# Multinationals, tax competition and outside options.\*

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#### Abstract

Lower barriers to entry and developments in world capital markets have increased the actual and potential mobility of multinational enterprises. This poses challenges for host countries' tax policies. The paper examines implications for such policies, for multinantionals' investment decisions and for host countries' welfare in cooperative and non-cooperative settings. An interesting finding is that more attractive outside options for firms may constitute a win-win situation; the firm as well as its present host countries may gain when this occurs. This means that better outside options for mutinational firms may reduce the gains from host countries' policy coordination and thus reduce those countries' incentives to coordinate their policies.

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# 1 Introduction

Lower barriers to entry and developments in world capital markets have increased the actual and potential mobility of multinational enterprises (MNEs). This poses challenges

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for host countries' tax and regulation policies. For a number of countries, such as, for example, the member countries of the European Union, the policy challenge is two-faceted. First, they are facing competition from other similar (e.g. EU member) countries, where national governments try to attract new corporate investments.<sup>1</sup> Second, many MNEs have attractive investment and localisation options in entirely different countries (outside the EU-area), e.g., in low cost countries. As global developments make such outside options more accessible and attractive for MNEs, how will host countries react? What will be the implications for their tax policies, for the MNEs' investment decisions and for host countries' welfare? In this paper we address these issues. An interesting finding is that more attractive outside options for MNEs may constitute a win-win situation; the MNE as well as its present host countries may gain when this occurs. The reason is that a more attractive outside option for the firm may affect the strategic tax and regulatory competition between its present host countries in such a way that a Pareto improvement is brought about.

In line with the complex characteristics of most multinational firms,<sup>2</sup> we assume that such a firm has better information than the governments about its efficiency.<sup>3</sup> Possessing private information about efficiency, the MNE has incentives to undertake strategic investments. On the one hand, to receive favorable treatment in terms of taxation, the firm may like to be conceived as a low-productivity type in the EU-countries. But it would also like to indicate that it is highly mobile, i.e., unless operating conditions in the EU-area are sufficiently favorable, it may reschedule investments or migrate altogether to another region where net costs are lower. To signal a credible threat of relocation, the firm would like to be conceived as having a high reservation profit, i.e., a high productivity on alterna-

<sup>&</sup>lt;sup>1</sup>In general, foreign direct investments have been rapidly increasing (see Markusen (1995)), and recent empirical research show that effective tax rates are important factors for determining the localisation decisions of multinational enterprises (see, e.g., Devereux and Freeman (1995)).

 $<sup>^{2}</sup>$ According to Markusen (1995), multinationals tend to be important in industries and firms that are characterised by: high levels of R&D relative to sales, a large share of professional and technical workers in their workforce, products that are new or technically complex, and high levels of product differentiation and advertising.

<sup>&</sup>lt;sup>3</sup>The international nature of an MNE and the high number of interfirm transactions make it hard for authorities to observe its true income and costs. Complex technology also implies obstacles for authorities to ascertain the firm's efficiency, and thereby derive its true operating profits. Many of the inputs are not standard commodities with established market prices, making it difficult to monitor costs or impose norm prices.

tive investments. However, under the reasonable assumption that the firm's productivities inside and outside the EU-area are positively correlated, the firm cannot at the same time indicate a low and a high productivity. So the firm has countervailing incentives vis-a-vis each government, but may still pitch governments against each other.

We model this setting as a common agency; the firm relates to several principals (governments) but has in addition an outside option. The paper thus analyses the combined effects of countervailing incentives (see Lewis and Sappington (1989), Maggi and Rodríguez-Clare (1995), Jullien (2000)) and common agency (Martimort (1992), Stole (1992), Martimort and Stole (2002, 2009)). Multiprincipal problems with countervailing incentives have previously been studied by Mezzetti (1997), but in a different and complementary setting.<sup>4</sup> There is by now a considerable literature analysing tax and regulatory competition in various settings, see Gresik (2001) for a general survey and Bond and Gresik (1996), Olsen and Osmundsen (2001, 2003), Laffont, and Pouyet (2004) and Calzolari (2001, 2004) for analyses in common agency frameworks. The novel feature considered here is the strategic implications of better outside options for firms, and in particular of outside options that are relatively more attractive for very efficient firms.

In several parts of the world countries work to coordinate and harmonize their tax policies. The EU is a prominent example. We analyse the effects of such measures by comparing outcomes for cooperating and competing countries, respectively. We show that with the presence of an outside option, tax competition - relative to coordination - may entail lower investments for inefficient firms and higher investments for efficient ones, and that the firm's profits may be lower or higher when the countries compete than when they cooperate. Whether the firm is better or worse off under policy competition relative to policy coordination, depends among other things on investment substitution possibilities and its ownership structure. The firm is better off under a cooperative relative to a competitive regime when the elasticity of substitution is low, or if owner shares held by residents of the cooperating countries are large. And as already mentioned, we also show that a higher outside option for the firm may actually be beneficial for the firm's

<sup>&</sup>lt;sup>4</sup>In Mezzetti (1997) the agent has private information about his *relative* productivity in the tasks he performs for two principals. With this informational assumption Mezzetti obtains a case of countervailing incentives and contract complements. In our model the agent has private information about his *absolute* efficiency level, the relevant actions are contract substitutes, and the presence of countervailing incentives is due to an outside option. The two models yield different implications; e.g. whereas Mezzetti obtains equilibria with pooling for a range of intermediate types, we obtain fully separating equilibria.

host countries when they are engaged in tax and regulatory competition with each other. This means that better outside options for the firm may reduce the gains from policy coordination and thus reduce host countries' incentives to coordinate their policies.

# 2 The model

The MNE invests  $K_1$  in country 1 and  $K_2$  in country 2,<sup>5</sup> yielding profits (before joint costs and taxes)  $N_1(K_1, \theta)$  and  $N_2(K_2, \theta)$ , where  $\theta$  is an efficiency parameter. The MNE also has an option of investing in another economic area. To simplify we assume that if the MNE exercises this option, it moves all its operations to this region.<sup>6</sup> We further assume that it is not optimal for the MNE to make all its investments only in country 1 or only in country 2.7 There are several examples that may motivate this assumption. First, consider a vertically integrated MNE which is located in two EU-countries (e.g., coal mining and natural gas extraction). Extraction levels exceed local demand, and excess output is exported to the neighbouring country, due to high transportation costs. Such a firm cannot credibly threaten to concentrate all its activities in only one of the countries. The outside option of the firm may be to extract natural resources and serve customers in another region. The second case is an MNE (e.g., in the food industry), that is presently located in two EU-countries.<sup>8</sup> The MNE is likely to maintain some activity in both countries due to irreversible investments that have been made in production facilities. Even without the presence of fixed factors, the firm may want to be present in both of the countries in order to be close to the customers and thus closely observe changing consumer patterns.<sup>9</sup> A third explanation for localisation in several countries is that the MNE is a

<sup>&</sup>lt;sup>5</sup>In addition there may be sunk investments in both countries.

<sup>&</sup>lt;sup>6</sup>Given a passive government in the outside region, this assumption mainly serves to simplify notation. An alternative setup would be to assume that the MNE in equilibrium actually invests in a third country, in which case the outside option would be to reschedule a larger fraction of its activities to this country. This alternative approach would generate the same qualitative results; see the appendix.

<sup>&</sup>lt;sup>7</sup>We thus assume intrinsic common agency. Calzolari and Scarpa (2008) and Martimort and Stole (2009) analyse both intrinsic and delegated agency, but assume a type-independent outside option. As a first step for the type dependent case, we limit the analysis to intrinsic common agency (except for a special case; see section 5.3).

<sup>&</sup>lt;sup>8</sup>The division of investments may have historical explanations, e.g., that the output is sold to consumers in both countries and that there used to be large transportation costs or other trade barriers.

<sup>&</sup>lt;sup>9</sup>This is important for products characterised by local variations in taste, and where product development, design and fashion are important. The food and furniture industries are examples.

multi-product firm, e.g., a producer of household appliances or semi-conductors, and that the countries differ with respect to the presence of industrial clusters for different types of products.<sup>10</sup> Lower trade costs may open up the possibility to locate in low cost or low tax regions, i.e., outside options may emerge.

Let  $\Pi$  and  $\pi$  denote, respectively, the pre- and post-tax global profits of the firm:

$$\Pi(K_1, K_2, \theta) = N_1(K_1, \theta) + N_2(K_2, \theta) - C(K),$$
(1)

$$\pi = \Pi - r_1 - r_2, \tag{2}$$

where  $K = K_1 + K_2$ , C(K) denotes joint costs for the two affiliates and  $r_1$  and  $r_2$  are the taxes paid to the two countries.<sup>11</sup> We assume that C'(K) > 0, C''(K) > 0. The convex costs C(K) imply economic interaction effects among the two affiliates; an increase in the investments in one of the countries implies a higher marginal joint cost, which again affects the investments of the other country. These joint costs may have different interpretations. First, K may represent scarce human capital, e.g., management resources or technical personnel, where we assume that the MNE faces convex recruitment and training costs. Second, K may represent real investments, where C(K) are management and monitoring costs of the MNE. Economic management and co-ordination often become more demanding as the scale of international operations increase, i.e., C(K) is likely to be convex. Third, instead of interpreting C(K) as joint costs, it may in the case of imperfect competition be perceived as measuring interaction effects in terms of market power. For example, if the two affiliates sell their output on the same market (e.g., in a third country), their activities are substitutes: high investments (and output) in affiliate 1 reduce the price obtained by affiliate 2. Another example of a market interaction effect is a case where  $K_1$  and  $K_2$ are investments in R&D; the marginal payoff on R&D-activities of affiliate 1 is lower the higher is the R&D activity of affiliate 2, e.g., due to a patent race.<sup>12</sup>

The countries compete to attract scarce real investments from the MNE, and the interaction of the principals is through the MNE's joint costs. Note that  $\frac{\partial^2 \Pi}{\partial K_1 \partial K_2} = -C''(K) < 0$ 

<sup>&</sup>lt;sup>10</sup>An example of a firm with such a dispersed manufacturing structure is Phillips. The value of the MNE may be closely linked to its business strategy of supplying multiple products. If this is common knowledge, a threat to become a niche producer that is located in only one country would not be credible.

<sup>&</sup>lt;sup>11</sup>Nothing substantial would change by using a more general (convex) cost function  $C(K_1, K_2)$ , with  $\partial^2 C / \partial K_1 \partial K_2 > 0$ .

<sup>&</sup>lt;sup>12</sup>Olsen (1993) analyses single-principal regulation of independent R&D units, and emphasizes the role of research activities as substitutes.

0, e.g., we address a case of contracting substitutes. The affiliates of the MNE are separate and independent entities, which means that they are subsidiaries and thus taxed at source. The firm has private information about  $\theta$  and net operating profits in the two countries. It is presumed that if the firm is efficient in one country it is also an efficient operator in the other country; for reasons of tractability we assume that the firm has the same efficiency in the two countries. Efficiency types are distributed according to the cumulative distribution function  $F(\theta)$  with density  $f(\theta)$  having the support  $[\underline{\theta}, \overline{\theta}]$ , where  $\underline{\theta}$  denotes the least and  $\overline{\theta}$  the most efficient type. The distribution satisfies the regularity conditions  $\frac{d}{d\theta} [F(\theta)/f(\theta)] \ge 0$  and  $\frac{d}{d\theta} [(1 - F(\theta))/f(\theta)] \le 0$ . Efficient types have higher net operating profits than less efficient types, both on average and at the margin:  $\frac{\partial N_j}{\partial \theta} (K_j, \theta) > 0$  and  $\frac{\partial^2 N_j}{\partial \theta \partial K_j} (K_j, \theta) > 0, j = 1, 2$ ; where the latter inequality is a single crossing condition.

The MNE and the governments are risk neutral. For all efficiency types the affiliate's net operating profits in each country are sufficiently high so that both governments always want to induce the domestic affiliate to make some investments in their home country. Domestic consumer surpluses in the two countries are unaffected by changes in the MNE's production level, since the firm is assumed to be a price taker (or its market is outside the two countries). The governments have utilitarian objective functions: the social domestic welfare generated by the MNE is a weighted sum of the domestic taxes paid by the firm and the firm's global profits:

$$W_j = (1+\lambda_j)r_j + \alpha_j \pi, \quad j = 1, 2,$$

where  $\lambda_j$  is the general equilibrium shadow cost of public funds in country j, and  $\alpha_j$  is the owner share of country j in the MNE. The shadow costs of public funds are taken as exogenously given in our partial analysis. We have  $\lambda_j > 0$ , j = 1, 2, since marginal public expenditure is financed by distortive taxes. By inserting for Eq.(1), the social welfare function for country 1 can be restated as

$$W_1 = (1 + \lambda_1) \left( \Pi(K_1, K_2, \theta) - r_2 \right) - (1 + \lambda_1 - \alpha_1) \pi.$$
(3)

The MNE has an additional localisation alternative: it has an option to move all its activity outside the EU area, e.g., to a low cost country or to a tax haven. This investment option would produce an after tax profit of  $n(\theta)$ , i.e., the firm has private information about the alternative return on its scarce resources. Assuming that firms that have high returns in the EU area also have high returns on outside options, we have  $n'(\theta) > 0$ . We consider

here the case where the participation constraint is binding for some type(s) other than the least productive one, i.e., for some type  $\theta \neq \underline{\theta}$ . In these cases there are typically countervailing incentives, where low-productivity types are tempted to claim to have high productivity in order to secure themselves high rents. To illustrate these effects, and yet have a fairly simple model, we confine ourselves to cases where the participation constraint is binding only for the least productive and the most productive type, i.e., only for  $\theta = \underline{\theta}$ and  $\theta = \overline{\theta}$ . This will occur, for example, if the outside returns function  $n(\theta)$  is 'sufficiently convex', in a sense to be made precise below.

# **3** Cooperating principals

To assess the benefits of cooperation, we consder first the case where the countries cooperatively design their tax policies. The countries (principals) then seek to maximise the cooperative welfare given by  $W = W_1 + W_2$  (we assume  $\lambda_1 = \lambda_2$ ) subject to incentive and participation constraints for the firm. Incentive compatibility requires that the firm's equilibrium profits (rents) satisfy<sup>13</sup>

$$\pi'(\theta) = \frac{\partial \Pi}{\partial \theta} \left( K_1(\theta), K_2(\theta), \theta \right) \tag{4}$$

The first-order condition (4) together with  $K'_j(\theta) \ge 0, j = 1, 2$  are sufficient for incentive compatibility.

The principals maximize expected welfare EW subject to the incentive compatibility (IC) and participation (IR) constraints. A comprehensive analysis of this problem has been given by Jullien (2000). Here we confine ourselves to the case of outside option functions  $n(\theta)$  that leave the IR constraints non-binding for interior types. There will then be at most one type (say  $\check{\theta}$ ) where the IC constraint is non-binding in the sense that the temptation to claim low  $\theta$  to indicate low productivity is exactly balanced by the temptation to claim high  $\theta$  to indicate favorable outside options. From Julien (2000) we have:

**Proposition 1** Suppose there is a  $\check{\theta} \in [\underline{\theta}, \overline{\theta}]$  such that  $K_1(\theta), K_2(\theta)$  given by

$$(K_1(\theta), K_2(\theta)) = \arg \max_{K_1, K_2} \left[ \Pi(K_1, K_2, \theta) - (1 - \frac{\alpha_1 + \alpha_2}{1 + \lambda}) \frac{\partial \Pi}{\partial \theta} (K_1, K_2, \theta) \frac{F(\theta) - F(\theta)}{f(\theta)} \right]$$

<sup>&</sup>lt;sup>13</sup>To interpret this condition, note that if type  $\theta + d\theta$  mimics the less efficient type  $\theta$  (by investing  $K_j(\theta)$  instead of  $K_j(\theta + d\theta)$ ), it obtains additional profits  $\Pi(K(\theta), \theta + d\theta) - \Pi(K(\theta), \theta)$  relative to type  $\theta$  in country j. To avoid such behavior the principal must allow for this rent differential in the tax scheme.

are increasing 
$$(K'_{j}(\theta) \geq 0)$$
. Suppose further that the associated rent  $\pi(\theta)$  given by (4),  
i.e.,  $\pi(\theta') = \int_{\underline{\theta}}^{\theta'} \frac{\partial \Pi}{\partial \theta} (K_{1}(\theta), K_{2}(\theta), \theta) d\theta + \pi(\underline{\theta})$ , satisfies  $\pi(\theta) \geq n(\theta)$  and  
(a)  $\pi(\underline{\theta}) = n(\underline{\theta})$  if  $\check{\theta} = \bar{\theta}$ .  
(b)  $\pi(\underline{\theta}) = n(\underline{\theta})$  and  $\pi(\bar{\theta}) = n(\bar{\theta})$  if  $\underline{\theta} < \check{\theta} < \bar{\theta}$ .  
(c)  $\pi(\bar{\theta}) = n(\bar{\theta})$  if  $\check{\theta} = \underline{\theta}$ .  
Then  $(K_{1}(\theta), K_{2}(\theta))$  together with the associated rent  $\pi(\theta)$  is the optimal solution.

To interpret the cooperative solution, note that the first order conditions for optimal investments take the form (double subscripts denote second-order partials)

$$(1+\lambda)\frac{\partial\Pi}{\partial K_j} - (1+\lambda - \alpha_1 - \alpha_2)\Pi_{\theta j}\frac{F(\dot{\theta}) - F(\theta)}{f(\theta)} = 0.$$
 (5)

A higher investment  $dK_j$  by type  $\theta$  will affect joint welfare  $W = (1 + \lambda)\Pi - (1 + \lambda - \alpha_1 - \alpha_2)\pi$  partly by its effect on the firm's pre-tax profits  $\Pi$  and partly by its effect on rents  $\pi$ . The two terms in (5) capture these effects. For  $\theta < \check{\theta}$  incentive constraints are binding downwards (the incentive to claim low productivity dominates the incentive to claim high outside options). A higher investment by such a type of firm will tighten incentive constraints for more efficient types (types in the range  $(\theta, \check{\theta})$ ), and this is costly in terms of increased rents to such firms. The second term in (5) accounts for these welfare costs.<sup>14</sup>

When  $\check{\theta} = \bar{\theta}$  - the conventional case - there is a welfare cost for all types except the most efficient one. Optimal investments are then (at least for symmetric returns) lower than their first-best levels. If on the other hand  $\check{\theta} < \bar{\theta}$ , the second term in (5) is negative for  $\theta > \check{\theta}$ , so the welfare effect associated with the firm's rents is positive. For such types the incentive constraints are binding upwards; the firm is tempted to mimic a more efficient type in order to make it appear that it has a higher outside option. By inducing such a firm to invest more, and thereby increase its "internal" profits,  $\pi(\theta)$ , the incentive constraints for firms with lower efficiency (types in the range  $(\check{\theta}, \theta)$ ) are relaxed. This leads (for symmetric returns) to overinvestments relative to the first-best solution for these types.

<sup>&</sup>lt;sup>14</sup>From (4) the rent differential  $\pi'(\theta)d\theta$  increases by  $\Pi_{\theta j}d\theta$ , and the same increase must be given to all types in  $(\theta, \check{\theta})$ , hence to a fraction  $F(\check{\theta}) - F(\theta)$  of all types.

# 4 Non-cooperative equilibrium

Consider now the case where the governments of the two countries compete (to attract the firm's investments) rather than cooperate. In this case the MNE relates to each government separately. The governments cannot credibly share information and they act non-cooperatively. In the present context it is natural to consider equilibria in tax functions.<sup>15</sup> Let  $R_j(K_j)$  denote the taxes that the firm pays to government j, based on the firm's investments in country j. For multinationals, profits are not observable to the tax authorities, due to among other things strategic transfer pricing. Taxes are therefore made contingent on investments, which are assumed here to be the key verifiable variables for such a firm.<sup>16</sup> We say that a pair  $K_1(\theta), K_2(\theta)$  of investment profiles is commonly implementable if there are tax schedules  $R_j(K_j)$ , one for each principal, such that for every type  $\theta$  the firm's profits are maximal for this pair of investments.

**Lemma 2** In any differentiable equilibrium where IR-constraints are binding only for types  $\underline{\theta}, \overline{\theta}$  we have: There exists  $\check{\theta}_1, \check{\theta}_2 \in [\underline{\theta}, \overline{\theta}]$  such that equilibrium investments and profits satisfy

$$\Pi_{i\theta}K'_{i} \ge -\Pi_{12}K'_{1}K'_{2}, i = 1, 2 \quad and \quad K'_{1}K'_{2}\left(\Pi_{1\theta}\Pi_{2\theta} + \Pi_{12}\left[\Pi_{1\theta}K'_{1} + \Pi_{2\theta}K'_{2}\right]\right) \ge 0$$
(6)

$$\frac{\partial \Pi}{\partial K_j} = \frac{1 + \lambda - \alpha_j}{1 + \lambda} \left[ \Pi_{\theta j} + \Pi_{\theta i} \frac{\Pi_{ij} K'_i(\theta)}{\Pi_{\theta i} + \Pi_{ij} K'_j(\theta)} \right] \frac{F(\check{\theta}_j) - F(\theta)}{f(\theta)}.$$
(7)

and

$$\int_{\underline{\theta}}^{\theta} \frac{\partial \Pi}{\partial \theta} (K_j(\theta'), K_i(\theta'), \theta') d\theta' + \pi(\underline{\theta}) \ge n(\theta), \quad all \ \theta, \ with \ equality \ for \ \theta = \underline{\theta}, \overline{\theta}. \tag{8}$$

Condition (6) is a well known necessary condition for common implementability, derived from the second-order conditions for the firm's maximization problem (see e.g. Stole (1992)). Except for the parameters  $(\check{\theta}_1, \check{\theta}_2)$ , the conditions (7) are analogous to the equilibrium conditions derived by Stole (1992) and others for the conventional case where the outside value is type independent. The conventional case corresponds to  $\check{\theta}_1 = \check{\theta}_2 = \bar{\theta}$ .

<sup>&</sup>lt;sup>15</sup>The Revelation Principle (in its usual form) doesn't hold for common agency games in general, see e.g. Martimort and Stole (2002), Peters (2001). Constraining both principals to use revelation mechanisms may thus be restrictive.

<sup>&</sup>lt;sup>16</sup>Profits may be less difficult to verify for purely domestic firms, and different tax schemes may thus be introduced for purely domestic and for multinational firms, reflecting the poorer information available for the latter.

To understand condition (7) note that the terms on the LHS represent the marginal effect of increased  $K_j$  on country j's surplus (adjusted by factor  $1 + \lambda$ ). The term on the RHS represents the marginal effects on rents (also adjusted by factor  $1 + \lambda$ ). This term has itself two components; the first is the conventional (direct) one, just like in the cooperative case; the second is a strategic effect, working through the change in foreign investments (say  $\frac{\partial \hat{K}_i}{\partial K_j}$ ) induced by the change in domestic investments. The foreign investment  $\hat{K}_i$  is given by  $\frac{\partial \Pi}{\partial K_i}(K_j, \hat{K}_i, \theta) = R'_i$  and hence satisfies  $(R''_i - \Pi_{ii})\frac{\partial \hat{K}_i}{\partial K_j} = \Pi_{ij}$ . In equilibrium the first-order condition for  $\hat{K}_i$  holds as an identity in  $\theta$ , and by differentiating this identity we obtain  $\frac{\partial \hat{K}_i}{\partial K_j} = \frac{\Pi_{ij}K'_i(\theta)}{\Pi_{\theta i} + \Pi_{ij}K'_j(\theta)}$ . This explains the formula (7). If investments are substitutes, increasing in both countries, and commonly implementable, the strategic effect will be negative.

Apart from the strategic effect, conditions (7) and (5) also differ in the way that condition (7) involves country-specific parameters  $\check{\theta}_j$  and only domestic owner shares  $(\alpha_j)$ . The latter reflects an equity externality; country j doesn't internalize the implications of its policy for the firm's foreign owners. This makes country j more aggressive with respect to extracting rents. The equity and strategic effects tend to have opposite effects on equilibrium investments.<sup>17</sup>

To derive sufficient conditions for an equilibrium we confine ourselves to quadratic versions (approximations) for the relevant functions. Then we have:

**Proposition 3** Suppose countries are symmetric,  $\theta$  is uniform and  $\Pi()$  has constant second-order partials with  $\Pi_{12} < 0$  (substitutes) and that  $\Pi(K_1, K_2, \theta)$  is concave in  $K_1, K_2$ . Then investments  $K_1(\theta), K_2(\theta)$  is a differentiable equilibrium with IR-constraints binding only for types  $\underline{\theta}, \overline{\theta}$  if and only if (6), (7) and (8) hold for some  $\check{\theta}_j, \check{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ .

In this case we obtain equilibrium investment schedules  $K_j(\theta)$  that are linear in the efficiency parameter  $\theta$ . Figure 1 provides an illustration. The first-best (full information) investment schedules are then symmetric across the countries, and so are the second-best (asymmetric information) schedules obtained in the cooperative regime. These are depicted as, respectively, the heavy line (first-best) and the broken line (second-best) in the figure. The thin (steepest) line represents the investment schedule for a symmetric

 $<sup>^{17}</sup>$  Olsen and Osmundsen (2001) analysed these effects for the pure tax/no type-dependent outside option case.

equilibrium in the non-cooperative regime.<sup>18</sup> Its qualitative properties are similar to those of the solution under cooperation; there is underinvestment relative to the first-best for low-efficiency types ( $\theta < \check{\theta}_j$ ) and overinvestment for high-efficiency types ( $\theta > \check{\theta}_j$ ). As discussed in the next section, the relative positions of the investment schedules for the two tax regimes will vary, depending on the parameters of the model. The figure depicts a case where competition exacerbates investment distortions: investments under competition are for low-efficiency types even lower and for high-efficiency types even higher than investments under cooperation.<sup>19</sup>



Figure 1. First-best (heavy line), cooperative (dotted line) and non-cooperative (thin line) equilibrium investments as functions of efficiency parameter  $\theta$ .

# 5 Properties of equilibria

In this section we will analyse properties of equilibria for the model. The following parametrization will be used

<sup>&</sup>lt;sup>18</sup>As discussed below, there will in this case also exist non-cooperative equilibria with linear investment schedules that are asymmetric between the two countries. The symmetric equilibrium Pareto dominates the other asymmetric equilibra.

<sup>&</sup>lt;sup>19</sup>Plot generated with model in Section 5; parameter values  $\lambda = .5$ ,  $\alpha = .5$ , m = 1, q = 4, a = .5, k = 2,  $n(\bar{\theta}) = .97$ ,  $n(\underline{\theta}) = 0$ .

$$N_j(K_j, \theta) = m\theta(K_j + h) + kK_j - \frac{1}{2}qK_j^2$$
$$C(K_1, K_2) = \frac{1}{2}a(K_1 + K_2)^2,$$
$$F(\theta) = \theta \text{ for } \theta \in [0, 1],$$

with m, k, q, a > 0. The assumption q > 0 guarantees concavity of  $\Pi$ . With this parametrization the second-order partials are

$$\Pi_{12} = -a, \quad \Pi_{jj} = -(q+a), \quad \Pi_{j\theta} = m.$$

As a reference point, the full information *first-best* solution is in this case given by  $\frac{\partial \Pi}{\partial K_i} = 0$ . This yields symmetric investment schedules that are linear in  $\theta$ . The first-order conditions (5) for the cooperative case also yield linear and symmetric solutions, and these exhibit underinvestment for low types (possibly overinvestment for high types) compared to first-best investments.

### 5.1 Equilibrium investments and profits

In the non-cooperative setting; the equilibrium equations (7) have linear solutions, say of the form  $K_j(\theta) = L_j + K'_j\theta$ , j = 1, 2, see the appendix. The slopes of the equilibrium schedules are seen to be independent of  $\check{\theta}_1, \check{\theta}_2$ , and therefore the same as in the case of a type-independent outside option. For symmetric countries (where  $\alpha_1 = \alpha_2$ ) they are also symmetric, so  $K'_1 = K'_2 = K'$ . While the slopes  $K'_j$  of the equilibrium schedules are uniquely determined (and equal), the intercepts  $L_j$  (or equivalently the parameters  $\check{\theta}_1, \check{\theta}_2$ ) are not unique and not necessarily equal, even when countries are symmetric. It turns out that aggregate equilibrium investment  $K_1(\theta) + K_2(\theta)$  is uniquely determined, but the model doesn't fully pin down how this investment is distributed between the countries.<sup>20</sup> But the Pareto-preferred equilibrium is the symmetric one, and we will concentrate on that equilibrium in the following.

**Proposition 4** (i) The slopes  $K'_j$  of the equilibrium investment schedules given in Proposition 3 are unique and equal, but the intercepts of these linear schedules are generally not unique. (ii) Aggregate equilibrium investment  $K_1(\theta) + K_2(\theta)$  and equilibrium profits  $\pi(\theta)$ are uniquely determined. (iii) For symmetric countries the equilibrium with the highest total expected welfare is the symmetric one.

 $<sup>^{20}</sup>$ In the (intrinsic) common agency framework we consider here, the equilibrium doesn't pin down the way that the countries divide between themselves the burden of providing rents for the firm, and this implies that equilibrium investments are not uniquely pinned down either.

We now turn to a comparison of resource allocations under the cooperative and the non-cooperative regimes. In the following we assume that the Pareto-preferred symmetric equilibrium is chosen under non-cooperation. We also assume that the outside value  $n(\theta)$  for the firm is such that participation constraints are binding only for the least efficient and/or most efficient types.

**Proposition 5** There is a critical number  $\Psi < 1$ ,  $(\Psi = 1/(1 + \frac{q}{4a}), \frac{q}{a} = \frac{\Pi_{11}}{\Pi_{12}} - 1)$ , such that for  $\frac{\alpha_1 + \alpha_2}{1 + \lambda} > \Psi$  we have: The firm's profits are for all types  $\theta \in (\underline{\theta}, \overline{\theta})$  lower when the countries compete than when they cooperate. Hence, the IR constraint for type  $\overline{\theta}$  is either (i) binding in both regimes, (ii) binding only in the competitive regime, or (iii) non-binding for both regimes. Investments are in case (iii) lower for all types (but type  $\overline{\theta}$ ) under competition compared to cooperation. In cases (i) and (ii), investments under competition are lower for inefficient types (all  $\theta < \overline{\theta}$ , some  $\overline{\theta} < \overline{\theta}$ ) and higher for efficient types ( $\theta > \overline{\theta}$ ) compared to investments under cooperation. For  $\frac{\alpha_1 + \alpha_2}{1 + \lambda} < \Psi$  the converse conclusions hold.<sup>21</sup>

The proposition implies that the firm's profits are lower in the competitive regime when the 'inside' owner share  $\alpha_1 + \alpha_2$  is large. This result parallels that given in Olsen and Osmundsen (2001) for the case of a constant (type-independent) outside option. When inside owner shares are large the equity externalities are large, and this leads to more aggressive rent extraction when countries compete compared to when they cooperate.

The type-dependent outside option yields however quite different implications for equilibrium investments. For a constant outside option, the more aggressive rent extraction associated with large equity externalities leads to equilibrium investments (under competition) that are for all types *lower* than investments under cooperation. This is covered by case (iii) in the proposition. But when outside options are type-dependent, and the most efficient types have sufficiently better outside options than less effcient types (cases (i) and (ii)), equilibrium investments are for the more effcient types *higher than investments under cooperation*. Figure 1 illustrates the investment comparisons for this case. The more

<sup>&</sup>lt;sup>21</sup>That is; the firm's profits are for all types  $\theta \in (\underline{\theta}, \overline{\theta})$  higher when the countries compete than when they cooperate. Hence, the IR constraint for type  $\overline{\theta}$  is either (i) binding in both regimes, (ii) binding only in the cooperative regime, or (iii) non-binding for both regimes. Investments are in the latter case (iii) higher for all types (but type  $\overline{\theta}$ ) under competition compared to cooperation. In cases (i) and (ii), investments under competition are higher for inefficient types (all  $\theta < \tilde{\theta}$ , some  $\tilde{\theta} < \overline{\theta}$ ) and lower for efficient types compared to investments under cooperation.

aggressive rent extraction associated with large equity externalities leads in this case to larger investments for high types and lower investments for low types.

The conditions in the proposition can also be related to the ease with which capital can be substituted between the two countries. The elasticity of substitution between  $K_1$ and  $K_2$  for the firm's symmetric pre-tax profit function  $\Pi(K_1, K_2, \theta)$ , evaluated at the point  $K_1 = K_2 = \frac{1}{2}K_F(\theta)$ , where  $K_F(\theta)$  is the first-best investment in each country, is  $\sigma = \frac{2a}{q} + 1.^{22}$  In view of this, the last proposition says that the firm's rents tend to be lower under competition compared to cooperation when the elasticity of substitution is 'small'.<sup>23</sup> Thus, it is when substitution is not too easy  $(\frac{a}{q} \text{ small})$  that the firm tends to be worse off when the countries compete compared to when they cooperate.

#### 5.2Implications of better outside options

We now consider comparative statics effects of variations in the outside value for the firm. This analysis is complicated by the fact that the equilibrium in principle depends on the whole profile of outside values (over all types), and hence that the exercise in general should involve comparisons of all such profiles. We limit ourselves to profiles that generate the type of equilibrium studied above, i.e. where the participation constraints are binding only for the most efficient and least efficient types. We will show that if  $n^1(\theta)$  and  $n^2(\theta)$ are two such profiles, and  $n^1(\theta) \ge n^2(\theta)$ , then under competition it may well be the case that the higher profile  $n^1(\theta)$  yields a greater social surplus than the lower profile  $n^2(\theta)$ . Hence all parties may gain when the firm's outside option becomes more favorable! This will not occur when the countries cooperate, since the higher profile implies a stricter set of participation constraints and therefore if anything a lower total surplus.

All else equal (technology, demand, owner shares etc.) an equilibrium of the form studied in this paper is determined by the outside option values for the most efficient and the least efficient types of the firm, or more precisely by the difference  $n(\bar{\theta}) - n(\theta)$ . This single number, which we will denote by  $\eta$ , determines how the equilibrium depends on the outside value profile. Normalizing  $n(\underline{\theta}) = 0$ , we have  $\eta = n(\overline{\theta})$ . Such an equilibrium is only feasible for  $\eta$  in some range  $(\eta_1, \eta_2)$ . The lower bound  $\eta_1$  of this range is the rent that would accrue to the best type in the conventional case with type-independent reservation

<sup>&</sup>lt;sup>22</sup>For the quadratic (and symmetric) functional form we find, for symmetric investments;  $\sigma =$  $\frac{q+2a}{q} \left( \frac{K_F(\theta)}{K_j} - 1 \right), \text{ where } K_F(\theta) = \frac{m\theta+k}{q+2a}.$ <sup>23</sup> For  $\alpha_1 = \alpha_2 = .5$  the condition is  $\lambda < \frac{q}{4a}$ , i.e.  $\sigma < 1 + \frac{1}{2\lambda}$ , hence  $\sigma < 3$  for  $\lambda = .25$ .

profit. This corresponds to the case  $\check{\theta}_1 = \check{\theta}_2 = \bar{\theta}$  in our model. The upper bound  $\eta_2$  is the profit that would accrue to the best type if on the other hand  $\check{\theta}_1 = \check{\theta}_2 = \underline{\theta}$ .

For  $\eta$  in this range, the firm's equilibrium profit is unique and given by a convex function  $\pi(\theta; \eta)$ . Here  $\eta$  is used as an indexing parameter; we have  $\pi(\bar{\theta}; \eta) = \eta$ . Note that any outside value profile that satisfies  $n(\underline{\theta}) = \pi(\underline{\theta}; \eta) = 0$ ,  $n(\bar{\theta}) = \pi(\bar{\theta}; \eta) = \eta$ , and  $n(\theta) \leq \pi(\theta; \eta)$ , will generate such an equilibrium. Let  $N(\eta)$  denote the family of all such profiles. Formally

**Definition.** For  $\eta$  in  $(\eta_1, \eta_2)$ , let  $N(\eta)$  be the family of all outside value profiles that satisfy  $n(\underline{\theta}) = 0$ ,  $n(\overline{\theta}) = \eta$  and  $n(\theta) \leq \pi(\theta; \eta)$ , where  $\pi(\theta; \eta)$  is (uniquely) given by  $\pi(\theta; \eta) = \int_{\underline{\theta}}^{\underline{\theta}} \frac{\partial \Pi}{\partial \theta} (K_1(\theta'), K_2(\theta'), \theta') d\theta'$ ,  $\pi(\overline{\theta}; \eta) = \eta$ , and  $K_j(\theta)$ , j = 1, 2 satisfy (7) and (8) with  $\check{\theta}_j \in (\underline{\theta}, \overline{\theta})$ , j = 1, 2.

We will study how the equilibrium outcome associated with an outside value profile in the family  $N(\eta)$  varies when  $\eta$  varies on the interval  $(\eta_1, \eta_2)$ . Each profile in  $N(\eta)$  yields equilibrium profits  $\pi(\theta; \eta)$ , and this function is increasing in  $\eta$ . A more favorable outside option, in the sense of one that yields an outside value that is higher for the best type and that belongs to the corresponding family  $N(\eta)$ , will thus lead to equilibrium profits that are more favorable for every type of firm.

**Proposition 6** Let  $\Psi = 1/(1 + \frac{q}{4a}) < 1$ . Then for  $\frac{\alpha_1 + \alpha_2}{1 + \lambda} > \Psi$  (respectively  $\frac{\alpha_1 + \alpha_2}{1 + \lambda} < \Psi$ ) we have: For the family  $N(\eta)$  it is the case that, as  $\eta$  (the outside value for the best type) increases on  $(\eta_1, \eta_2)$ , the total value  $E(W_1 + W_2)$  associated with the symmetric non-cooperative equilibrium first increases and then decreases (respectively decreases over the whole interval). In any case, every type of firm benefits as  $\eta$  increases.

The proposition shows that the total surplus under competition is either (i) first increasing and then decreasing, or (ii) monotone decreasing in the firm's outside value index  $\eta$ . More favorable outside opportunities for the firm will thus in a set of cases improve the social surplus, although only up to some point. But the improvement may be considerable; the efficiency loss relative to the first-best outcome may be reduced by as much as 75% when the outside value increases this way.<sup>24</sup>

Note also that the condition that defines case (i)  $\left(\frac{\alpha_1+\alpha_2}{1+\lambda}>\Psi\right)$ , is the same condition that makes the competitive tax regime less attractive for the firm than the cooperative regime. This is thus the case where domestic owner shares are large and substitution of

<sup>&</sup>lt;sup>24</sup>This reduction is obtained for  $\alpha_1 + \alpha_2 = 1$  and  $\lambda = 0$ ; proof available from the authors.

investments is not too easy for the firm.<sup>25</sup> Since the surplus under cooperation will if anything decline as  $\eta$  increases, we see that the relative performance of the competitive regime will then improve as the firm's outside opportunities become better. The total benefits of cooperation will thus become smaller when the MNE gets more attractive outside opportunities (e.g. in third-country tax havens), and the incentives to cooperate will diminish in such cases.

To obtain some intuition for the result, consider the case  $\alpha_1 + \alpha_2 = 1$  and  $\lambda = 0$ . Then  $W_1 + W_2 = \Pi$ , so only efficiency effects matter for total welfare.<sup>26</sup> But rents for the firm matter for each country's national welfare, since the rents accruing to foreign owners reduce domestic welfare. Suppose now that the IR-constraint for the high type is just binding initially ( $\eta = \eta_1$ ). Then we have underinvestment in both countries (thus  $\frac{\partial W}{\partial K_j} > 0$  holds in equilibrium). Consider a small increase of the outside value for highefficiency types. In order to accommodate higher rents for the firm, and in particular higher rents for the most efficient types, *investments must increase*. Since the aggregate welfare effect of increased rents is zero, while the effect of increased investments on the aggregate production surplus is positive (we had  $\frac{\partial W}{\partial K_j} > 0$  initially), it follows that the total welfare effect associated with the higher outside value will be positive. The two countries will thus in total benefit from the higher outside value offered to the firm in such cases.

### 5.3 Independent investments

It is instructive to briefly consider the case where investments in the two countries are independent in the sense that marginal profits for the firm in one country are not affected by its investments in the other one ( $\Pi_{12} = 0$ ). The essential implication of this assumption is that the strategic effect in the non-cooperative equilibrium disappears (the second term on the RHS of (7) vanishes). Intuitively, since the firm's marginal profitability in one country is independent of its investments in the other country, marginal changes in the tax schedule in one country will not have repercussions in the other country. The noncooperative equilibrium then differ from the cooperative one only because of the equity externalites: each country does not internalize the effects of its policy on the firm's foreign

<sup>&</sup>lt;sup>25</sup>As noted above, for reasonable parameter values (  $\alpha_1 = \alpha_1 = .5$ ,  $\lambda = .25$ ) the condition is that (the point measure of) the elasticity of substitution satisfies  $\sigma < 3$ .

<sup>&</sup>lt;sup>26</sup> In this particular case cooperative investments are thus first-best and hence independent of the outside option.

owners.

Formally, Propositions 4, 5 and 6 now go through with  $a = -\Pi_{12} = 0$ , implying in particular that the critical number  $\Psi$  in Propositions 5 and 6 is  $\Psi = 0$ . From the last proposition we see that the total surplus under competition (non-cooperation) will then always be first increasing and then decreasing in the firm's outside value index  $\eta$ . In this case there will thus always be som range where better outside options will improve both the firm's rents and the total surplus for the two countries.

We have so far assumed that the firm either operates in both of the taxing countries, or in none of them (intrinsic common agency). It is worth noting that the symmetric non-cooperative equilibrium derived under this assumption is in the independent case also an equilibrium if the firm can choose to operate in only one country. First, the firm obtains half of its net profits  $(\pi(\theta)/2)$  from each country, and cannot gain by moving all its operations to one country. (It would in fact lose  $\pi(\theta)/2$ , since nothing can be gained by modifying its investments in the country where it concentrates its operations.) Second, no principal can gain by unilaterally modifying its tax scheme so that the firm leaves the other country. To induce such a change, the principal would have to cover the firm's lost rents, which is costly. And since the firm's domestic operating surplus is not affected by a closure of its foreign operations, the principal cannot make up for the costly rents by inducing such a change. Hence no party can gain, and the given equilibrium remains an equilibrium also when the firm can freely concentrate is operations in one of the two countries.

The independent case illustrates that our results do not crucially hinge on the assumption that the firm must operate in either both or none of the two countries. It seems reasonable to conjecture that similar results will hold also if investments are substitutes  $(\Pi_{12} < 0)$  and the firm can operate in only one of the two countries, at least if the elasticity of substitution is relatively small.

# 6 Conclusion

We analyse a case where an MNE allocates investments between two countries (the home region), while also having an outside investment option, e.g. a low cost region or a tax haven. The two countries in the home region compete to attract the firm's investments and to extract rents from the firm. The ability to tax and regulate the MNE is limited by private information, e.g. facilitated by a large number of transfer prices for services provided among various affiliates of the MNE. The firm has private information about its efficiency and net operating profits in the two countries, and about the value of the outside investment option. It has an incentive to report a low productivity in the home region, and at the same time overstating its productivity on outside investments (exaggerating the value of its outside option). However, the productivity in the home region and the foreign region are likely to be correlated. Thus, the MNE faces countervailing incentives: it cannot at the same time claim to be efficient and inefficient.

In the symmetric equilibrium there is significant underinvestments (relative to the first best) for firms with low efficiency. If the participation constraint is binding for the most efficient type, there is overinvestment for the more efficient types. Policy competition may increase or decrease the firm's rents, relative to policy coordination. A higher value of the outside option is beneficial for the firm, and detrimental to the governments if they cooperate. However, the countries may well be positively affected by a higher outside option if they compete. Thus, enhanced outside options for the firm, e.g. due to reduced entry barriers in other regions, may actually benefit the home governments and represent a Pareto improvement for those countries and the firm. In such situations a development towards improved outside options for firms will reduce the incentives for governments to cooperate.

We have assumed that the firm has private information about its operating profits and about its efficiency level, whereas the investment levels are assumed to be subject to symmetric information. Observability of investments may be a reasonable description for physical capital, but not to the same extent for intangible assets. The latter may be important for MNEs, since they typically have high levels of R&D relative to sales.<sup>27</sup> Also, we assume that the MNE's efficiency levels are perfectly correlated in the countries of operation. Uncorrelated efficiency parameters, however, may be relevant if firms invest in different countries in order to diversify portfolios. Asymmetric information about investment levels, or uncorrelated information parameters, may represent interesting extensions

<sup>&</sup>lt;sup>27</sup>Privately observed investments that are undertaken *after* the tax system is in place (moral hazard) can be accomodated in the model by interpreting the profit function as an indirect function where such investments are chosen optimally, conditional on the observable  $K_j$ 's. Privately observed investments in place *ex ante* would, however, be a part of the firm's (multidimensional) private information. The model can be interpreted as representing a case where the aggregate effect of sveral such variables on profits can be captured by a one-dimensional parameter.

of the present model. However, each of these extensions would imply a multidimensional screening problem, which is not yet fully solved, not even in a single-principal setting; see Rochet and Chone (1998).

### Appendix

#### Simultaneous investments in all regions.

Consider the case where the MNE may operate also in the 'outside' country. The authorities in this country are assumed to be passive. We can then interpret the pre-transfer return function  $\Pi(K_1, K_2, \theta)$  in (1) as a 'reduced form' profit function that is the relevant one for the firm's operations in countries 1 and 2. To see this, let pre-transfer profits for the firm when it is active in all three countries be given by  $\tilde{\Pi}(K_1, K_2, K_3, \theta)$ . For any given investments  $K_1, K_2$  in the two 'inside' countries, the firm will choose its investments in the outside country so as to maximize  $\tilde{\Pi}(K_1, K_2, K_3, \theta)$ . We can then simply let  $\Pi(K_1, K_2, \theta)$ be defined as the maximum value function;  $\Pi(K_1, K_2, \theta) = \max_{K_3} \tilde{\Pi}(K_1, K_2, K_3, \theta)$ . Under reasonable assumptions regarding  $\tilde{\Pi}(K_1, K_2, K_3, \theta)$ , the indirect or reduced form function  $\Pi(K_1, K_2, \theta)$  will have the properties assumed in the main text.

The outside value is obtained when the firm completely withdraws from countries 1 and 2. We assume that the firm in that case is able to use an alternative technology that yields profits given by some function  $\hat{\Pi}(K_3, \theta)$ . For example, the firm may be able to better exploit economies of scale or scope. The outside value is then  $n(\theta) = \max_{K_3} \hat{\Pi}(K_3, \theta)$ , and under reasonable conditions the outside value will be increasing and convex in  $\theta$ . The kind of equilibrium we consider in this paper has participation constraints binding only for the least efficient and the most efficient types. A possible sufficient conditions for this to be the case is that the outside value to be more convex than the inside rent, i.e.  $n''(\theta) > \pi''(\theta)$ . The inside profit (rent) function will by incentive compatibility –under cooperation as well as non-cooperation–satisfy  $\pi'(\theta) = \frac{\partial \Pi}{\partial \theta}(K_1(\theta), K_2(\theta), \theta)$ , see (4), where  $K_1(\theta), K_2(\theta)$  are the equilibrium 'inside' investments. Since  $K_1(\theta), K_2(\theta)$  and therefore  $\pi(\theta)$  and its curvature are determined by the properties of the function  $\tilde{\Pi}()$ , while  $n(\theta)$ and its curvature are determined by the (different) function  $\hat{\Pi}()$ .

### Proof of Lemma 2:

Suppose principal *i* offers the tax schedule  $R_i(K_i)$ . Define

$$\hat{K}_i(K_j, \theta) = \arg\max_{K_i} \left[ \Pi(K_j, K_i, \theta) - R_i(K_i) \right]$$
(9)

By incentive compatibility the agent's maximal profit must satisfy

$$\pi'(\theta) = \frac{\partial \Pi}{\partial \theta} (K_j(\theta), \hat{K}_i(K_j(\theta), \theta), \theta)$$

Principal j's payoff is

$$EW_{j} = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ (1+\lambda) \left( \Pi(K_{j}(\theta), \hat{K}_{i}(K_{j}(\theta), \theta), \theta) - R_{i}(\hat{K}_{i}(K_{j}(\theta), \theta)) \right) - (1+\lambda - \alpha_{j}) \pi(\theta) \right\} dF(\theta)$$

By assumption  $K_j(\theta)$  maximizes this objective subject to the IC constraint and IRconstraints for the two end-types. The Hamiltionian for the problem is

$$H(K_j, \pi, \theta, p) = \left\{ \Pi(K_j, \hat{K}_i(K_j, \theta), \theta) - R_i(\hat{K}_i(K_j, \theta)) - \frac{1 + \lambda - \alpha_j}{1 + \lambda} \pi \right\} f(\theta) + p \frac{\partial \Pi}{\partial \theta} (K_j, \hat{K}_i(K_j, \theta), \theta)$$
(10)

The necessary conditions for an optimum include (Seierstad-Sydsaeter 1987, Thm 5 p 185)

$$p'(\theta) = -\frac{\partial H}{\partial \pi} = \frac{1+\lambda-\alpha_j}{1+\lambda}f(\theta), \quad p(\underline{\theta}) \le 0, \qquad p(\overline{\theta}) \ge 0$$

These conditions imply

$$p(\theta) = \frac{1 + \lambda - \alpha_j}{1 + \lambda} (F(\theta) - c), \qquad 0 \le c \le 1$$

So we may write

$$p(\theta) = \frac{1 + \lambda - \alpha_j}{1 + \lambda} (F(\theta) - F(\check{\theta}_j)), \quad \text{some } \check{\theta}_j \in [\underline{\theta}, \overline{\theta}]$$

It is further necessary that  $K_j(\theta)$  maximizes the Hamiltonian. The first-order condition for that is (using the envelope property for  $\hat{K}_i$ )

$$\Pi_j(K_j, \hat{K}_i(K_j, \theta), \theta) + \frac{p(\theta)}{f(\theta)} \left[ \Pi_{j\theta}(K_j, \hat{K}_i(K_j, \theta), \theta) + \Pi_{i\theta}(K_j, \hat{K}_i(K_j, \theta), \theta) \frac{\partial \hat{K}_i}{\partial K_j} \right] = 0$$

In equilibrium we must have  $\hat{K}_i(K_j(\theta), \theta) = K_i(\theta)$ . From the definition of  $\hat{K}_i$  we can then derive an (equilibrium) expression for  $\frac{\partial \hat{K}_i}{\partial K_j}$  (see the text following the lemma). Substituting this expression and the expression for  $p(\theta)$  into the first-order condition above yields the formula (7). This completes the proof.

### **Proof of Proposition 3**

It is well known that for the conventional case with type independent reservation utility (so  $\check{\theta}_1 = \check{\theta}_2 = \bar{\theta}$ ) and contract substitutes ( $\Pi_{12} < 0$ ) the system (7) has a unique solution that satisfies the necessary conditions (6) for common implementability. (Stole 1992, Martimort 1992) These necessary conditions for implementability are also sufficient in the case of quadratic functions and contract substitutes, provided both schedules  $K_1(\theta), K_2(\theta)$ are nondecreasing. The same reasoning shows that for given  $\check{\theta}_1, \check{\theta}_2 \in [\underline{\theta}, \overline{\theta}]$  the system (7) has a unique commonly implementable solution. For  $\theta$  uniform (so  $\frac{F(\theta) - F(\check{\theta}_j)}{f(\theta)}$  is linear) this solution has moreover schedules  $K_1(\theta), K_2(\theta)$  that are linear in  $\theta$ . From (7) we see (by symmetry) that the (constant) slopes are equal;  $K'_1 = K'_2 = K'$ . Moreover, we have (by symmetry and common implementability (6))  $0 \leq 2\frac{-\Pi_{12}}{\Pi_{1\theta}}K' \leq 1$ . (In fact it can be verified by explicit solution of (7) that both inequalities are strict when  $\Pi$  is strictly concave)

Let  $R_1(K_1), R_2(K_2)$  be a pair of tax schedules that implement the solution  $K_1(\theta), K_2(\theta)$ . For each investment level  $K_i$  in the range of  $K_i(\theta)$ , the tax function  $R_i(K_i)$  is uniquely determined up to an additive constant (by the firm's first-order condition). Moreover,  $R_i(K_i)$  is quadratic, and for  $\hat{K}_i$  given by (9) we have

$$\frac{\partial \hat{K}_i}{\partial K_j} = const = \frac{\Pi_{12}K'}{\Pi_{1\theta} + \Pi_{12}K'} \in [-1, 0]$$

For such a  $R_i(K_i)$  consider principal j's problem. The Hamiltonian for the relaxed program of maximizing her objective subject to (IC) and IR for the end-types is given by (10). This function is now quadratic in  $K_j$ , and we have

$$\frac{1}{f(\theta)}\frac{\partial^2 H}{\partial K_j^2} = \Pi_{jj} + \Pi_{ji}\frac{\partial \hat{K}_i}{\partial K_j} \le \Pi_{jj} - \Pi_{ji} < 0$$

where the first inequality follows from  $-1 \leq \frac{\partial \hat{K}_i}{\partial K_j} < 0$  and  $\Pi_{ij} < 0$ , and the second from concavity of  $\Pi$  and  $\Pi_{12} < 0$ . This shows that the Hamiltonian is concave in  $K_j$ , and hence is maximal for  $K_j = K_j(\theta)$ . (Stricly speaking, this argument demonstrates concavity of H for  $K_j$  in the range of  $K_j(\theta')$ ,  $\theta' \in [\underline{\theta}, \overline{\theta}]$ .  $R_i(.)$  can be extended (as in Martimort 1992) outside the equilibrium range such that the local maximum is also a global maximum for H.) Moreover, the maximized Hamiltonian is concave (in fact linear) in the state variable  $(\pi)$ , and this is then sufficient for  $K_j(\theta)$  to be optimal for the relaxed program. (Seierstad-Sydsaeter 1987, Thm. 6 p.186). Since this solution by (8) yields the agent a rent that satisfies the IR-constraints for all types, it is also a solution to the non-relaxed program. This completes the proof.

### **Proof of Proposition 4**

The equilibrium equations (7) now take the form:

$$m\theta + k - (q+a)K_j(\theta) - aK_i(\theta) = \frac{1+\lambda-\alpha_j}{1+\lambda} \left[m + \frac{maK_i'(\theta)}{aK_j'(\theta) - m}\right](\check{\theta}_j - \theta), \quad (11)$$

where  $i, j = 1, 2, i \neq j$ . The system has linear solutions of the form  $K_j(\theta) = L_j + K'_j \theta$ , j = 1, 2. Equations (11) yield four equations for the six parameters that characterize the solutions, i.e.,  $(L_j, K'_j, \check{\theta}_j), j = 1, 2$ :

$$m - (q+a)K'_j - aK'_i = -\frac{1+\lambda-\alpha_j}{1+\lambda} \left[m + \frac{maK'_i}{aK'_j - m}\right],$$
(12)

$$k - (q+a)L_j - aL_i = \frac{1+\lambda - \alpha_j}{1+\lambda} \left[ m + \frac{maK'_i}{aK'_j - m} \right] \check{\theta}_j, \tag{13}$$

The necessary implementability conditions (6) can be written, given  $K'_j > 0$  as

$$0 \le \frac{a}{m}K'_j \le 1 \ j = 1, 2$$
 and  $\frac{a}{m}K'_1 + \frac{a}{m}K'_2 \le 1.$  (14)

The slopes of the equilibrium schedules are seen to be independent of  $\check{\theta}_1, \check{\theta}_2$ , and therefore the same as in the case of no outside option. For symmetric countries (where  $\alpha_1 = \alpha_2$ ) they are also symmetric, so  $K'_1 = K'_2 = K'$ . An equilibrium as described in Proposition 3 must in addition satisfy  $\pi(\bar{\theta}) = n(\bar{\theta})$  and  $\pi(\underline{\theta}) = n(\underline{\theta})$ , hence we must have  $n(\bar{\theta}) - n(\underline{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial \Pi}{\partial \theta} d\theta$ , i.e.,

$$n(\bar{\theta}) - n(\underline{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \sum_{j=1}^{2} m(L_j + K'\theta + h)d\theta = m\left[(L_1 + L_2) + 2h + K'\right].$$
(15)

While the slopes  $K'_j$  of the equilibrium schedules are uniquely determined (and equal) under the conditions given in the last proposition, we note that there are only three equations to determine the remaining four parameters that characterize the equilibrium investment schedules. This leaves one degree of freedom, and we must therefore expect that these schedules are not uniquely determined. In fact, suppose we have an equilibrium solution  $(L_j, K', \check{\theta}_j)$ , j = 1, 2. According to (15), the solution must satisfy  $L_1 + L_2 = M$ , where M is a uniquely determined constant. We can then construct a new solution by letting the new intercepts satisfy this relation, and solve for the new  $\check{\theta}_j$ -parameters from (11). (This is feasible, at least for small variations in the intercept parameters.) This proves the first part of the proposition.

To verify part (ii), note that total investments are  $\Sigma_j K_j(\theta) = \Sigma_j (L_j + K'\theta)$  and that  $\pi'(\theta) = \frac{\partial \Pi}{\partial \theta} = \Sigma_j m (L_j + K'\theta + h)$ . Since the last sum is uniquely determined and  $\pi(\underline{\theta})$  is given, we see that  $\pi(\theta)$  as well as aggregate investments are uniquely determined for all  $\theta$ , as was to be shown.

To verify part (iii) note that total welfare is  $W_1 + W_2 = (1 + \lambda)\Pi(K_1, K_2, \theta) - (1 + \lambda - \Sigma \alpha_j)\pi(\theta)$ . Rents  $\pi(\theta)$  are constant across the relevant equilibria. In these equilibria investments are of the form  $K_1(\theta) = K(\theta) + \delta$ ,  $K_2(\theta) = K(\theta) - \delta$ . By symmetry and concavity of the objective  $\Pi(K_1, K_2, \theta)$  it is maximal for  $\delta = 0$ . This completes the proof.

### **Proof of Proposition 5.**

In the fully symmetric case one can easily solve for and compare the slope parameters  $(K'_{jC}, K'_j)$  of the investment schedules for the cooperative and the competitive regime, respectively. One finds that (as in Olsen and Osmundsen 2001)

 $K'_{jC} \leq K'_j$  iff  $\frac{1+\lambda}{\alpha_1+\alpha_2} \leq \Psi^{-1} = \frac{q}{4a} + 1$ . Consider the case  $\frac{1+\lambda}{\alpha_1+\alpha_2} < \Psi^{-1}$ . The investment schedule is then steeper in the competitive regime  $(K'_{jC} < K'_j)$ . If the outside value function is type-independent (the conventional case), then for both regimes the IR constraints are binding only for the low type  $\underline{\theta}$ , and there is 'no distortion at the top'  $(\underline{\theta} = \underline{\theta}_j = \overline{\theta}$  in our notation). Hence we have  $K_{jC}(\theta) > K_j(\theta)$  for all types but type  $\overline{\theta}$ . (The cooperative schedule is flatter, and investment levels are equal for  $\theta = \overline{\theta}$ .) It follows that investments are lower under competition, and hence that rents are lower in that regime too. Let  $\overline{\pi}_C$  and  $\overline{\pi}$  denote the rents accruing to type  $\overline{\theta}$  in this case, under cooperation and competition, respectively. We have  $\overline{\pi} < \overline{\pi}_C$ . The least efficient type obtains rents  $n(\underline{\theta})$  in both regimes.

In the following we fix  $n(\underline{\theta})$  and consider various forms that  $n(\theta)$  may take for  $\theta > \underline{\theta}$ . By assumption we consider only outside value functions that yield solutions with IR constraints binding for (at most) types  $\underline{\theta}$  and  $\overline{\theta}$ .<sup>28</sup> Holding  $n(\underline{\theta})$  fixed, the decisive factor is then the outside rent for the most efficient type, i.e.  $n(\overline{\theta})$ .

The IR constraints will continue to bind only for the least efficient type  $\underline{\theta}$  in both regimes as long as the outside value has  $n(\overline{\theta}) < \overline{\pi}$ . Investments and rents are then in both

<sup>&</sup>lt;sup>28</sup>Precise characterizations of such outside value functions can be derived by techniques similar to those used in section 5.2. This is left out for the sake of brevity.

regimes the same as when the outside value is type-independent. This covers case (iii) in the proposition.

Consider next  $\bar{\pi} < n(\bar{\theta}) < \bar{\pi}_C$ . The IR constraints for the cooperative case will then not be affected, while those for the competitive case will be affected in such a way that the IR constraint now becomes binding for type  $\bar{\theta}$  in addition to type  $\underline{\theta}$ . In the competitive symmetric equilibrium we then have  $\check{\theta}_j < \bar{\theta}$  and thus overinvestments compared to the first-best for  $\theta > \check{\theta}_j$ . Since cooperative investments are the same as in the conventional case considered above (IR constraints binding only for the low-efficiency type), and thus exhibit underinvestment relative to the first-best, they must also exhibit underinvestment relative to competitive investments ( $K_{jC}(\theta) < K_j(\theta)$ ) for  $\theta > \tilde{\theta}$ , for some  $\tilde{\theta} < \check{\theta}_j$ . We cannot have underinvestment for all types, since that would imply uniformly lower rents in the cooperative regime, and we have assumed  $\pi(\bar{\theta}) = n(\bar{\theta}) < \pi_C(\bar{\theta})$ . Hence we have  $K_{jC}(\theta) > K_j(\theta)$  for low-efficiency types ( $\theta < \tilde{\theta}$ ). This covers case (ii) in the proposition as far as investments are concerned.

To see that rents are for (almost) all types higher in the cooperative regime in this case, note that we have  $\pi_C(\underline{\theta}) = \pi(\underline{\theta}), \pi'_C(\theta) > \pi'(\theta)$  for  $\theta < \tilde{\theta}$ , and  $\pi_C(\bar{\theta}) > \pi(\bar{\theta})$ . Since both functions are quadratic (and therefore cannot cross more than twice), it follows that  $\pi_C(\theta) > \pi(\theta)$  for all  $\theta > \underline{\theta}$ . This proves the statements regarding case (ii).

Finally consider an outside value  $n(\theta)$  where  $n(\bar{\theta}) > \bar{\pi}_C > \bar{\pi}$ . The IR constraints will now be binding for types  $\bar{\theta}$  and  $\underline{\theta}$  under both regimes, so we have  $\pi_C(\theta) = \pi(\theta) = n(\theta)$ for  $\theta = \underline{\theta}, \bar{\theta}$ . It follows that the investment schedules  $K_{jC}(\theta)$  and  $K_j(\theta)$  must cross (once). Otherwise the highest schedule would generate higher rents for all types  $\theta > \underline{\theta}$ , and this would violate  $\pi_C(\bar{\theta}) = \pi(\bar{\theta})$ . Since  $K_j(\theta)$  is steepest, it must be below  $K_{jC}(\theta)$  for low-efficiency types, and this implies  $\pi'_C(\theta) > \pi'(\theta)$  for these types. This in turn yields  $\pi_C(\theta) > \pi(\theta)$  for all  $\theta$  in  $(\underline{\theta}, \overline{\theta})$ . The statements regarding case (i) are thereby proved.

This completes the proof for the parameter configuration  $\frac{1+\lambda}{\alpha_1+\alpha_2} < \Psi^{-1}$ . The complementary case can be handled similarly. QED.

### **Proof of Proposition 6**

Since the countries are symmetric with respect to technologies and owner shares, equations (12) admit unique solutions  $K'_j$ , with  $K'_1 = K'_2$ . For every  $\eta$  in  $(\eta_1, \eta_2)$ , and every outside value function in the family  $N(\eta)$ , there is a unique symmetric equilibrium of the form given in Proposition 4, with parameters  $L_1 = L_2$  and  $\check{\theta}_1 = \check{\theta}_2 \in (\underline{\theta}, \overline{\theta})$ . From (13,15) we see that these parameters are in fact linear functions of  $\eta$ ; with  $L_j(\eta)$  strictly increasing and  $\check{\theta}_j(\eta)$  strictly decreasing. The total value  $E(W_1 + W_2)$  associated with this equilibrium can (after an integration by parts) be written as

$$(1+\lambda)\int_{\underline{\theta}}^{\overline{\theta}} \left\{ \Pi(K_1, K_2, \theta) - (1 - \frac{\alpha_1 + \alpha_2}{1+\lambda}) \frac{\partial \Pi}{\partial \theta} (K_1, K_2, \theta) \frac{F(\check{\theta}_1) - F(\theta)}{f(\theta)} \right\} dF(\theta) - (1 + \lambda - \alpha_1 - \alpha_2) \left\{ \pi(\underline{\theta}) F(\check{\theta}_1) + \pi(\overline{\theta}) [1 - F(\check{\theta}_1)] \right\},$$

where  $K_j = K_j(\theta; \eta) = L_j(\eta) + K'_j\theta$ ,  $\check{\theta}_1 = \check{\theta}_1(\eta)$ ,  $\pi(\underline{\theta}) = 0$  (by our normalization) and  $\pi(\overline{\theta}) = \eta$ . Note that the partial derivative of this expression wrt.  $\check{\theta}_1$  is zero. Using the uniform distribution, the marginal effect on total expected welfare  $(\frac{\partial}{\partial \eta}E(W_1+W_2))$  can then be written as  $(1 + \lambda)$  times the following expression

$$\int_{\underline{\theta}}^{\overline{\theta}} \sum_{j} \left\{ \frac{\partial \Pi}{\partial K_{j}} - (1 - \frac{\alpha_{1} + \alpha_{2}}{1 + \lambda}) \frac{\partial^{2} \Pi}{\partial K_{j} \partial \theta} (\check{\theta}_{1} - \theta) \right\} \frac{\partial K_{j}}{\partial \eta} d\theta - (1 - \frac{\alpha_{1} + \alpha_{2}}{1 + \lambda}) [1 - \check{\theta}_{1}].$$

Using (7,11) and symmetry we can write this as

$$\left[2(1-\frac{\alpha_1}{1+\lambda})[m+\frac{maK_1'}{aK_1'-m}] - (1-\frac{2\alpha_1}{1+\lambda})2m\right]\int_{\underline{\theta}}^{\overline{\theta}}(\check{\theta}_1-\theta)d\theta\frac{\partial K_1}{\partial \eta} - (1-\frac{2\alpha_1}{1+\lambda})[1-\check{\theta}_1]$$

Note that  $\eta = \eta_1$  yields  $\check{\theta}_1 = \bar{\theta} = 1$ , and hence

$$sign\frac{\partial}{\partial\eta}E(W_1+W_2)_{\eta=\eta_1} = sign\left[(1-\frac{\alpha_1}{1+\lambda})\left[1+\frac{aK_1'}{aK_1'-m}\right] - (1-\frac{2\alpha_1}{1+\lambda})\right]$$

From (15) we see that  $\frac{\partial K_1}{\partial \eta} = \frac{\partial L_1}{\partial \eta} = \frac{1}{2m}$ . Differentiating once more we obtain

$$\frac{\partial^2}{\partial \eta^2} \frac{E(W_1 + W_2)}{(1+\lambda)} = \left\{ \left[ (1 - \frac{\alpha_1}{1+\lambda}) [1 + \frac{aK_1'}{aK_1' - m}] - (1 - \frac{2\alpha_1}{1+\lambda}) \right] + (1 - \frac{2\alpha_1}{1+\lambda}) \right\} \frac{\partial \check{\theta}_1}{\partial \eta} < 0$$

where the inequality follows from (14) and  $\frac{\partial \check{\theta}_1}{\partial \eta} < 0$ . Hence the total value  $E(W_1 + W_2)$  is strictly concave in  $\eta$ , and therefore increasing for some  $\eta$  if and only if  $\frac{\partial}{\partial \eta} E(W_1 + W_2) > 0$ for  $\eta = \eta_1$ . Using  $\gamma = 1 - \frac{\alpha_1}{1+\lambda}$ , we have

$$\frac{\partial}{\partial \eta} E(W_1 + W_2)_{\eta = \eta_1} > 0 \quad \text{iff} \quad \gamma [1 + \frac{aK_1'}{aK_1' - m}] - (1 - 2(1 - \gamma)) > 0.$$

Using (12), the condition is equivalent to  $-1 + (\frac{q}{a} + 2)\frac{a}{m}K'_1 + 1 - 2\gamma > 0.$ 

Since (12) can be solved explicitly for  $K'_1$  in this case, we find that the condition is equivalent to  $1 + \gamma + \frac{Q}{2} - \sqrt{\gamma + \gamma^2 + \frac{Q^2}{4}} - 2\gamma > 0$ , where  $Q = \frac{q}{a} + 1 > 1$ . This holds iff  $1 + Q > \gamma(Q + 3)$ . Substituting for  $\gamma = 1 - \frac{\alpha_1}{1+\lambda}$  and  $Q = \frac{q}{a} + 1$ , we see that the latter

condition is equivalent to  $\frac{\alpha_1}{1+\lambda} > \frac{2}{4+q/a}$ . This is again equivalent to the condition stated in the proposition.

Finally note that for  $\eta = \eta_2$  we have (by definition of  $\eta_2$ )  $\check{\theta}_j = \underline{\theta} = 0$ , and hence

$$\frac{\partial}{\partial \eta} \left[ \frac{E(W_1 + W_2)}{(1+\lambda)} \right]_{\eta = \eta_2} = [(1 - \frac{\alpha_1}{1+\lambda})[1 + \frac{aK_1'}{aK_1' - m}] - (1 - \frac{2\alpha_1}{1+\lambda})](-\frac{1}{2}) - (1 - \frac{2\alpha_1}{1+\lambda}) < 0$$

This completes the proof of the proposition.

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### **Supplement** (not for publication)

We prove here the assertion stated in the text following Proposition 6, namely that a higher outside value may reduce the efficiency loss by as much as 75 %. To this end consider the case  $\lambda = 0$ ,  $\alpha_i = .5$ . Note that for  $\eta = \eta_1$  (where  $\check{\theta}_1 = 1$ ) we have (see (7,11))

$$K_1(\theta) = L + K'_1 \theta = \frac{k+m}{q+2a} + K'_1(\theta-1)$$

For  $\eta > \eta_1$  we thus have

$$K(\theta;\eta) = \frac{\eta - \eta_1}{2m} + \frac{k + m}{q + 2a} + (\theta - 1)K_1'$$

We also have

$$\Pi(K, K, \theta) = 2\left[g + m(K+h)\theta + kK - \frac{q}{2}K^2\right] - \frac{a}{2}(2K)^2$$
  
=  $2(g + mh\theta) + \frac{(m\theta + k)^2}{q + 2a} - (q + 2a)\left[\frac{m\theta + k}{q + 2a} - K\right]^2$ 

So we may write

$$\Pi(K(\theta;\eta), K(\theta;\eta), \theta) = \Pi_F(\theta) - (q+2a) \left[ \left(\frac{m}{q+2a} - K_1'\right)(\theta-1) - \frac{\eta - \eta_1}{2m} \right]^2$$

where  $\Pi_F(\theta)$  is first-best profits. This yields

$$EW(\eta) = EW_F - (q+2a) \left[ \left(\frac{m}{q+2a} - K_1'\right)^2 \frac{1}{3} + \left(\frac{m}{q+2a} - K_1'\right) \frac{\eta - \eta_1}{2m} + \left(\frac{\eta - \eta_1}{2m}\right)^2 \right]$$

where  $EW_F$  is first-best total expected welfare (for  $\lambda = 0, \alpha = .5$ ). Under the stated conditions we have  $\frac{m}{q+2a} - K'_1 < 0$ , and it follows that we have

$$\max_{\eta} EW(\eta) = EW_F - (q+2a) \left(\frac{m}{q+2a} - K_1'\right)^2 \left[\frac{1}{3} - \frac{1}{4}\right] = EW_F - (EW_F - EW(\eta_1))[1 - \frac{3}{4}]$$

The efficiency loss is thus reduced by 75% when  $\eta$  increases from  $\eta_1$  to its optimal value.