Effective Political Contests

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Abstract

We study two-stage political contests with private entry costs. We show that these political contests could be ineffective, namely, the chance of low ability candidates participating in the contest might be higher than the chance of high ability candidates participating in the contest (and winning). However, by imposing a costly requirement (fee) on the winner of the contest, one can guarantee that the contest will be effective.

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JEL classification: D44, O31, O32

1 Introduction

In a primary election, parties select a nominee to run in a general election. In this process candidates first have to decide whether or not to enter a contest to be the party nominee. Afterwards, if there is more than one entrant, they must compete to determine the winner.

In this paper, we model this situation as a two-stage political contest where there is an entry stage and a campaigning stage. The cost of campaigning has two components: first, the

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opportunity (entry) costs of running a campaign and second, the expenditure used in campaigning. We model the former as privately known and fixed and the latter as publically known and variable. Also, each candidate has a publically known ability (charisma), either high or low.

The timing of the model is as follows. In entry stage, the candidates engage each other. They indicate their interest in running and every candidate learns the abilities of his potential opponents. Then, given his private cost of entry, he decides whether or not to participate in the second stage of the contest. The candidates who decide to participate pay their entry costs. After this stage, all candidates incur their entry costs and learn who has entered. In the second stage, the candidates compete against each other in what we model as an asymmetric all-pay auction under complete information. Each candidate chooses expenditure and the candidate with the highest expenditure/ability ratio wins the primary. Independent of success, all candidates bear the costs.

In the economic literature, all-pay auctions are studied under complete information where the players' valuations for the object are common knowledge (see, for example, Hillman and Riley, 1989; Baye et al., 1993, 1996; Che and Gale, 1998; and Kaplan et al., 2003) or under incomplete information where each player's valuation for the object is private information to that player and only the distribution of the players' valuations is common knowledge (see, for example, Amman and Leininger, 1996; Krishna and Morgan, 1997; and Moldovanu and Sela, 2001, 2006). In our model, each candidate has two private parameters: his ability, which is common knowledge, and his entry cost, which is private information.

We find that our model has cutoff equilibria, where any candidate with an entry cost higher than the cutoff for his type (ability) will decide to stay out of the contest and any candidate with an entry cost lower than the cutoff for his type will decide to participate in the contest.

We show that given these equilibrium entry decisions the contest may be ineffective; namely, the chance that a high ability candidate will participate may be lower than the chance that a low-ability candidate will participate. Consequently, there may be a higher chance that the party may choose the low ability candidate. We show that the party can overcome this problem and guarantee that the contest will be effective by imposing a requirement (task or fee) to be paid by the winner of the primary.

Finally, we consider the situation where the party wishes to minimize the total expenditures. In the classical all-pay contest without entry costs, if the number of candidates is endogenous, the contest designer should decrease the number of candidates if he wishes to minimize the total effort. In our model, however, we find that the total expenditures may either increase or decrease in the number of candidates. Therefore, manipulating the number of candidates in order to change the total expenditures may have unintended consequences in political contests with private entry costs.

2 The model

Consider n candidates competing in a political contest for one position. The candidates have the same value for winning the position (contest) which is normalized to be 1. Candidate i's ability, $\alpha_i \geq 0$, is common knowledge. Assume that there are n_1 candidates with high ability of α_1 and n_2 candidates with a low ability of $\alpha_2 < \alpha_1$.[1] Participating in the contest generates a (sunk) cost c_i/α_i for candidate i , where c_i is the entry cost which is private information and is drawn independently from the cumulative distribution function F which is on the interval $[\underline{c}, \overline{c}]$ where

[1] For simplicity, we assume two types of abilities. Our results can be generalized to the case with any number of types.

$0 \leq \underline{c} < \min \alpha_i$. We assume that F is continuously differentiable with $F(\underline{c}) = 0$ and is common knowledge.[2] In the first stage, all the candidates are engaged, they learn the valuations of their opponents and each one decides whether to stay out or participate in the second stage of the contest. The candidates who decide to participate pay their entry costs. Then, in the second stage, these candidates see who else has decided to participate and compete in an all-pay auction under complete information such that the candidate with the highest expenditure/ability ratio $\frac{x_i}{a_i}$ wins the nomination, while all the candidates pay their cost of effort. That is, if candidate i decides to participate at the second stage of the contest, pays his entry cost c_i, exerts an effort of x_i and wins the contest, then his payoff is given by $1 - \frac{(x_i + c_i)}{\alpha_i}$. On the other hand, if he does not win the contest his payoff is given by $-\frac{(x_i + c_i)}{\alpha_i}$.

3 Equilibrium

In our model there frequently are trivial equilibria strategies in which one of the candidates decides to always participate independent of his entry cost, and all the other candidates decide to stay out of the contest in the second stage. In order to prevent such equilibrium strategies (when $n_1, n_2 > 1$) we assume that candidates of the same type (same α) follow the same strategy. We say that an equilibrium is type-symmetric if all candidates of the same type follow the same strategy.

In the second stage the candidates compete in the all-pay auction where the candidates' abilities are common knowledge.[3] If there is only one entrant in the second stage, he will bid zero and win. If there is more than one entrant, there are three cases that need to be examined.

[2] To avoid a trivial solution assume that $F(\alpha_2) > 0$ (there is a chance that player i has a cost lower than α_2).

[3] The complete analysis of the equilibrium in the all-pay auction under complete information is given by Baye et al. (1996).

Let us denote e_i for the number of entrants of type i.

Case 1: There are two or more entrants with low abilities (type 2) only.

Then, these candidates randomize on the interval $[0, \alpha_2]$ according to their effort cumulative distribution functions $F_2(x)$, which is given by the indifference condition:

$$\alpha_2 F_2^{e_2 - 1}(x) - x = 0 \tag{1}$$

Thus, each candidate's effort is distributed according to $F_2(x) = \left(\frac{x}{\alpha_2}\right)^{\frac{1}{e_2-1}}$. Total effort is $e_2 \int_0^{\alpha_2} x dF_2(x) = \alpha_2$ and the expected payoff of every candidate is $u_2 = 0$.

Case 2: There are $e_1 \geq 2$ entrants with high abilities (type 1) and any number of entrants with low abilities.

In this case all the candidates of type 2 stay out and the candidates of type 1 enter in the second stage. These candidates randomize on the interval $[0, \alpha_1]$ according to their effort cumulative distribution functions $F_1(x)$, which is given by the indifference condition:

$$\alpha_1 F_1^{e_1 - 1}(x) - x = 0 \tag{2}$$

Thus, candidates' effort is distributed according to $F_1(x) = \left(\frac{x}{\alpha_1}\right)^{\frac{1}{e1-1}}$. The total expected effort is $e_1 \int_0^{\alpha_1} x dF_1(x) = \alpha_1$ and the expected payoff of every candidate is $u_1 = 0$.

Case 3: There is only one entrant with high ability and $e_2 \geq 1$ entrants with low abilities.

Then, the candidates randomize on the interval $[0, \alpha_2]$ according to their effort cumulative distribution functions, $F_1(x)$ and $F_2(x)$, which are given by the indifference conditions:

$$\begin{aligned} \alpha_1 F_2^{e_2}(x) - x &= \alpha_1 - \alpha_2 \\ \alpha_2 F_1(x) - x &= 0 \end{aligned} \tag{3}$$

Thus, type 1's effort is distributed according to $F_1(x) = \frac{x}{\alpha_1}$, while type 2's effort is distributed according to $F_2(x) = \left(\frac{x+\alpha_1-\alpha_2}{\alpha_1}\right)^{\frac{1}{e_2}}$. The total expected effort is $\int_0^{\alpha_2} x dF_1(x) +$

$e_2 \int_0^{\alpha_2} x dF_2(x) = \frac{\alpha_2+3e_2\alpha_2+2e_2^2(\alpha_1-\alpha_2)((1-\frac{\alpha_2}{\alpha_1})^{\frac{1}{e2}}-1)}{2(e_2+1)}$, and the respective expected payoffs are $u_1 = \alpha_1 - \alpha_2$ and $u_2 = 0$.

Now, given the analysis of the candidates' behavior in the second stage of the contest, we can analyze their entry decisions in the first stage. In the first stage, n_1 candidates with ability of α_1 and n_2 candidates with ability of α_2 are engaged and each of them decides whether to participate or not, and those who decide to participate pay their private entry costs. Denote by $d_i(c)$ the entry decision (the probability of entering) if one has entry cost c and ability $\alpha_i > 0$.

Proposition 1 *The entry decision (the probability of entering) of a candidate with cost c_i and ability $\alpha_i > 0$ in the first stage is*

$$d_i(c) = \begin{cases} 1 \quad \textit{if} \quad c \leq c_i^* \\ \\ 0 \quad \textit{if} \quad c > c_i^* \end{cases}$$

where the equilibrium cutoffs c_i^*, $i = 1, 2$ are given by[4]

$$c_1^* = (\alpha_1 - \alpha_2)(1 - F(c_1^*))^{n_1 - 1} + \alpha_2(1 - F(c_2^*))^{n_2}(1 - F(c_1^*))^{n_1 - 1} \tag{4}$$

$$c_2^* = \alpha_2(1 - F(c_1^*))^{n_1}(1 - F(c_2^*))^{n_2 - 1} \tag{5}$$

In the symmetric case where $\alpha_1 = \alpha_2$ and n is the total number of candidates, the symmetric

[4] Obviously, this equilibrium is for $n_1, n_2 \geq 1$. If $n_1 \geq 2, n_2 \geq 2$ and $\underline{c}= 0$, then any type-symmetric equilibrium must be interior. If $n_1 = 1$ or $n_2 = 1$ the type-symmetric equilibrium can be non interior with $c_1^* \geq \overline{c}, c_2^* \leq \underline{c}$ or $c_2^* \geq \overline{c}$, $\underline{c} < c_1^* < \overline{c}$ (and for $\underline{c} > v1 - v2$, non interior with $c_2^* \geq \overline{c}, c_1^* \leq \underline{c}$.) A cutoff $c_i > \overline{c}$ implies that everyone of type i would enter and a cutoff $c_i < \underline{c}$ implies that everyone of type i stays out.

entry decision is given by

$$d_i(c) = \begin{cases} 1 & \text{if } c \leq c^* \\ \\ 0 & \text{if } c > c^* \end{cases}$$

where the equilibrium cutoff $c^* > 0$ *is the solution of*[5]

$$c^* = \alpha(1 - F(c^*))^{n-1} \tag{6}$$

Proof. See Appendix.

The entry decision described by Proposition 1 is such that any candidate with ability α_i and an entry cost higher than the equilibrium cutoff c_i^* will stay out of the contest and any candidate with ability α_i and an entry cost lower than the equilibrium cutoff c_i^* will participate in the second stage of the contest. One may also notice that in our model, sometimes there is no entry and hence no nominee. This still fits many elections – often in US congressional elections, either the Democratic or Republican party does not select a nominee in the primary election and the other major party runs unoppossed.

4 Effectiveness

Given the equilibrium strategies, a candidate with ability α_2 has a positive payoff only if he is the only entrant. Thus, the payoff of a candidate with ability α_2 and entry cost $c \leq c_2^*$ is

$$\alpha_2(1 - F(c_1^*))^{n_1}(1 - F(c_2^*))^{n_2 - 1} - c = c_2^* - c$$

Similarly, a candidate with ability α_1 will profit α_1 when he is in the second stage of the contest alone and will profit the difference $\alpha_1 - \alpha_2$ when he is in the second stage with only

[5] For the symmetric case, any symmetric equilibrium is interior.

candidates with abilities of α_2. Thus, the payoff of a candidate with ability α_1 and entry cost $c \leq c_1^*$ is

$$(\alpha_1 - \alpha_2)(1 - F(c_1^*))^{n_1 - 1} + \alpha_2(1 - F(c_2^*))^{n_2}(1 - F(c_1^*))^{n_1 - 1} - c = c_1^* - c$$

Therefore, the expected payoff of a candidate with ability $\alpha_i, i = 1, 2$, is

$$\int_0^{c_i^*} (c_i^* - c) dF(c) \tag{7}$$

Definition 1 *We say that a contest is effective if the chance of participation of candidates is increasing in their abilities. That is, a contest is effective if $v_i > v_j$ implies $c_i^* > c_j^*$.*

In our model the contest is effective iff $c_1^* > c_2^*$. Below we show by an examples that a candidate with high ability and a relatively low entry cost may decide to stay out of the contest whereas a candidate with low ability and a relatively high entry cost may decide to participate in the contest. In other words, the contest is ineffective.

Example 1 *Consider a contest where $n_1 = 2$, $n_2 = 1$, $\alpha_1 = 2.25$, $\alpha_2 = 2$ and F is a uniform distribution on $[0, 1]$.*

By (4) and (5) the equilibrium interior cutoffs are given by:[6]

$$\begin{aligned} c_1^* &= (2.25 - 2)(1 - c_1^*) + 2(1 - c_2^*)(1 - c_1^*) \\ c_2^* &= 2(1 - c_1^*)^2 \end{aligned}$$

There are two solutions to this system of equations: 1. $c_1^* = 0.34255$ and $c_2^* = 0.8644$ 2. $c_1^* = 0.62993$ and $c_2^* = 0.2739$. Note that in the first solution the contest is ineffective. The

[6] Note that for simplicity of exposition, in our examples, we will write the equilibrium cutoff equations, (4) and (5), assuming there is an interior solution and then see if this is indeed the case.

equilibrium cutoff of the candidate with the low ability α_2 is higher than the equilibrium cutoff of the candidates with the higher ability α_1. This result implies that the expected payoff of the candidate with the low ability α_2 is larger than the expected payoff of his opponents with the higher abilities α_1. This then implies that the candidate with the low ability is more likely to be selected than the candidate with the high ability.

The intuition for why this is possible is that a candidate's willingness to enter depends upon his expected surplus of being in the contest. This surplus depends upon not only the candidate's ability but who else decides to enter the contest. Hence, if high-ability candidates are less likely to enter the contest, then it is indeed possible for low-ability candidates to be more willing to enter since they are more likely to be alone and reap all the profits.

The contest designer can guarantee that the contest will be effective by the following way

Proposition 2 *If the winner of the contest pays a constant fee equal to* $t = \max(0, 2\alpha_2 - \alpha_1)$ *then* $c_1^* > c_2^*$ *such that the contest is effective.*

Proof: By (4) and (5) if $\alpha_1 > 2\alpha_2$ we have,

$$\begin{aligned} c_1^* &= (\alpha_1 - \alpha_2)(1 - F(c_1^*))^{n_1 - 1} + \alpha_2 (1 - F(c_2^*))^{n_2} (1 - F(c_1^*))^{n_1 - 1} \\ &> \alpha_2 (1 - F(c_1^*))^{n_1 - 1} (1 + (1 - F(c_2^*))^{n_2}) > \alpha_2 (1 - F(c_1^*))^{n_1} (1 - F(c_2^*))^{n_2 - 1} = c_2^* \end{aligned}$$

If $\alpha_1 < 2\alpha_2$, by imposing a fee of $t = 2\alpha_2 - \alpha_1$ the candidates have actually new abilities given by

$$\widetilde{\alpha}_i = \alpha_i - t = \alpha_i - (2\alpha_2 - \alpha_1),\ i = 1, 2.$$

Since $\widetilde{\alpha_1} = 2\widetilde{\alpha_2}$ and $\widetilde{\alpha}_i > 0$ for all i, the result is obtained.

Hence by imposing a constant fee of $t = \max(0, 2\alpha_2 - \alpha_1)$ the party can guarantee that the chance of participation of the high-ability candidates will be larger than those of the low-ability candidates. However, it is important to note that by imposing a constant fee, the party lowers expected participation in the contest. Thus, before imposing a constant fee, the party should tradeoff having the benefit of higher participation versus desireability of an effective contest.

5 Total effort

So far we assumed that the number of potential candidates is exogenous. Suppose that the party is concerned with minimizing the total expenditure (total effort) of the candidates and it can determine the number of candidates. The reason why the party may be concerned with total expenditure is that perhaps this may affect future ability of the party to collect from donors or in the very least affect the ability to compete in the general election. We also assume that the candidates that are excluded will not pay entry costs. Usually in the standard all-pay auction under incomplete information the total effort increases in the number of candidates. In our model the effect of the number of candidates on the total effort is not clear. In order to demonstrate this point it is sufficient to consider the simpler case of symmetric contests. The following example consists of two cases and shows that an increase in the number of potential candidates has an ambiguous effect on the candidates' total effort.

Example 2 *Consider a contest where* $\alpha_1 = \alpha_2 = 1$.

Case 1: The candidates' entry costs are distributed according to a uniform distribution on $[0, 1]$. By (6) and (7) the equilibrium cutoff and the total effort are calculated, and as we can see below, an increase in the number of potential candidates yields an increase of the total effort.

number of candidates	equilibrium cutoff	total effort
2	0.5	0.25
3	0.381966	0.326238
4	0.317672	0.379581
5	0.275508	0.420873
10	0.175699	0.546468
1000	0.00524	0.9673

Case 2: The candidates' entry costs are distributed according to a uniform distribution on $[0.5, 0.75]$. As we can see below, in this case, an increase in the number of potential candidates yields a decrease of the total effort.

number of candidates	equilibrium cutoff	total effort
2	0.6	0.16
3	0.5625	0.15625
4	0.5457	0.15501
5	0.53608	0.15443
10	0.51764	0.15368
1000	0.50017	0.15335

Note that in both cases the candidates' abilities are uniformly distributed and the only difference is in the support. Thus, we can conclude that only a minor change in distribution of the candidates' abilities can dramatically change the effect of the number of candidates on the total expenditure. In that case, a party should be careful when it controls the number of candidates, if it wishes to increase or decrease the total expenditure in the contest.

6 Discussion

While our results are somewhat modest, we do point out two counter-intuitive possibilities of political contests. The first possibility is that in some situations there may a higher chance of selecting a low-ability candidate. This is not due to the difference in preferences between the party and the general public, but due to the structure of the primary. The second counter-intuitive possibility is that reducing the number of candidates may increase the total expenditure of the race. There is still work left to see if these occur with other contest success functions.

Finally, it is important to notice that in our symmetric model with private entry costs, the Revenue Equivalence Theorem (see Myerson, 1981; and Riley & Samuelson, 1981) holds whether or not candidates observe how many others have decided to enter the second stage of the contest. This implies that our results for the symmetric contest will hold for instance if the in the second stage the players compete through first-price auctions or second-price auctions instead of all-pay auctions. Moreover, in our asymmetric model similar results will hold for the second-price auction but not for the first-price auction. In particular, the first-price auction when the bidders are uninformed about who enters may generate lower revenue than the revenue in our model of all-pay auctions. This shows us that there is room to study other contest forms in our asymmetric environment.

7 Appendix

7.1 Proof of Proposition 1

Given the equilibrium in the second stage, a candidate with a low valuation α_2 will profit only when he is in the second stage of the contest alone. The probability of this is $(1 - F(c_1^*))^{n_1}(1 -$

$F(c_2^*))^{n_2-1}$ which implies equation (5). On the other hand, a candidate with a high valuation α_1 will profit α_1 when he is in the second stage of the contest alone and will profit the difference $\alpha_1 - \alpha_2$ when he is in the second stage with only candidates with valuations of α_2. These happen with probability $(1 - F(c_2^*))^{n_2}(1 - F(c_1^*))^{n_1-1}$ and $(1 - F(c_1^*))^{n_1-1}$ which implies equation (4). The existence of the equilibrium is derived by Brower's Fixed Point Theorem. The RHS of equations (4) and (5) form a bounded function from $[0, \alpha_1]\times$ $[0, \alpha_2]$ to $[0, \alpha_1]\times$ $[0, \alpha_2]$ that is continuous since F is continuous. Therefore, a fixed point must exist. (Note that if cutoff c_i^* of the fixed point is above b, then it would imply that everyone with value α_i enters. Likewise, if cutoff c_i^* of the fixed point is below a, then it would imply that everyone with value α_i stays out)

In the following we show that if n_1, $n_2 \geq 2$, and $a = 0$, then any fixed point is interior, that is $F(c_1^*), F(c_1^*)$ are from $(0, 1)$.[7] The RHS of equations (4) and (5) are decreasing in c_1^* and c_2^*. If $F(c_1^*) = 0$, then the RHS of (4) is greater than or equal to $\alpha_1 - \alpha_2 > 0$– a contradiction. If $F(c_1^*) = 1$, then the RHS of (4) is zero – also a contradiction. Hence, $0 < F(c_1^*) < 1$. A similar argument shows that $0 < F(c_2^*) < 1$ as well. The symmetric case can be shown in a similar, but simpler manner. ⊡

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