

Volume 30, Issue 2

Testing for an irrelevant regressor in a simple cointegration analysis

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Abstract

This paper investigates the asymptotic behavior of the t-ratio associated to an irrelevant variable in a three-variable cointegration analysis. It is proved that the t-ratio converges to a non-standard distribution suitable for statistical inference. Although the test-statistic is not pivotal when the innovations are serially correlated, Monte Carlo evidence suggests that the size distortion can be considerably mitigated by means of HAC standard errors.

I would like to thank the editor and the anonymous referee for very helpful advice. The usual disclaimer applies.

Citation: Daniel Ventosa-santaulària, (2010) "Testing for an irrelevant regressor in a simple cointegration analysis", *Economics Bulletin*, Vol. 30 no.2 pp. 1333-1345.

1. Introduction

The seminal concept of cointegration, first proposed by Granger (1981), Granger and Weiss (1983) and Engle and Granger (1987), lead to an impressive development of time-series econometrics. Following Stock and Watson (1988), a cointegrated relationship can be understood as the commonality of a stochastic trend amongst the variables. It is relatively well-known that standard inference by means of t-ratios and \mathcal{F} statistics cannot be drawn from a least squares-estimated (LS hereinafter) cointegrated relationship.¹ There are, however, several alternatives to deal with this issue and Fully-Modified LS (Phillips and Hansen 1990, Phillips 1995) is certainly one of the more important proposals.² This methodology allows for statistical inference throughout t-ratios and \mathcal{F} statistics in cointegrated relationships. To the best of our knowledge, little has been done in the study of underspecified cointegrated relationships. Moreover, the evidence is limited mainly to Monte Carlo experiments (Banerjee, Dolado, Hendry, and Smith 1986, Andrade, O'Brien, and Podivinsky 1994, Boswijk and Franses 1992, Cheung and Lai 1993, Podivinsky 1998). A relevant exception can be found in Pashourtidou (2003), who studies the-asymptotic-consequences of omitting a relevant variable in a Johansen cointegration test; the latter will "lead to either no detection of cointegrating relationships, if the true cointegrating rank is smaller than or equal to the number of omitted variables [...] or the detection of q < r cointegrating relationships [r is the cointegrating rank], if the true cointegrating rank is greater than the number of omitted variables."³

In this paper we prove that statistical inference about the significance of the LS estimates associated to irrelevant variables in a cointegrated regression can be drawn by means of the associated t-ratios, provided that the innovations are not autocorrelated. When the innovations happen to be serially correlated, statistical inference cannot be drawn since the test statistic is not pivotal anymore; however, this can be corrected—at least partially—simply by using Heteroskedasticity Autocorrelation Consistent (HAC) standard errors.

The paper is organized in the following way: section 2 introduces the data generating processes (DGPs) we work with as well as the specifications of the regressions

¹See, for example, Enders (2004), pp. 378-380.

²Fully-Modified LS may be succinctly described as the inclusion of lags and leads of the firstdifferenced explanatory variables as regressors. It was developed in the context of a cointegration estimation where the regressor is endogenous.

³Pashourtidou and O'Brien (2003) also studied the effect on the Johansen test when irrelevant variables are included; they conclude that inference about the cointegration rank is not affected.

under study. Section 3 develops the asymptotics for different settings: (i) A threevariable cointegrated relationship and an underspecified model that excludes a relevant variable; (ii) A three-variable cointegrated relationship and a correctly specified model that includes all the relevant variables, and; (iii) A two-variable cointegrated relationship and an overspecified model that includes an irrelevant variable. Monte Carlo evidence that supports our asymptotic results is discussed. Section 4 concludes.

2. Data Generating Processes and Specifications

We begin our study by specifying the data generating processes of the variables. Let x_t and z_t be generated as driftless unit roots:

$$x_t = X_0 + \xi_{xt} \tag{1}$$

$$z_t = Z_0 + \xi_{zt} \tag{2}$$

where X_0 and Z_0 are initial conditions, $\xi_{wt} = \sum_{i=1}^{t} u_{wi}$, and u_{wt} (for w = x, y, z) is an *iid* white noise. We then define the cointegrated relationship in eq. (3).

$$y_t = \mu_y + \beta_y x_t + \delta_y z_t + u_{yt} \tag{3}$$

On the one side, when $\delta_y \neq 0$, z_t is therefore a relevant variable in the cointegrated relationship. On the other side, when z_t is excluded (this is, when $\delta_y = 0$), this variable is irrelevant. Note that the variables x_t and y_t are always cointegrated. We further assume that the practitioner may include z_t or not in his estimation exercise, as marked in eqs. (5) and (4), respectively.

$$y_t = \alpha + \beta x_t + u_t \tag{4}$$

$$y_t = \alpha + \beta x_t + \gamma z_t + u_t \tag{5}$$

3. Asymptotics and Monte Carlo Evidence

In this section we prove that statistical inference on the estimated parameter associated to an irrelevant variable in a three-variable cointegration analysis can be drawn by means of its associated t-ratio. This is done by studying the asymptotics of a LS regression with two and/or three variables (see eqs. (4) and (5)), where x_t , y_t and z_t are generated by eqs. (1), (2) and (3), respectively (in the last DGP, the parameter δ_y may be equal to zero or not). It is straightforward to show that omitting z_t when the variable is in fact a relevant one, biases the estimates of the—incomplete cointegrating vector: **Proposition 1** Let x_t , z_t and y_t be generated by eqs. (1), (2), and (3) with $\delta_y \neq 0$, respectively, and use them to estimate specification (4). Denote $\hat{\alpha}$ and $\hat{\beta}$ the LS estimates of α and β and $t_{\hat{\alpha}}$, and $t_{\hat{\beta}}$ their associated t-ratios. Then, as $T \to \infty$:

•
$$T^{-\frac{1}{2}}\hat{\alpha} \xrightarrow{d} \delta_y \frac{\sigma_z \left[\int \omega_z \int \omega_x^2 - \int \omega_x \int \omega_x \omega_z\right]}{\int \omega_x^2 - \left(\int \omega_x\right)^2}; T^{-\frac{1}{2}} t_{\hat{\alpha}} = O_p(1)$$

• $\hat{\beta} - \beta_y \xrightarrow{d} \delta_y \frac{\sigma_z \left[\int \omega_z \int \omega_x - \int \omega_x \omega_z\right]}{\sigma_x \left[\int \omega_x^2 - \left(\int \omega_x\right)^2\right]}; T^{-\frac{1}{2}} t_{\hat{\beta}} = O_p(1)$

where $\stackrel{d}{\rightarrow}$ denotes convergence in distribution; ω_w , for w = x, z, accounts for a standard brownian motion, $\omega_w(r)$, and, $O_p(\cdot)$ refers to the order in convergence.

Proof: see appendix A.

The results in Proposition 1 are rather intuitive. When the model is underspecified (that is, when we omit a relevant variable), the cointegrating vector is poorly estimated; both estimates do not converge to their true value. This is in line with Pashourtidou's (2003) asymptotic results as well as with the Monte Carlo evidence obtained by Podivinsky (1998), who considers that:⁴ "(...) all the tests [Johansen, **Dickey-Fuller and Durbin-Watson**] can be misleading when the estimated model is underspecified because too few relevant variables are included in the analysis." Podivinsky (as well as Pashourtidou) was referring, however, to the obtention of evidence of cointegration. Yet, by adding Podivinsky's (1998) Monte Carlo evidence to our results, it could be said that, when the model is underspecified, not only the parameter estimates-associated to the cointegrated variables included in the specification–will not converge to their true values, but also there is a serious risk of finding no evidence at all of cointegration. Notwithstanding this, the ratios associated to the relevant (cointegrated) variables still diverge. If we were to use standard critical values to test the null hypothesis of no significance, we would eventually reject it for a sample size large enough.

On the other side, when we correctly specify the cointegrated regression, correct inference can be drawn:

Proposition 2 Let x_t , z_t and y_t be generated by eqs. (1), (2), and (3) with $\delta_y \neq 0$, respectively, and use them to estimate specification (5). Denote $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ the LS estimates of α , β and γ , and $t_{\hat{\alpha}}$, $t_{\hat{\beta}}$, and $t_{\hat{\gamma}}$ their associated t-ratios. Then, as $T \rightarrow \infty$:

⁴See Podivinsky (1998, p. 8).

- $\hat{\alpha} \mu_y \xrightarrow{p} 0; T^{-\frac{1}{2}} t_{\hat{\alpha}} = O_p(1)$
- $\hat{\beta} \beta_y \xrightarrow{p} 0$; $T^{-1}t_{\hat{\beta}} = O_p(1)$
- $\hat{\gamma} \delta_y \xrightarrow{p} 0$; $T^{-1}t_{\hat{\gamma}} = O_p(1)$

where \xrightarrow{p} denotes convergence in probability.

Proof: see appendix A.

Propositions 1 and 2 point to the fact that omitting a relevant variable in the cointegration equation flaws the statistical inference.

Furthermore, we prove that the inclusion of an irrelevant variable does not entail severe consequences (at least not when compared with the consequences in the previous scenario of an omitted relevant variable). This is in accordance to the guidelines of the General-to-Specific specification-design strategy. Omitting relevant variables is costlier that including irrelevant ones. Recall that Pashourtidou and O'Brien (2003) proved that overspecification does not affect the detection of the cointegrating rank. The following proposition further proves that the cointegrating parameters converge to their true value and the estimate of the parameter associated to the irrelevant variable collapses at rate T^{-1} :

Proposition 3 Let x_t , z_t and y_t be generated by eqs. (1), (2), and (3) with $\delta_y = 0$, respectively, and use them to estimate specification (5). Denote $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ the LS estimates of α , β and γ , and $t_{\hat{\alpha}}$, $t_{\hat{\beta}}$, and $t_{\hat{\gamma}}$ their associated t-ratios. Then, as $T \to \infty$:

- $\hat{\alpha} \mu_y \xrightarrow{p} 0; T^{-\frac{1}{2}} t_{\hat{\alpha}} = O_p(1)$
- $\hat{\beta} \beta_y \xrightarrow{p} 0; T^{-1}t_{\hat{\beta}} = O_p(1)$
- $T\hat{\gamma} \xrightarrow{d} \frac{\sigma_y \Gamma_n}{\sigma_z D}$

•
$$t_{\hat{\gamma}} \xrightarrow{d} \frac{\Gamma_n}{\sqrt{\left[\left(\int \omega_x\right)^2 - \int \omega_x^2\right]D}}$$

where:

1.
$$\Gamma_n = \int \omega_z d\omega_y \left[\left(\int \omega_x \right)^2 - \int \omega_x^2 \right] + \int \omega_x d\omega_y \left[\int \omega_x \int \omega_z - \int \omega_x \omega_z \right]$$

2. $D = \int \omega_z^2 \left[\left(\int \omega_x \right)^2 - \int \omega_x^2 \right] + \left(\int \omega_z \right)^2 \int \omega_x^2 - 2 \int \omega_z \int \omega_x \int \omega_x \omega_z + \left(\int \omega_x \omega_z \right)^2 \right]$

Proof: see appendix A.

The results of Proposition 3, together with those of Propositions 1 and 2, reinforce the general conclusion of Podivinsky (1998, p.7): "Underspecifying the possible number of variables in the cointegrating vector(s) seems to be a risky strategy, as it is more likely that if there are really more variables in the CI [Cointegrating] vector(s), these vectors may not all be detected, or no vectors detected at all. Alternatively, possible overspecification of the number of relevant variables generally is likely to result in detection of (at least) the true number of CI vectors." Our findings complement those of Podivinsky and suggest that the inclusion of such irrelevant variables may be tested by means of their associated t-ratios: the main result in Proposition 3 lies in the asymptotic expression for the t-ratio associated to $\hat{\gamma}$. The t-ratio, $t_{\hat{\gamma}}$, converges naturally to a nonstandard, nuisance-parameter-free distribution under the assumption that the innovations are stationary *iid* processes. This allows us to regard it as a potentially useful test statistic to test the null hypothesis \mathcal{H}_0 : $\delta_y = 0$. The asymptotic distribution has been non-parametrically estimated (see figure 1). Note that these results were obtained under the assumption of *iid* innovations in the DGPs. Figure 1 also depicts the estimated asymptotic distribution when the innovations, u_{xt} and u_{zt} , follow an AR(1) process.⁵ The presence of autocorrelation in the regressors' innovations does not seem to entail severe distortions of the asymptotic distribution of the t-ratio under the null hypothesis.

Critical values—assuming *iid* innovations in the DGP of y_t —under the null hypothesis have been therefore computed using the asymptotic expression of the t-ratio (Number of Replications: R = 100,000, see Table I).

	, 1	1	11
Lev	el Critical Value	Level	Critical Value
0.0	1 ± 2.57	0.20	± 1.28
0.0	2 ± 2.33	0.40	± 0.84
0.0	5 ± 1.96	0.60	± 0.52
0.1	0 ± 1.64	0.80	± 0.25

Table I: Asymptotic critical values for $t_{\hat{\gamma}}$ under the null hypothesis

The finite-sample evidence shows that autocorrelated innovations in the regressors' DGPs do not distort the level of the test. There is, however, a considerable size distortion when the innovations of y_t are not *iid*. Table (II) in appendix B shows the rejection rates of the t-ratio under the null hypothesis (panel a), as well as under the

⁵More precisely, $u_{wt} = \phi_w u_{w,t-1} + \epsilon_{wt}$ for $w = x, z, |\phi_w| < 1$ and $\epsilon_{wt} \sim \mathcal{N}(0, \sigma_{\epsilon, w}^2)$.

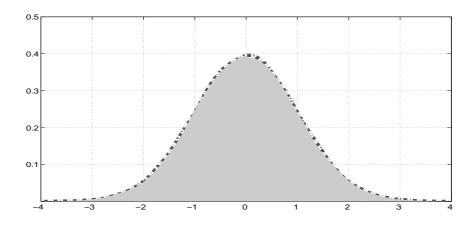


Figure 1: Asymptotic distribution under the null hypothesis: $\delta_y = 0$. Case 1 (area): innovations of x_t , y_t and z_t are $iid\mathcal{N}(0,1)$; Case 2 (grey line): innovations of x_t and z_t are generated as AR(1) processes, $\phi_x = 0.7$, $\phi_z = 0.5$; innovations of y_t are $iid\mathcal{N}(0,1)$. Number of replications: 10,000. Sample size: 100.

alternative hypothesis (panel b). Nevertheless, a second Monte Carlo experiment reveals that the autocorrelation problem can be considerably mitigated if we use a standard and well-known procedure; that is, if we compute HAC standard errors (Newey and West 1987).

This procedure is analogous to the one a practitioner would employ in a classical LS regression where the variables are stationary and there is evidence of autocorrelation. In fact, similar size distortions using stationary variables have been documented by Granger, Hyung, and Jeon (2001), both, theoretically and through Monte Carlo experiments.

4. Concluding Remarks

This paper proves that statistical inference in a three-variable cointegrating model can be drawn by means of the t-ratios of the estimates. We obtained the asymptotic distribution of the t-ratio associated to an irrelevant variable as well as the order in convergence of a relevant one. Critical values were tabulated in the former case. Curiously enough, Monte Carlo evidence suggests that the problem of autocorrelation in the specification can be controlled by employing HAC standard errors, a rather classical tool. The results presented here could eventually ease the making of statistical inference in cointegrated relationships. It is known that underspecification may lead to the failure in the detection of the cointegrating vector(s) whilst overspecification should help in the detection of such cointegrating vectors; our results suggest that the inclusion of an irrelevant variable in such a cointegrated system can be tested by means of its t-ratio.

Appendix A: Proof of Propositions 1, 2 and 3

We present a guide on how to obtain the order in probability of Proposition 2. The asymptotics of the remaining Propositions can be obtained by following these steps. Let x_t , z_t and y_t be generated by eqs. (1), (2), and (3) with $\delta_y \neq 0$, respectively, and use them to estimate specification (5). The expressions needed to compute the asymptotic value of $t_{\hat{\gamma}}$ statistic are:⁶

$$T^{-\frac{3}{2}} \sum w_t \stackrel{d}{\rightarrow} \sigma_w \int_0^1 \omega_w dr$$

$$T^{-2} \sum w_t^2 \stackrel{d}{\rightarrow} \sigma_w^2 \int_0^1 [\omega_w]^2 dr$$

$$T^{-2} \sum_{t=1}^T x_t z_t \stackrel{d}{\rightarrow} \sigma_x \sigma_z \int_0^1 \omega_x \omega_z dr$$

$$\sum y_t = \mu_y T + \beta_y \sum x_t + \delta_y \sum x_t + O_p \left(T^{\frac{1}{2}}\right)$$

$$\sum x_t y_t = \mu_y \sum x_t + \beta_y \sum x_t^2 + \delta_y \sum x_t z_t + \sum \xi_{x,t-1} u_{yt} + O_p \left(T^{\frac{1}{2}}\right)$$

⁶All sums are from t = 1 to T unless otherwise specified. w = x, z.

and,

$$\sum y_t^2 = \mu_y \sum y_t + \beta_y \sum x_t y_t + \delta_y \sum z_t y_t + \sum \xi_{x,t-1} u_{yt} + \sum \xi_{z,t-1} u_{yt}$$

As for the stochastic sums, most results can be found in Phillips (1986), Durlauf and Phillips (1988) and Phillips and Ouliaris (1990).

The previous elements allow for the programming of all those sums required to study the asymptotic behavior of the regression. These—*Mathematica*TM—programs can be downloaded from:

 $\label{eq:loss} \begin{array}{l} \mbox{http://dl.dropbox.com/u/1307356/Arxius&20en&20la&20web/Appendix_EB/OmVar.pdf} \\ \mbox{Lower-order terms } (O_p\left(T^{1/2}\right), \mbox{ for instance}) \mbox{ have been excluded in the expressions} \\ \mbox{ because their inclusion blocks the execution of the code.} \end{array}$

			el (a)					
Hypothesis		Parameters		Sample Size				
	δ_y	ρ_x, ρ_z	$ ho_y$	50	100	200	300	500
0			0.00	0.06	0.05	0.05	0.05	0.05
П		0.00;0.00	0.30	0.14	0.16	0.14	0.14	0.14
			0.70	0.36	0.37	0.38	0.39	0.40
$\mathcal{H}_0:\delta_y=$			0.00	0.05	0.05	0.04	0.04	0.04
δ	0.00	0.10;0.20	0.30	0.13	0.14	0.14	0.16	0.15
••			0.70	0.35	0.39	0.41	0.40	0.38
, O			0.00	0.04	0.05	0.04	0.04	0.04
ピ		0.50;0.60	0.30	0.14	0.13	0.16	0.14	0.14
<u> </u>			0.70	0.35	0.41	0.39	0.41	0.40
		Pan	el (b)					
			0.00	0.96	0.99	1.00	1.00	1.00
		0.00;0.00	0.30	0.93	1.00	1.00	1.00	1.00
			0.70	0.80	0.96	0.99	1.00	1.00
			0.00	0.98	1.00	1.00	1.00	1.00
	-0.50	0.10;0.20	0.30	0.97	1.00	1.00	1.00	1.00
			0.70	0.88	0.99	1.00	1.00	1.00
			0.00	0.99	1.00	1.00	1.00	1.00
		0.50;0.60	0.30	0.99	1.00	1.00	1.00	1.00
			0.70	0.97	1.00	1.00	1.00	1.00
0			0.00	1.00	1.00	1.00	1.00	1.00
\sim		0.00;0.00	0.30	1.00	1.00	1.00	1.00	1.00
\frown			0.70	0.99	1.00	1.00	1.00	1.00
y		0.10;0.20	0.00	1.00	1.00	1.00	1.00	1.00
δ	0.50		0.30	1.00	1.00	1.00	1.00	1.00
$\mathcal{H}_a:\delta_y\neq 0$			0.70	0.99	1.00	1.00	1.00	1.00
a		0.50;0.60	0.00	1.00	1.00	1.00	1.00	1.00
\mathcal{I}			0.30	1.00	1.00	1.00	1.00	1.00
			0.70	1.00	1.00	1.00	1.00	1.00
		0.00;0.00	0.00	1.00	1.00	1.00	1.00	1.00
			0.30	1.00	1.00	1.00	1.00	1.00
			0.70	1.00	1.00	1.00	1.00	1.00
			0.00	1.00	1.00	1.00	1.00	1.00
	1.00	0.10;0.20	0.30	1.00	1.00	1.00	1.00	1.00
			0.70	1.00	1.00	1.00	1.00	1.00
			0.00	1.00	1.00	1.00	1.00	1.00
		0.50;0.60	0.30	1.00	1.00	1.00	1.00	1.00
			0.70	1.00	1.00	1.00	1.00	1.00

Table II: Rejection rates of the test statistic: LS standard errors.

Appendix B: Monte Carlo Evidence

	Panel (a)									
Hypothesis		Sample Size								
	δ_y	ρ_x, ρ_z	ρ_y	50	100	200	300	500		
0			$\frac{\rho_y}{0.00}$	0.11	0.10	0.08	0.06	0.06		
<u> </u>		0.00;0.00	0.30	0.16	0.12	0.09	0.10	0.08		
			0.70	0.27	0.23	0.19	0.16	0.13		
		0.10;0.20	0.00	0.12	0.10	0.06	0.07	0.06		
δ_{i}	0.00		0.30	0.16	0.12	0.10	0.08	0.07		
$\mathcal{H}_0:\delta_y$			0.70	0.29	0.21	0.19	0.16	0.15		
0		0.50;0.60	0.00	0.12	0.10	0.07	0.07	0.07		
Z			0.30	0.16	0.12	0.10	0.09	0.09		
			0.70	0.29	0.26	0.19	0.19	0.15		
Panel (b)										
	-0.50	0.00;0.00	0.00	0.97	1.00	1.00	1.00	1.00		
			0.30	0.94	0.99	1.00	1.00	1.00		
			0.70	0.76	0.94	0.99	1.00	1.00		
		0.10;0.20	0.00	0.99	1.00	1.00	1.00	1.00		
			0.30	0.97	0.99	1.00	1.00	1.00		
			0.70	0.85	0.97	1.00	1.00	1.00		
			0.00	0.99	1.00	1.00	1.00	1.00		
		0.50;0.60	0.30	0.99	1.00	1.00	1.00	1.00		
			0.70	0.99	0.99	1.00	1.00	1.00		
0		0.00.0.00	0.00	1.00	1.00	1.00	1.00	1.00		
\mathcal{H}		0.00;0.00	0.30 0.70	1.00 0.99	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00		
			0.70	1.00	1.00	1.00	1.00	1.00		
Sy	0.50	0.10;0.20	0.00	1.00	1.00	1.00	1.00	1.00		
\mathbf{C}			0.30	0.99	1.00	1.00	1.00	1.00		
~			0.00	1.00	1.00	1.00	1.00	1.00		
$\mathcal{H}_a:\delta_y\neq 0$		0.50;0.60	0.00	1.00	1.00	1.00	1.00	1.00		
4			0.30	1.00	1.00	1.00	1.00	1.00		
		0.00;0.00	0.00	1.00	1.00	1.00	1.00	1.00		
			0.30	1.00	1.00	1.00	1.00	1.00		
			0.70	1.00	1.00	1.00	1.00	1.00		
			0.00	1.00	1.00	1.00	1.00	1.00		
	1.00	0.10;0.20	0.30	1.00	1.00	1.00	1.00	1.00		
	1.00		0.70	1.00	1.00	1.00	1.00	1.00		
			0.00	1.00	1.00	1.00	1.00	1.00		
		0.50;0.60	0.30	1.00	1.00	1.00	1.00	1.00		
			0.70	1.00	1.00	1.00	1.00	1.00		

Table III: Rejection rates of the test statistic: HAC standard errors. Panel (a)

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