Taxation of a Polluting Non-renewable Resource in the Heterogeneous World *

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Abstract

This paper extends the literature on the taxation of polluting exhaustible resources by taking international heterogeneities and national tax-setting into account. We propose a two-country Romer model of endogenous growth in which the South is endowed with the stock of an essential polluting non-renewable resource and world economic growth is driven by a northern research sector. We consider the stock of pollution as affecting global welfare.

First, we characterize the optimal environmental taxation policies. Second, we examine the impacts of national taxes. Their time profile determines the extraction path, the dynamics of pollution accumulation and that of world output. Their respective levels entail inter-country interactions by altering the efficiency of the world resource allocation, the tax revenues and the resource rents. We study isolatedly the distortional and distributional effects of local taxes. Then, we completely assess their overall impacts, shedding light on the divergent interests of heterogeneous regions regarding their national environmental taxes.

JEL classification: Q3; O4; H2

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1 Introduction

A current challenge for environmental economists is to advocate instruments to reduce the impact of climate change. Due to the global character of this phenomenon, whatever are the chosen instruments, participation of a large group of countries will be needed to implement an efficient policy. Hence, a particular aspect of economic instruments that deserves major attention is their international impacts. This paper aims at examining these impacts of taxes on the use of polluting non-renewable resources. This issue proves to be of a particular relevance when countries are heterogeneous along one or more dimensions.

A large literature investigates the optimal taxation of these resources. The first studies (in the 1990s) used partial equilibrium models of an exhaustible resource depletion where the flow of resource fills a stock of pollution. The optimal dynamics of depletion in presence of climate change was computed and compared to that in the absence of climate change by Withagen [26]. Sinclair [22], Ulph and Ulph [25] and Hoel and Kverndokk [13] analyzed the impacts of a carbon tax on the decentralized equilibrium and characterized the optimal tax schemes correcting the environmental distortion. More recently (in the 2000s), this issue has been addressed in dynamic general equilibrium, still one-country, models of endogenous growth. As a first step, some authors considered the flow of pollution from the resource consumption to be harmful (Schou [20] and [21], and Grimaud and Rougé [10]). A substantial theoretical improvement has been done by modeling pollution as in the partial equilibrium literature above, i.e. by assuming the stock of atmospheric pollution to have negative effects on the economy. Groth and Schou [12] and Grimaud and Rougé [11] represent this new generation of analytical studies. Overall, this literature highlights the requirement of a dynamic framework under perfect anticipations and of the explicit consideration of the resource exhaustibility (On this, see also Belgodere [3]). It then emphasizes the particular role of the time profile of the environmental tax rate. Precisely, extraction under laissez faire is shown to be faster than optimally, this distortion being corrected by a decreasing ad valorem tax on the resource use. This optimal policy fosters growth and slows down resource depletion.

On the one hand, these contributions are particularly relevant to address climate change. Indeed, it is now well known that carbon dioxide is the main anthropogenic greenhouse gas[1] and that a very large part of its emissions is due to combustion of exhaustible fossil fuels[2].

On the other hand, only aggregated models, representing a homogeneous world, have been used to study taxation of fossil fuels. However, the real world is very heterogeneous with respect to oil endowments[3], for example. Moreover, taxation of an exhaustible resource whose distribution among countries is heterogeneous entails inter-country transfers and thus conflicting interests.

Indeed, exploitation of a non-renewable resource generates pure rents that, as such, are partly captured through a tax on the resource (e.g. Dasgupta and Heal [7], Sinn [23] and Gaudet and Lasserre [9]): independently of the effect of the tax on the extraction path, it shifts much of the tax burden to the resource owners. Bergstrom [4] and Brander

[1] Carbon dioxide from energy represents 95% of the energy-related greenhouse gas emissions and about 80% of the world anthropogenic greenhouse gas emissions (Quadrelli and Peterson [18]).

[2] In 2004, fossil sources accounted for 81% of the global primary energy supply (Quadrelli and Peterson [18]).

[3] Combustion of oil products generates 40% of the world carbon dioxide emissions; it is the most important source of carbon dioxide emissions (Quadrelli and Peterson [18]). The 19 countries with the largest crude oil reserves per capita represent more than 80% of the world reserves (source: Oil & Gas Journal [17]).

and Djajic [6][4] highlighted that national taxes on these resources can be used strategically by resource consuming countries in order to extract the rents to be earned by the resource producing countries. These interesting contributions suggest that taxation of exhaustible resources, when their distribution is heterogeneous, is an international issue that must be addressed in a multi-country model.

The objective of this paper is to study the taxation of a polluting non-renewable resource in an international, heterogeneous framework. To this purpose, we simply divide into two regions a canonical – in the sense of the analytical literature cited above – endogenous growth model of climate change where the combustion of an exhaustible resource generates emissions accumulating into a world stock of pollution, representing the greenhouse effect. Moreover, because potential conflicting interests would arise from the different characteristics of countries, we take advantage of the two-country model to introduce realistic international heterogeneities. A surprising difference between the top oil producing and the top oil consuming countries is that the former are often poorer than the latter[5]. As a result of this endowment heterogeneity and the related heterogeneity in productivity levels throughout the world, the North largely consumes this resource, while the South mainly exports it. We will then divide the world economy into two regions: the North, representing the developed, top oil consuming countries, and the South, representing the relatively low-productive, top oil producing countries. Consistently, we also assume that world economic growth is driven by a northern research sector and that intellectual property rights (IPRs for short) are not enforced in the South.

This North-South division raises some issues regarding the international effects of regional environmental taxation and thus the possibility to solve a global problem with local tax-setting. First, we shall see that every environmental taxation policy implies a certain sharing of the world production. Second, in order to understand the benefit of one country from modifying its local tax rate, one need to examine how such a deviation alters the local tax revenues, relative competitiveness and the global efficiency of the polluting resource allocation. Third, since regional taxes change the location of productive activities and IPRs are not homogeneously enforced, the potential growth effect of environmental taxation should also be addressed. Finally, national taxes set by environment-conscious governments may solve a global environmental problem; however, if so, international conflicting interests may lead to a non environmental distortion.

The inter-country effects these issues rely on can be expected to be of a substantial magnitude. Indeed, although extremely heterogeneous, taxes on the use of fossil fuels are very high in top oil consuming regions[6], thus representing a large part of their fiscal revenues[7]. Understanding these effects and drawing their policy implications is all the more relevant as rising these taxes seem to be currently tempting for environmental and fiscal reasons.

The paper is organized as follows. In Section 2, we present our model and characterize the associated socially optimum allocation. Section 3 describes the solution of the decentralized equilibrium and examine qualitatively the effects of the time profile of the environmental tax rates as well as those of the environmental tax levels. In particular, we

[4] These studies do not deal with the environment. See Amundsen and Schöb [1] for an introduction in Bergstrom's model of a harmful flow of pollution.

[5] For instance, the per capita GNI over the 19 countries with the largest crude oil reserves per capita is lower than 5800 US$ in 2005 (sources: The World Bank, Oil & Gas Journal [17]). The same year, OECD represented 60% of the world oil consumption (source: EIA).

[6] On this, see Bacon [2] and IEA [14].

[7] Taxes on oil products constitute 6% of the total fiscal revenues of OECD member countries (Source: IEA). The G7 countries made $517 billion per year through these taxes over the period 2003-2007 (See OPEC [16]).

examine the potential growth effect of the tax levels under heterogeneous IPRs enforcement. In the same section, the optimal environmental taxation is determined. The results of Sections 2 and 3 are generalizations of those obtained in the literature on taxation of polluting exhaustible resources to the case of a multi-country world. These extensions emphasize the need that countries coordinate their environmental taxation policies. Section 4 studies the effects of the national tax levels on the output and consumption in the two countries and on the global efficiency of the resource allocation. We examine isolatedly a distributional rent transfer effect and a locational efficiency effect. Next, we compute the total impact of an increase in the northern environmental tax level. Finally, we shed light on the strategic interests of both countries in setting their environmental taxes at a lower or a greater rate, depending on their characteristics.

2 Model and Welfare

2.1 Model

At each date $t \in [0, +\infty)$, the final output is produced in both countries using the range of available intermediate goods, labor and a flow of resource. The aggregate production functions are[8]

$$Y_i = \Big(\int_0^{A_i} x_i(j)^\alpha \, dj \Big) {L_i^Y}^\beta {R_i}^\gamma, \ \alpha + \beta + \gamma = 1, \ i = N, S, \tag{2.1}$$

where $x_i(j)$ is the amount used of intermediate good j, L_i^Y is the quantity of labor employed in the production sector, R_i is the quantity of natural resource burnt in country i. The subscripts N and S refer respectively to the North and the South.

A_i, $i = N, S$, is an index of technological development which measures the range of the available innovations in each country. Only the North is engaged in a research activity. The derivative of a variable X with respect to time being denoted by $\dot{X}$, the production of innovations writes

$$\dot{A_N} = \psi A_N L_N^A, \ \psi > 0, \tag{2.2}$$

where L_N^A is the quantity of labor employed in the research sector. A constant fraction ϕ, $0 < \phi \leq 1$, of the ever discovered innovations diffuses naturally to the South while the remaining ones cannot be used in this country:

$$A_S = \phi A_N, \ 0 < \phi \leq 1. \tag{2.3}$$

ϕ can be interpreted as an index of southern development.

To each available innovation is associated an intermediate good produced in both countries through a one-for-one technology from the final output:

$$x_i(j) = y_i(j), \ j \in [0, A_i], \ i = N, S. \tag{2.4}$$

The resource is freely extracted from a finite initial stock (Q):

$$\dot{Q} = -R = -(R_N + R_S), \ Q(0) = Q_0 > 0, \ given, \tag{2.5}$$

and its use results in a proportional flow of pollution emptying a stock of environment quality (E)[9]:

$$\dot{E} = -hR = -h(R_N + R_S), \ h > 0, \ E(0) = E_0 > hQ_0, \ given. \tag{2.6}$$

[8] For simplicity, the time argument of each variable is dropped as long as this does not create ambiguity.

[9] Following Groth and Schou [12], we ignore the regeneration ability of the atmosphere. This is for simplicity since, from a control theoretic point of view, this definition of the level of environmental quality reduces the problem by one state variable. However, all our results are robust to the introduction of a linear auto-regeneration process.

Each household is endowed with one unit of labor. The total quantities of labor in North and South respectively are locally fixed and constant over time:

$$L_N^Y + L_N^A \leq L_N, \tag{2.7}$$

$$L_S^Y \leq L_S. \tag{2.8}$$

The households of both countries consume the amount of the final good remaining after the production of the intermediates so that the world level of consumption, C, must satisfy the world's constraint on the use of the final good:

$$C_N + C_S + \int_0^{A_N} x_N(j)\, dj + \int_0^{A_S} x_S(j)\, dj \leq Y_N + Y_S. \tag{2.9}$$

The preferences of the infinitely-lived representative households of North and South are identical and represented by the utility functions

$$U_i = \int_0^{+\infty} \ln\big(\frac{C_i}{L_i} E^\lambda\big) e^{-\rho t}\, dt,\ i = N, S,\ \lambda, \rho > 0, \tag{2.10}$$

where λ is an index of environmental concern and ρ the psychological discount rate.

2.2 Welfare

Let us characterize the Pareto optima of this economy. They are the solutions of the weighted utilitarian social planner's programm. This programm consists in maximizing

$$\int_0^{+\infty} \Big[\delta L_N \ln\big(\frac{C_N}{L_N} E^\lambda\big) + (1-\delta) L_S \ln\big(\frac{C_S}{L_S} E^\lambda\big)\Big] e^{-\rho t}\, dt,\ 0 < \delta < 1. \tag{2.11}$$

subject to equations (2.1) to (2.9) with respect to C_i, x_i, R_i, L_i^Y and L_N^A, $i = N, S$.

The results are formally given in Appendix A. Using these results and the phase diagram of Figure 1, we fully describe the optimal dynamics of the economy. The main findings are summarized in Proposition 1. The growth rate of any variable X is denoted by g_X. We define global variables as follows: $Y = Y_N + Y_S$, $C = C_N + C_S$, $R = R_N + R_S$ and $x = x_N + x_S$ while $A_N = A$ and $A_S = \phi A$. The upper-script o is used for optimum.

Proposition 1 *In the Pareto set:*
ı) ${L_N^Y}^o$ and ${L_N^A}^o = L_N - {L_N^Y}^o$ immediately jump to constant values. Thus, $g_{A_N^o} = g_{A_S^o} = g_{A^o} = \psi {L_N^A}^o$ is always constant. The relative output, $\frac{Y_N^o(t)}{Y_S^o(t)}$, is constant over time and is a decreasing function of the index of southern productivity, ϕ. The relative consumption level, $\frac{C_N^o(t)/L_N}{C_S^o(t)/L_S}$, is constant over time and is an increasing function of the relative weight of the North in the social welfare function, δ.
ıı) If households are indifferent to the environmental quality ($\lambda = 0$), the economy immediately jumps to its steady-state, in which

$$\begin{cases} g_{R_N^o} = g_{R_S^o} = g_{R^o} = -\rho \\ g_{C_N^o} = g_{C_S^o} = g_{C^o} = g_{Y^o} = g_{Y_N^o} = g_{Y_S^o} = g_{A^o} - \frac{\gamma\rho}{1-\alpha} \\ g_{x_N^o} = g_{x_S^o} = g_{x^o} = -\frac{\gamma\rho}{1-\alpha} \end{cases}.$$

ııı) In the case of environmental concern ($\lambda > 0$), the economy is always in transition while converging towards the steady-state where pollution does not matter ($\lambda = 0$). The flow of resource use decreases over time: $-\rho < g_R^o(t) < 0$ and $\lim_{t\mapsto+\infty} g_R^o(t) = -\rho$. Hence, the resource is extracted and used slower than with no environmental concern. The rates of growth of production and consumption levels, $g_Y^o(t)$ and $g_C^o(t)$, are also higher in this case.

Proof of Proposition 1 *See Appendix A.*

Let us give further details about these results.

First of all, note that, if households are indifferent to the environment, that is to say if pollution is not harmful ($\lambda = 0$), the economy immediately jumps to its steady-state and keeps growing regularly. Indeed, the growth rate of the stock of knowledge is always constant and in case of no pollution concerns, the resource is optimally depleted at a constant rate. Hence, the transitional dynamics of the model stems from the introduction of the environmental issue.

Let us now examine the optimal dynamics of the economy. To do so, we construct the phase diagram represented in Figure 1.

The optimal rate of extraction is shown in Appendix A to obey $g_R^o = -\rho - \lambda(1 - \alpha)g_E^o/\gamma$ (equation (A.25)). Obviously, g_R^o is related to g_E^o since it internalizes the effect of extracting and burning the resource R, on the environmental quality, E. This equation is represented in Figure 1 by the straight line (HC). Differentiating (A.25) with respect to time leads to $\dot{g_R^o} = -\lambda(1-\alpha)\dot{g_E^o}/\gamma$. From this, let us note that if $\dot{g_E^o} \geq 0$ then $\dot{g_R^o} \leq 0$ and inversely.

From the definition of pollution (2.6), $\dot{E}^o = -hR^o$, one gets $g_E^o = -hR^o/E^o$. Log-differentiating with respect to time leads to $\dot{g_E^o}/g_E^o = g_R^o - g_E^o$, i.e. $\dot{g_E^o} = g_E^o(g_R^o - g_E^o)$. This equation and the fact that $g_E^o < 0$ imply that, if $g_R^o \geq g_E^o$, then $\dot{g_E^o} \leq 0$, and thus $\dot{g_R^o} \geq 0$. Inversely, if $g_E^o \geq g_R^o$, then $\dot{g_E^o} \geq 0$ and $\dot{g_R^o} \leq 0$. This gives the dynamics of g_R^o and g_E^o on both sides of the $g_R = g_E$ line in Figure 1.

Before studying the phase diagram, we need to make two remarks. First, the flow of resource extraction is strictly positive at every date $t \geq 0$. Indeed, the resource is essential in the sense that the marginal productivity of the resource gets infinite as the resource use goes to 0, ($\lim_{R \mapsto 0} \partial Y/\partial R = +\infty$), and marginal utility gets infinite as consumption goes to 0 ($\lim_{C_i \mapsto 0} \partial U_i/\partial C_i = +\infty$, $i = N, S$). Hence, $R(t) > 0$ for all $t \geq 0$.

Second, the stock of the resource is completely depleted asymptotically. Indeed, each unit of resource consumed in the production process at any date can be used in such a way that it improves welfare. Its effect on utility is twofold. First, it allows an increase in consumption. Second, it increases the stock of pollution, thus being harmful from the date of its use on. The former positive effect can be marginally infinite due to the essentiality of the resource. However, due to the finite amount of resource, and thus the finiteness of the stock of pollution, and the continuity of damages in this stock, the latter effect is bounded: the marginal disutility of extraction at any date $t \geq 0$, and thus of pollution, formally writes $\int_t^{+\infty} (\delta L_N + (1-\delta)L_S)\lambda(1/E(s))(\partial E(s)/\partial R(t))e^{-\rho(s-t)}\, ds$, where $\partial E(s)/\partial R(t) = -h$ as $s \geq t$ and $1/E(s) < 1/(E_0 - hQ_0)$ since $E_0 - hQ_0$ is the positive lower bound of the environmental quality, reached after the polluting resource is fully exhausted. Hence, by spreading enough the use of the resource over time, more resource can be depleted in a utility improving way. Technically, this is to say that the costate variable associated to the exhaustibility constraint on the polluting resource (μ_Q in Appendix A) is strictly positive, thus meaning that the social value of each unit of resource is strictly positive.

Let us now study the phase diagram depicted in Figure 1. There are two steady-states, $SS1$ and $SS2$. $SS1$ is unstable. Along this steady-state, $\dot{E}$ is always strictly negative, even asymptotically. This would imply, by (2.6), that R is always strictly positive, even asymptotically. Since this contradicts $\lim_{t \mapsto +\infty} R(t) = 0$, this path can be ruled out. The divergence along (HC) towards a positive rate of extraction can also be ruled out. Indeed, a positive g_R would lead to an exhaustion in finite time. This would contradicts what is stated above.

Steady-state $SS2$ is stable. It is the path of the economy along which optimal extrac-

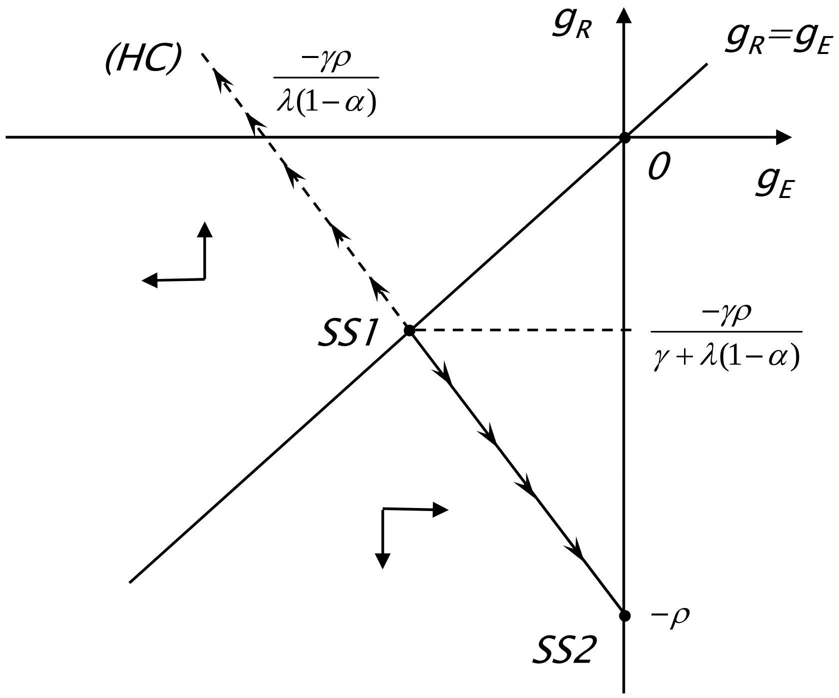

Figure 1: Phase diagram.

tion, R^o, and thus pollution, $\dot{E}^o$, tend to 0. It corresponds to the case of the absence of pollution ($\lambda = 0$). Therefore, it will also be the "laissez-faire" equilibrium, when environment is not internalized. We shall see it later in Subsection 3.2. The economy converges towards this path along the part of (HC) which is below $SS1$.

It is worth noting how evolve the main variables along this transition. The optimal growth rate of extraction, g_R^o, decreases over time and converges to its lower bound, $-\rho$. It means that this rate is always greater than without pollution. It also means that the introduction of pollution in the economy renders desirable to extract the resource more slowly. This is consistent with the findings of Withagen [26] and Grimaud and Rougé [11].

Environmental quality decreases unambiguously over time, g_E^o being negative, and reach asymptotically its lower limit, g_E^o converging to 0. Hence, the environmental quality decays slower and slower.

The dynamics of output is driven by both that of the resource extraction and that of the innovation side of the model. On the one hand, the allocation of labor being stable over time, the quantity of labor used in the production process, the stock of knowledge and the number of intermediates grow at constant rates. On the other hand, the use of the resource is not regular but asymptotically. Consistently, output grows faster along the transition than along the asymptotic steady-state.

Eventually, Proposition 1 emphasizes the continuum of Pareto optima depending on how is shared the output between the two countries. In our social planning approach, this will depend on how the planner weights both groups of households. Beyond the study of how to design policies to implement a social optimum in a decentralized economy, our two-country approach helps investigate how these policies affect the split of the output between North and South and thus their relative benefits from these policies.

The results about the optimal depletion of the resource and the path of pollution accumulation are consistent with the findings of the literature mentioned in the Introduction. Proposition 1 then offers a generalization of these results to the case of a two-country economy.

3 Decentralized Equilibrium for Given National Taxes

Given our examination of the optimal dynamics, we can characterize, in a decentralized model, the optimal taxation policy in the presence of a pollution externality. Moreover, this decentralized approach will render possible to investigate the effects of the national environmental taxes on the general equilibrium outcome.

On the one hand, we follow the literature regarding the basic assumptions relative to the growth engine, to the nature of the goods and to the markets' structures. Endogenous growth is modeled à la Romer and is driven by northern research. The final good, the intermediate goods and the extracted resource are private and freely tradable across countries. There is a world financial market[10]. The stock of atmospheric pollution is a pure public bad. The final sectors, the research sector and the extraction sector are perfectly competitive while the intermediate sector is monopolistic.

On the other hand, beyond the technological heterogeneities introduced in the previous section (international differences in labor productivities and countries' sizes), we can introduce other heterogeneities in the decentralized two-country framework. We assume that the southern households own the entire stock of the natural resource. Moreover, intellectual property rights (IPRs for short) are perfectly enforced in the North while they are not in the South.

The sources of inefficiency will thus be the standard public good character of knowledge and pollution and the monopolistic structure of the northern intermediate sector. Consistently, subsidies to the northern research sector and to the use of intermediates will be sufficient instruments to solve the distortions about the suboptimal investment in R&D. Furthermore, in order to solve the environmental problem, we assume the existence of two national taxes on the local use of the polluting resource.

3.1 Agents' Behavior

The Northern Final Sector. In what follows, the final good is chosen to be the numeraire of the economy and its price is normalized to unity.

The program of this sector consists in maximizing its profit, $\left(\int_0^A x_N(j)^\alpha \, dj\right) {L_N^Y}^\beta {R_N}^\gamma - \int_0^A p_N(j)(1-s_N)x_N(j)\, dj - w_N L_N^Y - p^R(1+\theta_N)R_N$, with respect to all $x_N(j)$, L_N^Y and R_N. In this expression, $p^N(j)$ is the unit price of intermediate good j, s_N is the unit subsidy to the use of intermediate goods in the North, w_N is the wage rate in the North[11], p^R is the unit price of the extracted resource and $\theta_N > -1$ is the unit ad valorem tax on the use of the resource. The behavior of this sector is summarized by the first-order conditions

$$\alpha x_N(j)^{\alpha-1} {L_N^Y}^\beta {R_N}^\gamma = p_N(j)(1-s_N),\ j \in [0, A], \tag{3.1}$$

$$\beta \frac{Y_N}{L_N^Y} = w_N, \tag{3.2}$$

$$\gamma \frac{Y_N}{R_N} = p^R \tau_N, \tag{3.3}$$

[10] Thanks to the Walras law, we can as usually avoid computing the equilibrium on this market. The details of the equilibrium on this market is available from the authors upon request.

[11] As the labor markets are segmented due to the fixity of this factor, there are two wages in the economy.

where[12] $\tau_N = 1 + \theta_N > 0$.

The Southern Final Sector. The program of this sector is similar to that of the northern final sector. However, there is no need of subsidy to the use of intermediate goods in the South since, due to the absence of intellectual property enforcement, the intermediate monopolists in competition with competitive pirates, are forced to sell their output at marginal cost[13].

The program of this sector is thus the maximization of $\left(\int_0^{\phi A} x_S(j)^\alpha \, dj\right) {L_S^Y}^\beta {R_S}^\gamma - \int_0^{\phi A} p_S(j) x_S(j) \, dj - w_S L_S^Y - p^R(1+\theta_S)R_S$, with respect to all $x_S(j)$, L_S^Y and R_S. Here, the variables introduced with a subscript S have the same meaning as those introduced in the previous subsection but transposed to the South.

The behavior of this sector is summarized by the first-order conditions

$$\alpha x_S(j)^{\alpha-1} {L_S^Y}^\beta {R_S}^\gamma = p_S(j), \; j \in [0, \phi A], \tag{3.4}$$

$$\beta \frac{Y_S}{L_S^Y} = w_S, \tag{3.5}$$

$$\gamma \frac{Y_S}{R_S} = p^R \tau_S, \tag{3.6}$$

where $\tau_S = 1 + \theta_S > 0$.

The Intermediate Sector. Each innovation is protected by a patent which gives rise to a monopoly position in the intermediate sector. Note that since there is no IPRs enforcement in the South, this monopoly cannot earn anything from its sales to the South. Therefore, the profit of the j^{th} monopolist is $\big(p_N(j) - 1\big) x_N\big(p_N(j)\big)$, where $x_N\big(p_N(j)\big)$ is the demand for the intermediate good j by the northern final sector.

The price chosen by the monopoly in the North is

$$p_N(j) = \frac{1}{\alpha}, \; \forall j \in [0, A], \tag{3.7}$$

which happens to be independent of j, while the intermediates are sold at their marginal cost in the South[14]:

$$p_S(j) = 1, \forall j \in [0, \phi A]. \tag{3.8}$$

As a result, the equilibrium is symmetric with respect to the quantities of intermediate goods:

$$x_N = \Big(\frac{\alpha^2 {L_N^Y}^\beta {R_N}^\gamma}{(1-s_N)}\Big)^{\frac{1}{1-\alpha}}, \; x_S = \Big(\alpha {L_S^Y}^\beta {R_S}^\gamma\Big)^{\frac{1}{1-\alpha}}. \tag{3.9}$$

No profit is made on x_S. The whole spot profit of an intermediate producer is made from its sales to the North (x_N):

$$\pi_{IG} = (\frac{1-\alpha}{\alpha}) x_N. \tag{3.10}$$

The Research Sector. The intermediate sector buys patents from the research sector at their market value,

$$V(t) = \int_t^{+\infty} \pi_{IG}(s) e^{-\int_t^s r(u)\,du} \, ds, \tag{3.11}$$

[12] In what follows, for notational convenience, we may prefer to use the multiplicative rate τ than the ad valorem rate θ.

[13] In other words, the optimal subsidy is zero. We will come back to this point in the next subsection.

[14] The northern monopolists are indifferent between letting the pirates produce and selling to the South at the marginal cost. For simplicity, we will assume all along that the intermediates are produced in the North. Because no rent can be extracted from the firms using intermediates in the South, no profit is made on the sales of intermediates to the South. Then, our results are robust to the alternative assumption that pirates supply competitively the intermediates used in the South.

where r is the interest rate. The existence of several assets (namely bonds and patents) implies that their rates of return must be equal in equilibrium. Indeed, by log-differentiating (3.11), we get

$$\forall t \in [0, +\infty),\ r(t) = \frac{\dot{V}(t)}{V(t)} + \frac{\pi_{IG}(t)}{V(t)}. \tag{3.12}$$

The profit function of the research sector is $\pi_R = \dot{A}V - w_A(1-\sigma)L_A^R$, where σ is the subsidy rate to the employment of researchers. Free-entry in this sector leads to the standard zero-profit condition:

$$V(t) = \frac{w_N(1-\sigma)}{\psi A}. \tag{3.13}$$

The Extraction Sector. The extraction sector maximizes its discounted profits, $\int_t^{+\infty} p^R(s)R(s)e^{-\int_t^s r(u)\,du}\,ds$, with respect to $R(t)$, $t \in [0, +\infty)$, under its stock constraint (2.5).

The latter program thus results in the Hotelling condition

$$r(t) = \frac{\dot{p^R}(t)}{p^R(t)},\ t \in [0, +\infty). \tag{3.14}$$

Households' Optimization. The households living in both countries, $i = N, S$, maximize their intertemporal utility, $\int_0^{+\infty} \ln\left(\frac{C_i}{L_i}E^{\lambda}\right)e^{-\rho t}\,dt$, with respect to $(C_i)_{t\in[0,+\infty)}$, subject to their budget constraint and the rule that there are no Ponzi games:

$$C_i + \dot{B}_i \leq w_i L_i + rB_i + H_i, \tag{3.15}$$

$$\lim_{t\to+\infty} B_i(t)e^{-\int_0^t r(s)\,ds} = 0, \tag{3.16}$$

where H_i captures all lump-sum transfers to the country i households[15]. This term includes funding of public subsidies and sharing of tax revenues and profits of local firms. Appendix B details all the flows of uses and resources of all agents and shows that $H_S = p^R(R_N + R_S) + \theta_S p^R R_S$ and $H_N = \theta_N p^R R_N - p_N s_N A x_N - \sigma w_N L_N^A$. More precisely, H_N represents the northern environmental tax revenues minus the subsidies to the research sector and to the use of intermediates by the northern final sector, while H_S represents the resource revenues and the southern environmental tax revenues. B_i is the country i's net stock of financial assets[16].

The first-order conditions of the above programm imply the standard Ramsey-Keynes conditions:

$$g_{c_i} = r - \rho,\ i = N, S. \tag{3.17}$$

3.2 Decentralized Equilibrium Outcome

Let us now characterize the general equilibrium of the economy. For simplicity, we will present it under some restrictions.

First, in order to focus on the environmental issue, we will correct the distortions related to the under-investment in research. Moreover, this will simplify the analysis

[15]The way these two terms enter the representative households' problems supposes that the sharing of net governments' budgets and positive local rents is symmetric. This assumption implies that the current paper does not consider local inequality and intra-country transfers.

[16]These assets are actually bonds demanded by the intermediate sector to finance the buying of patents.

a good deal[17]. Above, the optimal subsidy to the use of intermediates in the South is shown to be zero. As concerns the optimal subsidy in the North, equations (3.1) and (3.7) and the unity of the marginal cost of producing an intermediate good lead to the standard subsidy level: $s_N^o = 1 - \alpha$. Formula (C.12) in Appendix C, together with (A.24) in Appendix A, show that the optimal subsidy to employment by the research sector is $\sigma^o = 1 - (1-\alpha)\psi/[\beta(\rho + \psi L_N)/(\beta\rho/(\psi(1-\alpha)) - \phi^{(1-\alpha)/\beta} L_S) - \beta\rho]$. This is a generalization of the standard subsidy to the case of two countries: one being engaged in research and the other not enforcing IPRs. In particular, one can note that σ^o is increasing in the southern index of productivity, ϕ: because of the non-enforcement of IPRs in the South, the northern incentives to innovate do not depend on the diffusion of knowledge; the subsidy σ^o aims in particular at making the North internalize the usefulness of its innovations for the southern final sector. This usefulness is all the larger as the South is productive.

Second, we restrict the national taxes to evolve over time in the same way and not to be too decreasing, i.e. $g_{\tau_N}(t) = g_{\tau_S}(t) = g_\tau(t) > -\rho$, for all $t \in [0, +\infty)$. Later, we will show that it is a necessary condition of the implementation of the first-best equilibrium. Moreover, a difference in the growth rates of these taxes cannot be assumed for more than a finite period of time. Indeed, if these taxes diverge forever, the economy would become too unbalanced in the long run and one country would then collapse[18]. We will thus conserve this assumption even when studying the effects of changes in national tax levels.

Next, let us assume from now on that the southern households do not initially own bonds, i.e. $B_S(0) = 0$. The amount of bonds being equal to the financial demand of the intermediate sector to buy patents, it means that the southern households are not initially the creditors of the owners of patents[19].

The main findings about the decentralized equilibrium are summarized in Proposition 2 and detailed below.

Proposition 2 *In equilibrium of the decentralized economy, when the subsidies to the use of the intermediate goods and to employment in research are optimal:*
ı) L_N^Y and L_N^A immediately jump to constant values. Thus, $g_{A_N} = g_{A_S} = g_A = \psi L_N^A$ is always constant. The relative output, $\frac{Y_N(t)}{Y_S(t)}$, is constant over time, is a decreasing function of the index of southern productivity, ϕ, and a decreasing function of the relative tax, $\frac{\tau_N(t)}{\tau_S(t)}$. For given local outputs, $Y_N(t)$ and $Y_S(t)$, the consumption levels, $C_N(t)$ and $C_S(t)$, are respectively increasing and decreasing functions of the northern tax rate, $\tau_N(t)$.
ıı) If g_τ is constant over time, the economy immediately jumps to its steady-state, in

[17] Characterizing completely the equilibrium outcome only requires to set the subsidies to the use of intermediates at their optimal levels. However, solving the two Romer distortions from the beginning appears to be much more convenient as regards the presentation of the remainder of the paper. Later in this section, we will punctually lift the assumption of an optimal subsidy to research employment in order to examine the effect of environmental taxes on growth.

[18] In a particular case in which $g_{\tau_N} \neq g_{\tau_S}$ forever, one can show that one of the no-Ponzi game conditions is violated.

[19] In the equilibrium at date 0, only the total amount of bonds owned by northern and southern households is determined. The particular sharing of this amount between both kinds of households is an arbitrary assumption about initial endowments. This is made for simplicity. Under an alternative assumption on the sharing of initial financial endowments, our results would have been the same but conditional on this sharing. It would have thus given more complicated expressions of consumption levels.

which:

$$\begin{cases} g_{R_N} = g_{R_S} = g_R = -\rho - g_\tau \\ g_{C_N} = g_{C_S} = g_C = g_Y = g_{Y_N} = g_{Y_S} = g_A - \frac{\gamma(\rho+g_\tau)}{1-\alpha} \\ g_{x_N} = g_{x_S} = g_x = -\frac{\gamma(\rho+g_\tau)}{1-\alpha} \end{cases} .$$

If $g_\tau = 0$, this steady-state equilibrium is identical to the optimal one when households are indifferent to the environmental quality ($\lambda = 0$).
iii) If $g_\tau(t)$ is not constant over time, the economy is always in transition. At any date, the growth rates of $R_N(t)$, $R_S(t)$, $R(t)$, $C_N(t)$, $C_S(t)$, $C(t)$, $Y_N(t)$, $Y_S(t)$, $Y(t)$, $x_N(t)$, $x_S(t)$ and $x(t)$ are decreasing in $g_\tau(t)$.

Proof of Proposition 2 *See Appendix C.*

Let us first explain the role of the growth rate of the environmental taxes, g_τ, on the dynamics of the economy. An increase (decrease) in $g_\tau(t)$ implies a decrease (increase) in the rate of extraction g_R. Indeed, the evolution of the taxes distorts the evolution of the producer price of the resource. Formally, from the Hotelling rule (3.14), the Ramsey-Keynes conditions (3.17) and the growth rates given in (C.5) and (C.6), the law of motion of the resource price is given by $g_{p^R} = g_A + \beta\rho/(1-\alpha) - \gamma g_\tau/(1-\alpha)$. This affects the choice of the extraction sector about when to supply the resource. For instance, if this sector anticipates a higher tax in the future, and thus a relatively lower price, it will supply more resource immediately and less in the future, i.e. it will extract the resource faster. In turn, the speed of extraction will influence the growth rates of the other unfixed inputs, and thus of the total production and consumption. This effect has been cogently highlighted in some papers (e.g. Sinclair [22], Grimaud and Rougé [10], Sinn [24] and Daubanes [8]). Beyond the effects of the time profile of the environmental taxes, our two-country approach enables to study how the level of the tax in one region affects the global economy.

Let us now examine the effects of the absolute levels of the taxes. These are rather different from those of their time profile. First, they affect the allocation of the resource among the two final sectors. Because the producer price of the resource is the same wherever it is used, an increase in the tax rate of one country relative to the other renders the resource relatively more expensive there. The marginal productivities being equalized to the local final prices, this implies that a higher part of the flow of resource is used, and thus a higher part of the output is produced, in the latter country. In other words, the levels of the resource taxes imply some relocation of the economic activity.

Second, they imply transfers among countries. In Appendix C, (C.10) and (C.11) give the initial consumption levels in the two countries: $C_N(0) = (1-\alpha)Y_N(0) - \gamma\rho D Y_N(0)/\tau_N(0)$ and $C_S(0) = (1-\alpha)Y_S(0) + \gamma\rho D Y_N(0)/\tau_N(0)$, where D is a given positive scalar. In these equations, the northern tax rate, τ_N, affects directly, i.e. beyond its effects through the national outputs, the consumption levels. Precisely, taking as given the national productions, it affects positively the northern consumption and negatively the southern one. From condition (3.3), the total payment for the resource input by the northern final sector is $\tau_N p^R R_N = \gamma Y_N$. This is composed by the net resource revenues made in the North, $p^R R_N = \gamma Y_N/\tau_N$, and the tax revenues from the northern use of the resource, $(\tau_N - 1)p^R R_N = \gamma Y_N(\tau_N - 1)/\tau_N$. From these expressions, the northern output being taken as constant, an increase in the northern tax rate increases the tax revenues at the expense of the resource revenues. The former remaining in the North and the latter being earned by the South, this capture of some resource rents through taxation improves the consumption of northern households and deteriorates that of the southern ones. This rent transfer effect has already been mentioned formally out of the environmental literature by Bergstrom [4] and Brander and Djajic [6][20]. This distributional effect has much to do with the inelasticity of the asymptotic cumulated extraction of the resource (On this, see also Sinn [24]). As concerns the tax on the southern use of the resource, τ_S, it has not this distributional effect. Indeed, tax revenues made via this tax would be earned anyway by the South government through resource revenues. Hence, this tax only affects the relative output of both regions but do not enter directly their consumption.

Let us now evoke the potential dynamic effects of the tax levels. Contrary to their growth rates, g_τ, the levels of the tax, τ_N and τ_S, do not affect any growth rate. This is surprising. Indeed, if the tax levels influence the geographic repartition of the productive activities, and thus of the use of each intermediate good, and if IPRs are not homogeneously enforced across countries, one could expect them to affect the revenues from innovating, and thus growth. Actually, even if the subsidy to employment in research is suboptimal, the local taxes have no growth effect. In Appendix C (formula (C.4)), the allocation of labor is shown to be given by $L_N^Y = (\rho + \psi L_N)/[\psi(1 + (1-\alpha)/(\beta(1-\sigma)))]$ and $L_N^A = L_N - L_N^Y$ while growth is driven by $g_A = \psi L_N^A$ (formula (C.3)). There is no labor reallocation in the North after a change in national taxes. Let us consider an increase in τ_N. It implies a relocation of the mobile inputs (intermediates and resource) towards the South. On the one hand, the value of innovations decreases because the use of intermediates in the South is not profitable. This deteriorates the productivity of northern researchers. On the other hand, relocation of mobile inputs decreases the productivity of workers in the northern final sector as well. In a Cobb-Douglas world, the marginal productivities of labor in the two sectors decrease by the same proportion, thus not requiring any reallocation on the northern labor market. This explains the absence of the presumed growth effect of the tax levels[21]. Nevertheless, this effect would appear under other functional forms. Generally, because relocation deteriorates the productivity of northern labor in its two uses, this effect can be expected not to be substantial.

3.3 Optimal Environmental Taxation

In the previous subsection, the subsidies to the use of the intermediates and to the research sector have been set at their optimal levels. Here, they remain at these levels so that we keep focusing exclusively on the pollution externality. By comparing the rate of extraction in the welfare-maximizing allocation and in the decentralized outcome, we can solve for the resource taxation policy that implement an optimal use over time of the polluting resource.

The following proposition gives the main characteristics of this policy.

Proposition 3 *i) For the rate of extraction, R, and thus the rate of pollution accumulation, $\dot{E}$, to be efficient, the local tax rates on the resource use, τ_N and τ_S, must evolve at the same negative rate of growth, g_τ^o.*
ii) For the allocation of the flow of resource, R, between the two final sectors, N and S, to be efficient, these tax rates must be equalized.

Proof of Proposition 3 *See Appendix C.*

The equality of both tax rates, τ_N and τ_S, is required for efficiency. Indeed, it ensures that the resource final prices, and thus the resource marginal productivities, are equalized

[20] Nordhaus and Boyer [15] also mentioned it, but not formally. In their conclusion, they noted that if the resource supply is perfectly inelastic "(...) carbon taxes may have no economic effect at all and would simply redistribute rents from the resource owners to the government".

[21] Here, its absence will simplify substantially the analysis.

in both countries, which is a necessary condition for static efficiency. However, because the tax levels do not affect the dynamics of extraction, they play no role in the correction of the environmental externality.

To correct it, they simply should have the same optimal growth rate. As argued in the previous subsection, this rate is the relevant instrument to manipulate the extraction path.

It is worth noting that, even with different local tax levels, the global environmental problem could be theoretically solved by the use of this latter instrument. The equalization of the local taxes has thus nothing to do with the environmental correction but with the efficiency of the resource allocation.

Overall, the optimal international tax, inducing the efficiency of both the allocation of the resource over time and the split of its instantaneous flow, is defined up to an homothecy: if $\tau_N(t) = \tau_S(t) = \tau(t)$, $t \geq 0$ is optimal, then $k\tau_N(t) = k\tau_S(t) = k\tau(t)$, $t \geq 0$, $\forall k > 0$ is also optimal.

In Appendix C (formula (C.13)), the optimal time profile of the common tax rate is shown to be decreasing over time. This time profile would provide the society with an incentive to postpone the depletion of the polluting resource, i.e. to extract it slower. This is consistent with the remark in Section 2 that the introduction of pollution implies a slower optimal depletion. In Sinclair [22] and Groth and Schou [12], the same property arises. It extends to the case where the flow of pollution is harmful (Grimaud and Rougé [10]) and is robust to the introduction of a decaying stock of pollution (Ulph and Ulph [25] and Withagen [26]).

Compared to these results obtained in aggregated models, our two-country approach emphasizes the need that countries coordinate on an international tax level. The remainder of the paper focuses on the difficulty to achieve such a coordination by investigating the national conflicting viewpoints about the two environmental tax rates.

4 Effects of the National Tax Levels

In this section, we focus on the effects of the environmental taxes on the consumption levels and welfare of both countries. We keep assuming that the only distortion is environmental, i.e. $s_N = s_N^o$, $s_S = s_S^o$ and $\sigma = \sigma^o$. Moreover, we keep assuming that $g_{\tau_N} = g_{\tau_S} = g_\tau$, but we let this growth rate not be optimal and the environmental tax rates, τ_N and τ_S, not be equals. By effects of the tax levels, we mean effects of the mere fact that a country sets a higher or a lower tax rate, its time profile, given by $g_\tau(t)$, $t \geq 0$, remaining unchanged[22].

The previous section gave some insights about the effects of the tax levels on consumption. In particular, their distributional aspects provide heterogeneous countries with conflicting interests. This is why the taxation of polluting exhaustible resources should be addressed in a multi-country setting featuring some heterogeneity. However, stating this distributional effect is not sufficient since the tax rates also affect consumption through their relocational effect, likely to be related to global efficiency. Indeed, one country in the global economy, when setting its tax rate, faces the interference of these channels. Hence

[22] More formally, a tax policy is represented by a function of time, $\tau(t)$, for all $t \in [0, +\infty)$. Restricting attention to functions that are continuous, any tax profile $(\tau(t))_{t=0}^{+\infty}$ is completely characterized by $\imath$) an initial level, $\tau(0)$, and $\imath\imath$) a rate of growth profile, $(g_\tau(t))_{t=0}^{+\infty}$. From now on, by considering a change in the tax level, we refer to a change in the initial level, the growth rate profile remaining unchanged, that is to say a homothetic transformation of the whole tax profile, i.e. the substitution of $\tau(t)$ by $k\tau(t)$, $k > 0$, for all $t \in [0, +\infty)$.

the two kinds of effects on the decentralized equilibrium have to be examined separately and assessed together. This is the objective of this section.

4.1 Rent Capture

Isolating the rent transfer effect of the tax levels requires to rule out relocation, i.e. changes in national output. To this purpose, the following proposition considers an increase in both tax rates that does not modify their ratio.

Proposition 4 *An increase in both tax levels, τ_N and τ_S, by the same proportion, i.e. their ratio τ_N/τ_S remaining unchanged,*
ı) does not entail any relocation, the national outputs, Y_N and Y_S, being unaffected at all dates.
ıı) implies an inter-country transfer, increasing the northern consumption, C_N, at the expense of the southern one, C_S, at all dates.

Proof of Proposition 4 *See Appendix D.*

Since it can be isolated from any distortion, this rent transfer effect is purely distributional.

As detailed in Subsection 3.2, this effect is a transfer from the exploiters' rent to northern tax revenues. It thus benefits the resource poor economy and deteriorates the revenues of the resource rich country. Moreover, the tax rates play an asymmetric role: the only tax rate involved in this rent transfer effect is the northern one. Indeed, the southern tax rate would make the South collect tax revenues it would have earned anyway through resource rents.

As a consequence, this effect will provide only the resource poor North with an incentive to tax high.

4.2 Relocation and Global Inefficiency

Changes in the tax ratio entail, in equilibrium, a change in the relative marginal productivity of the resource. Their effects are twofold. First, it changes the geographic split of the resource flow, thus modifying national productions. Second, it necessarily affects the total output. The following proposition assesses these effects.

For simplicity, let us consider a change in the relative tax caused by a unilateral change in the northern tax level.

Proposition 5 *World production, $Y = Y_N + Y_S$, is maximal when the tax levels, τ_N and τ_S, are equalized across countries.*
A unilateral increase in the northern tax level, τ_N,
ı) leads to some relocation of the final production by decreasing the northern output, Y_N, and increasing the southern one, Y_S, at all dates.
ıı) yields a global efficiency distortion, the world output increasing at all dates if $\tau_N < \tau_S$ and decreasing if $\tau_N > \tau_S$.

Proof of Proposition 5 *See Appendix D.*

A change in the tax ratio improves the relative competitiveness of one country and plays an important role on the efficiency of the resource allocation among the two economies. This allocation is optimal when final prices, and thus marginal resource productivities, are equalized. A departure from this equality distorts the world economy. Proposition 5 tells in particular that the split of the resource flow between the two countries is all the less efficient as the tax rates are wide apart.

In Figure 2, $Ymax$ is the first bisecting line. The derivative of the world production level at any date with respect to the northern tax level, $\partial Y(t)/\partial \tau_N(0)$, $\forall t \geq 0$, is zero on this line, negative above (sectors 1, 2 and 6) and positive below (sectors 3, 4 and 5)[23].

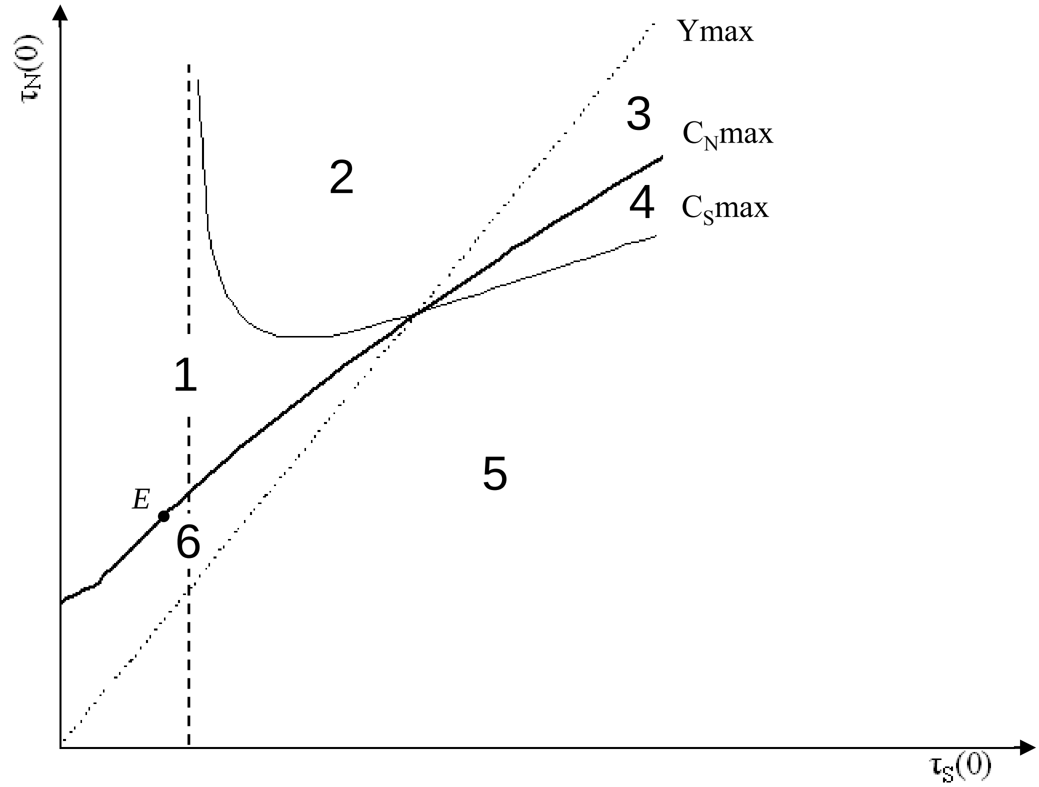

Figure 2: Effects of an increase in the northern tax level, $\tau_N(0)$, on national consumptions, C_N and C_S, and world production, Y.

Because relocation comes from changes in the tax ratio, the effect of an increase in the southern tax level is similar to that in the northern one. Hence, relocation affects both countries in a rather symmetric way. It implies that a higher tax in one region, whatever it is, reduces its output. Moreover, the impact of the tax rates on global efficiency gives a common interest in having close tax rates. However, we shall see later that the more totally productive one country is, the less affected it is by the relocation resulting from a relatively higher local tax.

4.3 Effects of a Unilateral Increase in the Tax Level

The national tax levels affect national consumptions and welfare through both rent capture and relocation. After characterizing these two effects separately, it is worth computing the overall change in national consumptions due to changes in the tax rates. As in the previous subsection, let us consider a unilateral change in the northern tax level. The impacts of such a change will depend on how high are the two national levels with respect to each other.

[23]Because the tax levels grow at the same rate, we could have represented the graph of Figure 2 in terms of the tax levels $\tau_N(t)$ and $\tau_S(t)$ at any date $t \geq 0$. For consistency with our definition of the tax levels in note 22 and with Appendix D, we choose a basis in terms of the taxes at date 0.

The results are formally given in Appendix D. The main findings are presented in Proposition 6 and illustrated and detailed below.

Proposition 6 *An increase in the northern tax level, τ_N, leads to an increase in the northern consumption, C_N, at all dates if the northern tax level is relatively low. It leads to an increase in the southern consumption, C_S, if both the southern tax level and the northern one are relatively high.*

Proof of Proposition 6 *See Appendix D.*

In Figure 2, along $C_N max$ and $C_S max$, the derivatives of the northern and the southern consumption levels at all dates with respect to the northern tax level, $\partial C_N(t)/\partial \tau_N(0)$ and $\partial C_S(t)/\partial \tau_N(0)$, $\forall t \geq 0$, respectively, are zero. These loci divide generally[24] the strictly positive orthant into six sectors. $\partial C_N(t)/\partial \tau_N(0)$ is positive below $C_N max$ (sectors 4, 5 and 6) and negative above (sectors 1, 2 and 3). $\partial C_S(t)/\partial \tau_N(0)$ is positive above the defined part of $C_S max$ (sectors 2, 3 and 4) and negative otherwise (sectors 1, 5 and 6).

The interference of the two effects analyzed above happens to be complicated: one can a priori hardly tell something about the effects of a unilateral tax increase on the two consumption levels, since they could be everything and anything. Let us refer to our previous results to understand how the tax rates affect consumption in both regions.

From the northern viewpoint, a greater tax rate would be beneficial if the tax rate is low and detrimental if it is already high. This results from the trade-off highlighted in the previous subsections. On the one hand, a greater environmental tax leads to more tax revenues. On the other hand, it leads to less competitiveness relatively to the other country, and thus relocation of its productive activities. Proposition 6 tells that the latter effect more than compensates the former when the North has a relatively high tax rate and inversely otherwise. This northern tax increase leads to a greater southern consumption if both the northern and the southern tax rates are high. In that case, the relocation effect benefits the South but fails to compensate the substantial rent capture effect.

What about the viewpoint of the South with respect to its own tax rate? A greater tax rate in the South would make it relatively less competitive. Moreover, as noted in Subsection 4.1, the South would not benefit from any rent capture effect.

As a result, if countries are not coordinated in the setting of their local taxes, the North would choose a relative high tax rate.

In order to illustrate this and the conflicting interests of the two different regions as concerns their environmental tax levels, let us consider that the two governments use strategically their tax levels to maximize their residents' utilities, subject to the decentralized decisions of firms. For simplicity, assume that the rate of growth of the taxes is given to them. In this context, the best-response of the North government (maximizing northern welfare) can be shown to be represented by $C_N max$. In Appendix D, the best-response of the South (maximizing southern welfare) is computed: it is independent of the northern tax rate. It is thus represented in Figure 2 by a vertical line that is shown to be located at the right of the vertical asymptote of $C_S max$. Hence, the Nash equilibrium of this simple game is along $C_N max$, at a point below the vertical asymptote of $C_S max$ (as for instance, point E in Figure 2). Let us give further details and make two remarks on this equilibrium.

[24] The form of Figure 2 and its sectors are general. Figure 2 is an illustration for a certain combination of parameters but its form is shown to be general in Appendix D.

First, in the Nash outcome, the North has indeed a greater tax rate than the South. Our examination of the rent transfer effect and of the relocation effect suggests that this gap is mainly due to the asymmetric rent transfer effect, the North benefiting more than the South from a high tax rate on the polluting resource. This implies that the economically relevant regions of Figure 2 are sectors 1 and 6. These regions are also relevant because this Nash equilibrium is consistent with empirical findings. Indeed, Bacon [2] shows that tax rates on petroleum products are significantly higher in oil importing countries than in oil exporting counties. In particular, taxes on petroleum products in developed oil consuming countries are very high (IEA [14]).

Second, the equation of $C_N max$, in (D.1), shows that the best-response of the North is decreasing in the index of development of the South, ϕ, whereas the best-response of the South is independent of ϕ. Hence, the poorer the South is, the higher point E is along the southern best-response, and the wider the gap between the northern and the southern tax rates. The explanation for this is the following: from the northern viewpoint, the negative effect of a high tax rate is its loss of competitiveness and the related relocation towards the South. The less totally productive the South is, the less affected by the threat of relocation the North is. As a result, the North chooses a tax rate that is all the greater as the South is low-productive. This is also supported by Bacon's observations. Indeed, he notes that higher-income countries set their average per unit taxes on gasoline and diesel two and a half times higher than developing countries do.

Eventually, these remarks imply that the current distortion due to the gap between the tax rates in the oil consuming countries and those in the oil producing ones is all the more serious as the latter have a relatively low productivity compared to the former. Moreover, it suggests that an increase in the tax rates on petroleum products in some oil consuming countries, where these rates are already very high (sector 1 in Figure 2), would deteriorate the efficiency of the world allocation of oil and would make people of the two regions worse-off.

5 Conclusion

In this paper, we have divided a model of a non-renewable resource depletion and of pollution accumulation into two regions in order to investigate the inter-country effects of local environmental taxes in a heterogeneous world. We have then constructed a two-country growth model in which the South is endowed with the entire stock of an essential polluting non-renewable resource and in which world economic growth is driven by a northern research sector.

First, the welfare maximizing allocation and the decentralized equilibrium outcome for given national tax rates have been fully characterized. Consistently with the one-country literature, we obtain that the time-profile of environmental taxes alters the extraction path and thus the speed of pollution accumulation. The socially optimal resource depletion is implemented through decreasing ad valorem tax rates on the use of the polluting resource in the two countries. Beyond the correction of the environmental problem, this extension to two countries emphasizes that their coordination on a common tax level is required to ensure the efficiency of the resource allocation. However, this coordination appears to be difficult given the divergent views of the two heterogeneous countries regarding their tax rates.

Our study of the international effects of national environmental tax rates highlights the reasons of this coordination problem. First, higher taxes lead to a partial capture of the southern resource rent by the North through tax revenues in a purely distributional way. Second, a higher environmental tax in one country deteriorates its relative competitiveness. The resulting relocation deteriorates global efficiency of the international split of the resource flow if it moves one tax rate further apart the other. However, in spite of this relocation and the heterogeneous IPRs protection, local tax levels do not affect the effort of research and thus growth.

As a result of these effects, the oil poor region is attracted by high environmental tax rates while the oil rich region is not. If the tax levels are set at the national scale, they will be used strategically, thus entailing heterogeneous tax rates. The resulting gap in environmental taxes appears to be all the wider as the oil-rich countries are relatively poor. Overall, a global distortion arises on the allocation of oil, which is of a particular concern as this resource is bound to get scarcer and scarcer.

Furthermore, we have assessed the effect of a hypothetical increase in the northern tax rate on the polluting exhaustible resource. From the current situation, with already very high environmental taxes in the North, the analysis suggests that increasing their level would deteriorate global efficiency and would make the two regions worse-off.

Addressing the strategic tax-setting issue in a satisfying way would actually require to apply dynamic game theoretic tools, which is extremely difficult in a resource depletion model. Further research must tackle this question in order to improve our understanding of environmental tax design. Indeed, whether taxes are used or not to implement international environmental policies, homogenizing the currently very heterogeneous taxes on oil will be needed to improve the allocation of this scarce resource.

A Appendix: Model and Welfare

Typically, the amount of each intermediate good does not depend on its identity. Due to the Inada conditions verified by the utility functions, the technical constraints will be binding and the non-negativity constraints can be dropped. Then the optimization problem of the social planner reduces to the maximization of (2.11) with respect to C_i, x_i, R_i, L_N^Y, $i = N, S$ subject to

$$C_N + C_S + Ax_N + \phi A x_S = A{x_N}^\alpha {L_N^Y}^\beta {R_N}^\gamma + \phi A {x_S}^\alpha {L_S}^\beta {R_S}^\gamma, \quad (A.1)$$

$$\dot{A} = \psi A(L_N - L_N^Y), \quad (A.2)$$

$$\dot{Q} = -(R_N + R_S), \quad (A.3)$$

$$\dot{E} = -h(R_N + R_S). \quad (A.4)$$

Let μ_Y, μ_A, μ_Q and μ_E denote the present-value multipliers associated respectively to the constraints (A.1)-(A.4). The first-order and transversality conditions that should be satisfied are

$$\begin{aligned}
\delta\frac{L_N}{C_N}e^{-\rho t} &= \mu_Y, && \text{(A.5)}\\
(1-\delta)\frac{L_S}{C_S}e^{-\rho t} &= \mu_Y, && \text{(A.6)}\\
\alpha\frac{Y_N}{x_N} &= A, && \text{(A.7)}\\
\alpha\frac{Y_S}{x_S} &= \phi A, && \text{(A.8)}\\
\mu_Y\gamma\frac{Y_N}{R_N} &= \mu_Q + h\mu_E, && \text{(A.9)}\\
\mu_Y\gamma\frac{Y_S}{R_S} &= \mu_Q + h\mu_E, && \text{(A.10)}\\
\mu_Y\beta\frac{Y_N}{L_N^Y} &= \mu_A\psi A, && \text{(A.11)}\\
\frac{\mu_Y}{\mu_A}\Big(\frac{Y_N+Y_S}{A} - x_N - \phi x_S\Big) + \psi(L_N - L_N^Y) &= -\frac{\dot{\mu_A}}{\mu_A}, && \text{(A.12)}\\
\dot{\mu_Q} &= 0, && \text{(A.13)}\\
\delta L_N\lambda e^{-\rho t} + (1-\delta)L_S\lambda e^{-\rho t} &= -E\dot{\mu_E}, && \text{(A.14)}\\
\lim_{t\to+\infty}\mu_A(t)A(t) = 0, & && \text{(A.15)}\\
\lim_{t\to+\infty}\mu_Q(t)Q(t) = 0, & && \text{(A.16)}\\
\lim_{t\to+\infty}\mu_E(t)E(t) = 0. & && \text{(A.17)}
\end{aligned}$$

a) From (A.7) and (A.8) we have $\frac{Y_S}{Y_N} = \frac{\phi x_S}{x_N}$, while from (A.9) and (A.10) we get $\frac{Y_S}{Y_N} = \frac{R_S}{R_N}$. Combining those two equations with the reduced forms $Y_N = A{x_N}^{\alpha}{L_N^Y}^{\beta}{R_N}^{\gamma}$ and $Y_S = \phi A{x_S}^{\alpha}{L_S}^{\beta}{R_S}^{\gamma}$, we get

$$\frac{Y_S^o(t)}{Y_N^o(t)} = \frac{R_S^o(t)}{R_N^o(t)} = \frac{\phi x_S^o(t)}{x_N^o(t)} = \phi^{\frac{1-\alpha}{\beta}}\frac{L_S}{{L_N^Y}^o}. \quad \text{(A.18)}$$

b) Rewrite (A.1) as

$$C_N + C_S + Ax_N + \phi Ax_S = Y_N + Y_S. \quad \text{(A.19)}$$

Substituting in (A.19) the conditions (A.7) and (A.8) and rearranging we get

$$\frac{C^o(t)}{Y^o(t)} = 1-\alpha \text{ and } \frac{A_N^o(t)x_N^o(t) + \phi A_N^o(t)x_S^o(t)}{Y^o(t)} = \alpha. \quad \text{(A.20)}$$

c) From (A.2), we have immediately

$$g_{A_N^o} = g_{A_S^o} = g_{A^o} = \psi {L_N^A}^o. \quad \text{(A.21)}$$

d) Now, we show that L_N^Y is constant. Log-differentiating (A.11), we get $\dot{\mu_Y}/\mu_Y - \dot{\mu_A}/\mu_A = g_A + g_{L_N^Y} - g_{Y_N}$. Log-differentiating (A.5), we get $\dot{\mu_Y}/\mu_Y = -g_{C_N} - \rho$. Plugging this into the previous equation, we obtain $-\dot{\mu_A}/\mu_A = g_{C_N} + \rho + g_A + g_{L_N^Y} - g_{Y_N}$. From (A.11), we also have $\mu_Y/\mu_A = \psi A L_N^Y/\beta Y_N$. Substituting the expressions of $-\dot{\mu_A}/\mu_A$ and μ_Y/μ_A in (A.12) and simplifying by (A.20), we obtain

$$g_{C_N} - g_{Y_N} + g_{L_N^Y} = -\rho + \frac{\psi}{\beta}(1-\alpha)L_N^Y\frac{(Y_N+Y_S)}{Y_N}, \quad \text{(A.22)}$$

where $(Y_N + Y_S)/Y_N = (L_N^Y + \phi^{(1-\alpha)/\beta} L_S)/L_N^Y$, from (A.18).

From (A.5), (A.6) and (A.20), we have $g_{C_N} = g_{C_S} = g_C = g_Y$. Thus, we have $g_{C_N} - g_{Y_N} = g_Y - g_{Y_N} = (\dot{Y}_N + \dot{Y}_S)/(Y_N + Y_S) - \dot{Y}_N/Y_N = (\dot{Y}_S Y_N - \dot{Y}_N Y_S)/((Y_N+Y_S)Y_N) = (\dot{Y_S/Y_N})/(1 + Y_S/Y_N)$. Using (A.18), $Y_S/Y_N = \phi^{(1-\alpha)/\beta} L_S/L_N^Y$, which gives $(\dot{Y_S/Y_N}) = -\phi^{(1-\alpha)/\beta} L_S g_{L_N^Y}/L_N^Y$, the previous equation reduces to $g_{C_N} - g_{Y_N} = -\phi^{(1-\alpha)/\beta} L_S g_{L_N^Y}/(L_N^Y + \phi^{(1-\alpha)/\beta} L_S)$. Substituting this later expression in (A.22) and defining $u = L_N^Y + \phi^{(1-\alpha)/\beta} L_S$, we obtain the differential equation $\dot{u} = -\rho u + (1-\alpha)\psi u^2/\beta$, whose solution is[25]

$$u = \frac{1}{e^{\rho t}\left(\frac{1}{u(0)} - (1-\alpha)\frac{\psi}{\beta\rho}\right) + (1-\alpha)\frac{\psi}{\beta\rho}}. \tag{A.23}$$

From (A.11), the transversality condition (A.15), writes $\lim_{t\to+\infty} \mu_Y Y_N/L_N^Y = 0$. Note that $(1-\delta)C_N(0)\mu_Y/L_N + \delta C_S(0)\mu_Y/L_S$ is a constant fraction of μ_Y so that the transversality condition rewrites $\lim_{t\to+\infty}[(1-\delta)C_N(0)\mu_Y/L_N + \delta C_S(0)\mu_Y/L_S]Y_N/L_N^Y = 0$. Using now (A.5) and (A.6), the term in brackets is $\delta(1-\delta)e^{-\rho t}[C_N(0)/C_N + C_S(0)/C_S]$. Since $g_{C_N} = g_{C_S} = g_C = g_Y$, this term rewrites $2\delta(1-\delta)e^{-\rho t}e^{-\int_0^t g_Y(s)ds}$, i.e. $2\delta(1-\delta)e^{-\rho t}Y(0)/Y$. Finally, the transversality condition becomes $\lim_{t\to+\infty} Y_N e^{-\rho t}/(Y L_N^Y) = 0$. Using that $Y = Y_N + Y_S$ and equation (A.18), this eventually yields $\lim_{t\to+\infty} e^{-\rho t}/u(t) = 0$.

From (A.23), $\lim_{t\to+\infty} e^{-\rho t}/u(t) = 1/u(0) - (1-\alpha)\psi/(\beta\rho)$. Then, from the transversality condition, $u(0) = \beta\rho/((1-\alpha)\psi)$, which is the steady-state value of u. Hence, L_N^Y jumps also at date 0 to its constant value given by

$$L_N^{Y\,o} = \frac{\beta\rho}{\psi(1-\alpha)} - \phi^{\frac{1-\alpha}{\beta}} L_S,\; L_N^{A\,o} = L_N - L_N^{Y\,o},\; L_S^{Y\,o} = L_S. \tag{A.24}$$

e) Substituting (A.5) in (A.9), we get $\mu_Q + h\mu_E = \gamma\delta L_N e^{-\rho t} Y_N/(C_N R_N)$. Combining this result with (A.14), we find $\dot{\mu_E}/(\mu_Q + h\mu_E) = -\lambda(\delta L_N + (1-\delta)L_S)C_N R_N/(\gamma\delta L_N Y_N E)$. Using now that from (A.5) and (A.6), $(1-\delta)L_S C_N/(\delta L_N) = C_S$, we finally have: $\dot{\mu_E}/(\mu_Q + h\mu_E) = -\lambda(C_N + C_S)R_N/(\gamma Y_N E)$. From (A.9) and (A.10), we have $Y_N/R_N = Y_S/R_S$, and thus $Y_N/R_N = Y/R$. Moreover, from (A.20), $C = (1-\alpha)Y$. Finally, the previous equation rewrites $\dot{\mu_E}/(\mu_Q + h\mu_E) = -\lambda(1-\alpha)R/(\gamma E)$.

Log-differentiating equation (A.9) after substituting μ_Y from (A.5), we get another expression of $\dot{\mu_E}/(\mu_Q + h\mu_E)$: $\dot{\mu_E}/(\mu_Q + h\mu_E) = (g_{Y_N} - g_{C_N} - g_{R_N} - \rho)$. Using (A.18) and the constancy of L_N^Y, we have $g_{Y_N} = g_{Y_S} = g_Y$ and $g_{R_N} = g_{R_S} = g_R$. Moreover, from (A.20), we have $g_Y = g_C$, while we already know that $g_{C_N} = g_{C_S} = g_C$. Then, equalizing the left-hand sides of both equations above, we get $g_R = (h\lambda(1-\alpha)/\gamma)R/E - \rho$. Using now (A.4), we obtain $g_R = -\lambda(1-\alpha)g_E/\gamma - \rho$. We then have

$$g_{R_N^o}(t) = g_{R_S^o}(t) = g_{R^o}(t) = -\rho - \frac{\lambda(1-\alpha)}{\gamma} g_{E^o}(t). \tag{A.25}$$

f) Log-differentiating the production functions (2.1) in which $x_i(j) = x_i$, $i = N, S$, one gets $g_{Y_i} = g_A + \alpha g_{x_i} + \gamma g_{R_i}$ since $g_{L_i^Y} = 0$. Log-differentiating (A.7) and (A.8), we have $g_{x_i} = g_{Y_i} - g_A$, $i = N, S$. It follows

$$g_{C_N^o}(t) = g_{C_S^o}(t) = g_{C^o}(t) = g_{Y^o}(t) = g_{Y_N^o}(t) = g_{Y_S^o}(t) = g_{A^o} + \frac{\gamma}{1-\alpha} g_{R^o}(t), \tag{A.26}$$

$$g_{x_N^o}(t) = g_{x_S^o}(t) = g_{x^o}(t) = \frac{\gamma}{1-\alpha} g_{R^o}(t). \tag{A.27}$$

[25] In order to transform this Ricatti differential equation into a linear first-order one, we consider a new variable $z \equiv 1/u$, which implies $\dot{z} = -\dot{u}/u^2$.

g) From (A.5) and (A.6), we get

$$\frac{C_N^o(t)/L_N}{C_S^o(t)/L_S} = \frac{\delta}{1-\delta},\ 0 < \delta < 1. \tag{A.28}$$

The previous equations give Proposition 1 and the other results of Section 2.

B Appendix: The Agents Uses and Resources

NORTH

Uses	Households	Resources
.consumption: C_N .accumulation of assets: $\dot{B}_N$		.$w_N(L_N^Y + L_N^A)$: wages .rB_N: interests .$\mathbf{H_N = -T}$ $= \theta_\mathbf{N} \mathbf{p^R R_N} - \mathbf{p_N s_N A x_N} - \sigma \mathbf{w_N L_N^A}$
Uses	**Firms**	**Resources**
.wages: $w_N L_N^Y$ .resource: $p^R R_N$ .taxes on resource use: $\theta_N p^R R_N$ .intermediates: Ax_N/α		.Y_N: sales .$p_N s_N A x_N$: subsidy to the use of intermediates
Uses	**Research and intermediate sectors**	**Resources**
.wages: $w_N L_N^A$ .buying of final good: Ax_N .interests: rB		.Ax_N/α: sales of intermediates .$\dot{B}$: debt variation .$\sigma w_N L_N^A$: subsidy to the employment of researchers
Uses	**Government**	**Resources**
.subsidy to the use of intermediates: $\mathbf{p_N s_N A x_N}$ .subsidy to the employment of researchers: $\sigma \mathbf{w_N L_N^A}$		.$\theta_\mathbf{N} \mathbf{p^R R_N}$: environmental taxes .$\mathbf{T}$: taxes on households

From the uses and resources of the households and the government, H_N is the environmental tax revenues of the North minus the public expenses which are the subsidies to the research sector and to the use of the intermediates. It can be negative or positive and represents the contribution of the northern households to the public expenses.

SOUTH

Uses	Households	Resources
.consumption: C_S .accumulation of assets: $\dot{B}_S$		.$w_S L_S$: wages .rB_S: interests .$\mathbf{H_S} = \mathbf{p^R(R_N + R_S)} + \theta_\mathbf{S} \mathbf{p^R R_S}$
Uses	**Firms**	**Resources**
.wages: $w_S L_S$ .resource: $p^R R_S$ .tax on resource use: $\theta_S p^R R_S$ .intermediates: $\phi A x_S$		.Y_S: sales
Uses	**Resource sector**	**Resources**
.profits: $\mathbf{p^R(R_N + R_S)}$		.$p^R(R_N + R_S)$: sales
Uses	**Intermediate sector**	**Resources**
.buying of final good: $\phi A x_S$		.$\phi A x_S$: sales
Uses	**Government**	**Resources**
.redistribution: $\theta_\mathbf{S} \mathbf{p^R R_S}$		.$\theta_S p^R R_S$: tax revenue

Here, we see that H_S is composed by the profits of the resource sector and the tax revenue from the environmental tax on the resource use. H_S is non ambiguously positive.

C Appendix: Decentralized Equilibrium for Given National Taxes

From (2.4), the marginal cost of producing intermediates is equal to unity. The left-hand side of (3.1) is the marginal productivity of intermediate goods used in the North. Thus, using (3.5), one gets that the subsidy to their use, ensuring that marginal cost equals marginal productivity, is $s_N^o = 1-\alpha$. On the contrary, from (3.6), the intermediates used in the South are sold at their marginal cost. Hence, no subsidy is needed in the South, i.e. $s_S^o = 0$. Let us assume that $s_N = s_N^o$ and $s_S = s_S^o$ all along.

Moreover, we assume that $g_{\tau_N}(t) = g_{\tau_S}(t) = g_\tau(t) > -\rho$, $\forall t \geq 0$. This implies that $\tau_N(t)/\tau_S(t)$ is constant over time.

Here, we only restrict σ to be constant and make no assumption on $B_S(0)$. We will fix the values of these parameters later in the proof.

a) Write the ratio of both production functions from (2.1) after noting that, from (3.9), $x_i(j) = x_i$, $i = N, S$. Compute the ratio of the resource quantities used by each final sector from equations (3.3) and (3.6). Using (3.1) and (3.4) after replacing the equilibrium prices given by (3.7) and (3.8), write the ratio of x_S/x_N. Combining these three ratios, we find

$$\frac{Y_S(t)}{Y_N(t)} = \frac{\tau_S(t)R_S(t)}{\tau_N(t)R_N(t)} = \frac{\phi x_S(t)}{x_N(t)} = \left(\frac{\tau_N(t)}{\tau_S(t)}\right)^{\frac{\gamma}{\beta}} \phi^{\frac{1-\alpha}{\beta}} \left(\frac{L_S}{L_N^Y}\right). \tag{C.1}$$

b) (3.9) can be written $\alpha Y_N/x_N = A$ and $\alpha Y_S/x_S = \phi A$. Substituting these conditions in (2.9) leads to $C = (1-\alpha)Y$, which gives

$$\frac{C(t)}{Y(t)} = 1-\alpha, \ \frac{A(t)x_N(t) + \phi A(t)x_S(t)}{Y(t)} = \alpha. \tag{C.2}$$

c) From (2.2), we immediately have

$$g_{A_N} = g_{A_S} = g_A = \psi L_N^A. \tag{C.3}$$

d) We now show that L_N^Y is constant over time. Combining (3.10) and (3.13) we get $\pi_{IG}/V = (1-\alpha)\psi A x_N/(\alpha(1-\sigma)w_N)$. Substituting w_N from (3.2) and using $Ax_N = \alpha Y_N$ from above, we find $\pi_{IG}/V = (1-\alpha)\psi L_N^Y/(\beta(1-\sigma))$. From (3.13), we also obtain that $g_V = g_{w_N} - g_A$ which gives from (3.2) $g_V = g_{Y_N} - g_{L_N^Y} - g_A$. Substituting the later expressions of g_V and π_{IG}/V in (3.12) and replacing the rate of interest with $r = g_C + \rho$ (from (3.17)) and rearranging, we get a first expression of $g_C - g_{Y_N}$: $g_C - g_{Y_N} = -\rho - g_{L_N^Y} - g_A + (1-\alpha)\psi L_N^Y/(\beta(1-\sigma))$. Due to equation (C.2), $g_C - g_{Y_N} = g_Y - g_{Y_N} = (\dot{Y_N} + \dot{Y_S})/(Y_N + Y_S) - \dot{Y}_N/Y_N$. Using (C.1), the previous equation reduces to $g_C - g_{Y_N} = -g_{L_N^Y}/[1 + \phi^{(\alpha-1)/\beta}(\tau_S/\tau_N)^{\gamma/\beta} L_N^Y/L_S]$ (in the same way as in d) of Appendix A). Replacing this second expression of $g_C - g_{Y_N}$ and $g_A = \psi(L_N - L_N^Y)$ together in the first expression of $g_C - g_{Y_N}$ above and rearranging, we get: $\dot{L}_N^Y = -(\rho + \psi L_N)[L_N^Y + \phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta} L_S] + [\psi + (1-\alpha)\psi/\beta(1-\sigma)][L_N^Y + \phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta} L_S]L_N^Y$. Defining $y = \left(L_N^Y + \phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta} L_S\right)^{-1}$, we eventually get the following differential equation: $\dot{y} = \left[\rho + \psi L_N + \left(\psi + (1-\alpha)\psi/\beta(1-\sigma)\right)\phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta} L_S\right] y - \left(\psi + (1-\alpha)\psi/\beta(1-\sigma)\right)$. The solution of this equation is $y(t) = e^{\left[\rho + \psi L_N + \left(\psi + \frac{(1-\alpha)\psi}{\beta(1-\sigma)}\right)\phi^{\frac{1-\alpha}{\beta}}(\tau_N/\tau_S)^{\frac{\gamma}{\beta}} L_S\right] t} \left[y(0) - \frac{\psi + \frac{(1-\alpha)\psi}{\beta(1-\sigma)}}{\rho + \psi L_N + \left(\psi + \frac{(1-\alpha)\psi}{\beta(1-\sigma)}\right)\phi^{\frac{1-\alpha}{\beta}}(\tau_N/\tau_S)^{\frac{\gamma}{\beta}} L_S}\right] + \frac{\psi + \frac{(1-\alpha)\psi}{\beta(1-\sigma)}}{\rho + \psi L_N + \left(\psi + \frac{(1-\alpha)\psi}{\beta(1-\sigma)}\right)\phi^{\frac{1-\alpha}{\beta}}(\tau_N/\tau_S)^{\frac{\gamma}{\beta}} L_S}$. We show now that y jumps immediately to its steady-state value. By (3.17), the transversality conditions (3.16) imply $\lim_{t\to+\infty} B_i(t)e^{-\rho t}e^{-\int_0^t g_C(s)\,ds} = 0$, $i = N, S$, what in turn implies

$\lim_{t\to+\infty}(B_N(t)+B_S(t))e^{-\rho t}e^{-\int_0^t g_C(s)\,ds} = 0$. Note now that $B_N+B_S = AV$ from the equilibrium of the financial market. Using this and the free-entry condition (3.13), we find that the transversality conditions imply $\lim_{t\to+\infty} e^{-\rho t}e^{-\int_0^t g_C(s)\,ds}Y_N/L_N^Y = 0$, and, since $g_C = g_Y$ from (C.2), this implies $\lim_{t\to+\infty} e^{-\rho t}Y_N/((Y_N+Y_S)L_N^Y) = 0$, what can be expressed after developing as $\lim_{t\to+\infty} e^{-\rho t}/y(t) = 0$, which is possible, from above, only if $y(t) = y(0) = [\psi+(1-\alpha)\psi/\beta(1-\sigma)]/[\rho+\psi L_N+(\psi+(1-\alpha)\psi/\beta(1-\sigma))\phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta}L_S]$, $\forall t \geq 0$. Eventually, this is equivalent to say that L_N^Y jumps also at date 0 to its constant value:

$$L_N^Y = \frac{\rho+\psi L_N}{\psi(1+\frac{1-\alpha}{\beta(1-\sigma)})},\ L_N^A = L_N - L_N^Y,\ L_S^Y = L_S. \tag{C.4}$$

e) Next, we show how to obtain the rate of extraction. Let us log-differentiate conditions (3.3) and (3.6). Using (3.17) and (3.14), we obtain $g_{Y_i} - g_{R_i} = g_{C_i} + \rho + g_\tau,\ i = N, S$. Combining those later equations and using that $g_{C_N} = g_{C_S} = g_C = g_Y$ from (3.17) and (C.2), we get $g_{Y_N} + g_{Y_S} - 2g_Y - g_{R_N} - g_{R_S} = 2\rho + 2g_\tau,\ i = N, S$. Since L_N^Y is constant, (C.1) implies that $g_{Y_N} = g_{Y_S} = g_Y$ and $g_{R_N} = g_{R_S} = g_R$. From the previous equation, one also gets $g_R = -\rho - g_\tau$, that is

$$g_{R_N}(t) = g_{R_S}(t) = g_R(t) = -\rho - g_\tau(t). \tag{C.5}$$

f) From (3.17), $g_{C_N}(t) = g_{C_S}(t) = g_C(t)$. (C.2) implies $g_C(t) = g_Y(t)$. (C.1) and the constancy of L_N^Y implies $g_{Y_N}(t) = g_{Y_S}(t) = g_Y(t)$. Let us show what the values of these growth rates and that of the quantity of intermediates are. The log-differentiation of the functional form of both production functions (2.1) in which $x_i(j) = x_i,\ i = N, S$ and L_N^Y is constant gives $g_{Y_i} = g_A + \alpha g_{x_i} + \gamma g_{R_i},\ i = N, S$. The log-differentiation of (3.9), $g_{x_i} = \gamma g_{R_i}/(1-\alpha),\ i = N, S$, and $g_{R_N} = g_{R_S} = g_R$ thus lead to

$$g_{C_N}(t) = g_{C_S}(t) = g_C(t) = g_Y(t) = g_{Y_N}(t) = g_{Y_S}(t) = g_A + \frac{\gamma}{1-\alpha}g_R(t) \tag{C.6}$$

$$\text{and} \quad g_{x_N}(t) = g_{x_S}(t) = g_x(t) = \frac{\gamma}{1-\alpha}g_R(t). \tag{C.7}$$

g) Let us now obtain the expressions of local final production levels. From equation (C.5), $R(0)$ is determined by $\int_0^{+\infty} R(0)e^{-\int_0^t(\rho+g_\tau(s))\,ds}\,dt = Q_0$. Then, $R(0)$ is increasing in Q_0 and independent from τ_N/τ_S. By (C.1), we get $R_N(0) = R(0)[1+\phi^{(1-\alpha)/\beta}(\tau_N/\tau_S)^{\gamma/\beta+1}L_S/L_N^Y]^{-1}$ and $R_S(0) = R(0)[1+\phi^{(\alpha-1)/\beta}(\tau_S/\tau_N)^{\gamma/\beta+1}L_N^Y/L_S]^{-1}$. Using the demands for intermediate goods (3.9) (recall that $1 - s_N = \alpha$), we can immediately compute $x_N(0) = \alpha^{1/(1-\alpha)}L_N^{Y\,\beta/(1-\alpha)}R_N(0)^{\gamma/(1-\alpha)}$ and $x_S(0) = \alpha^{1/(1-\alpha)}L_S^{\beta/(1-\alpha)}R_S(0)^{\gamma/(1-\alpha)}$. Putting these quantities in the production functions expressed at date 0, we finally get

$$Y_N(0) = A_0\alpha^{\frac{\alpha}{1-\alpha}}L_N^{Y\,\frac{\beta}{1-\alpha}}\Big(\frac{R(0)}{1+\phi^{\frac{1-\alpha}{\beta}}(\frac{\tau_N(0)}{\tau_S(0)})^{\frac{\gamma}{\beta}+1}\frac{L_S}{L_N^Y}}\Big)^{\frac{\gamma}{1-\alpha}} \tag{C.8}$$

$$\text{and} \quad Y_S(0) = \phi A_0\alpha^{\frac{\alpha}{1-\alpha}}L_S^{\frac{\beta}{1-\alpha}}\Big(\frac{R(0)}{1+\phi^{\frac{\alpha-1}{\beta}}(\frac{\tau_S(0)}{\tau_N(0)})^{\frac{\gamma}{\beta}+1}\frac{L_N^Y}{L_S}}\Big)^{\frac{\gamma}{1-\alpha}}. \tag{C.9}$$

h) Let us now obtain the expressions of the consumption levels. Substituting (3.5), (3.6) and (3.3) in the budget constraint of country S representative household (3.15) and using $\theta_i = \tau_i - 1,\ i = N, S$, we find $\dot{B}_S + C_S = (1-\alpha)Y_S + \gamma Y_N/\tau_N + rB_S$.

As the northern research sector is a net borrower and constitutes the only group of agents the households can trade with on the financial market, the equalization of its uses and resources from Appendix B writes $w_N L_N^A + Ax_N + rB = \sigma w_N L_N^A + \frac{1}{\alpha}Ax_N + \dot{B}$, where $B = B_N + B_S$. Using this condition to substitute the $\dot{B}_N - rB_N$ in the budget constraint of country N representative agent and simplifying with equations (3.2), (3.3), (3.7) and (3.9), we get: $-\dot{B}_S + C_N = (1-\alpha)Y_N - \gamma Y_N/\tau_N - rB_S$.

Solving those two instantaneous budget constraints as first-order linear differential equations in B_S, we obtain two intertemporal budget constraints that hold for any date $T \geq 0$: $-B_S(T)e^{-\int_0^T r(t)\,dt} + \int_0^T C_N(t)e^{-\int_0^t r(s)\,ds}\,dt$
$= (1-\alpha)\int_0^T Y_N(t)e^{-\int_0^t r(s)\,ds}\,dt - \gamma\int_0^T \frac{Y_N(t)}{\tau_N(t)}e^{-\int_0^t r(s)\,ds}\,dt - B_S(0)$
and $B_S(T)e^{-\int_0^T r(t)\,dt} + \int_0^T C_S(t)e^{-\int_0^t r(s)\,ds}\,dt$
$= (1-\alpha)\int_0^T Y_S(t)e^{-\int_0^t r(s)\,ds}\,dt + \gamma\int_0^T \frac{Y_N(t)}{\tau_N(t)}e^{-\int_0^t r(s)\,ds}\,dt + B_S(0)$. Taking the limit as $T \to +\infty$ of those two equations, the terms on the far left vanish (from the no-Ponzi-game conditions (3.16)) and the intertemporal budget constraints become
$-B_S(0) = \int_0^{+\infty}[C_N(t) + (1-\alpha)Y_N(t) + \gamma Y_N(t)/\tau_N(t)]e^{-\int_0^t r(s)\,ds}\,dt$ and
$-B_S(0) = \int_0^{+\infty}[(1-\alpha)Y_S(t) - C_S(t) + \gamma Y_N(t)/\tau_N(t)]e^{-\int_0^t r(s)\,ds}\,dt$. Finally, using from equations (3.17) and (C.6) that $g_{C_N} = g_{C_S} = g_{Y_N} = g_{Y_S} = r - \rho$ and reminding that $g_\tau(t) > -\rho$, $t \in [0, +\infty)$, these equations reduce to

$$C_N(0) = (1-\alpha)Y_N(0) - \gamma\rho D\frac{Y_N(0)}{\tau_N(0)} - \rho B_S(0) \tag{C.10}$$

$$\text{and} \quad C_S(0) = (1-\alpha)Y_S(0) + \gamma\rho D\frac{Y_N(0)}{\tau_N(0)} + \rho B_S(0), \tag{C.11}$$

where $D = \int_0^{+\infty} e^{-\int_0^t(\rho + g_\tau(s))\,ds}\,dt > 0$.

i) The optimal subsidy to the labor employment by the research sector can be simply found out by equalizing the optimal quantity of labor used by this sector, $L_N^{A\,o}$, given by equation (A.24), and the same quantity in equilibrium which is given by equation (C.4). We obtain

$$\sigma^o = 1 - \frac{(1-\alpha)\psi}{\frac{\beta(\rho+\psi L_N)}{L_N^{Y\,o}} - \beta\psi}. \tag{C.12}$$

The previous formulae give Proposition 2 and the results of Subsection 3.2 when the restrictions $B_S(0) = 0$ and $\sigma = \sigma^o$ are imposed.

j) Eventually, let us characterize the optimal environmental taxation policy. The fact that the environmental tax levels must equalize to ensure the efficiency of the world economy can be seen by comparing equations (C.1) and (A.18). Note that this results actually from the equality of the marginal resource productivities (see equations (3.3) and (3.6)). This implies that their time profiles must be identical. Their optimal common growth rate is obtained by equalizing the optimal rate of extraction, given by (A.25), and the equilibrium rate of extraction, given by (C.5):

$$g_\tau^o(t) = \frac{\lambda(1-\alpha)}{\gamma}g_{E^o}(t). \tag{C.13}$$

This rate is negative since g_E is technically bound to be negative. Moreover, as g_R^o, has been shown to be strictly negative, then, from equation (C.5), we get that $g_\tau^o > -\rho$.

This last point gives Proposition 3.

D Appendix: Effects of the National Tax Levels

a) From equations (C.8) and (C.9), it is straightforward that a proportional increase in both initial tax levels, $\tau_N(0)$ and $\tau_S(0)$, i.e. a change in these initial tax levels their ratio remaining constant, does not affect both national productions, $Y_N(0)$ and $Y_S(0)$. Since such an increase does not affect the growth rate of these productions, such a change affects neither the national productions nor the world production at all dates $t \geq 0$.

b) Using equations (C.10) and (C.11), let us now examine the effects of a proportional increase in both tax levels on the initial national consumption levels, $C_N(0)$ and $C_S(0)$. Since such a change does not affect the outputs, its effects on consumptions can only be through the direct presence of $\tau_N(0)$ in expressions (C.10) and (C.11). It turns out immediately that $C_N(0)$ increases and $C_S(0)$ decreases consequently to such an increase. In the same way as for productions, the considered change having no dynamic effect, both national consumptions react at all dates $t \geq 0$ just as initial national consumptions do.

The results of a) and b) above give Proposition 4.

In the following, let us use the variable $\chi = \phi^{(1-\alpha)/\beta}(\tau_N(0)/\tau_S(0))^{\gamma/\beta+1} L_S/L_N^Y$.

c) Let us now show how an increase in the northern tax level affects the national productions. From expression (C.8), we get $Y_N(0) = A_0\alpha^{\alpha/(1-\alpha)} {L_N^Y}^{\beta/(1-\alpha)} R(0)^{\gamma/(1-\alpha)}(1+\chi)^{-\gamma/(1-\alpha)}$. Hence, $\partial Y_N(0)/\partial\tau_N(0) = A_0\alpha^{\alpha/(1-\alpha)} {L_N^Y}^{\beta/(1-\alpha)} R(0)^{\gamma/(1-\alpha)}(-\gamma/(1-\alpha))(1+\chi)^{-\gamma/(1-\alpha)-1}(\gamma/\beta+1)\chi/\tau_N(0)$. Using $\alpha+\beta+\gamma$ and rearranging, this reduces to $\partial Y_N(0)/\partial\tau_N(0) = -(\gamma/\beta)(Y_N(0)/\tau_N(0))(\chi/(1+\chi)) < 0$. In the same way, from (C.9), $Y_S(0) = \phi A_0\alpha^{\alpha/(1-\alpha)} {L_S}^{\beta/(1-\alpha)} R(0)^{\gamma/(1-\alpha)}(1+\chi^{-1})^{-\gamma/(1-\alpha)}$ and we find $\partial Y_S(0)/\partial\tau_N(0) = (\gamma/\beta)(Y_S(0)/\tau_N(0))(\chi^{-1}/(1+\chi^{-1})) > 0$.

d) Here, we derive the effect of the northern tax rate on the world output.
i) From the previous results, $\partial Y(0)/\partial\tau_N(0) = \partial Y_N(0)/\partial\tau_N(0) + \partial Y_S(0)/\partial\tau_N(0)$
$= (\gamma/\beta)((Y_S(0)/\tau_N(0))(\chi^{-1}/(1+\chi^{-1})) - (Y_N(0)/\tau_N(0))(\chi/(1+\chi)))$.
Then, $\partial Y(0)/\partial\tau_N(0) > (resp <)0 \iff Y_S(0)(\chi^{-1}/(1+\chi^{-1})) > (resp <)Y_N(0)(\chi/(1+\chi)) \iff Y_S(0)/Y_N(0) > (resp <)\chi$. Finally, using equation (C.1), developing χ and simplifying, we find $\partial Y(0)/\partial\tau_N(0) > (resp <)0 \iff \tau_N(0) < (resp >)\tau_S(0)$.
ii) $Y(0)$ is strictly increasing in $\tau_N(0)$ on $(0, \tau_S(0))$ and strictly decreasing on $(\tau_S(0), +\infty)$ and is continuous in $\tau_N(0)$. Hence, $Y(0)$ is maximum in $\tau_N(0)$ at $\tau_N(0) = \tau_S(0)$.

Once again, since the initial levels, $\tau_N(0)$ and $\tau_S(0)$, have no dynamic effect, the previous results are valid for all dates $t \geq 0$: $\partial Y_N(t)/\partial\tau_N(0) < 0$, $\partial Y_S(t)/\partial\tau_N(0) > 0$ and $\partial Y(t)/\partial\tau_N(0) > (resp <)0 \iff \tau_N(0) < (resp >)\tau_S(0)$, $\forall t \in [0, +\infty)$.

The results of c) and d) above give Proposition 5.

e) Now, we solve for the effect of a change in the northern tax level on the northern consumption. From (C.10), we get $\partial C_N(0)/\partial\tau_N(0) = (1-\alpha)\partial Y_N(0)/\partial\tau_N(0) - \gamma\rho D(\partial Y_N(0)/\partial\tau_N(0))/\tau_N(0) + \gamma\rho D Y_N(0)/\tau_N(0)^2$. Using the definition of variable χ and the results of c) above, we have $\partial C_N(0)/\partial\tau_N(0) = -((1-\alpha)\gamma/\beta)(Y_N(0)/\tau_N(0))\chi/(1+\chi) + (\gamma/\beta)\gamma\rho D(Y_N(0)/\tau_N(0)^2)\chi/(1+\chi) + \gamma\rho D Y_N(0)/\tau_N(0)^2 = [(\chi/(1+\chi))Y_N(0)/\tau_N(0)^2][-(1-\alpha)\gamma\tau_N(0)/\beta + \gamma^2\rho D/\beta + \gamma\rho D(1+\chi)/\chi]$, where the sum of the last two terms between brackets yields $\gamma\rho D/\chi + (1-\alpha)\gamma\rho D/\beta$. Hence, $\partial C_N(0)/\partial\tau_N(0) > (resp <)0 \iff \rho D/\chi > (resp <)((1-\alpha)/\beta)(\tau_N(0) - \rho D)$. From this proposition, we can see that:
i) If $\tau_N(0) \leq \rho D$, then $\nexists \tau_S(0) > 0 : \partial C_N(0)/\partial\tau_N(0) < 0$, since $\chi > 0$. Hence, if $\tau_N(0) \leq \rho D$, then $\partial C_N(0)/\partial\tau_N(0) > 0$, $\forall\tau_S(0) > 0$.
ii) If $\tau_N(0) > \rho D$, $\partial C_N(0)/\partial\tau_N(0) > (resp <)0 \iff \rho D(\phi^{(1-\alpha)/\beta} L_S/L_N^Y)^{-1}\tau_S(0)^{(1-\alpha)/\beta} > (resp <)((1-\alpha)/\beta)(\tau_N(0)^{(1-\alpha)/\beta+1} - \rho D\tau_N(0)^{(1-\alpha)/\beta}) \iff \tau_S(0) > (resp <)\widehat{\tau_S}(\tau_N(0))$, where

$$\widehat{\tau_S}\big(\tau_N(0)\big) = \Big(\frac{(1-\alpha)}{\beta\rho D}\phi^{\frac{1-\alpha}{\beta}}\frac{L_S}{L_N^Y}\Big)^{\frac{\beta}{1-\alpha}}\Big(\tau_N(0)^{\frac{1-\alpha}{\beta}+1} - \rho D\tau_N(0)^{\frac{1-\alpha}{\beta}}\Big)^{\frac{\beta}{1-\alpha}}. \tag{D.1}$$

By continuity, the curve defined by equation $\tau_S(0) = \widehat{\tau_S}(\tau_N(0))$ is the set of pairs $(\tau_N(0), \tau_S(0))$ along which $\partial C_N(0)/\partial\tau_N(0) = 0$. Since the initial tax levels have no dynamic effect, $\partial C_N(t)/\partial\tau_N(0) = 0$ at all dates $t \geq 0$ along this curve and $\partial C_N(t)/\partial\tau_N(0)$ is of the same sign as $\partial C_N(0)/\partial\tau_N(0)$.

f) Now, we solve for the effect of a change in the northern tax level on the southern consumption. From (C.11), we get $\partial C_S(0)/\partial\tau_N(0) = (1-\alpha)\partial Y_S(0)/\partial\tau_N(0) + \gamma\rho D(\partial Y_N(0)/\partial\tau_N(0))/\tau_N(0) - \gamma\rho D Y_N(0)/\tau_N(0)^2$. Using the definition of variable χ and the results of c) above, we have $\partial C_S(0)/\partial\tau_N(0) = ((1-\alpha)\gamma/\beta)(Y_S(0)/\tau_N(0))\chi^{-1}/(1+\chi^{-1}) -$
$(\gamma/\beta)\gamma\rho D(Y_N(0)/\tau_N(0)^2)\chi/(1+\chi) - \gamma\rho D Y_N(0)/\tau_N(0)^2 = [(\chi/(1+\chi))Y_N(0)/\tau_N(0)^2][((1-\alpha)\gamma/\beta)(\tau_N(0)Y_S(0)/Y_N(0))(\chi^{-1}/(1+\chi^{-1}))((1+\chi)/\chi) - \gamma^2\rho D/\beta - \gamma\rho D(1+\chi)/\chi]$, where $Y_S(0)/Y_N(0) = \chi\tau_S(0)/\tau_N(0)$ by (C.1) and the sum of the last two terms between brackets is $-\gamma\rho D/\chi - (1-\alpha)\gamma\rho D/\beta$. Hence, $\partial C_S(0)/\partial\tau_N(0) > (resp <)0 \iff ((1-\alpha)\gamma/\beta)(\tau_S(0) - \rho D) > (resp <)\gamma\rho D/\chi$. From this proposition, we can see that:
i) If $\tau_S(0) \leq \rho D$, then $\nexists \tau_N(0) > 0 : \partial C_S(0)/\partial\tau_N(0) > 0$. Hence, if $\tau_S(0) \leq \rho D$, then $\partial C_S(0)/\partial\tau_N(0) < 0$, $\forall\tau_N(0) > 0$.
ii) If $\tau_S(0) > \rho D$, $\partial C_S(0)/\partial\tau_N(0) > (resp <)0$
$\iff [(1-\alpha)\phi^{(1-\alpha)/\beta}L_S/\beta\rho D L_N^Y][\tau_S(0)^{-(1-\alpha)/\beta+1} - \rho D\tau_S(0)^{-(1-\alpha)/\beta}] > (resp <)\tau_N(0)^{-(1-\alpha)/\beta}$
$\iff \tau_N(0) > (resp <)\widehat{\tau_N}(\tau_S(0))$, where

$$\widehat{\tau_N}\big(\tau_S(0)\big) = \Big(\frac{1-\alpha}{\beta\rho D}\phi^{\frac{1-\alpha}{\beta}}\frac{L_S}{L_N^Y}\Big)^{-\frac{\beta}{1-\alpha}}\Big(\tau_S(0)^{-\frac{1-\alpha}{\beta}+1} - \rho D\tau_S(0)^{-\frac{1-\alpha}{\beta}}\Big)^{-\frac{\beta}{1-\alpha}}. \qquad \text{(D.2)}$$

By continuity, the curve defined by equation $\tau_N(0) = \widehat{\tau_N}(\tau_S(0))$ is the set of pairs $(\tau_N(0), \tau_S(0))$ along which $\partial C_S(0)/\partial\tau_N(0) = 0$. Since the initial tax levels have no dynamic effect, $\partial C_S(t)/\partial\tau_N(0) = 0$ at all dates $t \geq 0$ along this curve and $\partial C_S(t)/\partial\tau_N(0)$ is of the same sign as $\partial C_S(0)/\partial\tau_N(0)$.

g) We explain now that both curves defined by $\tau_S(0) = \widehat{\tau_S}(\tau_N(0))$ and $\tau_N(0) = \widehat{\tau_N}(\tau_S(0))$, called respectively $C_N max$ and $C_S max$, cross only once and that their point of intersection is on the first bisecting line, i.e. such that $\tau_N(0) = \tau_S(0)$.

Both equations $\tau_S(0) = \widehat{\tau_S}(\tau_N(0))$ and $\tau_N(0) = \widehat{\tau_N}(\tau_S(0))$ form a system of equations with two unknowns. We look for its solution(s) in $(\tau_N(0), \tau_S(0)) \in [\rho D, +\infty)^2$. After computation, we can verify that the unique solution of the system is $\tau_N(0) = \tau_S(0) = \tau_X = (\beta\rho D/(1-\alpha))\phi^{-(1-\alpha)/\beta}L_N^Y/L_S + \rho D > \rho D$ (see Figure 2).

The results of e), f) and g) above give Proposition 6 and characterize the curves of Figure 2.

h) Let us compute the best response function of the South government. From (C.11), we can write $C_S(0) = (1-\alpha)Y_S(0) + \gamma\rho D Y_N(0)/\tau_N(0)$. Hence, $\partial C_S(0)/\partial\tau_S(0) = (1-\alpha)\partial Y_S(0)/\partial\tau_S(0) + (\gamma\rho D/\tau_N(0))\partial Y_N(0)/\partial\tau_S(0)$.

Using again the variable χ, we find that $\partial Y_N(0)/\partial\tau_S(0) = (\gamma/\beta)(Y_N(0)/\tau_S(0))\chi/(1+\chi)$ and $\partial Y_S(0)/\partial\tau_S(0) = -(\gamma/\beta)(Y_S(0)/\tau_S(0))\chi^{-1}/(1+\chi^{-1})$. Substituting these later results in the expression of $\partial C_S(0)/\partial\tau_S(0)$ above, we immediately have that
$\partial C_S(0)/\partial\tau_S(0) > (resp <)0 \iff -(1-\alpha)(\gamma/\beta)(Y_S(0)/\tau_S(0))\chi^{-1}/(1+\chi^{-1}) < (resp >)(\gamma\rho D/\tau_N(0))(\gamma/\beta)(Y_N(0)/\tau_S(0))\chi/(1+\chi)$. Multiplying both sides of the later inequality by $\tau_S(0)(1+\chi)\beta/\gamma$ and rearranging, we get $\partial C_S(0)/\partial\tau_S(0) > (resp <)0$
$\iff \chi^{-1}\tau_N(0)Y_S(0)/Y_N(0) < (resp >)\gamma\rho D/(1-\alpha)$. Recalling now from (C.1) that $Y_S(0)/Y_N(0) = \chi\tau_S(0)/\tau_N(0)$, we obtain: $\partial C_S(0)/\partial\tau_S(0) > (resp <)0 \iff \tau_S(0) < (resp >)\gamma\rho D/(1-\alpha)$.

By continuity, it appears that $C_S(0)$ is maximum in $\tau_S(0)$ when $\tau_S(0) = \gamma\rho D/(1-\alpha)$.

i) Let us compute the best response function of the North government. From e)$\imath$) above, the tax level maximizing $C_N(0)$ is larger than ρD, for any $\tau_S(0) > 0$. Then, from e)$\imath\imath$), the tax level $\tau_N(0) = \widehat{\tau_N}(\tau_S(0))$ maximizes $C_N(0)$.

j) Recalling that the initial tax levels have no effect on the dynamics of the economy, the initial tax level $\tau_i(0)$ maximizing $C_i(0)$ is also the one maximizing $C_i(t)$ at all dates $t \geq 0$, for any $i = N, S$. Moreover, looking at the utility functions of both representative agents in (2.10) and recalling that the tax levels do not affect the law of motion of E, note that it is equivalent for government i to maximize U_i and to maximize consumption in country i at all dates, $i = N, S$.

k) The Nash equilibrium of the game on the initial tax levels is at the intersection of both best response functions. That of the South is $\tau_S(0) = \gamma\rho D/(1-\alpha)$ that also writes $\tau_S(0) = \rho D - \beta\rho D/(1-\alpha)$, which is lower than ρD. Recalling that the vertical asymptote of $C_S max$ is $\tau_S(0) = \rho D$, this implies that the Nash equilibrium is on $C_N max$, below the asymptote.

Eventually, h), i), j) and k) give the best response of both governments and the Nash outcome of the game proposed in Subsection 4.3.

References

[1] Amundsen, E.S., R. Schöb (1999), Environmental Taxes on Exhaustible Resources, *European Journal of Political Economy*, 15, 311-329.

[2] Bacon, R., (2001), Petroleum Taxes: Trends in Fuel Taxes (and Subsidies) and the Implications, *World Bank, Public Policy for the Private Sector*, Note 240.

[3] Belgodere, A., (2008), On the Path of an Oil Pigovian Tax, *Manchester School*, forthcoming.

[4] Bergstrom, T.C., (1982), On Capturing Oil Rents with a National Excise Tax, *American Economic Review*, 72, 194-201.

[5] Blanchard, O.J., S. Fischer, (1989), *Lectures on Macroeconomics*, MIT Press.

[6] Brander, J., S. Djajic, (1983), Rent-Extracting Tariffs and the Management of Exhaustible Resources, *Canadian Journal of Economics*, 16, 288-298.

[7] Dasgupta, P.S., G.M. Heal, (1979), *Economic Theory and Exhaustible Resources*, Oxford University Press.

[8] Daubanes, J., (2008), Fossil Fuels Supplied by Oligopolies: On Optimal Taxation and Rent Capture, *Economics Bulletin*, 17, 13, 1-11.

[9] Gaudet, G., P. Lasserre, (1990) Dynamiques Comparées des Effets de la Taxation Minière, *L'Actualité Economique, Revue d'Analyse Economique*, 66, 467-497.

[10] Grimaud, A., L. Rougé, (2005), Polluting Non Renewable Resources, Innovation and Growth: Welfare and Environmental Policy, *Resource and Energy Economics*, 27, 109-129.

[11] Grimaud, A., L. Rougé, (2008), Environment, Directed Technical Change and Economic Policy, *Environmental and Resource Economics*, 41, 439-463.

[12] Groth, C., P. Schou, (2007), Growth and Non-renewable Resources: The Different Roles of Capital and Resource Taxes, *Journal of Environmental Economics and Management*, 53, 80-98.

[13] Hoel, M., S. Kverndokk, (1996), Depletion of Fossil Fuels and the Impacts of Global Warming, *Resource and Energy Economics*, 18, 115-136.

[14] IEA (International Energy Agency), (2001), Dealing with Climate Change: Policies & Measures in IEA Member Countries 2001 Edition.

[15] Nordhaus, W.D., J. Boyer, (2000), *Warming the World: Economic Models of Global Warming*, MIT Press.

[16] OPEC (Organization of Petroleum Exporting Countries), (2008), Who Gets What from Imported Oil?, Public Relations and Information Department.

[17] PennWell Corporation, (2004), *Oil & Gas Journal*, 112, 47.

[18] Quadrelli, R., S. Peterson, (2007), The Energy-Climate Challenge: Recent Trends in CO2 Emissions from Fuel Combustion, *Energy Policy*, 35, 5938-5952.

[19] Romer, P., (1990), Endogenous Technological Change, *Journal of Political Economy*, 98, S71-S102.

[20] Schou, P., (2000), Polluting Non-Renewable Resources and Growth, *Environmental and Resource Economics*, 16, 211-227.

[21] Schou, P., (2002), When Environmental Policy Is Superfluous: Growth and Polluting Resources, *Scandinavian Journal of Economics*, 104, 605-620.

[22] Sinclair, P.J.N., (1992), High Does Nothing and Rising is Worse: Carbon Taxes Should Keep Declining to Cut Harmful Emissions, *Manchester School*, 60, 41-52.

[23] Sinn, H.-W, (1982), Taxation, Growth, and Resource Extraction: A General Equilibrium Approach, *European Economic Review*, 19, 357-386.

[24] Sinn, H.-W, (2008), Public Policies Against Global Warming: A Supply Side Approach, *International Tax and Public Finance*, 15, 360-394.

[25] Ulph, A., D. Ulph, (1994), The Optimal Time Path of a Carbon Tax, *Oxford Economic Papers*, 46, 857-868.

[26] Withagen, C., (1994), Pollution and Exhaustibility of Fossil Fuels, *Resource and Energy Economics*, 16, 235-242.