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ASYMMETRIC DEMAND RESPONSES: A DEMAND SYSTEM APPROACH

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Asymmetric demand responses, as well as other asymmetric responses in economics, have been considered for some time, with early discussion provided by Marshall and one of the first empirical studies made by Farrell. Interest in estimation of asymmetric functions was sparked by studies by Tweeten and Quance (1969, 1971), and Wolffram, with the method of segmenting variables, suggested by Wolffram, becoming a popular means of dealing with asymmetry in empirical analyses.

Many of the empirical studies involving asymmetry have dealt with supply and demand (e.g., Houck, Traill et al., Young, Ward, and Nelson). In most of these studies, the focus of attention has been on a single asymmetric relationship. In the area of demand, much of the interest has been on asymmetry of the own-price response (e.g., Scitovsky, and Young). Little attention has been given to asymmetry relationships in context of systems of equations. Young has considered asymmetric cross-price and income demand responses but in context of a single equation; the proposed asymmetric cross-price specifications involve application of the segmenting method to price ratios.

In this paper, asymmetric responses are examined in a system of demand equations. A system of n demand equations has $n(1 + n)$ income and price responses (n income responses and n^2 price responses) to be considered as possibly asymmetric. How should one proceed? Should each price and income variable be segmented? How should the basic demand restrictions such as symmetry, homogeneity and adding-up be imposed? For example, consider the compensated cross-price effect s_{ij}--the change in demand for good i, given a change in the price of good j with real income constant. If dummy variable $d_j = 1$ when the price of good j is increasing, else $d_j = 0$, as typically defined when segmenting, then asymmetry might be specified as $s_{ij} = a_{ij} + b_{ij}\, d_j$, with a_{ij} being the substitution effect when the price is decreasing, and $a_{ij} + b_{ij}$ being the substitution effect when price is increasing. Given symmetry, we then have $s_{ij} = a_{ij} + b_{ij}\, d_j = a_{ji} + b_{ji}\, d_i = s_{ji}$. The latter, however, is obviously not consistent with all possible values of d_j and d_i, unless $b_{ij} = b_{ji} = 0$, and we have symmetry. Each s_{ij} might be specified as some function of all n dummy variables; but this approach may not be very attractive, as the number of effects to be estimated would be very large, in general.

In the present study, an approach is taken focusing on asymmetry of the income response and, in turn, asymmetry of the price responses through the specific and general substitution effects, as well as income effect of the Slutsky equation. This type of asymmetry can be empirically examined through the differential approach, using the Rotterdam model, and the basic restrictions of demand can be imposed straightforwardly. The asymmetry specification analyzed is essentially an extension of the method proposed by Wolffram. The Wolffram method directly attempts to measure asymmetry; factors which may be associated with asymmetry such as stocks or resource fixity are not modeled, in general. Inclusion of such factors in the model would make the analyses more realistic but may not be possible due to data limitations, as is the case in the present study.

The paper proceeds as follows. In the next section, the Rotterdam model with asymmetry is formally presented. Then, for illustration purposes, the model is applied to data on food and three other broadly defined goods. In the last section, concluding comments are given.

Model

The Rotterdam model is a first-order demand approximation developed by Theil and Barten (1966). Recent analyses by Barnett, Byron, and Mountain show the order of the approximation is no lower than that for other flexible functional forms. In this section, the Rotterdam model is extended to analyze asymmetry. Formally, the Rotterdam model can be written as

$$\omega_i \, d \log q_i = \mu_i \, d \log Q + \sum_j \Pi_{ij} \, d \log p_j \qquad (1)$$

where subscript i indicates a particular good; q and p are quantity and price, respectively, $\omega_i = \frac{p_i q_i}{x}$, the budget share for the good, x being total expenditure or income; $\mu_i = p_i \frac{\partial q_i}{\partial x}$, the marginal propensity to consume out of income (MPC); $d \log Q = \sum_j \omega_j \, d \log q_j$, the Divisia volume index in differential form; and $\Pi_{ij} = \frac{p_i p_j}{x} s_{ij}$, the Slutsky coefficient. Equation (1) is essentially a Hicksian demand approximation and can be derived straightforwardly from the total differential of either the Marshallian or Hicksian demand system (e.g., Theil et al.; Deaton and Muellbauer).

The Slutsky coefficient can further be written as (Barten, 1964)

$$\Pi_{ij} = \nu_{ij} - \phi \, \mu_i \, \mu_j$$
$$\nu_{ij} = \frac{p_i \, p_j}{x} \, \lambda \, u^{ij} \qquad (2)$$
$$\phi = \left(\frac{\partial \lambda}{\partial x} \cdot \frac{x}{\lambda} \right)^{-1}$$

where λ is the marginal utility of income and u^{ij} is the ij^{th} element of the matrix $\left[\frac{\partial^2 u}{\partial q_i \, \partial q_j} \right]^{-1}$, the inverse of the Hessian matrix for the utility maximization problem with u being utility. The term ϕ is referred to as the income flexibility, while ν_{ij} and $-\phi \, \mu_i \, \mu_j$ are the Slutsky coefficient terms corresponding to the specific and general substitution effects, $\lambda \, u^{ij}$ and $-\left(\frac{\lambda}{\left(\frac{\partial \lambda}{\partial x} \right)} \right) \frac{\partial q_i}{\partial x} \, \frac{\partial q_j}{\partial x}$, respectively.

In analyzing food demand and similarly broadly defined goods, the assumption of separability is frequently made. In the present study, the assumption of preference independence or strong separability is made with the analysis focusing on four broadly defined goods--services, food, other nondurables and durables. In this case, the Hessian matrix and its inverse are diagonal with $u^{ij} = 0$ for $i \neq j$, and the model described by (1) and (2) simplifies to

$$\omega_i \, d \log q_i = \mu_i \, d \log Q + \phi \, \mu_i \left(d \log p_i - \sum \mu_j \, d \log p_j \right) \qquad (3)$$

where $\Pi_{ij} = \phi \, \mu_i \, \Delta_{ij} - \phi \, \mu_i \, \mu_j$; $\Delta_{ij} = 1$ if $i = j$, else $\Delta_{ij} = 0$. (To obtain (3), note

$\sum_j \Pi_{ij} = \sum_j \left(v_{ij} - \phi\, \mu_i\, \mu_j\right) = 0$ or $\sum_j v_{ij} = \phi\, \mu_i$, based on homogeneity and adding-up; hence, for preference independence $v_{ii} = \phi\, \mu_i$.)

Consider the term $d \log Q$. By totally differentiating the budget constraint, it can be straightforwardly shown that $d \log Q = d \log x - \sum \omega_j\, d \log p_j$ and is a measure of the percentage change in real income--the percentage change in actual income minus the weighted average of the percentage changes in prices, with the weights being the budget shares. In this study, $d \log Q$ plays a central role in specifying asymmetry.

In general, the coefficients of the Rotterdam model depend on the levels of income and prices, and, as discussed by Theil et al., need not be constant as often assumed in empirical analyses. Two hypotheses about MPC are considered in this study.

The first hypothesis is that the MPC is asymmetric and takes two values--one when real income decreases and another when real income increases; i.e.,

$$\mu_i = \mu_{i0}\, d + \mu_{i1}\left(1 - d\right), \tag{4}$$

where $d = 1$ if $d \log Q \geq 0$, else $d = 0$; and μ_{i0} and μ_{i1} are the MPC's for an increase and decrease in real income, respectively. Note that, in turn, the Slutsky coefficients are asymmetric through the general substitution term, as well as the specific substitution term for the own-price in specification (3). That is, income asymmetry implies compensated price asymmetry; i.e., if $d \log Q \geq 0$, $\Pi_{ij} = \phi\, \mu_{i0}\, \Delta_{ij} - \phi\, \mu_{i0}\, \mu_{ij}$, else $\Pi_{ij} = \phi\, \mu_{i1} - \phi\, \mu_{i1}\, \mu_{j1}$. For the uncompensated price effects, asymmetry further involves the usual income effect of the Slutsky equation.

The second hypothesis considered is that the change in the MPC is proportional to the change in real income, i.e.,

$$\mu_{i,t} = \mu_{i,t-1} + a_i \, d \log Q_t \tag{5}$$

where subscript t indicates time. In this case, the MPC evolves over time and, by successive substitution, $u_{i,t} = u_{i,0} + a_i \sum_{k=1}^{t} d \log Q_k$. Under this hypothesis, the MPC may increase or decrease according to whether $d \log Q_t$ increases or decreases and the sign of parameter a_i.

These two hypotheses concerning the MPC's could, as well, apply to the other coefficient in model (3), the income flexibility, by replacing μ_i in (4) and (5) by ϕ. The latter possibility is examined in the empirical analysis, along with the hypotheses for the MPC.

The basic restrictions of demand require--(1) adding-up: $\sum_i \mu_i = 1$ and $\sum_i \Pi_{ij} = 0$; (2) homogeneity: $\sum_j \Pi_{ij} = 0$; and (3) symmetry: $\Pi_{ij} = \Pi_{ji}$. For model (3) with $\Pi_{ij} = \phi \, \mu_i \, \Delta_{ij} - \phi \, \mu_i \, \mu_j$, symmetry holds and, if adding-up holds, homogeneity also holds and vice versa. For the model defined by (3) and (4), adding-up requires $\sum_i \mu_{i0} = \sum_i \mu_{i1} = 1$; while for the model defined by (3) and (5), $\sum_i \mu_{i,0} = 1$ and $\sum_i a_i = 0$.

As demand over time may exhibit a trend, a constant term was added to specification (3). For adding-up, the constant terms sum to zero across goods.

For estimation, $\omega_{i,t}$, $d \log p_{i,t}$ and $d \log q_{i,t}$ can be approximated by $\frac{\omega_{i,t} + \omega_{i,t-1}}{2}$, $\log\left(\frac{p_{i,t}}{p_{i,t-1}}\right)$, and $\log\left(\frac{q_{i,t}}{q_{i,t-1}}\right)$, respectively. The demand elasticities for the Rotterdam model are (1) the income elasticity: $e_i = \frac{\mu_i}{\omega_i}$, (2) the compensated price elasticities: $e_{ij}^* = \frac{\Pi_{ij}}{\omega_i}$, and (3) the uncompensated price elasticities: $e_{ij} = e_{ij}^* - \omega_j\, e_i$.

Application

U.S. Department of Commerce data on personal consumption expenditures for services, food, other nondurables (hereafter simply called nondurables), and durables were analyzed.[1] The data are annual, and the sample period runs from 1929 through 1989. The expenditure data are measured in both actual and real (1982 = 100) dollars. Implicit prices were obtained by dividing actual expenditure by real expenditure for each of the four

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[1]The U.S. Department of Commerce product categories include the following types of goods.

(1) Services: housing, household operation, transportation, medical care, and other.

(2) Food: food purchased for off-premise consumption, purchased meals and beverages, food furnished employees, and food produced and consumed on farms.

(3) Nondurables: clothing and shoes, gasoline and oil, fuel oil and coal, and other.

(4) Durables: motor vehicles and parts, furniture and household equipment, other.

For further description of the types of goods included in the different categories, see pages 106-112 in "The National Income and Product Accounts of the United States, 1929-82," U.S. Department of Commerce.

expenditure groups. Quantities were measured by real expenditures. U.S. Department of Commerce data on the U.S. population were used to put demand on a per capita basis. Treatment of data in this study follows the approach taken by Johnson, Hassan and Green in analyzing similar data for Canada.

As the data add-up by construction--income in the model is total consumer expenditure on the four expenditure categories--the error covariance matrix is singular and the equation for durables was excluded (Barten, 1969). The errors across equations were assumed to be contemporaneously correlated, and the full information maximum likelihood procedure was used to estimate the model with homogeneity and symmetry imposed.

The estimates for the model defined by (3) and (4) suggest possible asymmetry in demand, while the estimates for the alternative specification defined by (3) and (5), the evolving parameter hypothesis, were not significant. Estimates of (3) with (4) and (5) defined for the income flexibility were also not significant. The following discussion focuses on the asymmetry suggested by specifications (3) and (4).

Nine out of 13 coefficient estimates for the latter model were more than twice their corresponding standard error estimates as shown in Table 1. The time trend for food and nondurables, along with the MPC's for a decrease in income for nondurables and durables were not significant. Wald tests for the equality of the MPC's for an increase and decrease in income indicate that, at the α = .05 level of significance, the MPC's for services and durables are asymmetric. For food and nondurables, the MPC's for an increase and decrease in income are statistically the same. The results for services indicate the MPC is .11 for an increase in income and .44 for a decrease in income, suggesting that when income falls services are cut relatively sharply at the margin. The MPC for durables was .44 for an increase in income and .22 for a decrease in income, suggesting just the opposite--when income expands, durables receive a relatively large part of marginal expenditures. The estimate of the income flexibility was negative at -.25 (an increase in income decreases the marginal utility of money λ).

As a result of the asymmetry of the MPC, the income and price elasticities for the model are also asymmetric as shown in Table 2. The estimated income elasticities follow the same general pattern as previously discussed for the MPC's. All own-price elasticity estimates are negative and about twice their corresponding standard errors, except the estimate for a decrease in real income for nondurables. The results indicate the demand for services becomes more price elastic with a decrease in income. On the other hand, the own-price elasticity for durables becomes more elastic with an increase in income. All own-price elasticity estimates are less than one in absolute value, indicating inelastic demands. All cross-price elasticity estimates are negative, indicating gross complementary relationships; however, a number of the cross-price elasticity estimates are insignificant. The estimates for the constant term indicate a trend towards services and away from durables.

Overall, the results suggest a number of possible asymmetry relationships. However, caution should be taken in making conclusions. Addition of stock and habit persistence variables to the model may offer alternative explanations. The trend variable attempts to capture changing consumption patterns, but this is a rough treatment of a problem that may require more precise measurements of variables indicating consumer preferences. Also, out of the 60 observations used in the analysis, ten had decreasing real income and half of those occurred during the 1930's depression period. The model might be reflecting, in part, consumption patterns peculiar to this period and afterwards. Nevertheless, the data analyzed do not refute the asymmetry hypothesis, and, lacking additional data for a more detailed analysis, the asymmetry specification is felt to be preferable to the usual constant coefficient model.

Concluding Comments

Asymmetry of the income response implies asymmetry of the price responses through the specific and general substitution effects, as well as the Slutsky income effects, of a demand system. The differential approach provides a straightforward means to model such asymmetry. Usually the basic coefficients of the differential demand model are assumed to be constant. Asymmetry hypotheses indicate how the constant coefficient assumption might be relaxed. The analysis in this paper suggests possible asymmetry related to the income effects for services and durables. However, caution needs to be taken in drawing strong conclusions, as more detailed analysis involving stock, habit persistence and other variables may provide alternative explanations for the particular data analyzed.

Table 1. Maximum likelihood estimates for the Rotterdam model with asymmetry.

Item	Parameter					
	MPC				Trend[c]	
	μ_{i0}[a]		μ_{i1}[b]			
Services	.114	(.050)[d]	.443	(.070)	.008	(.002)
Food	.209	(.024)	.214	(.051)	-.001	(.001)
Nondurables	.239	(.033)	.119	(.078)	-.002	(.002)
Durables	.437	(.558)	.223	(.145)	-.006	(.002)
			Income Flexibility			
			ϕ			
			-.254 (.079)			

[a] For an increase in real income: $d \log Q \geq 0$.

[b] For a decrease in real income: $d \log Q < 0$.

[c] Constant for time trend.

[d] Asymptotic standard errors in parentheses.

Table 2. Uncompensated elasticity estimates for Rotterdam model with asymmetry.[a]

Item	Change Real Income $d \log Q$	Elasticity				
		Income	Price			
			Services	Food	Nondurables	Durables
Services	≥ 0	0.277 (0.122)[b]	-0.177 (0.079)	-0.044 (0.019)	-0.052 (0.024)	-0.004 (0.012)
	< 0	1.074 (0.169)	-0.595 (0.083)	-0.170 (0.038)	-0.235 (0.044)	-0.072 (0.048)
Food	≥ 0	0.982 (0.115)	-0.377 (0.051)	-0.407 (0.053)	-0.185 (0.035)	-0.013 (0.041)
	< 0	1.005 (0.240)	-0.302 (0.057)	-0.415 (0.129)	-0.220 (0.042)	-0.068 (0.047)
Nondurables	≥ 0	0.961 (0.134)	-0.369 (0.059)	-0.154 (0.028)	-0.425 (0.068)	-0.012 (0.039)
	< 0	0.480 (0.315)	-0.144 (0.904)	-0.076 (0.046)	-0.227 (0.152)	-0.032 (0.035)
Durables	≥ 0	3.524 (0.450)	-1.353 (0.203)	-0.563 (0.083)	-0.663 (0.101)	-0.941 (0.187)
	< 0	1.800 (1.169)	-0.541 (0.392)	-0.285 (0.214)	-0.394 (0.289)	-0.578 (0.300)

[a] Evaluated at budget share sample means: .413 for Services, .213 for Food, .249 for Nondurables, and .124 for Durables.

[b] Asymptotic standard errors in parentheses.

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