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A Dynamic Model of Mesh Size Regulatory Compliance

Wisdom Akpalu

This paper employs a dynamic model for crimes that involve time and punishment to analyze the use of a net with illegal mesh size in a management regime where each community claims territorial use right over a fishery but has a discount rate that may differ from the social discount rate. The equilibrium stock and harvest levels are found to be much lower if the regulation is violated. Moreover, the optimal penalty for violation must be decreasing in the shadow cost of taking the risk to fish illegally, and increasing the risk of punishment increases the equilibrium stock level.

Key words: crime, dynamic model, fishery, regulation

Introduction

The use of illegal fishing technologies has played a major role in the depletion of fish stocks in many developing coastal countries where monitoring and enforcement of fishery regulations are far from complete. An illegal fishing technology generates a technological externality that may include the opportunity cost of larger and more valuable fish in the future. A typical example of such destructive technology, which occurs in all types of fisheries, is the use of fishing nets with very small mesh size. According to the Food and Agriculture Organization (FAO, 2001), the use of illegal nets which are highly destructive is popular in many African countries and is widely practiced along the coasts in lagoons, estuaries, and rivers. Similar practices are prevalent in other continents as well. For example, it has been reported that in a fishery in India, some fishers use stake nets with mesh sizes of less than 5 mm in contiguous rows to filter young prawns, while the prescribed minimum is 35 mm (Srinivasa, 2005). Yeboah (2002) documented that in the coastal countries of West Africa, there is evidence of illegal use of decreasing mesh sizes over the years, from the minimum legal size of 25 mm to about 5 mm.

Following the seminal paper by Becker (1968), considerable theoretical and empirical research has been conducted on violation of fishing regulations.[1] Based on Becker's model, fisheries economists have examined fishing regulations such as closed area and quantity restrictions, for which an illegal fisher is a rational, self-interested economic agent who

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[1] For the theoretical research, see, e.g., Sutinen and Anderson (1985), Anderson and Lee (1986), Charles, Mazany, and Cross (1999), Hatcher (2005), and Chavez and Salgado (2005). Empirical works include Furlong (1991), Hatcher et al. (2000), and Hatcher and Gordon (2005).

maximizes a one-period expected utility. Consequently, the fisher engages in illegal fishing if the expected gain from violation outweighs the gain from legal fishing, and to the extent that the expected marginal gain equates with the expected fine for violating the regulation. However, a fishery crime involving the use of an illegal fishing net is committed repeatedly until it is detected, especially in developing countries where a fisher uses one net. Thus, the rational fisher who uses the illegal net weighs the stream of benefits from fishing with the illegal mesh size against the expected penalty that he will receive if caught. This type of crime involving repeated use of illegal fishing gear and the potential negative impact of the use of such equipment on a fishery makes this a dynamic crime problem that has received little attention in the fisheries economics literature (see Akpalu, 2008).

Like the static models of Hatcher (2005) and Furlong (1991), the dynamic model presents a framework for analyzing the impact of changes in enforcement effort and penalty on the rate of violation. Moreover, employing a dynamic specification that involves time and punishment rather than using a static formulation provides some additional advantages. First, in the dynamic model of crime, the violator weighs the stream of potential net benefits obtainable from fishing illegally. Consequently, if, for example, a community has a territorial use right over a management area, its *effective discount rate* will determine the levels of exploitation and the optimum stock of the resource. The effective discount rate is the sum of the individual rates of time preference or benefit discount rate and the probability of detection (Davis, 1988). It is noteworthy that the probability of detection depends on the choice of mesh size and enforcement effort of inspectors, and a change in either of these two variables will affect both the short-run and optimum harvest and stock levels. Furthermore, since the use of illegal gear affects the intrinsic growth rate of the stock (Boyd, 1966; Escapa and Prellezo, 2003),[2] the dynamic model makes it possible to determine the impact of illegal fishing on the growth of the fish stock. Moreover, if the criminal activity is committed repeatedly, the optimal penalty can be evaluated in terms of the probability of detection and the marginal damage resulting from the illegal activity.

Catch levels of fisheries in many developing countries are not regulated. Typically, a community can harvest any quantity of the resource across the entire management area as long as fishing ordinances such as the mesh size regulation are obeyed, or as a common property resource where each community claims a territorial use right over a management area (Akpalu, 2008). Because the use of illegal fishing gear could impact fish stocks, this important topic merits investigation.

A fine is generally considered costless for the social planner and remains an important policy instrument for fishery management. The objective of this study is to derive and characterize an expression for an optimal fine. Our results show that the equilibrium fish stock and harvest levels are much lower if fishers use the illegal mesh size relative to a situation where they do not use it under the community use right management regime. The differences in the stock levels stem from both the effect of the illegal net use on the growth of the stock and the risk of punishment that scales up an *effective discount rate*. Further, with respect to the optimal fines, we find that the fine must be increasing in net marginal damage but decreasing in the shadow value of the cost of taking the risk of using the illegal net. In addition, increasing the risk of punishment increases the optimum level of stock.

[2] This specification is in contrast to Armstrong and Clark (1997), and Garza-Gil (1998), who assumed that different technologies impact harvest but not the growth function.

The Basic Model

In this section, definitions and assumptions of the model are presented, followed by derivations of results for the two management regimes that characterize fisheries management in developing countries: (*a*) a regulated open-access regime, and (*b*) a situation where a community has use rights over the fishery within a given management area.

Mesh Size and Stock Dynamics

Suppose a fishery has pelagic species such as mackerel, anchovy, sardines, etc., which have relatively short periods of maturity and do not fit well into the standard age-structured or cohort model of Beverton and Holt (1957). Following Boyd (1966) and Armstrong (1999), suppose the intrinsic growth rate of the stock depends on an index of the mesh size (α) whereby the natural growth function depends on fish biomass (x) and the mesh size which is a control variable [i.e., $\Lambda(x, \alpha)$ is the growth function].[3] It is expected that decreasing the mesh size may decrease the average size of fish caught and eventually decrease the number of egg-laying fish. Since decreasing the mesh size increases the mortality of juvenile fish and by-catches, the following partial derivatives apply: $\Lambda_x(x^*, \alpha) > 0$, $\Lambda_{xx}(x^*, \alpha) < 0$, and $\Lambda_\alpha(x, \alpha^*) > 0$, where x^* and α^* are local optimum values. The stock evolution or dynamics within a given management area is denoted by:

$$\text{(1)} \qquad \dot{x} = \Lambda(x, \alpha) - h,$$

where $\dot{x} \equiv dx / dt$, and h represents harvest, which is a control variable.

Harvest Function and Gross Revenue

If constant returns to scale are assumed between fishing capacity (i.e., all the inputs used in fishing) and effort [$E(t)$], the Schaefer harvest or production function of the fisher is written as:

$$\text{(2)} \qquad h = f(x, E, \alpha) = a(\alpha)Ex,$$

where $a(\alpha)$ is the catchability coefficient function and $a_\alpha < 0$, implying that for any given levels of effort and stock, reducing the mesh size will increase harvest. As reported by Mackinson, Sumaila, and Pitcher (1997), changes in a fishing technology affect the catchability coefficient. The simple Schaefer harvest function is assumed for tractability.

Market Structure and Total Revenue from Harvest

As noted above, two management regimes are considered: (*a*) a regulated open-access management regime, and (*b*) a situation where a community has the use right over the fishery within a given management area. In the latter case, although the community has some monopoly power over the use of the fishery resource within the management area, the area is generally comprised of several fishing communities, and one community cannot significantly influence the market price of the harvest. In addition, if the resource is managed as a regulated open access, it is also inevitable that no single community could influence the price

[3] Note that since the growth rate of the fish depends on the stock and index for mesh size, it is possible to solve for the stock as a function of mesh size and time from the growth function [i.e., $x = x(\alpha, t)$].

per unit of harvest. Consequently, the price per kilogram of harvest is assumed to be fixed at q so that the gross revenue obtained by the fisher from fishing is qh.

Cost of Harvest

Let the unit cost of harvest for each fisher be:

$$c(x,\alpha) = \frac{\varsigma}{a(\alpha)x}, \tag{3}$$

where ς is a constant per unit cost of effort. Thus, the illegal fishing net has the advantage of reducing the unit cost of harvest but not the cost per unit effort.

Expected Cost of Illegal Fishing

It is assumed that if the fisher is caught using the illegal net, he pays a fine (F) in addition to surrendering future fishing rents. Following the dynamic deterrence models of Davis (1988), Nash (1991), and Leung (1991, 1994), it is assumed the violator (i) does not know the exact time of detection, but only some probability distribution of the time of detection denoted by $g(t) \equiv dG(t)/dt$, which is the continuous time analogue of the probability in a one-period expected utility model (Becker, 1968). $G(t)$ is the cumulative density function (cdf) which defines the probability that detection would have occurred at time t in the future. The survivor function is therefore $(1 - G)$.

Furthermore, without loss of generality, we assume the violator will no longer be permitted to fish if caught, and will have zero exogenous income for the remainder of the planning horizon. It is the case that illegal nets are seized when detected, and we assume the user will lose his fishing license or will be barred from fishing. This harsh punishment reduces the propensity to recidivate (Smith and Gartin, 1989). Hence, if the fisher is caught, he pays an expected present value of a fine of

$$\int_0^T Fg(t)e^{-\delta t}dt$$

and receives nothing for the remainder of the planning horizon. Note that the future benefits and costs are discounted at a discount rate of δ, and since fishing nets are customarily bequeathed to subsequent generations and mended continuously, the fisher is assumed to have a planning horizon of $T \rightarrow \infty$.

Let the probability (i.e., the hazard rate or the instantaneous conditional probability, which may be subjective) that the offense will be detected within a very small interval of time (t) be $p(\alpha, \tau) = g/(1-G)$, given the offense had not been detected in the past. Here, τ is some exogenous enforcement effort of the management authorities, $p_\alpha < 0$ and $P_{\alpha\alpha} \leq 0$.[4] The assumption that the hazard rate depends on the illegal mesh size stems from the fact that the size composition of a fisher's catch could signal the use of illegal mesh size. Using $g = p(\alpha, \tau)(1 - G)$, the expected present value of the fine can be rewritten as:

$$\int_0^\infty Fp(\alpha, \tau)(1 - G)e^{-\delta t}dt.$$

[4] The specification of the (expected) probability as a function of violation level in a fishery and enforcement effort is consistent with Hatcher (2005).

Anti-Crime Policy Instruments and Property Rights Regimes

Policy makers employ two policy instruments to regulate illegal fishing—either increasing the risk of punishment and/or increasing the severity of punishment. Increasing the risk of punishment also implies increasing the *conditional* probability of detection [i.e., $p(\alpha, \tau)$] by increasing enforcement effort (i.e., τ). With regard to mesh size regulation, the fishery authorities either inspect nets at shore and at sea or inspect minimum landing size of the main species together with the existing mesh. In addition, the policy maker can increase the severity of punishment by increasing the fine or penalty. It is argued that while increasing the penalty is generally costless, increasing enforcement effort is unambiguously costly. As noted by Kuperan and Sutinen (1998), public expenditures on enforcement commonly constitute the largest cost element in regulatory programs. In many developing countries, enforcement efforts are very low due to the high cost of monitoring and surveillance. The high cost is unaffordable to governments, making the use of penalty a more attractive policy instrument (Akpalu, 2008). Moreover, since different property rights regimes in fisheries lead to different levels of capitalization and exploitation of fish stocks, the optimal fine is expected to differ across the various regimes.

The Social Planner's Technological Limit on Mesh Size

To determine the optimal *legal* mesh size, the policy maker maximizes a profit function with respect to catch and mesh size, subject to the stock dynamic equation [equation (1)]. The optimization problem is expressed as:

$$\text{(4)} \qquad \underset{\{\alpha,h\}}{\text{Max}} \int_0^{\infty} \big(q - c(x,\alpha)\big) h e^{-\rho t} dt,$$

subject to

$$\text{(5)} \qquad \dot{x} = \Lambda(x,\alpha) - h,$$

where h is total catch, with $x \geq 0$, $x(0) = x_0$, and $\alpha(0) = \alpha_0$.[5] Note that ρ is the social discount rate, and $\rho \leq \delta$, indicating the community could have a higher rate of time preference than the social planner. As argued by Sandal and Steinshamn (2004), a commercial fisher may often operate with a short-term horizon when making his decision. The corresponding current-value Hamiltonian is denoted by:

$$\text{(6)} \qquad \mathbf{H}(h, x, \alpha, \mu) = \big(q - c(x,\alpha)\big)h + \mu\big(\Lambda(x,\alpha) - h\big).$$

The first-order derivatives of the Hamiltonian with respect to harvest and the mesh size are given by equations (7) and (8), respectively. Equation (9) is the costate expression. In addition, the stock dynamic equation (10) must hold.

$$\text{(7)} \qquad \frac{\partial \mathbf{H}(\cdot)}{\partial h} = q - c(x,\alpha) - \mu = 0,$$

[5] As noted in the text, total catch (h) denotes catch by all fishers in the fishery. Since the social planner aims at setting an optimum total harvest level, h is the social planner's choice variable.

(8) $$\frac{\partial \mathbf{H}(\cdot)}{\partial \alpha} = -c_{\alpha}(x,\alpha)h + \mu\Lambda_{\alpha}(x,\alpha) = 0 \Rightarrow c_{\alpha}(x,\alpha)h = \mu\Lambda_{\alpha}(x,\alpha),$$

(9) $$\dot{\mu} - \rho\mu = -\frac{\partial \mathbf{H}(\cdot)}{\partial x} = c_x(x,\alpha)h - \mu\Lambda_x(x,\alpha) \Rightarrow \dot{\mu} - c_x(x,\alpha)h + \mu\Lambda_x(x,\alpha) = \rho\mu,$$

(10) $$\dot{x} = \Lambda(x,\alpha) - h.$$

In steady state, $\dot{x} = \dot{\mu} = 0$, which implies that from equations (5) and (9) we have $h = \Lambda(x, \alpha)$ and $\mu = -c_x(x, \alpha)h/(\rho - \Lambda_x(x, \alpha))$, respectively. Combining these with equations (7) and (8) yields equations (11) and (12):

(11) $$\left(\Lambda_x(x,\alpha) - \Lambda(x,\alpha)\frac{c_x(x,\alpha)}{(q - c(x,\alpha))}\right) = \rho,$$

(12) $$\frac{c_{\alpha}(x,\alpha)\Lambda(x,\alpha) + c(x,\alpha)\Lambda_{\alpha}(x,\alpha)}{\Lambda_{\alpha}(x,\alpha)} = q.$$

Using the specific functional forms of the growth and catchability coefficient functions, the social planner solves equations (11) and (12) simultaneously to obtain the optimum mesh size (i.e., $\alpha^* = \alpha^L$) and stock level (x^*).

For analytical purposes, suppose the specific functional forms are

$$\Lambda(x, \alpha) = rx(1 + \eta\alpha)(1 - x/k) \text{ and } a(\alpha) = a_0 - \upsilon\alpha,$$

where r is intrinsic growth rate; k is carrying capacity; and a_0, υ, and $\eta > 0$. The specific form of the growth function shows that a smaller mesh size decreases the growth of the stock due to harvesting of juvenile fish. Using parameter values chosen arbitrarily for computational convenience—$r = 0.04$, $\rho = 0.03$, $k = 100$, $\eta = 0.8$, $\varsigma = 5$, $a_0 = 10$, $q = 10$, and $\upsilon = 0.4$—we obtained $\alpha^L = 24$, $x^* = 49$, and $h^* = 20.3$. By implication, if a mesh size is illegal (i.e., α^v), then $\alpha^v < \alpha^L$ (i.e., $\alpha^v < 24$ in this specific example).

Figure 1 illustrates the graphs of equations (10) and (11). Equations (10) and (11) are written as implicit functions of the form $M(\alpha^v, x) = 0$ and $N(\alpha^v, x) = 0$, respectively, for the figure. The point at which the two curves intersect determines the optimum stock and mesh size levels, which are $x^* = 49$ and $\alpha^L = 24$. As can be seen from figure 1, a mesh size larger or smaller than 24 is inefficient since the curves cross only at one point. If the minimum mesh size is set by the fisheries management authority, it is inconceivable that fishers will use mesh sizes larger than the legal limitation.

Conversely, expected utility-maximizing fishers may fish illegally by using nets with mesh sizes smaller than the legal limitation. The marginal damage to the stock from using the smaller mesh size is the value of the marginal impact on the growth of the stock. However, since illegal catches are consumed and the smaller mesh size reduces the marginal cost of harvest, the net marginal damage is the difference between the marginal reduction in the cost of harvest and the marginal damage to the growth of the stock. In a Pigouvian paradigm, the net marginal damage could provide a measure of the penalty for violating the regulation.

Moreover, because the probability of detection depends on enforcement effort (τ), which is costly, the social planner must determine the socially optimal level of enforcement effort. We define $b(\tau)$ as the cost of enforcing the regulation, $p(\alpha^v, \tau)$ is the instantaneous conditional

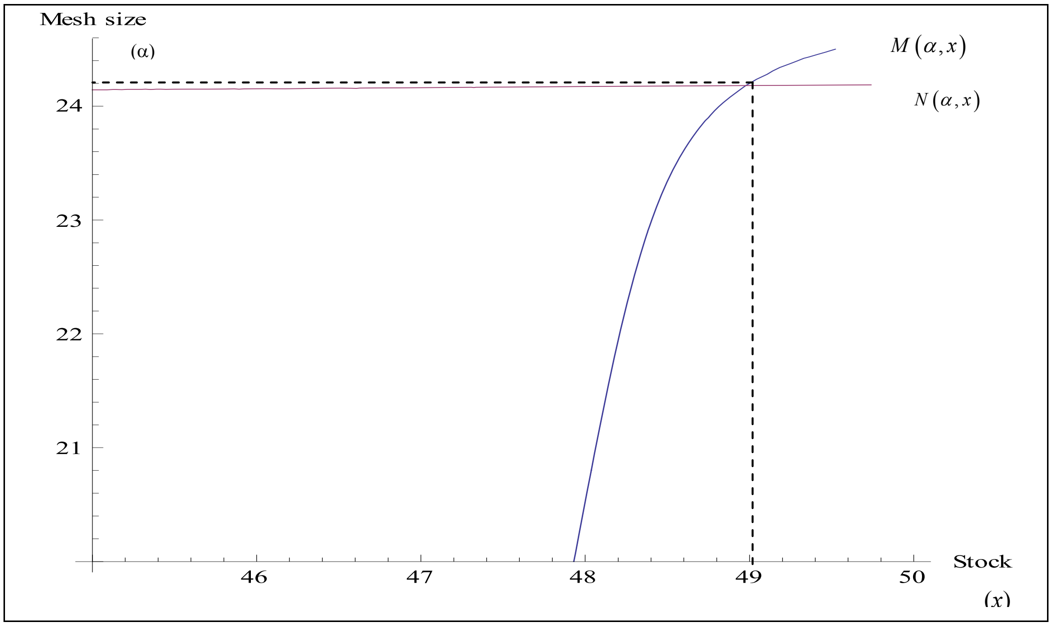

Figure 1. Optimum stock level and mesh size

probability of detection, and $D(\alpha^v)$ represents the social damage due to the use of net with illegal mesh size, where $D_\alpha > 0$. Then $p(\alpha^v, \tau)D(\alpha^v)$ is the expected social damage that is prevented by investing in enforcement. For the purpose of our analysis, we assume the probability distribution of detection is common knowledge between the enforcement officer and the fishers. We assume that the enforcement agency can always target a violating fisher. Since the probability distribution of enforcement is common knowledge and enforcement of violating and non-violating fishers has the same effect, we opt for the simplification that only violators are subject to enforcement effort. We believe that this assumption is validated by conditions in many developing countries. The social planner's optimization problem becomes:

(13) $$\max_{\{\alpha,h,\tau\}} \int_0^\infty \Big\langle \big(q - c(x,\alpha)\big)h - b(\tau) + p(\alpha^v,\tau)D(\alpha^v) \Big\rangle e^{-\rho t} dt,$$

subject to

(14) $$\dot{x} = \Lambda(x,\alpha) - h,$$

(15) $$\dot{G} = p(\cdot)(1 - G).$$

Equation (15) is the equation of motion of the state variable G. The corresponding current-value Hamiltonian is expressed as:

(16) $$\mathbf{H}(\cdot) = \big(q - c(x,\alpha)\big)h + \mu\big(\Lambda(x,\alpha) - h\big) - b(\tau) + p(\alpha^v,\tau)D(\alpha^v) + \vartheta p(\alpha^v,\tau)\big(1 - G\big),$$

where ϑ is the shadow value of the state variable G. In addition to equations (7)–(10), the first-order condition with respect to the enforcement effort is shown by equation (17) and the costate equation with respect to G is represented by equation (18):

$$\frac{\partial \mathbf{H}(\cdot)}{\partial \tau} = -b_\tau + p_\tau D(\alpha^v) + p_\tau \vartheta(1-G) = 0 \Rightarrow b_\tau = p_\tau\left(D(\alpha^v) + \vartheta(1-G)\right), \tag{17}$$

$$\begin{aligned} \frac{d}{dt}\left(\vartheta(1-G)\right) - \rho\vartheta(1-G) &= -\frac{\partial \mathbf{H}(\cdot)}{\partial G} \\ \Rightarrow \left(\dot{\vartheta} - \left(p(\cdot)+\rho\right)\vartheta\right)(1-G) &= -\frac{\partial \mathbf{H}(\cdot)}{\partial G} = \vartheta p(\cdot) \\ \Rightarrow \dot{\vartheta} - \left(p(\cdot)+\rho\right)\vartheta &= \frac{\vartheta p(\cdot)}{(1-G)}. \end{aligned} \tag{18}$$

From the first-order condition [equation (17)], we have $b_\tau = p_\tau(D(\alpha^v) + \vartheta)$, where $b_\tau > 0$. Thus, at equilibrium, the marginal cost of the enforcement (b_τ) must equal an adjusted expected damage resulting from fishing illegally [$p_\tau D(\alpha^v) + \vartheta$]. The enforcement officer could solve for the optimum enforcement effort for any given level of intensity of violation from the preceding equilibrium condition.

Territorial Use Right in a Fishery and Illegal Mesh Size

Consider a situation where each community has the use right over a fishery management area but enforcement of the mesh size regulation is incomplete. In a typical fishing community in a developing country, fishing activities are organized around a chief fisherman or the head of a beach management unit. A recent survey conducted in Ghana on violation of light attraction regulation showed that nearly all fishers within a fishing community violate the regulation if the chief fisherman does not comply with the regulation.[6]

This section therefore models the decision problem of the head of the community or the beach management unit (i.e., the community social planner). Let the community social planner, who is a representative agent, commit to violating the mesh size regulation at time $t = 0$.[7] Let $p(\cdot)$ be the hazard rate of the social planner, and h represent total catch of the community, assuming that net migration of the stock across the community management areas is zero. It is surmised that if the illegal activity is detected, the community is barred completely from fishing. Since we are interested in the impact of the illegal net on the levels of optimum stock and harvest, the analysis is limited to the segment of the value function associated with the illegal activity. Thus, neither of these two considerations will affect the outcome of our analysis and consequently will be ignored. From the time of the commitment, the objective of the planner will be to solve the following problem:

$$\underset{\{\alpha^v, h\}}{\text{Max}} \int_0^\infty \left\{\left(\left(q - c(x,\alpha^v)\right)h - p(\alpha^v,\tau)F\right)(1-G)\right\}e^{-\delta t}dt, \tag{19}$$

subject to

$$\dot{x} = \Lambda(x,\alpha^v) - h, \tag{20}$$

$$\dot{G} = p(\alpha^v,\tau)(1-G), \tag{21}$$

with $x \geq 0$, $x(0) = x_0$.

[6] The project on violation of light attraction regulation in an inshore fishery in Ghana was funded by the Center for Environmental Economics and Policy for Africa (CEEPA) and was undertaken by the author of this manuscript.

[7] Due to the divergence between the short- and long-term impacts of a change in mesh size on catch, fishing communities simply may not be convinced about the need for a mesh size regulation and may consequently violate it.

Thus, the constraints to equation (19) are the hazard rate [equation (21)] and the fish stock evolution [equation (20)]. These dynamics continue until the representative violator is caught. Hence, the value function [equation (19)] is multiplied by the survivor function.

The current-value Hamiltonian associated with equations (19)–(21) is given by equation (22). Following Johnston and Sutinen (1996), the shadow value of the fish stock $\mu(t)$ is multiplied by the survivor function [i.e., $\mu(t)(1-G)$] since the fisher does not benefit from the stock after the illegal activity is detected.

(22)
$$\mathbf{H}(h, x, \alpha, \mu) = \Big(\big(q - c(x, \alpha^v)\big)h - p(\alpha^v, \tau)F + \lambda\, p(\alpha^v, \tau) + \mu\big(\Lambda(x, \alpha^v) - h\big)\Big)(1-G).$$

The first-order conditions for the two control variables, h and α^v, are represented by equations (23) and (24), respectively. The costate equations with respect to the two stock variables, x and G, are equations (25) and (26), respectively. Similarly, the stock evolution equation must be binding; hence, we have equation (27), which is the same as equation (20).

(23)
$$\frac{\partial \mathbf{H}(\cdot)}{\partial h} = q - c(x, \alpha^v) - \mu = 0,$$

(24)
$$\frac{\partial \mathbf{H}(\cdot)}{\partial \alpha^v} = -c_\alpha(x, \alpha^v)h - p_\alpha(\alpha^v, \tau)\big(F - \lambda\big) + \mu\Lambda_\alpha(x, \alpha^v) = 0,$$
$$-c_\alpha(x, \alpha^v)h = p_\alpha(\alpha^v, \tau)\big(F - \lambda\big) + \mu\Lambda_\alpha(x, \alpha^v),$$

(25)
$$\frac{d}{dt}\big(\mu(1-G)\big) - \delta\mu(1-G) = -\frac{\partial \mathbf{H}(\cdot)}{\partial x} = \big(c_x(x, \alpha^v)h - \mu\Lambda_x(x, \alpha^v)\big)(1-G),$$

where
$$\frac{d}{dt}\big(\mu(1-G)\big) - \delta\mu(1-G) = \dot{\mu}(1-G) - \mu g - \delta\mu = (1-G)\big(\dot{\mu} - \big(p(\cdot) + \delta\big)\mu\big).$$

Hence,
$$(1-G)\big(\dot{\mu} - \big(p(\cdot) + \delta\big)\mu\big) = -\frac{\partial \mathbf{H}(\cdot)}{\partial x} = \big(c_x(x, \alpha^v)h - \mu\Lambda_x(x, \alpha^v)\big)(1-G)$$
$$\Rightarrow \dot{\mu} - \big(p(\cdot) + \delta\big)\mu = -\frac{\partial \mathbf{H}(\cdot)}{\partial x}\,\frac{1}{(1-G)} = c_x(x, \alpha^v)h - \mu\Lambda_x(x, \alpha^v)$$
$$\Rightarrow \dot{\mu} - c_x(x, \alpha^v)h + \mu\Lambda_x(x, \alpha^v) = \mu\big(\delta + p(\alpha^v, \tau)\big);$$

(26)
$$\frac{d}{dt}\big(\lambda(1-G)\big) - \delta\lambda(1-G) = -\frac{\partial \mathbf{H}(\cdot)}{\partial G},$$

where
$$\frac{d}{dt}\big(\lambda(1-G)\big) - \delta\lambda(1-G) = \dot{\lambda}(1-G) - \lambda g - \delta\lambda = (1-G)\big(\dot{\lambda} - \big(p(\cdot) + \delta\big)\lambda\big)$$
$$\Rightarrow \dot{\lambda} - \big(\delta + p(\alpha^v, \tau)\big)\lambda = -\frac{\partial \mathbf{H}(\cdot)}{\partial G}\,\frac{1}{1-G} = \mathbf{H}(\cdot)\frac{1}{\big(1-G\big)^2};$$

(27)
$$\dot{x} = \Lambda(x, \alpha^v) - h,$$

where $\lambda(t)$ is the shadow cost of the cumulative density function defining the time of detection, or simply the shadow cost of taking the risk. Equations (23) and (24) define the intertemporal profit-maximizing level of harvest and illegal mesh size, respectively. From (24), the fisher will choose the mesh size that equates the expected net marginal benefit from violation $[-c_\alpha(\cdot)h]$ to the marginal cost $[p_\alpha(\cdot)(F-\lambda) - \mu\Lambda_\alpha(\cdot)]$, which is the difference between an expected adjusted fine $[p_\alpha(\cdot)(F-\lambda)]$ and the shadow value of the growth of the stock $[\mu\Lambda_\alpha(x, \alpha^v)]$. The equation can be respecified as:

$$F(\alpha,\lambda) = \frac{-c_\alpha(\cdot)h + \mu\Lambda_\alpha(\cdot)}{p_\alpha(\cdot)} + \lambda. \tag{28}$$

Following Leung (1991) and Hatcher (2005), the society is assumed to derive benefits from the (illegal) harvest since the illegal catches are consumed. As a result, the fine necessary to internalize the technological externality should reflect the net marginal damage from illegal fishing [i.e., $-c_\alpha(\cdot)h + \mu\Lambda_\alpha(\cdot)$] and the perceived cost of taking the risk of fishing illegally. Thus, the fine can be set to discourage the use of the illegal mesh size.

Because the marginal cost of taking the risk of fishing illegally is not expected to increase exogenously over time, let $\dot{\lambda} = 0$. From equation (26), we have equation (29), which indicates that the fine must be decreasing in the shadow cost of fishing illegally:

$$\lambda = -\frac{\mathbf{H}(\cdot)}{(1-G)^2\left(\delta + p(\alpha^v,\tau)\right)} < 0. \tag{29}$$

Similarly, in intertemporal equilibrium [equation (25)], the marginal benefit of catching a unit of fish illegally now and earning an interest of $\mu(\delta + p(\alpha^v, \tau))$ must equate to the marginal opportunity cost of fishing illegally $[\dot{\mu} - c_x(x, \alpha^v)h + \mu\Lambda_x(x, \alpha^v)]$. In steady state, $\dot{\mu} = 0$ so that

$$\mu = \frac{-c_x(x,\alpha^v)h}{\left(\delta + p(\alpha^v,\tau) - \Lambda_x(x,\alpha^v)\right)}$$

from equation (25).

■ PROPOSITION 1. *If a community that has use right over a fishery fishes with a net of illegal mesh size, the equilibrium stock level will depend on the size of the mesh and an effective discount rate* (*i.e., pure rate of time preference plus probability of detection*). *In addition, the equilibrium stock and harvest are much lower than if the illegal mesh size is not used.*

The proof of proposition 1 is presented in the appendix. The basic idea here is to derive and compare the optimum stock and harvest levels for a situation where the community violates the regulation and the situation where it does not. In steady state, $\dot{x} = 0$, and the expression for the optimum stock derived from the costate equation and the first-order condition is:

$$\left(\Lambda_x\left(x^{**},\alpha^v\right) - \Lambda\left(x^{**},\alpha^v\right)\frac{c_x(x^{**},\alpha^v)}{\left(q - c(x^{**},\alpha^v)\right)}\right) = \delta + p(\alpha^v,\tau), \tag{30}$$

where x^{**} is the equilibrium level of the stock if the fisher violates the regulation, α^v is an index for the inverse of illegal mesh size, and $\delta + p(\alpha^v, \tau)$ is the *effective discount rate*. Note that the left-hand side of equation (30) captures the biological effect of the use of the illegal mesh size and the right-hand side is subjectively determined by the community. The corresponding equilibrium harvest is $h^{**} = \Lambda(x^{**}, \alpha^v)$. It could be inferred from equation (30) that $x^{**} = x(\tau, \alpha^v)$ and verified that $\partial x^{**}/\partial\tau > 0$ and $\partial x^{**}/\partial\alpha^v < 0$. Consequently, an increase in enforcement effort, all other things being equal, will increase the optimum stock while increased intensity of violation will decrease the optimum stock. Moreover, a decrease in the illegal mesh size will result in a decrease in the growth rate of the fish stock and the cost of harvest but will lead to an increase in the effective discount rate. The overall effect of this is a sharp decrease in the optimum level of stock.

Conversely, if the community does not violate the regulation, the corresponding steady-state interior solution for the stock is computed from maximizing

$$V = \int_0^\infty \Big(\big(q - c(x, \alpha^L)\big)h\Big)e^{-\delta t}dt$$

with respect to harvest, subject to equation (14). Thus,

$$\text{(31)} \qquad \left(\Lambda_x\left(x^*, \alpha^L\right) - \Lambda\left(x^*, \alpha^L\right)\frac{c_x(x^*, \alpha^L)}{\left(q - c(x^*, \alpha^L)\right)}\right) = \delta\,.$$

By comparing equations (30) and (31), if the two *effective* discount rates are different, it can be shown that even if the community does not use the illegal net, the equilibrium level of the stock will be lower than the optimum stock level desired by the management authority and will be much lower if the community violates the regulation (see appendix for the proof). The implication, as noted earlier, is that increased exploitation of the resource leads to a high reduction in the equilibrium stock. Moreover, since the fisher commits to violating the regulation continuously, a higher instantaneous conditional probability of detection [i.e., $p(\alpha^v, \tau)$] increases the effective discount rate and decreases the equilibrium stock level.

Stock Migration, Unrestricted Catch, and Illegal Mesh Size

Due to the high and uneven migration of pelagic fish stocks across territories, it is difficult to accurately define property rights over territorial stocks. As a result, fishing communities in developing countries usually do not set harvest quotas, but instead they regulate the use of fishing gears such as the mesh size of nets. However, if there is incomplete information about the rate and pattern of migration of the species, the optimum mesh size set by the social planner could be inefficient and may engender the use of smaller (illegal) mesh size by the community. Assume that the community has control over the stock within its territory but, due to the possibility of migration of the stock across boundaries, the fishery is managed by the community as a regulated open access. We define the fish stock dynamic equation as:

$$\text{(32)} \qquad \dot{x} = \Lambda(x, \alpha^v) - \varepsilon\big(n\big)x - h,$$

where $\varepsilon(n)$ [with $\varepsilon_n > 0$ and $\varepsilon(0) = 0$] is the community's private knowledge about net migration of the fish stock to other territories of which the social planner is ignorant, and n is the number of other communities that fish the stock at the extensive margin. The community leader is the local social planner, and his actions are copied by other members of the community. In addition, if the illegal activity is detected, the fisher pays a fine (F) and is barred from fishing. Assume the fishers in the community violate the regulation continuously until the illegal activity is detected. The optimization problem of the planner is specified as:

$$\underset{\{\alpha^v, h\}}{\text{Max}} \int_0^\infty \Big(qh - c(x, \alpha^v)h - p(\alpha^v, \tau)F \Big)(1-G)e^{-\delta t}, \tag{33}$$

subject to equations (21) and (32).

The corresponding current-value Hamiltonian is given by equation (34), and the first-order conditions with respect to harvest and the mesh size are designated by equations (35) and (36), respectively. The costate equations of the two stock variables are equations (37) and (38).

$$\begin{aligned}&\mathbf{Z}(h, x, \alpha, \omega) = \\ &\Big(\big(q - c(x, \alpha^v)\big)h - p(\alpha^v, \tau)F + \psi\, p(\alpha^v, \tau) + \omega\big(\Lambda(x, \alpha^v) - \varepsilon(n)x - h\big)\Big)(1-G),\end{aligned} \tag{34}$$

$$\frac{\partial \mathbf{Z}(\cdot)}{\partial h} = q - c(x, \alpha^v) - \omega = 0, \tag{35}$$

$$\frac{\partial \mathbf{Z}(\cdot)}{\partial \alpha} = -c_\alpha(\cdot)h - p_\alpha(\cdot)F + \omega\Lambda_\alpha(\cdot) + \psi\, p_\alpha(\cdot) = 0, \tag{36}$$

$$\frac{d}{dt}\big(\omega(1-G)\big) - \delta\omega(1-G) = -\frac{\partial \mathbf{Z}(\cdot)}{\partial x}, \tag{37}$$

where

$$\frac{d}{dt}\big(\omega(1-G)\big) - \delta\omega(1-G) = \dot{\omega}(1-G) - \omega g - \delta\omega = (1-G)\big(\dot{\omega} - \big(p(\cdot) + \delta\big)\omega\big),$$

$$\dot{\omega} - \big(\delta + p(\alpha^v, \tau)\big)\omega = -\frac{\partial \mathbf{Z}(\cdot)}{\partial x}\frac{1}{1-G} = c_x(x, \alpha^v)h - \omega\big(\Lambda_x(x, \alpha^v) - \varepsilon(n)\big);$$

$$\dot{\psi} - \big(\delta + p(\alpha^v, \tau)\big)\psi = -\frac{\partial \mathbf{Z}(\cdot)}{\partial G}\frac{1}{1-G} = \mathbf{Z}(\cdot)\frac{1}{(1-G)^2}, \tag{38}$$

$$\dot{x} = \Lambda(x, \alpha^v) - \varepsilon(n)x - h, \tag{39}$$

where ω is the marginal valuation of the stock to the community. In steady state, we have $\dot{x} = \dot{\omega}$, and the costate equation of the fish stock can be rewritten as:

$$\omega = -\frac{c_x(x, \alpha^v)h}{\big(\delta + p(\alpha^v, \tau) + \varepsilon(n) - \Lambda_x(x, \alpha^v)\big)}.$$

It is noteworthy that if $n \to \infty$, $\omega \to 0$ and $q = c(x, \alpha^v)$ from equation (35), which is the open-access equilibrium condition. This is because the competition for the stock by several users leaves the resource with no shadow value. Indeed, if there are many users, it becomes prohibitively expensive for individuals to exclude others and conserve the asset for future use (Gordon, 1954; Lueck, 1998; Edwards, Link, and Rountree, 2004).

From equation (36), the fine necessary to internalize the technological externality can be defined as:

(40)
$$F_0(\alpha, \psi) = \frac{-c_\alpha(\cdot)h + \omega\Lambda_\alpha(\cdot)}{p_\alpha} + \psi\,.$$

Similarly, if we assume that $\dot{\psi} = 0$, then from equation (38) we have:

(41)
$$\psi = -\frac{\mathbf{Z}(\cdot)}{(1-G)^2\left(\delta + p(\alpha^v, \tau)\right)} < 0.$$

As noted earlier, the fine must be decreasing in the shadow cost of taking the risk. In addition, assuming an interior solution exists [i.e., $q - c(x, \alpha^v) = \omega$] and combining this with equation (39) in steady state and using the steady-state expression for ω, we obtain:

(42)
$$\left(\Lambda_x(x_0^*, \alpha^v) - \varepsilon(n)\right) - \left(\Lambda(x_0^*, \alpha^v) - \varepsilon(n)x_0^*\right)\frac{c_x(x_0^*, \alpha^v)}{\left(q - c(x_0^*, \alpha^v)\right)} = \delta + p(\alpha^v, \tau) \Rightarrow$$

$$\left(\left(\Lambda_x(x_0^*, \alpha^v)\right) - \left(\Lambda(x_0^*, \alpha^v)\right)\frac{c_x(x_0^*, \alpha^v)}{\left(q - c(x_0^*, \alpha^v)\right)}\right)$$
$$+\,\varepsilon(n)\left(\frac{x_0^* c_x(x_0^*, \alpha^v)}{\left(q - c(x_0^*, \alpha^v)\right)} - 1\right) = \delta + p(\alpha^v, \tau)\,.$$

The components of the first term on the left-hand side of equation (42) are the same as those of equation (30). The second term on the left-hand side of equation (42) captures the stock effect of migration, which depends on the number of other communities harvesting the migrated stock. The term on the right-hand side, i.e., the effective discount rate, is determined by the community. The corresponding equilibrium harvest is $h_0^* = \Lambda(x_0^*, \alpha^v) - \varepsilon(n)x_0^*$. Comparing equations (30) and (42), we obtain proposition 2.

■ PROPOSITION 2. *If there is net migration of a fish stock to other territories and fishers use illegal mesh size at the source of migration, the equilibrium level of stock will be lower than for a situation where the fish do not migrate but the illegal mesh size is used.*

This proposition stems from the fact that the migration of the stock across the boundaries reduces the incentive to preserve the stock at the source, thereby reducing the marginal valuation of the stock by the community. Consequently, fishing effort may intensify, leading to a reduction in the equilibrium levels of catch and stock. The sketch of the proof is presented in the appendix. Note that if the social planner does not account for stock migration, his optimum stock and mesh size is inefficient.

By assuming the community has a discount rate that is higher than the social discount rate (e.g., 10% and 3%, respectively), graphs of equations (11), (30), and (42) are presented in figure 2. All other parameter values chosen arbitrarily for the illustration are the same as those used for figure 1, except the value of epsilon which is set as 0.01. From figure 2, $W^{SP}(\alpha, x^*)$, $W^{TUR}(\alpha, x^{**})$, and $W^{NM}(\alpha, x_0^*)$ denote graphs of equations (11), (30), and (42), respectively. The point at which each graph crosses the horizontal line [i.e., stock (x)] indicates the equilibrium level of stock corresponding to that equation. Clearly, the equilibrium stock level desired by the social planner is higher than that of the community, at the optimum mesh size. In our

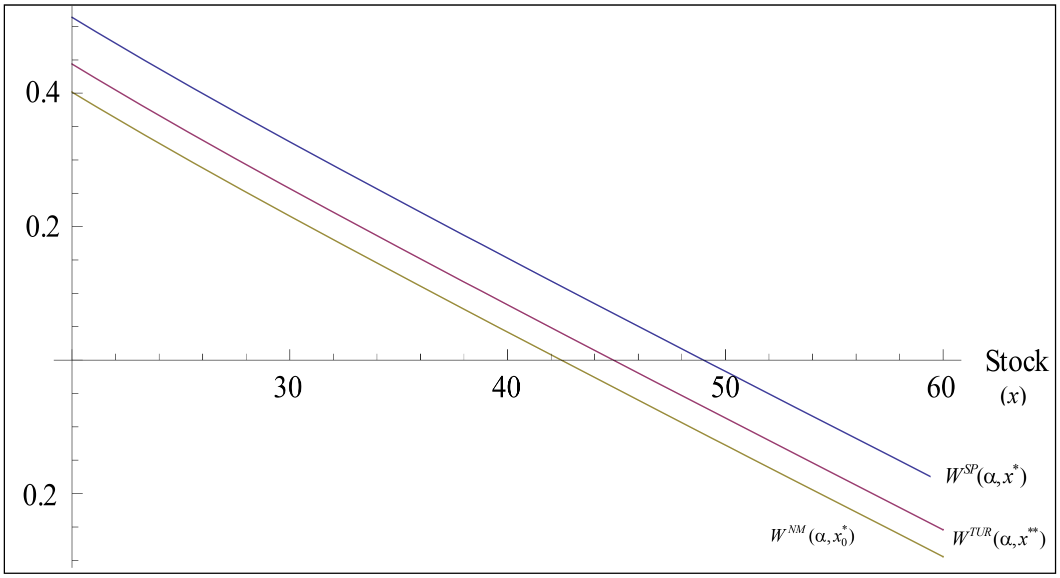

Figure 2. Impact of discount rate and migration on equilibrium stock

specific example, the stock levels for the social planner and the community with territorial use right are 49 and 45, respectively. Furthermore, by accounting for stock migration away from the management area, the equilibrium stock level of the community decreases even further to 42. An increase in the effective discount rate as a result of violating the regulation will increase the divergence between the equilibrium stock levels of the community and the social planner.

Conclusion

This paper extends the one-period or static expected utility model to a dynamic one to accommodate a chronic fishery crime problem in many (developing) countries. It incorporates time and punishment to analyze the effect of the use of fishing nets of illegal mesh size on fish stocks and harvest under a regime where each community claims a territorial use right over the resource in a management area but has a rate of time preference that may exceed the social discount rate. Two situations were considered: a situation where the stock migrates across the boundary to other territories, and a situation where no migration occurs. The optimum fine necessary to discourage the illegal activity was derived and characterized.

It has been shown that if the community has territorial use right but fishes with a net of illegal mesh size, the optimum stock and harvest will be much lower than what will prevail if there is no illegal fishing, irrespective of whether the stock migrates or not. Specifically, in addition to the fact that the illegal mesh size decreases the cost of harvest and intrinsic growth rate of the stock, it increases the effective discount rate. Note that a higher discount rate leads to increased harvest and stock levels. The effective discount rate is the sum of the benefit discount rate and the conditional probability of detection. Furthermore, if the stock migrates from a territory, the equilibrium stock will be lower than under a situation where it does not migrate if the fishers use the illegal net. In addition, we find that the fine must be set equal to some adjusted net marginal damage resulting from the use of the illegal net minus the shadow cost of taking the risk to fish illegally.

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Appendix: Proofs of Propositions 1 and 2

Proof of Proposition 1

The proof of proposition 1 requires deriving and comparing the harvest levels under the two situations. Let α^v and α^L denote indexes for the illegal and legal mesh sizes, respectively. In steady state, we have $\dot{x} = 0$ and $\dot{\mu} = 0$. The expression for the optimum stock which is derived from combining the costate equation and the first-order conditions is given by text equation (30) (i.e., A1):

$$\text{(A1)} \qquad \left(\Lambda_x\left(x^{**}, \alpha^v\right) - \Lambda\left(x^{**}, \alpha^v\right) \frac{c_x(x^{**}, \alpha^v)}{\left(q - c(x^{**}, \alpha^v)\right)} \right) = \delta + p(\alpha^v, \tau),$$

where x^{**} is the equilibrium level of the stock if the fisher violates the regulation. The corresponding equilibrium harvest is $h^{**} = \Lambda(x^{**}, \alpha^v)$. On the other hand, if the fisher does not violate the regulation, the equilibrium stock is computed by maximizing

$$\int_0^\infty \left(\left(q - c(x, \alpha^L)\right)h\right)e^{-\delta t}dt$$

with respect to h, subject to text equation (14). The solution gives text equation (31) (i.e., A2):

$$\text{(A2)} \qquad \left(\Lambda_x\left(x^{*}, \alpha^L\right) - \Lambda\left(x^{*}, \alpha^L\right) \frac{c_x(x^{*}, \alpha^L)}{\left(q - c(x^{*}, \alpha^L)\right)} \right) = \delta.$$

The corresponding steady-state harvest is $h^* = \Lambda(x^*, \alpha^L)$. From equation (A1), we know that $c_x < 0$. In addition, without violation, the growth of the stock is higher than what prevails in the presence of violation for any biomass level, and $\Lambda_x(x^*, \alpha^L) > \Lambda_x(x^{**}, \alpha^v)$. Furthermore, the last term in the bracket of equation (A1) is less than that of equation (A2), i.e.,

$$\frac{c_x(x^{**}, \alpha^v)}{q - c(x^{**}, \alpha^v)} < \frac{c_x(x^{*}, \alpha^L)}{q - c(x^{*}, \alpha^L)}.$$

This is because

$$\frac{c_x(x^{**}, \alpha^v)}{c(x^{**}, \alpha^v)} = \frac{c_x(x^{*}, \alpha^L)}{c(x^{*}, \alpha^L)} \quad \text{and} \quad q - c(x^{**}, \alpha^v) > 0.$$

Hence, $x^{**} << x^*$. □

Proof of Proposition 2

Citing text equation (30), we have:

(A3) $$\left(\Lambda_x\left(x^{**},\alpha^v\right)-\Lambda\left(x^{**},\alpha^v\right)\frac{c_x(x^{**},\alpha^v)}{\left(q-c(x^{**},\alpha^v)\right)}\right)=\delta+p(\alpha^v,\tau).$$

Also, from text equation (42), we have:

(A4) $$\left(\Lambda_x(x_0^*,\alpha^v)-\varepsilon(n)\right)-\left(\Lambda(x_0^*,\alpha^v)-\varepsilon(n)x_0^*\right)\frac{c_x(x_0^*,\alpha^v)}{\left(q-c(x_0^*,\alpha^v)\right)}=\delta+p(\alpha^v,\tau).$$

Equation (A4) can be rewritten as:

(A5) $$\left(\left(\Lambda_x(x_0^*,\alpha^v)\right)-\left(\Lambda(x_0^*,\alpha^v)\right)\frac{c_x(x_0^*,\alpha^v)}{\left(q-c(x_0^*,\alpha^v)\right)}\right)=\delta+p(\alpha^v,\tau)+\varepsilon(n)\left(1-x_0^*\frac{c_x(x_0^*,\alpha^v)}{\left(q-c(x_0^*,\alpha^v)\right)}\right).$$

The left-hand sides of equations (A3) and (A5) have the same terms, but we know that

$$\varepsilon(n)\left(1-x_0^*\frac{c_x(x_0^*,\alpha^v)}{\left(q-c(x_0^*,\alpha^v)\right)}\right)>0.$$

Hence, $x_0^*<x^{**}$. □