feedlot cattle and of cows and heifers is available. Feedlot animal inventory was assumed maintained over the year as animals reaching market-weight were replaced. Over the year, 43 and 57 percent of inventory were in the C1 and C2 livestock classes, respectively (*i.e.*, of the 14,685 feedlot cattle in the district, 6,315 were represented as 900 lb. feeders and 8,370 were represented as 1,120 lb. feeders during the 360 feeding days in the year). Sixty percent of beef cows that have calved were represented as 1,200 lb. cows (C3 and C4) and 40 percent were represented as 1,400 lb. cows (C5 and C6) (Greg Lardy and Marc Bauer,



personal communication, August 2000). Each was in a livestock class representing a cow, three months since calving (C3 or C5 for 1,200 or 1,400 lb. cows, respectively) for 3 months and representing a cow, 11 months since calving (C4 or C6 for 1,200 or 1,400 lb. cows, respectively) for 4 months. Beef cows were on pasture during the remaining five months of the year and were therefore not included in the inventory of any livestock class.

Half of the district's dairy cows were categorized as producing 66 lbs. (D1) of milk daily and half 88 lbs. (D2) (Greg Lardy and Marc Bauer, personal communication, August 2000). The lactating period for a dairy cow was 305 days. During the remainder of the year, dairy animals were on pasture and therefore not included in the inventory of either class (approximately 15 percent of the herd at any one time).

Swine were represented by growing pig (H1 through H4) or sow (H5 and H6) livestock classes. Growing pigs spend 15, 49, 42, and 46 days in livestock classes H1 through H4, respectively. Growing pig inventory was maintained over the year as animals reaching the next production stage were replaced. Continuous replacement of growing pigs results in approximately 2.37 inventory turns annually. Days on feed for gestating (H5) and lactating (H6) sows were 126 and 21, respectively (*i.e.*, the portion of sow inventory gestating and lactating, respectively, was .14 and .86).

Ewes older than one year and sheep and lambs on feed represented sheep. Sheep and lambs on feed were calculated by subtracting ewes older than one year from the sheep and lambs inventory. Ewe livestock classes represented flushing (S1), gestating (S2), and lactating (S3) ewes. Days on feed were 30, 148, and 60, respectively. During the remainder of the year, ewes were on pasture and therefore not considered among the inventory of any livestock class (Roger Haugen, personal communication, August 2000). Sheep and lambs on feed were represented by livestock classes S4 and S5. Days on feed were 60 and 100, respectively.

## RESULTS AND CONCLUSIONS

When competitively priced with more traditional feedstuffs, sugarbeet pulp, wheat middlings, and potato waste can be important components of livestock rations in North Dakota. The unique characteristics of each co-product influence their value in meeting livestock nutrient requirements. Identification of those livestock classes most important in the demand for a specific co-product facilitate efforts by processors to target and educate producers and may influence decisions such as co-product pricing, processing, and plant location. Alerted to the value of co-products in the rations of their livestock, producers may more carefully consider their inclusion as a means to reduce feed cost.

### *Demand Estimation*

A system of demand equations including each feed and co-product was estimated for the individual livestock classes. Model statistics are included in Appendix A. Feed demands for beef cows included only forages and the co-products, sugarbeet pulp, wheat middlings, and potato waste. Cereal grains and soybean meal did not enter least-cost rations for beef cows. Ingredients included in least-cost dairy cow rations were the three co-products and corn, barley, soybean

meal, corn silage, and alfalfa. Neither oats nor prairie hay entered the dairy rations. Ingredients in the demand system for ewes included all three co-products, alfalfa, prairie hay, and soybean meal. No cereal grains or corn silage entered least-cost ewe rations. Sugarbeet pulp and potato waste were not included in the demand system parameters for lactating ewes because these co-products were part of the least-cost ration less than 8 percent of the time.

Demand equations for 10 livestock classes (feedlot beef cattle, lambs, and swine) did not need to be estimated. Solving for least-cost rations resulted in a vertical or nearly vertical demand curve for each co-product within a livestock class. Co-product demand in these cases was limited by one or more nutrient constraints. High-energy requirements for growing beef, concentrate limits for growing lambs, and high protein requirements for swine constrained the diets to the inclusion of specific feeds and limited the inclusion of co-products.

### *Co-product Demand*

Demand for the individual co-products is reported in Tables 2-4. Results indicate quantities demanded holding constant prices of all other feeds. Prices of traditional feeds were fixed at their twenty-year average and of the other co-products at the mid-range price.

*Sugarbeet Pulp.* Beef cattle, especially cows and heifers with calves, are the main consumers of sugarbeet pulp (Table 2). Quantities demanded by beef cows for all co-products are large relative to other species because of their predominance in the district. In 1999, the local price of sugarbeet pulp was approximately \$65 per ton. At this price, quantity demanded by livestock in the Central Crop Reporting district is low. In fact, demand by all beef cows in the district drops to less than 1,000 tons at prices higher than \$60 per ton and to zero at prices higher than \$68 per ton. At prices higher than \$68 per ton, only beef feedlot animals consume sugarbeet pulp as part of a least-cost ration. Sugarbeet pulp does not enter a least-cost ration for any livestock in the district at prices higher than \$80 per ton.

Quantity demanded is price responsive (*i.e.*, demand is elastic) at prices lower than \$62 per ton. For example, quantity demanded at a price of \$40 per ton (100,810 tons) is nearly twice that demanded at a price of \$50 per ton (52,000 tons). In particular, demand by beef cows is elastic over most of the price range while that by ewes and dairy cows is elastic only at prices greater than \$36 and \$45 per ton, respectively. Demand by feedlot beef cattle and lambs is limited by an intake constraint and is constant over the range.

Ignoring transportation costs, at a price of \$40 per ton, ruminants in the Central Crop Reporting District alone would demand approximately 15 percent of the sugarbeet pulp produced annually by all seven processing plants in the adjacent region. Strong demand by local livestock at slightly lower prices may prove important should demand for sugarbeet pulp drop in other domestic or overseas markets. If, for example, local cooperatives approve the use of genetically modified sugarbeet varieties by growers, demand for sugarbeet pulp originating from the Red River Valley may fall in overseas markets. Existing overseas customers of locally-produced beet pulp are either unable (*e.g.*, Western Europe) or have expressed hesitancy (*e.g.*, Japan) to accept a co-product from the processing of transgenetic beets.

Table 2. Demand for Sugarbeet Pulp by Livestock in the Central Crop Reporting District of North Dakota <sup>a, b, c</sup>

Price		Quantity Demanded				Elasticity
Ton	Lb.	Beef Cows	Dairy Cows	Sheep	Aggregate	
30	0.015	147.04	12.15	0.66	192.55	-1.60
32	0.016	128.27	12.01	0.65	173.62	-1.97
34	0.017	108.83	11.88	0.63	154.04	-2.33
36	0.018	89.73	11.76	0.62	134.80	-2.64
38	0.019	71.94	11.63	0.59	116.85	-2.86
40	0.020	56.17	11.46	0.55	100.88	-2.98
42	0.021	42.79	11.22	0.49	87.19	-3.01
44	0.022	31.84	10.82	0.42	75.79	-2.95
46	0.023	23.17	10.24	0.37	66.47	-2.82
48	0.024	16.47	9.45	0.33	58.94	-2.65
50	0.025	11.41	8.47	0.30	52.88	-2.45
52	0.026	7.68	7.35	0.29	48.03	-2.23
54	0.027	4.99	6.18	0.29	44.15	-1.98
56	0.028	3.07	5.04	0.29	41.09	-1.71
58	0.029	1.74	3.98	0.29	38.70	-1.39
60	0.030	0.88	3.06	0.28	36.92	-1.07
62	0.031	0.39	2.28	0.28	35.65	-0.84
64	0.032	0.07	1.66	0.28	34.71	-0.50
66	0.033	0.02	1.18	0.28	34.18	-0.37
68	0.034	0.01	0.82	0.28	33.81	-0.28
70	0.035	0.00	0.56	0.28	33.54	-0.50
72	0.036	0.00	0.37	0.00	33.07	-0.14
74	0.037	0.00	0.24	0.00	32.93	-0.10
76	0.038	0.00	0.15	0.00	32.85	

<sup>a</sup> Sugarbeet pulp demand is estimated at the mean price of all other ingredients. Mean prices (\$/lb.) are 0.041 (barley), 0.01 (corn silage), 0.04 (corn), 0.03 (alfalfa), 0.02 (hay), 0.041 (oats), and 0.09 (soybean meal).

<sup>b</sup> Quantity demanded is reported in thousand tons. Quantity demanded at prices below \$40 per ton is extrapolated from data obtained from solving for least-cost rations based on demand schedules estimated using regression.

<sup>c</sup> Quantity demanded by feedlot beef cattle and lambs was 32.7 and .28 thousand tons, respectively, over the range of prices considered.

The responsiveness of quantity of sugarbeet pulp demanded by local livestock to changes in price is also important because the market is imperfectly competitive and availability of the product can, to some extent, be adjusted as processors attempt to maximize revenues from beet pulp sales. Members of the three sugar cooperatives in southern Minnesota and the Red River Valley of eastern North Dakota and northwestern Minnesota produce approximately 50 percent of the nation's sugarbeets. The combined quantity of beet pulp produced by these cooperatives is marketed jointly through a shared cooperative, Midwest AgriCommodities. More than 70 percent of the dried beet pulp currently produced is sold to customers in Japan and Western Europe.

Dried beet pulp can be stored but because Midwest AgriCommodities strategically maintains a presence in three distinctly separate markets, quantity available in any one can also be adjusted simply by shifting product between markets.

*Wheat Middlings.* Wheat middlings are abundant throughout North Dakota with approximately five wheat processing plants in operation. There is one plant in the Central Crop Reporting District (Carrington). The price of wheat middlings in the state generally ranges from \$35 to \$55 per ton.

Wheat middlings are a good source of protein compared to other concentrates commonly used in North Dakota livestock rations and enter rations as a substitute for corn and barley (Table 3). Demand is elastic over the range of prices considered because demand for inclusion in beef cow rations is price responsive. Elasticity increases at higher prices. Quantity demanded over the price range considered is constant for beef and lamb feeders and for swine and is inelastic for dairy cows and ewes.

Table 3. Demand for Wheat Middlings by Livestock in the Central Crop Reporting District of North Dakota <sup>a, b, c</sup>

Price		Quantity Demanded				Elasticity
Ton	Lb.	Beef Cows	Dairy Cows	Sheep	Aggregate	
20	0.010	283.73	17.01	2.21	306.58	-1.12
24	0.012	228.58	15.62	2.10	249.94	-1.26
28	0.014	185.72	14.55	2.02	205.92	-1.42
32	0.016	151.13	13.68	1.95	170.40	-1.61
36	0.018	122.38	12.96	1.90	140.88	-1.86
40	0.020	97.96	12.35	1.85	115.81	-2.16
44	0.022	76.96	11.83	1.81	94.24	-2.51
48	0.024	58.88	11.38	1.78	75.67	-2.92
52	0.026	43.48	10.97	1.75	59.84	-3.37
56	0.028	30.60	10.61	1.72	46.57	-3.84
60	0.030	20.06	10.29	1.70	35.68	-4.32
64	0.032	11.64	10.00	1.67	26.95	-4.05
68	0.034	6.03	9.74	1.65	21.06	-2.63
72	0.036	3.35	9.49	1.63	18.12	-5.89
76	0.038	1.31	9.27	1.62	13.14	-2.17
80	0.040	0.14	9.06	1.60	11.76	

<sup>a</sup> Wheat Middlings demand is estimated at the mean price of all other ingredients. Mean prices (\$/lb.) are 0.041 (barley), 0.01 (corn silage), 0.04 (corn), 0.03 (alfalfa), 0.02 (hay), 0.041 (oats), and 0.09 (soybean meal).

<sup>b</sup> Quantity demanded is reported in thousand tons. Quantity demanded at prices below \$35 and above \$75 per ton is extrapolated from data obtained from solving for least-cost rations based on demand schedules estimated using regression.

<sup>c</sup> Quantity demanded by feedlot beef cattle was 26.87 thousand tons, by lambs .58 thousand tons, and by all swine .95 thousand tons, over the range of prices considered.

Even at prices higher than those generally found in the region, all species continue to consume wheat middlings as part of their least-cost ration. At prices higher than \$65 per ton, quantity demanded by beef cows rapidly moves to zero and dairy cows become the most important consumers. Although beef cow rations are the highest value feed use for wheat middlings within the typical price range found in North Dakota, other species demand a notable amount of this feed ingredient proportionate to their specified diet.<sup>9</sup> Wheat middlings can be an important ingredient in dairy diets. However, wheat middlings cannot exceed 24 percent of dairy cow or sheep rations. The influence of this constraint on demand for wheat middlings by these species is reflected in the relatively constant quantity demanded by each animal class over a wide range of prices. Demand by these species is inelastic (elasticities range from -.47 to -.44 for dairy cows and from -.37 to -.30 for ewes). The inelastic nature of demand by individual dairy cows over a wide range of prices is an important result. Quantity demanded by dairy cows, even at higher prices, will change nearly proportionate with changes in the herd size.

The Dakota Growers Pasta Company located in Carrington produces approximately 90,000 tons of middlings per year. Livestock in the Central Crop Reporting District will use this quantity when prices are lower than approximately \$45 per ton. Demand by beef cattle alone will exhaust the quantity of wheat middlings produced by the region's pasta plant at a price of \$40 per ton. Because wheat middlings can be an important component of livestock rations, even at higher prices, and their value differs by livestock class, diversified market opportunities exist for processors. Educating livestock producers that most highly value this co-product will be beneficial. In particular, producers should be made aware of the role of wheat middlings as a strong substitute for feed grains and soybean meal because of its high energy and protein content. And, at higher prices, the district's dairy producers should specifically be targeted.

*Potato Waste.* Although characterized by a downward slope, the shape of the demand curve for potato waste is not consistent with the generalized Leontief functional form used to estimate demand systems. Specifically, it is concave at higher prices. This results from the method used to correct for bias introduced by data censoring.

Potato waste is important in beef cow rations at prices up to \$13 per ton and in dairy cow rations up to prices of \$11.80 per ton (Table 4). Because potato waste is a high moisture ingredient (e.g., 20 lbs. as fed equals 4.6 lbs. of dry matter), animals have to consume a large quantity to meet their nutritional requirements. Large ruminants have the ability to do so. Demand by individual dairy cows is similar to that by individual beef cows at lower prices and is always greater at prices higher than \$5.80 per ton. However, aggregate demand is much more dependent on the district's beef cow population because it exceeds that of dairy cows by a 20:1 ratio. Sheep, specifically flushing and gestating ewes, demand small amounts of potato waste and the co-product does not enter the ration for feedlot beef cattle or lambs because these animals cannot consume enough of the high-moisture ingredient to meet their nutrient requirements. Swine are unable to efficiently digest this feed.

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<sup>9</sup>As is true for each of the co-products considered, relatively low animal numbers further contribute to the lack of importance of dairy and sheep to overall demand for wheat middlings, particularly at lower prices.

Table 4. Demand for Potato Waste by Livestock in the Central Crop Reporting District of North Dakota<sup>a, b, c</sup>

Price		Quantity Demanded					Elasticity
Ton	Cwt.	Beef Cows	Dairy Cows	Ewes	Aggregate		
4	0.2	1,205.72	82.98	2.46	1,291.16	-0.82	
4.6	0.23	1,072.77	77.09	2.32	1,152.18	-0.85	
5.2	0.26	963.55	72.25	2.20	1,038.00	-0.89	
5.8	0.29	871.75	68.19	2.10	942.04	-0.93	
6.4	0.32	793.19	64.71	2.01	859.91	-0.97	
7	0.35	724.95	61.68	1.94	788.57	-1.01	
7.6	0.38	664.96	59.03	1.87	725.86	-1.05	
8.2	0.41	611.57	56.67	1.81	670.06	-1.12	
8.8	0.44	562.95	54.55	1.76	619.26	-1.26	
9.4	0.47	515.69	52.65	1.72	570.05	-1.62	
10	0.5	463.10	50.91	1.67	515.68	-2.50	
10.6	0.53	394.78	49.33	1.63	445.74	-4.17	
11.2	0.56	304.52	47.87	1.59	353.98	-6.47	
11.8	0.59	203.99	46.53	1.22	251.74	-14.59	
12.4	0.62	117.09	0.54	0.34	117.97	-14.37	
13	0.65	58.15	0.00	0.02	58.17	-17.54	
13.6	0.68	25.19	0.00	0.00	25.19	-21.52	
14.2	0.71	9.21	0.00	0.00	9.21		

<sup>a</sup> Potato waste demand is estimated at the mean price of all other ingredients. Mean prices (\$/lb.) are 0.041 (barley), 0.01 (corn silage), 0.04 (corn), 0.03 (alfalfa), 0.02 (hay), 0.041 (oats), and 0.09 (soybean meal).

<sup>b</sup> Quantity demanded is reported in thousand tons. Quantity demanded at prices below \$8.20 per ton is extrapolated from data obtained from solving for least-cost rations based on demand schedules estimated using regression.

<sup>c</sup> No potato waste comprises part of least-cost rations for feedlot cattle or lambs or swine.

The aggregate demand schedule for potato waste is inelastic at prices lower than \$7 per ton and elastic at higher prices. The elastic region reflects the price responsiveness of quantity demanded for use in beef cow rations. Demand for potato waste by dairy cows is much less sensitive to changes in price and is in fact inelastic over the entire price range. This co-product is cost effective in fulfilling the nutrient requirements of dairy cows over a wide price range. However, its high moisture content limits the quantity an individual dairy cow can consume. The high moisture content of potato waste also limits its inclusion in ewe rations. Demand for use in ewe rations is also inelastic over the range of prices.

Livestock markets for potato waste must be in close proximity to a potato processing plant. Its high-moisture content limits the distance it can be economically transported. Farm-delivered potato waste from Avico USA in Jamestown is priced accordingly. Producers within 20 miles of the plant are charged a base price that is a function of the local price for corn and barley. An additional \$2 per loaded mile (\$0.08/ton) is charged for the distance over 20 miles. In

addition to transportation difficulties, the high-moisture content of this co-product can create storage problems. The cold winters in North Dakota require special equipment (such as lined delivery trucks) to prevent freezing.

The district's only potato processor, Avico USA, produces approximately 52,000 tons of potato waste annually, well below the quantity demanded for district livestock rations over the price range considered. As prices fall, quantity demanded increases quickly.<sup>10</sup> Although beef cow demand comprises 80 to 90 percent of quantity demanded, at higher prices demand from the district's dairy cows becomes important. Dairy herds located near a potato processing plant may provide an excellent market for locally-produced potato waste, even at higher prices. Close proximity to a potato processing plant would, for example, allow a producer building or expanding a dairy operation to take advantage of the potato waste as a feed, particularly if a price below its value as a feed ingredient and a long-term contract can be negotiated. At this time, potato waste base price is as low as \$7 per ton. At this price, it could be transported up to 95 miles to beef cow operations, where the farm-gate cost would equal \$13 per ton including the \$6 transportation cost, and up to 80 miles to dairy operations, where the farm-gate cost would be \$11.80 per ton.

### *Concluding Comments*

Local livestock can be an important market for co-products produced in and nearby the Central Crop Reporting District of North Dakota. Distinct differences in the level and nature of co-product demand (*e.g.*, price elasticity) over a range of prices and, particularly, between species, makes demand estimates important for both.

Data and methods used in this research have certain limitations affecting application of the results to both producers and processors. The primary limitation of this research is that transportation costs are ignored. Demand is reported based on the price a producer would pay for a co-product as an on-farm component of a least-cost ration. Transportation costs, which vary by location of the producer relative to the processor and the characteristics of the co-product, are not explicitly considered. The accuracy with which demand is estimated depends on these transportation costs, as well as other factors related to on-farm co-product management (*e.g.*, storage). Consequently, estimated demand for the region is likely more accurate for wheat middlings, which have a relatively low transportation cost and are easy to store for extended periods in existing facilities, than for potato waste, which has a relatively high transportation cost and may be difficult and costly for the producer to store and handle. The results are applicable for individual livestock producers by adjusting for costs associated with transportation from the plant and co-product management. However, it is more challenging to correct estimated demand by livestock within a region for transportation and other costs so the results are more directly relevant for use by individual processors.

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<sup>10</sup>Caution is advised when interpreting demand for potato waste because distance from the plant to the farm was not considered (*i.e.*, price must be adjusted for transportation cost and other considerations related to the on-farm storage and management of potato waste).

To do so, demand would need to be estimated at the plant. Data used (*e.g.*, feed prices) would need to be refined for individual livestock classes based on explicit consideration of the distance between co-product origin and producers or groups of producers of livestock most important in demand for this co-product (*i.e.*, the transportation cost from the plant). Individual livestock classes considered would need to be defined not only by their nutritional requirements and constraints but also by their location relative to the plant and any other factors likely to influence demand for the co-product as an ingredient in the ration. This modification is particularly important when transportation or co-product management costs are high (*e.g.*, estimating demand for potato waste). These efforts need not be limited to estimating demand for co-products from existing plants but are also appropriate to evaluate the impact of plant location and other decisions.

A second limitation of this study is that the number of livestock classes was limited so that animals with varying nutritional requirements were represented within a single class. This limitation does not greatly affect the accuracy of the results because animal classes used correspond to livestock numbers in the Central Crop Reporting District in North Dakota. However, it does limit the usefulness of the results to producers who would benefit from more detailed information about co-product value for their specific herds. A similar modification as that previously suggested would help researchers or practitioners overcome this limitation and thereby improve application of the results to livestock producers. Specifically, more livestock classes might be considered within each species important to co-product demand (*e.g.*, four rather than two categories of dairy animals to better represent milk production levels typifying the region). Consideration of the specific nutritional components of the co-product produced from a particular plant will also help local producers better evaluate its value in their livestock rations.



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## APPENDIX – DEMAND SYSTEM EQUATIONS

The system of demand equations estimated for individual livestock classes are reported here. Each system is presented within a table; information provided includes parameter estimations and their standard error, t-statistic, and p-value. The t-test is used to measure the significance level associated with each parameter estimate. Model statistics presented include r-squared, standard error, and LM heteroscedasticity test statistic (HD). The HD test is not as meaningful as it otherwise might be because the system of equations were estimated using SUR. When errors are not normally distributed, as is the case when SUR is used, we expect a high HD in all models. Each model containing sufficient data for an econometrics application was estimated into a system of equations. Parameters in demand estimations are defined as follows:  $\beta_{BP}$  = Sugarbeet Pulp,  $\beta_P$  = Potato Waste,  $\beta_W$  = Wheat Middlings,  $\beta_A$  = Alfalfa,  $\beta_H$  = Prairie Hay,  $\beta_S$  = Corn Silage,  $\beta_C$  = Corn,  $\beta_B$  = Barley,  $\beta_O$  = Oats,  $\beta_{SM}$  = Soybean meal,  $\beta_F$  = Forages.

The equations also include a delta parameter associated with the correction procedure used for data censoring. Probit analysis was not necessary for ingredients present at consistent levels in individual rations. Tables A.1 through A.4 provide the system of equations estimated for the beef diets (C3, C4, C5, and C6) and individual model statistics for each of the three co-products: beet pulp, wheat middlings, and potato waste.

Table A.1. Estimated Parameters for System of Equations of C3, 1,200 Lb. Beef Cow, 3 Months Since Calving

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-16.872	1.860	-9.07	[.000]
$\beta_{BP,W}$	7.177	0.693	10.35	[.000]
$\beta_{BP,P}$	4.680	1.318	3.55	[.000]
$\beta_{BP,F}$	15.013	1.067	14.06	[.000]
$\delta_1$	-2.134	0.881	-2.42	[.015]
$\beta_{W,W}$	-28.24	1.091	-25.88	[.000]
$\beta_{W,P}$	20.93	1.069	19.58	[.000]
$\beta_{W,F}$	16.463	0.809	20.33	[.000]
$\delta_2$	2.632	0.810	3.25	[.001]
$\beta_{P,P}$	-66.106	5.188	-12.74	[.000]
$\beta_{P,F}$	20.48	1.266	16.16	[.000]
$\delta_3$	-8.426	2.757	-3.05	[.002]
$\beta_{F,F}$	-28.219	1.399	-20.16	[.000]

\*Note- $\beta_F$  is a weighted measure of alfalfa, corn silage, and hay from the least-cost model.

C3 Model Statistics			
	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	2.8	2.79	7.01
R-Squared	.609	.765	.920
LM Heteroscedasticity Test	29.1	.406	24.5

Table A.2. Estimated Parameters for System of Equations of C4, 1,200 Lb. Beef Cow, 11 Months Since Calving

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-7.707	1.907	-4.04	[.000]
$\beta_{BP,W}$	2.248	0.940	2.39	[.017]
$\beta_{BP,P}$	2.656	1.722	1.54	[.123]
$\beta_{BP,F}$	8.892	0.664	13.39	[.000]
$\beta_1$	-1.251	0.882	-1.42	[.156]
$\beta_{W,W}$	-26.807	1.722	-15.56	[.000]
$\beta_{W,P}$	18.904	1.448	13.05	[.000]
$\beta_{W,F}$	13.329	0.715	18.64	[.000]
$\beta_2$	3.006	1.028	2.92	[.003]
$\beta_{P,P}$	-52.869	7.468	-7.08	[.000]
$\beta_{P,F}$	13.866	0.928	14.94	[.000]
$\beta_3$	12.184	3.829	3.18	[.001]
$\beta_{F,F}$	-6.154	0.780	-7.88	[.000]

C4 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	1.97	3.12	10.25
R-Squared	.518	.640	.809
LM Heteroscedasticity Test	34.5	11.2	14.02

Table A.3. Estimated Parameters for System of Equations of C5, 1,400 Lb. Beef Cow, 3 Months Since Calving

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-18.439	2.078	-8.873	[.000]
$\beta_{BP,W}$	6.905	0.740	9.328	[.000]
$\beta_{BP,P}$	4.902	1.473	3.326	[.001]
$\beta_{BP,F}$	17.484	1.219	14.334	[.000]
$\beta_1$	-2.107	1.015	-2.074	[.038]
$\beta_{W,W}$	-30.504	1.155	-26.403	[.000]
$\beta_{W,P}$	22.257	1.162	19.139	[.000]
$\beta_{W,F}$	19.274	0.856	22.514	[.000]
$\beta_2$	1.981	0.855	2.316	[.021]
$\beta_{P,P}$	-68.557	5.785	-11.848	[.000]
$\beta_{P,F}$	22.818	1.411	16.170	[.000]
$\beta_3$	-13.517	3.079	-4.389	[.000]
$\beta_{F,F}$	-33.805	1.575	-21.457	[.000]

C5 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	3.27	2.90	7.78
R-Squared	.559	.790	.921
LM Heteroscedasticity Test	30.74	1.55	17.9

Table A.4. Estimated Parameters for System of Equations of C6, 1,400 Lb. Beef Cow, 11 Months Since Calving

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-8.080	2.144	-3.768	[.000]
$\beta_{BP,W}$	3.161	0.838	3.769	[.000]
$\beta_{BP,P}$	4.964	1.371	3.619	[.000]
$\beta_{BP,F}$	10.792	1.114	9.681	[.000]
$\beta_1$	-3.450	0.857	-4.022	[.000]
$\beta_{W,W}$	-24.794	1.436	-17.259	[.000]
$\beta_{W,P}$	13.786	1.220	11.294	[.000]
$\beta_{W,F}$	21.117	1.022	20.652	[.000]
$\beta_2$	-0.965	0.858	-1.124	[.261]
$\beta_{P,P}$	-56.732	5.922	-9.578	[.000]
$\beta_{P,F}$	27.947	1.410	19.811	[.000]
$\beta_3$	-13.25	3.684	-3.595	[.000]
$\beta_{F,F}$	-33.809	1.501	-22.516	[.000]

C6 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	2.03	2.90	7.91
R-Squared	.535	.746	.909
LM Heteroscedasticity Test	12.93	40.2	4.23

The second set of equations and statistics, representing dairy animals, are presented in Tables A.5 and A.6. The ingredients included in the systems' parameters are sugarbeet pulp, wheat middlings, potato waste, corn, corn silage, soybean meal, barley, and alfalfa.

Table A.5. Estimated Parameters for System of Equations of D1, 1,320 Lb. Dairy Cow, Producing 66 Lbs. of Milk Per Day

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	7.688	2.652	2.90	[.004]
$\beta_{BP,W}$	-1.101	0.505	-2.18	[.029]
$\beta_{BP,P}$	9.352	1.037	9.02	[.000]
$\beta_{BP,C}$	-8.046	2.118	-3.80	[.000]
$\beta_{BP,S}$	2.406	1.883	1.28	[.201]
$\beta_{BP,A}$	7.069	1.011	6.99	[.000]
$\beta_{BP,B}$	2.397	1.928	1.24	[.214]
$\beta_{BP,SM}$	-1.164	0.434	-2.68	[.007]
$\beta_1$	-3.828	1.068	-3.58	[.000]
$\beta_{W,W}$	1.250	0.534	2.34	[.019]
$\beta_{W,P}$	10.598	0.584	18.14	[.000]
$\beta_{W,C}$	3.406	1.033	3.29	[.001]
$\beta_{W,S}$	-5.314	1.549	-3.43	[.001]
$\beta_{W,A}$	-3.454	0.695	-4.96	[.000]
$\beta_{W,B}$	0.952	0.931	1.02	[.307]
$\beta_{W,SM}$	3.928	0.373	10.51	[.000]
$\beta_{P,P}$	-4.972	3.656	-1.36	[.174]
$\beta_{P,C}$	-8.294	2.154	-3.85	[.000]
$\beta_{P,S}$	24.327	2.993	8.13	[.000]
$\beta_{P,A}$	8.093	1.129	7.17	[.000]
$\beta_{P,B}$	2.767	1.780	1.55	[.120]
$\beta_{P,SM}$	-6.477	0.583	-11.09	[.000]
$\beta_3$	-14.641	4.705	-3.11	[.002]
$\beta_{C,C}$	2,488.59	726.275	3.43	[.001]
$\beta_{C,S}$	-4,887.49	1,442.38	-3.39	[.001]
$\beta_{C,A}$	-15.01	3.066	-4.89	[.000]
$\beta_{C,B}$	-3.356	5.335	-0.63	[.529]
$\beta_{C,SM}$	1.389	1.823	0.76	[.446]
$\beta_4$	-0.371	0.874	-0.42	[.671]
$\beta_{S,S}$	9,626.55	2,871.4	3.35	[.001]
$\beta_{S,A}$	59.379	6.120	9.70	[.000]
$\beta_{S,B}$	23.735	6.004	3.95	[.000]
$\beta_{S,SM}$	-17.627	3.471	-5.08	[.000]
$\beta_5$	6.2093	2.346	2.65	[.008]
$\beta_{A,A}$	-22.087	1.811	-12.19	[.000]
$\beta_{A,B}$	-1.657	1.939	-0.85	[.393]
$\beta_{A,SM}$	14.057	0.887	15.83	[.000]
$\beta_{B,B}$	-1.176	7.470	-0.16	[.875]
$\beta_{B,SM}$	1.719	0.937	1.83	[.067]
$\beta_7$	-3.195	1.252	-2.55	[.011]
$\beta_{SM,SM}$	-2.053	0.689	-2.98	[.003]
$\beta_7$	0.538	0.179	2.99	[.003]

D1 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	3.68	2.01	6.77
R-Squared	.680	.505	.948
LM Heteroscedasticity Test	.026	13.34	4.32

Table A.6. Estimated Parameters for System of Equations of D2, 1,320 Lb. Dairy Cow, Producing 88 Lbs. of Milk Per Day

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	9.040	2.682	3.37	[.001]
$\beta_{BP,W}$	-1.286	0.511	-2.52	[.012]
$\beta_{BP,P}$	9.210	1.029	8.95	[.000]
$\beta_{BP,C}$	-7.703	2.132	-3.61	[.000]
$\beta_{BP,S}$	2.645	1.893	1.40	[.163]
$\beta_{BP,A}$	6.507	1.073	6.06	[.000]
$\beta_{BP,B}$	2.612	1.960	1.33	[.183]
$\beta_{BP,SM}$	-1.766	0.435	-4.05	[.000]
61	-4.191	1.078	-3.88	[.000]
$\beta_{W,W}$	1.153	0.528	2.18	[.029]
$\beta_{W,P}$	10.662	0.579	18.39	[.000]
$\beta_{W,C}$	3.336	1.041	3.20	[.001]
$\beta_{W,S}$	-5.382	1.551	-3.47	[.001]
$\beta_{W,A}$	-3.489	0.729	-4.78	[.000]
$\beta_{W,B}$	1.189	0.945	1.26	[.208]
$\beta_{W,SM}$	3.994	0.355	11.24	[.000]
$\beta_{P,P}$	-4.194	3.594	-1.17	[.243]
$\beta_{P,C}$	-8.120	2.144	-3.79	[.000]
$\beta_{P,S}$	24.517	3.001	8.17	[.000]
$\beta_{P,A}$	8.058	1.164	6.92	[.000]
$\beta_{P,B}$	3.686	1.760	2.09	[.036]
$\beta_{P,SM}$	-7.411	0.537	-13.79	[.000]
63	-14.168	4.634	-3.06	[.002]
$\beta_{C,C}$	2,470.59	728.027	3.39	[.001]
$\beta_{C,S}$	-4,860.37	1,445.69	-3.36	[.001]
$\beta_{C,A}$	-14.29	3.099	-4.61	[.000]
$\beta_{C,B}$	-0.702	5.439	-0.13	[.897]
$\beta_{C,SM}$	1.625	1.826	0.89	[.373]
64	-0.264	0.874	-0.30	[.763]
$\beta_{S,S}$	9,566.96	2,877.7	3.32	[.001]
$\beta_{S,A}$	61.208	6.156	9.94	[.000]
$\beta_{S,B}$	26.033	6.070	4.29	[.000]
$\beta_{S,SM}$	-18.634	3.483	-5.35	[.000]
65	6.374	2.353	2.71	[.007]
$\beta_{A,A}$	-23.861	1.963	-12.15	[.000]
$\beta_{A,B}$	-2.814	2.083	-1.35	[.177]

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{A,SM}$	15.122	0.837	18.05	[.000]
$\beta_{B,B}$	-8.191	7.611	-1.08	[.282]
$\beta_{B,SM}$	3.995	0.908	4.40	[.000]
$\beta_7$	-3.309	1.306	-2.53	[.011]
$\beta_{SM,SM}$	-3.585	0.576	-6.22	[.000]

#### D2 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	3.68	2.02	6.81
R-Squared	.681	.505	.947
LM Heteroscedasticity Test	.030	11.87	4.72

Tables A.7 through A.9 list the system of equations and individual model statistics for 150 lb. ewes in the stages of flushing, gestating, and lactating. The ingredients that make up the parameters include sugarbeet pulp, wheat middlings, potato waste, alfalfa, prairie hay, and soybean meal. The equation system in Table A.9 does not include the co-products of sugarbeet pulp and potato waste. Sugarbeet pulp and potato waste entered the rations less than 8 percent of the time out of the 540 observations generated in GAMS for lactating ewes.

Table A.7. Estimated Parameters for System of Equations of S1, 150 lb. Ewe (Flushing)

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-0.201	0.065	-3.10	[.002]
$\beta_{BP,W}$	0.173	0.030	5.65	[.000]
$\beta_{BP,P}$	0.043	0.038	1.12	[.261]
$\beta_{BP,A}$	0.382	0.026	14.41	[.000]
$\beta_{BP,H}$	-0.023	0.030	-0.77	[.443]
$\beta_1$	0.062	0.036	1.70	[.089]
$\beta_{W,W}$	-0.117	0.039	-2.95	[.003]
$\beta_{W,P}$	0.354	0.032	11.05	[.000]
$\beta_{W,A}$	0.152	0.023	6.45	[.000]
$\beta_{W,H}$	0.207	0.028	7.16	[.000]
$\beta_{P,P}$	0.685	0.179	3.82	[.000]
$\beta_{P,A}$	-0.149	0.032	-4.67	[.000]
$\beta_{P,H}$	0.610	0.038	15.86	[.000]
$\beta_3$	-0.437	0.209	-2.09	[.036]
$\beta_{A,A}$	0.018	0.044	0.41	[.681]
$\beta_{A,H}$	0.525	0.043	12.12	[.000]
$\beta_{H,H}$	1.236	0.057	21.65	[.000]



S1 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	.215	.18	1.29
R-Squared	.80	.442	.97
LM Heteroscedasticity Test	10.68	236.4	7.58

Table A.8. Estimated Parameters for System of Equations of S2, 150 Lb. Ewe (Gestating)

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{BP,BP}$	-0.141	0.046	-3.03	[.002]
$\beta_{BP,W}$	0.102	0.020	5.02	[.000]
$\beta_{BP,P}$	0.028	0.028	1.00	[.315]
$\beta_{BP,A}$	0.283	0.030	9.31	[.000]
$\beta_{BP,H}$	-0.028	0.025	-1.11	[.268]
$\beta_1$	0.023	0.025	0.91	[.364]
$\beta_{W,W}$	0.312	0.027	11.31	[.000]
$\beta_{W,P}$	0.292	0.022	12.78	[.000]
$\beta_{W,A}$	0.258	0.028	9.13	[.000]
$\beta_{W,H}$	-0.013	0.025	-0.52	[.600]
$\beta_{P,P}$	0.278	0.129	2.15	[.031]
$\beta_{P,A}$	-0.137	0.035	-3.86	[.000]
$\beta_{P,H}$	0.438	0.033	13.17	[.000]
$\beta_3$	-0.386	0.154	-2.50	[.012]
$\beta_{A,A}$	0.133	0.060	2.20	[.028]
$\beta_{A,H}$	0.782	0.054	14.44	[.000]
$\beta_{H,H}$	1.073	0.063	16.77	[.000]

S2 Model Statistics

	Beet Pulp	Wheat Middlings	Potato Waste
Standard Error	.137	.117	.163
R-Squared	.802	.452	.962
LM Heteroscedasticity Test	12.51	124.9	6.25

Table A.9. Estimated Parameters for System of Equations of S3, 150 Lb. Ewe (Lactating)

Parameter	Estimate	Std. Error	T-Value	P-Value
$\beta_{W,W}$	0.670	0.048	13.72	[.000]
$\beta_{W,A}$	0.160	0.053	2.99	[.003]
$\beta_{W,SM}$	0.302	0.028	10.58	[.000]
$\beta_{W,H}$	0.108	0.030	3.53	[.000]
$\beta_{A,A}$	1.919	0.141	13.57	[.000]
$\beta_{A,SM}$	0.407	0.053	7.59	[.000]
$\beta_{A,H}$	0.659	0.075	8.75	[.000]
$\beta_{SM,SM}$	-0.172	0.027	-6.54	[.000]
$\beta_{SM,H}$	-0.239	0.030	-7.81	[.000]
$\beta_{H,H}$	1.374	0.053	25.71	[.000]

S3 Model Statistics

	Wheat Middlings
Standard Error	.240
R-Squared	.376
LM Heteroscedasticity Test	160.9

*Straight Line Estimation*

Quantity demanded was consistent, or nearly so, over a range of prices for 10 of the 19 diets (beef feedlot diets C1 and C2; lamb diets S4 and S5; and all swine diets, H1 through H6). That is, the resulting demand curve was or was nearly vertical for each of the co-products considered. The inelastic nature of demand over the range of prices considered resulted from one or more model constraints. High-energy requirements in beef feedlot situations, concentrate limits in sheep feedlot situations, and high protein requirements in swine situations constrain the diets to the inclusion of specific feedstuffs and specifically limit the inclusion of co-products.