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The Cost of Tractability and the Calvo Pricing Assumption

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The Cost of Tractability and the Calvo Pricing Assumption

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Abstract

This paper demonstrates that tractability gained from the Calvo pricing assumption is costly in terms of aggregate dynamics. I derive a generalized New Keynesian Phillips curve featuring a generalized hazard function, non-zero steady state inflation and real rigidity. Analytically, I find that important dynamics in the NKPC are canceled out due to the restrictive Calvo assumption. I also present a general result, showing that, under certain conditions, this generalized Calvo pricing model generates the same aggregate dynamics as the generalized Taylor model with heterogeneous price durations. The richer dynamic structure introduced by the non-constant hazards is also quantitatively important to the inflation dynamics. Incorporation of real rigidity and trend inflation strengthen this effect even further. With reasonable parameter values, the model accounts for hump-shaped impulse responses of inflation to the monetary shock, and the real effects of monetary shocks are 2-3 times higher than those in the Calvo model.

JEL classification: E12; E31
Key words: Hazard function, Nominal rigidity, Real rigidity, New Keynesian Phillips curve

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1 Introduction

The Calvo pricing assumption (Calvo, 1983) has become predominant in the world of applied monetary analysis under nominal rigidity. The main argument for using this approach, however, is solely based on its tractability. In recent years, detailed micro-level data sets have become available for researchers. Empirical work using these data sets[1] generally reach the consensus that, instead of having economy-wide uniform price stickiness, the frequency of price adjustments differs substantially within the economy. In addition, the Calvo assumption also implies a constant hazard function of price setting, meaning that the probability of adjusting prices is independent of the length of the time since last adjustment. Unfortunately, constant hazard functions are also largely rejected by empirical evidence from the micro level data. Cecchetti (1986) used newsstand prices of magazines in the U.S. and Goette et al. (2005) apply Swiss restaurant prices. Both studies find strong support for increasing hazard functions. By contrast, recent studies using more comprehensive micro data find that hazard functions are first downward sloping and then mostly flat, interrupted periodically by spikes (See, e.g.: Campbell and Eden, 2005, Alvarez, 2007 and Nakamura and Steinsson, 2008).

Given this conflict between theory and empirical evidence, it is important to understand to which extent the constant hazard function is innocuous for the inflation dynamics and implications of monetary policy.

To tackle this question, I construct a generalized time-dependent pricing model and derive the New Keynesian Phillips curve (NKPC) featuring an arbitrary hazard function, non-zero steady state inflation and real rigidity. The resulting NKPC includes components, such as lagged inflation, future and lagged expectations of inflation and real marginal costs. This version of the Phillips curve nests the Calvo case in the sense that, under a constant hazard function, effects of lagged inflation exactly cancel those of lagged expectations, so that, as in the Calvo NKPC, only current real marginal cost and expected future inflation remain in the expression. In the general case, however, both lagged inflation and inflation expectations should be presented in the dynamic structure of the Phillips curve. In light of this result, we learn that lagged inflation and lagged expectations are not extrinsic to the forward-looking pricing model. They are missing in the Calvo setup, only because the restrictive pricing assumption has them canceled out.

Furthermore, I present a general result that, up to log linearization approximation, the generalized Calvo pricing model based on a flexible hazard function implies the same aggregate dynamics as the general Taylor framework with heterogeneous price durations. In the literature, both frameworks have been commonly applied to study the effects of heterogeneous price stickiness on aggregate dynamics. Dixon and Kara (2005), for example, generalize the simple Taylor-wage-contract model to explicitly account for the presence of varying contract lengths, and Carvalho (2005) models heterogeneity in price stickiness by introducing continuous Calvo sticky price sectors. Both of these works find that the presence of a small portion of highly rigid sectors leads to more persistent inflation and larger real effects of monetary shocks. In this paper, I show that under a certain condition regarding the relationship between the distribution of price durations and the hazard function, these two frameworks imply the same aggregate dynamics. This result has an important empirical implication that the aggregate data can be

[1]See: e.g. Bils and Klenow (2004), Alvarez et al. (2006), Midrigan (2007), Nakamura and Steinsson (2008) among others.

used to uniquely identify both hazard functions and the distribution of sticky prices from either of these two frameworks.

When simulating the complete general equilibrium model, I combine the generalized NKPC with a standard IS curve and an exogenous nominal money growth rule. The simulation results show that, even without real rigidity and trend inflation, the increasing hazard function helps to increase both persistence of inflation and output gap. When introducing some degree of real rigidity, the generalized NKPC gives rise to substantially different inflation dynamics, namely, the impulse response of inflation to a nominal money growth shock becomes hump-shaped. Moreover, non-zero trend inflation amplifies this effect even further. The economic intuition behind these results is that, on the one hand, increasing hazard function postpones the timing of the price adjustment. On the other hand, strategic complementary makes earlier adjusting firms choose a small size for the adjustment, while the later adjusting firms make a larger price adjustment. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response. Trend inflation amplifies this effect even further, because high trend inflation causes relative prices to disperse quickly. Last but not least, when the real effects of monetary policy shocks are measured by the accumulative impulse responses of the real output gap, models with an increasing hazard function generate real effects of monetary policy which are 2-3 times larger than those in the corresponding Calvo model.

In the literature, some cases of this general hazard pricing model have been studied in different contexts. Wolman (1999) presents preliminary results, showing that inflation dynamics are sensitive to hazard functions under different pricing rules. Mash (2003) constructed a general pricing model that nests both the Calvo and Taylor models, and showed that implications for optimal monetary policy based on those limiting cases are not robust to the change in the hazard function. Whelan (2007) and Sheedy (2007) focus on the relationship between the shape of hazard functions and inflation persistence. The most closely related work is from Carvalho and Schwartzman (2008), who study the relationship between the heterogeneity of sticky prices (sticky information) and monetary non-neutrality. They find that heterogeneity in price stickiness leads to roughly 3 times larger monetary non-neutrality.

The remainder of the paper is organized as follows: in section 2, I present the model with the generalized time-dependent pricing and derive the New Keynesian Phillips curve; section 3 shows analytical results regarding new insights gained from relaxing the constant hazard function underlying the Calvo assumption; in section 4, I simulate the complete DSGE model with some commonly used parameter values in the literature and then present the simulation results; section 5 contains some concluding remarks.

2 The Model

In this section, I present a DSGE model of sticky prices based on both nominal and real rigidities. The scheme of nominal rigidity in the model allows for a general shape of the hazard function. A hazard function of price setting is defined as the probabilities of price adjustment conditional on the spell of time elapsed since the price was last set. Real rigidity is introduced similarly as in Sbordone (2002), who incorporates upward-sloping marginal cost as a source of strategic complementarity.

2.1 Representative Household

The representative infinitely-lived household deduces utilities from the composite consumption good C_t, its labor supply and the real money holding M_t^d/P_t, and it maximizes a discounted sum of utilities of the form:

$$\max_{\{C_t, M_t^d, L_t, B_{t+1}\}} \quad E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi_H \frac{L_t{}^{1+\phi}}{1+\phi} + \chi_M \log \left(\frac{M_t^d}{P_t} \right) \right) \right]$$

Here C_t denotes an index of the households‘s consumption of each of the individual goods $C_t(i)$ following a constant-elastisity-of-substitution aggregator (Dixit and Stiglitz, 1977).

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $\eta > 1$, and it follows that the corresponding cost-minimizing demand for $C_t(i)$ and the welfare based price index P_t are given by

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t \tag{2}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{3}$$

For simplicity, I assume that household supplies homogeneous labor units (L_t) in an enocomy-wide competitive labor market.

The flow budget constraint of the household at the beginning of period t is

$$P_t C_t + M_t^d + \frac{B_t}{R_t} \le M_{t-1}^d + W_t L_t + B_{t-1} + \int_0^1 \pi_t(i) di. \tag{4}$$

Where B_t is an Arrow-Debreu security of one-period bond and R_t denotes the gross nominal return on the bond. $\pi_t(i)$ represents the nominal profits of a firm that sells the good i. I assume that each household owns an equal share of all firms. Finally this sequence of period budget constraints is supplemented with a transversality condition of the form $\lim_{T\to\infty} E_t \left[\frac{B_T}{\Pi_{s=1}^T R_s} \right] \geqslant 0$.

The solution to the household's optimization problem can be expressed in three first order necessary conditions:

$$\chi_H L_t{}^{\phi} C_t^{\sigma} = \frac{W_t}{P_t}, \tag{5}$$

This equation gives the optimal labor supply as a function of real wage.

$$1 = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{R_t P_t}{P_{t+1}} \right], \tag{6}$$

The Euler equation tells us the relationship between the optimal consumption path and asset prices.

$$\chi_M \frac{M_t}{P_t} = \frac{C_t^{\sigma}}{1 - R_t^{-1}}, \tag{7}$$

Finally, the demand of real money balance is determined by weighting between the benefits and costs of holding money.

2.2 Firms in the Economy

In the economy, there is a continuum of monopolistic competitive firms, who use labor as the single input to produce good i.

$$Y_t(i) = Z_t L_t(i)^{1-a} \tag{8}$$

where Z_t denotes an aggregate productivity shock. Log deviations of the shock $\hat{z}_t$ follow an exogenous AR(1) process $\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t}$ is white noises and $\rho_z \in [0, 1)$. $L_t(i)$ is the demand of labor by firm i. Following equation (2), demand for intermediate goods is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t \tag{9}$$

2.2.1 Pricing Decisions under Real Rigidity

In Appendix (A), I derive the economy-wide optimal relative price, which is the ratio between the average optimal price chosen by the adjusting firms and aggregate price index. Note that even through the individual optimal prices are not the same due to the fact that marginal costs generally depend on the amount produced, we can still derive the aggregate optimal relative-price ratio at period t from the average marginal cost in the economy.

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta - 1} \frac{1}{1 - a} \right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi + \sigma(1-a) + a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{10}$$

To show how real rigidity affects price setting in this model, I log-linearize the relative price equation (10). Define $\hat{x}_t = logX_t - log\bar{X}$ as the log deviation from the steady state, up to a log linearization approximation, one can show that the log deviation of the relative price is equal to the log deviation of the economy-wide marginal cost, which in turn is a linear function of log deviations of output gap and the technology shock.

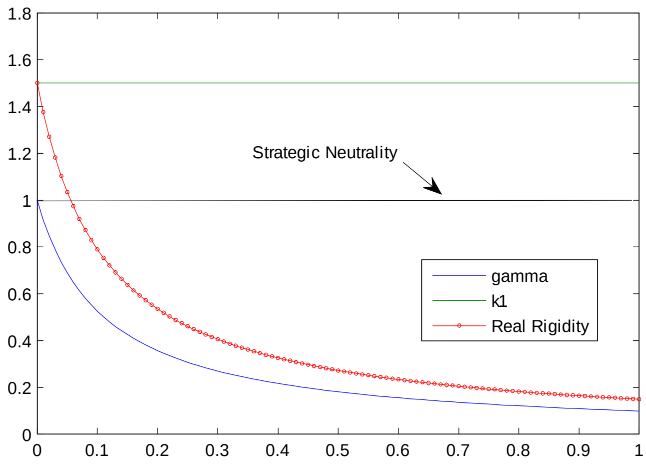


Figure 1: Real Rigidity, when $\sigma = 1, \phi = 0.5$ and $\eta = 10$

$$\begin{aligned}
\widehat{rp}_t &= \widehat{mc}_t = \gamma\left(\kappa_1 \hat{y}_t - \kappa_2 \hat{z}_t\right) \\
where &: \\
\gamma &= \frac{1}{1-a+a\eta} \\
\kappa_1 &= a+\phi+\sigma(1-a) \\
\kappa_2 &= 1+\phi
\end{aligned}$$

Parameters γ and κ_1 have the economic interpretation as the measure of real rigidity. γ is the elasticity of relative prices to the change in real marginal cost, while κ_1 measures the sensitivity of real marginal cost to the change in the output gap. Following Woodford (2003), price-setting decisions are called strategic complementarity when $\gamma\kappa_1 < 1$. When we assume that the monetary authority controls the growth rate of the nominal aggregate demand $\hat{d}_t$, then at equilibrium we have $\hat{y}_t = \hat{d}_t - \hat{p}_t$. In this case, price adjustments are "sticky" even under a flexible price setting, because relative price reacts less than one-to-one to a monetary shock. On the other hand, price setting decisions can be dubbed strategic substitutes when $\gamma\kappa_1 > 1$, so that relative price reacts strongly to monetary policy shocks.

Now we can discuss how changes in the labor share a affect the magnitude of real rigidity of price setting in the model. When setting a equal to zero, creating a linear production technology, then $\gamma = 1$ and $\kappa_1 = \delta + \phi$. Under the standard calibration values in the RBC literature ($\delta = 1$ and $\phi = 0.5$), the real rigidity parameter $\gamma\kappa_1$ is equal to 1.5 and price decisions are strategic substitutes. When the value of a rises, the real rigidity parameter becomes smaller, and price decisions turn into strategic complementarity.

In Figure (1), I plot values of γ and κ_1 against values of a, while setting $\sigma = 1, \phi = 0.5$ and $\eta = 10$. In this special case, the sensitivity of real marginal cost to the change in the output gap κ_1 is not affected by the labor share, while γ decreases fairly quickly as a becomes larger. This means that, given the parameter values, real rigidity is mainly driven by the sensitivity of the relative price to changes in real marginal cost, and the degree of real rigidity is decreasing in a. Only with a modest value of the labor share (around 0.1), real rigidity drops below the strategic neutrality threshold.

2.2.2 Pricing Decisions under Nominal Rigidity

In this section, I introduce a general form of nominal rigidity, which is characterized by an arbitrary hazard function. Many well known price setting models in the literature can be shown to have the incorporation of a hazard function of one form or another. The hazard function in this price setting is defined as the probability of price adjustment conditional on the spell of time elapsed since the price was last set. I assume that monopolistic competitive firms cannot adjust their price whenever they want. Instead, opportunities for re-optimizing prices depend on the hazard function h_j, where j denotes the time-since-last-adjustment and $j \in \{0, J\}$. J is the maximum number of periods in which a firm's price can be fixed. To keep the model general, I do not parameterize the hazard function, so that the relative magnitudes of hazard rates are totally free. As a result, this model is able to nest a wide range of staggered pricing New Keynesian models.

Dynamics of the vintage distribution In the economy, firms' prices are heterogeneous with respect to the time since their last price adjustment. I call them price vintages, while the vintage label j indicates the age of each price group. Table (1) summarizes key notations concerning the dynamics of vintages.

Vintage	Hazard Rate	Non-adj. Rate	Survival Rate	Distribution
j	h_j	α_j	S_j	$\theta(j)$
0	0	1	1	$\theta(0)$
1	h_1	$\alpha_1 = 1 - h_1$	$S_1 = \alpha_1$	$\theta(1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	h_j	$\alpha_j = 1 - h_j$	$S_j = \prod_{i=0}^{j} \alpha_i$	$\theta(j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	$h_J = 1$	$\alpha_J = 0$	$S_J = 0$	$\theta(J)$

Table 1: Notations of the dynamics of price-vintage-distribution.

Using the notation defined in Table (1), and also denoting the distribution of price durations at the beginning of each period by $\Theta_t = \{\theta_t(1), \theta_t(2) \cdots \theta_t(J)\}$, we can derive the ex post distribution of firms after price adjustments ($\tilde{\Theta}_t$)

$$\tilde{\theta}_t(j) = \begin{cases} \sum_{i=1}^{J} h_j \theta_t(i) \text{ , when } j = 0 \\ \alpha_j \theta_t(j) \text{ , when } j = 1 \cdots J \end{cases} \tag{11}$$

Intuitively, those firms that reoptimize their prices in period t are labeled as 'vintage 0', and the proportion of those firms is given by hazard rates from all vintages multiplied by their corresponding densities. The firm left in each vintage are the firms that do not adjust their prices. When period t is over, this ex post distribution $\tilde{\Theta}_t$ becomes the ex ante distribution for the new period Θ_{t+1}. All price vintages move to the next one, because all prices age by one period.

As long as the hazard rates lie between zero and one, dynamics of the vintage distribution can be viewed as a Markov process with an invariant distribution Θ, obtained by solving $\theta_t(j) = \theta_{t+1}(j)$ It yields the stationary vintage distribution $\theta(j)$ as follows:

$$\theta(j) = \frac{\prod_{i=0}^{j} \alpha_i}{\sum_{j=0}^{J} \prod_{i=0}^{j} \alpha_i} = \frac{S_j}{\sum_{j=0}^{J} S_j}, \text{ for } j = 0, 1 \cdots J \tag{12}$$

Let's assume the economy converges to this invariant distribution fairly quickly, so that regardless of the initial vintage distribution, I only consider the economy with the invariant distribution of price durations.

The Optimal Pricing under Nominal Rigidity In a given period when a firm is allowed to reoptimize its price, the optimal price chosen should reflect the possibility that it will not be re-adjusted in the near future. Consequently, adjusting firms choose optimal prices that maximize the discounted sum of real profits over the time horizon during which the new price is expected to be fixed. The probability that the new price is fixed is given by the survival function, S_j, defined in Table (1).

Here I setup the maximization problem of an average adjustor as follows:

$$\max_{P_t^*} E_t \sum_{j=0}^{J-1} S_j Q_{t,t+j} \left[Y_{t+j|t}^d \frac{P_t^*}{P_{t+j}} - \frac{TC_{t+j}}{P_{t+j}} \right]$$

Where E_t denotes the conditional expectation based on the information set in period t, and $Q_{t,t+j}$ is the stochastic discount factor appropriate for discounting real profits from t to $t+j$. Note that here P_t^* is defined as the average optimal price chosen by the average adjusting firm. Therefore TC_t denotes the average total costs of producing output Y_t^d. The representative adjusting Firm maximizes profits subject to demand for intermediate goods in period $t+j$ given that the firm resets the price in period t, $(Y_{t+j|t}^d)$.

$$Y_{t+j|t}^d = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\eta} Y_{t+j},$$

It yields the following first order necessary condition for the optimal price:

$$P_t^* = \frac{\eta}{\eta - 1} \frac{\sum_{j=0}^{J-1} S_j E_t [Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1} MC_{t+j}]}{\sum_{j=0}^{J-1} S_j E_t [Q_{t,t+j} Y_{t+j} P_{t+j}^{\eta-1}]} \tag{13}$$

MC_t denotes the average nominal marginal costs of adjusting firms. The optimal price is equal to the markup multiplied by a weighted sum of future marginal costs, where weights depend on the survival rates. In the Calvo case, where $S_j = \alpha^j$, this equation reduces to the Calvo optimal pricing condition.

Finally, given the stationary distribution $\theta(j)$, aggregate price can be written as a distributed sum of all vintage prices. I define the vintage price which was set j periods ago as P^*_{t-j}. Following the aggregate price index equation (3), the aggregate price is then obtained by:

$$P_t = \left(\sum_{j=0}^{J-1} \theta(j) P^{*1-\eta}_{t-j} \right)^{\frac{1}{1-\eta}} \tag{14}$$

2.2.3 Non-zero-inflation Steady State

If I assume that the gross growth rate of nominal money stock is g, then the steady state is characterized by constant real variables and a growing path of all nominal variables at the rate g. Because the aggregate price level increases with trend inflation in the steady state, firms need to adjust their prices so that the relative prices are close to the optimal ratio specified below. If we define $\overline{X}$ as the steady state value of variable X, then the optimality condition (13) can be rewritten as:

$$\begin{aligned} \bar{p}^*_t &= \frac{\eta}{\eta-1} \frac{\sum_{j=0}^{J} \beta^j S(j) \bar{Y} \bar{P}^{\eta}_{t+j}}{\sum_{j=0}^{J} \beta^j S(j) \bar{Y} \bar{P}^{\eta-1}_{t+j}} mc = \frac{\eta}{\eta-1} \frac{\sum_{j=0}^{J} \beta^j S(j) \bar{Y} \bar{P}^{\eta}_t g^{\eta j}}{\sum_{j=0}^{J} \beta^j S(j) \bar{Y} \bar{P}^{\eta-1}_t g^{(\eta-1)j}} mc \\ \frac{\bar{p}^*_t}{\bar{P}_t} &= \frac{\eta}{\eta-1} mc \left[\frac{\sum_{j=0}^{J} \beta^j S(j) g^{\eta j}}{\sum_{j=0}^{J} \beta^j S(j) g^{(\eta-1)j}} \right] \end{aligned} \tag{15}$$

As seen in Equation (15), the optimal relative price ratio is equal to the constant markup multiplied by the real marginal cost along with an extra term, which reflects how fast trend inflation erodes the relative prices in the economy. When the gross inflation rate is equal to one, this term is also equal one. In this case, we have the standard static price setting equation. However, when trend inflation is greater than one, it follows that the extra term is also greater than one, meaning that the adjusting firms want to 'front-load' their price adjustments in order to hedge the risk that they may not adjust again in the near future. As a result, they adjust their prices more than those in the case of zero inflation. The higher relative price, in turn, leads to lower steady state output and hence, induces an additional welfare loss caused by the steady state inflation.

2.3 Derivation of the New Keynesian Phillips Curve

In this section, I derive the New Keynesian Phillips curve for this generalized model. To do that, I first log-linearize equation (13) around the steady state with the trend inflation ($\bar{\pi}$). This is motivated by King and Wolman (1996) and Ascari (2004), who show that trend inflation plays an important role in both the long-run and the short-run dynamics.

The log-linearized optimal price equations are obtained by

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1}\frac{(\beta g^{\eta})^j\, S(j)}{\Omega}\left(\widehat{mc}_{t+j}+\hat{p}_{t+j}\right)\right], \tag{16}$$

$$where:$$

$$\Omega = \sum_{j=0}^{J-1}(\beta g^{\eta})^j\, S(j) \quad \text{and} \quad \widehat{mc}_t = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta}\hat{y}_t - \frac{1+\phi}{1-a+a\eta}\hat{z}_t$$

In a similar fashion, I derive the log deviation of the aggregate price by log linearizing equation (14).

$$\hat{p}_t = \sum_{k=0}^{J-1}\tau(k)\ \hat{p}_{t-k}^*, \qquad where \quad \tau(k) = \frac{\theta(k)g^{(\eta-1)k}}{\sum_{k=0}^{J-1}\theta(k)g^{(\eta-1)k}} \tag{17}$$

2.3.1 New Keynesian Phillips Curve

To reveal implications of the general hazard function on the inflation dynamics, I derive the generalized NKPC from equations (16) and (17). To keep the equation as simple as possible, I first derive it without trend inflation, i.e. $g = 1$. After some tedious algebra, I obtain the New Keynesian Phillips curve as follows[2]:

$$\begin{aligned}\hat{\pi}_t &= \sum_{k=0}^{J-1}\frac{\theta(k)}{1-\theta(0)}E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S(j)}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S(j)}{\Psi}\hat{\pi}_{t+i-k}\right)\\ &\quad - \sum_{k=2}^{J-1}\Phi(k)\hat{\pi}_{t-k+1}, \qquad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1}S(j)}{\sum_{j=1}^{J-1}S(j)}, \quad \Psi = \sum_{k=0}^{J-1}\beta^j S(j)\end{aligned} \tag{18}$$

At the first glance, this Phillips curve is quite different from the one in the Calvo model. It involves not only lagged inflation but also lagged expectations that were built into pricing decisions in the past. All coefficients in the NKPC are derived from structural parameters which are either the hazard function parameters or the preference parameters. When $J = 3$, for example, then the NKPC is of the following form:

[2] The detailed derivation of the NKPC can be found in the technical Appendix (B).

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \frac{1}{(\alpha_1+\alpha_1\alpha_2)\Psi}\widehat{mc}_t + \frac{\alpha_1}{(\alpha_1+\alpha_1\alpha_2)\Psi}\widehat{mc}_{t-1} + \frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_1\alpha_2)\Psi}\widehat{mc}_{t-2} \\
& + \frac{1}{\alpha_1+\alpha_1\alpha_2}E_t\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+2} + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+2}\right) \\
& + \frac{\alpha_1}{\alpha_1+\alpha_1\alpha_2}E_{t-1}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_t + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_{t+1} + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_t + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t+1}\right) \\
& + \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}E_{t-2}\left(\frac{\beta\alpha_1}{\Psi}\widehat{mc}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\widehat{mc}_t + \frac{\beta\alpha_1+\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2}{\Psi}\hat{\pi}_t\right) \\
& - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_1\alpha_2}\hat{\pi}_{t-1} \\
where \quad : \quad & \Psi = 1+\beta\alpha_1+\beta^2\alpha_1\alpha_2
\end{aligned}
$$

In this example, we see more clearly how current inflation depends on marginal costs, lagged inflation and a complex weighted sum of lagged expectations. All coefficients are expressed in terms of hazard rates ($\alpha_j = 1 - h_j$) and a preference parameter β.

2.3.2 The NKPC with Trend Inflation (g)

When I derive the NKPC by log-linearizing pricing equations around a steady state with non-zero trend inflation, it can be shown that the resulting Phillips curve has the exact same structure as the one without trend inflation. However, trend inflation affects the magnitude of all coefficients in the NKPC. Again, using the example where $J = 3$, we obtain

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \frac{1}{\Psi}mc_t + \frac{1}{\Psi}mc_{t-1} + \frac{1}{\Psi}mc_{t-2} \\
& + \gamma_1 E_t\left(\frac{\beta\alpha_1 g^\eta}{\Psi}mc_{t+1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_{t+2} + \frac{\Psi-1}{\Psi}\hat{\pi}_{t+1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_{t+2}\right) \\
& + \gamma_2 E_{t-1}\left(\frac{\beta\alpha_1 g^\eta}{\Psi}mc_t + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_{t+1} + \frac{\Psi-1}{\Psi}\hat{\pi}_t + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_{t+1}\right) \\
& + \gamma_3 E_{t-2}\left(\frac{\beta\alpha_1 g^\eta}{\Psi}mc_{t-1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}mc_t + \frac{\Psi-1}{\Psi}\hat{\pi}_{t-1} + \frac{\beta^2\alpha_1\alpha_2 g^{2\eta}}{\Psi}\hat{\pi}_t\right) && (19) \\
& - \gamma_3\hat{\pi}_{t-1} && (20) \\
& && (21) \\
\gamma_1 \quad = \quad & \frac{1}{\alpha_1 g^{\eta-1} + \alpha_2\alpha_1 g^{2\eta-2}}, \qquad \gamma_2 = \frac{\alpha_1 g^{\eta-1}}{\alpha_1 g^{\eta-1} + \alpha_2\alpha_1 g^{2\eta-2}} \\
\gamma_3 \quad = \quad & \frac{\alpha_1\alpha_2 g^{2\eta-2}}{\alpha_1 g^{\eta-1} + \alpha_2\alpha_1 g^{2\eta-2}}, \qquad \Psi = 1+\beta\alpha_1 g^\eta + \beta^2\alpha_1\alpha_2 g^{2\eta}
\end{aligned}
$$

In this case, trend inflation (g) enters every coefficient in the Phillips curve, and hence it has not only a significant impact on the steady state, but affects the inflation dynamics in a complex way as well. In general, γ_1 and γ_2 are decreasing in g, while γ_3 is increasing in g. So the changes in trend inflation alter the relative importance between the forward-looking and backward-looking terms in the Phillips curve.

3 Analytical Results

In this section, I explore the generalized NKPC (18) analytically to show which new insights we can learn from relaxing the constant hazard function underlying the Calvo assumption.

3.1 Relationship between the Calvo and the Generalized NKPC

The first question I want to address is why these lagged dynamic terms are absent in the Calvo NKPC? Are they new to the NKPC? The answer is No. Next, I use a proposition to illustrate this point.

Proposition 1 : *When assuming the hazard function is constant over the infinite horizon, the generalized NKPC (18) reduces to the standard Calvo NKPC and the following equation must also hold:*

$$\hat{\pi}_t = E_t\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i}\right) \tag{22}$$

Proof : *see Appendix (C).*

By iterating equation (22) backwards, the following equations hold

$$\hat{\pi}_{t-1} = E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$
$$\hat{\pi}_{t-2} = E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i mc_{t+i-1} + (1-\alpha)\sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right)$$
$$\vdots$$

In light of these results, we learn that the generalized NKPC nests the Calvo Phillips curve in the sense that, given the constant hazard function, the effects of lagged inflation terms exactly cancel the effects of lagged expectations, leaving only current variables and forward-looking expectations on inflation in the expression. Moreover, lagged inflation and lagged expectations are not extrinsic to the time-dependent nominal rigidity model. They are missing in the Calvo setup only because the constant hazard assumption causes them to be canceled out. Therefore, the fully written NKPC of the Calvo model should be of the form:

$$\begin{aligned}
\hat{\pi}_t &= \beta\ \hat{\pi}_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\ \widehat{mc}_t \\
&\quad - \alpha\hat{\pi}_{t-1}\ - \alpha^2\hat{\pi}_{t-2}\ - \cdots \\
&\quad +\alpha E_{t-1}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i\widehat{mc}_{t+i-1} + \sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-1}\right) \\
&\quad +\alpha^2 E_{t-2}\left((1-\alpha)(1-\alpha\beta)\sum_{i=0}^{\infty}\alpha^i\beta^i\widehat{mc}_{t+i-2} + \sum_{i=0}^{\infty}\alpha^i\beta^i\hat{\pi}_{t+i-2}\right) \\
&\quad \vdots
\end{aligned}$$

3.2 The Generalized Calvo Model and the Generalized Taylor Model

In the literature, modelers of nominal rigidity frequently use two mechanisms– the hazard function and the distribution of price durations– to characterize sticky prices or wages. One is following the idea of Calvo (1983), assuming that probabilities of nominal price adjustments are exogenously given, while the other modeling strategy has its origin from the staggered-contract model of Taylor (1980), who assumes some forms of the distribution over price durations. Depending on the purpose, the generalized Calvo model (GCM) tends to use a flexible hazard function (See, e.g. Wolman, 1999), while the generalized Taylor model (GTM) usually applies a flexible distribution of price durations (See, e.g. Jadresic, 1999). Here I present a general result, showing that, up to log linearization approximation, those two models generate the same aggregate dynamics under a certain regularity condition regarding the relationship between the distribution of price durations and the hazard function.

Proposition 2 : *Up to log linearization approximation, the generalized Taylor model (GTM) and the generalized Calvo model (GCM) imply the same aggregate dynamics, when* $\Theta(J)$ *the distribution of price durations in the GTM and* h_j *the hazard function in the GCM correspond according to Equation (40).*

Proof : *see Appendix (D).*

Here I prove that, given the same driving forces of inflation in both models, the aggregate price in the GCM is equal to the aggregate price in the GTM, so that these two models should generate the same aggregate dynamics. In light of this result, models assuming an exogenous distribution of price durations implies an aggregate hazard function, and vice versa. This result is not only theoretically interesting, but also has important implications for empirical work that uses those frameworks to study price stickiness (See: e.g. Coenen et al., 2007). It implies that the aggregate data can be used to uniquely identify both hazard functions and the distribution from either of these two frameworks. In another words, both models should extract same information out of the data about the price stickiness. Therefore one can choose to work on one of those models and safely draw conclusions about both distribution and hazard functions.

4 Numerical Results

4.1 The General Equilibrium Model

In the numerical experiment, I study the behavior of inflation dynamics in a general equilibrium setup. For this purpose, I close the model by adding a nominal money stock growth rule. The log-linearized equilibrium equations are summarized here:

$$
\begin{aligned}
\hat{\pi}_t &= \sum_{k=0}^{J-1} W_1(k,g) E_{t-k} \left(\sum_{j=0}^{J-1} W_2(j,g) \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} W_3(i,g) \hat{\pi}_{t+i-k} \right) - \sum_{k=2}^{J-1} W_4(k,g) \hat{\pi}_{t-k+1} \\
\widehat{mc}_t &= \frac{\phi+\sigma+a}{1+\eta\phi+\eta a} \hat{y}_t - \frac{1+\phi}{1+\eta\phi+\eta a} \hat{z}_t \\
\sigma E_t \left[\hat{y}_{t+1}\right] &= \sigma \hat{y}_t + \left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) \\
\hat{m}_t &= \sigma \hat{y}_t - \frac{\beta}{1-\beta} \hat{\imath}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + g_t \\
\hat{z}_t &= \rho_z * \hat{z}_{t-1} + \epsilon_t \quad where \quad \epsilon_t \backsim N(0, 0.007^2) \\
g_t &= g + u_t \quad where \quad u_t \backsim N(0, 0.0025^2)
\end{aligned}
$$

Where all variable are expressed in terms of log deviations from the non-stochastic steady state. The weights (W_1, W_2, W_3, W_4) in the NKPC are defined in the equation (18). $\hat{m}_t$ is the real money balance, and g_t denotes the growth rate of the nominal money stock, which consists of a constant g and a white-noise shock u_t, representing the regular and irregular parts of the standing monetary policy.

4.2 Calibration

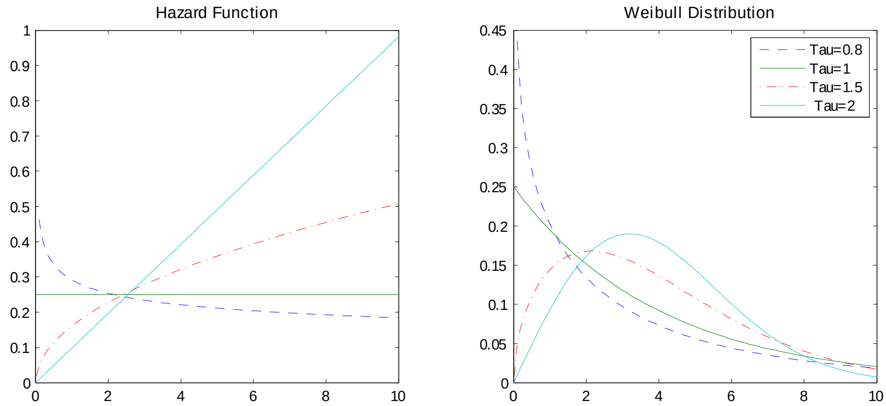
In the calibration, instead of referring to any microeconometric evidence on the hazard function, I parameterize the hazard function a parsimonious way. The reason is that, until now, there is not yet consensus on the shape of hazard functions in the empirical literature. As discussed in the introduction, it is evident that the shape of hazard functions is changing over time with the underlying economic conditions. Since the main purpose of the paper is to demonstrate the impact of varying hazard rates on the inflation dynamics, I choose to calibrate it based on the statistical theory of duration analysis. In particular, the functional form I apply is the hazard function of the Weibull distribution, which has two parameters:

$$
h(j) = \frac{\tau}{\lambda} \left(\frac{j}{\lambda} \right)^{\tau-1} \tag{23}
$$

λ is the scale parameter, which controls the average duration of the price adjustment, while τ is the shape parameter to determine the monotonic property of the hazard function. It enables the incorporation of a wide range of hazard functions by using various values for the shape parameter. In fact, any value of the shape parameter that is greater than one corresponds to an increasing hazard function, while values ranging between zero and one lead to a decreasing hazard function. By setting the shape parameter to one, we can retrieve the Poisson process from the Weibull distribution.

In this numerical experiment, I choose λ, such that it implies an average price duration of 3 quarters, which is largely consistent with the median price durations of 7 - 9 months documented by Nakamura and Steinsson (2008). The shape parameter is set in the interval between one and three, which covers a wide range of shapes of the hazard function[3]. As for

[3] This range only covers increasing hazard functions because it makes the maximum number of price duration J well defined.



the rest of the structural parameters, I use some common values in the literature to facilitate comparison the results. In the calibration of the preference parameters, I assume $\beta = 0.9902$, which implies a steady state real return on financial assets of about four percent per annum. I also assume the intertemporal elasticity of substitution $\sigma = 1$, implying log utility of consumption. I choose the Frisch elasticity of the labor supply to equal 0.5, an estimate commonly found in microeconometric studies (See: e.g. Blundell and Macurdy). As for the technology parameters, I set labor's share to be either 0 or 0.36 to show the effect of real rigidity. The elasticity of substitution between intermediate goods $\eta = 10$, which implies the desired markup over marginal cost should be about 11%. Finally, I set the standard deviation of the innovation to the nominal money growth rate to be 25 basic points per quarter. For the aggregate technology shock, I choose $\rho_z = 0.95$ and the standard deviation of 0.007, in line with commonly used values in the RBC literature, for example King and Rebelo (2000).

4.3 Simulation Results

To evaluate the quantitative implication for the aggregate dynamics, I apply the standard algorithm to solve for the log-linearized rational expectation model.

4.3.1 Effects of Increasing Hazard Functions

In the first experiment, I study the effects of varying the shape parameter on the equilibrium dynamics without any real rigidity and the trend inflation. In Table (2), I report second moments generated by the theoretical models, which are different with respect to the shape of the hazard function. Because I use the Weibull hazard function to calibrate the model, I can change the shape of the hazard function by varying the value of the shape parameter τ. In this experiment, I focus on the comparison between the baseline Calvo case, with a corresponding shape parameter of $\tau = 1$, and the increasing hazard models, where τ falls in the range between 1.6 and 3. In all cases, the moments are for a Hodrick-Prescott filtered time series. For each of these hazard functions, two sets of statistics are reported: first, the first-order autocorrelation coefficient of deviations on inflation, real marginal cost and output; and second, contemporaneous correlation coefficients between inflation and real marginal cost. In all models, I use a persistent technology shock and a transitory monetary shock, whose stochastic properties are specified above.

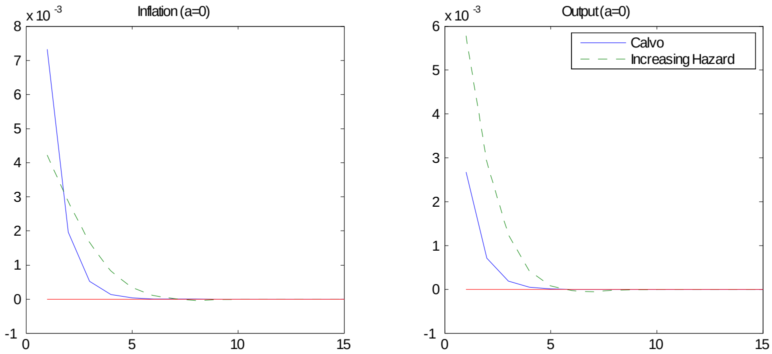


Figure 2: Comparing impulse responses functions

	Calvo Model	Increasing Hazard Models				
τ	1	1.6	1.8	2	2.5	3
$AR(1)\ \ \hat{\pi}$	0.166	0.524	0.537	0.549	0.567	0.576
$AR(1)\ \ \hat{y}$	0.811	0.876	0.874	0.873	0.870	0.868
$AR(1)\ \widehat{mc}$	0.169	0.362	0.338	0.318	0.280	0.264
$Corr(\hat{\pi}, \widehat{mc})$	0.998	0.977	0.965	0.950	0.915	0.891

Table 2: Second moments of the simulated data (HP filtered, lambda=1600)

The first noteworthy result from the table is that models with increasing hazard rates generate much higher persistence in inflation than in the Calvo model, *ceteris paribus*. Secondly, increases in the shape parameter reduces the persistence of real marginal cost and output. In the Calvo case, because inflation persistence is solely determined by the dynamics of real marginal cost, inflation persistence cannot exceed persistence of real marginal cost. In the increasing hazard model, however, the autoregressive terms of real marginal cost are brought into the Phillips curve through lagged expectations, and thus, in comparison to the Calvo model, this new transmission mechanism propagates more inflation persistence. Fuhrer (2006) presented empirical evidence showing that it is difficult to have a sizable coefficient on the driving process in the Calvo NKPC and that a reduced form shock in the NKPC explains a significant portion of the inflation persistence. We can understand this evidence through the lens of the generalized NKPC. The problem of the conventional NKPC is essentially caused by ignoring terms like lagged inflations and lagged expectations. As I show in the analytical result, this is not the case in the more general time-dependent pricing model. The misspecified Phillips curve fails to explain inflation persistence with its limited structure. Consequently, we either need to introduce the ad hoc backward-looking behavior or a persistent reduced-form shock to achieve a good fit to the data. Last but not least, as shown in the final row of the table, the increasing-hazard pricing model also helps to reduce the correlation between inflation and current real marginal cost, a rather robust feature of the data (See: e.g. Hornstein, 2007).

Figure 2 shows the impulse responses of the Calvo model compared to the increasing-hazard model with the shape parameter of 2. The left panel depicts the impulse responses of inflation

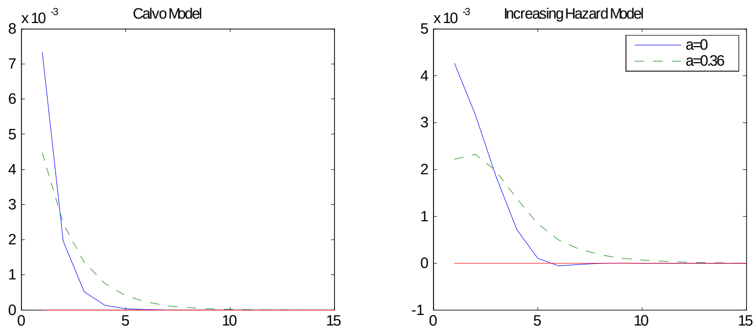


Figure 3: Impulse responses of inflation with real rigidity

while the right panel shows those of the output gap to a 1% increase in the annual nominal money growth rate. Without real rigidity and trend inflation, we observe that, even though the impulse response function of the increasing-hazard model is somewhat more persistent, the general pattern of the impulse responses are the same in both cases, namely, they drop monotonically back to the steady state.

4.3.2 Effects of Real Rigidity

As influentially argued in Woodford (2003), real price rigidity plays an important role in inflation dynamics in addition to nominal rigidity. In this model I introduce real rigidity in a parsimonious way, following Sbordone (2002). I now set the labor share parameter equal to 0.36. Combining this with other parameter values in the model, it implies that the real rigidity parameter ($\gamma\kappa_1 = \frac{a+\phi+\sigma(1-a)}{1-a+a\eta}$) equals 0.35, representing a modest level of strategic complementarity.

In Figure (3), I compare the impulse responses of inflation to a transitory money growth shock with and without real rigidity. The left panel shows the comparison in the Calvo model. Incorporation of real rigidity makes the impulse responses more long-lasting, but still monotonic. By contrast, in the right panel, impulse responses of inflation in the increasing hazard model change substantially with real rigidity. One can see that not only the persistence of the impulse response function gets improved, but, more importantly, the shape of it as well. In this case, the IRF becomes hump-shaped with a peak at around the second quarter.

The economic intuition behind this result is that, on the one hand, increasing hazard function postpones the timing of the price adjustment, i.e. only a few firms adjust their prices immediately after a shock, and more and more adjust later on. On the other hand, real rigidity helps to amplify this postponing effect even further. Because price decisions are strategic complementary, when fewer firms adjust their prices at the beginning phase of the IRF, even the adjusting firms choose a small size of the adjustment. Afterwards, however, when more firms reset their prices, the size of the price adjustment becomes also larger. In another words, the increasing-hazard pricing together with some degree of real rigidity not only affect the timing of the price adjustment, but also the average magnitude of firms' adjustments, leading to a hump-shaped response.

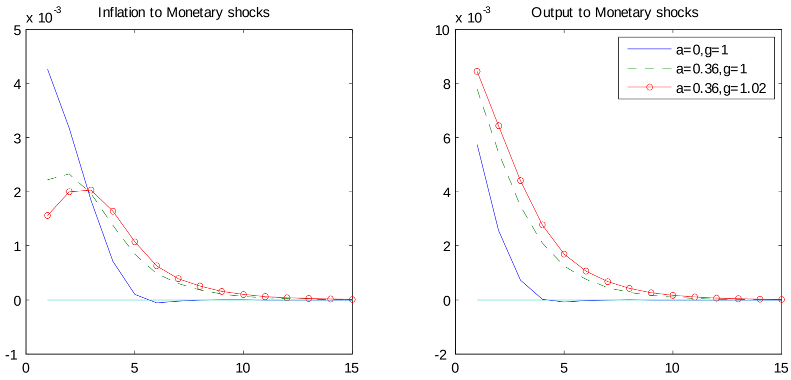


Figure 4: Impulse response functions with real rigidity and trend inflation

4.3.3 Effects of Trend Inflation

In his seminal paper, Ascari (2004) has shown that trend inflation has important implications for the model's dynamics when the Calvo pricing model is log-linearized around non-zero trend inflation. Here I analyze the dynamic effects of trend inflation in the increasing hazard pricing model. Combining these features is an interesting exercise, because, as I have shown in the previous section, introducing trend inflation affects all coefficients in the generalized NKPC (See Equation 19), and hence it changes the relative importance between the forward-looking and backward-looking terms in the Phillips curve. As a result, trend inflation exerts a larger impact on the dynamics of inflation in the increasing-hazard pricing model than in the Calvo case.

In Figure (4), I show the impulse responses of inflation and of the output gap to a transitory money growth shock in the increasing hazard model. In the left panel, inflation without real rigidity and trend inflation reacts to monetary shock monotonically (solid blue line), while the dashed green line depicts the impulse response of inflation when real rigidity is present. As shown earlier, this line becomes hump-shaped. Furthermore, when I add a non-zero trend inflation into the dynamic structure, the hump becomes even more salient and peaks later (red circled line). On the right panel, impulse responses of the output gap show that the real effect of the monetary shock is more persistent in the case when real rigidity and trend inflation are presented.

The reason why high trend inflation amplifies the effect of increasing hazard functions is, for one, that firms in the increasing hazard model are more likely to adjust when their prices are old. When presenting trend inflation, relative prices disperse quickly over time and, as a result, high trend inflation causes the size of a firm's first adjustment is increasing in the time since the shock occurred.

4.3.4 Real Effects of the Monetary Shock

In the previous sections, I have informally shown that the real effects of the monetary shock is larger in the increasing hazard model than in the Calvo case. Here I introduce a quantitative measure of the real effects of money. In Table (3), I report the accumulative IRF of the real output gap to a transitory 1% increase in the annual nominal money growth rate. The accumulative IRF is the area below the impulse response function over the whole horizon, and it is in the unit of percentage of the steady state level of real output.

Real Effects	Calvo Model		Increasing Hazard Model ($\tau = 2$)		
	a=0	a=0.36	a=0, g=1	a=0.36, g=1	a=0.36, g=1.2
Acc.IRF (%)	0.09	0.26	0.22	0.48	0.56

Table 3: Real Effects of A Transitory Monetary Shock) with varying trend inflation

In the Calvo model without any real rigidity, the real effect of money is only about 0.09% of real output in the steady state, while this figure rises by a factor of 3 when a modest level of real rigidity is present. On the other hand, the increasing hazard model can generate this level of real effects of the monetary shock even without any helping features. When adding real rigidity into the increasing hazard model, however, real effects rise to 0.48% of steady state real output, and presenting trend inflation reinforces real effects even further. All in all, the increasing hazard model implies 2-3 times more real effects of the monetary shock than the constant-hazard Calvo model.

5 Conclusion

The central theme of this study is to show that non-constant hazard functions underlying a pricing assumption implies very different aggregate dynamics. To illustrate this point, I derive a general New Keynesian Phillips curve, reflecting an arbitrary hazard function, trend inflation and real rigidity.

My main analytical results show that, first, the generalized NKPC involves components including lagged inflation, forward-looking and lagged expectations of inflations and real marginal cost, which nests the standard Calvo Phillips curve as a limiting case. When the hazard function is constant, the effect of lagged inflation exactly cancels the effects of the lagged expectation terms, so that only current variables and forward-looking expectations remain in the expression. Furthermore, I present a general result, showing that under a certain condition regarding the relationship between the distribution of price durations and the hazard function, the log deviation of the aggregate price in the GCM is equal to that in the GTM. In light of this result, hazard functions and random distribution of price durations are closely related concepts, and when setting them up accordingly, both models imply the same aggregate dynamics.

In the numerical exercise, I show that inflation and output are more persistent in the increasing hazard model than in the Calvo case. Introducing real rigidity and trend inflation strengthens the dynamic effects of the increasing hazard function on inflation even further. The model can account for hump-shaped impulse responses of inflation to the monetary shock. The real effects of the monetary shock implied by the increasing hazard model are 2-3 times higher than those in the Calvo model. However, due to the calibration strategy I choose in my paper, the numerical results are limited in the class of the monotonic shape of hazard functions. For future research, microfounded hazard functions of price setting behavior is clearly a favorable extension for further exploration of the topic.

References

Alvarez, L. J. (2007), What do micro price data tell us on the validity of the new keynesian phillips curve?, Kiel working papers, Kiel Institute for the World Economy.

Alvarez, L. J., E. Dhyne, M. Hoeberichts, C. Kwapil, H. L. Bihan, P. Lünnemann, F. Martins, R. Sabbatini, H. Stahl, P. Vermeulen, and J. Vilmunen (2006), Sticky prices in the euro area: A summary of new micro-evidence, *Journal of the European Economic Association*, 4(2-3), 575–584.

Ascari, G. (2004), Staggered prices and trend inflation: Some nuisances, *Review of Economic Dynamics*, 7(3), 642–667.

Bils, M. and P. J. Klenow (2004), Some evidence on the importance of sticky prices, *Journal of Political Economy*, 112(5), 947–985.

Blundell, R. and T. Macurdy (), Labor supply: A review of alternative approaches, in: O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*.

Calvo, G. A. (1983), Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, 12(3), 383–98.

Campbell, J. R. and B. Eden (2005), Rigid prices: evidence from u.s. scanner data, Working paper series, Federal Reserve Bank of Chicago.

Carvalho, C. (2005), Heterogeneity in price setting and the real effects of monetary shocks, Macroeconomics, EconWPA.

Carvalho, C. and F. Schwartzman (2008), Heterogeneous price setting behavior and aggregate dynamics: Some general results, Working paper.

Cecchetti, S. G. (1986), The frequency of price adjustment : A study of the newsstand prices of magazines, *Journal of Econometrics*, 31(3), 255–274.

Coenen, G., A. T. Levin, and K. Christoffel (2007), Identifying the influences of nominal and real rigidities in aggregate price-setting behavior, *Journal of Monetary Economics*, 54(8), 2439–2466.

Dixit, A. K. and J. E. Stiglitz (1977), Monopolistic competition and optimum product diversity, *American Economic Review*, 67(3), 297–308.

Dixon, H. and E. Kara (2005), Persistence and nominal inertia in a generalized taylor economy - how longer contracts dominate shorter contracts, Working paper series, European Central Bank.

Fuhrer, J. C. (2006), Intrinsic and inherited inflation persistence, *International Journal of Central Banking*, 2(3).

Goette, L., R. Minsch, and J.-R. Tyran (2005), Micro evidence on the adjustment of sticky-price goods: It's how often, not how much, Discussion papers, University of Copenhagen. Department of Economics.

Hornstein, A. (2007), Evolving inflation dynamics and the new keynesian phillips curve, *Economic Quarterly*, (Fall), 317–339.

Jadresic, E. (1999), Sticky prices - an empirical assessment of alternative models, Imf working papers, International Monetary Fund.

King, R. G. and S. T. Rebelo (2000), Resuscitating real business cycles, Nber working papers, National Bureau of Economic Research, Inc.

King, R. G. and A. L. Wolman (1996), Inflation targeting in a st. louis model of the 21st century, *Proceedings*, (May), 83–107.

Mash, R. (2003), New keynesian microfoundations revisited: A calvo-taylor-rule-of-thumb model and optimal monetary policy delegation, Tech. rep.

Midrigan, V. (2007), Menu costs, multi-product firms, and aggregate fluctuations, CFS Working Paper Series 2007/13, Center for Financial Studies.

Nakamura, E. and J. Steinsson (2008), Five facts about prices: A reevaluation of menu cost models, *The Quarterly Journal of Economics*, 123(4), 1415–1464.

Sbordone, A. M. (2002), Prices and unit labor costs: a new test of price stickiness, *Journal of Monetary Economics*, 49(2), 265–292.

Sheedy, K. D. (2007), Intrinsic inflation persistence, CEP Discussion Papers dp0837, Centre for Economic Performance, LSE.

Taylor, J. B. (1980), Aggregate dynamics and staggered contracts, *Journal of Political Economy*, 88(1), 1–23.

Whelan, K. (2007), Staggered price contracts and inflation persistence: Some general results, *International Economic Review*, 48(1), 111–145.

Wolman, A. L. (1999), Sticky prices, marginal cost, and the behavior of inflation, *Economic Quarterly*, (Fall), 29–48.

Woodford, M. (2003), *Interest and Prices*, Princeton University Press.

A Deviation of the Marginal Cost

I assume that there is an economy-wide competitive labor market, and hence intermediate firms are price takers in this market. In each period, firms choose optimal demands for labor inputs to maximize their real profits given wage and the production technology (8).

$$\max_{L_t(i)} \Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) \tag{24}$$

Real marginal cost can be derived from this maximization problem in the form:

$$mc_t(i) = \frac{W_t/P_t}{(1-a)\, Z_t L_t(i)^{-a}}$$

Using the production function (8), output demand equation (9), the labor supply condition (5) and the fact that at the equilibrium $C_t = Y_t$, we obtain the real marginal cost as follows:

$$mc_t(i) = \frac{1}{1-a} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a}} Z_t^{-\frac{1+\phi}{1-a}} \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\eta a}{1-a}} \tag{25}$$

Because marginal costs depend on the demand of the individual good, the price set by the firm also affects the marginal costs of the firm. Next, firms determine their optimal prices given marginal costs and the market demand for their goods (9)

$$\max_{P_t(i)} \Pi_t(i) = Y_t(i) \left(\frac{P_t(i)}{P_t} - mc_t(i)\right)$$

The first order condition for $P_t(i)$ yields:

$$\frac{P_t^*(i)}{P_t} = \frac{\eta}{\eta-1} mc_t(i)$$

The optimal relative price is equal to the markup multiplied by real marginal cost. By substituting the real marginal cost with equation (25), we get the economy-wide average relative price in the form:

$$\frac{P_t^*}{P_t} = \left(\frac{\eta}{\eta-1}\frac{1}{1-a}\right)^{\frac{1-a}{1-a+\eta a}} Y_t^{\frac{\phi+\sigma(1-a)+a}{1-a+\eta a}} Z_t^{-\frac{1+\phi}{1-a+\eta a}} \tag{26}$$

B Deviation of the New Keynesian Phillips Curve

Here I derive the NKPC for $g = 1$, Starting from 16

$$\hat{p}_t^* = E_t\left[\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\left(\widehat{mc}_{t+j} + \hat{p}_{t+j}\right)\right] \tag{27}$$

$$= E_t\left[\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{p}_{t+j}\right] \tag{28}$$

The last term can be further expressed in terms of future rates of inflation

$$\begin{aligned}
\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{p}_{t+j} &= \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\hat{p}_{t+J-1} \\
&= \frac{1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\hat{p}_t + \frac{\beta S_1}{\Psi}\left(\hat{p}_{t+1} - \hat{p}_t\right) + \cdots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\hat{p}_{t+J-1} \\
&= \left(\frac{1}{\Psi} + \frac{\beta S_1}{\Psi}\right)\hat{p}_t + \sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \frac{\beta^2 S_2}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\hat{p}_{t+J-2} \\
&= \left(\frac{1}{\Psi} + \frac{\beta S_1}{\Psi} + \frac{\beta^2 S_2}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} + \sum_{j=2}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1} \\
&\quad + \frac{\beta^3 S_3}{\Psi}\hat{p}_{t+1} + \cdots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\hat{p}_{t+J-2} \\
&\quad \vdots \\
&= \left(\frac{1}{\Psi} + \frac{\beta S_1}{\Psi} + \cdots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\right)\hat{p}_t + \sum_{j=1}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j} \\
&\quad + \sum_{j=2}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+j-1} + \ldots + \frac{\beta^{J-1} S_{J-1}}{\Psi}\hat{\pi}_{t+1} \\
&= \hat{p}_t + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}
\end{aligned}$$

The optimal price can be expressed in terms of inflation rates, real marginal cost and aggregate prices.

$$\hat{p}_t^* = \hat{p}_t + E_t\left[\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right] + E_t\left[\sum_{i=1}^{J-1}\sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right] \tag{29}$$

Next, I derive the aggregate price equation as the sum of past optimal prices. I lag equation

29 and substitute it for each $\hat{p}^*_{t-j}$ into equation **??**

$$
\begin{aligned}
\hat{p}_t &= \theta(0)\ \hat{p}^*_t + \theta(1)\ \hat{p}^*_{t-1} + \cdots + \theta(J-1)\hat{p}^*_{t-J+1} \\
&= \theta(0)\left[\hat{p}_t + E_t\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j}\right) + E_t\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i}\right)\right] \\
&+ \theta(1)\left[\hat{p}_{t-1} + E_{t-1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-1}\right) + E_{t-1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-1}\right)\right] \\
&\vdots \\
&+ \theta(J-1)\left[\hat{p}_{t-J+1} + E_{t-J+1}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-J+1}\right) + E_{t-J+1}\left(\sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-J+1}\right)\right] \\
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\left[\hat{p}_{t-k} + \underbrace{E_{t-k}\left(\sum_{j=0}^{J-1}\frac{\beta^j S_j}{\Psi}\widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1}\sum_{j=i}^{J-1}\frac{\beta^j S_j}{\Psi}\hat{\pi}_{t+i-k}\right)}_{F_{t-k}}\right] \qquad (30)
\end{aligned}
$$

Where F_t summarizes all current and lagged expectations formed at period t.
Finally, we derive the New Keynesian Phillips curve from equation 30.

$$
\begin{aligned}
\hat{p}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} + \underbrace{\sum_{k=0}^{J-1}\theta(k)F_{t-k}}_{Q_t} \\
\hat{\pi}_t &= \sum_{k=0}^{J-1}\theta(k)\ \hat{p}_{t-k} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + \theta(0)\hat{p}_{t-1} + \theta(1)\hat{p}_{t-1} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \theta(0)\ (\hat{p}_t - \hat{p}_{t-1}) + (\theta(0)+\theta(1))\,\hat{p}_{t-1} + \theta(2)\hat{p}_{t-2} + \cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= \underbrace{\theta(0)}_{W(0)}\hat{\pi}_t + \underbrace{(\theta(0)+\theta(1))}_{W(1)}\hat{\pi}_{t-1} + (\theta(0)+\theta(1)+\theta(2))\,\hat{p}_{t-2}\cdots + \theta(J-1)\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&\vdots \\
&= W(0)\ \hat{\pi}_t + W(1)\hat{\pi}_{t-1} + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{W(J-1)}_{=1}\hat{p}_{t-J+1} - \hat{p}_{t-1} + Q_t \\
&= W(0)\ \hat{\pi}_t + \cdots + W(J-2)\hat{\pi}_{t-J+2} + \underbrace{\hat{p}_{t-J+1} - \hat{p}_{t-J+2}}_{-\hat{\pi}_{t-J+2}} + \hat{p}_{t-J+2} - \cdots + \underbrace{\hat{p}_{t-2} - \hat{p}_{t-1}}_{-\hat{\pi}_{t-1}} + Q_t \\
(1-W(0))\hat{\pi}_t &= -(1-W(2))\hat{\pi}_{t-1} - \cdots - (1-W(J-1))\hat{\pi}_{t-J+2} + Q_t \\
\hat{\pi}_t &= -\sum_{k=2}^{J-1}\frac{1-W(k)}{1-\theta(0)}\hat{\pi}_{t-k+1} + \sum_{k=0}^{J-1}\frac{\theta(k)}{1-\theta(0)}F_{t-k}
\end{aligned}
$$

The generalized New Keynesian Phillips curve is:

$$
\begin{aligned}
\hat{\pi}_t &= \sum_{k=0}^{J-1} \frac{\theta(k)}{1-\theta(0)} E_{t-k} \left(\sum_{j=0}^{J-1} \frac{\beta^j S_j}{\Psi} \widehat{mc}_{t+j-k} + \sum_{i=1}^{J-1} \sum_{j=i}^{J-1} \frac{\beta^j S_j}{\Psi} \hat{\pi}_{t+i-k} \right) \\
&\quad - \sum_{k=2}^{J-1} \Phi(k) \hat{\pi}_{t-k+1}, \qquad where\ \Phi(k) = \frac{\sum_{j=k}^{J-1} S_j}{\sum_{j=1}^{J-1} S_j}, \quad \Psi = \sum_{j=0}^{J-1} \beta^j S_j
\end{aligned} \tag{31}
$$

C Proof for Proposition 1

In the Calvo pricing case, all hazards are equal to a constant between zero and one. Let's denote the constant hazard as $h = 1 - \alpha$. We can rearrange the NKPC 18 in the following way:

$$
\begin{aligned}
\hat{\pi}_t + \sum_{k=1}^{\infty} \alpha^k \hat{\pi}_{t-k} &= (1-\alpha) \sum_{k=0}^{\infty} \alpha^k E_{t-k} \left((1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-k} + \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-k} \right) \\
\hat{\pi}_t + \alpha\hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} + \cdots &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&+ \alpha^2 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right) \\
&\vdots
\end{aligned} \tag{32}
$$

Then iterating this equation one period forward,

$$
\begin{aligned}
\hat{\pi}_{t+1} + \alpha\hat{\pi}_t + \alpha^2 \hat{\pi}_{t-1} + \alpha^3 \hat{\pi}_{t-2} \cdots &= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+ \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&\vdots \\
\hat{\pi}_{t+1} + \alpha(\hat{\pi}_t + \alpha\hat{\pi}_{t-1} + \alpha^2 \hat{\pi}_{t-2} \cdots) &= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right) \\
&+ \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \\
&+ \alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right) \\
&\vdots
\end{aligned}
$$

Substitute equation **??** for the term in the brackets on the left hand side of this equation,

$$\hat{\pi}_{t+1} + \alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)$$
$$+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)$$
$$+\alpha^3 E_{t-2} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-2} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-2} \right)$$
$$\vdots$$
$$= E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)$$
$$+\alpha E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right)$$
$$+\alpha^2 E_{t-1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i-1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i-1} \right)$$
$$\vdots$$

After canceling out equal terms from both sides of the equation, we obtain the following equation:

$$\hat{\pi}_{t+1} = E_{t+1} \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i+1} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i+1} \right)$$

Iterate this equation backwards and rearrange it, we get the familiar NKPC of the Calvo model.

$$\begin{aligned} \hat{\pi}_t &= E_t \left((1-\alpha)(1-\alpha\beta) \sum_{i=0}^{\infty} \alpha^i \beta^i mc_{t+i} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \beta^i \hat{\pi}_{t+i} \right) \qquad (33) \\ \hat{\pi}_t &= (1-\alpha)(1-\alpha\beta) mc_t + (1-\alpha)\hat{\pi}_t + \alpha\beta E_t \left(\hat{\pi}_{t+1}\right) \\ \hat{\pi}_t &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t \left(\hat{\pi}_{t+1}\right) \end{aligned}$$

Proof done

D Proof for Proposition 2

D.1 The Generalized Calvo model (GCM)

Firstly, I denote j as the time-since-last-adjustment, which is the vintage label used in the generalized Calvo model. Furthermore, I define $\bar{J}$ as the maximum duration, in which a price can be fixed. As a result, in general prices differ across vintages ($j \in \{0, \bar{J}\}$), but, in each vintage, the average price shown in the paper is following the equation (13). After log-linearization, it yields:

$$\hat{p}_t^{GCM} = \sum_{j=0}^{\bar{J}-1} \frac{\beta^j S_j}{\sum_{j=0}^{\bar{J}} \beta^j S_j} E_t\left(\widehat{MC}_{t+j}\right) \tag{34}$$

Next, the log-linearized aggregate price in the GCM $\left(\hat{p}_t^C\right)$ is obtained by summing over all vintage prices, weighted by the stationary distribution $\theta(k)$,

$$\hat{p}_t^C = \sum_{k=0}^{J-1} \theta(k)\ \hat{p}_{t-k}^{GCM}, \qquad where \quad \theta(k) = \frac{S_k}{\sum_{k=0}^{J-1} S_k}, \text{ for } j = 0, 1 \cdots J$$

$$\hat{p}_t^C = \sum_{k=0}^{J-1} \theta(k) \left[\sum_{j=0}^{\bar{J}-1} \frac{\beta^j S_j}{\sum_{j=0}^{\bar{J}} \beta^j S_j} E_{t-k}\left(\widehat{MC}_{t+j-k}\right)\right] \tag{35}$$

D.2 The Generalized Taylor model (GTM)

In the GTM, there are $\bar{J}$ different price sectors in which prices are set exactly for J periods. As in the GCM, $\bar{J}$ is defined as the maximum duration in which a price can be fixed. As a result, J should range between 1 and $\bar{J}$. In addition, I denote the distribution of price sectors by $\Theta(J)$, with $J \in \{1, \bar{J}\}$.

In each price sector, the price choice is made to maximize the real profit over the next $J-1$ periods

$$\max_{P_t^J} \sum_{j=0}^{J-1} E_t\{Q_{t,t+j}\left[Y_{t+j|t}^d \frac{P_t^J}{P_{t+j}} - \frac{TC_{t+j}}{P_{t+j}}\right]\}$$

given that $Q_{t,t+j} = \beta E_t\left[\left(\frac{Y_t}{Y_{t+1}}\right)^{\sigma}\right]$ and $Y_{t+j|t}^d = \left(\frac{P_t^J}{P_{t+j}}\right)^{-\eta} Y_{t+j}$, which are consistent with those in the GCM.

It yields the following first order necessary condition for the optimal price in price sector J :

$$P_t^J = \frac{\eta}{\eta-1} \frac{\sum_{j=0}^{J-1} \beta^j E_t[Y_{t+j}^{1-\sigma} P_{t+j}^{\eta-1} MC_{t+j}]}{\sum_{j=0}^{J-1} \beta^j E_t[Y_{t+j}^{1-\sigma} P_{t+j}^{\eta-1}]} \tag{36}$$

After log-linearization, it yields:

$$\hat{p}_t^J = \sum_{j=0}^{J-1} \frac{\beta^j}{\sum_{j=0}^{J-1} \beta^j} E_t \left(\widehat{MC}_{t+j} \right) \tag{37}$$

Within each price sector J, there is exactly a $1/J$ fraction of firms that adjust to the new price $\hat{p}_t^J$. And there is also exactly a $1/J$ fraction of firms that still use the one-period-old price, $\hat{p}_{t-1}^J$, and so on. Therefore the average price in each sector can be expressed as follows:

$$\bar{p}_t^J = \frac{1}{J} \sum_{k=0}^{J-1} \hat{p}_{t-k}^J \tag{38}$$

Next, the aggregate price in the GTM can be obtained by summing over all average prices across sectors, weighted by the distribution of price sectors $\Theta(J)$,

$$\begin{aligned} \hat{p}_t^T &= \sum_{J=1}^{\bar{J}} \Theta(J)\ \bar{p}_t^J \\ &= \sum_{J=1}^{\bar{J}} \frac{\Theta(J)}{J} \left(\sum_{k=0}^{J-1} \hat{p}_{t-k}^J \right) \\ \hat{p}_t^T &= \sum_{J=1}^{\bar{J}} \frac{\Theta(J)}{J} \left\{ \sum_{k=0}^{J-1} \left[\sum_{j=0}^{J-1} \frac{\beta^j}{\sum_{j=0}^{J-1} \beta^j} E_{t-k} \left(\widehat{MC}_{t+j-k} \right) \right] \right\} \end{aligned} \tag{39}$$

When the aggregate dynamics in the GCM are the same as in the GTM, then the aggregate price obtained from both models should be equal,

$$\begin{aligned} \hat{p}_t^T &= \hat{p}_t^c \\ \sum_{J=1}^{\bar{J}} \frac{\Theta(J)}{J} \left\{ \sum_{k=0}^{J-1} \left[\sum_{j=0}^{J-1} \frac{\beta^j}{\sum_{j=0}^{J-1} \beta^j} E_{t-k} \left(\widehat{MC}_{t+j-k} \right) \right] \right\} &= \sum_{k=0}^{J-1} \theta(k) \left[\sum_{j=0}^{\bar{J}-1} \frac{\beta^j S_j}{\sum_{j=0}^{\bar{J}} \beta^j S_j} E_{t-k} \left(\widehat{MC}_{t+j-k} \right) \right] \end{aligned}$$

Given the same driving forces of inflation ($\widehat{MC}_t$) in both models, we have the same aggregate prices when the weights to the corresponding marginal costs are equal. This yields

$$\sum_{J=1}^{\bar{J}} \frac{\Theta(J)}{J} \left\{ \sum_{k=0}^{J-1} \left[\sum_{j=0}^{J-1} \frac{\beta^j}{\sum\limits_{j=0}^{J-1} \beta^j} E_{t-k} \left(\widehat{MC}_{t+j-k} \right) \right] \right\} = \sum_{k=0}^{J-1} \theta(k) \left[\sum_{j=0}^{\bar{J}-1} \frac{\beta^j S_j}{\sum\limits_{j=0}^{\bar{J}} \beta^j S_j} E_{t-k} \left(\widehat{MC}_{t+j-k} \right) \right] \tag{40}$$

$$where: \theta(k) = \frac{S_k}{\sum\limits_{k=0}^{J-1} S_k} \; S_J = \prod_{j=0}^{J} (1 - h_j)\,, \text{ for } j = 0, 1 \cdots \bar{J}$$

Equation (40) gives the exact correspondence between the distribution of price sector $\Theta(J)$ in the GTM and the hazard function h_j in the GCM. In principle, one can solve these $\bar{J} - 1$ number of equations of corresponding weights, plus the regularity condition that $\sum\limits_{J=1}^{\bar{J}} \Theta(J) = 1$, for $\Theta(J)$, then we get the expression of $\Theta(J)$ in terms of h_j. *Proof done*

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