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Bafile, Romina and Piergallini, Alessandro

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Romina Bafile† Alessandro Piergallini‡
University of L'Aquila University of Rome "Tor Vergata"

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Abstract

Standard New Keynesian models for monetary policy analysis are "cashless". When the nominal interest rate is the central bank's operating instrument, the LM equation is endogenous and, it is argued, can be ignored. The modern theoretical and quantitative debate on the importance of money for the conduct of monetary policy, however, overlooks firms' money demand. Working in an otherwise baseline New Keynesian setup, this paper shows that the monetary policy transmission mechanism is critically affected by the firms' money demand choice. Specifically, we prove that equilibrium determinacy may require either an active interest-rate policy (i.e., overreacting to inflation) or a passive interest-rate policy (i.e., underreacting to inflation), depending on the elasticity of production with respect to real money balances. We then calibrate the model to U.S. quarterly data and develop a sensitivity analysis in order to investigate the quantitative implications of our theoretical results. We find that macroeconomic stability is more likely to be guaranteed under an active, although not overly aggressive, monetary-policy stance.

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†Department of Economic Systems and Institutions, University of L'Aquila, Via Giovanni Falcone 25, 67100 Coppito - L'Aquila, Italy.

‡Corresponding author: Department of Economics, University of Rome "Tor Vergata", Via Columbia 2, 00133 Roma, Italy. E-mail: alessandro.piergallini@uniroma2.it. Phone: +390672595431. Fax: +39062020500.

1 Introduction

Standard macroeconomic theory of the New Keynesian type uses "cashless" models for monetary policy analysis (e.g., Rotemberg and Woodford, 1997, 1999; Clarida, Galí and Gertler, 1999; Taylor, 1999; Woodford, 2003; Galí, 2008). Under interest-rate policy rules, it is argued, it is not necessary to specify a money market equilibrium condition. The LM equation, typically derived employing the money-in-the-utility-function approach *à la* Sidrauski (1967), is endogenous when the interest rate is the policy instrument. Under a separable utility function, in particular, the LM equation is completely recursive to the equilibrium system. That is, the model solution would be unchanged by adding a money demand equation to the system. Under a non-separable utility function, the role of money in the IS equation resulting from the dependence of the marginal utility of consumption on real money balances has been proved to be quantitatively negligible (e.g., McCallum, 2001; Woodford, 2003; Ireland, 2004; Andrés, López-Salido and Vallés, 2006). Monetary aggregates can thus be ignored without altering policy implications (Woodford, 2008).

This approach to the theoretical analysis of monetary policy, with no explicit reference to money, arguably overlooks investigations on the role of money demand by firms. Empirically, in industrialized countries firms hold a considerable share of money supply. For instance, Mulligan (1997) documents that U.S. non-financial firms held at least 50% more demand deposit than households in the 1970-1990 period. In addition, firms' demand for money as a share of the aggregate appears to be increasing over time. For instance, Bover and Watson (2005) document that the U.S. firms' share of M1 was 35% of the non-financial private sector in the mid-1980s and 62% in 2000. In view of these remarkable stylized facts, the present paper attempts to evaluate the role of firms' money demand in the monetary policy transmission mechanism within an optimizing general equilibrium framework of the New Keynesian type.

To do this in a simple and intuitive way, we extend the baseline dynamic New Keynesian by employing the money-in-the-production-function approach. In the history of monetary theory and policy, a number of influencial economists have advocated that real money balances are a factor input and should, therefore, be included in the production function. Prominent examples include Friedman (1959, 1969), Levhari and Patinkin (1968), Johnson (1969), Bailey (1971), and Fischer (1974). The role of real balances in the production process has several theoretical rationales. In synthesis, money serves as an intermediate good for production, as a liquid reserve for investment, and as a way to enhance technical efficiency. There is also large empirical evidence showing that real

money balances have a significant role as an explicit input in the production process (e.g., Sinai and Stokes, 1972; Ben-Zion and Ruttan, 1975; Dennis and Smith, 1978; Short, 1979; Subrahmanyam, 1980; Simos, 1981; You, 1981; Khan and Ahmad, 1985; Nguyen, 1986; Hasan and Mahmud, 1993; Alexander, 1994; DeLorme, Thompson and Warren, 1995; Lotti and Marcucci, 2007; Apergis, 2010).

Monetary policy design turns out to be affected by the firms' money demand choice. Specifically, we prove that equilibrium determinacy may require either an active interest-rate policy (i.e., overreacting to inflation) or a passive interest-rate policy (i.e., underreacting to inflation), depending on the elasticity of production with respect to real money balances.

These theoretical results are in sharp contrast with the standard "cashless" New Keynesian framework, in which accommodating monetary policies, increasing the nominal interest rate to inflation with an elasticity lower than one, always incur undesirable sunspot fluctuations. The finding that the way in which money enters technology affects the stabilizing properties of feedback interest rate rules has first been demonstrated by Benhabib, Schmitt-Grohé and Uribe (2001) in the context of a continuous-time framework with either flexible prices or staggered price setting *à la* Rotemberg (1982). Our analytical results provide further theoretical support to these findings, for they are derived in the context of a canonical discrete-time New Keynesian model with staggered price setting *à la* Calvo (1983)-Yun (1996), extended to incorporate money in the production function.

The dispute over whether reacting to inflation aggressively is stabilizing or destabilizing has important implications for monetary theory and policy. Hence, quantitative investigations of the analytical findings derived in this paper are valuable. To this end we proceed by calibrating the model to U.S. quarterly data and performing a sensitivity analysis. Our results indicate that macroeconomic stability is more likely to be guaranteed under an active, although not overly aggressive, monetary-policy stance.

We shall conclude that when real balances facilitate the firms' production process, active interest-rate policies may well, in theory, bring about sunspot fluctuations; and conversely, passive interest-rate policies may well be compatible with equilibrium uniqueness and stability, as first emphasized by Benhabib, Schmitt-Grohé and Uribe (2001); nevertheless, calibration analysis appears to give support to the view that quantitatively active interest-rate policies still are necessary for equilibrium determinacy.

The reminder of the paper is organized in six sections. In Section 2, we set up the model. In Section 3, we log-linearize the equilibrium conditions. In Section 4, we analyze

equilibrium dynamics under flexible prices. In Section 5, we analyze equilibrium dynamics under sticky prices. In Section 6, we calibrate the model, derive policy implications and check their robustness. In Section 7, we present summary and concluding remarks.

2 The Model

We shall use a monetary model that has the baseline "cashless" New Keynesian setup as a particular case. To make the argument of the present paper as transparent as possibile, let us abstract from households' money demand and concentrate on firms' money demand.

The economy is populated by a large number of identical infinitely-lived households. The representative household maximizes the expected present discounted value of utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where $0 < \beta < 1$ is the subjective discount factor, C_t denotes a composite consumption goods defined as $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$, N_t denotes hours of work, and $\sigma, \varphi > 0$. The household's period budget constraint is given by

$$P_t C_t + \frac{B_t}{R_t} = B_{t-1} + W_t N_t + Z_t, \tag{2}$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$ is the price index, $P_t C_t = \int_0^1 P_t(i) C_t(i) di$ is total expenditure for consumption goods, B_t denotes nominal riskless bonds purchased in period t at price $1/R_t$ and paying one unit of numéraire in period $t+1$, R_t is the gross nominal interest rate on bonds, W_t is the nominal wage, and Z_t is a lump-sum component of income, including profits resulting from households' ownership of firms. The household is prevented from engaging in Ponzi's games. Optimality implies

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}, \tag{3}$$

$$\frac{1}{R_t} = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \Pi_{t+1}^{-1} \right\}, \tag{4}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$.

There is a continuum of monopolistically competitive firms, indexed by $i \in [0,1]$. Each

firm produces a differentiated good facing a production technology given by

$$Y_t(i) = \left[\frac{M_t(i)}{P_t}\right]^{\alpha} N_t(i)^{1-\alpha}, \tag{5}$$

where $0 < \alpha < 1$, and $M_t(i)$ denotes nominal money balances demanded by firm i for production purposes, in the spirit of Friedman (1959, 1969), Levhari and Patinkin (1968), Johnson (1969), Bailey (1971), and Fischer (1974). Total factor productivity is normalized to unity, for simplicity and without loss of generality.

Firm's total cost in nominal terms is $W_t N_t(i) + R_t M_t(i)$. Cost minimization, taking the nominal wage and the nominal interest rate as given, implies the following first order conditions:

$$\frac{R_t}{\alpha\left\{Y_t(i)/\left[M_t(i)/P_t\right]\right\}} = MC_t(i), \tag{6}$$

$$\frac{(W_t/P_t)}{\alpha\left[Y_t(i)/N_t(i)\right]} = MC_t(i), \tag{7}$$

where $MC_t(i)$ is the firm's real marginal cost. Combining (5), (6) and (7) yields

$$MC_t(i) = \frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}(R_t)^{\alpha}\left(\frac{W_t}{P_t}\right)^{1-\alpha}. \tag{8}$$

From (8), real marginal cost is identical across firms. We shall thus set $MC_t(i) = MC_t$.

According to the stochastic time dependent rule developed in Calvo (1983) and Yun (1996), each firm resets its price with a constant probability $1-\theta$. Then, a firm resetting its price in period t chooses the optimal price P_t^* that maximizes

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(i)\left(P_t^* - P_{t+k} MC_t\right), \tag{9}$$

subject to the sequence of demand constraints $Y_{t+k}(i) = \left(P_t^*/P_{t+k}\right)^{-\varepsilon} C_{t+k}$, where $Q_{t,t+k} \equiv \beta^k \left(C_{t+k}/C_t\right)^{-\sigma}\left(P_t/P_{t+k}\right)$ is the stochastic discount factor for nominal payoffs. The first order condition for this optimizing problem is given by

$$\sum_{k=0}^{\infty} \theta^k E_t\left\{Q_{t,t+k} Y_{t+k}(i)\left(P_t^* - \frac{\varepsilon}{\varepsilon-1} P_{t+k} MC_{t+k}\right)\right\} = 0. \tag{10}$$

According to (10), firms set their price equal to a markup over a weighted average of current and expected future nominal marginal costs.

The price level follows a law of motion given by

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{11}$$

To close the model, we need to specify the monetary policy regime. Many central banks conduct monetary policy by controlling a short-term nominal interest rate. We assume, in particular, that monetary policy takes the form of an interest-rate feedback rule whereby the nominal interest rate is set as an increasing function of the inflation rate:

$$R_t = \beta^{-1}\left(\Pi_t\right)^{\phi_\pi}, \tag{12}$$

where $\phi_\pi > 0$ measures the elasticity of R_t with respect to Π_t. Because the monetary policy rule pertains to the setting of nominal rate of interest, the nominal quantity of money supplied by the central bank, M_t, adjusts endogenously to satisfy firms' demand for money.

According to Leeper (1991), we use the following terminology.

Definition 1 *Monetary policy is active (passive) if and only if* $\phi_\pi > (<) 1$.

An active monetary policy is in the spirit of the so-called Taylor's (1993, 1999) principle, according to which the central bank should respond to an increase in inflation with a more-than-proportional increase in the nominal interest rate in order to ensure macroeconomic stability.

3 Linearized Equilibrium Conditions

Market clearing requires $M_t = \int_0^1 M_t(i)\,di$, $N_t = \int_0^1 N_t(i)\,di$, $Y_t(i) = C_t(i)$ for all $i \in [0,1]$, and so $Y_t = C_t$.

For a generic variable X_t, let $x_t \equiv \log\left(X_t/X\right)$, where X denotes its steady-state value. Using the market clearing conditions, log-linear approximations of (3), (4), (5), (6), (7), (10), (11), and (12) around a zero-inflation steady state yield

$$w_t - p_t = \sigma y_t + \varphi n_t, \tag{13}$$

$$y_t = E_t\left\{y_{t+1}\right\} - \frac{1}{\sigma}\left(r_t - E_t\left\{\pi_{t+1}\right\}\right), \tag{14}$$

$$y_t = \alpha\left(m_t - p_t\right) + (1-\alpha)\, n_t, \tag{15}$$

$$mc_t = (m_t - p_t - y_t) + r_t = (n_t - y_t) + (w_t - p_t), \tag{16}$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda mc_t, \tag{17}$$

$$r_t = \phi_\pi \pi_t. \tag{18}$$

where $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$.

4 Dynamics under Flexible Prices

Let us first study equilibrium dynamics under flexible prices. We shall demonstrate that even in the limiting case of no price rigidity, firms' money demand crucially affects uniqueness and stability of the rational expectations equilibrium under Taylor-type interest rate feedback rules.

If prices are flexible, i.e., if $\theta = 0$, all firms adjust each period. Equation (10) collapses to the familiar optimal price-setting condition with monopolistic competition, $P_t^* = [\varepsilon/(\varepsilon-1)] P_t MC_t$. That is, firms set the price for their differentiated good as a constant markup over marginal cost. Because in symmetric equilibrium all firms choose the same price, $P_t^* = P_t$, the flexible price equilibrium features a constant real marginal cost, $MC_t = (\varepsilon-1)/\varepsilon$. This implies $mc_t = 0$. Equation (16) thus becomes

$$(m_t - p_t - y_t) + r_t = (n_t - y_t) + (w_t - p_t) = 0. \tag{19}$$

Combining (13), (15) and (19) yields

$$y_t = -\frac{\alpha(1+\varphi)}{(1-\alpha)(\sigma+\varphi)} r_t. \tag{20}$$

Equation (20) reveals that even in the case of no price rigidity, monetary policy turns out to have real effects on output. This is beacuse an increase in the nominal interest rate brings about a decrease in firms' demand for money, thereby dampening output supply. Only in the standard cashless-economy paradigm, in which $\alpha = 0$, the level of output is independent of monetary factors and hence, recalling that we are abstracting from productivity shocks to render the model's implications as trasparent as possible, does not deviate from the trend, i.e., $y_t = 0$.

We shall make use of the following definitions.

Definition 2 *Under flexible prices ($\theta = 0$), a rational expextations equilibrium (REE)*

is a set of sequences $\{\pi_t, y_t, r_t\}$ *satisfying (14), (18), and (20) at all dates* $t \geq 0$.

Definition 3 *Under flexible prices* ($\theta = 0$)*, the model displays determinacy of the REE if there exists a unique set of stable sequences* $\{\pi_t, y_t, r_t\}$ *satisfying (14), (18), and (20) at all dates* $t \geq 0$*. The model displays indeterminacy of the REE if there exist infinite sets of stable sequences* $\{\pi_t, y_t, r_t\}$ *satisfying (14), (18), and (20) at all dates* $t \geq 0$.

Note that substituting (20) and (18) into (14) results in the following expectational first-order difference equation, capturing inflation dynamics:

$$E_t \{\pi_{t+1}\} = \gamma \pi_t, \tag{21}$$

where

$$\gamma \equiv \frac{[(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)]\phi_\pi}{(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)\phi_\pi}. \tag{22}$$

Because π_t is a jump variable, equilibrium determinacy requires that the coefficient γ is outside the unit circle, i.e., with modulus $|\gamma| > 1$ (Blanchard and Kahn, 1980; Woodford 2003).

To see how firms' money demand critically affects the dynamic properties of rational expectations equilibria and provide clear economic intuitions, it is convenient to consider two limiting cases, that shall be the object of Propositions 1-2.

Consider first what happens in the polar case of a cashless economy ($\alpha = 0$). From (22), it follows $\gamma \equiv \phi_\pi$. Hence, whether the equilibrium is determinate or indeterminate depends *only* on the monetary-policy stance. In particular, the existence of a unique stable solution for π_t, and as a consequence for all the other endogenous variables of the model, requires $\phi_\pi > 1$. This is summarized in the following proposition.

Proposition 1 *Suppose that prices are flexible* ($\theta = 0$) *and the economy is cashless* ($\alpha = 0$)*. Then, determinacy of the REE obtains if and only if*

$$\phi_\pi > 1. \tag{23}$$

Proof. From (21), the REE is determinate if and only if $|\gamma| > 1$. When $\alpha = 0$, we have $\gamma \equiv \phi_\pi$. Given the sign restriction $\phi_\pi > 0$, it follows that the REE is determinate if and only if condition (23) holds. ■

Condition (23) corresponds to the well-known Taylor principle, emphasized in modern monetary theory as a necessary condition for macroeconomic stability (e.g., Woodford,

2003; Galí, 2008). Intuitively, when the Taylor principle is satisfied, inflationary pressures are met by increases in real interest rates, which are necessary and sufficient to dampen aggregate demand and thus inflation.

Consider next the case of an economy in which firms' money demand for production purposes is taken into account ($\alpha > 0$). For now, let us focus on what occurs in the polar case of logarithmic preferences for consumption ($\sigma = 1$). This limiting case is often studied in Galí (2008) for balanced-growth considerations (e.g., Cooley and Prescott, 1995). From (22), it follows $\gamma \equiv [(1-\alpha) - \alpha\phi_\pi] / [(1-2\alpha)\,\phi_\pi]$. Whether $|\gamma| > 1$, so that the model exhibits equilibrium determinacy, now depends *not only* on the monetary policy feedback parameter ϕ_π, *but also* on the technology parameter α. In particular, examining the conditions under which $|\gamma| > 1$ enables us to state the following proposition.

Proposition 2 *Suppose that prices are flexible ($\theta = 0$) and preferences for consumption are logarithmic ($\sigma = 1$). If $0 < \alpha < 1/3$, then determinacy of the REE obtains if and only if*

$$\phi_\pi > 1;$$

if $1/3 < \alpha < 1/2$, then determinacy of the REE obtains if and only if

$$1 < \phi_\pi < \frac{1-\alpha}{3\alpha - 1};$$

if $1/2 < \alpha < 1$, then determinacy of the REE obtains if and only if

$$\frac{1-\alpha}{3\alpha - 1} < \phi_\pi < 1.$$

Proof. See Appendix A. ■

Figure 1 illustrates the results stated in Proposition 2. The general validity of the Taylor principle is limited to the cases in which $0 < \alpha < 1/3$. If $\alpha > 1/3$, the higher α, the less aggressive monetary policy must be to ensure macroeconomic stability, according to the hyperbolic frontier given by $\phi_\pi = (1-\alpha) / (3\alpha - 1)$. An intuitive economic interpretation is as follows. When monetary policy is active, inflationary pressures are offset by increases in the real interest rate, which dampen aggregate demand through (14). This negative demand-side effect tends to be deflationary over time. In the cashless-economy setup, in particular, this effect brought about by an active monetary-policy stance is necessary and sufficient to preserve macroeconomic stability. Once firms' demand for real money

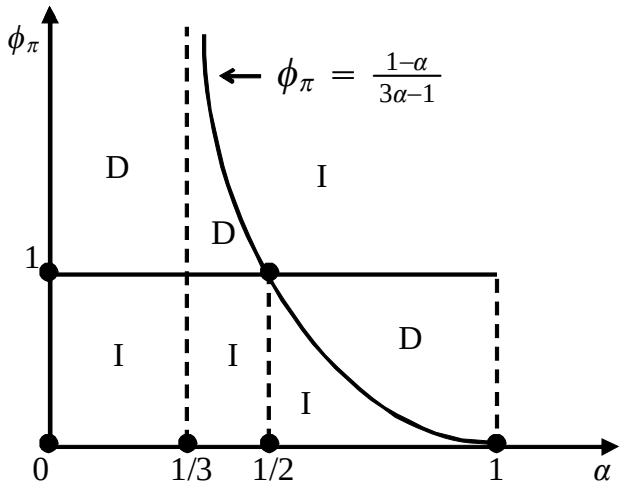

Figure 1: Regions of determinacy (D) and indeterminacy (I) under flexible prices ($\theta = 0$) and logarithmic preferences ($\sigma = 1$).

balances is taken into account, however, the rise in the nominal interest rate implied by the policy rule (18) also dampens output supply through (20). This negative supply-side effect tends, by contrast, to be inflationary over time. As a consequence, an active monetary-policy stance induces aggregate instability if it causes the supply-side, inflationary effect to prevail on the demand-side, deflationary effect. There exists, in particular, a threshold value of α, equal to $1/2$, beyond which the dynamic properties of Taylor-type interest rate rules are completely reversed, in a way that equilibrium determinacy requires a passive monetary-policy stance.

We shall now extend the foregoing results to the more general case of CRRA preferences for consumption ($\sigma \neq 1$). Specifically, the next proposition applies.

Proposition 3 *Suppose that prices are flexible ($\theta = 0$) and preferences for consumption are of the CRRA-type ($\sigma \neq 1$). If*

$$0 < \alpha < \frac{(\sigma + \varphi)}{(\sigma + \varphi) + 2\sigma\,(1 + \varphi)},$$

then determinacy of the REE obtains if and only if

$$\phi_\pi > 1;$$

if

$$\frac{(\sigma + \varphi)}{(\sigma + \varphi) + 2\sigma\,(1 + \varphi)} < \alpha < \frac{(\sigma + \varphi)}{(\sigma + \varphi) + \sigma\,(1 + \varphi)},$$

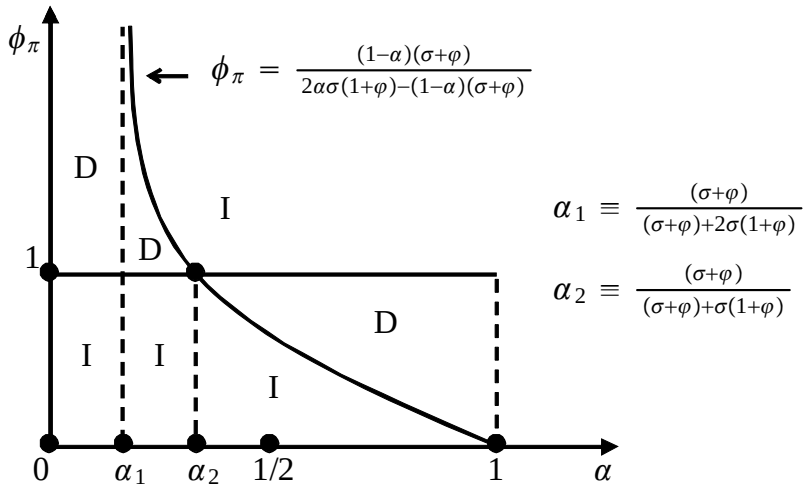

Figure 2: Regions of determinacy (D) and indeterminacy (I) under flexible prices ($\theta = 0$) and CRRA preferences ($\sigma > 1$).

then determinacy of the REE obtains if and only if

$$1 < \phi_\pi < \frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)};$$

if

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \alpha < 1,$$

then determinacy of the REE obtains if and only if

$$\frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)} < \phi_\pi < 1.$$

Proof. See Appendix B. ■

Figure 2 illustrates the results stated in Proposition 3 when the coefficient of relative risk aversion, σ, is greater than unity. The case $\sigma > 1$ is more empirically plausible than the case $\sigma < 1$. In fact estimates of σ, which equals the inverse of the intertemporal elasticity of substitution, fall in the range 3-10 (e.g., Hall, 1988; Barsky, Juster, Kimball and Shapiro, 1997; Rotemberg and Woodford, 1997). If $\sigma > 1$, the first threshold level of α, $\alpha_1 \equiv (\sigma+\varphi)/[(\sigma+\varphi)+2\sigma(1+\varphi)]$, beyond which the Taylor principle becomes necessary but not sufficient to ensure determinacy, and the second threshold level of α, $\alpha_2 \equiv (\sigma+\varphi)/[(\sigma+\varphi)+\sigma(1+\varphi)]$, beyond which the central bank should follow a passive stance to ensure determinacy, satisfy the following inequalities: $\alpha_1 < 1/3$ and

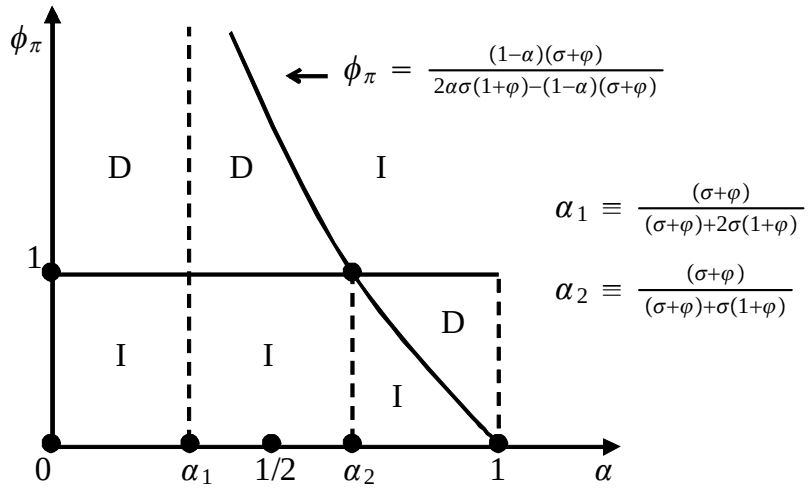

Figure 3: Regions of determinacy (D) and indeterminacy (I) under flexible prices ($\theta = 0$) and CRRA preferences ($\sigma < 1$).

$\alpha_2 < 1/2$.[1] This implies that the standard proposition that an active monetary-policy stance is a necessary and sufficient condition to stabilize the economy loses its general validity for lower values of α with respect to the case of logarithmic preferences.

In general, because $\partial\alpha_1/\partial\sigma, \partial\alpha_2/\partial\sigma < 0$,[2] the higher σ, the more plausible becomes the hypothesis that aggregate stability requires a less aggressive, even a passive, monetary-policy stance. The reason is the following. According to (14), a higher value of the relative risk aversion coefficient, that is, a lower value of the intertemporal elasticity of substitution, decreases the sensitivity of aggregate demand with respect to the real interest rate. Thus, under inflationary pressures, dampening aggregate demand would require a more aggressive interest-rate policy. But a more pronounced increase in the nominal interest rate does exacerbate the inflationary supply-side effect. In these circumstances, active monetary policies are more likely to generate indeterminacy. Conversely, the determinacy region under passive monetary policies increases.

[1] The opposite holds in the case in which $\sigma < 1$. See Figure 3.

[2] In fact, we have

$$\frac{\partial\alpha_1}{\partial\sigma} = -\frac{2\varphi\,(1+\varphi)}{[(\sigma+\varphi)+2\sigma\,(1+\varphi)]^2} < 0,$$

$$\frac{\partial\alpha_2}{\partial\sigma} = -\frac{\varphi\,(1+\varphi)}{[(\sigma+\varphi)+\sigma\,(1+\varphi)]^2} < 0.$$

5 Dynamics under Sticky Prices

We now analyze equilibrium dynamics under sticky prices and evaluate the robustness of the results obtained under flexible prices. Combining (13), (15) and (16), real marginal cost can be expressed as

$$mc_t = \frac{\alpha(1+\varphi)}{(1+\alpha\varphi)} r_t + \frac{(1-\alpha)(\sigma+\varphi)}{(1+\alpha\varphi)} y_t. \tag{24}$$

Substituting (24) into (17) results in the following forward-looking Phillips curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \frac{\lambda}{(1+\alpha\varphi)}\left[\alpha(1+\varphi) r_t + (1-\alpha)(\sigma+\varphi) y_t\right]. \tag{25}$$

We shall adopt the next definitions.

Definition 4 *Under sticky prices ($\theta > 0$), a REE is a set of sequences $\{\pi_t, y_t, r_t\}$ satisfying (14), (18), and (25) at all dates $t \geq 0$.*

Definition 5 *Under sticky prices ($\theta > 0$), the model displays determinacy of the REE if there exists a unique set of stable sequences $\{\pi_t, y_t, r_t\}$ satisfying (14), (18), and (25) at all dates $t \geq 0$. The model displays indeterminacy of the REE if there exist infinite sets of stable sequences $\{\pi_t, y_t, r_t\}$ satisfying (14), (18), and (25) at all dates $t \geq 0$.*

Using the policy rule (18) into *both* the expectational IS equation (14) and the expectational Phillips curve (25) enables us to get the equilibrium system involving the two endogenous variables π_t and y_t in matrix form, given by

$$\begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \end{bmatrix} = \Omega \begin{bmatrix} \pi_t \\ y_t \end{bmatrix}, \tag{26}$$

where

$$\Omega \equiv \begin{bmatrix} \frac{1}{\beta}\left[1 - \frac{\lambda\alpha(1+\varphi)}{(1+\alpha\varphi)}\phi_\pi\right] & -\frac{\lambda(1-\alpha)(\sigma+\varphi)}{\beta(1+\alpha\varphi)} \\ \frac{1}{\sigma}\left\{\left[1 + \frac{\lambda\alpha(1+\varphi)}{\beta(1+\alpha\varphi)}\right]\phi_\pi - \frac{1}{\beta}\right\} & 1 + \frac{\lambda(1-\alpha)(\sigma+\varphi)}{\sigma\beta(1+\alpha\varphi)} \end{bmatrix}. \tag{27}$$

Both π_t and y_t are jump variables. Consequently, equilibrium determinacy requires that both eigenvalues of matrix Ω are outside the unit circle (Blanchard and Kahn, 1980; Woodford 2003). This condition is verified if and only if *either* (Case I)

$$\det \Omega > 1, \tag{28}$$

$$\det \Omega - \mathrm{tr}\Omega > -1, \tag{29}$$

and

$$\det \Omega + \mathrm{tr}\Omega > -1, \tag{30}$$

or (Case II)

$$\det \Omega - \mathrm{tr}\Omega < -1, \tag{31}$$

and

$$\det \Omega + \mathrm{tr}\Omega < -1. \tag{32}$$

Within the cashless-economy paradigm, the next proposition applies.

Proposition 4 *Suppose that prices are sticky ($\theta > 0$) and the economy is cashless ($\alpha = 0$). Then, determinacy of the REE obtains if and only if*

$$\phi_\pi > 1.$$

Proof. See Appendix C. ■

Therefore, consistently with the standard literature (e.g., Woodford, 2003; Galí, 2008), a remarkable implication of Proposition 4 applies: in a cashless framework, the result that an active monetary policy is necessary and sufficient for determinacy holds *regardless* on whether prices are flexible or sticky.

Consider instead what happens in the presence of firms' demand for money. Our purpose, specifically, is to extend the results obtained in Propositions 1-3 under the assumption of flexible prices to the more general case of sticky prices. The following Propositions hold.

Proposition 5 *Suppose that prices are sticky ($\theta > 0$), preferences for consumption are logahritmic ($\sigma = 1$), and $1 + \beta < \lambda$. If $0 < \alpha < 1/3$, then determinacy of the REE obtains if and only if*

$$\phi_\pi > 1;$$

if $1/3 < \alpha < 1/2$, then determinacy of the REE obtains if and only if

$$1 < \phi_\pi < \frac{2\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(1+\varphi\right)\left(1-\alpha\right)}{\lambda\left(1+\varphi\right)\left(3\alpha-1\right)};$$

if

$$\frac{1}{2} < \alpha < \frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)},$$

then there is indeterminacy of the REE for any value of ϕ_π*; if*

$$\frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)} < \alpha < 1,$$

then determinacy of the REE obtains if and only if

$$\frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)} < \phi_\pi < 1.$$

Proof. See Appendix D. ■

Proposition 6 *Suppose that prices are sticky* ($\theta > 0$)*, preferences for consumption are logarithmic* ($\sigma = 1$) *and* $1+\beta > \lambda$*. If* $0 < \alpha < 1/3$*, then determinacy of the REE obtains if and only if*

$$\phi_\pi > 1;$$

if $1/3 < \alpha < 1/2$*, then determinacy of the REE obtains if and only if*

$$1 < \phi_\pi < \frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)};$$

if $1/2 < \alpha < 1$*, then there is indeterminacy of the REE for any value of* ϕ_π*.*

Proof. See Appendix E. ■

Proposition 7 *Suppose that prices are sticky* ($\theta > 0$)*, preferences for consumption are of the CRRA-type* ($\sigma \neq 1$)*, and* $1+\beta < \lambda$*. If*

$$0 < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)},$$

then determinacy of the REE obtains if and only if

$$\phi_\pi > 1;$$

if

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)} < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)},$$

then determinacy of the REE obtains if and only if

$$1 < \phi_\pi < \frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(\sigma+\varphi\right)\left(1-\alpha\right)}{\lambda\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}};$$

if

$$\frac{\left(\sigma+\varphi\right)}{\left(\sigma+\varphi\right)+\sigma\left(1+\varphi\right)} < \alpha < \frac{\sigma\left(1+\beta\right)+\lambda\left(\sigma+\varphi\right)}{\lambda\left(\sigma+\varphi\right)+\lambda\sigma\left(1+\varphi\right)-\sigma\varphi\left(1+\beta\right)},$$

then there is indeterminacy of the REE for any value of ϕ_π*; if*

$$\frac{\sigma\left(1+\beta\right)+\lambda\left(\sigma+\varphi\right)}{\lambda\left(\sigma+\varphi\right)+\lambda\sigma\left(1+\varphi\right)-\sigma\varphi\left(1+\beta\right)} < \alpha < 1,$$

then determinacy of the REE obtains if and only if

$$\frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(\sigma+\varphi\right)\left(1-\alpha\right)}{\lambda\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}} < \phi_\pi < 1.$$

Proof. See Appendix F. ■

Proposition 8 *Suppose that prices are sticky* ($\theta > 0$)*, preferences for consumption are of the CRRA-type* ($\sigma \neq 1$)*, and* $1+\beta > \lambda$*. If*

$$0 < \alpha < \frac{\left(\sigma+\varphi\right)}{\left(\sigma+\varphi\right)+2\sigma\left(1+\varphi\right)},$$

then determinacy of the REE obtains if and only if

$$\phi_\pi > 1;$$

if

$$\frac{\left(\sigma+\varphi\right)}{\left(\sigma+\varphi\right)+2\sigma\left(1+\varphi\right)} < \alpha < \frac{\left(\sigma+\varphi\right)}{\left(\sigma+\varphi\right)+\sigma\left(1+\varphi\right)},$$

then determinacy of the REE obtains if and only if

$$1 < \phi_\pi < \frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(\sigma+\varphi\right)\left(1-\alpha\right)}{\lambda\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}};$$

if

$$\frac{\left(\sigma+\varphi\right)}{\left(\sigma+\varphi\right)+\sigma\left(1+\varphi\right)} < \alpha < 1,$$

then there is indeterminacy of the REE for any value of ϕ_π*.*

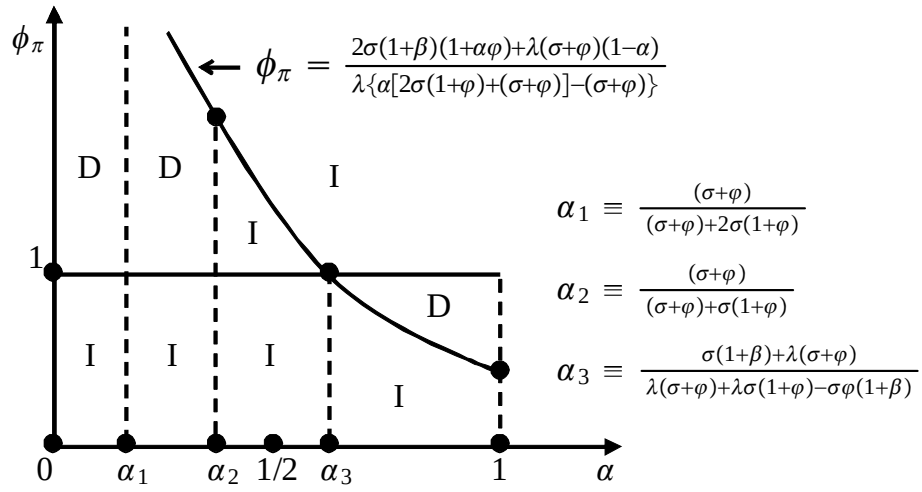

Figure 4: Regions of determinacy (D) and indeterminacy (I) under sticky prices ($\theta > 0$), CRRA preferences ($\sigma > 1$), and $1 + \beta < \lambda$.

Proof. See Appendix G. ■

Propositions 5-8 reveal that also under sticky prices, the standard finding that Taylor principle always ensures macroeconomic stability does not survive as soon as the monetary policy transmission mechanism is affected by firms' demand for money. There still exists, in particular, an hyperbolic frontier for the policy feedback parameter ϕ_π beyond which multiple equilibria take place.

However, there are two remarkable results that cause nominal rigidities to work in favor of the application of the Taylor principle. First, a passive monetary policy may be feasible only if $1 + \beta < \lambda$ (Figure 4);[3] on the other hand, if $1 + \beta > \lambda$, the Taylor principle is still necessary, though not sufficient, for determinacy (Figure 5). The value of λ, capturing the elasticity of inflation with respect to real marginal costs, is inversely related to the value of θ, capturing the degree of price stickyness. So the higher the degree of nominal rigidities, the lower the value of λ, the more likely an active monetary policy is a necessary, though not sufficient, condition to rule out sunspot fluctuations.

Second, consider the upper bound for ϕ_π, which in general is given by the hyperbolic

[3]It should be noted that in this case, there also exists an interval, given by

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \alpha < \frac{\sigma(1+\beta)+\lambda(\sigma+\varphi)}{\lambda(\sigma+\varphi)+\lambda\sigma(1+\varphi)-\sigma\varphi(1+\beta)},$$

in which indeterminacy pervails regardless of the value of ϕ_π, that is, both active and passive monetary policies are destabilizing.

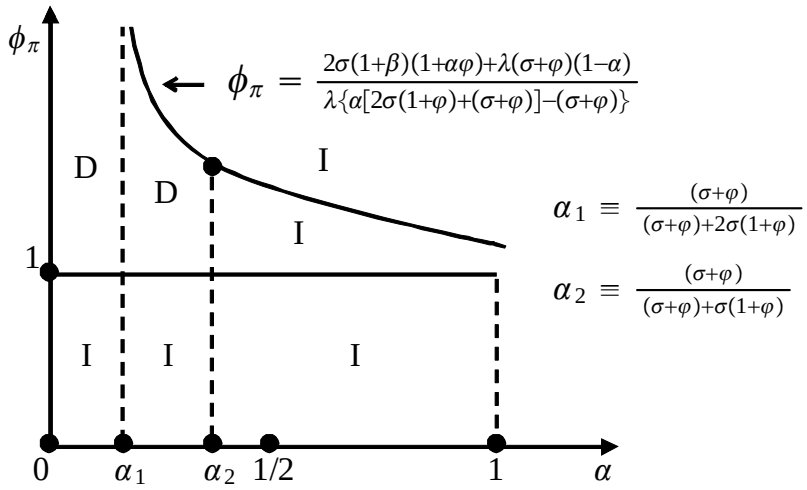

Figure 5: Regions of determinacy (D) and indeterminacy (I) under sticky prices ($\theta > 0$), CRRA preferences ($\sigma > 1$), and $1 + \beta > \lambda$.

frontier

$$\phi_\pi = \frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(\sigma+\varphi\right)\left(1-\alpha\right)}{\lambda\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}}. \tag{33}$$

From (33), it follows that $d\phi_\pi/d\lambda < 0$.[4] So the higher the degree of nominal rigidities, the lower the value of λ, the higher the upper bound that ϕ_π must satisfy.

The reason behind these results is as follows. From (17), a higher degree of price stickyness implies a decrease in the elasticity of inflation with respect to real marginal costs. From (25), it turns out that the supply-side, inflationary effect caused by an increase in the nominal interest rate becomes less pronounced. As a consequence, an active monetary policy is more likely to be stabilizing.

6 Calibration

To evaluate the theoretical results, we parameterize the model on a quarterly basis. To quantify the relevance of firms' money demand, we first employ a baseline calibration consistent with the standard New Keynesian literature. We then analyze the robustness of the results with respect to alternative parameter configurations.

[4] In fact, from (33) we have

$$\frac{d\phi_\pi}{d\lambda} = -\frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)}{\lambda^2\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}} < 0.$$

Baseline parameters		
Discount factor	β	0.99
CRRA coefficient	σ	5
Inverse of the (Frisch) labor supply elasticity	φ	0.5
Elasticity of substitution among differentiated goods	ε	11
Degree of price stickiness	θ	0.66
Steady-state firms' money balances to output ratio	M/PY	0.31
Implied parameters		
Steady-state price gross mark-up	$\varepsilon/(\varepsilon-1)$	1.1
Elasticity of inflation with respect to real marginal costs	λ	0.17
Elasticity of output with respect to real money balances	α	0.34

Table 1: Baseline calibration.

6.1 Baseline Calibration

The baseline parameter configuration is summarized in Table 1. As in Woodford (2003), we set the discount factor, β, equal to 0.99, which implies a steady-state annual real interest rate of about 4 percent. We set the coefficient of relative risk aversion, σ, equal to 5. This implies an intertemporal elasticity of substitution equal to 1/5, in line with the estimates in Hall (1988), Barsky, Juster, Kimball and Shapiro (1997), and Rotemberg and Woodford (1997, 1999). We set the the inverse of the Frisch elasticity, φ, equal to 1/2, in line with Rotemberg and Woodford (1997, 1999). We set the elasticity of substitution among differentiated goods, ε, equal to 11, as in Galí (2003). This implies a steady-state price mark-up of 10 percent. We calibrate the probability of keeping the price fixed between two consecutive periods, θ, to be 2/3, consistently with the estimates in Blinder, Canetti, Lebow and Rudd (1998), Rotemberg and Woodford (1997, 1999), and Sbordone (2002). The resulting elasticity of inflation with respect to real marginal costs, λ, is 0.17. Since we have $1+\beta > \lambda$, the regions of determinacy and indeterminacy depicted in Figure 5 apply, with $\alpha_1 = 0.27$, $\alpha_2 = 0.42$, and the upper bound for the monetary policy response

to inflation given by the function

$$\phi_{\pi}(\alpha) = \frac{20.84 + 9.01\alpha}{3.49\alpha - 0.94}. \tag{34}$$

Equation (6) enables us to calibrate the elasticity of output with respect to real money balances, α. In the steady state, real marginal cost equals the inverse of the price gross mark-up, $(\varepsilon - 1)/\varepsilon$. Therefore, equation (6) implies that

$$\alpha = \frac{1}{\beta}\left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{M}{PY}\right). \tag{35}$$

We use U.S. quarterly data to calibrate M/PY. The data are taken from the Federal Reserve Bank of St. Louis Database.[5] From 1959Q1 to 1981Q3, the M1 to nominal GDP ratio shows a clear downward trend, moving from 1.12 to 0.54. Therefore, we rule out this period to compute the steady state of the ratio. From 1981Q4 to 2010Q4, the ratio shows instead a relatively stationary dynamics, moving from 0.54 to 0.49, with a sample average equal to 0.52. As documented by Mulligan (1997) and Bover and Watson (2005), on average U.S. firms hold approximately 3/5 of the monetary aggregate M1. This implies a ratio of firms' money balances to output equal to 0.31. From (35), it thus follows $\alpha = 0.34$, so that $\alpha_1 < \alpha < \alpha_2$. From (34), the upper bound for the monetary policy feedback parameter is $\phi_{\pi} = 96.93$.

We are thus led to conclude that when real balances facilitate the firms' production process, active interest-rate policies may well, in theory, cause sunspot fluctuations; and conversely, passive interest-rate policies may well be compatible with equilibrium uniqueness and stability; however, calibration analysis lends support to the view that quantitatively active interest-rate policies still are necessary for equilibrium determinacy.

6.2 Robustness

Three critical parameters must be evaluated in order to check the robustness of the above numerical findings: the degree of nominal rigidities θ, the relative risk aversion coefficient σ, and the inverse of the Frisch elasticity of labor supply φ.

The degree of nominal rigidities θ crucially influences the restriction $1 + \beta > \lambda$, which rules out passive monetary policies according to Propositions 6 and 8. For $\beta = 0.99$, the threshold value of θ below which $1 + \beta < \lambda$ is 0.27. This value is arguably low if compared

[5] http://research.stlouisfed.org/fred.

with standard structural estimates of the New Keynesian Phillips curve (e.g., Galí and Gertler, 1999).

The relative risk aversion coefficient σ crucially affects the restriction $\alpha < \alpha_2$, which rules out passive monetary policies under both flexible and sticky prices according to Propositions 3, 7, and 8. In particular, we have demonstrated that $\partial\alpha_2/\partial\sigma < 0$. Estimates of σ suggests a value in the range 3-10 (e.g., Hall, 1988; Barsky, Juster, Kimball and Shapiro, 1997; Rotemberg and Woodford, 1997). However, even setting $\sigma = 10$ leads to $\alpha_2 = 0.41 > \alpha$.

The inverse of the Frisch elasticity of labor supply φ also affects the restriction $\alpha < \alpha_2$. In particular, when $\sigma > (<)\ 1$ we have $\partial\alpha_2/\partial\varphi < (>)\ 0$.[6] The New Keynesian literature suggests a value in the range 0.5-1 (e.g., Woodford, 2003; Galí, 2008) consistently with the business cycle literature (e.g., Cooley and Prescott, 1995). However, even setting $\varphi = 1$ yields $\alpha_2 = 0.38 > \alpha$.

Finally, the three parameters θ, σ, and φ influence the upper bound for the monetary policy feedback response to inflation, given by the frontier (33), beyond which an active monetary-policy stance brings about indeterminacy. Within the empirically plausible set of parameters $\theta \in [0.5, 1]$, $\sigma \in [1, 10]$, and $\varphi \in [0.1, 5]$, however, the upper bound is no less than 3.7. So indeterminacy only arises for overly aggressive monetary policies.

7 Conclusions

As emphasized by McCallum (2008), "there is hardly any issue of a more fundamental nature, with regard to monetary policy analysis, than whether such analysis can coherently be conducted in models that make no explicit reference whatsoever to any monetary aggregate".

The theoretical status of research on monetary policy is in favor of "cashless" models. The main reason is twofold. First, the central bank is assumed to adopt the nominal interest rate as operating instrument rather than money supply; so the money market equlibrium condition is endogenous and, under a standard separable households' utility function with real balances as one of the arguments, does not affect inflation and output determination. Second, even potential cross-derivative terms resulting from a

[6] In fact, we have

$$\frac{\partial\alpha_2}{\partial\varphi} = \frac{1-\sigma}{[\sigma\,(1+\varphi)]}.$$

non-separable utility function are found to be quantitatively negligible.

The foregoing arguments make no reference to the role played by firms' demand for money. In the U.S., however, firms hold approximately 60 percent of the monetary aggregate M1.

Within the New Keynesian literature, much has been said about households' money demand following the money-in-the-utility-function approach, but very little has been investigated about firms' money demand following the money-in-the-production-function approach. Several empirical studies, nevertheless, indicate that it is theoretically appropriate to incorporate the real money balances variable as a factor input in a production function, in order to capture the productivity gains derived from using money.

Along these lines, the subject of this paper is to evaluate the implications of firms' money demand for the design of interest rate rules within a canonical discrete-time New Keynesian framework. The paper's main results can be summarized as follows. (*i*) The existence of a unique stable rational expectations equilibrium may occur under either an active interest-rate policy (i.e., overreacting to inflation) or a passive interest-rate policy (i.e., underreacting to inflation), depending on the elasticity of production with respect to real money balances; thus, the Taylor principle, stressed in the New Keynesian literature as a prescription for dynamic uniqueness and stability, relies on the strict assumption of a "cashless" macroeconomic framework; and it vanishes as a necessary and sufficient condition to rule out multiple equilibria as soon as the monetary policy transmission mechanism is influenced by firms' demand for money. (*ii*) Quantitatively, however, calibration analysis using U.S. quarterly data supports the view that macroeconomic stability is more likely to be guaranteed when an active, although not overly aggressive, monetary policy is implemented by the central bank.

Appendix A: Proof of Proposition 2

When $\sigma = 1$, (21) and (22) imply that the flexible-price REE is determinate if and only if

$$\left| \frac{(1-2\alpha)\,\phi_\pi}{(1-\alpha)-\alpha\phi_\pi} \right| > 1. \tag{A.1}$$

When *either* $\phi_\pi < (1-\alpha)/\alpha$ and $0 < \alpha < 1/2$, *or* $\phi_\pi > (1-\alpha)/\alpha$ and $1/2 < \alpha < 1$, condition (A.1) holds if (Case I)

$$\frac{(1-2\alpha)\,\phi_\pi}{(1-\alpha)-\alpha\phi_\pi} > 1. \tag{A.2}$$

When *either* $\phi_\pi > (1-\alpha)/\alpha$ and $0 < \alpha < 1/2$, *or* $\phi_\pi < (1-\alpha)/\alpha$ and $1/2 < \alpha < 1$, condition (A.1) holds if (Case II)

$$\frac{(1-2\alpha)\,\phi_\pi}{(1-\alpha)-\alpha\phi_\pi} < -1. \tag{A.3}$$

First, when $0 < \alpha < 1/3$, Case I yields

$$\phi_\pi > 1, \tag{A.4}$$

while Case II yields

$$\phi_\pi > -\frac{(1-\alpha)}{(1-3\alpha)}. \tag{A.5}$$

Under the sign restriction $\phi_\pi > 0$, condition (A.5) is necessarily satisfied. This proves that for $0 < \alpha < 1/3$, determinacy obtains if and only if (A.4) applies, i.e., if the Taylor principle is verified.

Second, when $1/3 < \alpha < 1/2$, Case I again yields (A.4), while Case II yields

$$\phi_\pi < \frac{1-\alpha}{3\alpha-1}. \tag{A.6}$$

Condition (A.6) now implies the existence of an *upper* bound for the monetary policy feedback parameter ϕ_π, because when $1/3 < \alpha < 1/2$, we have that $(1-\alpha)/(3\alpha-1) > (1-\alpha)/\alpha > 1$. This proves that for $1/3 < \alpha < 1/2$, determinacy obtains if and only if

$$1 < \phi_\pi < \frac{1-\alpha}{3\alpha-1}. \tag{A.7}$$

Third, when $1/2 < \alpha < 1$, Case I yields

$$\phi_\pi < 1, \tag{A.8}$$

while Case II yields

$$\phi_\pi > \frac{1-\alpha}{3\alpha-1}. \tag{A.9}$$

Condition (A.8) now implies the *violation* of the Taylor principle, while condition (A.9) implies the existence of a *lower* bound for the monetary policy feedback parameter ϕ_π, because when $1/2 < \alpha < 1$, we have that $0 < (1-\alpha)/(3\alpha-1) < (1-\alpha)/\alpha < 1$. This proves that for $1/2 < \alpha < 1$, determinacy obtains if and only if

$$\frac{1-\alpha}{3\alpha-1} < \phi_\pi < 1. \tag{A.10}$$

Appendix B: Proof of Proposition 3

In the general case in which $\sigma \neq 1$, (21) and (22) imply that the flexible-price REE is determinate if and only if

$$\left| \frac{[(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)]\phi_\pi}{(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)\phi_\pi} \right| > 1. \tag{B.1}$$

When *either*

$$\phi_\pi < \frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)} \quad \text{and} \quad 0 < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)},$$

or

$$\phi_\pi > \frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)} \quad \text{and} \quad \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \alpha < 1,$$

condition (B.1) holds if (Case I)

$$\frac{[(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)]\phi_\pi}{(1-\alpha)(\sigma+\varphi) - \alpha\sigma(1+\varphi)\phi_\pi} > 1. \tag{B.2}$$

When *either*

$$\phi_\pi > \frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)} \quad \text{and} \quad 0 < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)},$$

or

$$\phi_\pi < \frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)} \quad \text{and} \quad \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \alpha < 1,$$

condition (B.1) holds if (Case II)

$$\frac{[(1-\alpha)(\sigma+\varphi)-\alpha\sigma(1+\varphi)]\phi_\pi}{(1-\alpha)(\sigma+\varphi)-\alpha\sigma(1+\varphi)\phi_\pi} < -1. \tag{B.3}$$

First, when

$$0 < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)}, \tag{B.4}$$

Case I yields

$$\phi_\pi > 1, \tag{B.5}$$

while Case II yields

$$\phi_\pi > -\frac{(1-\alpha)(\sigma+\varphi)}{(1-\alpha)(\sigma+\varphi)-2\alpha\sigma(1+\varphi)}. \tag{B.6}$$

Under the sign restriction $\phi_\pi > 0$, condition (B.6) is necessarily satisfied. This proves that in the interval (B.4), determinacy obtains if and only if (B.5) applies, i.e., if the Taylor principle is verified.

Second, when

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)} < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)}, \tag{B.7}$$

Case I again yields (B.5), while Case II yields

$$\phi_\pi < \frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)}. \tag{B.8}$$

Condition (B.8) constitutes an *upper* bound for the monetary policy feedback parameter ϕ_π, because in the interval (B.7), we have that

$$\frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)} > \frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)} > 1.$$

This proves that in the interval (B.7), determinacy obtains if and only if

$$1 < \phi_\pi < \frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)}. \tag{B.9}$$

Third, when

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)}<\alpha<1, \tag{B.10}$$

Case I yields

$$\phi_\pi<1, \tag{B.11}$$

while Case II yields

$$\phi_\pi>\frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)}. \tag{B.12}$$

Condition (B.11) implies the *violation* of the Taylor principle, while condition (B.12) constitues a *lower* bound for the monetary policy feedback parameter ϕ_π, because in the interval (B.10), we have that

$$0<\frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)}<\frac{(1-\alpha)(\sigma+\varphi)}{\alpha\sigma(1+\varphi)}<1.$$

This proves that in the interval (B.10), determinacy obtains if and only if

$$\frac{(1-\alpha)(\sigma+\varphi)}{2\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)}<\phi_\pi<1. \tag{B.13}$$

Appendix C: Proof of Proposition 4

In the case in which prices are sticky ($\theta>0$) and the economy is cashless ($\alpha=0$), conditions (28)-(32) give the following constraints for the monetary-policy feedback parameter ϕ_π:

(Case I)

$$\phi_\pi>-\frac{\sigma(1-\beta)}{\lambda(\sigma+\varphi)}, \tag{C.1}$$

$$\phi_\pi>1, \tag{C.2}$$

$$\phi_\pi>-\frac{2\sigma\beta+2\sigma+\lambda(\sigma+\varphi)}{\lambda(\sigma+\varphi)}, \tag{C.3}$$

or (Case II)

$$\phi_\pi<1, \tag{C.4}$$

$$\phi_\pi<-\frac{2\sigma\beta+2\sigma+\lambda(\sigma+\varphi)}{\lambda(\sigma+\varphi)}. \tag{C.5}$$

Under the sign restriction $\phi_\pi > 0$, condition (C.5) is necessarily violated, which implies that Case II is never validated. Case I, on the other hand, is always satisfied for $\phi_\pi > 1$. This proves Proposition 4.

Appendix D: Proof of Proposition 5

Consider the case in which prices are sticky ($\theta > 0$), preferences for consumption are logarithmic ($\sigma = 1$), and $1 + \beta < \lambda$.

Consider Case I. Condition (29) yields

$$\phi_\pi > 1. \tag{D.1}$$

Condition (30) yields

$$\lambda (1 + \varphi)(1 - 3\alpha)\phi_\pi > -2(1 + \beta)(1 + \alpha\varphi) - \lambda(1 + \varphi)(1 - \alpha). \tag{D.2}$$

When $0 < \alpha < 1/3$, the inequality in (D.2) implies

$$\phi_\pi > -\frac{2(1 + \beta)(1 + \alpha\varphi) + \lambda(1 + \varphi)(1 - \alpha)}{\lambda(1 + \varphi)(1 - 3\alpha)}. \tag{D.3}$$

Under the sign restriction $\phi_\pi > 0$, condition (D.3) is necessarily satisfied. Condition (28) yields

$$\phi_\pi > -\frac{(1 - \beta)(1 + \alpha\varphi)}{\lambda(1 + \varphi)(1 - 2\alpha)}, \tag{D.4}$$

that is also always satisfied when $0 < \alpha < 1/3$. At the same time, when $0 < \alpha < 1/3$, condition (32) is necessarily violated, so that Case II is not possible. This proves that in the interval $0 < \alpha < 1/3$, determinacy obtains if and only if (D.1) applies, i.e., if the Taylor principle is verified.

When $1/3 < \alpha < 1/2$, the inequality in (D.2) yields

$$\phi_\pi < \frac{2(1 + \beta)(1 + \alpha\varphi) + \lambda(1 + \varphi)(1 - \alpha)}{\lambda(1 + \varphi)(3\alpha - 1)}. \tag{D.5}$$

Note that in the case in which $1 + \beta < \lambda$, the frontier

$$\phi_\pi = \frac{2(1 + \beta)(1 + \alpha\varphi) + \lambda(1 + \varphi)(1 - \alpha)}{\lambda(1 + \varphi)(3\alpha - 1)} \tag{D.6}$$

is above unity as long as

$$\alpha < \frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)}, \tag{D.7}$$

where

$$\frac{1}{2} < \frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)} < 1. \tag{D.8}$$

This proves that for $1/3 < \alpha < 1/2$, determinacy obtains if and only if

$$1 < \phi_\pi < \frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)}. \tag{D.9}$$

When

$$\frac{1}{2} < \alpha < \frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)}, \tag{D.10}$$

condition (28) yields

$$\phi_\pi < -\frac{(1-\beta)(1+\alpha\varphi)}{\lambda(1+\varphi)(2\alpha-1)}, \tag{D.11}$$

which is never satisfied. At the same time, in the interval (D.10), Case II yields is eliminated by conditions (31)-(32). This proves that in the interval (D.10), inderminacy prevails.

Instead, when

$$\frac{(1+\beta)+\lambda(1+\varphi)}{2\lambda(1+\varphi)-\varphi(1+\beta)} < \alpha < 1, \tag{D.12}$$

Case I is ruled out by condition (28). For Case II, condition (31) yields

$$\phi_\pi < 1, \tag{D.13}$$

and condition (32) yields

$$\phi_\pi > \frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)}. \tag{D.14}$$

This proves that in the interval (D.12), determinacy obtains if and only if

$$\frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)} < \phi_\pi < 1. \tag{D.15}$$

Appendix E: Proof of Proposition 6

Consider the case in which prices are sticky ($\theta > 0$), preferences for consumption are logarithmic ($\sigma = 1$), and $1 + \beta > \lambda$.

Consider Case I. Condition (29) yields

$$\phi_\pi > 1. \tag{E.1}$$

Condition (30) yields

$$\lambda (1 + \varphi) (1 - 3\alpha) \phi_\pi > -2 (1 + \beta) (1 + \alpha\varphi) - \lambda (1 + \varphi) (1 - \alpha). \tag{E.2}$$

When $0 < \alpha < 1/3$, the inequality in (E.2) implies

$$\phi_\pi > -\frac{2 (1 + \beta) (1 + \alpha\varphi) + \lambda (1 + \varphi) (1 - \alpha)}{\lambda (1 + \varphi) (1 - 3\alpha)}. \tag{E.3}$$

Under the sign restriction $\phi_\pi > 0$, condition (E.3) is necessarily satisfied. Condition (28) yields

$$\phi_\pi > -\frac{(1 - \beta) (1 + \alpha\varphi)}{\lambda (1 + \varphi) (1 - 2\alpha)}, \tag{E.4}$$

that is also always satisfied when $0 < \alpha < 1/3$. At the same time, when $0 < \alpha < 1/3$, condition (32) is necessarily violated, so that Case II is not possible. This proves that in the interval $0 < \alpha < 1/3$, determinacy obtains if and only if (E.1) applies, i.e., if the Taylor principle is verified.

When $1/3 < \alpha < 1/2$, the inequality in (D.2) yields

$$\phi_\pi < \frac{2 (1 + \beta) (1 + \alpha\varphi) + \lambda (1 + \varphi) (1 - \alpha)}{\lambda (1 + \varphi) (3\alpha - 1)}. \tag{E.5}$$

Note that in the case in which $1 + \beta > \lambda$ the frontier

$$\phi_\pi = \frac{2 (1 + \beta) (1 + \alpha\varphi) + \lambda (1 + \varphi) (1 - \alpha)}{\lambda (1 + \varphi) (3\alpha - 1)} \tag{E.6}$$

is always above unity. This proves that for $1/3 < \alpha < 1/2$, determinacy obtains if and only if

$$1 < \phi_\pi < \frac{2 (1 + \beta) (1 + \alpha\varphi) + \lambda (1 + \varphi) (1 - \alpha)}{\lambda (1 + \varphi) (3\alpha - 1)}. \tag{E.7}$$

When $1/2 < \alpha < 1$, condition (28) yields

$$\phi_\pi < -\frac{(1-\beta)(1+\alpha\varphi)}{\lambda(1+\varphi)(2\alpha-1)}, \tag{E.8}$$

which is never satisfied. For Case II, condition (31) yields

$$\phi_\pi < 1, \tag{E.9}$$

while condition (32) yields

$$\phi_\pi > \frac{2(1+\beta)(1+\alpha\varphi)+\lambda(1+\varphi)(1-\alpha)}{\lambda(1+\varphi)(3\alpha-1)}. \tag{E.10}$$

Because the frontier (E.6) is above unity, conditions (E.9)-(E.10) are not compatible. This proves that for $1/2 < \alpha < 1$, indeterminacy applies.

Appendix F: Proof of Proposition 7

Consider the case in which prices are sticky ($\theta > 0$), preferences for consumption are of the CRRA-type ($\sigma \neq 1$), and $1 + \beta < \lambda$.

Consider Case I. Condition (29) yields

$$\phi_\pi > 1. \tag{F.1}$$

Condition (30) yields

$$\lambda\{(\sigma+\varphi)-\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]\}\phi_\pi > -2\sigma(1+\beta)(1+\alpha\varphi)-\lambda(\sigma+\varphi)(1-\alpha). \tag{F.2}$$

When

$$0 < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)}, \tag{F.3}$$

the inequality in (F.2) implies

$$\phi_\pi > -\frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{(\sigma+\varphi)-\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]\}}. \tag{F.4}$$

Under the sign restriction $\phi_\pi > 0$, condition (F.4) is necessarily satisfied. Condition (28) yields

$$\phi_\pi > -\frac{\sigma(1-\beta)(1+\alpha\varphi)}{\lambda[(1-\alpha)(\sigma+\varphi)-\alpha\sigma(1+\varphi)]}, \tag{F.5}$$

that is also always satisfied in the interval (F.3). At the same time, in the interval (F.3), condition (32) is necessarily violated, so that Case II is not possible. This proves that in the interval (F.3), determinacy obtains if and only if (F.1) applies, i.e., if the Taylor principle is verified.

When

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+2\sigma(1+\varphi)} < \alpha < \frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)}, \tag{F.6}$$

the inequality in (F.2) yields

$$\phi_\pi < \frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]-(\sigma+\varphi)\}}. \tag{F.7}$$

Note that in the case in which $1+\beta < \lambda$, the frontier

$$\phi_\pi = \frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]-(\sigma+\varphi)\}} \tag{F.8}$$

is above unity as long as

$$\alpha < \frac{\sigma(1+\beta)+\lambda(\sigma+\varphi)}{\lambda[(\sigma+\varphi)+\sigma(1+\varphi)]-\varphi\sigma(1+\beta)}, \tag{F.9}$$

where

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \frac{\sigma(1+\beta)+\lambda(\sigma+\varphi)}{\lambda[(\sigma+\varphi)+\sigma(1+\varphi)]-\varphi\sigma(1+\beta)} < 1. \tag{F.10}$$

This proves that in the interval (F.6), determinacy obtains if and only if

$$1 < \phi_\pi < \frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]-(\sigma+\varphi)\}}. \tag{F.11}$$

When

$$\frac{(\sigma+\varphi)}{(\sigma+\varphi)+\sigma(1+\varphi)} < \alpha < \frac{\sigma(1+\beta)+\lambda(\sigma+\varphi)}{\lambda[(\sigma+\varphi)+\sigma(1+\varphi)]-\varphi\sigma(1+\beta)}, \tag{F.12}$$

condition (28) yields

$$\phi_\pi < -\frac{\sigma(1-\beta)(1+\alpha\varphi)}{\lambda[\alpha\sigma(1+\varphi)-(1-\alpha)(\sigma+\varphi)]}, \tag{F.13}$$

which is never satisfied. At the same time, in the interval (F.11), Case II is eliminated by conditions (31)-(32). This proves that in the interval (F.11), indeterminacy prevails.

Instead, when

$$\frac{\sigma(1+\beta)+\lambda(\sigma+\varphi)}{\lambda[(\sigma+\varphi)+\sigma(1+\varphi)]-\varphi\sigma(1+\beta)} < \alpha < 1, \tag{F.14}$$

Case I is ruled out by condition (28). For Case II, condition (31) yields

$$\phi_\pi < 1, \tag{F.15}$$

and condition (32) yields

$$\phi_\pi > \frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]-(\sigma+\varphi)\}}. \tag{F.16}$$

This proves that in the interval (F.14), determinacy obtains if and only if

$$\frac{2\sigma(1+\beta)(1+\alpha\varphi)+\lambda(\sigma+\varphi)(1-\alpha)}{\lambda\{\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]-(\sigma+\varphi)\}} < \phi_\pi < 1. \tag{F.17}$$

Appendix G: Proof of Proposition 8

Consider the case in which prices are sticky ($\theta > 0$), preferences for consumption are of the CRRA-type ($\sigma \neq 1$), and $1+\beta > \lambda$.

Consider Case I. Condition (29) yields

$$\phi_\pi > 1. \tag{G.1}$$

Condition (30) yields

$$\lambda\{(\sigma+\varphi)-\alpha[2\sigma(1+\varphi)+(\sigma+\varphi)]\}\phi_\pi > -2\sigma(1+\beta)(1+\alpha\varphi)-\lambda(\sigma+\varphi)(1-\alpha). \tag{G.2}$$

When

$$0 < \alpha < \frac{(\sigma + \varphi)}{(\sigma + \varphi) + 2\sigma (1 + \varphi)}, \tag{G.3}$$

the inequality in (G.2) implies

$$\phi_\pi > -\frac{2\sigma (1 + \beta)(1 + \alpha\varphi) + \lambda (\sigma + \varphi)(1 - \alpha)}{\lambda \{(\sigma + \varphi) - \alpha [2\sigma (1 + \varphi) + (\sigma + \varphi)]\}}. \tag{G.4}$$

Under the sign restriction $\phi_\pi > 0$, condition (G.4) is necessarily satisfied. Condition (28) yields

$$\phi_\pi > -\frac{\sigma (1 - \beta)(1 + \alpha\varphi)}{\lambda [(1 - \alpha)(\sigma + \varphi) - \alpha\sigma (1 + \varphi)]}, \tag{G.5}$$

that is also always satisfied in the interval (G.3). At the same time, in the interval (G.3), condition (32) is necessarily violated, so that Case II is not possible. This proves that in the interval (G.3), determinacy obtains if and only if (G.1) applies, i.e., if the Taylor principle is verified.

When

$$\frac{(\sigma + \varphi)}{(\sigma + \varphi) + 2\sigma (1 + \varphi)} < \alpha < \frac{(\sigma + \varphi)}{(\sigma + \varphi) + \sigma (1 + \varphi)}, \tag{G.6}$$

the inequality in (G.2) yields

$$\phi_\pi < \frac{2\sigma (1 + \beta)(1 + \alpha\varphi) + \lambda (\sigma + \varphi)(1 - \alpha)}{\lambda \{\alpha [2\sigma (1 + \varphi) + (\sigma + \varphi)] - (\sigma + \varphi)\}}. \tag{G.7}$$

Note that in the case in which $1 + \beta > \lambda$ the frontier

$$\phi_\pi = \frac{2\sigma (1 + \beta)(1 + \alpha\varphi) + \lambda (\sigma + \varphi)(1 - \alpha)}{\lambda \{\alpha [2\sigma (1 + \varphi) + (\sigma + \varphi)] - (\sigma + \varphi)\}} \tag{G.8}$$

is always above unity. This proves that in the interval (G.6), determinacy of obtains if and only if

$$1 < \phi_\pi < \frac{2\sigma (1 + \beta)(1 + \alpha\varphi) + \lambda (\sigma + \varphi)(1 - \alpha)}{\lambda \{\alpha [2\sigma (1 + \varphi) + (\sigma + \varphi)] - (\sigma + \varphi)\}}. \tag{G.9}$$

When

$$\frac{(\sigma + \varphi)}{(\sigma + \varphi) + \sigma (1 + \varphi)} < \alpha < 1, \tag{G.10}$$

condition (28) yields

$$\phi_\pi < -\frac{\sigma (1 - \beta)(1 + \alpha\varphi)}{\lambda [\alpha\sigma (1 + \varphi) - (1 - \alpha)(\sigma + \varphi)]}, \tag{G.11}$$

which is never satisfied. For Case II, condition (31) yields

$$\phi_\pi < 1, \tag{G.12}$$

while condition (32) yields

$$\phi_\pi > \frac{2\sigma\left(1+\beta\right)\left(1+\alpha\varphi\right)+\lambda\left(\sigma+\varphi\right)\left(1-\alpha\right)}{\lambda\left\{\alpha\left[2\sigma\left(1+\varphi\right)+\left(\sigma+\varphi\right)\right]-\left(\sigma+\varphi\right)\right\}}. \tag{G.13}$$

Because the frontier (G.8) is above unity, conditions (G.12)-(G.13) are not compatible. This proves that in the interval (G.10), indeterminacy applies.

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