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Abstract

The literature on the effects of market concentration in platform industries or two-sided markets often compares the competitive outcome against a benchmark. This benchmark is either the "joint management" solution in which one decision maker runs all platforms or a "pure" monopoly with just one platform. Literature has not generally discussed, which benchmark is the appropriate one. We show that the appropriate benchmark, i.e. how many platforms the monopolist will operate, depends on whether agents multi- or singlehome, whether the externalities are positive or negative, and in some cases on the properties of the demand functions. Different situations require different benchmarks. Our results also help to anticipate the effects of proposed platform mergers, where the assessment might crucially depend on the number of platforms after a merger.

1. Introduction

In the last decade, an enormous amount of research has been done on “two-sided markets” or “platform industries”, starting with the seminal articles of Caillaud & Jullien (2003); Rochet & Tirole (2003). Many contributions are interested in the impact of market concentration on prices, quantities, and welfare. One approach to study such effects is to compare the competitive outcome against the monopoly benchmark.

In doing so some contributions assume the monopoly to be a “joint management” of the existing platforms, i.e. they assume that all platforms continue to operate and are controlled by a single decision maker who maximizes joint profits [e.g. Ambrus & Reisinger (2006); Anderson & Coate (2005); Chandra & Collard-Wexler (2009); Weyl (2010)]. Other authors assume that a monopolist operates just one platform [e.g. Chaudhri (1998); McCabe & Snyder (2007)].

While sometimes a footnote points the reader to the fact that one could have used another benchmark [e.g. Anderson & Coate (2005); Chandra & Collard-Wexler (2009)], to our knowledge, only Ambrus & Argenziano (2009)] actually discuss, how many platforms a monopolist optimally chooses. They consider a situation with purely positive network externalities and singlehoming agents. In their model, the monopolist can choose between opening one or two homogeneous platforms. Consistent with our results, they find that a monopolist will always prefer to operate just one platform, if agents on each market side are homogeneous (their Theorem 2). They add that if agents are heterogeneous, there might be an incentive to operate a second platform in order to implement second-degree price discrimination.

Other reasons why a monopolist may find it more profitable to run just a single platform, instead of multiple ones (or vice versa) could be the well-known ones from traditional markets, e.g. the presence (or absence) of economies of scale or high fixed costs. However, on two-sided markets, the monopolist must also take the impact of the number of platforms on the magnitude of the indirect network effects into account, when deciding how many platforms to operate. To understand the economic intuition, consider the case of singlehoming agents on both market sides, i.e. agents join at most one platform, and positive externalities that is, both market sides benefit from a high number of agents on the other market side. Then, because of singlehoming, operating two platforms would mean that the “joint manager” faces cannibalization effects in the sense that platforms steal members from each other, making both of them less attractive to agents than a large, unified platform. Agents would experience a higher gross utility on a unified platform that translates into a higher willingness to pay, which results in increasing profits. As a consequence, in this case using the "joint management" benchmark would be of academic interest only. For practical purposes, e.g. when assessing the impact of potential mergers, such a benchmark would be misleading, as the merger would result in one platform being voluntarily closed by the newly formed monopolist.

Real-world examples are manifold and can be found across many different industries. For instance, in 1999 eBay bought German rival Alando, and closed it. Since 2001, one of the major German cinema operators, Cinestar, took over movie theaters of two rivals in Chemnitz, Germany, just to close these locations shortly after.

Of course, there are also many examples of platforms coexisting after merger, illustrating that it is not trivial to choose the appropriate benchmark. In 2009, eBay bought South Korean rival Gmarket, and both platforms coexist. In Rostock, Germany, Cinestar currently operates three movie theaters. After entering the market in 1991 buying two locations, they opened a third one in 1996. NASCAR operates multiple racing series. Next to their top-level series Sprint Cup, the Nationwide series fields similar cars and some drivers even compete in both series simultaneously, so that both series are almost homogeneous.

The basic argument also applies to differentiated platforms, although product heterogeneity relaxes competition (or cannibalization), and hence, heterogeneous platforms might well coexist, were homogeneous ones would not.

Since most two-sided market models focus on specific settings regarding the direction of network effects and regarding multi- or singlehoming of agents, this paper contributes to the literature by discussing the appropriate choice of a benchmark along these dimensions. In a rather general homogeneous two-sided market setting, we will compare a one-platform monopoly (OPM) with a two-platform monopoly (TPM).[1] Whether the platform operator on a TPM is actually a monopolist or two platform operators with "joint management" or colluding oligopolists, is irrelevant as long as decision making is as if a monopolist owns both platforms. Profits will, ceteris paribus, differ in OPM and TPM for any given number of agents on each market side. If the OPM profit exceeds the TPM profit, the monopolist will rationally decide to close one platform. In this case, using a TPM as a benchmark to study competitive effects would be of theoretical interest only, because a TPM would be a fictitious case. For practical considerations, the OPM would be the appropriate benchmark. Vice versa, the TPM would be appropriate, if profits on a TPM exceeded profits on the OPM, because the monopolist would opt to open (or keep) a second platform.

[1] As a "two-platform" monopoly is sufficient to illustrate our point, we abstain from generalizing the model to a "n-platform" monopoly. The exposition would become more complex without adding to our argument.

Indeed, we find that for most cases, one profit function is unambiguously above or below the other. In other cases, we determine sufficient conditions the inverse demand functions must satisfy in order to draw clear-cut conclusions. Turning to the case of heterogeneous platforms, we built on the widely used model of Armstrong (2006) to study whether our results hold for heterogeneous platforms as well. This model has been used a lot in the literature, for instance recently by Weyl (2010), who generalizes the model with regard to the distribution of agents and compares the competitive outcome with a TPM. We find that in the specific setting of the Armstrong model, OPM profits exceed TPM profits, which highlights the importance of choosing the right benchmark in each of the many possible settings, and for each specific model.

Our model and its discussion remain academic to some extent, as we assume away a number of empirically relevant aspects, which might either strengthen or weaken our results. The point is, however, that the effects of these left out aspects are intuitively straightforward and well-known from traditional markets without indirect network effects, so including them does not yield any additional insights, but makes the exposition more complex. One such aspect is binding capacity constraints. In a two-sided market context, the existence of such constraints ceteris paribus favors the existence of two platforms rather than one. Another aspect is the cost function, which might favor either a TPM, if there are low or no fixed costs or increasing marginal costs or an OPM, if there are high fixed costs or decreasing marginal costs.

The paper is structured as follows: First we present a rather general model for homogeneous platforms, and discuss whether OPM or TPM is the appropriate benchmark depending on whether both market sides multihome, only one markt side multihomes, or both sides singlehome. Then we illustrate our results using a slightly modified version of Armstrong's singlehoming model Armstrong (2006), in which we will also study the case of heterogeneous platforms. The last section concludes.

2. Model Setup

Consider a two-sided market in which there are N_i agents on market side i, where $i = 1,2$. Prices p_i represent the prices charged to agents on market side i, and are determined by the inverse demand functions $p_i\left(q_1^i, q_2^i\right)$, where $q_1^i, q_2^i > 0$ represents the relevant number of agents from the perspective of market side *i*. The relevant number of agents may or may not be equal to the total number of agents, depending on the market structure and other factors to be discussed later. Inverse demand functions are assumed to be strictly decreasing in their own argument that is $\frac{\partial p_i}{\partial q_i^i} < 0$, $i = 1,2$.

In case of an OPM the profit function of the monopolist is defined as

$$\Pi_{OPM}(N_1, N_2) = N_1 \cdot p_1(N_1, N_2) + N_2 \cdot p_2(N_1, N_2).^2$$

Since there is just one platform, the relevant number of agents is the total number of agents that is $q_1^i = N_1$ and $q_2^i = N_2$. However, in this case the total number of agents is also equal to the number of agents on the platform n_i, that is, $N_i = n_i$. Therefore, OPM profit can also be denoted as

$$\Pi_{OPM}(n_1, n_2) = n_1 \cdot p_1(n_1, n_2) + n_2 \cdot p_2(n_1, n_2).$$

Now assume, in contrast, that there are two perfectly homogeneous platforms on the market, which are jointly managed (TPM). Since these platforms are perfectly homogeneous, agents do not have any intrinsic preference for either one. We therefore focus on perfect symmetry that is both platforms are identical twins in terms of prices and patronage. Hence, each platform earns a profit defined by

$$\Pi_{TPM/2} = n_1 \cdot p_1(q_1^1, q_2^1) + n_2 \cdot p_2(q_1^2, q_2^2)$$

and because of symmetry, joint profits are

$$\begin{aligned} \Pi_{TPM} &= 2 \cdot \Pi_{TPM/2} \\ &= 2 \cdot \left(n_1 \cdot p_1(q_1^1, q_2^1) + n_2 \cdot p_2(q_1^2, q_2^2)\right). \\ &= N_1 \cdot p_1(q_1^1, q_2^1) + N_2 \cdot p_2(q_1^2, q_2^2) \end{aligned}$$

The relevant number of agents depends on whether agents are allowed to multihome or restricted to singlehoming. Throughout the paper, we will use the following assumption:

Assumption 1: Whether agents multihome or singlehome is an exogenous constraint.

This assumption simplifies our analysis substantially, because multihoming agents will always join all or none of the available platforms. They are neither restricted by income nor do they voluntarily singlehome for other reasons. Independence of income is a common assumption in the literature. For instance Rochet & Tirole (2003) assume a demand function $D_i(p_i)$, in which demand of market side i only depends on the price charged to market side i, but not on income. The model of Armstrong (2006) that will be used in subsequent sections, also excludes income effects.

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[2] As we discussed above, the impact of costs is straight forward.

We are now able to define the relevant number of agents (q_1^i, q_2^i) in $p_i(q_1^i, q_2^i)$ for the TPM. If agents on market side i are allowed to multihome, they will join a platform, if their net benefit is positive. By Assumption 1, the decision to join a platform is independent of the decision to join the other platform at the same time. Independence of decisions implies that $q_i^i = n_i$. Since both platforms are perfectly homogeneous, an agent will ceteris paribus always join either both or none of the platforms so that total demand on the market is $2 \cdot n_i$.

If agents on market side i are restricted to singlehoming, they face the trade-off which platform to join. Joining Platform 1 implies that Platform 2 cannot be joined and vice versa. Since the decision to join a platform depends on the decision to join the other platform, the inverse demand function depends on the total number of agents on market side i, reflecting rivalry for singlehoming agents, i.e. $q_i^i = N_i$.

Independent of single- and multihoming, the relevant number of agents from the other market side, q_{-i}^i, is always the number of agents that join the considered platform, i.e. $q_{-i}^i = n_{-i}$.

In our analysis, we consider three cases. If both market sides are allowed to multihome (Case 1), the corresponding inverse demand functions are $p_i(n_1, n_2)$. If side i is allowed to multihome, while side *-i* is restricted to singlehoming (Case 2), the corresponsing inverse demand functions are $p_i(n_i, n_{-i})$ and $p_{-i}(N_{-i}, n_i)$. If both sides are restricted to singlehoming (Case 3), the corresponding inverse demand functions are $p_i(N_i, n_{-i})$.

3. Model Analysis

Case 1

First, we study the case, where agents on both market sides are allowed to multihome. Since we assumed perfect symmetry of the two platforms, we know that all agents, who join one platform, will join the other platform as well. The implications of this case are summarized in Proposition 1.

Proposition 1: In case that the agents on either side of the market are allowed to multihome, and if at (n_1, n_2) OPM profit is positive, TPM profit strictly exceeds OPM profit. In particular, we have $\Pi_{TPM}(n_1, n_2) = 2 \cdot \Pi_{OPM}(n_1, n_2)$.

Proof: Denote the OPM profit function as

$$\Pi_{OPM}(n_1, n_2) = n_1 \cdot p_1(n_1, n_2) + n_2 \cdot p_2(n_1, n_2).$$

Both market sides are allowed to multihome, therefore each platform in the TPM faces a willingness to pay given by $p_1(n_1, n_2)$ and $p_2(n_1, n_2)$. Under TPM, agents simultaneously patronize both platforms, which yields

$$\Pi_{TPM}(n_1,n_2) = 2\cdot n_1 \cdot p_1(n_1,n_2) + 2\cdot n_2 \cdot p_2(n_1,n_2).$$

Comparing both profit functions easily reveals that $\Pi_{TPM}(n_1,n_2) = 2\cdot\Pi_{OPM}(n_1,n_2)$. Obviously, we find that $2\cdot\Pi_{OPM}(n_1,n_2) > \Pi_{OPM}(n_1,n_2)\forall(n_1,n_2)$, given $\Pi_{OPM}(n_1,n_2) > 0$.

(q.e.d.)

Proposition 1 is not surprising given the stylizing assumption of our model. Since by Assumption 1 all agents who join the single platform in an OPM, also join a second, identical platform, if available, the monopolist is able to double her profits by offering a second platform. In fact, the monopolist could realize infinite profits by offering infinitely many platforms. Relaxing Assumption 1 would result in TPM profit being less than twice the OPM profit. However, the economic intuition of Proposition 1 continues to hold: If agents on both market sides multihome, a monopolistic platform operator has an incentive to operate more than just one platform, because it generates additional revenues. Different from the case of singlehoming agents, which we will discuss later, platforms are not rival in patronage; they do not cannibalize.

Case 2

A lot of applications require that one market side is restricted to singlehoming, while the other one is allowed to multihome. Armstrong (2006) coined the term "competitive bottlenecks" for this case. Without loss of generality, we will assume market side 1 to be the multihoming one, and market side 2 to be the singlehoming one.

Proposition 2: On a competitive bottleneck two-sided market with homogeneous platforms, the OPM profit strictly exceeds the TPM profit if $p_1(n_1, 0.5\cdot N_2) < 0.5\cdot p_1(n_1,N_2)\forall(n_1,N_2)$. Vice versa, the TPM profit strictly exceeds the OPM profit if $p_1(n_1, 0.5\cdot N_2) > 0.5\cdot p_1(n_1,N_2)\forall(n_1,N_2)$ and $\frac{\partial p_1}{\partial q_2^1} > 0$, or in any case if $\frac{\partial p_1}{\partial q_2^1} < 0$.

Proof: By perfect symmetry on the TPM, we know that the total number of agents on the singlehoming side is equally shared among both platforms as prices and the numbers of agents per platform on the multihoming side will be identical on both platforms. Hence, we know that the number of agents per platform on the singlehoming side is given by $n_2 = 0.5\cdot N_2$. Multihoming agents will focus on the price and the number of singlehoming agents on *each* platform. Therefore, the TPM profit can be described as

$$\Pi_{TPM}(n_1,n_2) = 2\cdot n_1 \cdot p_1(n_1,n_2) + 2\cdot n_2 \cdot p_2(n_1, 2\cdot n_2).$$

Taking into account that $n_2 = 0.5\cdot N_2$, we can rewrite the TPM profit as

(1) $$\Pi_{TPM}(n_1, N_2) = 2 \cdot n_1 \cdot p_1(n_1, 0.5 \cdot N_2) + N_2 \cdot p_2(n_1, N_2).$$

Under OPM $n_i = N_i$, hence the OPM profit can be rewritten as

(2) $$\Pi_{OPM}(n_1, N_2) = n_1 \cdot p_1(n_1, N_2) + N_2 \cdot p_2(n_1, N_2).$$

Comparing equations (1) and (2), it is easy to see that the second term on the right-hand side of both equations is identical for any given (n_1, N_2), which implies that the profit generated from the singlehoming side is equal under both markets structures. Focusing on the profit from the multihoming side, we find that $\Pi_{OPM}(n_1, N_2) > \Pi_{TPM}(n_1, N_2)$, if $p_1(n_1, N_2) > 2 \cdot p_1(n_1, 0.5 \cdot N_2) \forall (n_1, N_2)$. The opposite result – that is $\Pi_{OPM}(n_1, N_2) < \Pi_{TPM}(n_1, N_2)$ – holds, if $p_1(n_1, N_2) < 2 \cdot p_1(n_1, 0.5 \cdot N_2) \forall (n_1, N_2)$ and $\frac{\partial p_1}{\partial q_2^1} > 0$. In case that $\frac{\partial p_1}{\partial q_2^1} < 0$, we can immediately see that $p_1(n_1, N_2) < p_1(n_1, 0.5 \cdot N_2) \forall (n_1, N_2)$, and therefore $\Pi_{OPM}(n_1, N_2) < \Pi_{TPM}(n_1, N_2)$.

(q.e.d.)

To understand the economic intuition of Proposition 2, let us first remember that the agents' willingness to pay depends on the number of agents from the other market side that are on the same platform. In other words, the willingness to pay, represented by the inverse demand functions, depends on the magnitude of the indirect network effect agents are exposed to. Taking into account that under TPM those multihoming agents, who join the one platform, also join the other platform, singlehoming agents are exposed to the same magnitude of the externality in TPM and OPM for any given n_1. Their willingness to pay is therefore unaffected with respect to the externality, when comparing both market structures. If we denote the profit functions in the (n_1, N_2) space, this is reflected by the equality of the terms that represent revenues from the singlehoming market side 2. Multihoming agents are under TPM only exposed to the network effects caused by half of the total number of agents from side 2. If multihoming agents are negatively affected by the presence of singlehoming agents, i.e. $\frac{\partial p_1}{\partial q_2^1} < 0$, the magnitude of this negative effect is lower under TPM than under OPM, and therefore the willingness to pay of side 1 agents is higher for the TPM than for the OPM, which translates into TPM profit exceeding OPM profit.

If the multihoming agents' willingness to pay is positively affected by the number of singlehoming agents, i.e. if $\frac{\partial p_1}{\partial q_2^1} > 0$, there are two effects in opposite directions: Firstly, if there are two platforms, multihoming agents join both of them, and hence, the platform operator can multiply revenues by offering more platforms. This effect has already been discussed, when giving the intuition for Proposition 1. Secondly, because multihoming agents are exposed to an externality of lower magnitude in the TPM, their willingness to pay is lower than in the OPM. Proposition 2 states that if the first effect is stronger than the second effect, a TPM is more profitable than an OPM, while the OPM is more profitable if the second effect outweighs that is, if network effects are strong enough.

Case 3

Now we assume that both market sides singlehome. Proposition 3 summarizes the corresponding implications.

Proposition 3: In case that agents on either side of the market are restricted to singlehoming, OPM profit strictly exceeds TPM profit, if $\frac{\partial p_i}{\partial q_{-i}^i} > 0$, where $i = 1{,}2$. Vice versa, TPM profit strictly exceeds OPM profit, if $\frac{\partial p_i}{\partial q_{-i}^i} < 0$.

Proof: OPM profit can be denoted as

(3) $$\Pi_{OPM}(N_1,N_2) = N_1 \cdot p_1(N_1,N_2) + N_2 \cdot p_2(N_1,N_2) = \sum_{i=1}^{2} N_i \cdot p_i(N_i,N_{-i}).$$

By the assumption of perfect symmetry on the TPM, we know that the total number of agents of each market side will be equally shared between both platforms, i.e. $n_i = 0.5 \cdot N_i$, $i = 1{,}2$. Hence, singlehoming agents of market side i meet $0.5 \cdot N_{-i}$ agents of market side $-i$ on each platform. Therefore, in (N_1, N_2) space TPM profit can be denoted as

(4) $$\Pi_{TPM}(N_1,N_2) = N_1 \cdot p_1(N_1, 0.5 \cdot N_2) + N_2 \cdot p_2(0.5 \cdot N_1, N_2) = \sum_{i=1}^{2} N_i \cdot p_i(N_i, 0.5 \cdot N_{-i}).$$

Comparing (3) and (4), we see that both terms on the right hand side of (3) strictly exceed their counterparts in (4), if $\frac{\partial p_i}{\partial q_{-i}^i} > 0$. Vice versa, both terms on the right hand side of (4) exceed their counterparts in (3), if $\frac{\partial p_i}{\partial q_{-i}^i} < 0$, which constitutes the result.

(q.e.d.)

Proposition 3 describes the effect of rivalry in patronage on both market sides. When discussing the economic intuition of Proposition 2, we stated that there are two opposing effects, if the indirect network externality is positive for both market sides, i.e. if $\frac{\partial p_i}{\partial q_{-i}^i} > 0$. The first effect originated in the ability of side 1 agents to multihome. This effect, of course, vanishes if agents are restricted to singlehoming. The second effect was a downward-shift in the willingness to pay due to the lower magnitude of the externality under TPM, because on each platform there is only half the number of agents from the other side, while under OPM all agents are on the same platform. Hence, different from the discussion of Proposition 2, revenues from market side 1 are strictly lower for the TPM than for the OPM. Another difference is that this argument now holds for both market sides, while in Case 2 market side 2 was unaffected, because market side 1 was able to multihome. Therefore, given positive externalities, OPM profit strictly exceeds TPM profit. Analogously, if network effects are negative for both market sides, i.e. if $\frac{\partial p_i}{\partial q_{-i}^i} < 0$, agents on each market side are exposed to a negative externality that is smaller in magnitude in TPM than in OPM, which translates to TPM profits strictly exceeding OPM profits.

In case of mixed indirect network effects, i.e. $\frac{\partial p_i}{\partial q_{-i}^i} > 0, \frac{\partial p_{-i}}{\partial q_i^{-i}} < 0$, a simple comparison of general profit functions does not suffice to draw conclusions. Whether TPM or OPM yields higher profits for a given (N_1, N_2), depends on the price difference between both market forms, weighted by the corresponding N_i. Assume, for instance, that market side 1 experiences positive externalities and market side 2 experiences negative externalities. Then, if

$$N_1 \cdot [p_1(N_1, N_2) - p_1(N_1, 0.5 \cdot N_2)] > N_2 \cdot [p_2(0.5 \cdot N_1, N_2) - p_2(N_1, N_2)]$$

it follows that OPM profit exceeds TPM profit. This illustrates the key message of our paper, which is: Whether OPM or TPM is the appropriate benchmark, depends on the specific situation studied. It depends not only on whether agents are restricted to singlehoming or are allowed to multihome, as well as on the direction and magnitude of the indirect externality, but also on size (represented by N_i), and willingness to pay (represented by $p_i(q_1^i, q_2^i)$) of each market side.

So far, we only studied general profit functions as a whole, and not their optimal values. The reason is simply the fact that with regard to the message of our paper, we do not gain from comparing first-order conditions of a general environment. In specific settings with specific functional forms, it might however be possible and helpful to determine optimal solutions and compare them in OPM and TPM, as we will demonstrate using the model of Armstrong (2006) in a subsequent section.

4. Heterogeneous Platforms

In this section, we expand our analysis to heterogeneous platforms. The probably most widely used framework used in the literature builds on the Hotelling (1929) model. We build our analysis on Armstrong (2006), which is a seminal paper that studies heterogeneous platforms in a Hotelling-type setting. Armstrong's singlehoming model assumes specific (symmetrical) functional forms for each market side's demand function that are derived from an additive-separable utility function. While such a specific formulation has of course some disadvantages[3], it is not only tractable analytically, but also allows us to derive both OPM

[3] For a discussion, see Armstrong (2006).

and TPM demand functions as coming from the same utility function. Note, however, that Armstrong's OPM model is not directly comparable to his setting of duopolistic competition, because his OPM model is not built into a Hotelling setting, while his duopoly model is. In order to directly compare OPM and TPM, we therefore assume that in the OPM case the platform is located in the middle of the Hotelling line that is at location 0.5. We consider two cases of the TPM: To show that our results from the previous section transfer to Armstrong's model, we first assume that both platforms in the TPM are also located at 0.5 that is, platforms are homogeneous. Afterwards, we study the effects of platforms being located at points 0 and 1 on the Hotelling line, as in Armstrong (2006).

Under OPM assume that the utility of agents on market side i, U_i, can be described by the utility function

(5) $$U_i = \alpha_i n_j - p_i - (0.5 - x)t_i \quad i,j = 1,2 \quad i \neq j,$$

if the agent's preferred location x on the Hotelling line is to the left, and by

(6) $$U_i = \alpha_i n_j - p_i - (x - 0.5)t_i \quad i,j = 1,2 \quad i \neq j,$$

if the agent's preferred location is to the right of the platform's location, which is 0.5 by the assumption made above.

Parameter α_i represents the constant marginal indirect network effect of one agent of market side *j* being present on the same platform, and t_i is the transportation cost parameter.

Since Armstrong does not impose a nonnegativity constraint on utility – actually the indifferent consumer obtains negative utility in his duopoly equilibrium – we assume a reservation utility $\underline{u_i}$, which represents the minimum utility, an agent requires to join the platform. Setting (5) and (6) equal to $\underline{u_i}$ and solving for *x* yields the locations of those agents, who just obtain their reservation utility, and are hence indifferent between joining and not joining the platform. The marginal agent on the left hand side of the platform is located at

$$x_i^l = \frac{1}{2} - \frac{\alpha_i n_j - p_i - \underline{u_i}}{t_i},$$

while the marginal agent on the right hand side of the platform is located at

$$x_i^r = \frac{1}{2} + \frac{\alpha_i n_j - p_i - \underline{u_i}}{t_i}.$$

Following Armstrong, we assume that agents are uniformly distributed on the Hotelling line, which yields the demand function

(7) $$n_i = \frac{2(\alpha_i n_j - p_i - \underline{u_i})}{t_i},$$

From (7), we can easily obtain the inverse demand function as,

$$p_i = \alpha_i n_j - \frac{n_i t_i}{2} - \underline{u_i},$$

and hence OPM profit is given by

(8) $$\Pi_{OPM} = \sum_{\substack{i,j=1\\ i\neq j}}^{2} n_i\left(\alpha_i n_j - \underline{u_i} - \frac{n_i t_i}{2}\right) = \sum_{\substack{i,j=1\\ i\neq j}}^{2} N_i\left(\alpha_i N_j - \underline{u_i} - \frac{N_i t_i}{2}\right).$$

This setting can easily be transferred to represent a TPM with homogeneous platforms located at 0.5. As platforms are homogeneous, agents obtain ceteris paribus the same utility on each platform. Since both market sides must singlehome, the market will be equally shared between both platforms, and hence, an agent, who joins a platform is exposed to the indirect network effect of half the number of agents in the market, which yields demand

(9) $$N_i = \frac{\alpha_i n_j - 2\left(p_i + \underline{u_i}\right)}{t_i},$$

inverse demand

$$p_i = \alpha_i n_j - n_i t_i - \underline{u_i},$$

and a total profit of

(10) $$\Pi_{TPM}^{\text{hom}} = \sum_{\substack{i,j=1\\ i\neq j}}^{2} 2n_i\left(\alpha_i n_j - \underline{u_i} - n_i t_i\right) = \sum_{\substack{i,j=1\\ i\neq j}}^{2} N_i\left(\alpha_i \frac{N_j}{2} - \underline{u_i} - \frac{N_i}{2} t_i\right).$$

Subtracting (10) from (8), it is easy to see that for every $\left(N_1, N_2\right)$ $\Pi_{OPM} < \Pi_{TPM}^{\text{hom}}$, if $\alpha_i < 0$, $i = 1,2$ and $\Pi_{OPM} > \Pi_{TPM}^{\text{hom}}$, if $\alpha_i > 0$, $i = 1,2$, which is in line with Proposition 3. If $\alpha_1 > 0$ and $\alpha_2 < 0$, OPM profit strictly exceeds TPM profit, if α_1 is greater than α_2 in absolute terms, while reservation utility levels are sufficiently high. The opposite holds, if α_1 is smaller than α_2 in absolute terms.

To study platform heterogeneity, we now assume that under TPM one of the platforms is located at point 0 on the Hotelling line, while the other platform is located at point 1. This situation is equal to the setting Armstrong uses to study duopolistic competition, except for the minor modification of reservation utility levels introduced above. As already mentioned, the equilibrium results imply negative utility of the indifferent agent. While competition between the platform operators ensures that agents will not be forced to accept infinitely negative utility, this property of the model will under certain conditions, result in infinitely high profits and infinitely low utility, if there is no reservation utility level binding the monopolistic platform operator. Also note that for the sake of unambiguous results, we do not compare profits for any (q_1, q_2) anymore, but optimal solutions.

By the assumptions made so far, specifically by the assumptions of uniform distribution of agents on the Hotelling line and by symmetry, we know that the indifferent agent will be located at point 0.5 on the Hotelling line, and that in optimum the total market is equally divided between both platforms. While under platform competition, platform operators try to steal consumers from the competitor, a monopolist who operates both platforms will set her price so that the indifferent agent on each market side will just receive their reservation utility. The resulting optimal price is

$$p^*_{het,i} = \frac{\alpha_i - t_i}{2} - \underline{u}_i,$$

which yields a profit of

$$\Pi^{het*}_{TPM} = \frac{\alpha_1 + \alpha_2 - t_1 - t_2}{2} - \underline{u}_1 - \underline{u}_2. \tag{11}$$

Armstrong's equilibrium solution implies a utility value of $0.5 \cdot (\alpha_i + 2\alpha_j - 3t_i)$ for the indifferent agent on market side i. We consider this utility level to be the upper bound of reservation utility that is, we assume $\underline{u}_i < 0.5 \cdot (\alpha_i + 2\alpha_j - 3t_i)$. Note that for $\underline{u}_i = 0.5 \cdot (\alpha_i + 2\alpha_j - 3t_i)$, (11) becomes $\Pi^{het*}_{TPM} = t_1 + t_2 - \alpha_1 - \alpha_2$, which is exactly the equilibrium profit, if both platforms compete (Armstrong's equation (13), p. 675).

Optimizing (8) with respect to n_1 and n_2 yields an OPM profit of

$$\Pi^*_{OPM} = \frac{t_2(\underline{u}_1)^2 + 2\underline{u}_1\underline{u}_2(\alpha_1 + \alpha_2) + t_1(\underline{u}_2)^2}{2\left(t_1 t_2 - (\alpha_1 + \alpha_2)^2\right)}. \tag{12}$$

For simplicity, assume that $\underline{u}_1 = \underline{u}_2$. Then for $\alpha_i > 0$, $i = 1,2$, it still holds that $\Pi^*_{OPM} > \Pi^{het*}_{TPM}$, as was the case with homogeneous platforms. If we additionally assume that $t_1 = t_2$, then for $\alpha_i < 0$ it holds that $\Pi^*_{OPM} < \Pi^{het*}_{TPM}$.[4] In case that $\alpha_1 > 0$ and $\alpha_2 < 0$, we obtain ambiguous results. Summarizing our findings, we see that all results are in line with Proposition 3. Hence, we can conclude that platform heterogeneity is per se not sufficient to disprove the results of our paper.

5. Conclusions

Selecting the appropriate benchmark is the basic step when analyzing the effects of market concentration. On two-sided markets it is therefore of essential interest to know, how many platforms a monopolist will operate. She has to trade-off additional revenues that could be gained when operating an additional platform and the loss of revenues due to cannibalization. Determinants in this respect are whether agents single- or multihome on one or on both market sides and whether agents perceive the indirect network externality as positive or negative. Our analysis follows these situations systematically and shows that in some of them

[4] These results were established using Mathematica. The code will be provided from the authors upon request.

there is an unambiguous tendency towards one (or multiple) platforms, while in other situations general analysis remains ambiguous. In some cases, ambiguity can be resolved, if details like characteristics of the demand functions and the relative sizes of the market sides are known. We also demonstrated that platform heterogeneity does not resolve the problem per se. Table 1 summarizes the results for homogeneous platforms.

	+/+	+/-	-/+	-/-
multi/multi	TPM	TPM	TPM	TPM
multi/single	ambiguous	ambiguous	TPM	TPM
single/single	OPM	ambiguous	ambiguous	TPM

Table 1: Market characteristics and number of platforms under monopoly. Columns: sign of the externality on market side 1 / market side 2. Rows: multi- or singlehoming agents on market side 1 / market side 2.

In the introduction we gave the example of eBay buying a German rival and closing this platform. This was in 1999, when German internet users had smallband access available only,[5] and internet service providers offered two-part tariffs, i.e. users had to pay extra for every minute they spent online. With slow and expensive internet access, users are less likely to use more than one internet auction platform, especially, if buyers find the same sellers on both platforms and vice versa. Therefore, we interpret this example as being a "single/single; +/+" situation. Our analysis finds that in this situation the OPM solution is more profitable. The case is different for the South Korean eBay example, where the acquisition took place 10 years later. South Korea is among those countries with the highest broadband penetration rate,[6] so using more than one platform, e.g. to compare prices, is easy for buyers, and offering on more than one platform is easy for sellers. We should therefore interpret this as an example for a "multi/multi; +/+" situation for which our analysis finds strong incentives to keep the additional platform. The examples of the movie theaters represent a "multi/single; +/-" situation, as advertisers are able to advertise in multiple cinemas, while moviegoers can only be in one movie theater at the same time. Advertisers obviously benefit from many people seeing their ads, while moviegoers are likely to be ad-averse. For such a situation we found that either OPM or TPM can be more profitable. The decisive criterion is by Proposition 2 the intensity of the indirect network effect on the multihoming side, i.e. advertisers' valuation of moviegoers, which might well differ between the two cities considered. NASCAR allows fans to multihome by scheduling Nationwide races on

[5] Two years later, by the end of 2001, only 2.4% of the population had broadband access to the internet (Source: OECD Broadband Portal on http://www.oecd.org/).

[6] In 2008, one third of the population had broadband internet access (Source: OECD Broadband Portal on http://www.oecd.org/).

Saturdays and Sprint Cup races on Sundays. Sponsors are of course able to participate in both series, so that there is a "multi/multi" case, and the coexistence of the two series is consistent with our analysis.

Our analysis provides some guidelines in which direction – one or more platforms – a specific model is likely to tend. As a rule of thumb, we can say that factors relaxing competition for patronage make way for additional platforms, while factors that increase cannibalization render a single platform solution more attractive. It is, however, not our intention to give a complete "cooking recipe", which puts out the appropriate benchmark, as there are numerous additional determinants well-known from traditional industries, e.g. economies of scale and fixed costs, which all have to be taken into account. Identifying the appropriate benchmark will in most cases probably require (numerical) evaluation of the corresponding optimal solutions, and therefore specific functional forms that allow for such an analysis. Due to this complexity, we also cannot generally answer the question of how using an inappropriate benchmark influences results qualitatively. However, we emphasize that it is important to be aware of this problem in theory as well as in applied research. In theory, results would be at least biased quantitatively in terms of prices, quantities, and welfare, when using an inappropriate benchmark. Models that encompass several situations, for instance all-positive and mixed perception of the externality, might need to use different benchmarks for each situation. In practice, assessment of proposed mergers needs to anticipate the post-merger situation in order to give meaningful recommendations.

6. References

Ambrus, A., & Argenziano, R. (2009). Asymmetric Networks in Two-Sided Markets. *American Economic Journal: Microeconomics, 1*(1), 17-52.

Ambrus, A., & Reisinger, M. (2006). Exclusive vs overlapping viewers in media markets. Unpublished Working Paper. Department of Economics, Ludwig-Maximilians-University.

Anderson, S. P., & Coate, S. (2005). Market provision of broadcasting: a welfare analysis. *Review of Economic Studies, 72*(4), 947-972.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics, 37*(3), 668-691.

Caillaud, B., & Jullien, B. (2003). Chicken & egg: competition among intermediation service providers. *RAND Journal of Economics, 34*(2), 309-328.

Chandra, A., & Collard-Wexler, A. (2009). Mergers in two-sided markets: an application to the Canadian newspaper industry. *Journal of Economics and Management Strategy, 18*(4), 1045-1070.

Chaudhri, V. (1998). Pricing and efficiency of a circulation industry: The case of newspapers. *Information Economics and Policy, 10*(1), 59-76.

Hotelling, H. (1929). Stability in competition. *Economic Journal, 39*, 41-57.

McCabe, M. J., & Snyder, C. M. (2007). Academic journal prices in a digital age: a two-sided market model. *B.E. Journal of Economic Analysis & Policy, 7*(1).

Rochet, J.-C., & Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association, 1*(4), 990-1029.

Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review, 100*(4), 1642-1672.