International Diversification: An Extreme Value Approach

Lorán Chollete, Victor de la Peña, and Ching-Chih Lu*

June 30, 2009

Abstract

International diversification has costs and benefits, depending on the degree of asset dependence. In light of theoretical research linking diversification and dependence, we examine international diversification with two dependence measures: correlations and extreme dependence. We document several findings. First, dependence has generally increased over time. Second, there is evidence of asymmetric dependence or downside risk in all regions, albeit at different times. Surprisingly, recent Latin American returns exhibit little downside risk. Third, Latin America exhibits a great deal of correlation complexity. Fourth, extreme dependence is related to returns. Our results suggest international limits to diversification. They are also consistent with a possible tradeoff between international diversification and systemic risk.

Keywords: Diversification; Downside Risk; Correlation Complexity; Extreme Value; Systemic Risk

JEL Classification: C14, F30, G15

*Chollete is at University of Stavanger; De la Peña is at Columbia University; Lu is at National Chengchi University. De la Peña and Chollete acknowledge support of NSF Grant #DMS-02-05791, de la Peña PI. Chollete acknowledges support from Finansmarkedsfondet Grant #185339. We are grateful for comments from Bruno Gerard, Philipp Hartmann, Chris Heyde, Robert Hodrick, Thomas Mikosch, Oyvind Norli, Bernt-Arne Odegaard, Elvira Sojli, Assaf Zeevi, and participants at BI, Columbia, Federal Reserve Bank of Boston, International Conference on Finance, and MIT. Corresponding author is Chollete, loran.chollete@nhh.no. UiS Department of Business Administration, Stavanger N-4036, Norway.

1 Introduction

Understanding the net benefits of international diversification is important in today's economic climate. In general, the balance between diversification's benefits and costs hinges on the degree of dependence across securities, as observed by Samuelson (1967), Veldkamp and Van Nieuwerburgh (2008), Ibragimov, Jaffee, and Walden (2009b), and Shin (2009), among others. Diversification benefits are typically assessed using a measure of dependence, such as correlation.[1] It is therefore vital for investors to have accurate measures of dependence. There are several measures available in finance, including the traditional correlation and extreme dependence. While each approach has advantages and disadvantages, they rarely have been compared in the same empirical study.[2] Such reliance on one dependence measure prevents easy assessment of the degree of international diversification opportunities, and how they differ over time or across regions.

The main goal of this paper is to assess diversification opportunities available in international stock markets, using both correlations and extreme dependence. The recent history of international markets is interesting in itself, due to the large number of financial crises, increasingly globalized markets, and financial contagion.[3] We also examine some basic implications for international asset pricing. In particular, we investigate whether the diversification measures are related to international stock returns. This research is valuable because considerations of diversification and dependence should affect risk premia.

A secondary focus of our paper is the relation between diversification and systemic risk. This is motivated by theoretical research such as Brumelle (1974), Ibragimov, Jaffee, and Walden (2009b), and Shin (2009). When portfolio distributions are heavy tailed, not only do they represent limited diversification, they may also suggest existence of a wedge between individual risk and systemic risk. Most empirical research on extreme dependence of markets takes it for granted that larger tail dependence leads to poorer investor diversification in practice. While this may be true, what is arguably more important from an economic point of view is that there are aggregate ramifications for elevated levels of asset dependence. Specifically, in a heavy-tailed portfolio environment, diversification may yield

[1] See Solnik (1974); Ingersoll (1987) Chapter 4; and Carrieri, Errunza, and Sarkissian (2008).

[2] Throughout, we use the word dependence as an umbrella to cover any situation where two or more variables move together. We adopt this practice because there are numerous words in use (e.g. correlation, concordance, co-dependency, comovement), and we wish to use a general term. We do not assume that any dependence measure is ideal, and throughout we indicate advantages and disadvantages as the case may be.

[3] See Dungey and Tambakis (2005); Reinhart (2008); and Reinhart and Rogoff (2009).

both individual benefits and aggregate systemic costs. If systemic costs are too severe, a coordinating agency may be needed to improve the economy's resource allocation.[4] Such policy considerations are absent from previous empirical research on international asset dependence, and provide a further motivation for our paper.

The remaining structure of the paper is as follows. In Section 2 we review theoretical and empirical literature on diversification and dependence. In Section 3 we compare and contrast diversification measures used in empirical finance. Section 4 discusses our data and main results. Section 5 illustrates some financial implications, and Section 6 concludes.

2 Diversification, dependence, and systemic risk

The notion that diversification improves portfolio performance is pervasive in economics, and appears in asset pricing, insurance, and international finance. A central precept is that, based on the law of large numbers, a group of securities carries a lower variance than any single security.[5] An important caveat, noted as early as Samuelson (1967), concerns the dependence structure of security returns, as we discuss below. This theoretical importance of dependence structure motivates our use of extreme dependence in the empirical analysis.

2.1 Theoretical background

When assets have substantial dependence in their tails, diversification may not be optimal.[6] In an early important paper, Samuelson (1967) examines the restrictive conditions necessary to ensure that diversification is optimal.[7] He underscores the need for a general definition of negative dependence, framed in terms of the distribution function of security

[4] For related work, see Ibragimov, Jaffee, and Walden (2009a); Chollete (2008); and Shin (2009).

[5] Aspects of this precept have been formalized by Markowitz (1952); Sharpe (1964); Lintner (1965); Mossin (1966); and Samuelson (1967).

[6] See Embrechts, McNeil, and Frey (2005), and Ibragimov (2009).

[7] Samuelson (1967) discusses several approaches to obtain uniform diversification, as well as positive diversification in at least one asset. The distributional assumptions on security returns involve i.i.d. and strict independence of at least one security. Although both utility functions and distributional assumptions are relevant, Samuelson focuses on distributional concerns. A special case of dependence when diversification may be optimal is that of perfect negative correlation. However, if a portfolio consists of more than 2 assets, some of which are negatively correlated, then at least 2 must be positively correlated. This could still result in suboptimality of diversification for at least one asset, when there are short sale constraints. See Ibragimov (2009), and Samuelson (1967), page 7.

returns. In a significant development, Brumelle (1974) proves that negative correlation is neither necessary nor sufficient for diversification, except in special cases such as normal distributions or quadratic preferences. Brumelle uses a form of dependence as a sufficient condition for diversification in the following result:[8]

Background Result 1 *(Brumelle, 1974). Suppose X and Y are random variables with $E(X) = E(Y)$ and that the utility function U is strictly concave. Suppose that derivatives exist. Then a sufficient condition for the investor to hold both asset X and Y is:*

$$\frac{\partial \Pr[Y \leq y | X = x]}{\partial x} > 0 \text{ and } \frac{\partial \Pr[X \leq x | Y = y]}{\partial y} > 0. \tag{1}$$

Intuitively, increasing X leads to a lower return on Y probabilistically and vice versa, so it makes sense for a risk averse investor to hold some of each asset. The conditions in (1) resemble negative correlation, but unlike correlation, involve nonlinear derivatives defined over the entire distribution. Thus, shortly after the inception of modern portfolio theory, both Brumelle (1974) and Samuelson (1967) realize and discuss the need for restrictions on the joint distribution, in order to obtain diversification. However, that discussion has a gap: it stops short of examining multivariate ($n > 2$) asset returns, and the practical difficulty of imposing a condition like (1) on empirical data. The use of extreme value theory may be one way to fill this gap.[9] The research of Poon, Rockinger, and Tawn (2004) provides a good application of multivariate extreme value theory to finance. The authors discuss the inaccuracies of the standard Pearson correlation as a risk signal for joint extreme events. They suggest using measures of extreme dependence to capture the likelihood of rare events in financial markets. An important building block for their results is the nonlinear correlation measure we term left or **downside risk**,[10] $\lambda^L(u)$. This function measures the conditional probability of an extreme event below some threshold u. For simplicity, normalize variables to the unit interval $[0, 1]$. Then

$$\lambda^L(u) \equiv \Pr(F_X(x) \leq u \mid F_Y(y) \leq u). \tag{2}$$

[8]This result is stated by Brumelle (1974), although not formulated as a theorem.

[9] Another approach involves copulas, which we explore elsewhere.

[10]The concept of downside risk appears in a number of settings without being explicitly named. It is the basis for many measures of systemic risk, see Hartmann, Straetmans, and de Vries (2003); Poon, Rockinger, and Tawn (2004) equation 2; Cherubini, Luciano, and Vecchiato (2004) page 43; and Adrian and Brunnermeier (2008).

The main concept that Poon, Rockinger, and Tawn (2004) use is based on left **tail dependence**, $\chi^L(u)$, which is the limit of downside risk as losses become extreme,

$$\chi^L(u) \equiv \lim_{u \downarrow 0} \Pr(F_X(x) \le u \mid F_Y(y) \le u). \tag{3}$$

The authors construct parametric and nonparametric measures of extreme dependence based on (3), which they use to examine G5 countries for evidence of tail dependence.[11] Since these multivariate measures represent dependence in the tails for arbitrary distributions, in principle they allow us to examine diversification effects for heavy-tailed joint distributions, in a part of the data that might not be captured by correlations. This development therefore accords with the logic of Brumelle (1974) and Samuelson (1967).

The above approaches analyze investor decisions, and say little about systemic risk. Evidently investors' decisions, in aggregate, may have an externality effect on financial and economic markets. The existence of externalities related to ”excessive” diversification has been emphasized by several recent papers. We discuss the following three articles, since their results focus on distributional dependence.[12] Ibragimov, Jaffee, and Walden (2009b) develop a model of catastrophic risks. They characterize the existence of *non-diversification traps*: situations where insurance providers may not insure catastrophic risks nor participate in reinsurance even though there is a large enough market for complete risk sharing. Conditions for this market failure to occur comprise limited liability or heavy left-tailedness of risk distributions. Below we state a central result, where $\aleph$ is the set of relevant risks:[13]

Background Result 2 *(Ibragimov, Jaffee, and Walden (2009b)). Suppose insurers' liability is finite, the risks $X \in \aleph$ have $E(X) = 0$, and $E(X^2) = \infty$. Then a nondiversification trap may occur. This result continues to hold for distributions with moderately heavy left tails.*

Economically speaking, if assets have infinite second moments, this represents potentially unbounded downside risk and upside gain. In the face of this, insurers prefer to ration insurance rather than decide coverage unilaterally.[14] The authors go on to say that, if the

[11] We use the terms tail dependence and extreme dependence interchangeably. For more details, see de Haan and Ferreira (2006). We discuss the extreme dependence measures in more detail in Section 3.

[12] Other papers include Chollete (2008), Krishnamurthy (2009), Shin (2009); and Danielsson, Shin, and Zigrand (2009). A closely related paper is that of Zhou (2009), who analyzes the dependence of risk factors in a multivariate setting. He realistically accounts for non-independent risk factors and shows that the optimality of diversification for heavy tailed distributions is highly sensitive to the degree of tail dependence.

[13] This result is a partial converse that we derive from part iii) of their Proposition 6.

[14] This parallels the credit rationing literature of Jaffee and Russell (1976) and Stiglitz and Weiss (1981).

number of insurance providers is large but finite, then nondiversification traps can arise only with distributions that have moderately heavy left tails. In a related paper, Ibragimov and Walden (2007) examine distributional considerations that limit the optimality of diversification. They show that non-diversification may be optimal when the number of assets is small relative to their distributional support. They suggest that such considerations can explain market failures in markets for assets with possibly large negative outcomes. They also identify theoretical non-diversification regions, where risk-sharing will be difficult to create, and risk premia may appear anomalously large. In preparation for presenting their results, let r be the lower bound on the tail index α_j, let $\bar{a}$ denote a bound that depends on portfolio moments and r, and let $Y_1(a)$ and $Y_w(a)$ denote losses on asset 1 and on the portfolio w of (independent) risks, respectively. The authors obtain results on nondiversification, which we summarize below:[15]

Background Result 3 *(Ibragimov and Walden (2007)). Let $n \geq 2$ and let $w \in I_n$ be a portfolio of weights with $w_{[1]} \neq 1$. Then, for any $z > 0$ and all $a > \bar{a}$, the following inequality holds: $Pr(Y_w(a) > z) > Pr(Y_1(a) > z)$. In this nondiversification region, risk premia may be unusually high. The result continues to hold for some dependent risks, which exhibit tail dependence.*

In economic terms, diversification is disadvantageous under some heavy-tailed distributions because they exhibit large downside dependence. Thus, the likelihood and impact of several catastrophes exceeds that of a single catastrophe. The second part of the above theorem says that this result hold for many dependent risks as well, in particular convolutions of dependent risks with joint truncated $\alpha-$symmetric distributions. This class contains spherical distributions, including multinormal, multivariate t, and multivariate spherically symmetric $\alpha-$stable distributions. Since these convolutions exhibit heavy-tailedness in dependence, copula models are potentially useful in empirical applications of this result, by extracting the dependence structure of portfolio risks. In a recent working paper, Ibragimov, Jaffee, and Walden (2009a) discuss the importance of characterizing the potential for externalities transmitted from individual bank risks to the distribution of systemic risk. Their model highlights the phenomenon of *diversification disasters*: for some distributions, there is a wedge between the optimal level of diversification for individual agents and for society. This wedge depends crucially on the degree of heavy-tailedness: for very small or very large heavy-tailedness, individual rationality and social optimality agree, and the

[15] This result is a simplified summary of key parts from Theorems 1 and 4 of the authors. For more details, see Ibragimov and Walden (2007).

wedge is small. The wedge is potentially largest for moderately heavy tailed risks.[16] They consider an economy with M different risk classes and M risk neutral agents, and show the following:[17]

Background Result 4 *(Ibragimov, Jaffee, and Walden (2009a)). For moderately heavy-tailed distributions, there is a wedge between individually and socially desirable levels of diversification. This result continues to hold for risky returns with uncertain dependence or correlation complexity.*

The intuition for this result is that when risk distributions are moderately heavy tailed, this represents potentially unbounded downside risk and upside gain. In such a situation, some investors might wish to invest in several asset classes, even though this contributes to an increased fragility of the entire financial system. Thus, individual and social incentives are not aligned. A similar situation exists when the structure of asset correlations is complex and uncertain.[18] The authors provide a calibration illustrating a diversification disaster where society prefers concentration, while individuals prefer diversification. As in Ibragimov, Jaffee, and Walden (2009b), they explain that their results hold for general distributions, including the student's t, logistic, and symmetric stable distributions, all of which generally exhibit tail dependence.

2.2 Relation of theoretical results to extreme dependence

The research above emphasizes on theoretical grounds the importance of isolating dependence in the joint distribution of asset returns in order to say something concrete about diversification. At first glance, it may seem that the Background Results can be examined empirically using an extreme value approach since such measures based on (3) apply to arbitrary distributions. However, some of these theoretical results are phrased in terms of the distributions, not the extremes directly. Therefore, extreme value theory can at times only help an empirical study by showing that the dependence in the data satisfies a nec-

[16]The authors define a distribution $F(x)$ to be moderately heavy-tailed if it satisfies the following relation, for $1 < \alpha < \infty : \lim_{x \to +\infty} F(-x) = \frac{c+o(1)}{x^{\alpha}} l(x)$. Here c and α are positive constants and $l(x)$ is a slowly varying function at infinity. The parameter α is the tail index, and characterizes the heavy-tailedness of F. α is a parameter in many copula functions. For more details, see de Haan and Ferreira (2006) and Embrechts, Kluppelberg, and Mikosch (1997).

[17]This result is based on Theorem 2, Implication 2 and Equation (4) of the authors. For further details, see Ibragimov, Jaffee, and Walden (2009a).

[18] Individuals have an incentive to diversify because they do not bear all the costs in the event of systemic crises. That is, the aggregate risk is an externality, as examined by Chollete (2008) and Shin (2009).

essary condition. For example, if the estimated functions exhibit tail dependence, then it is possible for limited diversification, diversification traps and diversification disasters to occur.

We now discuss how the Background Results relate to tail dependence functions. Result 1 is not directly related, since (1) involves conditioning on an equality, $\Pr[X \leq x|Y = y]$, whereas tail dependence involves two weak inequalities, corresponding to $\Pr[X \leq x|Y \leq y]$. For Result 2, the key conditions are $E(X^2) = \infty$ and heavy left tails. This relates to our discussion on dependence, since if X represents returns on a portfolio of assets with infinite variance and heavy left tails, it will have asymmetric dependence. This property can be detected by estimation of tail dependence. For Results 3 and 4, the possibility of non-diversification and diversification disasters relates to joint distributions. These symmetric $\alpha-$stable and moderately heavy tailed distributions exhibit tail dependence. For both Results 3 and 4, therefore, a necessary condition is that there be tail dependence. Result 4 also relates to correlations and extreme dependence: if different measures of dependence disagree, and if they change over time, it signals that dependence may have a complex structure, which we denote correlation complexity. We therefore summarize empirical implications of the Background Results in the following observations:[19]

Observation 1. (correlation complexity) *If the extreme value-based dependence and correlation estimates disagree, or if the dependence changes over time, then the set of returns may be prone to diversification disasters. That is, investors' levels of diversification can lead to systemic risk.*

Observation 2. (asymmetric dependence) *If the estimated data exhibit heavy tailed asymmetric dependence, then non-diversification may be optimal. Further, there may be nondiversification traps and diversification disasters in the particular dataset. That is, it is not optimal to diversify, and investors' levels of diversification can lead to systemic risk.*

[19]These observations merely summarize necessary conditions that extreme dependence must satisfy in order to obtain non-diversification results discussed above.

2.3 Related empirical research

Previous research generally falls into either correlation or extreme value frameworks.[20] The literature in each area applied to international finance is vast and growing, so we summarize only some key contributions.[21] With regard to correlation, a major finding of Longin and Solnik (1995) and Ang and Bekaert (2002) is that international stock correlations tend to increase over time. Moreover, Cappiello, Engle, and Sheppard (2006) document that international stock and bond correlations increase in response to negative returns, although part of this apparent increase may be due to an inherent volatility-induced bias.[22] Regarding extreme value-based studies of dependence, in two early studies, Mandelbrot (1963) and Fama (1965) show that US stocks are not gaussian and have univariate heavy tails. Fama (1965) also shows that stock crashes occur more frequently than booms. Jansen and de Vries (1991) investigate the distribution of extreme stock prices. This study is motivated by the 1987 stock market crash, and calculates the tail index using a univariate, nonparametric approach. They use daily data from 10 stocks on the S&P500 list, from 1962 to 1986. Jansen and de Vries (1991) document that the magnitude of 1987's crash was somewhat exceptional, occurring once in 6 to 15 years. Susmel (2001) uses extreme value theory to investigate the univariate tail distributions for international stock returns. He analyzes weekly returns from industrialized economies including US, UK, Australia, Canada, Germany, Japan. He also analyzes the Latin American markets of Argentina, Brazil, Chile and Mexico. Susmel (2001) documents that Latin American markets have significantly heavier left tails than do the industrialized economies. Further, he combines the extreme value approach with the safety-first criterion of Roy (1952), and demonstrates improved asset allocation relative to that of the mean-variance approach. Longin and Solnik (2001) use a parametric multivariate approach to derive a general distribution of extreme correlation. They use equity index data for US, UK, France, Germany and Japan from 1959 to 1996 to test for multivariate normality in both positive and negative tails. They document that tail correlations may go to zero (multivariate normality) in the positive tail but

[20] There is also a related literature that examines dependence using copulas, as well as threshold correlations or dynamic skewness. These papers all find evidence that dependence is nonlinear, increasing more during market downturns for many countries, and for bank assets as well as stock returns. For copula approaches, see Patton (2006); Rosenberg and Schuermann (2006); Ning (2006); Ning (2008); and Chollete, Heinen, and Valdesogo (2009). Also see the surveys of Embrechts (2009) and Patton (2009). For threshold correlations, see Ang and Chen (2002). For dynamic skewness, see Harvey and Siddique (1999).

[21] For summaries of extreme value approaches to finance, see Embrechts, McNeil, and Frey (2005); and Jondeau, Poon, and Rockinger (2007). For more general information on dependence in finance, see Embrechts, Kluppelberg, and Mikosch (1997), and Cherubini, Luciano, and Vecchiato (2004).

[22] See Forbes and Rigobon (2002).

not the negative tail. Further, Longin and Solnik (2001) show that correlations increase during market downturns. This constitutes evidence of asymmetric heavy tails. Hartmann, Straetmans, and de Vries (2003) use an extreme value approach to analyze the behavior of currencies during crisis periods. They develop a co-crash measure that is related to tail dependence, and analyze markets for industrialized nations including US, UK, Germany and Japan. They also include 10 nations in east Asia and Latin America. Their data comprise weekly returns from 1980 to 2001. Their results show that Latin American currencies have less extreme dependence than in east Asia, and that the developing markets often have less likelihood of joint extremes than do the industrialized nations. Hartmann, Straetmans, and de Vries (2004) develop a nonparametric measure of asset market dependence during extreme periods. This measure is based on quantiles of joint failure probability, and hence relates to tail dependence. The authors construct a test statistic and estimate the likelihood of simultaneous crashes in G5 countries, using weekly stock and bond data from 1987 to 1999. Hartmann, Straetmans, and de Vries (2004) document that stock markets crash together in one out of five to eight crashes, and that G5 markets are statistically dependent during crises. They also show that bond markets are less likely to co-crash, and that stock and bond markets in the same country are even less likely to co-crash. Nevertheless, the likelihood of asset dependence during extremes is found to be statistically significant. Poon, Rockinger, and Tawn (2004) use a multivariate extreme value approach to model the tails of stock index returns. They utilize both parametric and nonparametric models for the joint tail distribution. They then use daily stock index data from US, UK, Germany, France and Japan, from 1968 to 2001. Poon, Rockinger, and Tawn (2004) divide the data into several subperiods and country pairs, and document that in only 13 of 84 cases is there evidence of asymptotic dependence. They argue therefore that the probability of systemic risk may be over-estimated in financial literature. The authors also discuss how their methods can be used to modify VaR and other risk management concepts. Longin (2005) uses extreme value theory to develop hypothesis tests that differentiate between candidates for the distribution of stock returns, including the gaussian and stable Paretian. He then tests the distribution of daily returns from the S&P500, from 1954 to 2003. Longin (2005) documents that only the student-t distribution and ARCH processes can plausibly characterize the data. Harvey and de Rossi (2009) construct a model of time-varying quantiles, which allow them to focus on the expectation of different parts of the distribution. This model is also general enough to accommodate irregularly spaced data. A recent working paper by Adrian and Brunnermeier (2008) builds on an extreme value framework to analyze a measure named CoVar. This measure summarizes the dependence of Value at Risk for different institutions, and represents the conditional likelihood of an institution's experiencing a tail event, given that other institutions are in distress. They estimate CoVar by quantile regression, and also identify economic variables that help to predict CoVar. Empirically the authors study commercial banks, investment banks and hedge funds in the US. They document statistically significant spillover risk across institutions. This risk may be hedged at a cost of reduced returns, using traded risk factors such as market factors, VIX straddle, variance swaps, liquidity spreads, yield spreads and credit spreads. These papers all contribute to the mounting evidence on significant asymmetric dependence in joint asset returns.[23]

2.4 Contribution of our paper

Our paper has similarities and differences with the previous literature. The main similarity is that, with the aim of gleaning insight on market returns and diversification, we estimate dependence of international financial markets. There are several main differences. First, we assess diversification using both correlation and extreme value techniques, and we are agnostic ex ante about which technique is appropriate. To the best of our knowledge, ours is the first paper to analyze international dependence using both methods.[24] Second, with the exception of Hartmann, Straetmans, and de Vries (2003), who analyze foreign exchange, our work uses a broader range of countries than most previous studies, comprising both developed and emerging markets. Third, we undertake a preliminary analysis to explore the link between diversification and regional returns.

Finally, our paper builds on specific economic theories of diversification and dependence. Previous empirical research focuses very justifiably on establishing the existence of extreme or asymmetric dependence, and dynamic dependence. Understandably, these empirical studies are generally motivated by implications for individual market participants and risk management benchmarks such as VaR. By contrast, our work builds on theoretical diversification research, and discusses both individual and systemic implications of asset dependence structure. Most empirical research assessing market dependence takes it for granted that larger dependence leads to poorer diversification in practice. While this can be true, what is arguably more important from an economic point of view is that there are

[23] Evidence on asymmetric tail dependence is also found using the copula approach. See Patton (2006); Ning (2006); Ning (2008); and Chollete, Heinen, and Valdesogo (2009).

[24] We assume time-invariant dependence in this study. While a natural next step is time-varying conditional dependence, we start at the unconditional case, since there has been little or no comparative research even at this level. Furthermore, we do analyze whether dependence changes in different parts of the sample.

aggregate ramifications for elevated asset dependence. Therefore, we present the average dependence across regions and over time, in order to obtain empirical insight on the possibility of a wedge between individual and social desiderata. Such considerations are absent from most previous empirical research on extreme dependence.

We position our paper transparently in terms of what our methodology can and cannot do. In particular, in Observations 1 and 2, we make it clear that the extreme dependence approach typically allows us to assess only necessary conditions about diversification.

3 Measuring diversification

Diversification is assessed with various dependence measures. If two assets have relatively lower dependence, they offer better diversification than otherwise. In light of the above discussion, we estimate dependence in two ways, using correlations and extreme dependence.[25] The extent of discrepancy between the two can suggest correlation complexity. It can also be informative if we wish to obtain a sense of possible mistakes from using correlations alone. We now define the dependence measures. Throughout, we consider X and Y to be two random variables, with a joint distribution $F_{X,Y}(x, y)$, and marginals $F_X(x)$ and $F_Y(y)$, respectively.

3.1 Correlations

Correlations are the most familiar measures of dependence in finance. If properly specified, correlations tell us about average diversification opportunities over the entire distribution. The Pearson **correlation** coefficient ρ is the covariance divided by the product of the standard deviations:

$$\rho = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}} \tag{4}$$

The main advantage of correlation is its tractability. There are, however, a number of theoretical shortcomings, especially in finance settings.[26] First, a major disadvantage is that correlation is not invariant to monotonic transformations. Thus, the correlation of two return series may differ from the correlation of the squared returns or log returns. Second,

[25]Readers already familiar with dependence concepts may proceed to Section 4.

[26]Disadvantages of correlation are discussed by Embrechts, McNeil, and Straumann (2002).

there is substantial evidence of infinite variance in financial data.[27] From equation (4), if either X or Y has infinite variance, the estimated correlation may give little information on dependence, since it will be undefined or close to zero. A third drawback concerns estimation bias: by definition the conditional correlation is biased and spuriously increases during volatile periods.[28] Fourth, correlation is a linear measure and therefore may overlook important nonlinear dependence. It does not distinguish, for example, between dependence during up and down markets.[29] Whether these shortcomings matter in practice is an empirical question that we approach in this paper.

A related, nonlinear measure is the **rank** (or Spearman) **correlation**, ρ_S. This is more robust than the traditional correlation. ρ_S measures dependence of the ranks, and can be expressed as $\rho_S = \frac{\text{Cov}(F_X(x),F_Y(y))}{\sqrt{\text{Var}(F_X(x))\text{Var}(F_Y(y))}}$.[30] The rank correlation is especially useful when analyzing data with a number of extreme observations, since it is independent of the levels of the variables, and therefore robust to outliers. A more general nonlinear correlation measure is the **copula function** $C(u, v)$, defined as a joint distribution with uniform marginals U and V.[31] In the bivariate case, that means

$$C(u, v) = \Pr[U \leq u, V \leq v]. \tag{5}$$

The intuition behind copulas is that they ”couple” or join marginals into a joint distribution. Copulas summarize the dependence structure between variables.[32] Specifically, for any joint distribution $F_{X,Y}(x, y)$ with marginals $F_X(x)$ and $F_Y(y)$, we can write the distribution as

$$F_{X,Y}(x, y) = C(F_X(x), F_Y(y)). \tag{6}$$

[27] See Mandelbrot (1963); Fama (1965); Gabaix, Gopikrishnan, Plerou, and Stanley (2003); and Rachev (2003).

[28] See Forbes and Rigobon (2002). After adjusting for such bias, Forbes and Rigobon (2002) document that prior findings of international dependence (contagion) are reversed.

[29] Such nonlinearity may be substantial, as illustrated by Ang and Chen (2002) in the domestic context. These researchers document significant asymmetry in downside and upside correlations of US stock returns.

[30] See Cherubini, Luciano, and Vecchiato (2004), page 100.

[31] See de la Peña, Ibragimov, and Sharakhmetov (2006), Definition 3.1. It is typical to express the copula in terms of the marginal distributions $F_X(x)$ and $F_Y(y)$. In general, the transformations from X and Y to their distributions F_X and F_Y are known as probability integral transforms, and F_X and F_Y can be shown to be uniformly distributed. See Cherubini, Luciano, and Vecchiato (2004), page 52; and Embrechts (2009).

[32] This result holds for multivariate settings. It is due to Sklar (1959), who proves that copulas uniquely characterize continuous distributions. For non-continuous distributions, the copula will not necessarily be unique. In such situations, the empirical copula approach of Deheuvels (1979) helps narrow down admissible copulas.

The usefulness of (6) is that we can sometimes simplify analysis of dependence in a return distribution $F_{X,Y}(x,y)$ by studying instead a parametric copula C.[33] There are a number of parametric copula specifications, which have different distributional shapes and tail dependence. There are several advantages of using copulas in finance. First, they are a convenient choice for modeling potentially nonlinear portfolio dependence, such as correlated defaults. A second advantage is that copulas can aggregate portfolio risk from disparate sources, such as credit and operational risk.[34] A third advantage is invariance. Since the copula is based on ranks, it is invariant under strictly increasing transforms.[35] There are two drawbacks to using copulas. First, from a finance perspective, a major disadvantage is that many copulas do not have moments that are directly related to Pearson correlation. It may therefore be difficult to compare copula results to those of financial models based on correlations or variances. Second, the best-fitting parametric copulas may fit other parts of the distribution better than the tail. Thus, extreme dependence measures may help ameliorate drawbacks of the copula approach, since they are of similar scale to correlation, and fit the tails of return distributions.

3.2 Extreme dependence

Extreme dependence measures diversification opportunities during extreme periods. Measurement of extreme dependence is particularly relevant for regulators concerned with systemic banking failures, or investors and risk managers wishing to hedge large systematic losses on their portfolios. Extreme value theory is an increasingly common approach used to evaluate such large joint losses. For convenience we discuss two cases, asymptotic dependence and asymptotic independence, as is fairly common in finance applications.[36] Throughout, we normalize variables to the unit interval $[0,1]$ for simplicity. As mentioned in section 2 and equation (2), the building block for extreme dependence is downside risk $\lambda^L(u)$, which measures dependence below some threshold u. This function therefore quan-

[33]For further information, see Embrechts, McNeil, and Straumann (2002).

[34]See Rosenberg and Schuermann (2006).

[35]See Schweizer and Wolff (1981). For more details on copula properties, see Nelsen (1998), Chapter 2.

[36]See Poon, Rockinger, and Tawn (2004). For more information on extreme value theory and applications, see Embrechts, Kluppelberg, and Mikosch (1997); Berliant, Goegebeur, Segers, and Teugels (2005); de Haan and Ferreira (2006); and Jondeau, Poon, and Rockinger (2007).

tifies the probability of a *joint* extreme event in two asset markets. By reversing the inequalities, we obtain the counterpart of upside or right potential,

$$\lambda^R(u) \equiv \Pr(F_X(x) \geq u \mid F_Y(y) \geq u). \tag{7}$$

Formally, extreme dependence is measured by the limit of upside and downside risk. Specifically, left extreme dependence is the limit of $\lambda^L(u)$ and right extreme dependence is the limit of $\lambda^R(u)$.[37] Following the tradition in extreme value theory, we phrase our discussion in terms of right extreme dependence. When $\lambda^R(u)$ converges to zero, the variables are asymptotically independent. Otherwise they are asymptotically dependent. In economic terms, if two asset markets are asymptotically independent, then they are unlikely to experience joint extreme returns. On the other hand, if they are asymptotically dependent, they can experience joint booms or crashes.

In practice, we use two measures to assess extreme dependence. First, in accordance with the above paragraph, we estimate right extreme dependence χ as the limiting value of $\lambda^R(u)$,

$$\chi \equiv \lim_{u\uparrow 1} \lambda^R(u). \tag{8}$$

The function χ, if it exists, lies between 0 and 1. As mentioned above, if χ is zero, then there is asymptotic independence. Otherwise there is asymptotic dependence, which increases with the value of χ.[38] Furthermore, as in Poon, Rockinger, and Tawn (2004) we also compute another measure of dependence, $\bar{\chi}$, which indicates the strength of dependence when $\chi = 0$. First denote as $\bar{C}(u,u)$ the survivor copula of $C(u,u)$. Then we define $\bar{\chi}$ as

$$\bar{\chi} \equiv \lim_{u\uparrow 1} \frac{2\log(1-u)}{\log \bar{C}(u,u)} - 1, \tag{9}$$

which lies between -1 and $+1$. The function $\bar{\chi}$ is identically equal to $+1$ for asymptotically dependent variables. Otherwise $-1 \leq \bar{\chi} < 1$, and the variables are asymptotically independent. In this latter case of asymptotic independence, the function $\bar{\chi}$ measures the

[37] When the context is clear, we will omit the middle word and just use the terms ”left dependence” and ”right dependence”. Extreme dependence is sometimes called tail dependence or extremal dependence. We use the term extreme dependence to underscore its relation to other dependence measures, and in order to distinguish from other extremal properties.

[38] However, some variables can be asymptotically independent and still be correlated in the bulk of the distribution. An example is the normal distribution with nonzero correlation. For a discussion of this point, see Poon, Rockinger, and Tawn (2004); and Berliant, Goegebeur, Segers, and Teugels (2005), Chapter 9.

amount of dependence towards the center of the distribution.[39] In order to estimate χ and $\bar{\chi}$ consistently, we use the same method as Poon, Rockinger, and Tawn (2004).[40] In terms of utilizing these two measures of dependence χ and $\bar{\chi}$, we consider two basic cases. The first case involves *asymptotic dependence*, when $\bar{\chi} = 1$ and $0 < \chi \leq 1$. Here, χ measures extreme dependence for the class of distributions. In the second case, we have *asymptotic independence*, where $\chi = 0$ and $-1 \leq \bar{\chi} < 1$. In this situation, we use $\bar{\chi}$ to measure extreme dependence. Consequently, the procedure involves first checking whether $\bar{\chi} = 1$. If this equality occurs, we then assess extreme dependence with χ.

There are several important advantages to using extreme dependence measures in finance. First, a major advantage is that, in principle, they allow financial market participants to assess the likelihood of large portfolio losses due to simultaneous losses on individual assets in the portfolio. Second, extreme dependence can be used in a theoretically-based model to quantify the risk faced by safety-first or downside risk averse agents.[41] Third, the extreme dependence measures are based on the ranks and are therefore invariant to monotonic transforms. Fourth, since extreme dependence measures are rank-based and can incorporate asymmetry, they are also natural dependence measures from a theoretical perspective. The reason is that a growing body of research recognizes that investors care a great deal about the ranks and downside performance of their investment returns.[42] A disadvantage of the classic extreme value framework, based on Hill (1975), is that estimation involves a challenging tradeoff between bias and inefficiency. If the threshold u is too far in the tails there is inefficiency, and if the threshold is too far in the center there is bias. We attempt to address this issue by using fairly recent methods developed by Hall (1990) and Danielsson and DeVries (1997) to compute optimal thresholds.

[39] Our expressions for χ and $\bar{\chi}$ are phrased in terms of u in $[0, 1]$ for ease of comparison with copula formulations, which characterize the dependence structure as seen in section 3.1. This is equivalent to the approach of Poon, Rockinger, and Tawn (2004), which is the basis of our empirical method. For more details, see also Berliant, Goegebeur, Segers, and Teugels (2005), Chapter 9.

[40] In particular, we estimate χ with $\hat{\chi} = \frac{un_u}{n}$, where u is the threshold, n_u is the number of observations that exceed u, and n is the sample size. We estimate $\bar{\chi}$ with $\hat{\bar{\chi}} = \frac{2}{n_u}\left[\sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u}\right)\right] - 1$, where $z_{(j)}$ are the values of the n_u observations that exceed u. For more details on the estimation procedure, see Poon, Rockinger, and Tawn (2004); and Jondeau, Poon, and Rockinger (2007).

[41] See Susmel (2001) and Jondeau, Poon, and Rockinger (2007), chapter 9.

[42] See Polkovnichenko (2005) and Barberis, Huang, and Santos (2001).

3.3 Relationship of diversification measures

We briefly outline the relationship of the diversification measures.[43] If the true joint distribution is bivariate normal, then the correlation and copula give the same information, and correlation coincides with the second extreme dependence measure: $\rho = \bar{\chi}$. Once we move far away from normality, there is no clear relation between correlation and the other measures. However, all the other, more robust measures of dependence are pure distributional properties and can be phrased in terms of copulas. We describe the relations for rank correlation ρ_S, downside risk $\lambda^L(u)$, $\bar{\chi}$, and χ, in turn. The relation between copulas and rank correlation is given by

$$\rho_S = 12 \int_0^1 \int_0^1 C(u,v) dC(u,v) - 3. \tag{10}$$

This means that if we know the correct copula, we can recover rank correlation, and vice versa. Therefore, rank correlation is a pure copula property. Regarding downside risk, it can be shown that $\lambda^L(u)$ satisfies

$$\begin{aligned} \lambda^L(u) &\equiv \Pr(F_X(x) \leq u \mid F_Y(y) \leq u) \\ &= \frac{\Pr(F_X(x) \leq u, F_Y(y) \leq u)}{\Pr(F_Y(y) \leq u)} \\ &= \frac{C(u,u)}{u}, \end{aligned} \tag{11}$$

where the third line uses definition (5) and the fact since $F_Y(y)$ is uniform, $\Pr[F_Y(y) \leq u] = u$. Third, the relation between $\bar{\chi}$ and the copula is already expressed in equation (9). Fourth, since our measure χ is the limit of upside risk as in (7), it can also be rewritten using (11) as

$$\chi = \lim_{u \uparrow 1} \frac{C(u,u)}{u}. \tag{12}$$

To summarize the relation of the different diversification measures, all of the nonlinear measures are directly related, as expressed in equations (9) through (12); and $\bar{\chi}$, ρ and the normal copula coincide when the data are jointly normal. While the above discussion describes how to link the various concepts in theory, there is little empirical work comparing the different diversification measures. This provides a rationale for our empirical study.

[43] For background and proofs on the relations between dependence measures, see Cherubini, Luciano, and Vecchiato (2004) Chapter 3; Embrechts, McNeil, and Frey (2005); and Jondeau, Poon, and Rockinger (2007).

4 Data and results

We use security market data from fourteen national stock market indices, for a sample period of January 11, 1990 to May 31, 2006. These countries are chosen because they all have daily data available for a relatively long sample period.[44] The countries are from the G5, east Asia and Latin America. The G5 countries are France (FR), Germany (DE), Japan (JP), the UK and the US. The east Asian countries are Hong Kong (HK), South Korea (KR), Singapore (SI), Taiwan (TW) and Thailand (TH). The Latin American countries include Argentina (AR), Brazil (BR), Chile (CH) and Mexico (ME). We aggregate the data to a weekly frequency (Wednesday - Wednesday returns) in order to avoid time zone differences. Therefore the total number of observations is 831 for the full sample.[45] We briefly overview summary statistics, then discuss the correlation and copula estimates.

Table 1 summarizes our data. From an investment perspective, the most striking point is US dominance, since it has the lowest volatility in each sample. The US also has one of the largest mean returns in the full sample and during the 1990s, dominating all other G5 and east Asian countries. This suggests that recent stock market history is markedly different from previous times such as those examined by Lewis (1999), when US investment overseas had clearer diversification benefits. For the full sample, across all countries mean returns are between 3 and 16 percent. The smallest and largest returns are for Thailand (-3.7) and Brazil (15.24), respectively. Generally standard deviations are high, at least twice the magnitude of the mean, and often much larger. In the first part of the sample, 1990-2001, average returns are roughly the same as for the entire sample. As in the full sample, the smallest and largest returns are for Thailand (-14.88) and Brazil (15.37), respectively. In the latter sample, 2001 to 2006, average returns are similar in magnitude to the first sample. However, there is some evidence of a shift upwards: the smallest return is now positive, for the US (0.09), and the maximal return, for Thailand (19.16) is larger than the preceding period. Notably, the US shifted dramatically from having the largest G5 returns in the 1990s to having the lowest of all countries after 2001. Another indication of a dramatic shift in international returns is that Thailand went from having the lowest returns in the 1990s to having the largest returns after the turn of the century.

[44] Moreover, many of them are considered integrated with the world market by Bekaert and Harvey (1995).

[45] We also split the sample in two, from 1991 to 2001 and 2001 to 2006. This division of the sample was chosen so that at least one part of the sample, the first part, covers a complete business cycle in the US, as described by the National Bureau of Economic Research.

4.1 Correlation estimates of dependence

Table 2 presents correlation and rank correlation estimates. We first consider G5 countries. Panel A shows results for the entire sample, where the average correlation is 0.545. Panel B shows results for the first part of the sample, which features a slightly lower correlation of 0.487. Panel C displays results from the latter part of the sample, where average correlations are much larger, at 0.637. In all sample periods, the maximum and minimum correlations are for the same countries, France-Germany, and Japan-US, respectively. Similar patterns are detected by the rank correlation. Thus, for the G5 average dependence has increased (diversification has fallen) for every country pair over time, the countries affording maximal and minimal diversification benefits are stable over time, and the dependence measures agree on which countries offer the best and worst diversification.

Now we consider the east Asian economies. For the entire sample, in Panel A, the average Pearson correlation of 0.406 is considerably lower than for the G5 economies. Panel B shows results for the first sample. Here, average correlation is slightly lower than for the full sample, at 0.379. The maximum and minimum are also smaller than for the full sample. Panel C shows the latter sample, where correlation has increased substantially to 0.511. Throughout, the country pair with maximal correlation is that of Hong Kong-Singapore. However, the minimal correlation (best diversification pair) switches from Korea-Taiwan in the first half to Hong Kong-Thailand in the latter half, and is Taiwan-Thailand for the entire sample. Therefore the best countries for diversification differ depending on investors' holding periods. Moreover, the dependence measures disagree in the latter sample with regard to the best diversification: ρ picks Hong Kong-Thailand, whereas ρ_S chooses Taiwan-Thailand. Thus, for east Asian economies, average dependence has increased over time, the two-country portfolios affording best diversification are not stable, and the dependence measures disagree for the more recent periods.

Finally, we consider the Latin American economies. Panel A shows the full sample estimates, which feature an average correlation of 0.414. Panel B presents the first sample, with an average correlation of 0.416. Panel C shows the latter sample, with a similar correlation of 0.423. The two dependence measures do not agree with regard to which countries have maximal and minimal dependence in the early sample. They also do not agree on maximal dependence in the full sample. Further, there is a switch in the coutries offering best dependence: for the early sample it is Argentina-Brazil according to ρ, which switches to Argentina-Chile for the later sample. Thus, for Latin American countries, dependence increases only slightly, the countries with best diversification are not stable over time, and dependence measures disagree in the early and full sample.

In terms of general comparison, the lowest average dependence (best diversification) for the full sample and early period are for east Asia, and for Latin America in the latter period. The specific countries with the very minimum dependence are ambiguous for the full sample: using ρ it is in the G5, while ρ_S selects east Asia. In the early and late periods, the countries with minimal dependence are in east Asia and Latin America, respectively. In purely economic terms, an investor who invests solely in east Asia or Latin America has enhanced diversification benefits, relative to an investor who invests solely in the G5. However, given that the dependence measures sometimes disagree in Latin America and east Asia, this suggests correlation complexity, which may mitigate the apparent benefits.[46]

4.2 Extreme value estimates of asymmetric dependence

We now discuss estimates of extreme dependence. For simplicity we use the terms 'left dependence' and 'right dependence' to denote dependence of returns in the left and right tails of the joint distribution. In keeping with our discussion in Section 3, we first look at the estimates of $\bar{\chi}$ as a screen to check whether they equal 1, then for such countries, examine their asymptotic dependence with χ. Table 3 presents estimates of $\bar{\chi}$. Let us discuss the G5 countries first. Regarding left dependence, for the full sample and early sample, there is only small evidence of such dependence. However, in the later sample almost all country pairs have left tail dependence. Regarding right dependence, there is almost no evidence in the full and early samples. For the latter sample, there is some evidence of right dependence in 6 of the 10 country pairs. Now we turn to east Asia. Only 3 country pairs have left dependence in the full and early samples, while almost all countries have left dependence in the late sample. In terms of right dependence, all except 1 country pair have no such dependence in the full and early samples. In the late sample, all country pairs have significant right dependence. Finally, for Latin America, for all samples almost every country pair has left dependence. However, only one country pair has right dependence. Therefore, Latin America is very asymmetric. In general, in all regions there is little or no tendency to comove for positive extreme returns, except east Asia in the recent

[46]We assume an investor holds stock market indices. A separate approach involves holding industry portfolios to diversify sectorally, see Berben and Jansen (2005) and Flavin (2004).

period. There is stronger evidence of left dependence in all regions, especially in the late sample for all regions. Latin America has strong left dependence in all periods.

In the second step, Table 4, we examine the size of χ for those countries that exhibit extreme dependence in the previous table. First consider the G5 region. There is moderate evidence of left dependence for the full and early samples. In the full sample, 4 country pairs have left dependence, while 3 do in the early sample. However, almost all countries have left dependence in the later sample. Average dependence increases from 0.504 in the early sample to 0.535 in the late sample. Regarding right dependence, there is not much in the full and early sample, except for the France-Germany pair. However, in the later sample, 6 of the 10 pairs have right dependence. Further, the best diversification (lowest dependence) countries change from the early to late period. Now we consider east Asia. Regarding left dependence, in the full and early sample, there are only 3 country pairs. However, in the late sample, 9 of 10 countries have left dependence. Regarding right dependence, there is only 1 country for full and early samples, but 5 country pairs have it in the later sample. Moreover, average dependence has increased over time, from 0.454 in the first sample to 0.467 in the latter. Finally, we examine Latin America. There is strong left dependence in nearly every country in the full and early samples, but only for one country pair in the latter sample. There is never any right dependence for Latin America. Also, there is a switch in the best diversification country pair, which is Brazil-Chile for the early sample. However the best country pair becomes Argentina-Chile for the late sample and for the full sample.[47] The most risky (highest left dependence) country pairs are consistent for the G5 and east Asia in all samples, France-Germany and Hong Kong-Singapore, respectively. For Latin America, the most risky pair is Brazil-Mexico for the full and early samples, switching to Argentina-Brazil for the late sample. These results have implications for investment decisions during extreme periods. Consider an investor in the G5 in the period 2001-2006. If she invests in most of these countries, she faces an average risk of joint extreme downturns of 0.535.[48] In most cases, this is offset by a likelihood of upside dependence of similar magnitude.

To summarize, a number of countries in all three regions exhibit extreme dependence or downside risk. There are a number of regional differences. First, for the G5 and east Asia, left dependence increases over time, but not in Latin America. To the best of our knowledge, this finding of limited downside risk in Latin American stock markets has

[47] Since several country pairs have zero tail dependence, they have to be compared using $\bar{\chi}$ from Table 3.3

[48] The exception is Japan-US, which has no extreme dependence.

not been documented before.[49] Second, for the G5 and east Asia there is some offsetting upside dependence, but not for Latin America. Third, there is heterogeneity in the timing of extreme dependence: G5 and east Asian countries experience left dependence in the later sample, while Latin American countries experience it in the earlier sample. Fourth, across regions, the largest average left dependence is for the G5. Overall, the lowest exposure to left tail dependence is in east Asia for the full sample, with only three countries having left tail dependence. In the early sample both the G5 and east Asia have the lowest number, three countries. In the late sample the lowest exposure to left dependence is Latin America, with only one country. In economic terms, an investor concerned about extreme downside risk is afforded the best hedging in Latin America after 2001.

4.3 Comparing correlation and extreme dependence results

We summarize the results from correlations in section 4.1 and extreme dependence in section 4.2. Both correlation and extreme dependence agree that average dependence has increased over time for the G5 and east Asia. However, the extreme approach disagrees with regard to Latin America, which has exhibited a fall in left dependence in recent years. Regarding the best diversification (lowest dependence) country pairs, correlations and extreme dependence again disagree. Correlations are ambiguous for the full sample, while the extreme approach selects east Asia. For the early sample, correlations definitely choose east Asia as the lowest dependence region, while the extreme approach is ambiguous between the G5 and east Asia. For the latter sample, both measures of dependence agree on Latin America as the best region to obtain diversification. Regarding the most risky countries, both correlations and extreme dependence agree on which country pair has the worst diversification (highest dependence) for the the G5 and east Asia. For the G5 this pair is France-Germany, while for east Asia it is Hong Kong-Singapore. However, the Latin American region again features disagreement. Correlations choose Brazil-Mexico as the worst diversifiers for all periods, while extreme dependence chooses Argentina-Brazil. Furthermore, the correlation and extreme dependence measures disagree on the minimum dependence countries in Latin America. Such disagreement is evidence of correlation complexity. Thus, according to Observation 1, Latin American countries are potentially susceptible to systemic risk, through the channel of correlation complexity.

[49]This does not mean that overall risk is small in Latin America, just that the risk of spillovers during extreme periods is relatively low. A similar finding has been documented in currency markets by Hartmann, Straetmans, and de Vries (2003).

The extreme dependence approach allows us to examine asymmetric dependence or downside risk, which cannot be analyzed by correlations. All the regions exhibit relatively large downside risk. The G5 and east Asia have downside risk in the later sample, while Latin America does so in the earlier. According to Observation 2, therefore, all the regions are susceptible to limited diversification and systemic risk, through the channel of asymmetric dependence. Latin America is the least susceptible in recent times, while east Asia and the G5 are the most susceptible recently.

More broadly, our results show that dependence signals often disagree on important international finance issues. This empirical evidence bolsters the theoretical reasons of Embrechts, McNeil, and Straumann (2002) for using more robust dependence measures in risk management. Comparatively speaking, the G5 and east Asia have only one major channel for diversification problems, namely, asymmetric dependence. By contrast, Latin America is susceptible to nondiversification and systemic risk through two channels at different times, correlation complexity and downside risk. In purely economic terms, an investor who invests solely in Latin America has enhanced diversification benefits in recent years, relative to an investor who invests solely in the G5 or east Asia. However, the strong correlation complexity in Latin America may mitigate the apparent benefits.

5 Implications for international finance

As discussed in Section 3, higher dependence corresponds to reduced diversification. Investors should therefore demand higher returns to compensate for increased dependence.[50]

5.1 Relationship between returns and international diversification

If investors require higher returns for lower diversification, it is natural to explore which of our dependence measures more closely relates to returns over our sample period. Table 5 displays the relation between average returns and average diversification measures in each

[50] A classic example in finance is the CAPM, which under some conditions, says that for any stock i, its return R_i relates to its dependence (covariance) with the market return R_m:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f], \tag{13}$$

where $\beta = \text{Cov}(R_m, R_i)/\text{Var}(R_m)$. Therefore, the greater its dependence with the market, the higher an asset's own return.

region. For simplicity each variable is ranked from low (L) to high (H). Panel A shows the results for the full sample. Interestingly, even though Latin America has more than double the returns of the others, its world market beta is not the largest. This indicates that a world CAPM will not tell the full story. The only diversification measure that has the same relation across the regions is the left $\bar{\chi}$, which measures joint downside risk. Panel B shows the first half of the sample, which has the same pattern. Panel C shows the second half, where none of the diversification measures has the same pattern as returns, although $\bar{\chi}$ still has its highest rankings for the region with highest returns.

To summarize, the only diversification measure for which there is a monotonic relationship to returns, is left dependence $\bar{\chi}$. This monotonic relation exists for our sample as a whole, and for the early part of our sample, although not for the more recent sample. Left extreme dependence is also the only measure that is always largest for the region with largest returns. Economically speaking, this finding is consistent with the idea that investors are averse to (and therefore demand returns for) exposure to downside risk during extreme periods. Therefore, a simple international CAPM model where returns depend on the world market beta, as in equation (13), may be augmented with a risk factor related to downside dependence. In economic terms, this reflects *joint* downside risk, and differs from previous studies that focus on univariate downside risk.[51] Our findings, while suggestive and related to theoretical work on investor behavior during exuberant or costly-information times, are evidently preliminary.[52] These findings may therefore merit further study in a conditional setting with a wider group of countries.

6 Conclusions

The net effect of diversification involves involves benefits and costs, as noted by a growing body of theoretical literature. When assets have extreme dependence, diversification may not be optimal. Moreover, individually optimal diversification may differ from social optimality, since investors undervalue systemic risk. These observations motivate our empirical study. We examine diversification opportunities in international markets, using two different diversification measures, correlations and extreme dependence.

[51] See Post and van Vliet (2004) for a domestic CAPM version of downside risk factors. See Karni (1979) for a theoretical examination of joint downside risk.

[52] For related theoretical work, see Gul (1991); Abreu and Brunnermeier (2003); Polkovnichenko (2005); Pavlov and Wachter (2006); and Veldkamp (2006).

Empirically, we have several findings. First, correlations and extreme dependence sometimes deliver different or ambiguous risk management signals regarding countries with maximal and minimal risk of being undiversified. This correlation complexity bolsters extant theoretical reasons for using robust dependence measures in risk management. Second, dependence has increased over time for the G5 and east Asia, but not in Latin America. Third, all regions exhibit asymmetric dependence or downside risk at different times. There is little evidence of downside risk in recent Latin American stock markets, a finding that to the best of our knowledge is previously undocumented. This regional increase in diversification opportunities during extreme periods is of practical value to international investors. In economic terms, an investor concerned about extreme downside risk obtains substantial diversification benefits in Latin America after 2001. However, the investor has difficulty identifying the most risky country pairs therein.

More broadly, the fact that international returns exhibit asymmetric dependence and correlation complexity implies that they not only represent limited diversification, they are also consistent with the possibility of a wedge between investor diversification and international systemic risk. Such aggregate implications are largely absent from previous empirical research on diversification and dependence in international markets. In a simple application, we find a link between extreme dependence and regional stock returns. In particular, the only diversification measure for which there is ever a monotonic relationship to returns, is left dependence. The monotonic relation exists for our sample as a whole, and for the bigger part of the sample. Left dependence is also the only measure that is always largest for the region with largest returns. This finding relates to several branches of theoretical literature on investor behavior, including loss aversion, downside risk, bubbles, and costly information constraints. The observed relation between returns and dependence is consistent with an international CAPM model augmented with a risk factor related to extreme dependence. Since returns mirror joint downside risk, this concept differs from previous single-asset work, and suggests that international investors are compensated for exposure to extreme downside risk.

References

Abreu, D., and M. Brunnermeier, 2003, Bubbles and Crashes, *Econometrica* 71, 173–204.

Adrian, T., and M. Brunnermeier, 2008, CoVaR: A systemic risk contribution measure, Working paper, Princeton University.

Ang, Andrew, and Geert Bekaert, 2002, International Asset Allocation with Regime Shifts, *Review of Financial Studies* 15, 1137–87.

Ang, Andrew, and Joseph Chen, 2002, Asymmetric Correlations of Equity Portfolios, *Journal of Financial Economics* 63, 443–94.

Barberis, N., M. Huang, and T. Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* CXVI, 1–53.

Bekaert, Geert, and Campbell R. Harvey, 1995, Time-Varying World Market Integration, *Journal of Finance* 50, 403–44.

Berben, R., and W. Jansen, 2005, Comovement in international equity markets: A sectoral view, *Journal of International Money and Finance* 24, 832–857.

Berliant, J., Y Goegebeur, J. Segers, and J. Teugels, 2005, *Statistics of Extremes: Theory and Applications*. (John Wiley & Sons, New York).

Brumelle, S., 1974, When does diversification between two investments pay?, *Journal of Financial and Quantitative Analysis* IX, 473–483.

Cappiello, L., R. F. Engle, and K. Sheppard, 2006, Asymmetric dynamics in the correlations of global equity and bond returns, *Journal of Financial Econometrics* 4, 537–572.

Carrieri, F., V. Errunza, and S. Sarkissian, 2008, Economic integration, industrial structure, and international portfolio diversification, Working paper, McGill University.

Cherubini, Umberto, Elisa Luciano, and Walter Vecchiato, 2004, *Copula Methods in Finance*. (Wiley West Sussex, England).

Chollete, L., 2008, The propagation of financial extremes, Working paper, Norwegian School of Economics and Business Administration.

Chollete, L., A. Heinen, and A. Valdesogo, 2009, Modeling international financial returns with a multivariate regime-switching copula, *Journal of Financial Econometrics* forthcoming.

Danielsson, J., and C. DeVries, 1997, Tail index and quantile estimation with very high frequency data, *Journal of Empirical Finance* 4, 241–257.

Danielsson, J., H. Shin, and J. Zigrand, 2009, Risk appetite and endogenous risk, Working paper, Princeton University.

de Haan, L., and A. Ferreira, 2006, *Extreme Value Theory: An Introduction*. (Springer).

de la Peña, V., R. Ibragimov, and S. Sharakhmetov, 2006, Characterizations of joint distributions, copulas, information, dependence and decoupling, with applications to time series, in J. Rojo, eds.: *2nd Erich Lehmann Symposium – Optimality: IMS Lecture Notes, Monograph Series 49* (Institute of Mathematical Statistics, Beachwood, OH).

Deheuvels, G., 1979, La function de dependance empirique et ses proprietes. Un test non parametriquen d'independance, *Acad. Roy. Belg. Bull. C1. Sci.* 65, 274–292.

Dungey, M., and D. Tambakis, 2005, *Identifying International Financial Contagion: Progress and Challenges*. (Oxford Press).

Embrechts, P., 2009, Copulas: A personal view, *Journal of Risk and Insurance* forthcoming.

Embrechts, P., C. Kluppelberg, and T. Mikosch, 1997, *Modelling Extremal Events for Insurance and Finance*. (Springer, Berlin).

Embrechts, P., A. McNeil, and R. Frey, 2005, *Quantitative Risk Management: Concepts, Techniques and Tools*. (Princeton University Press).

Embrechts, P., A. McNeil, and D. Straumann, 2002, Correlation and dependence in risk managament: Properties and pitfalls, in M. Dempster, eds.: *Risk Management: Value at Risk and Beyond* (Cambridge University Press, Cambridge, UK).

Fama, E., 1965, The behavior of stock market prices, *Journal of Business* 38, 34–105.

Flavin, T., 2004, The effect of the Euro on country versus industry portfolio diversification, *Journal of International Money and Finance* 23, 1137–1158.

Forbes, Kristin J., and Roberto Rigobon, 2002, No Contagion, Only Interdependence: Measuring Stock Market Comovements, *Journal of Finance* 57, 2223–61.

Gabaix, X., P. Gopikrishnan, V. Plerou, and H. Stanley, 2003, A theory of power-law distributions in financial market fluctuations, *Nature* 423, 267–270.

Gul, Faruk, 1991, A Theory of Disappointment Aversion, *Econometrica* 59, 667–686.

Hall, P., 1990, Using the Bootstrap to Estimate Mean Squared Error and Select Smoothing Parameter in Nonparametric Problems, *Journal of Multivariate Analysis* 32, 177–203.

Hartmann, P., S. Straetmans, and C. de Vries, 2003, A Global Perspective on Extreme Currency Linkages, in W. C. Hunter, G. G. Kaufman, and M. Pomerleano, eds.: *Asset Price Bubbles: Implications for Monetary, Regulatory and International Policies* (MIT Press, Cambridge).

Hartmann, P., S. Straetmans, and C. de Vries, 2004, Asset market linkages in crisis periods, *The Review of Economics and Statistics* 86, 313–326.

Harvey, A., and G. de Rossi, 2009, Quantiles, expectiles and splines, *Journal of Econometrics* forthcoming.

Harvey, Campbell R., and Akhtar Siddique, 1999, Autoregressive Conditional Skewness, *Journal of Financial and Quantitative Analysis* 34, 465–87.

Hill, B., 1975, A Simple General Approach to Inference about the Tail of a Distribution, *The Annals of Statistics* 3, 1163–1174.

Ibragimov, R., 2009, Heavy-tailed densities, in S. Durlauf, and L. Blume, eds.: *The New Palgrave Dictionary of Economics Online* (Palgrave Macmillan,).

Ibragimov, R., D. Jaffee, and J. Walden, 2009a, Diversification disasters, Working paper, University of California at Berkeley.

Ibragimov, R., D. Jaffee, and J. Walden, 2009b, Non-diversification traps in catastrophe insurance markets, *Review of Financial Studies* 22, 959–993.

Ibragimov, R., and J. Walden, 2007, The limits of diversification when losses may be large, *Journal of Banking and Finance* 31, 2551–2569.

Ingersoll, J., 1987, *Theory of Financial Decision Making*. (Rowman and Littlefield Publishers).

Jaffee, D., and T. Russell, 1976, Imperfect information, uncertainty, and credit rationing, *Quarterly Journal of Economics* XC, 651–666.

Jansen, D., and C. de Vries, 1991, On the frequency of large stock returns: Putting booms and busts into perspective, *The Review of Economics and Statistics* 73, 18–24.

Jondeau, E., S. Poon, and M. Rockinger, 2007, *Financial Modeling under Non-Gaussian Distributions*. (Springer London).

Karni, E., 1979, On multivariate risk aversion, *Econometrica* 47, 1391–1401.

Krishnamurthy, A., 2009, Amplification mechanisms in liquidity crises, Working paper, Northwestern University.

Lewis, Karen K., 1999, Trying to Explain Home Bias in Equities and Consumption, *Journal of Economic Literature* 37, 571–608.

Lintner, J., 1965, Security prices, risk, and maximal gains from diversification, *Journal of Finance* 20, 587–615.

Longin, F., 2005, The choice of the distribution of asset returns: How extreme value theory can help?, *Journal of Banking and Finance* 29, 1017–1035.

Longin, F., and B. Solnik, 1995, Is the Correlation in International Equity Returns Constant: 1960-1990?, *Journal of International Money and Finance* 14, 3–26.

Longin, Francois, and Bruno Solnik, 2001, Extreme Correlation of International Equity Markets, *Journal of Finance* 56, 649–76.

Mandelbrot, B., 1963, The variation of certain speculative prices, *Journal of Business* 36, 394–419.

Markowitz, H., 1952, Portfolio selection, *Journal of Finance* 7, 77–91.

Mossin, J., 1966, Equilibrium in a capital asset market, *Econometrica* 34, 261–276.

Nelsen, Roger B., 1998, *An Introduction to Copulas*. (Springer-Verlag New York, Inc. New York).

Ning, C., 2006, Dependence structure between the equity market and the foreign exchange market–a copula approach, Working paper, Ryerson University.

Ning, C., 2008, Extreme dependence of international stock market, Working paper, Ryerson University.

Patton, A., 2006, Modelling Asymmetric Exchange Rate Dependence, *International Economic Review* 47, 527–556.

Patton, A., 2009, Copula-based models for financial time series, in T. Andersen, R. Davies, J. Kreiss, and T. Mikosch, eds.: *Handbook of Financial Time Series* (Springer,).

Pavlov, A., and S. Wachter, 2006, The inevitability of marketwide underpricing of mortgage default risk, *Real Estate Economics* 34, 479–496.

Polkovnichenko, V., 2005, Household Portfolio Diversification: A Case for Rank-Dependent Preferences, *Review of Financial Studies* 18, 1467–1501.

Poon, S., M. Rockinger, and J. Tawn, 2004, Extreme value dependence in financial markets: Diagnostics, models, and financial implications, *Review of Financial Studies* 17, 581–610.

Post, T., and P. van Vliet, 2004, Conditional downside risk and the CAPM, Working paper, Erasmus University Rotterdam.

Rachev, S., 2003, *Handbook of Heavy Tailed Distributions in Finance*. (North Holland).

Reinhart, C., 2008, 800 years of financial folly, Working paper, University of Maryland.

Reinhart, C., and K. Rogoff, 2009, The aftermath of financial crises, *American Economic Review* forthcoming.

Rosenberg, J., and T. Schuermann, 2006, A general approach to integrated risk management with skewed, fat-tailed risks, *Journal of Financial Economics* 79, 569–614.

Roy, A., 1952, Safety first and the holding of assets, *Econometrica* 20, 431–449.

Samuelson, P., 1967, General proof that diversification pays, *Journal of Financial and Quantitative Analysis* March, 1–13.

Schweizer, B., and E. F. Wolff, 1981, On Nonparametric Measures of Dependence for Random Variables, *The Annals of Statistics* 9, 879–885.

Sharpe, W., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Shin, H., 2009, Securitisation and system stability, *Economic Journal* 119, 309–322.

Sklar, Abraham, 1959, Fonctions de repartition a n dimensions et leurs marges, *Pub. Inst. Statist. Univ. Paris* 8, 229–231.

Solnik, B., 1974, Why Not Diversify Internationally Rather Than Domestically?, *Financial Analysts Journal* 30, 48–54.

Stiglitz, J., and A. Weiss, 1981, Credit rationing in markets with imperfect information, *American Economic Review* 71, 393–410.

Susmel, R., 2001, Extreme observations and diversification in Latin American emerging equity markets, *Journal of International Money and Finance* 20, 971–986.

Veldkamp, L., 2006, Information markets and the comovement of asset prices, *Review of Economic Studies* 73, 823–845.

Veldkamp, L., and S. Van Nieuwerburgh, 2008, Information acquisition and under-diversification, Working paper, New York University, Stern School.

Zhou, C., 2009, Dependence structure of risk factors and diversification effects, Working paper, De Nederlandsche Bank.

Table 1: Average Returns for International Indices

	1990-2006	1990-2001	2001-2006
FR	7.10	8.31	4.64
	(20.38)	(18.99)	(22.99)
DE	5.49	6.85	2.69
	(21.97)	(19.92)	(25.69)
JP	0.09	-2.52	5.43
	(22.58)	(23.30)	(21.04)
UK	5.96	6.90	4.05
	(16.38)	(15.81)	(17.52)
US	8.10	12.03	0.09
	(15.49)	(14.69)	(17.00)
HK	7.76	10.61	1.93
	(24.64)	(27.03)	(18.85)
KR	4.68	-4.49	23.41
	(36.60)	(39.38)	(30.03)
SI	3.48	2.78	4.91
	(25.19)	(27.75)	(18.95)
TW	1.16	0.98	1.53
	(32.62)	(34.90)	(27.45)
TH	-3.70	-14.88	19.16
	(37.85)	(42.24)	(26.51)
AR	12.95	14.70	9.35
	(40.53)	(41.38)	(38.81)
BR	15.24	15.37	14.98
	(44.32)	(48.59)	(34.07)
CH	11.16	10.33	12.86
	(22.61)	(24.28)	(18.79)
ME	13.61	12.18	16.54
	(31.80)	(35.14)	(23.58)

The average country portfolio returns are annualized and in percentage points. Standard deviations are in parentheses. Source: MSCI.

Table 2: Correlation Estimates of International Dependence

	G5			**East Asia**			**Latin America**		
Panel A: 1990-2006									
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
ρ	0.545	0.822	0.303	0.406	0.588	0.315	0.414	0.506	0.355
		(FR-DE)	(JP-US)		(HK-SI)	(TW-TH)		(BR-ME)	(AR-CH)
ρ_S	0.523	0.772	0.304	0.373	0.539	0.271	0.376	0.447	0.299
		(FR-DE)	(JP-US)		(HK-SI)	(TW-TH)		(AR-ME)	(AR-CH)
Panel B: 1990-2001									
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
ρ	0.487	0.762	0.281	0.379	0.577	0.237	0.416	0.493	0.359
		(FR-DE)	(JP-US)		(HK-SI)	(KR-TW)		(BR-ME)	(AR-BR)
ρ_S	0.471	0.709	0.267	0.322	0.511	0.176	0.366	0.480	0.307
		(FR-DE)	(JP-US)		(HK-SI)	(KR-TW)		(AR-ME)	(BR-CH)
Panel C: 2001-2006									
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
ρ	0.637	0.901	0.355	0.511	0.639	0.353	0.423	0.561	0.310
		(FR-DE)	(JP-US)		(HK-SI)	(HK-TH)		(BR-ME)	(AR-CH)
ρ_S	0.624	0.887	0.389	0.512	0.641	0.376	0.405	0.520	0.266
		(FR-DE)	(JP-US)		(HK-SI)	(TW-TH)		(BR-ME)	(AR-CH)

ρ and ρ_S denote the Pearson and rank correlations, defined in Section 3 of the text. Avg, Max and Min denote the average, maximum and minimum dependence for each region. Further details on individual countries are available from the authors upon request.

Table 3: Extreme-Value Estimates of International Dependence: $\bar{\chi}$

	Full Sample				**1990-2001**				**2001-2006**			
	Left Tail		Right Tail		Left Tail		Right Tail		Left Tail		Right Tail	
	$\bar{\chi}$		$\bar{\chi}$		$\bar{\chi}$		$\bar{\chi}$		$\bar{\chi}$		$\bar{\chi}$	
G5												
FR-DE	1.028	(0.146)	0.769	(0.151)	1.002	(0.167)	0.634*	(0.170)	1.038	(0.255)	0.959	(0.253)
FR-JP	0.677*	(0.132)	0.243*	(0.150)	0.616*	(0.153)	0.317*	(0.137)	0.748	(0.218)	0.524*	(0.225)
FR-UK	0.853	(0.135)	0.537*	(0.140)	0.766	(0.155)	0.473*	(0.163)	0.909	(0.239)	0.740	(0.202)
FR-US	0.653*	(0.134)	0.558*	(0.126)	0.578*	(0.150)	0.381*	(0.173)	0.696	(0.221)	0.749	(0.211)
DE-JP	0.766	(0.132)	0.533*	(0.106)	0.684*	(0.140)	0.515*	(0.133)	0.787	(0.215)	0.388*	(0.196)
DE-UK	0.931	(0.136)	0.490*	(0.139)	0.917	(0.166)	0.279*	(0.167)	0.936	(0.219)	0.617	(0.229)
DE-US	0.702*	(0.134)	0.472*	(0.127)	0.683*	(0.160)	0.443*	(0.130)	0.681	(0.210)	0.731	(0.223)
JP-UK	0.667*	(0.124)	0.348*	(0.119)	0.557*	(0.158)	0.473*	(0.140)	0.738	(0.217)	0.108*	(0.185)
JP-US	0.530*	(0.123)	0.479*	(0.116)	0.634*	(0.139)	0.441*	(0.132)	0.475*	(0.192)	0.563*	(0.211)
UK-US	0.698*	(0.150)	0.558*	(0.122)	0.652*	(0.172)	0.470*	(0.158)	0.668	(0.217)	0.684	(0.211)
East Asia												
HK-KR	0.592*	(0.124)	0.402*	(0.128)	0.435*	(0.158)	0.357*	(0.146)	0.696	(0.221)	0.398	(0.206)
HK-SI	0.840	(0.132)	0.492*	(0.127)	0.901	(0.154)	0.650*	(0.148)	0.595	(0.215)	0.415	(0.200)
HK-TW	0.559*	(0.126)	0.418*	(0.110)	0.720*	(0.139)	0.341*	(0.137)	0.343*	(0.224)	0.529	(0.206)
HK-TH	0.752	(0.127)	0.510*	(0.108)	0.824	(0.152)	0.613*	(0.137)	0.817	(0.199)	0.640	(0.191)
KR-SI	0.588*	(0.118)	0.485*	(0.117)	0.424*	(0.145)	0.495*	(0.137)	0.962	(0.236)	0.554	(0.176)
KR-TW	0.654*	(0.115)	0.457*	(0.113)	0.558*	(0.135)	0.450*	(0.133)	0.856	(0.242)	0.599	(0.206)
KR-TH	0.548*	(0.119)	0.568*	(0.111)	0.526*	(0.134)	0.602*	(0.144)	0.781	(0.209)	0.685	(0.196)
SI-TW	0.664*	(0.124)	0.551*	(0.119)	0.702*	(0.135)	0.463*	(0.129)	0.664	(0.200)	0.645	(0.222)
SI-TH	0.825	(0.116)	0.672*	(0.128)	0.825	(0.137)	0.745	(0.157)	0.828	(0.214)	0.633	(0.204)
TW-TH	0.689*	(0.116)	0.541*	(0.116)	0.601*	(0.138)	0.457*	(0.136)	0.880	(0.220)	0.585	(0.191)
Latin America												
AR-BR	0.789	(0.127)	0.504*	(0.124)	0.814	(0.144)	0.477*	(0.143)	0.700	(0.205)	0.612	(0.217)
AR-CH	0.824	(0.120)	0.577*	(0.113)	0.831	(0.143)	0.612*	(0.140)	0.744	(0.198)	0.290*	(0.183)
AR-ME	0.801	(0.133)	0.467*	(0.129)	0.814	(0.159)	0.488*	(0.164)	0.761	(0.220)	0.451*	(0.227)
BR-CH	0.772	(0.120)	0.421*	(0.115)	0.711*	(0.138)	0.532*	(0.138)	0.909	(0.230)	0.469*	(0.198)
BR-ME	0.893	(0.133)	0.381*	(0.109)	0.808	(0.153)	0.319*	(0.131)	1.133	(0.229)	0.523*	(0.190)
CH-ME	0.923	(0.128)	0.520*	(0.109)	0.894	(0.151)	0.569*	(0.136)	0.909	(0.224)	0.391*	(0.174)

The $\bar{\chi}$ statistics marked with an asterisk are significantly different from 1 at a 95% level. Standard errors are in parentheses. Estimates are computed using the methods described in section 3, following the approach of Poon, Rockinger, and Tawn (2004).

Table 4: Extreme-Value Estimates of International Dependence: χ

	Full Sample				**1990-2001**				**2001-2006**			
	Left Tail		Right Tail		Left Tail		Right Tail		Left Tail		Right Tail	
	χ		χ		χ		χ		χ		χ	
Panel A: G5												
FR-DE	0.577	(0.036)	0.547	(0.043)	0.525	(0.038)			0.650	(0.071)	0.618	(0.071)
FR-JP									0.451	(0.049)		
FR-UK	0.547	(0.035)			0.510	(0.039)			0.614	(0.067)	0.612	(0.061)
FR-US									0.535	(0.062)	0.523	(0.054)
DE-JP	0.422	(0.028)							0.475	(0.050)		
DE-UK	0.516	(0.032)			0.477	(0.036)			0.570	(0.055)	0.555	(0.071)
DE-US									0.565	(0.062)	0.530	(0.060)
JP-UK									0.441	(0.048)		
JP-US												
UK-US									0.514	(0.059)	0.497	(0.054)
Average	0.515				0.504				0.535		0.556	
Max	0.577				0.525				0.650		0.618	
Min	0.422				0.477				0.441		0.497	
Range	0.155				0.048				0.209		0.121	
Panel B: East Asia												
HK-KR									0.488	(0.056)		
HK-SI	0.487	(0.031)			0.478	(0.033)			0.513	(0.062)		
HK-TW												
HK-TH	0.438	(0.028)			0.433	(0.031)			0.425	(0.039)	0.412	(0.041)
KR-SI									0.464	(0.048)		
KR-TW									0.501	(0.058)	0.472	(0.054)
KR-TH									0.464	(0.047)	0.435	(0.043)
SI-TW									0.464	(0.048)	0.473	(0.057)
SI-TH	0.459	(0.024)			0.450	(0.028)	0.432	(0.034)	0.469	(0.047)	0.446	(0.049)
TW-TH									0.416	(0.042)		
Average	0.461				0.454				0.467		0.448	
Max	0.487				0.478				0.513		0.473	
Min	0.438				0.433				0.416		0.412	
Range	0.049				0.045				0.097		0.061	
Panel C: Latin America												
AR-BR	0.448	(0.028)			0.460	(0.031)			0.449	(0.047)		
AR-CH	0.417	(0.023)			0.420	(0.028)						
AR-ME	0.444	(0.029)			0.458	(0.035)						
BR-CH	0.446	(0.026)										
BR-ME	0.472	(0.029)			0.462	(0.034)						
CH-ME	0.421	(0.024)			0.416	(0.028)						
Average	0.441				0.443							
Max	0.472				0.462							
Min	0.417				0.416							
Range	0.056				0.046							

We report the χ statistics which are significantly different from 1. Standard errors are in parentheses. Estimates are calculated using the statistical method described in section 3, following Poon, Rockinger, and Tawn (2004).

Table 5: Regional Returns and International Dependence

Panel A: Full Sample

	Return	World Beta	ρ	Left χ	Left $\bar{\chi}$	Right χ	Right $\bar{\chi}$
East Asia	2.68 (L)	0.416 (L)	0.406 (L)	0.461 (M)	0.671 (L)		0.510 (H)
G5	5.35 (M)	0.739 (H)	0.545 (H)	0.515 (H)	0.750 (M)	0.547	0.499 (M)
Latin	13.24 (H)	0.426 (M)	0.414 (M)	0.441 (L)	0.834 (H)		0.478 (L)

Panel B: 1990-2001

	Return	World Beta	ρ	Left χ	Left $\bar{\chi}$	Right χ	Right $\bar{\chi}$
East Asia	-1.00 (L)	0.358 (L)	0.379 (L)	0.454 (M)	0.652 (L)	0.432	0.517 (H)
G5	6.31 (M)	0.701 (H)	0.487 (H)	0.504 (H)	0.709 (M)		0.443 (L)
Latin	13.15 (H)	0.370 (M)	0.416 (M)	0.443 (L)	0.812 (H)		0.499 (M)

Panel C: 2001-2006

	Return	World Beta	ρ	Left χ	Left $\bar{\chi}$	Right χ	Right $\bar{\chi}$
East Asia	10.19 (M)	0.537 (L)	0.511 (M)	0.467 (M)	0.742 (L)	0.448 (M)	0.568 (M)
G5	3.38 (L)	0.812 (H)	0.637 (H)	0.535 (H)	0.768 (M)	0.556 (H)	0.606 (H)
Latin	13.43 (H)	0.544 (M)	0.423 (L)	0.443 (L)	0.862 (H)	0.412 (L)	0.456 (L)

The table presents average returns and average dependence for different regions. The world beta is computed on filtered returns in similar fashion to equation (13). L, M and H denote the lowest, middle and highest returns or dependence, compared across regions. χ and $\bar{\chi}$ denote the tail dependence parameters defined in Section 3.