# Informed Principal and Countervailing Incentives

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#### Abstract

It has been shown by Maskin and Tirole (1990, proposition 11) that with quasi-linear preferences and private values, an informed principal neither gains nor loses if her private information is revealed before contracting takes place. The note shows that this result may not hold when the agent faces countervailing incentives.

*Keywords:* Informed Principal; Countervailing Incentives; Risk Neutrality

JEL Classification: D82

#### 1 Introduction

In a well-known contribution, Maskin and Tirole (1990) study the case of bilateral adverse selection in contracting with private values. Their last result (Maskin and Tirole, 1990, proposition 11) has an important implication for the theory of adverse selection: they show that when the agent (he) is risk-neutral, the principal (she) can not benefit from her private information. This amounts to say that all the results obtained in the standard quasi-linear setting with an uninformed principal translate directly to the informed principal case, up to a parameterization. The aim of this note is to show that this result is no more true when one removes the assumption that one type of agent is always more efficient than the other. Relatedly, Cella (2004) shows that if the types are correlated, the principal is better off if her information is not revealed.

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Following Lewis and Sappington (1989), we consider the Baron and Myerson (1982) framework with only two types, and allow for countervailing incentives that arise when fixed and marginal costs are negatively related. Then, depending on the level of the characteristic (say, quantity) traded, one type of agent or the other may be more efficient. When the agent is not sure about the principal's type, he does not know what quantity will be traded, and he may not know whether he is the efficient type or not. This sometimes softens the incentive constraints, and therefore makes pooling desirable for the principal. We illustrate this idea in a setting where such effect may even allow to implement the first-best allocation.

## 2 Setting

The notations are borrowed from Maskin and Tirole (1990) whenever possible. The principal and the agent exchange some good with contractible attribute  $y \in \mathbb{R}^+$  (say, quantity or observable quality) and a monetary transfer  $t \in \mathbb{R}^+$ . The agent has type  $\theta_i$  with probability  $p_i$ , i = 1, 2, such that  $p_1 + p_2 = 1$ , and a type-independent reservation utility equal to 0. The principal has two possible types  $\alpha^j$ , j = 1, 2, with probability  $\pi^j$  such that  $\pi^1 + \pi^2 = 1$ . The utility function of the principal is

$$V(y, t, \alpha) = \alpha v(y) - t$$

where v is increasing and strictly concave, and without loss of generality,  $\alpha_1 < \alpha_2$ . The utility of the agent is

$$U(y, t, \theta) = t - (F(\theta) + \theta y)$$

In the following, the notations are shortened with indices pertaining to types: Subscripts refer to the agent and superscripts to the principal. We assume the following regarding fixed costs  $F(\theta)$  and marginal costs  $\theta$ :

Assumption 1  $\theta_1 < \theta_2$  and  $F(\theta_1) = F_1 > F_2 = F(\theta_2)$ .

In addition, we assume that the principal's preferred levels if he knew the agent's type,  $y_i^{j*} = (v')^{-1} \frac{\theta_i}{\alpha^j}$ , satisfy:

Assumption 2  $y_i^{j*} > 0$  for all i, j.

Given assumption 1, technology 1 performs better for high levels of y than technology 2, while the converse is true for low levels. Indeed, let  $\hat{y} = \frac{F_1 - F_2}{\theta_2 - \theta_1}$ , then the following holds:

$$F_1 + \theta_1 y \le F_2 + \theta_2 y$$
 if and only if  $y \ge \hat{y}$  (1)

This is the source of countervailing incentives in this model: for a given (y, t), one or the other type of agent may have an incentive to misrepresent his type, but which one depends on y. In turn, the level that the principal wants to implement is also a function of her own private valuation<sup>1</sup>.

We know from Myerson (1983) that we can restrict attention to mechanisms where first the principal offers a full contingent contract, then the agent and the principal simultaneously announce their type, under incentive and interim participation constraints, and finally the corresponding allocation is implemented. The informed principal must thus solve simultaneously the program  $(P^j)$  for each type  $\alpha^j$ , which is

$$\begin{array}{l} \underset{\{y_{i}^{j},t_{i}^{j}\}}{Max} & p_{1}V^{j}(y_{1}^{j},t_{1}^{j}) + p_{2}V^{j}(y_{2}^{j},t_{2}^{j}) \\ & \pi^{1}U_{i}(y_{i}^{1},t_{i}^{1}) + \pi^{2}U_{i}(y_{i}^{2},t_{i}^{2}) \geq 0 \quad (IR_{i}) \\ & \sum_{j=1}^{2}\pi^{j}U_{1}(y_{1}^{j},t_{1}^{j}) \geq \sum_{j=1}^{2}\pi^{j}U_{1}(y_{2}^{j},t_{2}^{j}) \quad (IC_{1}) \\ & \sum_{j=1}^{2}\pi^{j}U_{2}(y_{2}^{j},t_{2}^{j}) \geq \sum_{j=1}^{2}\pi^{j}U_{2}(y_{1}^{j},t_{1}^{j}) \quad (IC_{2}) \\ & \sum_{i=1}^{2}p_{i}V^{1}(y_{i}^{1},t_{i}^{1}) \geq \sum_{i=1}^{2}p_{i}V^{1}(y_{i}^{2},t_{i}^{2}) \quad (IC^{1}) \\ & \sum_{i=1}^{2}p_{i}V^{2}(y_{i}^{2},t_{i}^{2}) \geq \sum_{i=1}^{2}p_{i}V^{2}(y_{i}^{1},t_{i}^{1}) \quad (IC^{2}) \end{array}$$

<sup>&</sup>lt;sup>1</sup>Maskin and Tirole (1990) assume that  $U(y, t, \theta_1) > U(y, t, \theta_2)$  for all (y, t), and a Spence-Mirrlees property for some results. Here, the first assumption is relaxed, as (1) illustrates, and that is the essential point. A Spence-Mirrlees condition holds here (that boils down to  $c_1 < c_2$ ), but is not important in itself. The same results obtain with the following specification, that entails no fixed cost and no single-crossing:  $U_i(y, t) = t - c_i(y)$ , with  $c_i(0) = c_i(0^+) = 0$ ,  $0 < c'_2(0) < c'_1(0)$  and  $0 \le c''_1 < c''_2$ . A property analogous to (1) would then hold, namely, there is some  $\hat{y}$  such that  $c_1(y) \le c_2(y)$  if and only if  $y \ge \hat{y}$ . Similarly, a type-dependent reservation utility - as in Maggi and Rodriguez-Clare (1995) and Jullien (2000) - could also generate the same results, since reservation utility would play the same role as fixed cost.

where the first two incentive constraints concern the agent and the last two concern the principal. Depending on the parameters, the solutions to this program have (very) different structures. The next section focuses on the case in which the principal may benefit from countervailing incentives.

## 3 Results

An important implication of countervailing incentives for the principal is the following:

**Proposition 1** The informed principal can achieve her first-best payoff if and only if:

$$\pi^1 y_2^{1*} + \pi^2 y_2^{2*} \le \hat{y} \le \pi^1 y_1^{1*} + \pi^2 y_1^{2*} \tag{2}$$

**Proof.** Sufficiency. Assume that (2) holds. We want to show that the allocation  $\{y_i^{j*}, t_i^{j*} = F_i + \theta_i y_i^{j*}\}$  is implementable.

- a. The participation constraint of the agent is binding.
- b. The incentive constraints of the principal are necessarily satisfied: by definition,  $\{y_i^{j*}\}$  are her preferred levels.
- c. Consider now the incentive constraint of agent 1; because  $(IR_1)$  is binding, it writes:

$$\sum_{j=1}^{2} \pi^{j} U_{1}(y_{2}^{j}, t_{2}^{j}) \leq 0$$

or:

$$F_1 + \theta_1(\pi^1 y_2^{1*} + \pi^2 y_2^{2*}) \ge \pi^1 t_2^{1*} + \pi^2 t_2^{2*} = F_2 + \theta_2(\pi^1 y_2^{1*} + \pi^2 y_2^{2*})$$

But since  $\pi^1 y_2^{1*} + \pi^2 y_2^{2*} \leq \hat{y}$ , we know from (1) that the inequality is satisfied.

d. The case of agent 2 is symmetric.

*Necessity.* Now, assume that her preferred outcome is implementable by the principal. Then it must be the case that the incentive constraints are satisfied. It is straightforward to see by reverting the preceding calculations that this implies (2).  $\blacksquare$ 

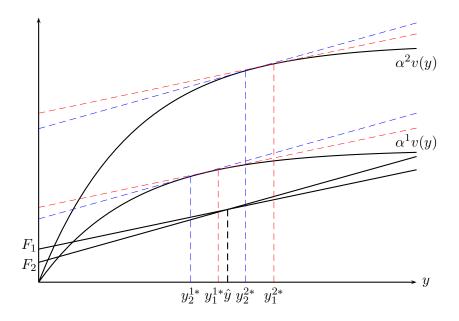


Figure 1: A setting satisfying the conditions of corollary 1

Under some circumstances, the principal can implement her first-best choice. However, this may simply come from the countervailing effect, and not be a consequence of the principal's private information. Indeed, if for  $\alpha^j$  one has

$$y_2^{j*} < \hat{y} < y_1^{j*}$$

then, by applying proposition 1 to  $\pi^j = 1$ , we know the principal  $\alpha^j$  can implement the first-best. If both  $\alpha^1$  and  $\alpha^2$  satisfy the inequality, both types can implement the first-best independently and the fact that the principal is privately informed has no consequence. As the next corollary shows, however, this is true in broader circumstances.

**Corollary 1** If  $y_1^{1*} < \hat{y} < y_2^{2*}$ , no principal could implement the first-best if the agent knew her type. However, if  $v''' \leq 0$ , there always exist beliefs  $(\pi^1, \pi^2)$  of the agent such that both types of principal can implement their preferred allocation.

**Proof.** Consider  $y^*(\alpha, \theta)$ , the mapping from both types to the principal's preferred solution. By the first-order condition, we have  $v'(y^*) = \frac{\theta}{\alpha}$  for any  $\alpha, \theta$ . It is clear that  $\frac{\partial y^*}{\partial \alpha} > 0$  and  $\frac{\partial y^*}{\partial \theta} < 0$ . Since  $\alpha_1 < \alpha_2$ , under the given

condition, we have  $y_2^{1*} \le y_1^{1*} < \hat{y} < y_2^{2*} \le y_1^{2*}$ .

Assume the agents knows  $\alpha$ , then  $\pi^j = 0$  or 1, and condition (2) is violated. Thus no principal can implement the first-best here if  $\alpha$  is common knowledge.

Now we want to find  $(\pi_1, \pi_2)$  that verify condition (2). Satisfying it is equivalent to be able to find a  $\pi^1$  such that

$$\frac{y_2^{2^*} - \hat{y}}{y_2^{2^*} - y_2^{1^*}} \le \pi_1 \le \frac{y_1^{2^*} - \hat{y}}{y_1^{2^*} - y_1^{1^*}} \tag{3}$$

which is feasible when the corresponding interval is non empty (it is included in (0, 1) in any case). From the study of  $y^*$ , we already have  $y_2^{2*} - \hat{y} < y_1^{2*} - \hat{y}$ . We now show that  $y^*(\alpha, \theta)$  has increasing differences when  $v''' \leq 0$  to complete the proof. We can differentiate twice the first-order condition because it holds for any  $\alpha, \theta$ , so that:

$$\frac{\partial^2 y^*}{\partial \alpha \partial \theta} v''(y^*) = -\frac{1}{\alpha^2} - \frac{\partial y^*}{\partial \alpha} \frac{\partial y^*}{\partial \theta} v'''(y^*)$$

The second term of the RHS is negative when  $v''' \leq 0$ ; also, v'' < 0 given strict concavity. Thus necessarily  $\frac{\partial^2 y^*}{\partial \alpha \partial \theta} \geq 0$ , which means  $y^*$  has increasing differences, implying  $y_2^{2^*} - y_2^{1^*} \geq y_1^{2^*} - y_1^{1^*}$ , so we can conclude that there always exists a  $\pi^1$  satisfying (3).

The corollary is illustrated in figure 1. We know from Lewis and Sappington (1989, Section 4) that a principal may want to create countervailing incentives. When the principal is informed, it may be strictly desirable for her to preserve *Bayesian* countervailing incentives (by keeping private her information as long as possible), or even attempt to manipulate beliefs to create them.

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