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FISCAL SHOCKS IN A TWO SECTOR OPEN ECONOMY*

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Abstract

We use a two-sector neoclassical open economy model with traded and non-traded goods to investigate both the aggregate and the sectoral effects of temporary fiscal shocks. One central finding is that both sectoral capital intensities and labor supply elasticity matter in determining the response of key economic variables. In particular, the model can produce a drop in investment and in the current account, in line with empirical evidence, only if the traded sector is more capital intensive than the non-traded sector, and labor is supplied elastically. Irrespective of sectoral capital intensities, a fiscal shock raises the relative size of the non-traded sector substantially in the short-run. Additionally, allowing for the markup to depend on the number of competitors, the two-sector model can produce the real exchange rate depreciation found in the data. Finally, markup variations triggered by firm entry modify substantially the response of the real wage and the sectoral composition of GDP in the short-run.
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1 Introduction

There has recently been a revival of interest among policy makers in the fiscal policy tool. The fiscal transmission mechanism has also attracted considerable attention in the academic literature. A number of papers have explored the ability of quantitative business cycle models, both of the neoclassical and of the new Keynesian variety, to account for the data, see Burnside, see e.g. Eichenbaum and Fisher [2004] and Gali, Lopez-Salido and Valles [2007], respectively. However, most of the analyses have been confined to closed economy models and to one-sector frameworks. In the present paper we take up the following question instead: to what extent can an open economy version of the two-sector neoclassical model account for the time-series evidence on fiscal policy transmission mechanism?

Assuming that government spending is predetermined relative to the other variables included in the vector autoregression (VAR) model, as suggested by Blanchard and Perotti [2002], Cardi and Müller [2010] establish a first set of main findings: an exogenous increase in government spending raises output, and lowers both investment and the current account.[1] The second set of main findings relates to the labor market. Perotti [2007] documents an increase in hours worked and in the real wage.[2] The third empirical fact relates to the impact of fiscal shocks on the sectoral composition of output. Estimates by Bénétrix and Lane [2010] reveal that a boost to government spending benefits disproportionately the non-traded sector. The fourth empirical fact relates to the shift in the relative price of home goods. Monacelli and Perotti [2010] and Enders et al. [2011] find that government spending yields a real exchange rate depreciation.[3]

To address these empirical evidence, we use an open economy with a traded and a non-traded sector. Our neoclassical framework builds on Turnovsky and Sen [1995] and Coto-Martinez and Dixon [2003]. Like Coto-Martinez and Dixon, we allow for the non-traded sector to be imperfectly competitive. Our work differs from that of Turnovsky and Sen [1995] and Coto-Martinez and Dixon [2003] in one major respect.[4] They consider the

[1]Such findings are consistent with the conclusions reached by Corsetti and Müller [2006], Beetsma, Giuliodori and Klaassen [2008], and Monacelli and Perotti [2010].

[2]While Perotti's [2007] conclusions are in line with those of Rotemberg and Woodford [1992] and Pappa [2009] for the U.S., Ramey [2011] finds that hours worked increase but real wages can rise or decline on impact, depending on the period considered. Yet, in both cases, the real wage exceed its initial level after two years.

[3]The sample of countries considered by Cardi and Müller [2010], Perotti [2007], Monacelli and Perotti [2010], comprises four countries: the U.S., the U.K., Canada and Australia. In Cardi and Müller [2010], the period runs from 1980:1 to 2007:4, in Perotti [2007] from 1954:1 to 2003:1 for the U.S., and in Monacelli and Perotti [2010] from 1980:1 to 2006:4. Bénétrix and Lane [2010] consider a panel of eleven member countries of the euro area over the period 1970-2005. All these papers adopt the identification procedure of fiscal shocks developed by Blanchard and Perotti [2002].

[4]Turnovsky and Sen [1995] investigate the effects of permanent government spending shocks by assuming fixed labor supply. Coto-Martinez and Dixon [2003] introduce an elastic labor supply but restrict their analysis to the effects of a permanent rise in public spending by assuming that the traded sector is more

effects of permanent fiscal shocks while we examine the impact of temporary fiscal shocks of different degrees of persistence. Beyond the fact that considering a transitory increase in public spending allows us to address the VAR evidence, the effects of temporary fiscal shocks can be different to those of a permanent shock.[5]

One attractive feature of a two-sector model with tradable and non tradable goods is to cover both the closed economy and the open economy dimensions of contemporary industrialized countries. In particular, the empirical evidence shows that the non-tradable content of GDP and employment is substantial, at around two-thirds.[6] From an analytical point of view, as in a closed economy model, capital accumulation clears the home good market, see e.g., Baxter and King [1993]. As in a small open economy, external borrowing allows households to smooth consumption intertemporally. A second key feature of our framework is that we can investigate the sectoral effects of fiscal shocks. Such a sectoral decomposition of output is pivotal to understanding the fiscal transmission mechanism in an open economy. In particular, such a model enables us to connect sectoral output responses to the trade balance adjustment. Third, we are able to address the depreciation in the real exchange rate which has recently been documented in the empirical literature.

Beyond the relevancy of a two-sector model for investigating the fiscal transmission mechanism, such a framework can accommodate most of the empirical evidence mentioned above, albeit under certain conditions. By contrast, the open-economy version of Baxter and King's [1993] model fails to account for the first set of observations, particularly for the simultaneous decline of investment and the current account in response to an exogenous and temporary increase in government spending, see e.g., Karayalçin [1999].[7] As stressed in the classic paper by Baxter and King [1993], a representative household responds to the higher tax burden (which we assume to be lump-sum) by lowering consumption and increasing labor supply. This raises the return on capital and triggers a rise in investment, which, in turn, drives the current account into deficit. Furthermore, as the marginal product of labor falls dramatically, the real wage fails to exceed its initial value during the transition.

One prominent feature of the time series of government spending is that its non tradable content is substantial, at around 90%.[8] We therefore concentrate on the effects of a rise in public purchases of non-traded goods. We find that a two-sector model with traded

capital intensive. Additionally, neither Turnovsky and Sen [1995] nor Coto-Martinez and Dixon [2003] solve the model numerically.

[5]The reason behind this result is that after a temporary fiscal shock, consumption falls much less than after a permanent fiscal shock due to consumption smoothing behavior. Hence, if the traded sector is more capital intensive, an excess demand (rather than excess supply) arises in the non-traded good market so that investment is crowded out rather than being crowded in as long as the shock is transitory.

[6]Non tradable proportions are given in Table 3 (see Appendix A).

[7]Karaylalçin [1999] analyzes the effects of permanent and temporary fiscal shocks by using an open economy version of the Baxter and King's [1993] model. Regardless of its persistence, a fiscal expansion triggers an investment boom and a current account deficit.

[8]The non-tradable content of government spending is reported in Table 3 for thirteen OECD countries.

and non-traded goods can accommodate the two first sets of empirical findings, with the exception of the rising real wage, as long as the traded sector is more capital intensive than the non-traded sector. The explanation is intuitive. Following a fiscal expansion, agents cut real expenditure and raise labor supply. In contrast to a small open economy model, investment clears the home goods (or non-traded goods) market. Assuming that the traded sector is more capital intensive, an excess demand arises in the non-traded goods market which triggers a drop in investment. In addition, according to Rybczynski's theorem, higher labor supply produces a fall in traded output which in turn lowers net exports and thereby drives the current account into deficit. Assuming that the non-traded sector is more capital intensive, the real exchange rate appreciates which shifts resources towards the non-traded sector so that investment is crowded in while the current account enters in deficit as a result of the fall in traded output.

We also estimate numerically the sectoral effects of fiscal shocks to address the third empirical fact outlined above. Regardless of sectoral capital intensities, a temporary fiscal shock benefits the non-traded sector substantially in the short-run, in line with the empirical evidence reported by Bénétrix and Lane [2010]. Furthermore, our numerical results show that, in the long run, GDP growth is mostly driven by expansion in the traded output. The steady-state rise in traded output is necessary to compensate for the short-run current account deficits.

Keeping the markup fixed, the predictions of the two-sector model run counter to two of the stylized facts outlined above: the real wage does not rise and the real exchange rate does not depreciate, irrespective of whether the traded sector is more or less capital intensive than the non-traded sector. To address the real exchange rate depreciation and the increase in the real wage following government spending shocks, we follow Jaimovich and Floetotto [2008] in allowing for the markup to be endogenous. Considering that only a limited number of intermediate good producers operate in the non-traded sector, the price-elasticity of demand and thereby the markup faced by each firm depends on the number of competitors.[9] As the rise in government spending is expected to boost non-traded output, a fiscal shock triggers the entry of new firms. Hence, the markup falls, regardless of sectoral capital intensities. If the traded sector is more capital intensive, the real exchange rate must depreciate so that the return on domestic capital equalizes the return on foreign bonds. By exerting a positive influence on the wage rate and triggering a depreciation in the real exchange rate, the decline in the markup can boost the real wage, although only under certain circumstances. More precisely, the cumulative response of the real wage rate two years after the fiscal shock becomes positive only if the fiscal shock is either short-lived or long-lived.[10] If the non-traded sector is more capital intensive, the real exchange must

[9] Coto-Martinez and Dixon [2003] consider the case of a fixed markup rather than an endogenous markup.

[10] A long-lived and a short-lived fiscal shock last 32 quarters and 8 quarters, respectively. In the baseline

appreciate to equalize the return on domestic and foreign assets. As a consequence, the real wage falls dramatically and remains below its original level over the transition towards the steady-state.

Closely related to our paper is the study by Ramey and Shapiro [1998] who simulate a two-sector neoclassical model with costly capital reallocation. In a similar spirit, we achieve a better understanding of aggregate effects of fiscal shocks by investigating sectoral effects. In contrast to our study, they consider a closed economy so that they do not address the behavior of the current account or the real exchange rate. In addition, they do not discuss the role of sectoral capital intensities. Finally, whereas Ramey and Shapiro analyze the implications of costly capital mobility, we rather conduct a sensitivity analysis with respect to the duration of the fiscal shock and the elasticity of labor supply, considering a traded sector alternatively more or less capital intensive than the non-traded sector, and contrasting the case of a fixed markup with that of an endogenous markup.

The remainder of this paper is organized as follows. Section 2 outlines the specification of a two-sector model with traded and non-traded goods. Section 3 provides an analytical exploration of the short-run and long-run effects of fiscal shocks. In section 4, we report the results of our numerical simulations and discuss the sectoral effects of a temporary fiscal expansion. Section 5 explores the case of an endogenous markup quantitatively, focusing on the reactions of the real exchange rate and the real wage. In Section 6, we summarize our main results and present our conclusions.

2 The Framework

We consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever.[11] The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^\star$. A perfectly competitive sector produces a traded good denoted by the superscript T that can be exported and consumed domestically. An imperfectly competitive sector produces a non-traded good denoted by the superscript N which is devoted to physical capital accumulation and domestic consumption.[12] The traded good is chosen as the numeraire.[13]

scenario, the fiscal shock lasts 16 quarters, in line with estimates by Cardi and Müller [2010] for the U.S. Ramey [2011] also find that a fiscal shock lasts 4 years.

[11] More details on the model as well as the derivations of the results which are stated below are provided in an Appendix which is available from the authors on request.

[12] As stressed by Turnovsky and Sen [1995], allowing for traded capital investment would not affect the results (qualitatively). Furthermore, like Burstein et al. [2004], we find that the non tradable content of investment accounts for the lion's share of total investment expenditure (averaging to 60%).

[13] The price of the traded good is determined on the world market and exogenously given for the small open economy.

2.1 Households

At each instant the representative agent consumes traded goods and non-traded goods denoted by C^T and C^N, respectively, which are aggregated by a constant elasticity of substitution function:

$$C\left(C^T, C^N\right) = \left[\varphi^{\frac{1}{\phi}}\left(C^T\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}}\left(C^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{1}$$

where φ is the weight attached to the traded good in the overall consumption bundle $(0 < \varphi < 1)$ and ϕ is the intratemporal elasticity of substitution $(\phi > 0)$.

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $l \equiv 1 - L$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$U = \int_0^\infty \left\{\frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} - \gamma \frac{1}{1+\frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}}\right\} e^{-\beta t} \mathrm{d}t, \tag{2}$$

where β is the consumer's discount rate, $\sigma_C > 0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_L > 0$ is the Frisch elasticity of labor supply.

Factor income is derived by supplying labor L at a wage rate W, and capital K at a rental rate r^K. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^\star B(t)$. Denoting lump-sum taxes by Z, the households' flow budget constraint can be written as:

$$\dot{B}(t) = r^\star B(t) + r^K(t)K(t) + W(t)L(t) - Z - P_C\left(P(t)\right)C(t) - P(t)I(t), \tag{3}$$

where P_C is the consumption price index which is a function of the relative price of non-traded goods P. The last two terms represent households' expenditure which includes purchases of consumption goods and investment expenditure PI. Aggregate investment gives rise to overall capital accumulation according to the dynamic equation

$$\dot{K}(t) = I(t) - \delta_K K(t), \tag{4}$$

where we assume that physical capital depreciates at rate δ_K. In the rest of this paper, the time-argument is suppressed to increase clarity.

Denoting the co-state variable associated with eq. (3) by λ the first-order conditions characterizing the representative household's optimal plans are:

$$C = \left(P_C \lambda\right)^{\sigma_C}, \tag{5a}$$

$$L = \left(\frac{\lambda}{\gamma_L} W\right)^{\sigma_L}, \tag{5b}$$

$$\dot{\lambda} = \lambda\left(\beta - r^\star\right), \tag{5c}$$

$$\frac{r^K}{P} - \delta_K + \frac{\dot{P}}{P} = r^\star, \tag{5d}$$

plus the appropriate transversality conditions. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta = r^{\star}$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ, will undergo a discrete jump when individuals receive new information and must remain constant over time from thereon, i.e. $\lambda = \bar{\lambda}$.

The homogeneity of $C(.)$ allows a two-stage consumption decision: in the first stage, consumption is determined, and the intratemporal allocation between traded and non-traded goods is decided at the second stage. Applying Shephard's lemma gives $C^T = (1-\alpha_C) P_C C$ and $PC^N = \alpha_C P_C C$, with α_C being the share of non-traded goods in the consumption expenditure.

2.2 Firms

Both the traded and non-traded sectors use physical capital, K^T and K^N, and labor, L^T and L^N, according to constant returns to scale production functions, $Y^T = F\left(K^T, L^T\right)$ and $Y^N = H\left(K^N, L^N\right)$, which are assumed to have the usual neoclassical properties of positive and diminishing marginal products. Both sectors face two cost components: a capital rental cost equal to r^K, and a labor cost equal to the wage rate W. The traded sector is assumed to be perfectly competitive. As described in more details below, the non-traded sector contains a large number of industries and each industry is comprised of differentiated monopolistically competitive intermediate firms.[14]

The final non-traded output, Y^N, is produced in a competitive retail sector with a constant-returns-to-scale production which aggregates a continuum measure one of sectoral non-traded goods.[15] We denote the elasticity of substitution between any two different sectoral goods by $\omega > 0$. In each sector, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good. The elasticity of substitution between any two varieties within a sector is denoted by $\epsilon > 0$, and we assume that this is higher than the elasticity of substitution across sectors, i.e. $\epsilon > \omega$ (see e.g., Jaimovich and Floetotto [2008]). Within each sector, there is monopolistic competition; each firm that produces one variety is a price setter. Output $\mathcal{X}_{i,j}$ of firm i in sector j is produced using capital and labor, i.e. $\mathcal{X}_{i,j} = H\left(\mathcal{K}_{i,j}, \mathcal{L}_{i,j}\right)$. Each firm chooses capital and labor by equaliz-

[14]This assumption relies upon observed empirical facts. The markups in the traded sector we estimated for a sample of 13 OECD economies average to 1.2 with small dispersion across countries whereas for the non-traded sector, the markups average to 1.4 with large dispersion across countries. Additionally, assuming that the traded sector is imperfectly competitive would not affect qualitatively the results, as long as the markup is fixed. Estimates of the markups charged by the traded sector are available on request while estimates for the non-traded sector are reported in Table 3.

[15]This setup builds on Jaimovich and Floetotto's [2008] model. Details of its derivation are therefore relegated to the Appendix, which is available on request.

ing markup-adjusted marginal products to the marginal cost of inputs, i. e. $H_K/\mu = r^K$, and $H_L/\mu = W$, where μ is the markup over the marginal costs. At a symmetric equilibrium, non-traded output is equal to $Y^N = N\mathcal{X} = H\left(K^N, L^N\right)$. We assume that there are a large number of firms within each sector, so that each single intermediate producer is small relative to the economy. In this set up each producer in a sector faces the same price elasticity of demand, ϵ. Hence, the producer of a variety charges a constant markup $\mu = \frac{e}{e-1}$, where e is the price-elasticity of demand. Because the number of competitors is large, e is equal to ϵ. In section 5, we relax this assumption and assume instead that a finite number of firms operate within each sector producing non-tradable varieties.[16] Whether the markup is fixed or endogenous, we assume instantaneous entry, which implies that the zero profit condition holds at each instant of time.

Denoting by $k^i \equiv K^i/L^i$ the capital-labor ratio for sector $i = T, N$, enables us to express the production functions in intensive form, i.e. $f\left(k^T\right) \equiv F\left(K^T, L^T\right)/L^T$ and $h\left(k^N\right) \equiv H\left(K^N, L^N\right)/L^N$. Production functions are supposed to take a Cobb-Douglas form: $f\left(k^T\right) = \left(k^T\right)^{\theta^T}$, and $h\left(k^N\right) = \left(k^N\right)^{\theta^N}$, where θ^T and θ^N represent the capital income share in output in the traded and non-traded sectors respectively. Since inputs can move freely between the two sectors, marginal products in the traded and the non-traded sector equalize:

$$\theta^T \left(k^T\right)^{\theta^T-1} = \frac{P}{\mu}\theta^N \left(k^N\right)^{\theta^N-1} \equiv r^K, \tag{6a}$$

$$\left(1-\theta^T\right)\left(k^T\right)^{\theta^T} = \frac{P}{\mu}\left(1-\theta^N\right)\left(k^N\right)^{\theta^N} \equiv W. \tag{6b}$$

These static efficiency conditions state that the sectoral marginal products must equal the labor cost W and capital rental rate r^K.

Aggregating labor and capital over the two sectors, gives us the resource constraints for the two inputs:

$$L^T + L^N = L, \quad K^T + K^N = K. \tag{7}$$

2.3 Government

The final agent in the economy is the government which finances government expenditure by raising lump-sum taxes Z in accordance with the balanced condition:[17]

$$G^T + PG^N = Z. \tag{8}$$

Public spending consists of purchases of traded goods, G^T, and non-traded goods, G^N. As the data suggests that government expenditure tends to be mainly on non-traded goods,

[16] As stressed by Yang and Heijdra [1993], departing from the usual assumption made by Dixit and Stiglitz [1977] implies that the price elasticity of demand becomes an increasing function of the number of firms and that the markup is endogenous.

[17] Government spending on traded goods G^T is considered for calibration purpose. The effects of a permanent and temporary fiscal expansion on G^T are explored in the Appendix, available on request.

we concentrate on the effects of an increase in G^N.[18]

2.4 Short-Run Static Solutions

System (6a)-(6b) can be solved for sector capital intensity ratios: $k^T = k^T(P)$ and $k^N = k^N(P)$. Using the fact that $W \equiv \theta^T \left(k^T\right)^{\theta^T-1}$, the wage rate also depends on P, i.e. $W = W(P)$, with $W_P \gtrless 0$. An increase in the relative price P raises or lowers W depending on whether the traded sector is more or less capital intensive than the non-traded sector.

Plugging sectoral capital-labor ratios into the resource constraints and production functions leads to short-term static solutions for sectoral output: $Y^T = Y^T(K, L, P)$ and $Y^N = Y^N(K, L, P)$. According to the Rybczynski effect, a rise in K raises the output of the sector which is more capital intensive, while a rise in L raises the output of the sector which is more labor intensive. An increase in the relative price of non tradables exerts opposite effects on sectoral outputs by shifting resources away from the traded sector towards the non-traded output.

By substituting first $W = W(P)$, eqs. (5a)-(5b) can be solved for consumption and labor supply as follows: $C = C\left(\bar{\lambda}, P\right)$ with $C_{\bar{\lambda}} < 0$, $C_P < 0$, and $L = L\left(\bar{\lambda}, P\right)$ with $L_{\bar{\lambda}} > 0$ and $L_P \gtrless 0$. A rise in the shadow value of wealth induces agents to cut their real expenditure and to supply more labor. By raising the consumption price index, an appreciation in the relative price of non tradables drives down consumption. Finally, depending on whether $k^T \gtrless k^N$, a rise in P stimulates or depresses labor supply by raising or lowering W.

2.5 Macroeconomic Dynamics

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises two equations. First, the dynamic equation for the relative price of non-traded goods (5d) equalizes the return on domestic capital and traded bonds $r^\star$. Second, the accumulation equation for physical capital clears the non-traded goods market along the transitional path. This can be written as:

$$\dot{K} = \frac{Y^N(K, L, P)}{\mu} - C^N\left(\bar{\lambda}, P\right) - G^N - \delta_K K. \tag{9}$$

Dynamic equations (5d) and (9) form a separate subsystem in P and K. Inserting short-run static solutions, linearizing these two equations around the steady-state, and denoting the long-term values with a tilde, we obtain in a matrix form:

$$\begin{pmatrix} \dot{K} \\ \dot{P} \end{pmatrix} = \begin{pmatrix} \frac{Y_K^N}{\mu} - \delta_K & \frac{Y_P^N}{\mu} - C_P^N \\ 0 & \frac{Y_K^T}{\tilde{P}} \end{pmatrix} \begin{pmatrix} K(t) - \tilde{K} \\ P(t) - \tilde{P} \end{pmatrix} \tag{10}$$

[18]The data summarized in Table 3 reveal that the non-tradable content of government spending averages about 90%.

The determinant of the linearized 2×2 matrix is unambiguously negative and the trace is equal to $r^\star$.[19] Hence, the equilibrium yields a unique one-dimensional stable saddle-path, irrespective of the relative sizes of the sectoral capital-labor ratios. Denoting the negative eigenvalue by ν_1 and the positive eigenvalue by ν_2, the general solutions for K and P are

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \quad P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{11}$$

where B_1 and B_2 are constants to be determined and ω_2^i is the eigenvector associated with the eigenvalue ν_i (with $i = 1, 2$). Two features of the two-sector economy's equilibrium dynamics deserve special attention. First, as long as the markup is fixed, if $k^T > k^N$, the temporal path for the relative price remains flat for the no-arbitrage condition (5d) to be fulfilled. Hence, in this case, $\omega_2^1 = 0$. If capital intensities are reversed, then $\omega_2^1 < 0$. As a consequence, the relative price exhibits transitional dynamics; P and K move in opposite directions. Second, after a permanent fiscal shock, to ultimately approach the steady-state $(\tilde{K}, \tilde{P})$ and to satisfy the transversality condition $\lim_{t \to \infty} P(t) K(t) e^{-r^\star t} = 0$, it is necessary to set the arbitrary constant B_2 equal to zero. When the expansionary policy is only transitorily implemented (i.e. the fiscal shock only lasts for $\mathcal{T}$ periods), two periods have to be considered, namely a first period (labelled period 1) over which the temporary policy is in effect, and a second period (labelled period 2) after the policy has been removed. While the small country converges towards its new long run equilibrium over period 2, i. e. B_2 must be set to zero, the economy follows unstable paths over period 1. These are described by eqs. (11).

Substituting eqs. (9) and (8) into eq. (3), we obtain the dynamic equation for the current account (denoted by $CA \equiv \dot{B}$):

$$\dot{B} = r^\star B + Y^T\left(K, L, P\right) - C^T\left(\bar{\lambda}, P\right) - G^T, \tag{12}$$

where $Y^T - C^T - G^T$ correspond to net exports. Eq. (12) states that the current account is equal to the balance of trade denoted by NX plus interest receipts on outstanding assets. Linearizing (12) around the steady-state and substituting (11), the general solution for the stock of foreign assets is given by:[20]

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}. \tag{13}$$

When the disturbance is temporary, we must take into account that assets (i.e. domestic capital and foreign bonds) have been accumulated (or decumulated) over the period 1. The time path for net foreign assets is described by eq. (13) during this unstable period.

[19] See the Appendix for further details.

[20] If $k^T > k^N$, then $\Phi_1 = -\tilde{P} < 0$ and $\Phi_2 = \tilde{P}\nu_1 \left\{1 + \frac{\omega_2^2}{\tilde{P}\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_1 + \delta_k\right)\right]\right\}$. If $k^N > k^T$, then $\Phi_1 = \tilde{P}\nu_2 \left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right)\right]\right\}$ and $\Phi_2 = -\tilde{P}$.

As stocks of assets are modified over period 1 (i.e. $(0,\mathcal{T})$), we have to take new initial conditions (i.e. $B_{\mathcal{T}}$ and $K_{\mathcal{T}}$) into account when the fiscal policy is removed.[21]

2.6 Steady-State

We will now discuss the salient features of the steady-state. Setting $\dot{P}=0$ into eq. (5d), we obtain the equality between the rate of return on domestic capital income and the exogenous world interest rate, i.e.

$$\frac{h_k\left[k^N\left(\tilde{P}\right)\right]}{\mu}-\delta_K=r^\star. \tag{14}$$

This equality determines the steady-state value of the relative price of non-tradables, i.e. $\tilde{P}$. Hence, the long-run level of P remains unaffected by a rise in government spending, as long as the markup is fixed.

Setting $\dot{K}=0$ into eq. (9) yields the market-clearing condition for the non-traded good:

$$\frac{Y^N\left(\tilde{K},\tilde{L},\tilde{P},\right)}{\mu}=C^N\left(\bar{\lambda},\tilde{P}\right)+\tilde{I}+G^N, \tag{15}$$

where $\tilde{I}=\delta_K\tilde{K}$.

Setting $\dot{B}=0$ into eq. (12) leads to the market-clearing condition for the traded good:

$$Y^T\left(\tilde{K},\tilde{L},\tilde{P}\right)=-r^\star\tilde{B}+C^T\left(\bar{\lambda},\tilde{P}\right)+G^T. \tag{16}$$

For the country to remain ultimately solvent, we have to impose one single and overall intertemporal budget constraint:[22]

$$B_0-\tilde{B}=\Phi_1\left(K_0-\tilde{K}\right). \tag{17}$$

where $\Phi_1<0$ describes the effect of capital accumulation on the the external asset position and K_0 and B_0 are the initial conditions.[23] The four equations (14)-(17) jointly determine $\tilde{P}$, $\tilde{K}$, $\tilde{B}$ and $\bar{\lambda}$.

3 Temporary Fiscal Expansion: An Analytical Exploration

In this section, we explore analytically the macroeconomic effects of a temporary fiscal expansion, emphasizing how traded and non-traded goods modify the propagation mech-

[21] Following a permanent budget policy, the economy moves along a stable path; hence, the trajectory for $B(t)$ is obtained by invoking the transversality condition $\lim_{t\to\infty}\bar{\lambda}B(t)e^{-r^\star t}=0$ which implies that $B_2=0$.

[22] Substituting first the short-run solutions, then linearizing the dynamic equation of the internationally traded bonds (12) in the neighborhood of the steady-state, substituting the solutions for $K(t)$ and $P(t)$ and finally invoking the transversality condition, we obtain the linearized version of the nation's intertemporal budget constraint (17).

[23] Since for all parameterizations, Φ_1 is always negative, we assume $\Phi_1<0$ from now thereon. Hence, capital accumulation deteriorates the current account along the transitional path.

anism.[24] Suppose that at time $t = 0$, the government raises public spending on the non-traded good and at time $\mathcal{T}$ it removes the expansionary budget policy.[25] The higher $\mathcal{T}$, the stronger the persistence of the shock. For ease of computation, we first consider the labor supply to be inelastically supplied; only in this case are we able to derive analytical expressions for the impact effects of a temporary fiscal expansion, regardless of sectoral capital intensities.[26] At the end of this section, we discuss the implications of elastic labor supply, when analytical expressions can be derived only if $k^T > k^N$.[27]

3.1 The Case of Inelastic labor Supply

We investigate both the impact and long-run effects of a temporary rise in G^N by assuming that the labor supply is fixed. In particular, we provide analytical expressions of impact effects on key economic variables, namely investment and the current account.

Impact Effects

Let fist consider the situation when the traded sector is more capital intensive than the non-traded sector. Since taxes must be raised to balance the budget, the subsequent fall in the real disposable income induces agents to lower their consumption. Because the reduction in real expenditure is spread over the two goods, the fall in consumption of the non-traded good is not large enough to compensate for the rise in public spending G^N. Hence, an excess demand arises in the non-traded good market which produces a drop in investment (see eq. (9)). Formally, the initial reaction of investment is given by

$$\frac{\mathrm{d}I(0)}{\mathrm{d}G^N} = \alpha_C \left(1 - e^{-r^\star \mathcal{T}}\right) - 1 < 0, \tag{18}$$

where α_C is the non tradable content of consumption. The first term on the RHS of (18) reflects the positive effect of the drop in C^N on capital accumulation. As shown by the second term on the RHS, the rise in G^N withdraws resources from investment. Since $\alpha_C \left(1 - e^{-r^\star \mathcal{T}}\right)$ is smaller than one, a rise in G^N unambiguously crowds-out investment on impact. If public spending is raised over a short period (i.e. if $\mathcal{T}$ is small), agents reduce their real expenditure by a small amount because the tax burden is low. Consequently, the excess of demand in the non-traded good market increases, which drives down further investment.

[24]As the shocks identified in the VAR literature are transitory, we focus the theoretical analysis on temporary increases in government spending.

[25]We assume further that all agents perfectly understand at the outset the temporary nature of the policy change. Hence, at time $\mathcal{T}$, there is no new information and thereby no jump in the marginal utility of wealth at this date.

[26]To derive formal solutions after a temporary fiscal shock, we applied the procedure developed by Schubert and Turnovsky [2002].

[27]In deriving formal solutions after temporary fiscal shocks, without loss of generality, we assume that the non-traded sector is perfectly competitive so that the markup is one, and that the rate of depreciation of physical capital is zero. In the numerical analysis, we relax these two assumptions.

The adjustment of the current account is described by the market-clearing condition for the traded good, i.e. eq (12). Since consumption in the traded good falls, net exports increase which yields a current account surplus as the stock of traded bonds is initially predetermined. After tedious computations, it can be shown that the initial reaction of the current account is given by:

$$\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N} = \tilde{P}\left(1-\alpha_C\right)\left(1-e^{-r^\star \mathcal{T}}\right) > 0. \tag{19}$$

where $1-\alpha_C$ is the tradable content of consumption. The current account surplus increases with the length of the shock $\mathcal{T}$ as consumption in the traded good C^T falls by a larger amount.

If the traded sector is more capital intensive than the non-traded sector, sectoral outputs remain unaffected as the relative price P is unchanged and the capital stock is predetermined.

The responses of investment and the current account change dramatically when the capital intensities are reversed. The reason is that now, the relative price of non tradables appreciates on impact as a result of the excess of demand in the non-traded good market. The increase in P influences sectoral outputs by shifting resources from the traded sector towards the non-traded sector. Since Y^N now expands, the response of investment becomes ambiguous as resources are devoted to capital accumulation. More formally, the reaction of investment is given by:

$$\frac{\mathrm{d}I(0)}{\mathrm{d}G^N} = \left(\frac{\nu_2-\nu_1}{\nu_2}\right)\left(1-e^{-\nu_2\mathcal{T}}\right) + \frac{\sigma_C \tilde{C}^N}{\bar{\lambda}}\frac{\nu_1}{\nu_2}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} - 1, \tag{20}$$

where

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\alpha_C\bar{\lambda}}{\sigma_C\tilde{C}^N}\left(\frac{1-\tilde{\Psi}}{1-\alpha_C\tilde{\Psi}}\right)\left[\left(1-e^{-\nu_2\mathcal{T}}\right) - \frac{\left(e^{-r^\star\mathcal{T}}-e^{-\nu_2\mathcal{T}}\right)}{\left(1-\tilde{\Psi}\right)}\right] > 0,$$

with $0 < \tilde{\Psi} \equiv -\frac{r^\star}{\nu_2^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1 < 1$. The first term on the RHS of eq. (20) reflects the positive influence on investment of the initial appreciation in the relative price of non tradables. This term is positive and increasing with $\mathcal{T}$. As shown by the second term on the RHS, the wealth effect reflected by a rise in the marginal utility of wealth $\bar{\lambda}$, now exerts a negative impact on capital accumulation. The reason is that the drop in real disposable income induces agents to reduce C^N which in turn moderates the excess of demand in the non-traded good market and thereby the appreciation in P. The last term on the RHS of eq. (20) reflects the rise in G^N that withdraws resources from physical capital accumulation. Hence, investment may now respond positively to a fiscal shock as a result of the initial rise in P.

Whereas the relative price appreciation of non-traded goods exerts a positive impact on investment by raising non-traded output, it drives down net exports by depressing traded output on impact, due to the shift of resources towards the non-traded sector. Regardless of the length of the fiscal shock, we find that the current account unambiguously deteriorates after an increase in G^N:

$$\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N} = -\tilde{P}\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \nu_1 \Phi_1 \frac{(1-\alpha_C)}{\nu_2\left(1-\alpha_C \tilde{\Psi}\right)}\left[\left(1-e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] < 0. \tag{21}$$

where $\Phi_1 = -\tilde{P}\left(1 + \frac{1}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right) < 0$.

When $k^N > k^T$, the initial rise in the relative price of non tradables causes a shifting of labor away from the traded to the non-traded sector. Hence, Y^N rises in the short-run while Y^T falls on impact.

Long-Run Effects

A temporary increase in government spending has permanent or long-run effects, because the model features the zero-root property.[28] Since government spending reverts back to its initial level at time $\mathcal{T}$, in the long-run (i.e. in the steady-state) changes are only driven by the change in the equilibrium value of the marginal utility of wealth. Confronted with a fall in their disposable income, agents are induced to permanently lower their real consumption. As consumption in the non-traded sector is reduced, non-traded output is decreased as well in the long-run, regardless of sectoral capital intensities. Hence, labor shifts from the non-traded to the traded sector, and traded output expands in the long-run. The reason is that the small open economy decumulates traded bonds as a result of the investment boom once the fiscal shock ends. To service the debt accumulated over the transition, the economy must run a trade balance surplus in the long-run, which is achieved through a drop in C^T and a rise in Y^T.

As labor is fixed, the long-run adjustment of GDP is driven by capital accumulation which depends on sectoral capital intensities. Since Y^N declines in the long-run to meet lower demand, capital falls or rises depending on whether the non-traded sector is more or less capital intensive, and so GDP decreases or increases.[29]

3.2 The Case of Elastic Labor Supply

We now clarify the role of labor supply in the transmission of fiscal policy.

Impact Effects

Let us first assume that the traded sector is more capital intensive than the non-traded sector. Following the drop in their disposable income, agents always cut their real expenditure but also supply more labor. According to Rybczynski's theorem, a rise in labor supply raises the output of the sector which is more labor intensive.

The labor supply channel now makes the responses of investment and the current ac-

[28] Technically, this follows from the assumption that $\beta = r^\star$, which requires the joint determination of the transition and the steady-state.

[29] Formal expressions are derived by setting $\sigma_L = 0$ into (24).

count ambiguous by affecting sectoral outputs. More precisely, the rise in non-traded output implies that higher public spending G^N may crowd in or crowd out capital investment. Formally, the initial reaction of investment should now be rewritten as:

$$\frac{\mathrm{d}I(0)}{\mathrm{d}G^N} = -\left[1 + \left(1 - e^{-r^\star \mathcal{T}}\right) \frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\nu_1 - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \lessgtr 0. \tag{22}$$

Setting $\sigma_L = 0$ in eq. (22) yields eq. (18). As long as $\sigma_L > 0$, the sign of eq. (22) is no longer clear-cut. The more responsive the labor supply, the larger the increase in employment on impact, and thereby the less likely it is that investment is crowded out by public spending.

Turning to the initial response of the current account, we obtain after computation:

$$\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N} = \tilde{P}\left(1 - e^{-r^\star \mathcal{T}}\right)\left[1 + \frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\nu_1 - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \gtrless 0. \tag{23}$$

By depressing traded output, the increase in labor now makes a deterioration in the current account possible. The higher the elasticity of labor supply σ_L, the more likely an initial drop in the current account.

When $k^N > k^T$, the initial change in labor supply is the result of two opposite forces: the wealth effect which induces agents to supply more labor and the appreciation in the relative price of non tradable goods that counteracts this effect by driving down the wage rate. Hence, employment may increase or decrease on impact. As will become clear in Section 4, the elasticity σ_L plays a crucial role in driving the initial reaction of labor supply, and thereby the change in GDP on impact. For clarity, let us assume that agents supply more labor. Higher employment exerts a negative impact on non-traded output and a positive influence on traded output.[30] Hence, the increase in L makes a drop in investment more likely as resources are shifted towards the traded sector. However, the relative price of non-tradables exerts a positive impact on investment. Finally, the initial current account reaction is no longer clear cut as net exports may fall or increase on impact. More precisely, while a higher labor supply boosts traded output, the appreciation of the real exchange rate exerts a negative impact on Y^T. To sum up, if the relative price channel predominates, investment is crowded in while the current account falls into deficit.

Long-Run Effects

The conclusions established in the case of an inelastic labor supply hold in the long run, with the exception of the steady-state changes in physical capital and GDP. Regardless of sectoral capital intensities, it is found that the open economy accumulates physical capital as long as the labor supply is elastic enough. Using the fact that the steady-state capital stock can be expressed as a function of the marginal utility of wealth and government

[30] We were unable to obtain analytical results for $k^N > k^T$. Yet, as shown numerically, the analytical results for an inelastic labor supply hold.

spending G^N, and remembering than in the long-run, G^N remains unchanged, the long-run adjustment of physical capital is given by:[31]

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = K_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{1}{\bar{\lambda}\nu_1}\left(\sigma_C\tilde{C}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_1\right)\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} > 0, \quad k^T > k^N, \tag{24a}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = K_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{1}{\bar{\lambda}\nu_2}\left(\sigma_C\tilde{C}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\right)\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} \gtreqless 0, \quad k^N > k^T, \tag{24b}$$

where $K_{\bar{\lambda}}$ represents the partial derivative of the steady-state capital stock w.r.t. the shadow value of wealth $\bar{\lambda}$, $\nu_1 < 0$ and $\nu_2 > 0$ are the stable and unstable roots, and $\mathrm{d}\bar{\lambda}/\mathrm{d}G^N > 0$ reflects the wealth effect. If $k^T > k^N$, capital stock and labor supply are positively correlated in the long-run, and a temporary fiscal expansion crowds in investment in the long run. With the reversal of capital intensities, $\tilde{K}$ and $\tilde{L}$ move together only if $\sigma_C\tilde{C}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2 < 0$. More precisely, if the labor supply is elastic enough, higher employment triggers a drop in non-traded output to such an extent that capital stock must increase to clear the non-traded good market. As we shall see in the next section, $\tilde{K}$ increases in all scenarios, even if σ_L is small.

4 Temporary Fiscal Expansion: A Quantitative Exploration

In this section, we analyze the effects of a temporary rise in government spending quantitatively. For this purpose we solve the model numerically. We therefore discuss parameter values first, before turning to the long- and short-term effects of the fiscal shock

4.1 Baseline Parametrization

We start by describing the calibration of consumption-side parameters that we use as a baseline. The world interest rate which is equal to the subjective time discount rate β is set to 1%. One period of time corresponds to a quarter. The elasticity of substitution between traded and non-traded goods ϕ is set to 1.5 (see e.g. Cashin and Mc Dermott [2003]). An additional critical parameter is φ which is set to 0.5 in the baseline calibration to target a non tradable content in total consumption expenditure (i.e., α_C) of 45%, in line with our empirical evidence.[32] The intertemporal elasticity of substitution for consumption σ_C is set to 0.5 because empirical evidence overwhelmingly suggest values smaller than one.[33] One critical parameter is the intertemporal elasticity of substitution for labor supply σ_L. In our baseline parametrization, we set $\sigma_L = 0.5$, in line with evidence reported by Domeij and Flodén [2006].

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate $\delta_K = 1.5\%$ to target an investment-GDP ratio of 20%. The

[31] Solving (14)-(16) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ yields: $\tilde{P}$ = constant, $\tilde{K} = K\left(\bar{\lambda}, G^N\right)$ and $\tilde{B} = B\left(\bar{\lambda}, G^N\right)$. See the Appendix for further details.

[32] Table 3 shows the non tradable content of GDP components for thirteen OECD countries.

[33] Consumption expenditure is 55% of GDP for our baseline calibration.

shares of sectoral capital income in output take two different values depending on whether the traded sector is more or less capital intensive than the non-traded sector. In line with our estimates, if $k^T > k^N$, θ^T and θ^N are set to 0.4 and 0.3, respectively.[34] Alternatively, when $k^N > k^T$, we choose $\theta^T = 0.3$ and $\theta^N = 0.4$. Setting the elasticity of substitution between sectoral goods ω to 1 and the elasticity of substitution between varieties, ϵ to 4 yields a markup charged by the non-traded sector of 1.35, which is close to our estimates (see Table 3).

We set G^N and G^T so as to yield a non-tradable share of government spending of 90%, and government spending as a share of GDP of 20%.[35] We consider three different scenarios for the duration of the fiscal shock: a short-lived ($\mathcal{T} = 8$), a medium-lived ($\mathcal{T} = 16$), and a long-lived ($\mathcal{T} = 32$) fiscal shock. As the baseline scenario, we take the medium-lived fiscal shock, i.e. a shock that lasts 16 quarters. In this case, the cumulative increase in government spending corresponds approximately to the cumulative increase in US government spending six years after an exogenous spending shock by one percentage point of GDP according to the estimates reported by Cardi and Müller [2010]. For $\mathcal{T} = 16$, we also conduct a sensitivity analysis with respect to the elasticity of labor supply (i.e. we set σ_L to 0.1 and 1).

4.2 Long-Run Effects

Panel A of Table 1 gives the numerical results for the long-run effects of a temporary fiscal expansion. In the baseline scenario, agents cut real expenditure by 0.07% of GDP while they raise labor supply by 0.13% as a result of the decrease in real disposable income. The open economy accumulates physical capital in the long-run, regardless of sectoral capital intensities. This is because when $k^T > k^N$, the increase in employment produces an excess of supply in the non-traded good market which calls for a long-run increase in capital that yields a shift of labor towards the more capital intensive sector. This in turn raises traded output and thereby net exports which allows the external debt accumulated in the short-run to be serviced. If the capital intensities are reversed, the long-run rise in labor drives down non-traded output. To clear the market, capital must be raised. Interestingly, while GDP increases in all scenarios, the rise in G^N always benefits the traded sector in the long-run, irrespective of sectoral capital intensities. More precisely, remembering that the relative price P remains unaffected by the fiscal shock in the long-run, traded output, i.e. $Y^T = L^T f\left(k^T(\tilde{P})\right)$, is only driven by employment L^T. As the open economy must run a

[34] Table 3 gives the values of θ^j ($j = T, N$) for thirteen OECD countries. The values of θ^T and θ^N we have chosen correspond roughly to the averages for countries with $k^T > k^N$. For these values, the non tradable content of GDP and labor are 63% and 66%, respectively. When $k^N > k^T$, we can use reverse but symmetric values for θ^N so that the size of $k^T - k^N$ remains unchanged. For $\theta^T = 0.3$ and $\theta^N = 0.4$, the non tradable content of GDP and labor are 69% and 65%, respectively.

[35] Close to the average of the values reported in Table 3, the ratios G^T/Y^T and G^N/Y^N are 6% and 28% in the baseline calibration.

trade balance surplus in the long-run, traded output and thereby labor in that sector must increase. With regard to non-traded output, Y^N decreases in all scenarios if $k^N > k^T$ since the open country experiences sizeable current account deficits in the short-run that require significant long-run improvement in the balance of trade.

4.3 Short-Run Effects

We now turn to the short-run effects of the fiscal expansion. We take the medium-lived spending shock as our baseline scenario, but we also refer to short-lived and long-lived fiscal shocks, as the length of fiscal stimulus may vary across countries. Panels B and C of Table 1 show the results for this situation, as well as for a number of alternative scenarios. While panel B gives the response on impact, panel C displays the cumulative responses over the first two years (i.e. eight quarters) after the shock.

The transitional paths of key variables under the baseline scenario are displayed in Figures 1. The responses of GDP, investment and current account are expressed in percentage of the initial steady-state output, while the real exchange rate is given as the percentage deviation from the initial steady state. Horizontal axes measure quarters. When the reaction of the variable is sensitive to the elasticity of labor supply, we compare the baseline scenario (solid line) to alternative scenarios. The dashed-dotted line gives the results for a low labor supply elasticity (i.e. $\sigma_L = 0.1$), while the dotted line shows those for a high labor supply elasticity (i.e. $\sigma_L = 1$).

Before analyzing in the detail the role of sectoral reallocation in shaping the short-run dynamics in response to a temporary increase in government spending, we should mention the set of empirical evidence established by Cardi and Müller [2010]. It is found that in all the countries in their sample, an exogenous increase in government spending raises output, and induces a simultaneous decline of investment and the current account. In the following, we discuss the predictions of our model for the behavior of these variables when $k^T > k^N$ and when $k^N > k^T$.

While employment and thereby GDP increase in all the scenarios where $k^T > k^N$, labor supply and output increase slightly or decrease when the capital intensities are reversed. The reason is that when $k^T > k^N$, agents are induced to supply more labor as a result of the wealth effect. By contrast, when $k^N > k^T$, the appreciation of the real exchange rate drives down the wage rate which in turn counteracts the wealth effect. Interestingly, we find that employment and thereby GDP falls on impact if σ_L is raised from 0.5 to 1. The reason is that for a given change in the shadow value of wealth, the relative price must appreciate more as a result of the larger labor outflow. Hence, the consecutive decrease in W is large enough to induce agents to supply less labor, which reduces GDP by 0.05% on impact.

In the model, the initial reaction of investment is ambiguous as long as labor supply is elastic. Numerically, we find that its short-run response depends heavily on sectoral capital intensities. On impact, an increase in G^N crowds out investment only if the traded sector is more capital intensive. While non-traded output expands as a result of the increase in labor supply, the rise in public spending G^N produces an excess of demand which must be eliminated by a drop in investment. As shown in the seventh line of panel B of Table 1, the less elastic labor supply is, the larger the crowding-out effect of investment. By contrast, if the non-traded sector is more capital intensive, the increase in G^N triggers an appreciation in the relative price of non tradables P which stimulates Y^N and thereby investment, in all scenarios. The cumulative responses reported in the third line of panel C of Table 1 show that a fiscal expansion crowds in investment by about 3.22% of initial GDP on impact if $k^N > k^T$, while investment is crowded out by 3.16% if $k^T > k^N$. The investment boom when $k^N > k^T$ triggers a positive cumulative response of output, as summarized in the fifth line of panel C in Table 1. By contrast, the decline in investment when $k^T > k^N$ implies a smaller cumulative response of GDP, across all scenarios.[36]

As shown in the eight line of panel B of Table 1, the open economy experiences a current account deficit, regardless of sectoral capital intensities. When $k^T > k^N$, a larger labor supply induces a shift of labor towards the non-traded sector, and the subsequent decrease in traded output drives down net exports. When the capital intensities are reversed, the appreciation in the real exchange rate more than offsets the positive effect of the labor inflow on Y^T. As a consequence, traded output falls which yields a current account deficit.

4.4 Transitional Adjustment

We now discuss the dynamic effects which are depicted in Figures 1, starting with the adjustment of labor which is displayed in the third line. The dashed-dotted line shows the results for a weakly responsive labor supply (i.e. $\sigma_L = 0.1$), while the dotted line shows the results for a highly elastic labor supply (i.e. $\sigma_L = 1$). If the traded sector is more capital intensive, the temporal path for L is flat as the relative price P remains unaffected. When $k^N > k^T$, the dynamics for L no longer degenerate as a result of the depreciation in the real exchange rate (after its initial appreciation) along the transitional path. The consequent increase in the wage rate W induces agents to supply more labor during the transitional period.

The transitional path of investment is also quite distinct, depending on whether the traded sector is more or less capital intensive than the non-traded sector. Along the transi-

[36] As shown in the fifth line of panel C in Table 1, the cumulative response of GDP at a two-year horizon is negative in two scenarios when $k^T > k^N$: when σ_L is low and when the fiscal shock is short-lived. In these two scenarios, the response of labor supply is limited, Y^N rises less, and the excess demand in the non-traded good market becomes larger, which in turn produces a larger decline in investment.

tional path, capital accumulation clears the non-traded good market. When $k^T > k^N$, the size of the crowding-out of investment reduces over time, but when $k^T > k^N$, investment decreases monotonically as the depreciation in the relative price P lowers non-traded output. After about 2 years, the investment flow becomes negative and the open economy decumulates physical capital until the fiscal policy is removed. At time $\mathcal{T}$, government spending G^N reverts back to its initial level which releases resources for capital accumulation. Regardless of sectoral capital intensities, investment is crowded in.

The temporal path for GDP is driven by the adjustments in both labor and capital. In the case $k^T > k^N$, the dynamics for GDP are the mirror image of capital accumulation: the slowdown in GDP growth as government spending is raised originates from the crowding out of investment. By contrast, when $k^N > k^T$, the temporal path of output is hump-shaped: GDP growth first increases as labor supply rises, and then slows down as a result of the decline in investment which starts after about two years. At the time the fiscal policy is removed, the economy experiences an investment boom which boosts GDP in both cases. While in a one-sector model, the response of output increases with labor supply responsiveness (as stressed by Baxter and King [1993]), this is not the case when we consider a two-sector model. Considering that $k^T > k^N$ and raising σ_L from 0.5 to 1 increases the cumulative GDP response from 0.32 to 0.55. By contrast, when $k^N > k^T$, the reaction of GDP decreases from 0.69 to 0.58, as a result of the drop in the wage rate which depresses labor supply.

Regardless of sectoral capital intensities, the current account stays in deficit while government spending is raised. In the case $k^T > k^N$, the decumulation of physical capital drives down traded output, which in turn amplifies the current account deficit along the transitional path. If the sectoral capital intensities are reversed, the depreciation in the relative price of non tradables moderates the decrease in Y^T and thereby the worsening in the foreign asset position. Yet, in the latter case, the current account deficit at an horizon of two years is almost three times larger, as shown in the fourth line of panel C of Table 1.

4.5 Sectoral Decomposition of the Effects of Fiscal Shocks

The sectoral decomposition of the effects of fiscal shocks sheds light on the propagation mechanism in an open economy. The impact and cumulative responses of sectoral outputs are summarized in the last two lines of panels B and C of Table 1, respectively. Interestingly, the sectoral outputs change in opposite directions, both on impact and along the transitional path. In the benchmark scenario, assuming that $k^T > k^N$, agents raises the labor supply by 0.12% which induces a shift of employment towards the more labor intensive sector. As a result, non-traded output increases by 0.32% of GDP while traded output declines by 0.24% of GDP. If sectoral capital intensities are reversed, the appreciation in the relative price of non-tradables is large enough to more than offset the Rybczynski effect which boosts non-traded output by 1% of initial GDP while the traded sector experiences a decline by the same amount. Hence, GDP remains unchanged on impact in the case $k^N > k^T$. Interestingly, raising σ_L amplifies the dispersion of sectoral output responses as agents supply less labor.

The fifth line of Figure 1 depicts the transitional paths of sectoral outputs expressed as percentage deviations from the initial steady-state values scaled by the initial GDP. The solid line depicts the transitional path for traded output while the dotted line shows the dynamics for Y^N. Along the transitional path, sectoral outputs vary in opposite directions as a result of the reallocation of inputs across sectors. When $k^T > k^N$, capital decumulation produces a fall in traded output while non-traded output expands. Whereas sectoral outputs diverge in this configuration, Y^T and Y^N converge when $k^N > k^T$. More precisely, the relative price depreciation raises traded output whereas the fall in P drives down non-traded output. Finally, as shown in the two last lines of panel A of Table 1, long-run GDP growth is driven by traded output growth. The rise in traded output is required in the long-run to produce an improvement in the balance of trade, regardless of sectoral capital intensities.

4.6 Taking the Model to the Data

Since time-series evidence on the effects of fiscal shocks, in particular on key variables like investment, current account, and GDP, is now available, we decided to compare our model's predictions withe the empirical results.

Three notable papers have estimated the effects of fiscal shocks on the trade balance: Beetsma, Giuludori and Klassen [2008], Cardi and Müller [2010], Monacelli and Perotti [2010]. While the first paper includes only GDP and trade variables in its VAR model, the other two also include components of GDP such as investment. All these papers use the Blanchard-Perotti identification scheme that assumes that government spending is predetermined within the quarter relative to the other variables included in the VAR model. Yet, they differ in their sample of countries: Beetsma et al. [2008] consider fourteen European Union countries and use a panel vector auto-regression approach; Cardi and Müller [2010] and Monacelli and Perotti [2010] estimate the effects of fiscal shocks for four countries: Canada, Australia, the UK and the US. All three papers find that an exogenous increase in government spending raises output and lowers the current account. Additionally, Cardi and Müller and Monacelli and Perotti report a substantial decline in investment following a fiscal expansion. The ability of our model to predict such empirical facts is mixed, as it relies upon sectoral capital intensities.

A rise in government spending crowds out investment only if the traded sector is more capital intensive than the non-traded sector. The reason is that when $k^N > k^T$, the relative price of non-tradables appreciates on impact, which produces a reallocation of resources towards the non-traded sector so that investment is crowded in on impact, irrespective of the shock's duration or the elasticity of labor supply. It is worthwhile noting that a one-sector small open economy model (see e.g., Karayalçin [1999]) cannot produce a drop in investment after a fiscal shock because the increased labor supply raises the marginal product of capital which leads to more investment.

We find that the current account deteriorates in all our model scenarios, in line with empirical evidence. In the model, the short-run worsening in the foreign asset position originates from the drop in the traded output caused by the labor shift towards the non-traded sector. When $k^T > k^N$, the fall in Y^T is triggered by the Rybczynski effect. If sectoral capital intensities are reversed, the real exchange rate appreciation produces the decline in Y^T.

Empirical studies generally find that a fiscal expansion tends to raise output. Our model produces a significant increase in GDP on impact in the benchmark scenario if $k^T > k^N$ since the real wage does not decrease in this case. If $k^N > k^T$, output is almost unaffected. Yet, in this case, the cumulative response of GDP at an horizon of two years becomes substantial across all scenarios, as shown in the fifth line of panel C of Table 1.

It is interesting to compare our results when $k^T > k^N$ (panel C of Table 2) with the numbers documented in empirical studies. By estimating a VAR model on quarterly time-series data for the U.S., Australia, the U.K, and Canada, covering the period 1980-2007, Cardi and Müller [2010] find that cumulative impulse responses after two years range between 0.3 and 1.1 for output, between -0.1 and -1.1 for investment, and -0.1 and -1.8 for the current account. While our model overpredicts both the crowding out of investment and the current account deficit, it predicts pretty well the GDP response, falling in the range of VAR evidence.

Finally, since our model predicts the sectoral impact of fiscal shocks, it is interesting to compare our results with empirical data in this area. Only a few previous studies have estimated the effects of a boost to government spending on sectoral outputs. Among these, Bénétrix and Lane [2010] find that fiscal spending shocks generate a shift in the sectoral composition of output as public purchases disproportionately benefit the non-traded sector. This finding is in line with our numerical results reported in the two last lines of panel B of Table 1. Regardless of sectoral capital intensities and across all the scenarios, a rise in government spending boosts non-traded output, more so if the non-traded sector is more capital intensive.

< Please insert Table 1 about here >

< Please insert Figure 1 about here >

5 Temporary Fiscal Expansion: The Case of Endogenous Markup

Several papers have stressed that the variation in the number of competitors and the consecutive change in the markup provides an important magnification mechanism, see e.g., Jaimovich and, Floetotto [2008], Wu and Zhang [2000], Zhang [2007], all of whom consider one-sector models. We therefore decide to revisit quantitatively the effects of temporary fiscal shocks by allowing for the markup to be endogenous. Since the long-run effects remain almost unchanged compared to those the case of fixed markup, we will not discuss them further. Rather, we will concentrate on how an endogenous markup modifies the short-run adjustment of key variables and influences the sectoral composition of GDP.

Before analyzing in detail the role of sectoral reallocation in shaping the short-run dynamics in response to a temporary increase in government spending, we recall the conclusions of empirical studies. Perotti [2007] finds that the real wage responds positively to a fiscal shock. Estimates by Monacelli and Perotti [2010] show that the real exchange rate depreciates in the U.S., Australia, the U.K. and Canada, while Enders et al. [2011] confirm this finding for the U.S. As shown in section 4, a two-sector model can produce the positive impact on output and the simultaneous drop in investment and the current account after a fiscal shock as long as the traded sector is more capital intensive than the non-traded sector. However, it fails to produce the real exchange rate depreciation or the rise in the real wage. Since markup variations affect the relative price P and the wage rate W, we decide to investigate whether the predictive power of the two-sector model would improve if the markup were endogenous.

5.1 Extending the Model to Endogenous Markup

So far, we adopted the Dixit-Stiglitz assumption according to which the number of competitors is large enough within each sector to yield a fixed price-elasticity of demand. Yet, as emphasized by Yang and Heijdra [1993], the Dixit and Stiglitz's [1977] assumption is an approximation when the sectoral good is aggregated from a finite number of intermediate goods. Following Jaimovich and Floetotto [2008], we depart from the usual practice by assuming that the number of firms is large enough that the strategic effects can be ignored, but not so large that the effect of entry on the firm's demand curve is minuscule. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms N. Taking into account that output of one variety does not affect the general price index P, but does influence the sectoral price level, in a symmetric equilibrium the resulting price elasticity of demand is given by:[37]

$$e(N) = \epsilon - \frac{(\epsilon - \omega)}{N}, \quad N \in (1, \infty). \tag{25}$$

Assuming that $\epsilon > \omega$, the price elasticity of demand faced by any single firm is an increasing function of the number of firms N within a sector. Henceforth, the markup $\mu = \frac{e}{e-1}$ decreases as the number of competitors increases.

In the interest of space, we restrict our attention to the major changes in deriving the macroeconomic equilibrium. First, the zero-profit condition in the intermediate good sector can be solved for the number of firms, i.e. $N = N(K, L, P)$. Bearing in mind that $\mu = \mu(N)$, the equalities of marginal products between sectors (i.e., eqs. (6)) imply that capital-labor ratios k^j ($j = T, N$) are affected by the markup and so by the number of firms, i.e. $k^j = k^j(P, \mu)$. Substituting the capital-labor ratios into $\theta^T \left(k^T\right)^{\theta^T - 1} \equiv W$ to solve for the wage rate, and into the resource constraints (i.e., eqs. (7)) and the production functions to solve for the sectoral outputs, short-run static solutions become:

$$W = W(P, \mu), \quad Y^T = Y^T(K, P, L, \mu), \quad Y^N = Y^N(K, P, L, \mu), \tag{26}$$

where $W_\mu \lesseqgtr 0$ depending on whether $k^T \gtreqless k^N$, $Y^T_\mu > 0$ and $Y^N_\mu < 0$. To understand this result intuitively, i.e. the impact of markup variations, let us consider that the number of competitors increases so that μ falls. All things being equal, since the ratio P/μ rises, non-traded output Y^N increases while traded output Y^T falls. Additionally, if $k^T > k^N$, a fall in the markup μ raises the capital-labor ratios and thereby the wage rate. As a consequence, Y^N rises further and Y^T declines more because the increased real wage induces households to raise labor supply which shifts towards the more labor intensive sector. The same logic applies in the case $k^N > k^T$ but W falls.

5.2 Short-Run Effects

We now investigate the short-term effects of fiscal shocks when the markup is endogenous, focusing on the shift in the real exchange rate and the adjustment of the real wage. The latter has been estimated as the ratio of the wage rate to the consumption price index. Numerical results for impact and cumulative effects are summarized in panels B and C of Table 2.[38] The baseline calibration is identical to that described in section 4.1.

Case $k^T > k^N$

We first consider the situation when the traded sector is more capital intensive. As the number of firms, and thereby the markup, adjusts over time, the dynamics for the real

[37] Details of the derivation can be found in the Appendix.

[38] To aid comprehension, panel B of Table 2 also shows the initial reaction of the wage rate W.

exchange rate are restored and driven by the no-arbitrage equation according to which the return on domestic capital must be equalized with the return on traded bonds:

$$\frac{h_k \left\{k^N \left[P, \mu\left(N\right)\right]\right\}}{\mu\left(N\right)} + \frac{\dot{P}}{P} - \delta_K = r^{\star}. \tag{27}$$

The markup μ depends on the number of firms N which drives profits down towards zero in the non-traded sector at each instant of time. Since non-traded output is expected to increase while government spending G^N is raised, it creates profit opportunities in that sector. Hence, the number of firms increases over time, which lowers the markup. The subsequent decrease in the return of capital h_k/μ triggered by the rise in k^N requires a fall in the relative price of non-traded goods. As shown in the second line of panel B of Table 2, P drops on impact across all scenarios, as long as $k^T > k^N$. While the initial depreciation in P is fairly small, the first line of panel C reveals that the real exchange rate depreciation becomes substantial at an horizon of two years.

The reaction of the wage rate is the result of two opposite forces: the fall in the markup which raises W and the decline in P that lowers it. As shown in the third line of panel B of Table 2, the wage rate decreases on impact as the relative price channel predominates. The second line of panel C shows that the two-year horizon cumulative response of the real wage is negative for the baseline scenario. Yet, as displayed in Figure 1, the dynamic path for the real wage shows that it increases along the transitional path and exceeds its initial level after about 6 quarters. Only if the fiscal shock is short-lived or long-lived (i.e., G^N is raised over 8 or 32 quarters), does the cumulative response of the real wage becomes positive. After a long-lived fiscal shock, both non-traded output expansion and, as a consequence, firm entry are larger. Hence, the decline in the markup is large enough to produce a positive cumulative response of the real wage. Following a short-lived fiscal shock, the real exchange rate appreciates rapidly after its short-term depreciation, and it has a positive impact on the wage rate.

Let now investigate how the markup variations modify the responses of key economic variables, relative to those obtained with a fixed markup. First, as a result of the initial drop in the wage rate, labor supply increases more moderately. Second, the real exchange rate depreciation induces a shift of resources towards the traded sector. As shown in the two last lines of panel B of Table 2, while non-traded output increases very slightly on impact in the baseline scenario, Y^N decreases substantially if the fiscal shock is short-lived (i.e., $T = 8$) or the labor supply is weakly responsive (i.e., $\sigma_L = 0.1$), because the wealth effect is smaller or the labor supply reacts less to the wealth effect. As a consequence, investment is crowded out by a larger amount (almost 1% of initial GDP rather than 0.66% when the markup is fixed). Third, as the relative price P depreciates, traded output now expands (instead of declining) in all scenarios, except that of a long-lived fiscal shock. While the initial increase in Y^T triggers a small current account surplus on impact, panel C of

Table 2 reveals that the external asset position worsens very rapidly and dramatically in the short-run. Fourth, as labor the supply increases less, the cumulative response of GDP summarized in panel C remains smaller than when the markup is fixed.

To summarize, the two-sector model can produce a real exchange rate depreciation but fails to trigger a positive cumulative response of the wage rate for the baseline duration of the fiscal shock, i.e. $T = 16$.

Case $k^N > k^T$

While the two-sector model does a fairly good job of accommodating most of the evidence reported by empirical studies if the traded sector is more capital intensive than the non-traded sector, the predictive power of the two-sector model is weak if sectoral capital intensities are reversed.

When $k^N > k^T$, non-traded output is expected to increase sizeably after a temporary increase in G^N. The consequent flow of entries triggers a substantial decline in the markup. The drop in μ now raises the return on domestic capital, which requires a real exchange rate appreciation. This prediction contradicts the empirical evidence. The real exchange rate appreciation, together with the decline in the markup, drive down the wage rate. As the cost of consumption goods increases, the real wage falls substantially. The last line of Figure 1 reveals that the real wage fails to exceed its original value along the transitional path.

The GDP response to a fiscal shock when $k^N > k^T$ is negative in most of scenarios, due the substantial decline in the real wage which exerts a negative impact on labor supply. More precisely, L increases only if σ_L is low or the fiscal shock is long-lived. Furthermore, the competition channel amplifies the increase in investment. The reason is that, as intermediate good producers in the non-traded sector perceive a more elastic demand, they are induced to produce more. Since Y^N increases by a larger amount than in the case of a fixed markup, investment is further crowded in.

Whereas non-traded output expands substantially, traded output falls dramatically across all scenarios as a result of the appreciation in the real exchange rate and the smaller markup. The subsequent decline in net exports drives the current account into a larger deficit than with a fixed markup.

< Please insert Table 2 about here >

5.3 Sectoral Effects

We now turn to the sectoral impact of fiscal policy. This will allow us to investigate whether the competition channel amplifies or reduces the heterogeneity in sectoral output responses.

When $k^T > k^N$, the competition channel modifies the distribution of the increase in GDP across sectors substantially, as summarized in the two last lines of panel B of Table 2. With a fixed markup, traded output falls in all scenarios, while the fiscal shock boosts non-traded output. But if μ is endogenous, the real exchange rate depreciation is strong enough to boost traded output on impact, as long as the fiscal shock does not last too long. The reason is that when the fiscal shock is long-lived, the wealth effect is substantial and thereby counteracts the negative impact on L of the decline in W. As a consequence, the labor supply increases substantially and shifts towards the non-traded sector.

With regard to the transitional dynamics, as shown in the sixth line of Figure 1, while the real exchange rate continues to depreciate, the crowding-out of investment together with the rise in labor supply boost Y^N but depress Y^T. In a nutshell, due to the intersectoral reallocation of resources, the transitional adjustment of Y^T is the mirror image of the dynamics of Y^N. The results displayed in the two last lines of panel C of Table 2 show that with an endogenous markup, the cumulative responses of traded and non-traded output are -3.84% and 4.05% of initial GDP respectively, while the cumulative responses are -4.11% and 4.42%, respectively, with a fixed markup. Hence, when $k^T > k^N$, the decline in μ reduces the heterogeneity in the responses of sectoral outputs. Nevertheless, the fall in traded output remains substantial after two years. Once the fiscal shock ends, non-traded output starts decreasing (as investment is crowded in), while traded output rises. In the long-run, GDP growth is mostly driven by the rise in traded output.

If $k^N > k^T$, the patterns of the transitional adjustment of sectoral output remain approximately the same as those found with a fixed markup. Yet, the competition channel amplifies the dispersion of sectoral output responses. More precisely, non-traded output rises by more than 2% of initial GDP (rather than about 1% when the markup is fixed).

In conclusion, regardless of sectoral capital intensities and whether the markup is fixed or not, non-traded output always expands significantly following a temporary increase in public spending. Allowing for the markup to depend on the number of competitors, the numerical analysis reveals that the competition channel moderates or amplifies the responses of sectoral outputs, depending on whether the traded sector is more or less capital intensive than the non-traded sector.

6 Conclusion

In this paper we have shown that the open economy version of the two-sector neoclassical model with traded and non-traded goods can account for the empirical evidence on the effects of fiscal shocks, but only if the traded sector is more capital intensive than the non-traded sector. In particular, a robust conclusion emerging from empirical papers is that government spending tends to crowd out both investment and the current account. Considering both traded and non-traded goods enables the model to account for this finding, whereas the standard one-sector small open-economy framework cannot. In addition, by enabling the markup to depend negatively on the number of competitors, the model can generate a counter-cyclical markup which is pivotal to producing the real exchange rate depreciation which has recently been documented in the empirical literature. The subsequent decline in the consumption price index and the positive impact of the lower markup on the wage rate produces an increase in the real wage, although only if the fiscal shock is short- or long-lived, not if it holds for a medium term.

In addition to the ability of the two-sector economy model to provide a better understanding of the fiscal transmission mechanism in an open economy, it delivers interesting insights into the sectoral effects of fiscal shocks. The numerical analysis reveals that the relative size of the non-traded sector increases substantially in the short-run, in line with the evidence reported by Bénétrix and Lane [2010]. Our numerical results also show that in the long-run, the relative size of the traded sector increases to service the debt accumulated in the short-run. Hence, GDP growth is mostly driven by the rise in traded output in the long-run. Along the transitional path, in all scenarios, the outputs of the two sectors move in opposite direction. More precisely, traded output can either fall or rise during the transition period when government spending is raised, depending on whether the traded sector is more or less capital intensive than the non-traded sector.

The duration of the fiscal shock plays also a pivotal role in driving the responses of both aggregate and sectoral variables. In all the scenarios, both labor supply and GDP increase more when the fiscal expansion is implemented over a long rather than a short period. The multiplier can exceed one only in this case, while the dispersion of responses of sectoral outputs is amplified.

In conclusion, we must stress a number of caveats. If the non-traded sector is assumed to be the more capital intensive sector, the model fails to match the evidence along a number of dimensions. Notably, in this case, the two-sector model fails to account for the crowding-out of investment which is one of the most consistent responses to a fiscal shock documented in the empirical literature. Additionally, if the traded sector is more capital intensive than the non-traded sector, the model cannot produce a positive cumulative response of the real wage in the baseline scenario. Finally, due to our assumption of perfect mobility across sectors, traded and non-traded output vary in opposite direction while evidence by Bénétrix and Lane [2010] mostly predict that sectoral outputs co-vary. Further analysis of these issues has to be left for future research.

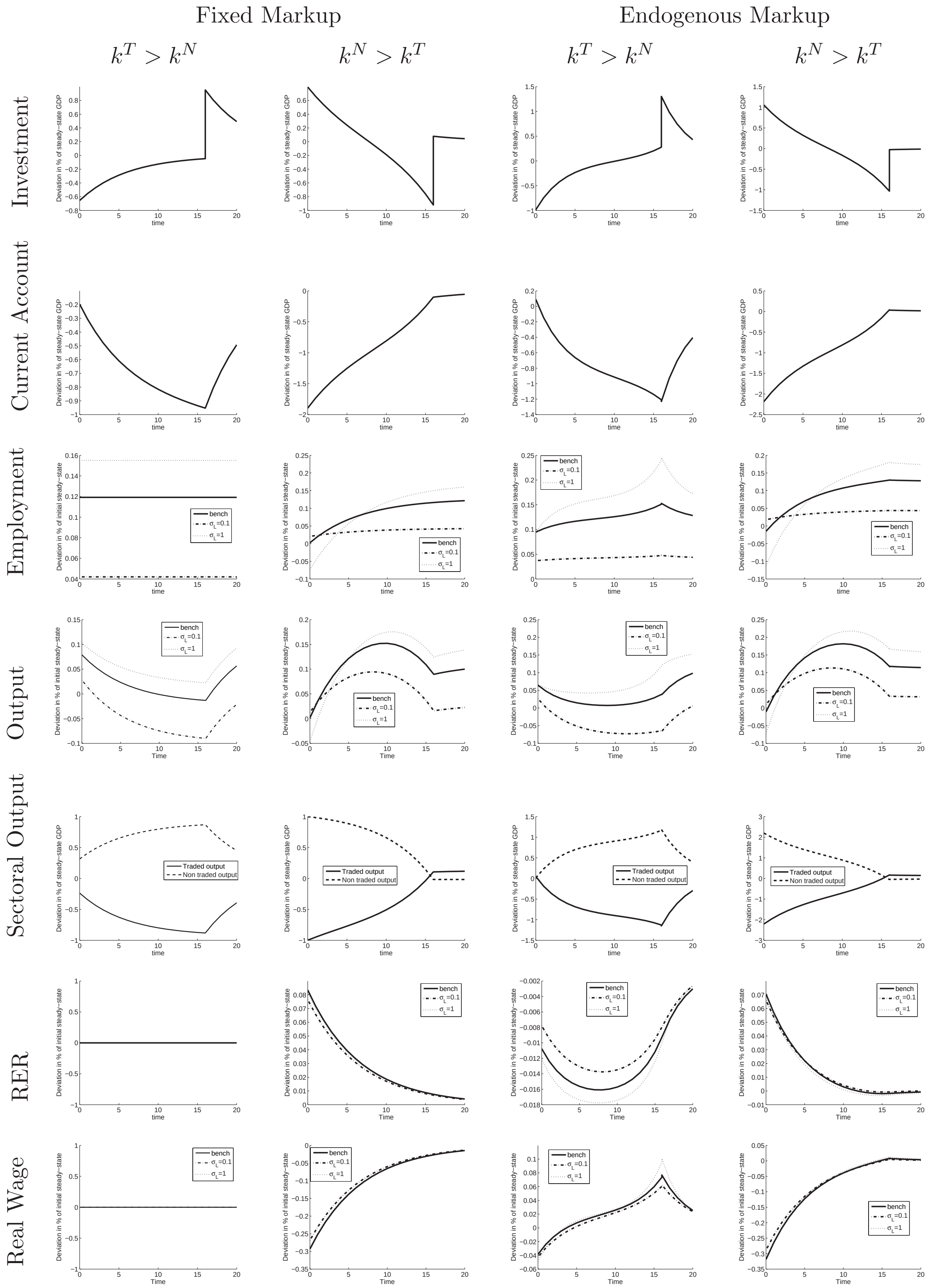

Figure 1: Effect of government spending shocks. Notes: variables are measured in percentage points of output, with the exception of employment, the real exchange rate and real wage which are scaled by their initial steady-state values.

Table 1: Quantitative Effects of a Temporary Fiscal Expansion (in %): The Case of a Fixed Markup

Variables	$k^T > k^N$					$k^N > k^T$				
	Bench $\mathcal{T} = 16$			Short $\mathcal{T} = 8$	Long $\mathcal{T} = 32$	Bench $\mathcal{T} = 16$			Short $\mathcal{T} = 8$	Long $\mathcal{T} = 32$
	$(\sigma_L = 0.5)$	$(\sigma_L = 0.1)$	$(\sigma_L = 1)$	$(\sigma_L = 0.5)$	$(\sigma_L = 0.5)$	$(\sigma_L = 0.5)$	$(\sigma_L = 0.1)$	$(\sigma_L = 1)$	$(\sigma_L = 0.5)$	$(\sigma_L = 0.5)$
A.Long-Term										
Consumption, $d\tilde{C}$	-0.07	-0.12	-0.05	-0.04	-0.13	-0.07	-0.12	-0.05	-0.04	-0.13
Labor, $d\tilde{L}$	0.12	0.04	0.16	0.06	0.22	0.13	0.04	0.17	0.07	0.23
Capital, $d\tilde{K}$	0.15	0.07	0.19	0.08	0.29	0.09	0.01	0.13	0.05	0.16
GDP, $d\tilde{Y}$	0.13	0.05	0.17	0.07	0.24	0.11	0.03	0.16	0.06	0.21
Traded output, $d\tilde{YT}$	0.13	0.09	0.15	0.07	0.24	0.13	0.09	0.15	0.07	0.24
Non traded output, $d\tilde{YN}$	0.00	-0.04	0.02	0.00	0.00	-0.02	-0.06	0.00	-0.01	-0.03
B.Impact										
Consumption, $dC(0)$	-0.07	-0.12	-0.05	-0.04	-0.13	-0.08	-0.13	-0.06	-0.05	-0.15
RER, $dP(0)$	0.00	0.00	0.00	0.00	0.00	0.08	0.08	0.08	0.06	0.10
Wage, $dW(0)$	0.00	0.00	0.00	0.00	0.00	-0.25	-0.23	-0.25	-0.18	-0.30
Real wage, $dW(0)/P_C(0)$	0.00	0.00	0.00	0.00	0.00	-0.29	-0.27	-0.29	-0.22	-0.36
Labor, $dL(0)$	0.12	0.04	0.16	0.06	0.22	0.00	0.02	-0.07	-0.02	0.08
Savings, $dS(0)$	-0.85	-0.85	-0.85	-0.92	-0.73	-1.10	-1.02	-1.17	-1.11	-1.03
Investment, $dI(0)$	-0.66	-0.84	-0.57	-0.82	-0.36	0.80	0.64	0.89	0.36	1.05
Current Account, $dCA(0)$	-0.20	-0.01	-0.28	-0.10	-0.37	-1.90	-1.66	-2.06	-1.47	-2.08
GDP, $dY(0)$	0.08	0.03	0.10	0.04	0.15	0.00	0.01	-0.05	-0.02	0.05
Traded output, $dYT(0)$	-0.24	-0.08	-0.31	-0.12	-0.44	-1.00	-0.87	-1.17	-0.80	-1.01
Non traded output, $dYN(0)$	0.32	0.11	0.41	0.16	0.59	1.00	0.88	1.12	0.79	1.07
C.Cumulative Response										
RER, dP	0.00	0.00	0.00	0.00	0.00	0.42	0.38	0.41	0.31	0.51
Real wage, dW/P_C	0.00	0.00	0.00	0.00	0.00	-1.47	-1.35	-1.44	-1.08	-1.79
Investment, dI	-3.16	-4.04	-2.75	-3.95	-1.74	3.22	2.43	3.70	-0.94	5.23
Current account, dCA	-3.91	-3.02	-4.32	-3.70	-4.28	-11.53	-10.36	-12.34	-7.62	-12.76
GDP, dY	0.32	-0.18	0.55	-0.07	1.00	0.69	0.50	0.58	0.13	1.33
Traded output, dYT	-4.11	-3.50	-4.39	-3.75	-4.73	-6.70	-6.17	-7.49	-4.02	-7.11
Non traded output, dYN	4.42	3.32	4.94	3.69	5.73	7.39	6.67	8.07	4.15	8.45

Notes: Notes: We consider a temporary rise in G^N which raises total government spending by one percentage point of GDP. Impact and steady-state deviations are scaled by initial GDP, exception with the real exchange rate, wage rate, labor, and capital which are scaled by their initial steady-state values. A short-lived, medium-lived and long-lived shock lasts 8, 16, 32 quarters respectively.

Table 2: Quantitative Effects of a Temporary Fiscal Expansion (in %): The Case of an Endogenous Markup

Variables	$k^T > k^N$					$k^N > k^T$				
	Bench $\mathcal{T}=16$			Short $\mathcal{T}=8$	Long $\mathcal{T}=32$	Bench $\mathcal{T}=16$			Short $\mathcal{T}=8$	Long $\mathcal{T}=32$
	$(\sigma_L=0.5)$	$(\sigma_L=0.1)$	$(\sigma_L=1)$	$(\sigma_L=0.5)$	$(\sigma_L=0.5)$	$(\sigma_L=0.5)$	$(\sigma_L=0.1)$	$(\sigma_L=1)$	$(\sigma_L=0.5)$	$(\sigma_L=0.5)$
A.Long-Term										
Consumption, $d\tilde{C}$	-0.07	-0.12	-0.04	-0.04	-0.13	-0.07	-0.12	-0.05	-0.04	-0.13
Labor, $d\tilde{L}$	0.12	0.04	0.15	0.06	0.22	0.13	0.04	0.17	0.07	0.23
Capital, $d\tilde{K}$	0.15	0.07	0.19	0.08	0.28	0.09	0.01	0.13	0.04	0.16
GDP, $d\tilde{Y}$	0.13	0.05	0.16	0.07	0.24	0.11	0.03	0.15	0.06	0.20
Traded output, $d\tilde{YT}$	0.13	0.09	0.14	0.07	0.23	0.13	0.09	0.15	0.07	0.24
Non traded output, $d\tilde{YN}$	0.00	-0.04	0.02	0.00	0.00	-0.02	-0.06	0.00	-0.01	-0.03
B.Impact										
Consumption, $dC(0)$	-0.07	-0.12	-0.04	-0.03	-0.12	-0.08	-0.13	-0.06	-0.04	-0.14
RER, $dP(0)$	-0.01	-0.01	-0.01	-0.01	-0.01	0.07	0.07	0.07	0.05	0.09
Wage, $dW(0)$	-0.04	-0.05	-0.05	-0.05	-0.02	-0.28	-0.25	-0.28	-0.22	-0.33
Real wage, $dW(0)/P_C(0)$	-0.04	-0.04	-0.05	-0.05	-0.01	-0.32	-0.29	-0.32	-0.24	-0.38
Labor, $dL(0)$	0.09	0.04	0.09	0.03	0.21	-0.02	0.02	-0.11	-0.04	0.06
Savings, $dS(0)$	-0.90	-0.89	-0.93	-0.98	-0.74	-1.13	-1.04	-1.21	-1.14	-1.06
Investment, $dI(0)$	-0.99	-1.14	-1.07	-1.22	-0.49	1.06	0.82	1.23	0.63	1.31
Current Account, $dCA(0)$	0.09	0.25	0.14	0.24	-0.25	-2.19	-1.86	-2.44	-1.76	-2.36
GDP, $dY(0)$	0.06	0.03	0.06	0.02	0.14	-0.01	0.01	-0.07	-0.03	0.04
Traded output, $dYT(0)$	0.05	0.18	0.11	0.22	-0.33	-2.21	-1.91	-2.46	-1.78	-2.41
Non traded output, $dYN(0)$	0.01	-0.16	-0.05	-0.20	0.46	2.20	1.93	2.38	1.75	2.45
Number of firms, $dN(0)$	0.02	-0.24	-0.07	-0.30	0.71	3.13	2.75	3.39	2.48	3.49
C.Cumulative Response										
RER, dP	-0.11	-0.09	-0.13	-0.09	-0.13	0.29	0.28	0.26	0.22	0.35
Real wage, dW/P_C	-0.04	-0.08	-0.05	0.04	0.05	-1.44	-1.32	-1.40	-1.06	-1.73
Investment, dI	-3.55	-4.48	-3.27	-3.78	-1.81	4.28	3.22	5.01	-0.05	6.06
Current account, dCA	-3.61	-2.67	-3.94	-3.87	-4.23	-12.60	-11.15	-13.69	-8.52	-13.60
GDP, dY	0.23	-0.23	0.39	-0.09	0.97	0.78	0.58	0.68	0.22	1.41
Traded output, dYT	-3.84	-3.17	-4.05	-3.95	-4.70	-12.36	-11.16	-13.31	-8.28	-13.54
Non traded output, dYN	4.05	2.92	4.43	3.84	5.65	13.15	11.75	14.00	8.52	14.96

Notes: Notes: We consider a temporary rise in G^N which raises total government spending by one percentage point of GDP. Impact and steady-state deviations are scaled by initial GDP, exception with the real exchange rate, wage rate, labor, and capital which are scaled by their initial steady-state values. A short-lived, medium-lived and long-lived shock lasts 8, 16, 32 quarters respectively.

A Data

In this Appendix, we describe how we split output, labor and GDP components into a traded sector and a non-traded sector. Table 3 shows the non-tradable content of GDP, employment, consumption, gross fixed capital formation and government spending. Table 3 also shows the share of government spending on the traded and non-traded good in the sectoral output, the shares of capital income in output in both sectors, and the markup charged by the non-traded sector for 13 OECD countries. The choice of these countries has been dictated by data availability. For the countries of our sample, the period runs from 1970 to 2004.[39]

For output and employment, we used the methodology proposed by De Gregorio et al. [1994], who treat Agriculture, Hunting, Forestry and Fishing, Mining and Quarrying, Total Manufacturing, Transport and Storage and Communication as traded goods. Electricity, Gas and Water Supply, Construction, Wholesale and Retail Trade, Hotels and Restaurants, Finance, Insurance, Real Estate and Business Services, Community Social and Personal Services are classified as non-traded sectors (Source: EU KLEMS [2007]). The non-tradable shares of output and labor, shown in the first and second column of Table 3, average to 65% and 63%, respectively.

To split consumption expenditure into consumption in traded and non-traded goods, we made use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2007]). Among the twelve items, the following ones are treated as consumption in traded goods: Food and Non-Alcoholic Beverages, Alcoholic Beverages, Tobacco and Narcotics, Clothing and Footwear, Furnishings, Household Equipment, Transport, Miscellaneous Goods and Services. The remaining items are treated as consumption in non-traded goods: Housing, Water, Electricity, Gas and Fuels, Health, Communication, Recreation and Culture, Education, Restaurants and Hotels. The non-tradable share of consumption shown in the third column of Table 3 averages to 45%, in line with the share reported by Stockman and Tesar [1995].

With regard to investment, we follow the methodology proposed by Burstein et al. [2004] who treat Housing and Other Construction as non-tradable investment and Products of agriculture, forestry, fisheries and aquaculture, Metal products and machinery, Transport Equipment as tradable investment expenditure (Source: OECD Input-Output database [2008a]). Non tradable share of investment shown in the fourth column of Table 3 averages to 60%, in line with estimates provided by Burstein et al. [2004].

Sectoral government expenditure data were obtained from the Government Finance Statistics Yearbook (Source: IMF [2007]) and the OECD General Government Accounts database (Source: OECD [2008b]). Adopting Morshed and Turnovsky's [2004] methodology, the following four sectors were treated as traded: Fuel and Energy; Agriculture,

[39] The exception is consumption expenditure. Data start in 1976 for Austria, in 1995 for Belgium, in 1975 for Finland, in 1991 for Germany, in 1987 for Netherlands, in 1995 for Spain and in 1993 for Sweden. Data end in 2004 for all countries except Japan (1999) and the U.S. (2000).

Forestry, Fishing, and Hunting; Mining, Manufacturing, and Construction; Transport and Communications. The sectors treated as non-traded are: Government Public Services; Defense; Public Order and Safety; Education; Health; Social Security and Welfare; Housing and Community Amenities; Recreation Cultural and Community Affairs. The non tradable component of government spending shown in the fifth column of Table 3 averages to 90%. The proportion of government spending on the traded and non-traded good (i.e., G^T/Y^T and G^N/Y^N) are shown in the sixth and seventh column of Table 3. They average to 7% and 32%, respectively.

Markups in the non-traded sector were estimated at the industry level in each country and aggregated as follows to construct the markup: $\mu = \sum_{j=1}^{6} \omega_j \, \mu_j$ where ω_j is the nominal value-added weight of industry j in the non-traded sector. Estimates of μ_j were obtained by applying the methodology developed by Roeger [1995]. The testable equation is:

$$y_{j,t} = \beta_j \, x_{j,t} + \varepsilon_{j,t}, \tag{28}$$

where the dependent variable $y_{j,t}$ is the Solow residual - percentage change in output less the percentage change in inputs (each input is weighted by the corresponding income share in output) - and $x_{j,t}$ is the output growth minus capital growth.[40] Estimate of μ_j is equal to $1/(1-\hat{\beta}_j)$. Variables required to apply the Roeger's method are the following: gross output (at basic current prices), compensation of employees, intermediate inputs at current purchasers prices, and capital services (volume) indices. All these variables are compiled from the EU KLEMS database (Source: EU KLEMS [2007]), with the exception of the user cost of capital r_t. No sector-specific information was available to construct r_t; hence, the capital user cost is calculated as $r_t (\equiv r_{j,t}) = p_I \, (i - \pi_{GDP} + \delta_K)$, with p_I the deflator for business non residential investment, i the long-term nominal interest rate, π_{GDP} the GDP deflator based inflation rate; the rate of depreciation δ_K is set to 5%; p_I, i and π_{GDP} were taken from the OECD Annual National Accounts database (Source OECD [2008c]). To tackle the potential endogeneity of the regressor and the heteroskedasticity and autocorrelation of the error term when estimating (28), we use the correction of Newey and West [1993].[41] According to the estimates given in the last column of Table 3, the markup charged by the non-traded sector averages to 1.39.

[40]Formally, $y_t = \Delta \left(p_{j,t} Y_{j,t}\right) - \alpha_{L,t} \Delta \left(w_{j,t} L_{j,t}\right) - \alpha_{M,t} \Delta \left(m_{j,t} M_{j,t}\right) - \left(1 - \alpha_{L,t} - \alpha_{M,t}\right) \Delta \left(r_t K_{j,t}\right)$ and $x_{j,t} = \Delta \left(p_{j,t} Y_{j,t}\right) - \Delta \left(r_t K_{j,t}\right)$. We denote by $\alpha_{i,t}$ for $i = L, M, K$ the share of a generic input (labor, material and capital) on total output, $\Delta \left(p_{j,t} Y_{j,t}\right)$ the nominal output growth in industry j, $\Delta \left(w_{j,t} L_{j,t}\right)$ the nominal labor cost growth, $\Delta \left(m_{j,t} M_{j,t}\right)$ the growth in nominal intermediate input costs and $\Delta \left(r_t K_{j,t}\right)$ the nominal capital cost growth.

[41]Countries estimates for each $\hat{\mu}_j$ are not reported here to save space, but are available on request.

Table 3: Data to Calibrate the Two-Sector Model (1970-2004)

Countries	Non tradable Share					G^j/Y^j		Capital Share		Markup
	Output	Labor	Consumption	Investment	Gov. spending	G^N/Y^N	G^T/Y^T	θ^T	θ^N	μ
AUT	0.65	0.60	0.44	0.59	0.90	0.28	0.07	0.28	0.32	1.42
BEL	0.67	0.65	0.44	n.d.	0.85	0.30	0.09	0.33	0.35	1.34
DEU	0.64	0.61	0.44	0.54	0.91	0.30	0.06	0.22	0.33	1.45
DNK	0.70	0.67	0.43	0.58	0.93	0.40	0.07	0.32	0.32	1.40
FIN	0.58	0.57	0.44	0.63	0.84	0.34	0.09	0.27	0.30	1.32
FRA	0.69	0.64	0.40	0.61	0.93	0.33	0.06	0.22	0.35	1.35
GBR	0.62	0.66	0.52	0.52	0.93	0.33	0.05	0.30	0.28	1.37
ITA	0.63	0.56	0.36	0.59	0.91	0.29	0.06	0.42	0.39	1.60
JPN	0.64	0.61	0.39	0.63	n.a.	n.a.	n.a.	0.37	0.29	1.51
NLD	0.67	0.69	0.45	0.64	0.91	0.34	0.08	0.41	0.33	1.32
SPA	0.61	0.59	0.50	0.63	0.90	0.25	0.05	0.35	0.26	1.33
SWE	0.65	0.67	0.51	0.47	0.90	0.43	0.09	0.30	0.30	1.31
USA	0.68	0.72	0.49	0.59	0.90	0.22	0.06	0.36	0.32	1.43

Notes: G^j/Y^j is the share of government spending on good j in output of sector j; θ^j is the share of capital income in output of sector $j = T, N$; μ is the markup charged by the non-traded sector.

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FISCAL SHOCKS IN A TWO SECTOR OPEN ECONOMY

TECHNICAL APPENDIX

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A Short-Run Static Solutions

In this section, we compute short-run static solutions. It is worthwhile noting that in this paper, we assume that the non traded sector is imperfectly competitive and charges a markup denoted by μ. We also allow for the markup to be endogenous in section 5 in the text. In order to isolate the influence of markup variations on variables, i.e. the competition channel, we express variables in terms of the markup; hence, we treat μ as an exogenous variable in computing short-run static solutions. For example, if a short-run static solution is given by $x = x\left(\bar{\lambda}, P, \mu\right)$ with $\bar{\lambda}$ the shadow value of wealth, P the relative price of non tradables and μ the markup, the variable x is only affected by $\bar{\lambda}$ and P in the case of fixed markup while x is influenced also by the competition channel when we allow for the markup to be endogenous. In section K, we set out the model with an imperfectly competitive non-traded sector, assuming that a limited number of competitors operate within each sector. When the number of competitors is large, the imperfectly competitive non-traded sector charges a fixed markup.

A.1 Short-Run Static Solutions for Consumption-Side

In this subsection, we compute short-run static solutions for real consumption and labor supply. Static efficiency conditions (5a) and (5b) can be solved for consumption and labor which of course must hold at any point of time:

$$C = C\left(\bar{\lambda}, P\right), \quad L = L\left(\bar{\lambda}, P, \mu\right), \tag{29}$$

with

$$C_{\bar{\lambda}} = \frac{\partial C}{\partial \bar{\lambda}} = -\sigma_C \frac{C}{\bar{\lambda}} < 0, \tag{30a}$$

$$C_P = \frac{\partial C}{\partial P} = -\alpha_C \sigma_C \frac{C}{P} < 0, \tag{30b}$$

$$L_{\bar{\lambda}} = \frac{\partial L}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} > 0, \tag{30c}$$

$$L_P = \frac{\partial L}{\partial P} = \sigma_L L \frac{W_P}{W} = -\sigma_L L \frac{1}{W} \frac{k^T h}{\mu\left(k^N - k^T\right)} \lessgtr 0, \tag{30d}$$

$$L_\mu = \frac{\partial L}{\partial \mu} = \sigma_L L \frac{W_\mu}{W} = \sigma_L L \frac{1}{W} \frac{k^T P h}{(\mu)^2\left(k^N - k^T\right)} \gtrless 0, \tag{30e}$$

where σ_C and σ_L correspond to the intertemporal elasticity of substitution for consumption and labor, respectively.

Denoting by ϕ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (29) into intra-temporal allocations between non tradable and tradable goods, we solve for C^T and C^N:

$$C^T = C^T\left(\bar{\lambda}, P\right), \quad C^N = C^N\left(\bar{\lambda}, P\right), \tag{31}$$

with

$$\begin{aligned} C^T_{\bar{\lambda}} &= -\sigma_C \frac{C^T}{\bar{\lambda}} < 0, && \text{(32a)} \\ C^T_P &= \alpha_C \frac{C^T}{P} \left(\phi - \sigma_C\right) \lesseqgtr 0, && \text{(32b)} \\ C^N_{\bar{\lambda}} &= -\sigma_C \frac{C^N}{\bar{\lambda}} < 0, && \text{(32c)} \\ C^N_P &= -\frac{C^N}{P} \left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right] < 0, && \text{(32d)} \end{aligned}$$

where we used the fact that $-\frac{P''_C p}{P'_C} = \phi\left(1-\alpha_C\right) > 0$ and $P'_C C = C^N$.

A.2 Short-Run Static Solutions for Production-Side

Capital-Labor Ratios

From static optimality conditions (6a) and (6b), we may express sector capital-labor ratios as functions of the real exchange rate:

$$k^T = k^T\left(P, \mu\right), \qquad k^N = k^N\left(P, \mu\right), \tag{33}$$

with

$$\begin{aligned} k^T_P &= \frac{\partial k^T}{\partial P} = \frac{h}{\mu f_{kk}\left(k^N - k^T\right)}, && \text{(34a)} \\ k^T_\mu &= \frac{\partial k^T}{\partial \mu} = -\frac{Ph}{\left(\mu\right)^2 f_{kk}\left(k^N - k^T\right)}, && \text{(34b)} \\ k^N_P &= \frac{\partial k^N}{\partial P} = \frac{\mu f}{P^2 h_{kk}\left(k^N - k^T\right)}. && \text{(34c)} \\ k^N_\mu &= \frac{\partial k^N}{\partial \mu} = -\frac{f}{P h_{kk}\left(k^N - k^T\right)}. && \text{(34d)} \end{aligned}$$

Wage

Equality $\left[f\left(k^T\right) - k^T f_k\left(k^T\right)\right] \equiv W$ can be solved for the wage rate:

$$W = W\left(P, \mu\right), \tag{35}$$

with

$$\begin{aligned} W_P &= \frac{\partial W}{\partial P} = -k^T f_{kk} k^T_P = -k^T \frac{h}{\mu\left(k^N - k^T\right)} \lesseqgtr 0, && \text{(36a)} \\ W_\mu &= -\frac{\partial W}{\partial \mu} = -k^T f_{kk} k^T_\mu = k^T \frac{Ph}{\left(\mu\right)^2\left(k^N - k^T\right)} \gtreqless 0. && \text{(36b)} \end{aligned}$$

Labor

Substituting short-run static solutions for labor (29) and capital-labor ratios (33) into the resource constraints for capital and labor (7), we can solve for traded and non-traded labor as follows:

$$L^T = L^T\left(K, P, \bar{\lambda}, \mu\right), \quad L^N = L^N\left(K, P, \bar{\lambda}, \mu\right), \tag{37}$$

with

$$L_K^T = \frac{\partial L^T}{\partial K} = \frac{1}{k^T - k^N} \lesseqgtr 0, \tag{38a}$$

$$L_P^T = \frac{\partial L^T}{\partial P} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{P^2 h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N h\right] < 0, \tag{38b}$$

$$L_\mu^T = \frac{\partial L^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T P h}{f_{kk}} + \frac{\mu^2 L^N f}{P h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N P h\right] > 0, \tag{38c}$$

$$L_{\bar{\lambda}}^T = \frac{\partial L^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N}{k^N - k^T} \gtreqless 0, \tag{38d}$$

$$L_K^N = \frac{\partial L^N}{\partial K} = \frac{1}{k^N - k^T} \gtreqless 0, \tag{38e}$$

$$L_P^N = \frac{\partial L^N}{\partial P} = -\frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{L^T h}{f_{kk}} + \frac{\mu^2 L^N f}{P^2 h_{kk}} - \sigma_L L \frac{1}{W}\left(k^T\right)^2 h\right] > 0, \tag{38f}$$

$$L_\mu^N = \frac{\partial L^N}{\partial \mu} = \frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T P h}{f_{kk}} + \frac{\mu^2 L^N f}{P h_{kk}} - \sigma_L L \frac{1}{W}\left(k^T\right)^2 P h\right] < 0, \tag{38g}$$

$$L_{\bar{\lambda}}^N = \frac{\partial L^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T}{k^N - k^T} \lesseqgtr 0. \tag{38h}$$

$$\tag{38i}$$

Output

Inserting short-run static solutions for capital-labor ratios (33) and for labor (38) into the production functions, we can solve for traded output, $Y^T = L^T f\left(k^T\right)$, and non-traded output, $Y^N = L^N h\left(k^N\right)$:

$$Y^T = Y^T\left(K, P, \bar{\lambda}, \mu\right), \qquad Y^N = Y^N\left(K, P, \bar{\lambda}, \mu\right), \tag{39}$$

with

$$Y_K^T = \frac{\partial Y^T}{\partial K} = -\frac{f}{k^N - k^T} \lesseqgtr 0, \tag{40a}$$

$$Y_P^T = \frac{\partial Y^T}{\partial P} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{(P)^2 h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N h f\right] < 0, \tag{40b}$$

$$Y_\mu^T = \frac{\partial Y^T}{\partial \mu} = -\frac{1}{\left[\mu\left(k^N - k^T\right)\right]^2}\left[\frac{L^T (P h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P h_{kk}} - \sigma_L L \frac{1}{W} k^T k^N P h f\right] > 0, \tag{40c}$$

$$Y_{\bar{\lambda}}^T = \frac{\partial Y^T}{\partial \bar{\lambda}} = \sigma_L \frac{L}{\bar{\lambda}} \frac{k^N f}{k^N - k^T} \gtreqless 0, \tag{40d}$$

$$Y_K^N = \frac{\partial Y^N}{\partial K} = \frac{h}{k^N - k^T} \gtreqless 0, \tag{40e}$$

$$Y_P^N = \frac{\partial Y^N}{\partial P} = -\frac{1}{P\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P^2 h_{kk}} - \frac{P}{\mu} \sigma_L L \frac{1}{W}\left(k^T h\right)^2\right] > 0 \tag{40f}$$

$$Y_\mu^N = \frac{\partial Y^N}{\partial \mu} = \frac{1}{\mu\left(k^N - k^T\right)^2}\left[\frac{P L^T (h)^2}{\mu f_{kk}} + \frac{L^N (\mu f)^2}{P^2 h_{kk}} - \frac{P}{\mu} \sigma_L L \frac{1}{W}\left(k^T h\right)^2\right] < 0, \tag{40g}$$

$$Y_{\bar{\lambda}}^N = \frac{\partial Y^N}{\partial \bar{\lambda}} = -\sigma_L \frac{L}{\bar{\lambda}} \frac{k^T h}{k^N - k^T} \lesseqgtr 0, \tag{40h}$$

From (40b) and (40f), an appreciation in the real exchange rate attracts resources from the traded to the non-traded sector which in turn raises the output of the latter. From (40a) and (40e), a rise in the capital stock raises the output of the sector which is relatively more capital intensive. From (40d) and (40h), an increase in the marginal utility of wealth raises labor supply and thereby increases output in the sector which is more labor intensive.

For clarity purpose, in the text, we write out short-run static solutions by expressing output in terms of labor supply, i.e. $Y^T = Y^T(K, L, P)$ and $Y^N = Y^N(K, L, P)$. The partial derivatives of sectoral output w. r. t. to labor are:

$$Y_L^T = \frac{\partial Y^T}{\partial L} = \frac{k^N f}{k^N - k^T} \gtrless 0, \quad Y_L^N = \frac{\partial Y^N}{\partial L} = -\frac{k^T h}{k^N - k^T} \lessgtr 0. \tag{41}$$

Useful Properties

Making use of (40b) and (40f), (40a) and (40e), we deduce the following useful properties:

$$Y_P^T + P\frac{Y_P^N}{\mu} = -\sigma_L L \frac{k^T h}{\mu(k^N - k^T)} \lessgtr 0, \tag{42a}$$

$$Y_K^T + \frac{P}{\mu}Y_K^N = \frac{\mu f - Ph}{\mu(k^T - k^N)} = \frac{P}{\mu}h_k = f_k, \tag{42b}$$

$$Y_L^T + P\frac{Y_L^N}{\mu} = W, \tag{42c}$$

$$Y_\mu^T + P\frac{Y_\mu^N}{\mu} = \sigma_L L k^T \frac{Ph}{\mu^2(k^N - k^T)} \gtrless 0, \tag{42d}$$

$$Y_{\bar\lambda}^T + P\frac{Y_{\bar\lambda}^N}{\mu} = \sigma_L \frac{L}{\bar\lambda}\frac{(k^N \mu f - k^T Ph)}{\mu(k^N - k^T)} = \sigma_L \frac{L}{\bar\lambda} W > 0, \tag{42e}$$

where we used the fact that $\mu f \equiv P\left[h - h_k\left(k^N - k^T\right)\right]$ and $k^N\mu f - k^T Ph = P\left(h - h^K k^N\right)\left(k^N - k^T\right) = \mu W\left(k^N - k^T\right)$.

In addition, using the fact that $r^K = f_k\left[k^T(P, \mu)\right]$, the rental rate of capital denoted by r^K can be expressed as a function of the real exchange rate P and the mark-up μ:

$$r^K = r^K(P, \mu), \tag{43}$$

with partial derivatives given by:

$$r_P^K \equiv \frac{\partial r^K}{\partial P} = \frac{h}{\mu(k^N - k^T)} \gtrless 0, \tag{44a}$$

$$r_P^\mu \equiv \frac{\partial r^K}{\partial \mu} = -\frac{Ph}{\mu^2(k^N - k^T)} \lessgtr 0. \tag{44b}$$

B Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions (29), (31) and (39) into (5d) and (18), we obtain:

$$\dot K = \frac{1}{\mu}Y^N\left(K, P, \bar\lambda\right) - C^N\left(\bar\lambda, P\right) - \delta_K K - G^N, \tag{45a}$$

$$\dot P = P\left[r^\star + \delta_K - \frac{h_k(P)}{\mu}\right]. \tag{45b}$$

Linearizing these two equations around the steady-state, and denoting $\tilde x = \tilde K, \tilde P$ the long-term values of $x = K, P$, we obtain in a matrix form:

$$\left(\dot K, \dot P\right)^T = J\left(K(t) - \tilde K, P(t) - \tilde P\right)^T, \tag{46}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{47}$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \quad b_{12} = \frac{Y_P^N}{\mu} - C_P^N > 0, \tag{48a}$$

$$b_{21} = 0, \quad b_{22} = -\tilde{P}\frac{h_{kk}k_P^N}{\mu} = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^T}{\tilde{P}} \lessgtr 0. \tag{48b}$$

Equilibrium Dynamics

By denoting ν the eigenvalue of matrix J, the characteristic equation for the matrix of the linearized system (46) can be written as follows:

$$\nu^2 - \frac{1}{\tilde{P}}\left(Y_K^T + \frac{\tilde{P}}{\tilde{\mu}}Y_K^N - \delta_K\tilde{P}\right)\nu + \frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) = 0. \tag{49}$$

The determinant denoted by Det of the linearized 2×2 matrix (47) is unambiguously negative:[42]

$$\text{Det J} = b_{11}b_{22} = \frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right) = -\frac{\tilde{f}\tilde{h}}{\mu\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)^2} - \delta_K\frac{Y_K^T}{\tilde{P}} < 0, \tag{50}$$

and the trace denoted by Tr is given by

$$\text{Tr J} = b_{11} + b_{22} = \frac{1}{\tilde{P}}\left(Y_K^T + \frac{\tilde{P}}{\tilde{\mu}}Y_K^N\right) - -\delta_K = \frac{h_k}{\mu} - \delta_K = r^\star > 0, \tag{51}$$

where we used the fact that at the long-run equilibrium $\frac{h_k}{\mu} = r^\star + \delta_K$.

From (49), the characteristic root reads as:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \sqrt{(r^\star)^2 - 4\frac{Y_K^T}{\tilde{P}}\left(\frac{Y_K^N}{\mu} - \delta_K\right)}\right\} \gtrless 0, \quad i = 1, 2. \tag{52}$$

Using (51), then (52) can be rewritten as follows:

$$\nu_i \equiv \frac{1}{2}\left\{r^\star \pm \left[\frac{Y_K^T}{\tilde{P}} - \left(\frac{Y_K^N}{\mu} - \delta_K\right)\right]\right\} \gtrless 0, \quad i = 1, 2. \tag{53}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{54}$$

Since the system features one state variable, K, and one jump variable, P, the equilibrium yields a unique one-dimensional stable saddle-path.

Formal Solutions

General solutions paths are given by :

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{55a}$$

$$P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{55b}$$

[42] Starting with the equality of labor marginal products across sectors, using the fact that $f_k = \frac{P}{\mu}h_k$ and $h_k/\mu = r^\star + \delta_K$, it is straightforward to prove that b_{11} is positive in the case $k^N > k^T$.

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \tag{56}$$

with

$$b_{11} = \frac{Y_K^N}{\mu} - \delta_K = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}_T\right)} - \delta_K \gtrless 0, \tag{57a}$$

$$b_{12} = \frac{Y_P^N}{\mu} - C_P^N > 0, \tag{57b}$$

where C_P^N is given by (32d).

Case $k^N > k^T$

This assumption reflects the fact that the capital-labor ratio of the non-traded good sector exceeds the capital-labor of the traded sector. From (54), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} < 0, \tag{58a}$$

$$\nu_2 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K > 0, \tag{58b}$$

since we suppose that $k^N > k^T$.

We can deduce the signs of several useful expressions:

$$Y_K^N = \mu\left(\nu_2 + \delta_K\right) > 0, \tag{59a}$$

$$Y_K^T = \tilde{P}\nu_1 < 0, \tag{59b}$$

$$\frac{\tilde{P}h_{kk}k_P^N}{\mu} = -\nu_1 > 0, \tag{59c}$$

$$Y_{\bar{\lambda}}^N = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{k}^T\mu\left(\nu_2 + \delta_K\right) < 0, \tag{59d}$$

$$Y_{\bar{\lambda}}^T = -\frac{1}{\bar{\lambda}}\sigma_L\tilde{L}\tilde{P}\tilde{k}^N\nu_1 > 0. \tag{59e}$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ \frac{\nu_1 - \nu_2}{\left(\frac{Y_P^N}{\mu} - C_P^N\right)} & (-) \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}. \tag{60}$$

Case $k^T > k^N$

This assumption reflects the fact that the capital-labor ratio of the traded good sector exceeds the capital-labor ratio of the non traded sector. From (54), the stable and unstable eigenvalues can be rewritten as follows:

$$\nu_1 = \frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} - \delta_K < 0, \tag{61a}$$

$$\nu_2 = -\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} > 0, \tag{61b}$$

since we suppose that $k^T > k^N$.

We can deduce the signs of several useful expressions:

$$\begin{aligned} Y_K^N &= \mu\left(\nu_1 + \delta_K\right) < 0, && \text{(62a)} \\ Y_K^T &= \tilde{P}\nu_2 > 0, && \text{(62b)} \\ \frac{\tilde{P} h_{kk} k_P^N}{\mu} &= -\nu_2 < 0, && \text{(62c)} \\ Y_\lambda^N &= -\frac{1}{\lambda}\sigma_L \tilde{L}\tilde{k}^T \mu\left(\nu_1 + \delta_K\right) > 0, && \text{(62d)} \\ Y_\lambda^T &= -\frac{1}{\lambda}\sigma_L \tilde{L}\tilde{P}\tilde{k}^N \nu_2 < 0. && \text{(62e)} \end{aligned}$$

We write out eigenvector ω^i associated with eigenvalue ν_i (with $i = 1, 2$), to determine their signs:

$$\omega^1 = \begin{pmatrix} 1 & (+) \\ 0 & \end{pmatrix}, \quad \omega^2 = \begin{pmatrix} 0 & \\ \frac{\nu_2 - \nu_1}{\left(\frac{Y_P^N}{\mu} - C_P^N\right)} & (+) \end{pmatrix}. \tag{63}$$

Formal Solution for the Stock of Foreign Assets

We first linearize equation (19) around the steady-state:

$$\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right) + Y_K^T\left(K(t) - \tilde{K}\right) + \left[Y_P^T - C_P^T\right]\left(P(t) - \tilde{P}\right). \tag{64}$$

where C_P^T is given by (32b).

Inserting general solutions for $K(t)$ and $P(t)$, the solution for the stock of international assets is given by follows:

$$\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right) + Y_K^T \sum_{i=1}^{2} B_i e^{\nu_i t} + \left[Y_P^T - C_P^T\right] \sum_{i=1}^{2} B_i \omega_2^i e^{\nu_i t}. \tag{65}$$

Solving the differential equation leads to the following expression:

$$B(t) - \tilde{B} = \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{66}$$

with

$$\Phi_i = \frac{N_i}{\nu_i - r^\star} = \frac{Y_K^T + \left[Y_P^T - C_P^T\right]\omega_2^i}{\nu_i - r^\star}, \quad i = 1, 2. \tag{67}$$

Invoking the transversality condition for intertemporal solvency, the terms in brackets of equation (56) must be null and we must set $B_2 = 0$. We obtain the linearized version of the nation's intertemporal budget constraint:

$$B_0 - \tilde{B} = \Phi_1\left(K_0 - \tilde{K}\right). \tag{68}$$

The stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi_1\left(K(t) - \tilde{K}\right). \tag{69}$$

Case $k^N > k^T$

$$
\begin{aligned}
N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1, \\
&= \tilde{P}\nu_2 \left\{1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]\right\} \gtrless 0, \quad (70a) \\
N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2, \quad (70b) \\
&= Y_K^T = \tilde{P}\nu_1 < 0, \quad (70c)
\end{aligned}
$$

where (70c) follows from the fact that $\omega_2^2 = 0$. We made use of property (42a) together with the fact that $C_P^T = P_C C_P - P C_P^N$ to compute $Y_P^T - C_P^T = -\tilde{P}\left(\frac{Y_P^N}{\mu} - C_P^N\right) - P_C C_P - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right) \gtrless 0$.

The sign of Φ_1 is ambiguous and reflects the impact of capital accumulation on the foreign asset accumulation along a stable transitional path:

$$
\dot{B}(t) = \Phi_1 \dot{K}(t).
$$

where $\dot{K}(t) = \nu_1 B_1 e^{\nu_1 t}$. Following empirical evidence suggesting that the current account and investment are negatively correlated (see e. g. Glick and Rogoff [1995]), we will impose thereafter:

Assumption 1 $\Phi_1 < 0$ *which implies that* $N_1 > 0$.

The condition for the assumption to hold, i. e. $N_1 > 0$, may be rewritten as follows:

$$
\nu_2 > -\frac{\omega_2^1}{\tilde{P}}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\right]. \quad (71)
$$

Note that, for all parametrization, we find $\Phi_1 < 0$.

Case $k^T > k^N$

$$
\begin{aligned}
N_1 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^1, \\
&= Y_K^T = \tilde{P}\nu_2 > 0, \quad (72a) \\
N_2 &= Y_K^T + \left(Y_P^T - C_P^T\right)\omega_2^2, \\
&= \tilde{P}\nu_1 \left\{1 + \frac{\omega_2^2}{\tilde{P}\nu_1}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_k\right)\right]\right\}, \lessgtr 0, \quad (72b)
\end{aligned}
$$

where (72b) follows from the fact that $\omega_2^1 = 0$. We made use of property (42a) together with $C_P^T = P_C C_P - P C_P^N$ to compute $Y_P^T - C_P^T = -\tilde{P}\left(\frac{Y_P^N}{\mu} - C_P^N\right) - P_C C_P - \sigma_L \tilde{L}\tilde{k}^T\left(\nu_1 + \delta_K\right) \gtrless 0$.

C Derivation of the Current Account Equation

In this section, we derive the current account equation. Substituting the definition of lump-sum taxes Z by using (8), the market clearing condition for non-traded goods (9) into (3) we get:

$$
\begin{aligned}
\dot{B} &= r^\star B + r^K K(t) + WL - P_C C - PI - Z, \\
&= r^\star B + \left(r^K K + WL\right) - P_C C - P\left(\frac{Y^N}{\mu} - C^N - G^N\right).
\end{aligned}
$$

Using the fact that $L^T + L^N = L$, $K^T + K^N = K$, , the dynamic equation for the current account can be rewritten as follows:

$$\begin{aligned}\dot{B} &= r^\star B + \left[WL^T + r^K K^T\right] + \left[WL^N + r^K K^N\right] - P\frac{Y^N}{\mu} - C^T - G^T, \\ &= r^\star B + Y^T - C^T - G^T,\end{aligned}$$

where variable cost $WL^N + r^K K^N$ in the non-traded sector and output net of fixed cost in that sector, i. e. $\frac{Y^N}{\mu} = Z^N$, cancel each other.[43]

D Long-Run Effects of Permanent Fiscal Shocks: The Case of Inelastic Labor Supply

In this section, we derive the steady-state effects of permanent fiscal shocks by assuming that labor supply is inelastically supplied. For clarity purpose, we further assume that the non traded sector is perfectly competitive so that $\mu = 1$ and abstract from physical capital depreciation, i.e. $\delta_K = 0$. These two assumptions will be relaxed in numerical analysis but do not affect qualitatively the results.

The steady-state is given by follows:

$$h_k\left[k^N\left(\tilde{P}\right)\right] = r^\star, \tag{73a}$$

$$\frac{Y^N\left(\tilde{K}, \tilde{P},\right)}{\mu} = C^N\left(\bar{\lambda}, \tilde{P}\right) + \tilde{I} + G^N, \tag{73b}$$

$$r^\star\tilde{B} + Y^T\left(\tilde{K}, \tilde{P}\right) - C^T\left(\bar{\lambda}, \tilde{P}\right) = G^T, \tag{73c}$$

and the intertemporal solvency condition

$$\left(\tilde{B} - B_0\right) = \Phi_1\left(\tilde{K} - K_0\right). \tag{73d}$$

We totally differentiate the steady-state (73) which yields in a matrix form:

$$\begin{pmatrix} h_{kk}k^N_P & 0 & 0 & 0 \\ \left(Y^N_P - C^N_P\right) & Y^N_K & 0 & -C^N_{\bar{\lambda}} \\ \left(Y^T_P - C^T_P\right) & Y^T_K & r^\star & -C^T_{\bar{\lambda}} \\ 0 & -\Phi_1 & 1 & 0 \end{pmatrix}\begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B} \\ \mathrm{d}\bar{\lambda}\end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{d}G^N \\ \mathrm{d}G^T \\ 0\end{pmatrix} \tag{74}$$

Determinant of matrix of coefficients is given by

$$D = h_{kk}k^N_P\frac{P_C\tilde{C}\sigma_C}{\bar{\lambda}}\left[Y^N_K - r^\star\frac{\alpha_C}{\tilde{P}}\left(\tilde{P} + \Phi_1\right)\right]. \tag{75}$$

We have to consider two cases, depending on whether the non-traded sector is more or less capital intensive than the traded sector :

$$D = -\frac{\nu_1\nu_2 P_C\tilde{C}\sigma_C}{\tilde{P}\bar{\lambda}} > 0, \text{ if } k^T > k^N, \tag{76a}$$

$$D = -\frac{\nu_1 P_C\tilde{C}\sigma_C}{\tilde{P}\bar{\lambda}}\left[\nu_2 + \alpha_c\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] > 0, \text{ if } k^N > k^T. \tag{76b}$$

[43] In the traded sector which is perfectly competitive, we have : $Y^T = F_L L^T + r^K K^T = WL^T + r^K K^T$. Instead, in the non-traded sector which is imperfectly competitive we have: $PZ^N = P\frac{H_L}{\mu}L^N + P\frac{H_K}{\mu}K^N$ or $P\mu Z^N = PY^N = PH_L L^N + PH_K K^N = WL^N + r^K K^N$.

The term in square brackets on the right-hand side of (76b) is positive if the following inequality holds

$$\nu_2 > -\alpha_C \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1. \tag{77}$$

From (38f), this inequality is satisfied since $\alpha_C \frac{r^\star}{\nu_2} < 1$.

D.1 Long-Run Effects of a Rise in G^T

Case $k^N > k^T$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = -\frac{1}{\tilde{P}_c \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} < 0, \tag{78a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = \frac{\alpha_C \bar{\lambda}}{\sigma_C \tilde{P} \tilde{C}^N \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} > 0, \tag{78b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{78c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^T} = \frac{\alpha_C}{\tilde{P}\tilde{h} \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} > 0, \tag{78d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = -\frac{\alpha_C}{\tilde{P}\nu_2 \left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} < 0, \tag{78e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \frac{\alpha_C}{\nu_2} \frac{\left[1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]}{\left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} > 0. \tag{78f}$$

Case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = -\frac{1}{\tilde{P}_c} < 0, \tag{79a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = \frac{\alpha_C \bar{\lambda}}{\sigma_C \tilde{P} \tilde{C}^N} > 0, \tag{79b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{79c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^T} = \frac{\alpha_C}{\tilde{P}\tilde{h}} > 0, \tag{79d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = -\frac{\alpha_C}{\tilde{P}\nu_1} > 0, \tag{79e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \frac{\alpha_C}{\nu_1} < 0. \tag{79f}$$

D.2 Long-Run Effects of a Rise in G^N

Case $k^N > k^T$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\tilde{P}}{\tilde{P}_C}\frac{\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, \tag{80a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\alpha_c\bar{\lambda}}{\sigma_C\tilde{C}^N}\frac{\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, \tag{80b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{80c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^N} = -\frac{(1-\alpha_C)}{\tilde{h}\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0, \tag{80d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{(1-\alpha_C)}{\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, \tag{80e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = -\frac{\tilde{P}(1-\alpha_C)\left[1+\frac{1}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}{\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0. \tag{80f}$$

Case $k^T > k^N$

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\tilde{P}}{\tilde{P}_C} < 0, \tag{81a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\alpha_c\bar{\lambda}}{\sigma_C\tilde{P}\tilde{C}^N} > 0, \tag{81b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{81c}$$

$$\frac{\mathrm{d}\tilde{L^T}}{\mathrm{d}G^N} = -\frac{(1-\alpha_C)}{\tilde{h}} < 0, \tag{81d}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = \frac{(1-\alpha_C)}{\nu_1} < 0, \tag{81e}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = -\frac{\tilde{P}(1-\alpha_C)}{\nu_1} > 0. \tag{81f}$$

E Long-Run Effects of Permanent Fiscal Shocks: The Case of Elastic Labor Supply

In this section, we derive the steady-state effects of permanent fiscal shocks by assuming that labor supply is elastically supplied. For simplicity purpose, we further assume that the non traded sector is perfectly competitive so that $\mu = 1$ and abstract from capital depreciation, i.e. $\delta_K = 0$. These two assumptions will be relaxed in numerical analysis but do not affect qualitatively the results.

Substituting first the appropriate short-un static solutions, the steady-state of the economy is obtained by setting $\dot{K}, \dot{P}, \dot{B} = 0$ and is defined by the following set of equations:

$$h_k\left[k^N\left(\tilde{P}\right)\right] = r^\star, \tag{82a}$$

$$Y^N\left(\tilde{K},\tilde{P},\bar{\lambda}\right) - C^N\left(\bar{\lambda},\tilde{P}\right) - G^N = 0, \tag{82b}$$

$$r^\star\tilde{B} + Y^T\left(\tilde{K},\tilde{P},\bar{\lambda}\right) - C^T\left(\bar{\lambda},\tilde{P}\right) - G^T = 0, \tag{82c}$$

and the intertemporal solvency condition

$$\left(B_0 - \tilde{B}\right) = \Phi\left(K_0 - \tilde{K}\right). \tag{82d}$$

The steady-state equilibrium composed by these four equations jointly determine $\tilde{P}$, $\tilde{K}$, $\tilde{B}$ and $\bar{\lambda}$.

We totally differentiate the system (82) evaluated at the steady-state which yields in a matrix form:

$$\begin{pmatrix} h_{kk}k_P^N & 0 & 0 & 0 \\ \left(Y_P^N - C_P^N\right) & Y_K^N & \left(Y_{\bar{\lambda}}^N - C_{\bar{\lambda}}^N\right) & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & \left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) & r^\star \\ 0 & -\Phi_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{B} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{d}G^N \\ \mathrm{d}G^T \\ 0 \end{pmatrix} \tag{83}$$

The determinant denoted by D of the matrix of coefficients is given by:

$$D \equiv h_{kk}k_P^N\left\{Y_K^N\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right) - \left(Y_{\bar{\lambda}}^N - C_{\bar{\lambda}}^N\right)\left[Y_K^T + r^\star\Phi_1\right]\right\} \tag{84}$$

We have to consider two cases, depending on wether the non-traded sector is more or less capital intensive than the traded sector:

$$D = -\frac{\nu_1\nu_2}{\tilde{P}\bar{\lambda}}\left(\sigma_L\tilde{W}\tilde{L} + \sigma_C P_C\tilde{C}\right) > 0, \quad \text{if} \quad k^T > k^N, \tag{85a}$$

$$D = -\frac{\nu_1\nu_2}{\tilde{P}\bar{\lambda}}\left\{\left(\sigma_L\tilde{W}\tilde{L} + \sigma_C P_C\tilde{C}\right) + \frac{r^\star}{\nu_2}\frac{\omega_2^1}{\nu_2}\left(\sigma_C\tilde{C}^N - \sigma_L\tilde{L}\tilde{k}^T\nu_2\right)^2\right\} > 0, \quad \text{if} \quad k^N > k^T, \tag{85b}$$

where we used the fact that and $fk^N - Phk^T = W\left(k^N - k^T\right)$ together with $-P\left(k^N\nu_2 + k^T\nu_1\right) \equiv W$ if $k^T > k^N$ or $-P\left(k^N\nu_1 + k^T\nu_2\right) \equiv W$ if $k^N > k^T$.

E.1 A Permanent Rise in G^T

We computed useful expressions:

$$Y_K^N Y_{\bar{\lambda}}^T - Y_K^T Y_{\bar{\lambda}}^N = \sigma_L\frac{\tilde{L}}{\bar{\lambda}}\frac{\tilde{h}\tilde{f}}{\left(\tilde{k}^N - \tilde{k}^T\right)}, \tag{86a}$$

$$P_C' Y_K^T - (1-\alpha_C)P_C Y_K^N = -\frac{P_C}{\tilde{P}}\left[\frac{\alpha_C\tilde{f} + (1-\alpha_C)\tilde{P}\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)}\right], \tag{86b}$$

$$Y_{\bar{\lambda}}^N - P_C' C_{\bar{\lambda}} = \frac{1}{\bar{\lambda}}\left[-\sigma_L\tilde{L}\tilde{k}^T\frac{\tilde{h}}{\left(\tilde{k}^N - \tilde{k}^T\right)} + \sigma_C\tilde{C}^N\right], \tag{86c}$$

$$Y_K^T - (1-\alpha_C)P_C C_{\bar{\lambda}} = \frac{\tilde{P}}{\bar{\lambda}}\left[\sigma_L\tilde{L}\tilde{k}^N\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^N - \tilde{k}^T\right)} + \sigma_C\frac{\tilde{C}^T}{\tilde{P}}\right]. \tag{86d}$$

If $k^N > k^T$, eqs. (86) rewrite as follows:

$$Y_K^N Y_{\bar{\lambda}} - Y_K^T Y_{\bar{\lambda}}^N = -\tilde{P}\nu_1\nu_2\sigma_L \frac{\tilde{L}}{\bar{\lambda}}\left(\tilde{k}^N - \tilde{k}^T\right) > 0, \tag{87a}$$

$$P_C' Y_K^T - (1-\alpha_C) P_C Y_K^N = -P_C\left(\nu_2 - \alpha_C r^\star\right) < 0, \tag{87b}$$

$$Y_{\bar{\lambda}}^N - P_C' C_{\bar{\lambda}} = \frac{1}{\bar{\lambda}}\left[\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right] \lessgtr 0, \tag{87c}$$

$$Y_K^T - (1-\alpha_C) P_C C_{\bar{\lambda}} = \frac{\tilde{P}}{\bar{\lambda}}\left[\sigma_C \frac{\tilde{C}^T}{\tilde{P}} - \sigma_L \tilde{L}\tilde{k}^N \nu_1\right] > 0. \tag{87d}$$

If $k^T > k^N$, eqs. (86) write as follows:

$$Y_K^N Y_{\bar{\lambda}}^T - Y_K^T Y_{\bar{\lambda}}^N = \sigma_L \frac{\tilde{L}}{\bar{\lambda}}\tilde{f}\nu_1 < 0, \tag{88a}$$

$$P_C' Y_K^T - (1-\alpha_C) P_C Y_K^N = -P_C\left(\nu_1 - \alpha_C r^\star\right) > 0, \tag{88b}$$

$$Y_{\bar{\lambda}}^N - P_C' C_{\bar{\lambda}} = \frac{1}{\bar{\lambda}}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_1\right) > 0, \tag{88c}$$

$$Y_K^T - (1-\alpha_C) P_C C_{\bar{\lambda}} = \frac{\tilde{P}}{\bar{\lambda}}\left[\sigma_C \frac{\tilde{C}^T}{\tilde{P}} - \sigma_L \tilde{L}\tilde{k}^N \nu_2\right] \gtrless 0. \tag{88d}$$

Case $k^N > k^T$

If $k^N > k^T$, the steady-state changes after a permanent rise in G^T are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = \frac{\sigma_C \tilde{C}}{\tilde{P}\bar{\lambda}}\frac{\nu_1\nu_2}{D} < 0, \tag{89a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = -\frac{\nu_1\nu_2}{\tilde{P}D} > 0, \tag{89b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{89c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = \frac{\nu_1}{\tilde{P}\bar{\lambda}D}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right) \lessgtr 0, \tag{89d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = -\frac{\nu_1}{\bar{\lambda}D}\left[1 + \frac{\omega_2^1}{\tilde{P}\nu_2}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right)\right]\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right) \gtrless 0, \tag{89e}$$

where $\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}$.

Case $k^T > k^N$

If $k^T > k^N$, the steady-state changes after a permanent rise in G^T are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T} = -\frac{\sigma_C \tilde{C}}{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0, \tag{90a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T} = \frac{\bar{\lambda}}{\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{90b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^T} = 0, \tag{90c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T} = \frac{\left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^N\right)}{\nu_1\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{90d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = -\frac{\tilde{P}\left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^N\right)}{\nu_1\left(\sigma_L \tilde{W}\tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0. \tag{90e}$$

E.2 A Permanent Rise in G^N

Case $k^N > k^T$

If $k^N > k^T$, the steady-state changes after a permanent rise in G^N are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = \frac{\sigma_C \tilde{C}}{\bar{\lambda}} \frac{\nu_1 \nu_2}{D} \left[1 + \frac{r^\star}{\nu_2} \frac{\omega_2^1}{\tilde{P}\nu_2} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \nu_2\right)\right] < 0, \tag{91a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = -\frac{\nu_1 \nu_2}{D} \left[1 + \frac{r^\star}{\nu_2} \frac{\omega_2^1}{\tilde{P}\nu_2} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \nu_2\right)\right] > 0, \tag{91b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{91c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\nu_1}{\bar{\lambda}D} \left(\sigma_C \frac{\tilde{C}^T}{\tilde{P}} - \sigma_L \tilde{L} \tilde{k}^N \nu_1\right) > 0, \tag{91d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = \frac{\nu_1 \tilde{P}}{\bar{\lambda}D} \left[1 + \frac{\omega_2^1}{\tilde{P}\nu_2} \left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L} \tilde{k}^T \nu_2\right)\right] \left(\sigma_C \frac{\tilde{C}^T}{\tilde{P}} - \sigma_L \tilde{L} \tilde{k}^N \nu_1\right) < 0. \tag{91e}$$

Case $k^T > k^N$

If $k^T > k^N$, the steady-state changes after a permanent rise in G^N are:

$$\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N} = -\frac{\sigma_C \tilde{C} \tilde{P}}{\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} < 0, \tag{92a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} = \frac{\bar{\lambda} \tilde{P}}{\left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} > 0, \tag{92b}$$

$$\frac{\mathrm{d}\tilde{P}}{\mathrm{d}G^N} = 0, \tag{92c}$$

$$\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} = -\frac{\tilde{P}\left(\sigma_L \tilde{L} \tilde{k}^N \nu_2 - \sigma_C \frac{\tilde{C}^T}{\tilde{P}}\right)}{\nu_1 \left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} \lesseqgtr 0, \tag{92d}$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N} = \frac{\tilde{P}^2\left(\sigma_L \tilde{L} \tilde{k}^N \nu_2 - \sigma_C \frac{\tilde{C}^T}{\tilde{P}}\right)}{\nu_1 \left(\sigma_L \tilde{W} \tilde{L} + \sigma_C P_C \tilde{C}\right)} \gtreqless 0. \tag{92e}$$

F Derivation of Formal Solutions after Temporary Fiscal Shocks

In this section, we provide the main steps to derive formal solutions for key variables after temporary fiscal shocks, by applying the procedure developed by Schubert and Turnovsky [2002]. For simplicity purpose, we assume that $\mu = 1$ and $\delta_K = 0$.

F.1 Steady-State

As in Schubert and Turnovsky [2002], we define a viable steady-state i starting at time $\mathcal{T}_i$ to be one that is consistent with long run solvency, given the stocks of capital, $K_{\mathcal{T}_i}$ and foreign bonds, $B_{\mathcal{T}_i}$. We rewrite the system of steady-state equations for an arbitrary period

i (with $i = 0, 1, 2$):

$$h_k \left[\tilde{k}^N \left(\tilde{P}_i \right) \right] = r^\star, \tag{93a}$$

$$Y^N \left(\tilde{K}_i, \tilde{P}_i \right) - \tilde{C}_i^N - G_i^N = 0, \tag{93b}$$

$$r^\star \tilde{B}_i + Y^T \left(\tilde{K}_i, \tilde{P}_i \right) - \tilde{C}_i^T - G_i^T = 0, \tag{93c}$$

together with the intertemporal solvency condition

$$\left(\tilde{B}_i - B_{\mathcal{T}_i} \right) = \Phi_1 \left(\tilde{K}_i - K_{\mathcal{T}_i} \right). \tag{93d}$$

F.2 Steady-State Functions

The new consistent procedure consists in two steps. In a **first step**, we solve the system (93a)-(93c) for $\tilde{P}_i$, $\tilde{K}_i$ and $\tilde{B}_i$ as functions of the marginal utility of wealth, $\bar{\lambda}_i$, the government expenditure on the traded and non-traded goods, i.e. G^T and G^N. Totally differentiating equations (93a)-(93c) yields in matrix form:

$$\begin{pmatrix} h_{kk} k_P^N & 0 & 0 \\ \left(Y_P^N - C_P^N \right) & Y_K^N & 0 \\ \left(Y_P^T - C_P^T \right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P}_i \\ \mathrm{d}\tilde{K}_i \\ \mathrm{d}\tilde{B}_i \end{pmatrix} = \begin{pmatrix} 0 \\ P_C' C_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_i + \mathrm{d}G_i^N \\ \left(1 - \alpha_C\right) P_C C_{\bar{\lambda}} \mathrm{d}\bar{\lambda}_i + \mathrm{d}G_i^T \end{pmatrix} \tag{94}$$

The equilibrium value of the marginal utility of wealth $\bar{\lambda}_i$ and fiscal policy parameters, G_i^T, G_i^N, determine the following steady-state values:

$$\tilde{P}_i = \text{constant}, \tag{95a}$$

$$\tilde{K}_i = K\left(\bar{\lambda}_i, G_i^N\right), \tag{95b}$$

$$\tilde{B}_i = B\left(\bar{\lambda}_i, G_i^T, G_i^N\right), \tag{95c}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}_i}{\partial \bar{\lambda}_i} = \frac{h_{kk} k_P^N P_C P_C' r^\star}{G} = -\sigma_C \frac{\tilde{C}_i^N}{\bar{\lambda}_i} \frac{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)}{\tilde{h}_i} \lessgtr 0, \tag{96a}$$

$$\begin{aligned} B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}_i}{\partial \bar{\lambda}_i} &= \frac{h_{kk} k_P^N P_C \left(-P_C' Y_K^T + \left(1 - \alpha_C\right) P_C Y_K^N \right)}{G}, \\ &= \frac{P_C \tilde{C}_i}{\bar{\lambda}_i} \frac{\sigma_C}{Y_K^N r^\star} \left[\alpha_C r^\star - \frac{\tilde{h}_i}{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)} \right], \\ &= -\frac{P_C \tilde{C}_i}{\bar{\lambda}_i} \frac{\sigma_C}{r^\star \tilde{P} \tilde{h}_i} \left[\alpha_C \tilde{f}_i + \left(1 - \alpha_C\right) \tilde{P}_i \tilde{h}_i \right] < 0, \end{aligned} \tag{96b}$$

and

$$K_{G^T} \equiv \frac{\partial \tilde{K}_i}{\partial G_i^T} = 0, \tag{97a}$$

$$B_{G^T} \equiv \frac{\partial \tilde{B}_i}{\partial G_i^T} = \frac{1}{r^\star} > 0, \tag{97b}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}_i}{\partial G_i^N} = \frac{h_{kk} k_P^N u_{cc} r^\star}{G} = \frac{\left(\tilde{k}_i^N - \tilde{k}_i^T \right)}{\tilde{h}_i} \gtrless 0, \tag{98a}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}_i}{\partial G_i^N} = -\frac{h_{kk} k_P^N u_{cc} Y_K^T}{G} = \frac{\tilde{f}_i}{\tilde{h}_i} \frac{1}{r^\star} > 0, \tag{98b}$$

where $G \equiv h_{kk} k_P^N u_{cc} Y_K^N r^\star$ which simplifies as follows :

$$G \equiv \frac{\tilde{f}\tilde{h}}{\tilde{P}^2 \left(\tilde{k}^N - \tilde{k}^T\right)^2} u_{cc} r^\star < 0. \tag{99}$$

The **second step** consists to determine the equilibrium change of $\bar{\lambda}_i$ by taking the total differential of the intertemporal solvency condition (93d):

$$\left[B_{\bar{\lambda}} - \Phi_1 K_\lambda\right] \mathrm{d}\bar{\lambda}_i = \mathrm{d}B_{\mathcal{T}_i} - \Phi_1 \mathrm{d}K_{\mathcal{T}_i} - \left[B_{G^N} - \Phi_1 K_{G^N}\right] \mathrm{d}G_i^N - B_{G^T} \mathrm{d}G_i^T, \tag{100}$$

from which may solve for the equilibrium value of $\bar{\lambda}_i$ as a function of initial stocks at time $\mathcal{T}_i$ and government spending:

$$\bar{\lambda} = \lambda\left(K_{\mathcal{T}_i}, B_{\mathcal{T}_i}, G^T, G^N\right), \tag{101}$$

with

$$\lambda_K \equiv \frac{\partial \bar{\lambda}_i}{\partial K_{\mathcal{T}_i}} = -\frac{\Phi_1}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} < 0, \tag{102a}$$

$$\lambda_B \equiv \frac{\partial \bar{\lambda}_i}{\partial B_{\mathcal{T}_i}} = \frac{1}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} < 0, \tag{102b}$$

$$\lambda_{G^T} \equiv \frac{\partial \bar{\lambda}_i}{\partial G_i^T} = -\frac{B_{G^T}}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} > 0, \tag{102c}$$

$$\lambda_{G^N} \equiv \frac{\partial \bar{\lambda}_i}{\partial G_i^N} = -\frac{\left[B_{G^N} - \Phi_1 K_{G^N}\right]}{\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]} > 0. \tag{102d}$$

From (102), we obtain the following properties:

$$\lambda_B \left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right] = 1, \tag{103a}$$

$$\lambda_B B_{G^T} = -\lambda_{G^T}, \tag{103b}$$

$$\lambda_B \left[B_{G^N} - \Phi_1 K_{G^N}\right] = -\lambda_{G^N}. \tag{103c}$$

F.3 Formal Solutions for Temporary Fiscal Shocks

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $i = 0$:

$$K_0 = \tilde{K}_0 = K\left(\bar{\lambda}_0, G_0^N\right) = K\left(\lambda\left(K_0, B_0, G_0^T, G_0^N\right), G_0^N\right), \tag{104a}$$

$$B_0 = \tilde{B}_0 = B\left(\bar{\lambda}_0, G_0^T, G_0^N\right) = B\left(\lambda\left(K_0, B_0, G_0^T, G_0^N\right), G_0^T, G_0^N\right), \tag{104b}$$

$$\lambda_0 = \bar{\lambda}_0 = \lambda\left(K_0, B_0, G_0^T, G_0^N\right). \tag{104c}$$

We suppose now that government expenditure changes unexpectedly at time $t = 0$ from the original level G_0^T (resp. G_0^N) to level G_1^T (resp. G_1^N) over the period $0 \leq t < \mathcal{T}$, and reverts back at time $\mathcal{T}$ permanently to its initial level, $G_{\mathcal{T}}^T = G_2^T = G_0^T$ (resp. $G_{\mathcal{T}}^N = G_2^N = G_0^N$).

Period 1 $(0 \leq t < \mathcal{T})$

Whereas the fiscal expansion is implemented, the economy follows unstable transitional paths:

$$K(t) = \tilde{K}_1 + B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \quad (105a)$$

$$P(t) = \tilde{P}_1 + \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \quad (105b)$$

$$B(t) = \tilde{B}_1 + \left[\left(B_0 - \tilde{B}_1\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + +\Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \quad (105c)$$

with the steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ given by the following functions (set $i = 1$ into (95b)-(95c)):

$$\tilde{K}_1 = K\left(\bar{\lambda}, G_1^N\right), \quad (106a)$$

$$\tilde{B}_1 = B\left(\bar{\lambda}, G_1^T, G_1^N\right), \quad (106b)$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 ($t \geq \mathcal{T}$)

Once government spending reverts back to its initial level, the economy follows stable paths

$$K(t) = \tilde{K}_2 + B_1' e^{\nu_1 t}, \quad (107a)$$

$$P(t) = \tilde{P}_2 + \omega_2^1 B_1' e^{\nu_1 t}, \quad (107b)$$

$$B(t) = \tilde{B}_2 + \Phi_1 B_1' e^{\nu_1 t}, \quad (107c)$$

with the steady-state values $\tilde{K}_2$ and $\tilde{B}_2$ given by the following functions (set $i = 2$ into (95b)-(95c)):

$$\tilde{K}_2 = K\left(\bar{\lambda}, G_2^N\right), \quad (108a)$$

$$\tilde{B}_2 = B\left(\bar{\lambda}, G_2^T, G_2^N\right). \quad (108b)$$

During the transition period 1, the economy accumulates capital and foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the *two-step method* is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions, $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$, prevailing when the shock ends and accumulated over the unstable period. Therefore, for the country to remain intertemporally solvent, we require:

$$B_{\mathcal{T}} - \tilde{B}_2 = \Phi_1\left(K_{\mathcal{T}} - \tilde{K}_2\right). \quad (109)$$

In order to determine the three constants B_1, B_2, and B_1', and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $K(0) = K_0$, $B(0) = B_0$ must be met.

2. Economic aggregates K and P remain continuous at time $\mathcal{T}$.

3. The intertemporal solvency constraint (109) must hold implying that the net foreign assets remain continuous at time $\mathcal{T}$.

Set $t = 0$ in solution (105a), and evaluating first at time $t = \mathcal{T}$, equate (105a) and (107a), (105b) and (107b):

$$\tilde{K}_1 + B_1 + B_2 = K_0, \tag{110a}$$

$$\tilde{K}_1 + B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} = \tilde{K}_2 + B_1' e^{\nu_1 \mathcal{T}}, \tag{110b}$$

$$\tilde{P}_1 + \omega_2^1 B_1 e^{\nu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\nu_2 \mathcal{T}} = \tilde{P}_2 + \omega_2^1 B_1' e^{\nu_1 \mathcal{T}}, \tag{110c}$$

where we used the continuity condition.

Evaluating $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$ from respectively (105a) and (105c), substituting into (109), and using functions of steady-state values $\tilde{K}_i$ and $\tilde{B}_i$ given by (104) (for $i = 0$), (106) (for $i = 1$), and (108) (for $i = 2$), the intertemporal solvency condition can be rewritten as

$$B\left(\bar{\lambda}, G_1^T, G_1^N\right) + \left[\left(B\left(\lambda_0, G_0^T, G_0^N\right) - B\left(\bar{\lambda}, G_1^T, G_1^N\right)\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star \mathcal{T}} + \Phi_1 B_1 e^{\nu_1 \mathcal{T}}$$
$$+ \Phi_2 B_2 e^{\nu_2 \mathcal{T}} - B\left(\bar{\lambda}, G_2^T, G_2^N\right) = \Phi_1 \left[K\left(\bar{\lambda}, G_1^N\right) + B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} - K\left(\bar{\lambda}, G_2^N\right)\right]. \tag{111}$$

Then, we approximate the steady-state changes with the differentials:

$$\tilde{K}_1 - \tilde{K}_0 \equiv K\left(\bar{\lambda}, G_1^N\right) - K\left(\lambda_0, G_0^N\right) = K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + K_{G^N} \mathrm{d}G^N, \tag{112a}$$

$$\tilde{K}_2 - \tilde{K}_1 \equiv K\left(\bar{\lambda}, G_2^N\right) - K\left(\bar{\lambda}, G_1^N\right) = -K_{G^N} \mathrm{d}G^N, \tag{112b}$$

$$\tilde{B}_1 - \tilde{B}_0 \equiv B\left(\bar{\lambda}, G_1^T, G_1^N\right) - B\left(\lambda_0, G_0^T, G_0^N\right) = B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + B_{G^T} \mathrm{d}G^T + B_{G^N} \mathrm{d}G^N, \tag{112c}$$

$$\tilde{B}_2 - \tilde{B}_1 \equiv B\left(\bar{\lambda}, G_2^T, G_2^N\right) - B\left(\bar{\lambda}, G_1^T, G_1^N\right) = -B_{G^T} \mathrm{d}G^T - B_{G^N} \mathrm{d}G^N, \tag{112d}$$

where $\mathrm{d}\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$.

By substituting these expressions in (110) and (111), we obtain finally

$$B_1 + B_2 = -K_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - K_{G^N} \mathrm{d}G^N, \tag{113a}$$

$$B_1 e^{\nu_1 \mathcal{T}} + B_2 e^{\nu_2 \mathcal{T}} - B_1' e^{\nu_1 \mathcal{T}} = -K_{G^N} \mathrm{d}G^N, \tag{113b}$$

$$\omega_2^1 B_1 e^{\nu_1 \mathcal{T}} + \omega_2^2 B_2 e^{\nu_2 \mathcal{T}} - \omega_2^1 B_1' e^{\nu_1 \mathcal{T}} = 0, \tag{113c}$$

and

$$B_1 \Upsilon_1 + B_2 \Upsilon_2 + B_{\bar{\lambda}} \mathrm{d}\bar{\lambda} = \Omega_1, \tag{114}$$

where we set

$$\Upsilon_1 \equiv \Phi_1, \tag{115a}$$

$$\Upsilon_2 \equiv \Phi_2 + \left(\Phi_1 - \Phi_2\right) e^{-\nu_1 \mathcal{T}}, \tag{115b}$$

$$\Omega_1 \equiv \left[\left(v_{g^j} - \Phi_1 K_{g^j}\right) e^{-r^\star \mathcal{T}} - v_{g^j}\right] \mathrm{d}g^j \quad j = T, N, \tag{115c}$$

where $K_{G^T} = 0$.

Case $k^N > k^T$

We write out some useful expressions

$$K_{\bar{\lambda}} = -\frac{\tilde{C}^N}{\bar{\lambda}}\frac{\sigma_C}{\nu_2} < 0, \tag{116a}$$

$$K_{G^N} = \frac{1}{\nu_2} > 0, \tag{116b}$$

$$B_{\bar{\lambda}} = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_2 r^\star}\left[(1-\alpha_C)\,\nu_2 - \alpha_C\nu_1\right] < 0, \tag{116c}$$

$$B_{G^N} = -\frac{\tilde{P}\nu_1}{\nu_2 r^\star} > 0, \tag{116d}$$

$$(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}) = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_2 r^\star}\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] < 0, \tag{116e}$$

$$(B_{G^N} - \Phi_1 K_{G^N}) = \frac{\tilde{P}}{\nu_2 r^\star}\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right] > 0, \tag{116f}$$

$$\Upsilon_2 = -\tilde{P}\left[1 + \frac{\tilde{C}^N}{\tilde{P}}\frac{\sigma_C}{\nu_2}\omega_2^1 e^{-\nu_1\mathcal{T}}\right], \tag{116g}$$

$$B_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_2}\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1 e^{-\nu_1\mathcal{T}}\right] < 0, \tag{116h}$$

$$\Omega_1 K_{\bar{\lambda}} + B_{\bar{\lambda}}K_{G^N}\mathrm{d}G^N = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star(\nu_2)^2}\left\{\alpha_C\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]e^{-r^\star\mathcal{T}} + (1-\alpha_C)\,\nu_2\right\}\mathrm{d}G^N < 0, \tag{116i}$$

and $B_{G^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\nu_2 + \tilde{k}^N\nu_1 = -\frac{W}{\tilde{P}}$ and the following expression:

$$\Omega_1 = -\frac{1}{r^\star}\left(1 - e^{-r^\star\mathcal{T}}\right)\mathrm{d}G^T + \frac{\tilde{P}}{r^\star\nu_2}\left\{\nu_1 + \left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]e^{-r^\star\mathcal{T}}\right\}\mathrm{d}G^N. \tag{117}$$

Case $k^T > k^N$

We write out some useful expressions

$$K_{\bar{\lambda}} = -\frac{\tilde{C}^N}{\bar{\lambda}}\frac{\sigma_C}{\nu_1} > 0, \tag{118a}$$

$$K_{G^N} = \frac{1}{\nu_1} < 0, \tag{118b}$$

$$B_{\bar{\lambda}} = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{\nu_1 r^\star}\left[(1-\alpha_C)\,\nu_1 - \alpha_C\nu_2\right] < 0, \tag{118c}$$

$$B_{G^N} = -\frac{\tilde{P}\nu_2}{\nu_1 r^\star} > 0, \tag{118d}$$

$$(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}) = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star} < 0 \tag{118e}$$

$$(B_{G^N} - \Phi_1 K_{G^N}) = \frac{\tilde{P}}{r^\star} > 0, \tag{118f}$$

$$\Upsilon_2 = -\tilde{P}\left[1 + \frac{\tilde{C}^N}{\tilde{P}}\frac{\sigma_C}{\nu_1}\omega_2^2\left(1 - e^{-\nu_1\mathcal{T}}\right)\right] < 0, \tag{118g}$$

$$B_{\bar{\lambda}} - \Upsilon_2 K_{\bar{\lambda}} = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_1}\left[\nu_1 + \alpha_C\frac{r^\star}{\nu_1}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^2\left(1 - e^{-\nu_1\mathcal{T}}\right)\right] \gtreqless 0, \tag{118h}$$

$$\Omega_1 K_{\bar{\lambda}} + B_{\bar{\lambda}}K_{G^N}\mathrm{d}G^N = -\frac{P_C\tilde{C}}{\bar{\lambda}}\frac{\sigma_C}{r^\star\nu_1}\left[(1-\alpha_C) + \alpha_C e^{-r^\star\mathcal{T}}\right] > 0, \tag{118i}$$

and $B_{G^T} = 1/r^\star > 0$. We used the fact that $\tilde{k}^T\nu_1 + \tilde{k}^N\nu_2 = -\frac{W}{\tilde{P}}$ and the following expression:

$$\Omega_1 = -\frac{1}{r^\star}\left(1 - e^{-r^\star \mathcal{T}}\right)\mathrm{d}G^T + \frac{\tilde{P}}{r^\star\nu_1}\left(\nu_2 + \nu_1 e^{-r^\star \mathcal{T}}\right)\mathrm{d}G^N. \tag{119}$$

Case $k^N > k^T$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\frac{B_1}{\mathrm{d}G^T} = \frac{\alpha_C\left(1 - e^{-r^\star\mathcal{T}}\right)}{\tilde{P}\nu_2\left[1 + \alpha_C \frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0, \tag{120a}$$

$$\frac{B_2}{\mathrm{d}G^T} = 0, \tag{120b}$$

$$\frac{B_1'}{\mathrm{d}G^T} = \frac{B_1}{\mathrm{d}G^T}, \tag{120c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = \lambda_{G^T}\left(1 - e^{-r^\star\mathcal{T}}\right) > 0, \tag{120d}$$

where, from (113a), $\frac{B_1}{\mathrm{d}G^T}$ can be written also as follows

$$\frac{B_1}{\mathrm{d}G^T} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp}. \tag{121}$$

The solutions for a rise in the government expenditure on the non traded good are given by:

$$\begin{aligned}\frac{B_1}{\mathrm{d}G^N} &= -\frac{\left[\left(1 - e^{-\nu_2\mathcal{T}}\right) - \alpha_C\left(1 - e^{-r^\star\mathcal{T}}\right)\right]}{\nu_2\left[1 + \alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} \\ &= -\frac{\left(1-\alpha_C\right)\left(1 - e^{-\nu_2\mathcal{T}}\right) + \alpha_C\left(e^{-r^\star\mathcal{T}} - e^{-\nu_2\mathcal{T}}\right)}{\nu_2\left[1 + \alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0,\end{aligned} \tag{122a}$$

$$\frac{B_2}{\mathrm{d}G^N} = -\frac{e^{-\nu_2\mathcal{T}}}{\nu_2} < 0, \tag{122b}$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} < 0, \tag{122c}$$

$$\begin{aligned}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} &= \left(1 - e^{-\nu_2\mathcal{T}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + \frac{u_{cc}\tilde{P}}{(P_C)^2}\frac{\nu_2\left(e^{-r^\star\mathcal{T}} - e^{-\nu_2\mathcal{T}}\right)}{\left[\nu_2 + \alpha_C\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} \\ &= \lambda_{G^N}\left\{\left(1 - e^{-\nu_2\mathcal{T}}\right) - \frac{\nu_2\left(e^{-r^\star\mathcal{T}} - e^{-\nu_2\mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}\right\} \lessgtr 0,\end{aligned} \tag{122d}$$

where we used expression (80b) to obtain (122d). From (113a), $\frac{B_1}{\mathrm{d}G^T}$ and $\frac{B_2}{\mathrm{d}G^T}$ can also be written as follows:

$$\frac{B_1}{\mathrm{d}G^N} + \frac{B_2}{\mathrm{d}G^N} = -K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N} \quad \text{and} \quad \frac{B_2}{\mathrm{d}G^N} = -K_{G^N}e^{-\nu_2\mathcal{T}}. \tag{123}$$

Case $k^T > k^N$

The solutions for a rise in the government expenditure on the traded good are given by:

$$\frac{B_1}{\mathrm{d}G^T} = \frac{\alpha_C}{\nu_1 \tilde{P}}\left(1 - e^{-r^\star \mathcal{T}}\right) < 0, \tag{124a}$$

$$\frac{B_2}{\mathrm{d}G^T} = 0, \tag{124b}$$

$$\frac{B_1'}{\mathrm{d}G^T} = \frac{B_1}{\mathrm{d}G^T}, \tag{124c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = \lambda_{G^T}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{124d}$$

The solutions for a rise in the government expenditure on the non traded good are given by:

$$\begin{aligned}\frac{B_1}{\mathrm{d}G^N} &= -\frac{1}{\nu_1}\left[(1-\alpha_C) + \alpha_C e^{-r^\star \mathcal{T}}\right], \\ &= -\frac{1}{\nu_1}\left[(1-\alpha_C)\left(1 - e^{-r^\star \mathcal{T}}\right) + e^{-r^\star \mathcal{T}}\right] > 0,\end{aligned} \tag{125a}$$

$$\frac{B_2}{\mathrm{d}G^N} = 0, \tag{125b}$$

$$\begin{aligned}\frac{B_1'}{\mathrm{d}G^N} &= \frac{B_1}{\mathrm{d}G^N} + K_{G^N} e^{-\nu_1 \mathcal{T}} \\ &= -\frac{1}{\nu_1}\left[\left(1 - e^{-\nu_1 \mathcal{T}}\right) - \alpha_C\left(1 - e^{-r^\star \mathcal{T}}\right)\right] < 0,\end{aligned} \tag{125c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{125d}$$

G Transitional Dynamics after a Rise in G^N

In this section, we investigate in details the dynamics of key variables after a permanent and temporary rise in G^N, considering both cases: $k^T > k^N$ and $k^N > k^T$. Transitional paths are depicted in Figures 2 and 4 for $k^T > k^N$ and $k^N > k^T$, respectively. To keep analytical tractability, we assume that labor supply is fixed, i.e. we set $\sigma_L = 0$. Since these two parameters do no affect qualitatively the results, we further assume that the non-traded sector is perfectly competitive, i.e. we set $\mu = 1$, and we set the rate of depreciation of physical capital to zero.

G.1 Long-Run Effects

We derive the ultimate steady-state changes of the economic key variables after a permanent rise in government spending on the non-traded good by differentiating the functions (95) w.r.t G^N:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} < 0, \tag{126a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + K_{G^N} \gtreqless 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{126b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + B_{G^N} \lesseqgtr 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{126c}$$

where analytical expressions are given by the set of equations (80) and (81).

We turn now to the long run changes of macroeconomic aggregates after a temporary fiscal expansion by considering two cases.

Case $k^N > k^T$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left\{\left(1-e^{-\nu_2\mathcal{T}}\right) - \frac{\nu_2\left(e^{-r^\star\mathcal{T}}-e^{-\nu_2\mathcal{T}}\right)}{\left[\nu_2+\frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}\right\} < 0. \tag{127}$$

The sign of the change in the equilibrium value of the marginal utility of wealth can be determined by noting that expression (127) tends towards zero whenever the parameter $\mathcal{T}$ tends towards *zero* and tends towards λ_{G^N} when the parameter tends towards ∞. In addition, the term in square brackets is an increasing and monotonic function of parameter $\mathcal{T}$. Therefore, the change in $\bar{\lambda}$ after a temporary rise in government spending lies in the range $[0, \lambda_{G^N}]$. Consequently, we can deduce that expression (127) has a positive sign.

Using the functions (95), we deduce the long run changes for the real consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{128a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{128b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp} = B_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{128c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$, and $B_{\bar{\lambda}} < 0$.

The change of the period 1 steady-state value $\tilde{K}_1$ compared to its initial (given) value $\tilde{K}_0$ is given by:

$$\begin{aligned}\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^N}\right|_{temp} &= K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + K_{G^N}, \\ &= \frac{(1-\alpha_C)+\alpha_C\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1 e^{\nu_1\mathcal{T}}\right]e^{-r^\star\mathcal{T}}}{\nu_2\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} > 0,\end{aligned} \tag{129}$$

where we have substituted the expressions of $K_{\bar{\lambda}} < 0$ given by (116a), $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} > 0$ given by (127) and $K_{G^N} > 0$ given by (116b).

The change of the period 1 steady-state value $\tilde{B}_1$ compared to its initial (given) value $\tilde{B}_0$ is given by :

$$\begin{aligned}\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} &= B_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + B_{G^N}, \\ &= -\frac{\tilde{P}}{r^\star\nu_2}\frac{1}{\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]}\left\{\left((1-\alpha_C)\nu_2-\alpha_C\nu_1\right)\left[1+\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]\left(1-e^{-r^\star\mathcal{T}}\right)\right. \\ &\quad \left.+\left[1+\alpha_C\frac{r^\star}{(\nu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]\nu_1\right\} \gtrless 0,\end{aligned} \tag{130}$$

where we have substituted the expressions of $B_{\bar{\lambda}} < 0$ given by (116c), $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} > 0$ given by (127) and $B_{G^N} > 0$ given by (116d). The sign of expression (130) remains indeterminate because it is the sum of two terms of opposite signs. The first term on the right-hand side of (130) is negative and is an increasing function of parameter $\mathcal{T}$ and may be dominated by the second term B_{G^N} which is positive. We may infer that the shorter-lasting the rise in government expenditure, the more likely a higher steady-state value $\tilde{B}_1$ compared to its initial (given) value $\tilde{B}_0$.

It is interesting to compare the magnitudes of the long run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + B_{G^N} \gtreqless B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp}, \tag{131}$$

where $B_{G^N} > 0$, $B_{\bar{\lambda}} < 0$ and $\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} = \lambda_{G^N} > 0$. The key factor that determines the magnitude of the long run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, simulations indicate that there exists a time $\mathcal{T} = \acute{\mathcal{T}}$ for which the two changes are equal. For high durations of the policy, i. e. $\mathcal{T} > \acute{\mathcal{T}}$, the deterioration of the net foreign asset position features a greater magnitude after a temporary fiscal expansion compared to a permanent policy. This result is reversed when the public policy is implemented over a short period, say $\mathcal{T} < \acute{\mathcal{T}}$.

From steady-state changes following permanent and temporary rise in government expenditure falling on the non-traded good, we can deduce the following inequalities whatever the duration of the public policy:

$$\tilde{K}_{temp} < K_0 < \tilde{K}_{perm} < \tilde{K}_1, \tag{132a}$$

$$\tilde{B}_{temp} < \tilde{B}_{perm} < B_0, \quad \text{if} \quad \mathcal{T} > \acute{\mathcal{T}}, \tag{132b}$$

$$\tilde{B}_{perm} < \tilde{B}_{temp} < B_0, \quad \text{if} \quad \mathcal{T} < \acute{\mathcal{T}}. \tag{132c}$$

Case $k^T > k^N$

The equilibrium change of $\bar{\lambda}$ is:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N} \left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{133}$$

From (133), we see that that the change of λ after a temporary change in G^N is smaller than that after a permanent increase in G^N but goes in the same direction. Hence we deduce the following inequality:

$$0 < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm}. \tag{134}$$

From the functions (95), we deduce long run changes of real consumption, the stock of physical capital, and the stock of traded bonds:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{135a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} > 0, \tag{135b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^N}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \tag{135c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} > 0$, and $B_{\bar{\lambda}} < 0$.

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values K_0 and B_0 are given by :

$$\begin{aligned}
\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^N}\right|_{temp} &= K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + K_{G^N}, \\
&= \frac{(1-\alpha_C) + \alpha_C e^{-r^\star \mathcal{T}}}{\nu_1} < 0, \qquad (136a) \\
\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} &= B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + B_{G^N}, \\
&= -\frac{\tilde{P}}{r^\star \nu_1}\left\{(1-\alpha_C)\, r^\star - \left[(1-\alpha_C)\,\nu_1 - \alpha_C \nu_2\right] e^{-r^\star \mathcal{T}}\right\} > 0. \qquad (136b)
\end{aligned}$$

where we have evaluated the signs of (136a)-(136b) by making use of (118a)-(118d) and (81b).

From (134), because the change in the equilibrium value of $\bar{\lambda}$ following a temporary change in G^N is smaller compared with that after a permanent increase in G^N, by making use of (135b)-(135c), (126b)-(126c), and (136a)-(136b), we are able to deduce the following inequalities:

$$\tilde{K}_1 < \tilde{K}_{perm} < K_0 < \tilde{K}_{temp}, \qquad (137a)$$

$$\tilde{B}_{temp} < B_0 < \tilde{B}_{perm} < \tilde{B}_1. \qquad (137b)$$

G.2 Transitional Dynamics after a Permanent Increase in G^N

Case $k^N > k^T$

The initial jump of P is obtained by setting $t = 0$ in (105b) and then by differentiating with respect to G^N:

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} > 0. \qquad (138)$$

From the short run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of the relative price of the non-traded good, we get the initial jump of real consumption:

$$\begin{aligned}
\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} &= C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} + C_P \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm} = -\frac{\tilde{P}\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] - \tilde{C}^N \sigma_C \omega_2^1}{P_C\left[\nu_2 + \alpha_C \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} \\
&= \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} + \frac{(1-\alpha_C)}{P_C}\frac{\tilde{C}^N \sigma_C \omega_2^1}{\left[\nu_2 + \alpha_C \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C\omega_2^1\right]} < 0. \qquad (139)
\end{aligned}$$

From (139), we deduce the following inequality

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} < \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} < 0. \qquad (140)$$

The rise in the marginal utility of wealth and the initial appreciation in the relative price of the non-traded good lowers $C(0)$ below its steady-state value. Along the stable adjustment, real consumption rises:

$$\dot{C}(t) = -C\sigma_C\alpha_C \frac{\dot{P}(t)}{P(t)} > 0, \qquad (141)$$

where the relative price of the non-traded good depreciates along the stable adjustment when the non-traded sector is relatively more capital intensive. Otherwise, the relative price of the non-traded good's and thus the real consumption's temporal paths are flat.

The dynamics of the key economic variables after a permanent rise in government spending falling on the non-traded good are as follows:

$$\dot{K}(t) = -\nu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N > 0, \tag{142a}$$

$$\dot{P}(t) = -\nu_1 \omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0, \tag{142b}$$

$$\dot{B}(t) = -\nu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0. \tag{142c}$$

Note that the long run changes of $\tilde{K}$ and $\tilde{B}$ are opposite to those after a permanent rise G^T.

Case $k^T > k^N$

If $k^T > k^N$, the initial change in the real consumption is solely affected by the change in the equilibrium value of the marginal utility of wealth and jumps immediately to its new lower steady-state level:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm} = \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^N}\right|_{perm} < 0. \tag{143}$$

Over time, investment decreases and the stock of international assets rises:

$$I(t) = -\nu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N < 0, \tag{144a}$$

$$CA(t) = -\nu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} e^{\nu_1 t}\mathrm{d}G^N > 0. \tag{144b}$$

As it shall be useful later, we calculate the slope of the trajectory after a permanent fiscal expansion in the (K, B)-space by differentiating the solutions for $B(t)$ and for $K(t)$ w.r.t time:

$$\frac{\mathrm{d}B(t)}{\mathrm{d}K(t)} = \frac{\nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}}{\nu_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}} = -\tilde{P} < 0. \tag{145}$$

where we used the fact that $\Phi_1 = -\tilde{P}$.

G.3 Transitional Dynamics after a Temporary Increase in G^N

Case $k^N > k^T$

First, we evaluate the constants $B_1/\mathrm{d}G^N$ and $B_2/\mathrm{d}G^N$:

$$\begin{aligned}\frac{B_1}{\mathrm{d}G^N} &= -\frac{B_2}{\mathrm{d}G^N} - K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - K_{G^N}, \\ &= -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] < 0. \end{aligned} \tag{146a}$$

$$\frac{B_2}{\mathrm{d}G^N} = -K_{G^N} e^{-\nu_2 \mathcal{T}} = -\frac{e^{-\nu_2 \mathcal{T}}}{\nu_2} < 0. \tag{146b}$$

By evaluating the formal solution for $P(t)$ at time $t=0$, differentiating with respect to G^N, and remembering that $\mathrm{d}\tilde{P}_1/\mathrm{d}G^N = 0$, we get the initial jump of P:

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} = \omega_2^1 \frac{B_1}{\mathrm{d}G^N} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left(1-e^{-\nu_2 \mathcal{T}}\right) - \omega_2^1 \frac{\alpha_C \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}$$

$$= -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1-e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] > 0, \quad (147)$$

where we have inserted the steady-state change of the capital stock after a permanent fiscal expansion falling on the non-traded good given by (80e). From (147), we can see that the magnitude of the initial appreciation in the real exchange after a temporary fiscal expansion may be magnified if the policy is implemented during a long period, i. e. for $\mathcal{T} > \frac{1}{\nu_1} \ln\left[\alpha_C\right]$.

By making use of the short run static solution (29) for C, we obtain the response of real consumption at time $t=0$:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + C_P \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} < 0. \quad (148)$$

It is now convenient to evaluate the magnitude of the downward jump of real consumption after a temporary rise in G^N compared with that after a permanent fiscal expansion by computing the following expression:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} = C_{\bar{\lambda}} \left[\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{perm}\right]$$

$$+ C_P \left[\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} - \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{perm}\right] \gtrless 0. \quad (149)$$

(150)

From (149), we deduce the following inequality:

$$\left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{perm} < \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^N}\right|_{temp} < 0. \quad (151)$$

The initial response of the investment flow following a temporary rise in G^N is given by:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}$$

$$= -\nu_1 \left\{\frac{\left(1-\alpha_C\right)\left(1-e^{-\nu_2 \mathcal{T}}\right) + \alpha_C \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \alpha_C \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}\right\} - e^{-\nu_2 \mathcal{T}},$$

$$= -\nu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left[\left(1-e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1-\alpha_C}\right)\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)\right] - e^{-\nu_2 \mathcal{T}} \gtrless 0. \quad (152)$$

The sign of expression (152) is not clear-cut. As investment plays the role of clearing the non-traded goods market, its sign depends on the jumps of the relative price of the non-traded good and of the marginal utility of wealth. On the one hand, the relative price of the non-traded good appreciates which raises the return on domestic capital by reducing k^N. On the other hand, the increase in P raises the capital user cost. The latter effect is larger, the shorter-living the fiscal shock.

To derive a more easily interpretable expression for the initial reaction of investment after a temporary rise in G^N, we first linearize the non-traded good market clearing condition in the neighborhood of the steady-state:

$$I(t) - \tilde{I} = Y_K^N \left(K(t) - \tilde{K}\right) + \left(Y_P^N - C_P^N\right)\left(P(t) - \tilde{P}\right).$$

Using the fact that $\mathrm{d}\tilde{I} = Y_K^N \mathrm{d}\tilde{K} + \left(Y_P^N - C_P^N\right)\mathrm{d}\tilde{P} - C_{\bar{\lambda}}^N \mathrm{d}\bar{\lambda}|_{temp} - \mathrm{d}G^N$, and evaluating the expression above at time $t = 0$, we get:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(Y_P^N - C_P^N\right)\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} + \sigma_C \frac{\tilde{C}^N}{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \tag{153}$$

Using the fact that $\mathrm{d}\tilde{P} = 0$, we evaluate the initial jump of P which is given by:

$$\begin{aligned}
\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^N}\right|_{temp} &= \omega_2^1 \frac{\mathrm{d}B_1}{\mathrm{d}G^N} = -\omega_2^1\left[K_{G^N}\left(1 - e^{-\nu_2 \mathcal{T}}\right) + K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right], \\
&= \omega_2^1\left[-\frac{\left(1 - e^{-\nu_2 \mathcal{T}}\right)}{\nu_2} + \frac{\sigma_C}{\nu_2}\frac{\tilde{C}^N}{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp}\right],
\end{aligned} \tag{154}$$

where we substituted $K_{G^N} = 1/\nu_2$ and $K_{\bar{\lambda}} = -\frac{\sigma_C}{\nu_2}\frac{\tilde{C}^N}{\bar{\lambda}}$. Substituting (154) into (153) and using the fact that $\omega_2^1 = \frac{\nu_1 - \nu_2}{\left(Y_P^N - C_P^N\right)}$, the initial reaction of investment finally rewrites as:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \left(\frac{\nu_2 - \nu_1}{\nu_2}\right)\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \frac{\sigma_C \tilde{C}^N}{\bar{\lambda}}\frac{\nu_1}{\nu_2}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - 1. \tag{155}$$

Eq. (155) corresponds to **eq. (20) in the text**.

By differentiating the formal solution (105c) over period 1 for $B(t)$ with respect to time, then evaluating the resulting expressions at $t = 0$, and differentiating with respect to G^N, we obtain the initial response of the current account following a temporary fiscal expansion:

$$\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= r^\star\left\{-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right\} \\
&\quad + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \Phi_2 \frac{B_2}{\mathrm{d}G^N}.
\end{aligned} \tag{156}$$

In order to simplify the solution (156), we rewrite the term in square brackets as follows

$$\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \left[\Phi_1 \frac{B_1}{\mathrm{d}G^N} + \Phi_2 \frac{B_2}{\mathrm{d}G^N}\right] \\
= & -\left[B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right]\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} - \left[B_{G^N} - \Phi_1 K_{G^N}\right] + \left[\Phi_1 - \Phi_2\right]\frac{B_2}{\mathrm{d}G^N}, \\
= & -\frac{\lambda_{G^N}}{\lambda_B}\left\{\left(1 - e^{-\nu_2 \mathcal{T}}\right) - \frac{\nu_2\left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}\right\} + \frac{\lambda_{G^N}}{\lambda_B} + \frac{1}{\nu_2}\tilde{C}^N \sigma_C \omega_2^1 K_{G^N} e^{-\nu_2 \mathcal{T}}, \\
= & \frac{\lambda_{G^N}}{\lambda_B} e^{\nu_2 \mathcal{T}} - \frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} + \frac{\tilde{P}}{\nu_2 r^\star}\left[\nu_2 + \frac{r^\star}{\nu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] e^{-\nu_2 \mathcal{T}}, \\
= & -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} < 0,
\end{aligned} \tag{157}$$

where we have substituted the expression of the change in the equilibrium value of the marginal utility of wealth given by (122d), we made use of properties (103), expression

(116f) and inserted these useful expressions:

$$
\begin{aligned}
\frac{B_1}{\mathrm{d}G^N} &= -\frac{B_2}{\mathrm{d}G^N} - K_{\bar{\lambda}} \left. \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N} \right|_{temp} - K_{G^N} < 0, \\
\Phi_1 - \Phi_2 &= -\frac{1}{\nu_2} \tilde{C}^N \sigma_C \omega_2^1 > 0, \\
\frac{B_2}{\mathrm{d}G^N} &= -K_{G^N} e^{-\nu_2 \mathcal{T}} < 0, \\
\frac{\left(B_{G^N} - \Phi_1 K_{G^N}\right)}{\left[\nu_2 + \frac{r^\star}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1\right]} &= \frac{\tilde{P}}{\nu_2 r^\star} > 0.
\end{aligned}
$$

By inserting (157) into (156), the expression of the initial response of the current account reduces to:

$$
\begin{aligned}
\left. \frac{\mathrm{d}CA(0)}{\mathrm{d}G^N} \right|_{temp} &= \nu_1 \tilde{P} \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1 \right) \left. \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} \right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C} \right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) \right] \\
&\quad - \tilde{P} e^{-r^\star \mathcal{T}} + \tilde{P} e^{-\nu_2 \mathcal{T}}, \\
&= -\tilde{P} \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1 \right) \left. \frac{\mathrm{d}I(0)}{\mathrm{d}G^N} \right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C} \right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) \right] \\
&\quad - \tilde{P} \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) < 0,
\end{aligned}
\tag{158}
$$

where we simplified several expressions as follows:

$$
\begin{aligned}
K_{\bar{\lambda}} \frac{u_{CC} \tilde{P}}{P_C^2} \nu_2 &= \frac{\tilde{P} \tilde{C}^N}{P_C \tilde{C}} = \alpha_C > 0, \\
\nu_2 \Phi_2 - \nu_1 \Phi_1 &= -\tilde{P} \nu_2 + \tilde{P} \nu_1 \left(1 + \frac{1}{\nu_2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1 \right) < 0.
\end{aligned}
$$

To derive a more easily interpretable expression for the initial reaction of the current account after a temporary rise in G^N, we use eq. (146a):

$$
\begin{aligned}
\left. \frac{\mathrm{d}CA(0)}{\mathrm{d}G^N} \right|_{temp} &= -\tilde{P} \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N}, \\
&= -\tilde{P} \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) - \nu_1 \Phi_1 \left. \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N} \right|_{perm} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C} \right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) \right], \\
&= -\tilde{P} \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) - \nu_1 \Phi_1 \frac{(1 - \alpha_C)}{\nu_2 \left(1 - \alpha_C \tilde{\Psi} \right)} \left[\left(1 - e^{-\nu_2 \mathcal{T}}\right) + \left(\frac{\alpha_C}{1 - \alpha_C} \right) \left(e^{-r^\star \mathcal{T}} - e^{-\nu_2 \mathcal{T}} \right) \right]
\end{aligned}
\tag{159}
$$

where $0 < \tilde{\Psi} \equiv -\frac{r^\star}{\nu_2^2} \frac{\tilde{C}^N}{\tilde{P}} \sigma_C \omega_2^1 < 1$. Eq. (159) corresponds to **eq. (21) in the text**.

Now, we investigate the dynamics for $K(t)$ and $P(t)$ over the unstable period $(0, \mathcal{T})$, say period 1:

$$
\dot{K}(t) = \nu_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t} \mathrm{d}G^N + \nu_2 \frac{B_2}{\mathrm{d}G^N} e^{\nu_2 t} \mathrm{d}G^N \gtreqless 0, \tag{160a}
$$

$$
\dot{P}(t) = \nu_1 \omega_2^1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t} \mathrm{d}G^N < 0, \tag{160b}
$$

where $B_1/\mathrm{d}G^N < 0$, $B_2/\mathrm{d}G^N < 0$, and $\omega_2^1 < 0$. As it can be seen from (160a), investment dynamics are the result of two opposite forces. If the initial investment flow is positive, it must be negative at time $\tilde{t}$ along the trajectory:

$$
\tilde{t} = \frac{1}{\nu_1 - \nu_2} \ln \left[-\frac{\nu_2 B_2/\mathrm{d}G^N}{\nu_1 B_1/\mathrm{d}G^N} \right], \tag{161}
$$

where the term in square brackets is less than one under the condition that the initial investment flow is positive (see eq. (152)), otherwise the trajectory for investment is monotonic.

The current account dynamics over period 1 are described by the following equation:

$$CA(t) = \left[\tilde{P}e^{-\nu_2(\mathcal{T}-t)}\left(1-e^{-\nu_1(\mathcal{T}-t)}\right) + \nu_1\Phi_1\frac{B_1}{\mathrm{d}G^N}e^{\nu_1 t}\right]\mathrm{d}G^N < 0. \tag{162}$$

We turn now to the analysis of transitional dynamics over the stable period 2. By making use of standard methods, the adjustments of the stock of physical capital, the relative price of non tradables P and the stock of international assets are driven by the following equations:

$$\dot{K}(t) = \nu_1\frac{B_1'}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t} > 0, \tag{163a}$$

$$\dot{P}(t) = \nu_1\omega_2^1\frac{B_1'}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t} < 0, \tag{163b}$$

$$\dot{B}(t) = \nu_1\Phi_1\frac{B_1'}{\mathrm{d}G^N}\mathrm{d}G^N e^{\nu_1 t} < 0. \tag{163c}$$

Evaluate (163c) at time t^+, and calculate $\mathrm{d}CA\left(\mathcal{T}\right) = CA\left(\mathcal{T}^+\right) - CA\left(\mathcal{T}^-\right)$, we can see that the current account is continuous in the neighborhood of time $\mathcal{T}$. Thus we have $CA\left(\mathcal{T}^-\right) = CA\left(\mathcal{T}^+\right)$. Performing the same procedure of investment, we obtain:

$$\frac{\mathrm{d}I\left(\mathcal{T}\right)}{\mathrm{d}G^N} = -\nu_2\frac{B_2}{\mathrm{d}G^N}e^{\nu_2\mathcal{T}} = 1. \tag{164}$$

When the policy is removed at time $\mathcal{T}$, i. e. government spending falls by an amount equals to $\mathrm{d}G^N\left(\mathcal{T}\right) \equiv G_2^N - G_1^N \equiv -\mathrm{d}G^N$, investment must rise to guarantee that the market-clearing condition holds at time $\mathcal{T}$.

Case $k^T > k^N$

Like after a permanent fiscal expansion, an unexpected transitory rise in government spending on the non-traded good leaves unaffected the relative price of the non-traded good both in the short run and in the long run. To evaluate the investment dynamics, we differentiate the solution for $K(t)$ given by (105a) with respect to time, evaluate the resulting expression at time $t = 0$, and then differentiate with respect to G^N, keeping in mind that $B_2/\mathrm{d}G^N = 0$ if $k^T > k^N$:

$$\begin{aligned}\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} &= \nu_1\frac{B_1}{\mathrm{d}G^N} = -\nu_1\frac{1}{\nu_1}\left[(1-\alpha_C)\left(1-e^{-r^\star\mathcal{T}}\right)+e^{-r^\star\mathcal{T}}\right], \\ &= \alpha_C\left(1-e^{-r^\star\mathcal{T}}\right) - 1 < 0, \\ &= \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{perm}\left(1-e^{-r^\star\mathcal{T}}\right) - e^{-r^\star\mathcal{T}} < 0. \end{aligned} \tag{165}$$

The second line of eq. (165) corresponds to **eq. (18) in the text**

Applying standard methods, the initial response of the current account following a temporary fiscal expansion on the non-traded good is given by:

$$\begin{aligned}\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} &= r^\star\left\{-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1\frac{B_1}{\mathrm{d}G^N}\right\} + \nu_1\Phi_1\frac{B_1}{\mathrm{d}G^N}, \\ &= \tilde{P}\left(1-\alpha_C\right)\left(1-e^{-r^\star\mathcal{T}}\right) > 0, \end{aligned} \tag{166}$$

where $\nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} = \tilde{P}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}\right]$. Eq. (166) corresponds to **eq. (19) in the text**.

In deriving (166), we have also simplified the term in square braces as follows:

$$\begin{aligned}
& -\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} \\
= & -\left\{\left[\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \left(B_{G^N} - \Phi_1 K_{G^N}\right)\right]\right\}, \\
= & \frac{\lambda_{G^N}}{\lambda_B} e^{-r^\star \mathcal{T}} = -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}} < 0. \qquad (167)
\end{aligned}$$

We investigate the dynamics of the stocks of physical capital and traded bonds by taking the time derivative of formal solutions prevailing over period 1:

$$\begin{aligned}
I(t) &= \dot{K}(t) = \nu_1 \frac{B_1}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t}, \\
&= -\nu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^N}\right|_{perm} \left(1-e^{-r^\star \mathcal{T}}\right) \mathrm{d}G^N e^{\nu_1 t} - e^{-r^\star \mathcal{T}} \mathrm{d}G^N e^{\nu_1 t} < 0, \qquad (168)
\end{aligned}$$

and

$$\begin{aligned}
CA(t) &= -r^\star \left[\left(n\left(\bar{\lambda}, G_1^N\right) - B\left(\lambda_0, G_0^N\right)\right) + \Phi_1 \frac{B_1}{\mathrm{d}G^N}\right] \mathrm{d}G^N e^{r^\star t} + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t}, \\
&= \tilde{P}\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right) e^{\nu_1 t} - e^{-r^\star \mathcal{T}}\left(e^{r^\star t} - e^{\nu_1 t}\right)\right] \mathrm{d}G^N \gtrless 0. \qquad (169)
\end{aligned}$$

There exists a time $t = \acute{t}$ such that the current account changes of sign:

$$\acute{t} = -\frac{1}{\nu_2} \ln\left[\frac{e^{-r^\star \mathcal{T}}}{(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}}\right], \qquad (170)$$

where the term in square brackets is positive and lower than one. Over period 1, the current account improves first while the negative investment flow more than outweighs the *smoothing* effect. At time $\acute{t}$, these two effects cancel each other and after this date, the current account deteriorates as the smoothing behavior predominates, such that $CA\left(\mathcal{T}^-\right) < 0$. To see it more formally, we evaluate (169) at time $\mathcal{T}^-$:

$$CA\left(T^-\right) = \tilde{P} e^{\nu_1 \mathcal{T}}\left[\left(1-e^{-\nu_1 \mathcal{T}}\right) - \alpha_C\left(1-e^{-r^\star \mathcal{T}}\right)\right] \mathrm{d}G^N < 0. \qquad (171)$$

At time $\mathcal{T}^-$, the investment flow is also negative:

$$I\left(\mathcal{T}^-\right) = -e^{-\nu_2 \mathcal{T}}\left[1-(1-\alpha_C)\left(1-e^{r^\star \mathcal{T}}\right)\right] < 0. \qquad (172)$$

We have now to compare the slope of the trajectory after a transitory fiscal expansion over period $0 \leq t < \acute{t}$ in the (K, B)-space with the slope of the trajectory after a permanent fiscal expansion:

$$\begin{aligned}
\frac{\mathrm{d}B(t)}{\mathrm{d}K(t)} &= \frac{-\tilde{P} e^{-r^\star(\mathcal{T}-t)} + \nu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}}{\nu_1 \frac{B_1}{\mathrm{d}G^N} e^{\nu_1 t}}, \\
&= -\frac{\tilde{P}\left\{\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}\right] e^{\nu_1 t} - e^{-r^\star(\mathcal{T}-t)}\right\}}{\left[(1-\alpha_C)\left(1-e^{-r^\star \mathcal{T}}\right)+e^{-r^\star \mathcal{T}}\right] e^{\nu_1 t}}, \qquad (173)
\end{aligned}$$

where we have substituted the expression of the constant $B_1/\mathrm{d}G^N$. Over period $0 \leq \acute{t} < t$, the numerator is positive and the denominator is negative. Thus the slope of the trajectory is negative in the (K, B)-space. Comparing the terms in numerator and in denominator of (173), it is straightforward to show that the slope in absolute terms is lower than $\tilde{P}$. Therefore, the slope is negative and lower (in absolute terms) than the slope of the trajectory after a permanent fiscal expansion (equal to $-\tilde{P}$).

We turn now to the investigation of transitional dynamics of key macroeconomic variables over the stable period, say period 2. By adopting the standard procedure, we get:

$$I(t) = \dot{K}(t) = \nu_1 \frac{B_1'}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t} > 0 \tag{174a}$$

$$CA(t) = \dot{B}(t) = \nu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^N} \mathrm{d}G^N e^{\nu_1 t} < 0. \tag{174b}$$

Since the period 2 is a stable period, the dynamics are monotonic. If we can determine the sign of (174) at time $t = \mathcal{T}^+$, we are able to evaluate the transitional dynamics over the entire period:

$$I\left(\mathcal{T}^+\right) = -\left[(1-\alpha_C)\left(e^{\nu_1 \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \left(1 - e^{-\nu_2 \mathcal{T}}\right)\right] \mathrm{d}G^N > 0, \tag{175a}$$

$$CA\left(\mathcal{T}^+\right) = \tilde{P}\left[(1-\alpha_C)\left(e^{\nu_1 \mathcal{T}} - e^{-\nu_2 \mathcal{T}}\right) - \left(1 - e^{-\nu_2 \mathcal{T}}\right)\right] \mathrm{d}G^N < 0. \tag{175b}$$

From (171) and (175b), we deduce that the current account is continuous in the neighborhood of $\mathcal{T}$, such that $CA\left(\mathcal{T}^-\right) = CA\left(\mathcal{T}^+\right) < 0$. At the opposite, from (172) and (175a), we see that investment is not continuous in the neighborhood of T since at this date, it must clear the non tradable market. To see it formally, we write the non tradable clearing market condition at time $\mathcal{T}^-$ and at time $\mathcal{T}^+$:

$$I\left(\mathcal{T}^-\right) = Y^N\left[K\left(\mathcal{T}^-\right), P\left(\mathcal{T}^-\right)\right] - C^N\left[\lambda\left(\mathcal{T}^-\right), P\left(\mathcal{T}^-\right)\right] - G_1^N < 0, \tag{176a}$$

$$I\left(\mathcal{T}^+\right) = Y^N\left[K\left(\mathcal{T}^+\right), P\left(\mathcal{T}^+\right)\right] - C^N\left[\lambda\left(\mathcal{T}^+\right), P\left(\mathcal{T}^+\right)\right] - G_2^N > 0, \tag{176b}$$

where $G_2^N = G_0^N$. Goods market equilibrium is subject to two discrete perturbations: one at time $t = 0$ when the government raises the public spending, the other at time $t = \mathcal{T}$ when the policy is permanently removed. Since capital is a predetermined variable, it cannot jump neither at time $t = 0$ or at time $t = \mathcal{T}$. In addition, the marginal utility of wealth jumps at time $t = 0$ and remains constant from thereon. So we get $\bar{\lambda} = \lambda\left(\mathcal{T}^-\right) = \lambda\left(\mathcal{T}^+\right)$. Finally, when the tradable good sector is relatively more capital intensive, a rise in government spending leaves unaffected the relative price of the non-traded good both in the sort-run and in the long run, such that $\tilde{P} = P\left(\mathcal{T}^-\right) = P\left(\mathcal{T}^+\right)$. With output constrained at time $\mathcal{T}$ by the capital stock and by the relative price of the non-traded good, it therefore follows from (176) that for the market-clearing condition to hold, we must have

$$\mathrm{d}I\left(\mathcal{T}\right) = \mathrm{d}\dot{K}\left(\mathcal{T}\right) = -\mathrm{d}G^N\left(\mathcal{T}\right) = \mathrm{d}G^N > 0, \tag{177}$$

where $\mathrm{d}G^N\left(\mathcal{T}\right) \equiv G_2^N - G_1^N \equiv G_0^N - G_1^N \equiv -\mathrm{d}G^N$. Thus, the non-traded goods market equilibrium is maintained though the investment in physical capital, $\dot{K}\left(\mathcal{T}\right)$. Since at time $\mathcal{T}$, government expenditure reverts back to its original level, the investment flow changes

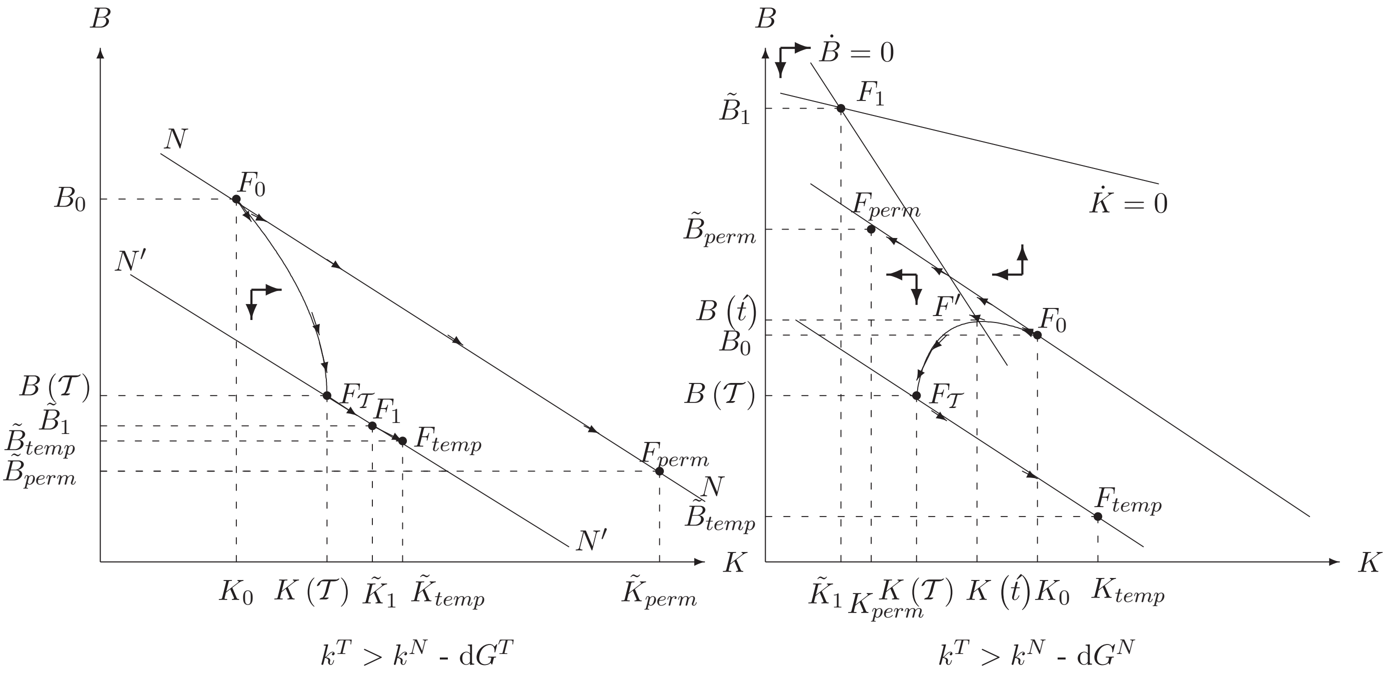


Figure 2: Permanent Vs. Temporary Increase in G^j - $k^T > k^N$

of sign and turns out to be positive as a greater share of the non tradable production (Y^N) may be allocated to investment (I) since the global consumption $(C^N + G^N)$ falls.

H Transitional Dynamics after a Rise in G^T

In the text, we consider only an increase in G^N. In this section, we analyze the effects of an increase in G^T. Hence, we provide details on the dynamics of key variables after a permanent and temporary rise in G^T, considering both cases: $k^T > k^N$ and $k^N > k^T$. Transitional paths are depicted in Figures 2 and 3 for $k^T > k^N$ and $k^N > k^T$, respectively. To keep analytical tractability, we assume that labor supply is fixed, i.e. we set $\sigma_L = 0$. Since these two parameters do no affect qualitatively the results, we further assume that the non-traded sector is perfectly competitive, i.e. we set $\mu = 1$, and we set the rate of depreciation of physical capital to zero.

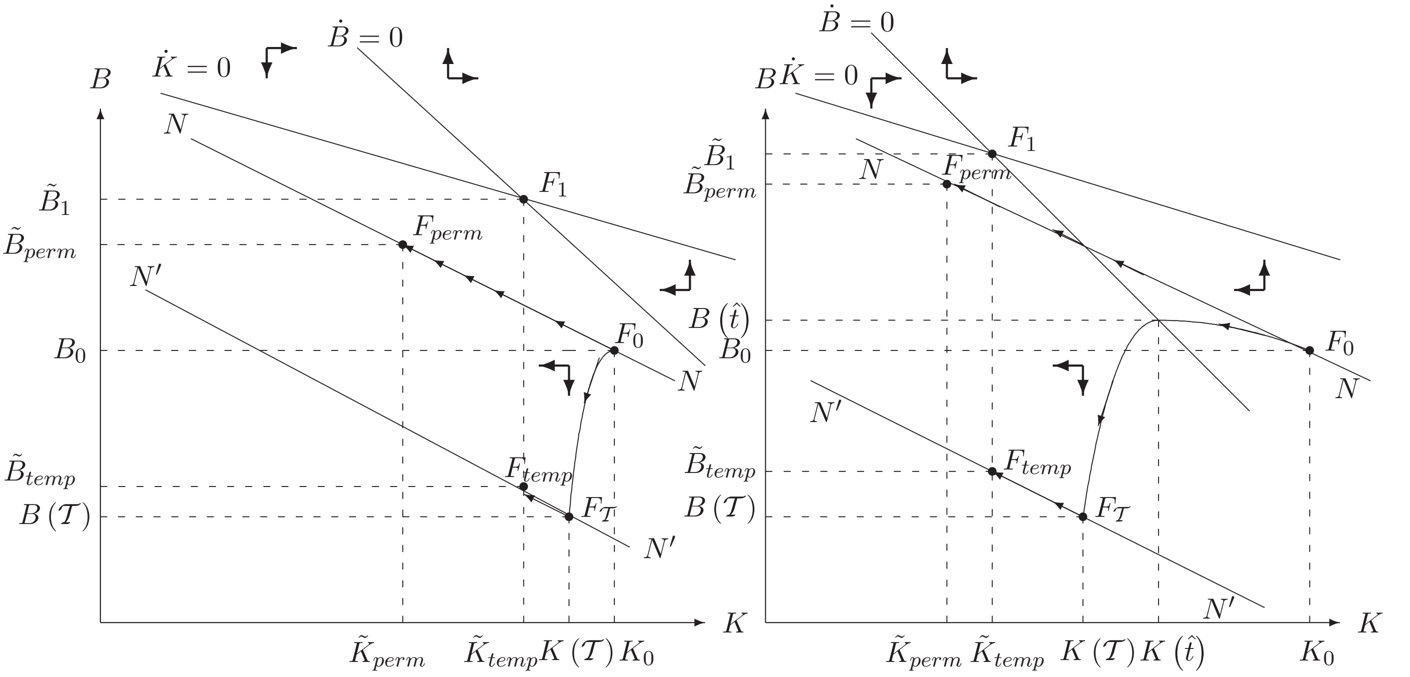
H.1 Long-Run Effects

It is convenient to determine first the long run changes of the real consumption, the stock of physical capital and the stock of foreign assets following a permanent rise in government spending on the traded good by differentiating (29) and (95):

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{178a}$$

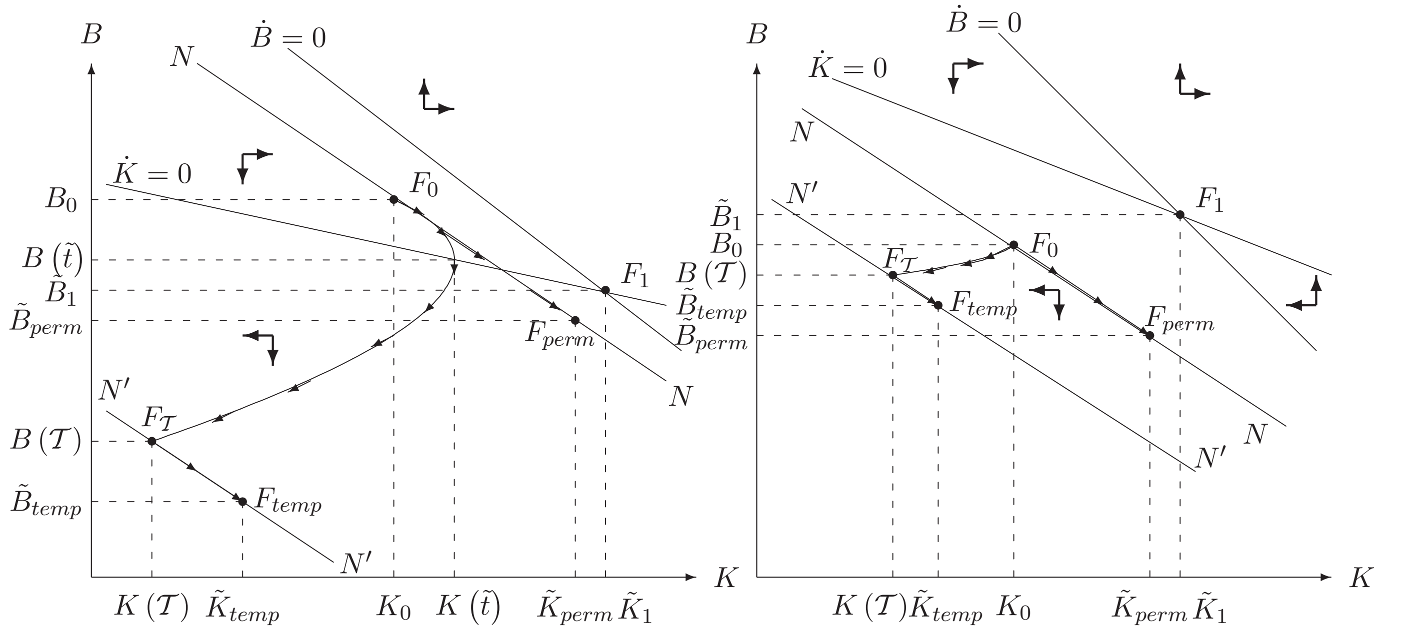
$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} \lesseqgtr 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{178b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} + B_{G^T} \gtreqless 0 \quad \text{depending on whether} \quad k^N \gtreqless k^T, \tag{178c}$$



$k^N > k^T$ - dG^T - $\mathcal{T} < \hat{\mathcal{T}}$ $\qquad$ $k^N > k^T$ - dG^T - $\mathcal{T} > \hat{\mathcal{T}}$

Figure 3: Permanent Vs. Temporary increase in G^T - $k^N > k^T$



$k^N > k^T$ - dG^N - $\mathcal{T} > \acute{\mathcal{T}}$ $\qquad$ $k^N > k^T$ - dG^N - $\mathcal{T} < \acute{\mathcal{T}}$

Figure 4: Permanent Vs. Temporary Increase in G^N - $k^N > k^T$

where $C_{G^T} = 0$ and $K_{G^T} = 0$. Expressions of the steady-state changes are given by the set of equations (78) and (79).

We compare the once-for-all jump of the marginal utility of wealth after a permanent increase in public spending on the traded good with respect to its change after a permanent rise:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) = \lambda_{G^T} \left(1 - e^{-r^\star \mathcal{T}}\right) > 0. \tag{179}$$

We now evaluate the long run changes of key economic variables after a temporary fiscal shock by differentiating (29) and (95). Since the signs of expressions depend crucially on the sectoral capital intensities, we consider two cases.

Case $k^N > k^T$

When the non-traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long run are given by:

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{180a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{180b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{180c}$$

where $C_{\bar{\lambda}} < 0$, $K_{\bar{\lambda}} < 0$ (if $k^N > k^T$), and $B_{\bar{\lambda}} < 0$.

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values K_0 and B_0 are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \tag{181a}$$

$$\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} > 0, \tag{181b}$$

where $K_{\bar{\lambda}} < 0$, $B_{\bar{\lambda}} < 0$ and $B_{G^T} > 0$. From (178b)-(178c), (180b)-(180c), and (181a)-(181b), we are able to deduce the following inequalities:

$$\tilde{K}_{perm} < \tilde{K}_1 = \tilde{K}_{temp} < K_0, \tag{182a}$$

$$\tilde{B}_{temp} < B_0 < \tilde{B}_{perm} < \tilde{B}_1. \tag{182b}$$

Case $k^T > k^N$

When the traded sector is relatively more capital intensive, the variations of macroeconomic aggregates in the long run are given by

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = C_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0, \tag{183a}$$

$$\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} > 0, \tag{183b}$$

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{183c}$$

It is interesting to compare the magnitudes of the long run changes in the stock of international assets between a permanent and a temporary fiscal expansion:

$$\left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} = B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T} \gtreqless B_{\bar{\lambda}}\lambda_{G^T}\left(1 - e^{-r^\star \mathcal{T}}\right) = \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{temp}. \tag{184}$$

The key factor that determines the magnitude of the long run change in the stock of foreign assets is the period of implementation of the government policy. More specifically, there exists a time $\mathcal{T} = \widetilde{\mathcal{T}}$ for which the two changes are equal which is given by

$$\widetilde{\mathcal{T}} = \frac{1}{r^\star}\ln\left[-\frac{B_{\bar{\lambda}}\lambda_{G^T}}{B_{G^T}}\right]. \tag{185}$$

As the fiscal shock is more persistent, i. e. $\mathcal{T} > \widetilde{\mathcal{T}}$, the external asset position deteriorates more than after a permanent fiscal shock. We can summarize our results as follows:

$$\tilde{B}_{temp} < \tilde{B}_{perm} < B_0 \quad \text{if} \quad \mathcal{T} > \widetilde{\mathcal{T}}, \tag{186a}$$

$$\tilde{B}_{perm} < \tilde{B}_{temp} < B_0 \quad \text{if} \quad \mathcal{T} < \widetilde{\mathcal{T}}. \tag{186b}$$

The changes of the period 1 steady-state values $\tilde{K}_1$ and $\tilde{B}_1$ compared to their initial (given) values $\tilde{K}_0$ and $\tilde{B}_0$ are given by :

$$\left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} = K_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} > 0, \tag{187a}$$

$$\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} = B_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} \gtrless 0, \tag{187b}$$

where $K_{\bar{\lambda}} > 0$, $B_{\bar{\lambda}} < 0$ and $B_{G^T} > 0$. The sign of (187b) is indeterminate but we are able to determine the length of fiscal shock, denoted by $\bar{\mathcal{T}}$, for which the steady-state change (187b) is equal to zero:

$$\bar{\mathcal{T}} = -\frac{1}{r^\star}\ln\left[\frac{B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T}}{B_{\bar{\lambda}}\lambda_{G^T}}\right]. \tag{188}$$

The existence of time $\bar{\mathcal{T}}$ relies upon inequality $B_{\bar{\lambda}}\lambda_{G^T} < B_{\bar{\lambda}}\lambda_{G^T} + B_{G^T} < 0$ which in turn implies that the term in square brackets is positive and less than unity. Consequently, we get the following inequality:

$$\tilde{B}_1 \lesseqgtr B_0 \quad \text{depending on whether} \quad \mathcal{T} \gtreqless \bar{\mathcal{T}}. \tag{189}$$

From (178b)-(178c), (183b)-(183c), (186) and (187a)-(187b), we are able to deduce the following inequalities:

$$K_0 < \tilde{K}_1 = \tilde{K}_{temp} < \tilde{K}_{perm}, \tag{190a}$$

$$\tilde{B}_{perm} < \tilde{B}_{temp} < B_0 \quad \text{if} \quad \mathcal{T} < \widetilde{\mathcal{T}}, \tag{190b}$$

$$\tilde{B}_{temp} < \tilde{B}_{perm} < \tilde{B}_0 \quad \text{if} \quad \mathcal{T} > \widetilde{\mathcal{T}}, \tag{190c}$$

where we assume that $\widetilde{\mathcal{T}} < \bar{\mathcal{T}}$.

H.2 Transitional Dynamics after a Permanent Increase in G^T

As shown previously, the stable adjustment of the economy is described by a saddle-path in (K, P)-space. The capital stock, the relative price of the non-traded good, and the stock of traded bonds evolve according to:

$$K(t) = \tilde{K} + B_1 e^{\mu_1 t}, \tag{191a}$$

$$P(t) = \tilde{P} + \omega_2^1 B_1 e^{\mu_1 t}, \tag{191b}$$

$$B(t) = \tilde{B} + \Phi_1 B_1 e^{\mu_1 t}, \tag{191c}$$

where $\omega_2^1 = 0$ if $k^T > k^N$ and with

$$B_1 = K_0 - \tilde{K} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\mathrm{d}G^T,$$

where we made use of the constancy of K at time $t = 0$ (i. e. K_0 is predetermined).

Case $k^N > k^T$

Using the fact that the steady-state value of the relative price of the non-traded good remains affected by a permanent rise in G^T, the initial jump of P is given by

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{perm} = -\omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{192}$$

From the short run static solutions, and by substituting the change in the equilibrium value of the marginal utility of wealth and the initial jump of P, we get the response of real consumption at time $t = 0$:

$$\begin{aligned} \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^T}\right|_{perm} &= C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} + C_P \left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{perm} = -\frac{\left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}{P_C\left[1 + \alpha_C \frac{r^\star}{(\mu_2)^2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right]}, \\ &= \left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] \left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} < 0, \end{aligned} \tag{193}$$

where $0 < \left[1 + \alpha_C \frac{1}{\mu_2}\frac{\tilde{C}^N}{\tilde{P}}\sigma_C \omega_2^1\right] < 1$. Therefore, we deduce the following inequality

$$\left.\frac{\mathrm{d}\tilde{C}}{\mathrm{d}G^T}\right|_{perm} = C_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{perm} < \left.\frac{\mathrm{d}C(0)}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{194}$$

Irrespective of sectoral capital intensities, a rise in G^T induces a once-for-all upward jump of the marginal utility of wealth which reduces real consumption. If $k^N > k^T$, the initial fall of C is moderated by the depreciation in P at time $t = 0$ and falls by less than in the long run.

Differentiating solutions (191), with respect to time, one obtains:

$$\dot{K}(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T < 0, \tag{195a}$$

$$\dot{P}(t) = -\mu_1 \omega_2^1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{195b}$$

$$\dot{B}(t) = -\mu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{195c}$$

where $\Phi_1 < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0$.

Along the stable adjustment, real consumption decreases:

$$\dot{C} = -\sigma_C C \alpha_C \frac{\dot{P}}{P} < 0, \tag{196}$$

where $\left(r^\star - \alpha_C \frac{\dot{P}}{P}\right)$ corresponds to the consumption-based real interest rate. After its initial depreciation, the relative price of the non-traded good appreciates to revert back to its initial value. This appreciation lowers the consumption-based real interest rate below the world interest rate which stimulates real consumption.

Case $k^T > k^N$

Differentiating solutions (191), with respect to time, one obtains

$$\dot{K}(t) = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{197a}$$

$$\dot{P}(t) = 0, \tag{197b}$$

$$\dot{B}(t) = -\mu_1 \Phi_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} e^{\mu_1 t}\mathrm{d}G^T < 0, \tag{197c}$$

where $\Phi_1 < 0$ and $\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} > 0$.

H.3 Transitional Dynamics after a Temporary Increase in G^T

Case $k^N > k^T$

By evaluating formal solution for $P(t)$ and differentiating with respect to G^T, we get the initial jump of P

$$\left.\frac{\mathrm{d}P(0)}{\mathrm{d}G^T}\right|_{temp} = -\omega_2^1 \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{198}$$

By adopting a similar procedure, we obtain the initial response of the investment flow following a temporary rise in government spending on the traded good :

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^T}\right|_{temp} = \left(1 - e^{-r^\star \mathcal{T}}\right) \left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^T}\right|_{perm} < 0. \tag{199}$$

By differentiating the formal solution (105c) over period 1 for $B(t)$ with respect to time, remembering that $B_2/\mathrm{d}G^T = 0$, then evaluating this at $t = 0$, and differentiating with respect to G^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp}\right] + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T}.$$

The expression in brackets can be evaluated by using properties (103), and the fact that $B_{G^T} = -\lambda_{G^T}/\lambda_B$:

$$\begin{aligned} -\left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp}\right] &= -\left[B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} - \Phi_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp}\right], \\ &= -\left[\frac{\lambda_{G^T}}{\lambda_B}\left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{G^T}}{\lambda_B}\right], \\ &= -B_{G^T} e^{-r^\star \mathcal{T}}. \end{aligned} \tag{200}$$

Inserting this expression, and remembering that $\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}$, we obtain the reaction of the current account at time $t = 0$:

$$\begin{aligned}\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1 \Phi_1 K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \lambda_{G^T}, \\ &= -e^{-r^\star \mathcal{T}} - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \lessgtr 0. \end{aligned} \tag{201}$$

The initial current account response is the result of two conflictory forces: (i) a *smoothing* effect which deteriorates the current account, and (ii) the negative investment flow which improves the external asset position. From (201), there exists a critical value of shock's length, $\hat{\mathcal{T}} > 0$, such that the current account response is zero on impact, i. e. $\dot{B}(0) = 0$. Solving (201) for $\hat{\mathcal{T}}$, we get:

$$\hat{\mathcal{T}} = \frac{1}{r^\star} \ln \left[\frac{1 - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm}}{-\mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm}} \right], \tag{202}$$

where the term in square brackets is higher than one.

The dynamics for K and P over period 1 are derived by taking the time derivative of equations (105a) and (105b):

$$\dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T < 0, \tag{203a}$$

$$\dot{P}(t) = \mu_1 \omega_2^1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\omega_2^1 \mu_1 \left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T > 0, \tag{203b}$$

where we used the fact that $B_1/\mathrm{d}G^T = -\left.\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right)$.

While the P and K go in the same direction as after a permanent rise in G^T, differentiation with respect to time of eq. (105c) shows that the current account may change of sign over period 1:

$$CA(t) = \dot{B}(t) = -e^{-r^\star(\mathcal{T}-t)} \mathrm{d}G^T - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t} \mathrm{d}G^T \lessgtr 0. \tag{204}$$

We have now to determine the conditions under which the current account dynamics displays a non monotonic behavior. Equation (204) reveals that the stock of international assets reaches a turning point during its transitional adjustment at time $\hat{\mathcal{T}}$ given by

$$\hat{\mathcal{T}} = \frac{1}{\mu_2} \ln \left[-\mu_1 \frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\Big|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{r^\star \mathcal{T}} \right]. \tag{205}$$

The necessary condition for $\hat{\mathcal{T}} > 0$, corresponds to:

$$0 < e^{-r^\star \mathcal{T}} < -\mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \quad \Leftrightarrow \quad \left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} > 0. \tag{206}$$

If the fiscal expansion lasts a short period, i. e. $\mathcal{T} < \hat{\mathcal{T}}$, the current account initially deteriorates and the stock of foreign assets decreases monotonically until time $\mathcal{T}$. If the fiscal expansion lasts a time period longer than $\hat{\mathcal{T}}$, the current account initially improves before reaching a turning point at time $\hat{\mathcal{T}}$. Subsequently, the current account deteriorates until time $\mathcal{T}$.

Once the government policy has been removed at time $\mathcal{T}$, the relative price of the non-traded good keeps on depreciating and the capital stock converges towards its new lower steady-state value:

$$\dot{K}(t) = \mu_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T < 0, \tag{207a}$$

$$\dot{P}(t) = \mu_1 \omega_2^1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T > 0, \tag{207b}$$

where $B_1'/\mathrm{d}G^T = B_1/\mathrm{d}G^T > 0$. Over period 2, the current account improves unambiguously as it can be seen from the time derivative of solution (107c):

$$\dot{B}(t) = \mu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T > 0. \tag{208}$$

Case $k^T > k^N$

If $k^T > k^N$, the dynamics for P are flat as after a permanent fiscal expansion since the constant $B_2/\mathrm{d}G^T$ is zero, i.e. $\dot{P}(t) = 0$. The investment flow is positive over period 1

$$I(t) = \dot{K}(t) = \mu_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T = -\mu_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} e^{\mu_1 t} \mathrm{d}G^T > 0. \tag{209}$$

Differentiating eq. (105c) with respect to time and remembering that $B_2/\mathrm{d}G^T = 0$ yields the transitional path for $B(t)$:

$$CA(t) = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] e^{r^\star t} \mathrm{d}G^T + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T} e^{\mu_1 t} \mathrm{d}G^T. \tag{210}$$

By evaluating this expressions at $t = 0$, and differentiating with respect to G^T, we obtain the initial response of the current account following a fiscal expansion:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} = -r^\star \left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] + \mu_1 \Phi_1 \frac{B_1}{\mathrm{d}G^T}.$$

The expression in brackets can be evaluated by using properties (103), and the fact that $B_{G^T} = -\lambda_{G^T}/\lambda_B$:

$$\begin{aligned}
-\left[\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^T}\right|_{temp} - \Phi_1 \left.\frac{\mathrm{d}\tilde{K}_1}{\mathrm{d}G^T}\right|_{temp} \right] &= -\left[B_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} + B_{G^T} - \Phi_1 K_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} \right], \\
&= -\left[\frac{\lambda_{G^T}}{\lambda_B} \left(1 - e^{-r^\star \mathcal{T}}\right) - \frac{\lambda_{G^T}}{\lambda_B} \right], \\
&= -B_{G^T} e^{-r^\star \mathcal{T}}.
\end{aligned} \tag{211}$$

Inserting expression (211) and remembering that $\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T} = \Phi_1 \frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^T}$, we obtain the reaction of the current account at time $t = 0$:

$$\begin{aligned}
\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^T}\right|_{temp} &= -e^{-r^\star \mathcal{T}} - \mu_1 \Phi_1 K_{\bar{\lambda}} \left(1 - e^{-r^\star \mathcal{T}}\right) \lambda_{G^T}, \\
&= -\left[e^{-r^\star \mathcal{T}} + \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) \right] < 0.
\end{aligned} \tag{212}$$

If $k^T > k^N$, both the *smoothing* effect and the positive investment flow lead to a decumulation of foreign assets. Consequently, the current account deteriorates initially and the stock of internationally traded bonds keeps on decreasing over period 1:

$$CA(t) = \dot{B}(t) = -e^{-r^\star(\mathcal{T}-t)}\mathrm{d}G^T - \mu_1 \left.\frac{\mathrm{d}\tilde{B}}{\mathrm{d}G^T}\right|_{perm} \left(1 - e^{-r^\star \mathcal{T}}\right) e^{\mu_1 t}\mathrm{d}G^T < 0. \tag{213}$$

Over period 2, the stocks of physical capital keeps on decreasing and the current account deteriorates monotonically:

$$I(t) = \mu_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t}\mathrm{d}G^T > 0, \tag{214a}$$

$$CA(t) = \mu_1 \Phi_1 \frac{B_1'}{\mathrm{d}G^T} e^{\mu_1 t}\mathrm{d}G^T < 0, \tag{214b}$$

where $B_1'/\mathrm{d}G^T = B_1/\mathrm{d}G^T < 0$.

I The Effects of Temporary Fiscal Shocks: The Case of Elastic Labor Supply

In this section, we derive formal solutions by assuming elastic labor supply. Yet, the derivations of formal solutions are only possible in the case $k^T > k^N$ since if sectoral capital intensities are reversed, expressions become analytically untractable.

We first solve the system (82a)-(82c) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$ and government spending G^N. Totally differentiating equations (82a)-(82c) yields in matrix form:

$$\begin{pmatrix} h_{kk}k_P^N & 0 & 0 \\ \left(Y_P^N - C_P^N\right) & Y_K^N & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B} \end{pmatrix} = \begin{pmatrix} 0 \\ -\left(Y_{\bar{\lambda}}^N - C_{\bar{\lambda}}^N\right)\mathrm{d}\bar{\lambda} + \mathrm{d}G^N \\ -\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right)\mathrm{d}\bar{\lambda} \end{pmatrix}. \tag{215}$$

Steady-state values of K and B can be expressed as functions of the shadow value of wealth and government spending G^N:

$$\tilde{K} = K\left(\bar{\lambda}, G^N\right), \tag{216a}$$

$$\tilde{B} = v\left(\bar{\lambda}, G^N\right), \tag{216b}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\nu_1}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_1\right) > 0 \quad \text{if} \quad k^T > k^N, \tag{217a}$$

$$= -\frac{1}{\bar{\lambda}}\frac{1}{\nu_2}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right) \gtreqless 0 \quad \text{if} \quad k^N > k^T, \tag{217b}$$

$$v_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{r^\star \tilde{h}}\left[\sigma_C\left(\tilde{f}\tilde{C}^N + \tilde{h}\tilde{C}^T\right) + \sigma_L \tilde{L}\tilde{h}\tilde{f}\right] < 0, \tag{217c}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}}{\partial G^N} = \frac{1}{Y_K^N} = \frac{1}{\nu_1} < 0 \quad \text{if} \quad k^T > k^N, \tag{218a}$$

$$= \frac{1}{Y_K^N} = \frac{1}{\nu_2} > 0 \quad \text{if} \quad k^N > k^T, \tag{218b}$$

$$\upsilon_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = -\frac{Y_K^T}{Y_K^N r^\star} = \frac{\tilde{f}}{\tilde{h} r^\star} > 0. \tag{218c}$$

Adopting the same procedure described in section (K.7), we provide formal expressions of the constants B_1, B_2 and B_1' only in the case $k^T > k^N$. We were unable to derive useful formal expressions with the reversal of capital intensities. Yet, in the latter case, analytical results derived by assuming inelastic labor supply are in line with numerical results and thereby elastic labor supply does not affect qualitatively the results.

Case $k^T > k^N$

The solutions after a rise in G^N are:

$$\frac{B_1}{\mathrm{d}G^N} = -\frac{\left[\sigma_C\left(\tilde{P}\tilde{C}^N e^{-r^\star T} + \tilde{C}^T\right) - \sigma_L \tilde{L}\tilde{P}\left(\nu_2 \tilde{k}^N + \nu_1 \tilde{k}^T e^{-r^\star T}\right)\right]}{\nu_1\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)},$$

$$= -\frac{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \left(1 - e^{-r^\star T}\right)\tilde{P}\left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^N\right)\right]}{\nu_1\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)} \gtrless 0, \tag{219a}$$

$$\frac{B_2}{\mathrm{d}G^N} = 0, \tag{219b}$$

$$\frac{B_1'}{\mathrm{d}G^N} = \frac{B_1}{\mathrm{d}G^N} + K_{G^N} e^{-\nu_1 \mathcal{T}}$$

$$= -\frac{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)\left(1 - e^{-\nu_1 T}\right) + \left(1 - e^{-r^\star T}\right)\tilde{P}\left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^N\right)\right]}{\nu_1\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)} \gtrless 0, \tag{219c}$$

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} = \lambda_{G^N}\left(1 - e^{-r^\star \mathcal{T}}\right) > 0, \tag{219d}$$

where λ_{G^N} represents the change in the equilibrium value of the shadow value of wealth after a permanent increase in G^N (see eq. (92b)).

General solutions for K and P are:

$$K(t) - \tilde{K} = B_1 e^{\nu_1 t} + B_2 e^{\nu_2 t}, \tag{220a}$$

$$P(t) - \tilde{P} = \omega_2^1 B_1 e^{\nu_1 t} + \omega_2^2 B_2 e^{\nu_2 t}, \tag{220b}$$

Differentiating eq. (220a) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of investment following a temporary rise in government spending on the non-traded good:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = \nu_1 \frac{B_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2}{\mathrm{d}G^N}.$$

Substituting (219a) and using the fact that $\frac{B_2}{\mathrm{d}G^N} = 0$, the initial reaction of investment rewrites as:

$$\left.\frac{\mathrm{d}I(0)}{\mathrm{d}G^N}\right|_{temp} = -\nu_1 \frac{\left[\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right) + \left(1-e^{-r^\star T}\right)\tilde{P}\left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \sigma_C \tilde{C}^N\right)\right]}{\nu_1\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)},$$

$$= -\left[1 + \left(1-e^{-r^\star T}\right)\frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\nu_1 - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \lesseqgtr 0. \tag{221}$$

Eq. (221) corresponds to **eq. (22) in the text**

The general solution for the stock of foreign assets is given by:

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right] e^{r^\star t} + \Phi_1 B_1 e^{\nu_1 t} + \Phi_2 B_2 e^{\nu_2 t}, \tag{222}$$

Differentiating eq. (222) w.r.t. time, evaluating at time $t = 0$ and differentiating w.r.t. G^N, we obtain the initial response of the current account after a temporary rise in G^N:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = r^\star\left[\left(B_0 - \tilde{B}\right) - \Phi_1 B_1 - \Phi_2 B_2\right]\nu_1 \frac{B_1\Phi_1}{\mathrm{d}G^N} + \nu_2 \frac{B_2\Phi_2}{\mathrm{d}G^N}.$$

Using the fact that

$$-\left.\frac{\mathrm{d}\tilde{B}_1}{\mathrm{d}G^N}\right|_{temp} - \Phi_1 \frac{B_1}{\mathrm{d}G^N} - \Phi_2 B_2$$

$$= -\left\{\left[\left(B_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}\right)\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} + \left(B_{G^N} - \Phi_1 K_{G^N}\right)\right]\right\},$$

$$= \frac{\lambda_{G^N}}{\lambda_B} e^{-r^\star \mathcal{T}} = -\frac{\tilde{P}}{r^\star} e^{-r^\star \mathcal{T}}, \tag{223}$$

the initial reaction of the current account rewrites as:

$$\left.\frac{\mathrm{d}CA(0)}{\mathrm{d}G^N}\right|_{temp} = -\tilde{P}e^{-r^\star \mathcal{T}} - \nu_1 \tilde{P}\frac{B_1}{\mathrm{d}G^N},$$

$$= \tilde{P}\left(1-e^{-r^\star T}\right)\left[1 + \frac{\left(\sigma_L \tilde{L}\tilde{k}^T \tilde{P}\nu_1 - \sigma_C \tilde{P}\tilde{C}^N\right)}{\left(\sigma_C P_C \tilde{C} + \sigma_L \tilde{W}\tilde{L}\right)}\right] \gtreqless 0, \tag{224}$$

where we used the fact that $\Phi_1 = -\tilde{P}$. Eq. (224) corresponds to **eq. (23) in the text**.

J Savings

Since the current account can be alternatively expressed as net exports plus interest earnings from traded bond holding, or as the savings less investment, we provide details for the derivation of steady-state and dynamic effects of fiscal shocks on savings.

J.1 Formal Solution for Financial Wealth

The law of motion for financial wealth ($S(t) = \dot{A}(t)$) is given by:

$$\dot{A}(t) = r^\star A(t) + W(P)\,L\left(\bar{\lambda}, P\right) - P_C(P)\,C\left(\bar{\lambda}, P\right) - Z, \tag{225}$$

with $Z = G^T + PG^N$.

The linearized version of (225) writes as follows:

$$\dot{A}(t) = r^{\star}\left(A(t) - \tilde{A}\right) + M\left(P(t) - \tilde{P}\right), \tag{226}$$

with M given by

$$\begin{aligned} M &= \left(W_P \tilde{L} + \tilde{W} L_P\right) - \left(\tilde{C}^N + P_C C_P + G^N\right), \\ &= \tilde{L} W_P \left(1 + \sigma_L\right) - \left[\tilde{C}^N \left(1 - \sigma_C\right) + G^N\right], \\ &= -\left\{\tilde{K}\left(\nu_2 + \delta_K\right) + \left[\sigma_L \tilde{L} \tilde{k}^T \left(\nu_2 + \delta_K\right) - \sigma_C \tilde{C}^N\right]\right\} < 0. \end{aligned} \tag{227}$$

From the second line of (227), if $\sigma_C < 1$ as empirical studies suggest, then the term in square brackets is positive and M is negative. The last line has been computed by using the fact that $\tilde{L} = \tilde{L}^N + \tilde{L}^T$ and $\tilde{K} = \tilde{k}^T \tilde{L}^T + \tilde{k}^N \tilde{L}^N$ which allows to simplify $\frac{1}{\mu}\left[\tilde{Y}^N + \tilde{L}\tilde{k}^T\left(\nu_2 + \delta_K\right)\mu\right]$ to $\tilde{K}\left(\nu_2 + \delta_K\right)$.

The general solution for the stock of financial wealth writes as follows:

$$A(t) = \tilde{A} + \left[\left(A_0 - \tilde{A}\right) - \frac{M\omega_2^1}{\nu_1 - r^{\star}} B_1 - \frac{M\omega_2^2}{\nu_2 - r^{\star}} B_2\right] e^{r^{\star} t} + \frac{M\omega_2^1}{\nu_1 - r^{\star}} B_1 e^{\nu_1 t} + \frac{M\omega_2^2}{\nu_2 - r^{\star}} B_2 e^{\nu_1 t}. \tag{228}$$

Invoking the transversality condition, we obtain the stable solution for financial wealth:

$$A(t) = \tilde{A} + \frac{M\omega_2^1}{\nu_1 - r^{\star}} B_1 e^{\nu_1 t}, \tag{229}$$

and the intertemporal solvency condition

$$\tilde{A} - A_0 = \frac{M\omega_2^1}{\nu_1 - r^{\star}}\left(\tilde{K} - K_0\right). \tag{230}$$

J.2 Steady-State and Dynamic Effects of a Permanent Fiscal Shock

Differentiating (230) w. r. t. G^i $(i = T, N)$, long-term changes of financial wealth are given by:

$$\frac{\mathrm{d}\tilde{A}}{\mathrm{d}\tau^j} = \frac{\omega_2^1}{\nu_2}\left(\tilde{K}\nu_2 + \sigma_L \tilde{L}\tilde{k}^T \nu_2 - \sigma_C \tilde{C}^N\right)\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^i}. \tag{231}$$

Differentiating (229) w. r. t. G^i $(i = T, N)$, we get the dynamics of savings:

$$S(t) = \dot{A}(t) = \nu_1 \frac{M\omega_2^1}{\nu_1 - r^{\star}} \frac{B_1}{\mathrm{d}G^i} \mathrm{d}G^i e^{\nu_1 t}, \tag{232}$$

where $\frac{B_1}{\mathrm{d}G^i} = -\frac{\mathrm{d}\tilde{K}}{\mathrm{d}G^i}$.

J.3 Steady-State and Dynamic Effects of a Temporary Fiscal Shock

We now evaluate the transitional dynamics of saving after a temporary shock, $\mathrm{d}G_i$ $(i = T, N)$.

Case $k^N > k^T$

Over the unstable period 1, savings evolve as follows:

$$S(t) = \dot{A}(t) = r^{\star}\left[\left(A_0 - \tilde{A}_1\right) - \frac{M\omega_2^1}{\nu_1 - r^{\star}} B_1\right] e^{r^{\star} t} + \nu_1 \frac{M\omega_2^1}{\nu_1 - r^{\star}} B_1 e^{\nu_1 t}, \tag{233}$$

with

$$\left(A_0 - \tilde{A}_1\right) = \left(B_0 - \tilde{B}_1\right) + \tilde{P}_0\left(K_0 - \tilde{K}_1\right) + K_0\left(P_0 - \tilde{P}_1\right). \quad (234)$$

Over the stable period 2, savings evolve as follows:

$$S(t) = \dot{A}(t) = \nu_1 \frac{M\omega_2^1}{\nu_1 - r^\star} B_1' e^{\nu_1 t}. \quad (235)$$

To compute the long-term changes of financial wealth, we linearize its expression, i. e. $A(t) = B(t) + P(t)K(t)$ in the neighborhood of the final steady-state:

$$A(t) - \tilde{A}_2 = \left(B(t) - \tilde{B}_2\right) + \tilde{P}\left(K(t) - \tilde{K}_2\right) + \tilde{K}\left(P(t) - \tilde{P}_2\right)$$

Then we evaluate at time $t = 0$:

$$A_0 - \tilde{A}_2 = \left(B_0 - \tilde{B}_2\right) + \tilde{P}_0\left(K_0 - \tilde{K}_2\right) + \tilde{K}_0\left(P(0) - \tilde{P}_2\right),$$

where we used the fact that $A(0) = A_0$, $B(0) = B_0$, $K(0) = K_0$ and assumed that the small open economy starts initially from the steady-state, i. e. $A_0 = \tilde{A}_0 = \tilde{A}$, $B_0 = \tilde{B}_0 = \tilde{B}$, $K_0 = \tilde{K}_0 = \tilde{K}$. Substituting $P(0) - \tilde{P}_2 = \omega_2^1 B_1$ into the expression above and differentiating w.r.t G^i ($i = T, N$), long-term changes of financial wealth are given by:

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^T}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \quad (236a)$$

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^N}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \quad (236b)$$

with

$$\left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) = -\frac{\sigma_C P_C \tilde{C}}{\bar{\lambda} r^\star} < 0. \quad (237)$$

Case $k^T > k^N$

Since $\omega_2^1 = 0$ whenever the traded good sector is relatively more capital intensive, and because $B_2/\mathrm{d}G^i = 0$, the transitional dynamics for saving degenerate and the financial wealth jumps immediately to its new steady-state level.

Adopting a similar procedure than previously (i. e. in the case $k^N > k^T$), we can calculate the long-term changes of financial wealth as follows:

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^T}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^T}\right|_{temp} < 0, \quad (238a)$$

$$\left.\frac{\mathrm{d}\tilde{A}}{\mathrm{d}G^N}\right|_{temp} = \left(B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}}\right) \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}G^N}\right|_{temp} < 0, \quad (238b)$$

with

$$B_{\bar{\lambda}} + \tilde{P}K_{\bar{\lambda}} = -\frac{\sigma_C P_C \tilde{C}}{\bar{\lambda} r^\star} < 0. \quad (239)$$

K The Case of Endogenous Markup

The framework builds on Jaimovich and Floetotto [2008]. While we consider the case of an endogenous markup, it holds for an exogenous markup, though in the latter case the number of competitors is large enough so that the price-elasticity of demand is not affected by firm entry. There are two sectors in the economy: a perfectly competitive sector which produces a traded good denoted by the superscript T and an imperfectly competitive sector which produces a non-traded good denoted by the superscript N. We assume that each producer of a unique variety of the non-traded good has the following technology $X_j^N = H\left(\mathcal{K}_j, \mathcal{L}_j\right)$ with $\mathcal{K}_j$ the capital stock and $\mathcal{L}_j$ labor.

K.1 Framework

The final non-traded output, Y^N, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral non-traded goods:

$$Y^N = \left[\int_0^1 \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}}, \tag{240}$$

where $\omega > 0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_j^N$ stands for intermediate consumption of sector'j variety (with $j \in [0, N]$). The final good producers behave competitively, and the households use the final good for both consumption and investment.

In each of the j sectors, there are $N > 1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good by a CES aggregating function. The non-traded output sectoral good j writes as:[44]

$$\mathcal{Q}_j^N = N^{-\frac{1}{\epsilon-1}} \left[\int_0^N \left(\mathcal{X}_{i,j}^N\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{241}$$

where $\mathcal{X}_{i,j}^N$ stands for output of firm i in sector j and ϵ is the elasticity of substitution between any two varieties.

Denoting by P and $\mathcal{P}_j$ the relative price of the final good and of the jth variety of the intermediate good, respectively, the profit the final good producer is written as follows:

$$\Pi^N = P\left[\int_0^N \left(\mathcal{Q}_j^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}j\right]^{\frac{\omega}{\omega-1}} - \int_0^1 \mathcal{P}_j \mathcal{Q}_j^N \mathrm{d}j. \tag{242}$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$\mathcal{Q}_j^N = \left(\frac{\mathcal{P}_j}{P}\right)^{-\omega} Y^N, \tag{243}$$

and the price of the final output is given by:

$$P = \left(\int_0^1 \mathcal{P}_j^{1-\omega} \mathrm{d}j\right)^{\frac{1}{1-\omega}}. \tag{244}$$

where $\mathcal{P}_j$ is the price index of sector j and P is the price of the final good.

Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i,j}^N$ is a price setter. Intermediate output $\mathcal{X}_{i,j}^N$ is produced using capital $\mathcal{K}_{i,j}^N$ and labor $\mathcal{L}_{i,j}^N$:

$$\mathcal{X}_{i,j}^N = H\left(\mathcal{K}_{i,j}^N, \mathcal{L}_{i,j}^N\right). \tag{245}$$

[44] By having the term $N^{-\frac{1}{\epsilon-1}}$ in (241), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

Denoting by $\mathcal{P}_{i,j}$ the price of good i in sector j, the profit function for the jth sector good producer denoted by π_j^N is:

$$\pi_j^N \equiv \mathcal{P}_j N^{-\frac{1}{\epsilon-1}} \left(\int_0^N \left(\mathcal{X}_{i,j}^N\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^N \mathcal{P}_{i,j} \mathcal{X}_{i,j}^N \mathrm{d}i. \tag{246}$$

The demand faced by each producer $\mathcal{X}_{i,j}^N$ is defined as :

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \frac{\mathcal{Q}_j^N}{N}, \tag{247}$$

and the price index of sector j is given by:

$$\mathcal{P}_j = N^{-\frac{1}{1-\epsilon}} \left(\int_0^N \mathcal{P}_{i,j}^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}. \tag{248}$$

Combining (243) and (247), the demand for variety $\mathcal{X}_{i,j}^N$ can be expressed in terms of the relative price of the final non-traded good:

$$\mathcal{X}_{i,j}^N = \left(\frac{\mathcal{P}_{i,j}}{\mathcal{P}_j} \right)^{-\epsilon} \left(\frac{\mathcal{P}_j}{P} \right)^{-\omega} \frac{Y^N}{N}. \tag{249}$$

In order to operate, each intermediate good producer must pay a fixed cost denoted by FC measured in terms of the final good which is assumed to be symmetric across firms. Each firm j chooses capital and labor to maximize profits. The profit function for the ith producer in sector j denoted by $\pi_{i,j}^N$ is:

$$\pi_{i,j}^N \equiv \mathcal{P}_j H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - r^K \mathcal{K}_j^N - W \mathcal{L}_j^N - pFC. \tag{250}$$

The demands for capital and hours worked are given by the equalities of the markup-adjusted marginal revenues of capital $\frac{\mathcal{P}_j H_K}{\mu}$ and labor $\frac{\mathcal{P}_j H_L}{\mu}$, to the capital rental rate r^K and the producer wage W, respectively.

K.2 First-Order Conditions

The current-value Hamiltonian for the j-th firm's optimization problem in the non-traded sector writes as follows:

$$\mathcal{H}_j^N = \mathcal{P}_j H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - r^K \mathcal{K}_j^N - W \mathcal{L}_j^N - pFC + \eta_j \left[H\left(\mathcal{K}_j^N, \mathcal{L}_j^N\right) - \mathcal{X}_{i,j}^N \right], \tag{251}$$

where $\mathcal{X}_j^N$ stands for the demand for variety j; firm j chooses its price ϱ_j to maximize profits treating the factor prices as given. First-order conditions for the non-traded sector write as follows:

$$\mathcal{P}_j H_K + \eta H_K = r^K, \quad , \tag{252a}$$

$$\mathcal{P}_j H_L + \eta H_L = W, \tag{252b}$$

$$\eta_j = \mathcal{P}_j' H_j, \tag{252c}$$

Combining (252a)-(252b) with (252c), by assuming that firms j are symmetric, yields:

$$\mathcal{P}_j H_K \left(1 - \frac{1}{e_j} \right) = r^K, \tag{253a}$$

$$\mathcal{P}_j H_L \left(1 - \frac{1}{e_j} \right) = W, \tag{253b}$$

where we used the fact that $\frac{\mathcal{P}'_j}{\mathcal{P}_j X^N_{i,j}} = -\frac{1}{e_j}$.

We consider a symmetric equilibrium where all firms in the intermediate good sector produce the output level $\mathcal{X}^N_{i,j} = \mathcal{X}^N$ with the same quantities of labor $\mathcal{L}^N_{i,j} = \mathcal{L}^N$ and capital $\mathcal{K}^N_{i,j} = \mathcal{K}^N$. Hence, the aggregate stock of physical capital and hours worked are $K^N = N\mathcal{K}^N$ and $L^N = N\mathcal{L}^N$, respectively. They also set the same price $\mathcal{P}_{i,j} = \mathcal{P}$. Hence, eqs. (244) and (248) imply that $\mathcal{P} = P$.

Defining the markup $\mu = \frac{e}{e-1}$, first-order conditions rewrite as follows:

$$P\frac{H_K}{\mu} = r^K, \tag{254a}$$

$$P\frac{H_L}{\mu} = W. \tag{254b}$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account the influence of the individual price on the sectoral price index:

$$e\left(N\right) = \epsilon - \frac{\left(\epsilon - \omega\right)}{N}, \quad N \in \left(1, \infty\right). \tag{255}$$

As it will be useful later, we calculate expressions of the partial derivatives of the price-elasticity of demand and the markup with respect to the number of firms:

$$e_N = \frac{\partial e}{\partial N} = \frac{\epsilon - \omega}{N^2} > 0, \quad \mu_N = \frac{\partial \mu}{\partial N} = -\frac{e_N}{\left(e-1\right)^2} = -\frac{e_N}{e-1}\frac{\mu}{e} < 0, \tag{256}$$

where we let $\mu = \frac{e}{e-1}$.

We further assume that free entry drives profits down to zero in every non-traded sector at each instant of time. Using constant returns to scale in production, i. e. $X = H\left(K, L\right) = H_K K + H_L L$, and the zero profit condition, in the aggregate, we have

$$PH\left(K^N, L^N\right) - r^K K^N - WL^N - PNFC = 0. \tag{257}$$

Substituting the short-run static solution for non-traded output (39), the zero-profit condition (257) can be rewritten as:

$$Y^N\left(K, P, \bar{\lambda}, \mu\left(N\right)\right)\left(1 - \frac{1}{\mu\left(N\right)}\right) = NFC. \tag{258}$$

K.3 Short-Run Static Solution for the Number of Firms

The zero profit condition can be solved for the number of producers in the non-traded sector:

$$N = N\left(K, P, \bar{\lambda}\right), \tag{259}$$

with partial derivatives given by:

$$N_x \equiv \frac{\partial N}{\partial x} = -\frac{Y^N_x \omega_{FC}}{\chi} \gtrless 0, \tag{260}$$

where $x = K, P, \bar{\lambda}$, $\omega_{FC} \equiv NFC/Y^N$ stands for the share of fixed costs in markup adjusted output and we set

$$\chi = \frac{Y^N}{N}\left\{\left[\eta_{Y^N,\mu}\left(\mu - 1\right) + 1\right]\frac{\eta_{\mu,N}}{\mu} - \omega_{FC}\right\}, \tag{261}$$

Inspection of (261) shows that $\chi < 0$ if $\eta_{\mu,N}$ is not too large. This implies that an input inflow ion the non-traded sector that raises Y^N and thereby yields to profit opportunities stimulates entry of firms.

K.4 Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions for non-traded output and consumption, given by (39) and (31) respectively, into the non-traded good market-clearing condition (18), and inserting short-run static solution for capital-labor ratio in the non-traded good sector (33) into the dynamic equation for the real exchange rate (5d), and substituting the short-run static solution for the number of firms (259) yields:

$$\dot{K} = \frac{Y^N \{K, P, \mu [N(K,P)]\}}{\mu [N(K,P)]} - C^N(P) - \delta_K K - G^N, \tag{262a}$$

$$\dot{P} = P \left\{ r^\star + \delta_K - \frac{h_k \left(k^N \{p, \mu [N(K,P)]\}\right)}{\mu [N(K,P)]} \right\}. \tag{262b}$$

For clarity purpose, we dropped variables which are constant over time from short-run static solutions.

Linearizing these two equations around the steady-state, and denoting $\tilde{x} = \tilde{K}, \tilde{P}$ the long-term values of $x = K, p$, we obtain in a matrix form:

$$\left(\dot{K}, \dot{P}\right)^T = J \left(K(t) - \tilde{K}, P(t) - \tilde{P}\right)^T, \tag{263}$$

where J is given by

$$J \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{264}$$

with

$$b_{11} = \frac{Y^N}{\mu} \left[\frac{Y_K^N}{Y^N} - \frac{\mu_N}{\mu} N_K \left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - \delta_K, \tag{265a}$$

$$b_1 2 = \frac{Y^N}{\mu} \left[\frac{Y_P^N}{Y^N} - \frac{\mu_N}{\mu} N_P \left(1 - \frac{Y_\mu^N \mu}{Y^N}\right)\right] - c_p^N, \tag{265b}$$

$$b_{21} = \frac{P}{\mu} h_{kk} \frac{\mu_N N_K}{\mu} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right), \tag{265c}$$

$$b_{22} = -\frac{P}{\mu} h_{kk} \left[k_p^N - \frac{\mu_N N_p}{\mu} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right)\right], \tag{265d}$$

Equilibrium Dynamics

The sing of the determinant denoted by Det of the 2×2 Jacobian matrix (264) is ambiguous:

$$\begin{aligned} \text{Det J} &= b_{11} b_{22} - b_{12} b_{21} \\ &= \left(\frac{Y_K^N}{\mu} - \delta_K\right) \left[\frac{Y_K^T}{\tilde{P}} + \frac{P}{\mu} h_{kk} k^N \frac{\mu_N N_p}{\mu} \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right)\right] \\ &\quad - \frac{\mu_N}{\mu} N_K \left[\frac{Y^N}{\mu} \left(1 - \frac{Y_\mu^N \mu}{Y^N}\right) \frac{Y_K^T}{\tilde{P}} + \left(\frac{Y_P^N}{\mu} - C_P^N\right) \frac{P}{\mu} h_{kk} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right)\right] \end{aligned} \tag{266}$$

and the trace denoted by Tr given by

$$\begin{aligned} \text{Tr J} &= b_{11} + b_{22} = \frac{Y_K^T}{\mu} + \frac{Y_K^N}{P} - \delta_K \\ & -\frac{\mu_N}{\mu} \left[N_K \frac{Y^N}{\mu} \left(1 - \frac{Y_\mu^N \mu}{Y^N} \right) - N_P \frac{P}{\mu} h_{kk} k^N \left(\frac{h_k}{h_{kk} k^N} - \frac{k_\mu^N \mu}{k^N} \right) \right], \\ &= r^\star - \frac{\mu_N}{\mu} N_K \frac{Y^N}{\mu} > 0, \end{aligned} \tag{267}$$

where we used the fact that $\frac{Y_K^T}{\mu} + \frac{Y_K^N}{P} = \frac{h_k}{\mu} = r^\star + \delta_K$; the positive sign follows from $N_K > 0$ and $\mu_N < 0$. If the elasticity of the markup to the flow of entry is not too large, the determinant (266) is negative so that the condition for saddle-path stability with real-valued roots holds. Such a condition requires that the markup must be initially not too large.

Characteristic roots from J write as follows:

$$\nu_i \equiv \frac{1}{2} \left\{ \text{Tr J} \pm \sqrt{(\text{Tr J})^2 - 4\text{Det J}} \right\} \gtrless 0, \quad i = 1, 2. \tag{268}$$

We denote by $\nu_1 < 0$ and $\nu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\nu_1 < 0 < r^\star < \nu_2. \tag{269}$$

Since the system features one state variable, K, and one jump variable, P, the equilibrium yields a unique one-dimensional stable saddle-path.

General solutions are those described by (55) with eigenvector ω_2^i associated with eigenvalue μ_i given by

$$\omega_2^i = \frac{\nu_i - b_{11}}{b_{12}}, \tag{270}$$

K.5 Formal Solution for the Stock of Foreign Assets

We first linearize equation (21) around the steady-state:

$$\dot{B}(t) = r^\star \left(B(t) - \tilde{B} \right) + \left[Y_K^T + Y_\mu^T \mu_N N_K \right] \left(K(t) - \tilde{K} \right) + \left[\left(Y_P^T + Y_\mu^T \mu_N N_P \right) - C_P^T \right] \left(P(t) - \tilde{P} \right). \tag{271}$$

where C_P^T is given by (32b).

Using the fact that $P(t) - \tilde{P} = \omega_2^1 \left(K(t) - \tilde{K} \right)$, setting

$$N_1 = \left[Y_K^T + Y_\mu^T \mu_N N_K \right] + \left[\left(Y_P^T + Y_\mu^T \mu_N N_P \right) - C_P^T \right] \omega_2^1, \tag{272}$$

solving for the differential equation and invoking the transversality condition for intertemporal solvency, the stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi_1 \left(K(t) - \tilde{K} \right), \tag{273}$$

and the linearized version of the nation's intertemporal budget constraint:

$$\tilde{B} - B_0 = \Phi_1 \left(\tilde{K} - K_0 \right) \tag{274}$$

where we substituted $B_1 \equiv K_0 - \tilde{K}$.

K.6 Solutions for L, N, and W

Linearizing the short-run static solution $N = N(K, P)$ yields the solution for the number of firms:

$$\begin{aligned} N(t) &= \tilde{N} + N_K \left(K(t) - \tilde{K}\right) + N_P \left(P(t) - \tilde{P}\right), \\ &= \tilde{N} + \left(N_K + N_P \omega_2^1\right) B_1 e^{\nu_1 t} + \left(N_K + N_P \omega_2^2\right) B_2 e^{\nu_2 t}. \end{aligned} \tag{275}$$

Linearizing the short-run static solution for labor $L = L(P, \mu)$, using the fact that $\mu = \mu(N)$, and substituting the appropriate solutions, the solution for $L(t)$ reads:

$$L(t) = \tilde{L} + L_P \left(P(t) - \tilde{P}\right) + L_\mu \left(\mu(t) - \tilde{\mu}\right), \tag{276}$$

$$= \tilde{L} + L_P \left[\omega_2^1 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^1\right)\right] B_1 e^{\nu_1 t} + L_P \left[\omega_2^2 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^2\right)\right] B_2 e^{\nu_2 t}, \tag{277}$$

where we used the fact that $L_\mu = -\frac{L_P P}{\mu}$.

Linearizing the short-run static solution for the wage rate $W = W(P, \mu)$ and substituting appropriate solutions yields:

$$\begin{aligned} W(t) &= \tilde{W} + W_P \omega_2^1 \left(K(t) - \tilde{K}\right) + W_\mu \mu_N \left(N(t) - \tilde{N}\right), \\ &= \tilde{W} + W_P \left[\omega_2^1 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^1\right)\right] B_1 e^{\nu_1 t} + W_P \left[\omega_2^2 - \frac{\tilde{P}}{\tilde{\mu}} \mu_N \left(N_K + N_P \omega_2^2\right)\right] B_2 e^{\nu_2 t}, \end{aligned} \tag{278}$$

where we used the fact that $W_\mu = -\frac{W_P P}{\mu}$.

K.7 The Two-Step Procedure: Wealth Effect and Tax Effects

By analytical convenience, we rewrite the system of steady-state equations, assuming that $\delta_K = 0$:

$$\frac{h_k \left[k^N \left(\tilde{P}\right)\right]}{\mu} = r^\star, \tag{279a}$$

$$\frac{1}{\mu} Y^N \left(\tilde{K}, \tilde{P}, \bar{\lambda}\right) - C^N \left(\bar{\lambda}, \tilde{P}\right) - G^N = 0, \tag{279b}$$

$$r^\star \tilde{B} + Y^T \left(\tilde{K}, \tilde{P}, \bar{\lambda}\right) - C^T \left(\bar{\lambda}, \tilde{P}\right) - G^T = 0, \tag{279c}$$

together with the intertemporal solvency condition

$$\left(\tilde{B} - B_0\right) = \Phi_1 \left(\tilde{K} - K_0\right). \tag{279d}$$

where K_0 and B_0 correspond to the initially predetermined stocks of physical capital and foreign assets.

Derivation of Steady-State Functions

In a **first step**, we solve the system (279a)-(279c) for $\tilde{P}$, $\tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$, government spending G^N together with the mark-up. Totally differentiating equations (279a)-(279c) yields in matrix form:

$$\begin{pmatrix} h_{kk}k_P^N & 0 & 0 \\ \left(\frac{Y_P^N}{\mu} - C_P^N\right) & \frac{Y_K^N}{\mu} & 0 \\ \left(Y_P^T - C_P^T\right) & Y_K^T & r^\star \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{P} \\ \mathrm{d}\tilde{K} \\ \mathrm{d}\tilde{B} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{Y_K^N}{\mu}\mathrm{d}\mu \\ -\left(\frac{Y_{\bar{\lambda}}^N}{\mu} - C_{\bar{\lambda}}^N\right)\mathrm{d}\bar{\lambda} - \left(\frac{Y_\mu^N}{\mu} - \frac{Y^N}{\mu^2}\right)\mathrm{d}\mu + \mathrm{d}G^N \\ -\left(Y_{\bar{\lambda}}^T - C_{\bar{\lambda}}^T\right)\mathrm{d}\bar{\lambda} - Y_\mu^T \mathrm{d}\mu \end{pmatrix}, \tag{280}$$

where we used the fact that $\mu f = P\left[h - h_k\left(k^N - k^T\right)\right]$ and $\frac{h_k}{\mu} = r^\star$ at the steady-state to rewrite $r^\star - h_{kk}k_\mu^N$ as $\frac{\tilde{h}}{\mu\left(\tilde{k}^N - \tilde{k}^T\right)} = \frac{Y_K^N}{\mu}$.

The equilibrium value of the marginal utility of wealth $\bar{\lambda}$, government spending G^N and the markup μ determine the following steady-state values:

$$\tilde{P} = P(\mu), \tag{281a}$$

$$\tilde{K} = K\left(\bar{\lambda}, G^N, \mu\right), \tag{281b}$$

$$\tilde{B} = B\left(\bar{\lambda}, G^N, \mu\right), \tag{281c}$$

with partial derivatives given by:

$$K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{\nu_1}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_1\right) > 0 \quad \text{if} \quad k^T > k^N, \tag{282a}$$

$$= -\frac{1}{\bar{\lambda}}\frac{1}{\nu_2}\left(\sigma_C \tilde{C}^N - \sigma_L \tilde{L}\tilde{k}^T \nu_2\right) > 0 \quad \text{if} \quad k^N > k^T, \tag{282b}$$

$$B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} = -\frac{1}{\bar{\lambda}}\frac{1}{r^\star \tilde{h}}\left[\sigma_C\left(\tilde{f}\tilde{C}^N + \tilde{h}\tilde{C}^T\right) + \sigma_L \tilde{L}\tilde{h}\tilde{f}\right] < 0, \tag{282c}$$

and

$$K_{G^N} \equiv \frac{\partial \tilde{K}}{\partial G^N} = \frac{1}{Y_K^N/\mu} = \frac{1}{\nu_1} < 0 \quad \text{if} \quad k^T > k^N, \tag{283a}$$

$$= \frac{1}{Y_K^N/\mu} = \frac{1}{\nu_2} > 0 \quad \text{if} \quad k^N > k^T, \tag{283b}$$

$$B_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = -\frac{Y_K^T \mu}{Y_K^N r^\star} = \frac{\tilde{f}}{\tilde{h} r^\star} > 0. \tag{283c}$$

and

$$P_\mu \equiv \frac{\partial \tilde{P}}{\partial \mu} = -\frac{\tilde{P}}{\mu}\frac{\tilde{P}Y_K^N}{\mu Y_K^T} = -\frac{\tilde{P}\nu_1}{\mu\nu_2} > 0, \quad \text{if} \quad k^T > k^N, \tag{284a}$$

$$= -\frac{\tilde{P}\nu_2}{\mu\nu_1} > 0, \quad \text{if} \quad k^N > k^T, \tag{284b}$$

$$K_\mu \equiv \frac{\partial \tilde{K}}{\partial \mu} = \frac{\tilde{P}}{\mu\nu_1\nu_2}\left[\frac{Y_P^N}{\mu} - \nu_1 C_P^N\right] + \frac{Y^N}{\mu^2 \nu_1} < 0, \quad \text{if} \quad k^T > k^N, \tag{284c}$$

$$= \frac{\tilde{P}}{\mu\nu_1\nu_2}\left[\frac{Y_P^N}{\mu} - \nu_2 C_P^N\right] + \frac{Y^N}{\mu^2 \nu_2} \lesseqgtr 0, \quad \text{if} \quad k^N > k^T, \tag{284d}$$

$$B_\mu \equiv \frac{\partial \tilde{B}}{\partial \mu} = -\frac{\tilde{P}}{\mu\nu_2}\left[\tilde{P}\left(\frac{Y_P^N}{\mu}\frac{r^\star}{\nu_1} - C_P^N\right) + \left(\sigma_L \tilde{L}\tilde{k}^T \nu_1 - \frac{\nu_1}{r^\star}\sigma_C \tilde{C}^N\right)\right] + \frac{\tilde{L}^N \tilde{f}}{\mu r^\star} \gtrless 0, \quad \text{if} \quad k^T > k^N \tag{284e}$$

$$= -\frac{\tilde{P}}{\mu\nu_1}\left[\tilde{P}\left(\frac{Y_P^N}{\mu}\frac{r^\star}{\nu_2} - C_P^N\right) + \left(\sigma_L \tilde{L}\tilde{k}^T \nu_2 - \frac{\nu_2}{r^\star}\sigma_C \tilde{C}^N\right)\right] + \frac{\tilde{L}^N \tilde{f}}{\mu r^\star} \gtrless 0, \quad \text{if} \quad k^N > k^T \tag{284f}$$

where we used the fact that $h_{kk}k_P^N = -\frac{\mu}{P}\frac{Y_K^T}{P}$ to derive the first equality of (284a). In addition, we made use of the following property $Y_\mu^N = -\frac{P}{\mu}Y_P^N$ and $Y_\mu^T = -\frac{P}{\mu}Y_P^T$ to determine (284c)-(284d) and (284e)-(284f). Finally, use has been made of property (42a) to rewrite $Y_P^T - C_P^T$ and property (42b) to simplify $\mu Y_K^T + \mu Y_K^N$ which is equal to $\tilde{P}\mu r^\star$ in the long-run.

Since the change in the markup modifies the long-run levels of real consumption and labor supply through the steady-state change in the relative price of non tradables, it is convenient to write their steady-state functions by substituting into their static solutions (29) that hold in the long-run:

$$C = m\left(\bar{\lambda}, \mu\right), \quad L = n\left(\bar{\lambda}, \mu\right), \tag{285}$$

where partial derivatives are given by (30) evaluated at the steady-state (that's why we substitute respectively the notations m and n for C and L) and

$$m_\mu \equiv \frac{\partial \tilde{C}}{\partial \mu} = \alpha_C \sigma_C \tilde{C}\frac{\nu_1}{\nu_2} < 0, \quad \text{if} \quad k^T > k^N, \tag{286a}$$

$$= \alpha_C \sigma_C \tilde{C}\frac{\nu_2}{\nu_1} < 0, \quad \text{if} \quad k^N > k^T, \tag{286b}$$

$$n_\mu \equiv \frac{\partial \tilde{L}}{\partial \mu} = -\frac{\sigma_L \tilde{L}\tilde{k}^T}{\tilde{W}}\frac{\tilde{P}\tilde{h}}{\tilde{f}}\frac{\tilde{P}r^\star}{\mu^2} < 0. \tag{286c}$$

We computed (286c) as follows: $n_\mu = \frac{\sigma_L \tilde{L}\tilde{k}^T}{\tilde{W}}\frac{\tilde{P}Y_K^N}{\mu Y_K^T}\frac{\tilde{P}r^\star}{\mu}$.

Following the same procedure, i. e. substituting the steady-state function for the real exchange rate into the static solution for wage evaluated at the steady-state, the steady-state function for wage writes as follows:

$$W = W\left(\mu\right), \tag{287}$$

where the partial derivative w. r. t. μ is given by:

$$W_\mu \equiv \frac{\partial \tilde{W}}{\partial \mu} = -\tilde{k}^T\frac{\tilde{P}\tilde{h}}{\tilde{f}}\frac{\tilde{P}r^\star}{\mu^2} < 0, \tag{288}$$

where $W_\mu = \tilde{k}^T\frac{\tilde{P}Y_K^N}{\mu Y_K^T}\frac{\tilde{P}r^\star}{\mu}$ with $\frac{Y_K^N}{Y_K^T} = -\frac{\tilde{h}}{\tilde{f}} < 0$.

Finally, following a similar procedure, we may express the rental rate of physical capital as a function of μ:

$$r^K = r^K\left(\mu\right), \tag{289}$$

where the partial derivative w. r. t. μ is given by:

$$r_\mu^K \equiv \frac{\partial \tilde{r}^K}{\partial \mu} = -r^\star\frac{\tilde{P}}{\mu}\frac{\nu_1}{\nu_2} > 0, \quad \text{if} \quad k^T > k^N, \tag{290}$$

$$r_\mu^K \equiv \frac{\partial \tilde{r}^K}{\partial \mu} = -r^\star\frac{\tilde{P}}{\mu}\frac{\nu_2}{\nu_1} > 0, \quad \text{if} \quad k^N > k^T. \tag{291}$$

Derivation of the Equilibrium Value of the Marginal Utility of Wealth

The **second step** consists to determine the equilibrium change of $\bar{\lambda}$ by taking the total differential of the intertemporal solvency condition (279d):

$$\left[v_{\bar{\lambda}} - \Phi_1 K_\lambda\right]\mathrm{d}\bar{\lambda} = -\left[v_{G^N} - \Phi_1 K_{G^N}\right]\mathrm{d}G^N, \tag{292}$$

from which may solve for the equilibrium value of $\bar{\lambda}$ as a function of tax rates:

$$\bar{\lambda} = \lambda\left(G^N\right), \tag{293}$$

with

$$\lambda_{G^N} \equiv \frac{\partial\bar{\lambda}}{\partial G^N} = -\frac{[v_{G^N} - \Phi_1 K_{G^N}]}{[v_{\bar{\lambda}} - \Phi_1 K_{\bar{\lambda}}]}. \tag{294}$$

References

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