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Re-Interpreting Sub-Group Inequality Decompositions

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Abstract

We propose a modification to the conventional approach of decomposing income inequality by population sub-groups. Specifically, we propose a measure that evaluates observed between-group inequality against a benchmark of maximum between-group inequality that can be attained when the number and relative sizes of groups under examination are fixed. We argue that such a modification can provide a complementary perspective on the question of whether a particular population breakdown is salient to an assessment of inequality in a country. As our measure normalizes between-group inequality by the number and relative sizes of groups, it is also less subject to problems of comparability across different settings. We show that for a large set of countries our assessment of the importance of group differences typically increases substantially on the basis of this approach. The ranking of countries (or different population groups) can also differ from that obtained using traditional decomposition methods. Finally, we observe an interesting pattern of higher levels of overall inequality in countries where our measure finds higher between-group contributions.

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I. Introduction

The significance of group differences in wellbeing is often at the center of the study of inequality. Roemer (1998) suggests that inequality of opportunity occurs when the ability of people to pursue lives of their own choosing depends on predetermined characteristics, such as gender, race, social group, or family background.[2] This perspective implies that it can be instructive to disentangle inequalities due to differences between groups, defined in terms of such predetermined characteristics, from those due to, say, individual differences in effort, talent, or luck. Given two countries with the same overall income inequality, one might worry more about social stability and prospects for inclusive long-term prosperity in the country with higher inequality between groups.

Statistical methods are often used to 'decompose' economic inequality into constituent parts. Sub-group decomposable measures of inequality can be written as the sum of inequality that is attributable to differences in mean outcomes across population sub-groups and that which is due to inequality *within* those sub-groups.[3] Many have used such decompositions to 'understand' economic inequality and guide the design of economic policy. Indeed, Cowell (2000) argues: "It is almost essential to attempt to 'account for' the level of, or trend in, inequality by components of the population."

Although decompositions of inequality, as described above, have long been the workhorse in this literature, empirical implementation has tended to find little evidence of significant between group differences. For example, in a classic reference, Anand (1983)

[2] World Development Report 2006, entitled "Equity and Development," adopts a notion of equity that combines the concept of equality of opportunities with the avoidance of absolute deprivation – a Rawlsian form of inequality aversion in the space of outcomes.

[3] See Bourguignon (1979), Shorrocks (1980, 1984) and Cowell (1980). Cowell (2000) provides a recent survey of methods of inequality measurement, including a discussion of the various approaches to sub-group decomposition.

showed that inequality between ethnic groups in Malaysia accounted for only 15% of total inequality in the 1970s. This led to his recommendation that government strategy should focus on inequality within ethnic groups rather than that between them. Cowell and Jenkins (1995), who find that most income inequality remains unexplained even after taking into account the age, sex, race and earner status of the household head in the U.S., argue that the real story of inequality is to be found within these population groups and point to the importance of chance.

Not everyone is comfortable with such interpretations, however. Kanbur (2000) states that the use of such decompositions "…assists the easy slide into a neglect of inter-group inequality in the current literature." He argues that finding a relatively small share of inequality between groups does not mean that the mean differences between them are less important than inequalities within such groupings. In particular, he argues that social stability and racial harmony can break down once the average differences between groups go beyond a certain threshold, with the threshold varying from country to country.[4]

There are also difficulties with comparisons of such decompositions across settings (e.g among countries or over time). This is because underlying population structures often vary. Consider three countries where the issue of racial differences in income features prominently in public discourse: the United States, Brazil and South Africa. The shares of income inequality attributable to differences between racial groups

[4] Foster and Sen (1997) point to the 'separatist' view implicit in these sub-group consistent measures, which they claim ignores potentially relevant information when making inequality comparisons. For example, should a change in inequality within a certain group (while the means and population shares remain unchanged) when that group is richer than a second group affect inequality in exactly the same manner as in the presence of a much wealthier second group? Sub-group consistency requires this to be true. Kanbur (2000) builds on this argument and suggests that invoking such separatist axioms "…go[es] against basic intuition and considerable evidence which suggest that individuals do indeed pay special attention to outcomes for their particular racial, ethnic, or regional group."

in these countries are 8%, 16%, and 38%, respectively.[5] Do these numbers provide a good yardstick with which to judge the relevance of race to an understanding of inequality in these countries? Should South African and Brazilian policy-makers worry much more about racial differences in incomes than do their American counterparts? Does the small percentage of income inequality attributable to race in the U.S. mean that racial inequality is not a pertinent economic and social issue?

Conventionally, between-group inequality is calculated as a function of two arguments: differences among groups in mean incomes and the relative size and number of the groups. The figures above are based on four population groups for Brazil, three for South Africa, and five for the U.S., but the population shares of the white groups versus non-white groups differ tremendously.[6] In each country, the mean income of the non-white groups is much below that of the white group, but the non-white groups form the majority in South Africa (80%), half of the population in Brazil (50%), and a minority in the U.S. (28%). The difference in between-group inequality observed between these three countries could in fact be due largely to the difference in population shares of the racial groups instead of the differences in relative mean incomes of these groups.[7]

The conventional between-group share is calculated by taking the ratio of observed between-group inequality to total inequality. Total inequality, however, can be viewed as the between-group inequality that would be observed if every household in the

[5] These figures have been calculated by the authors using data from PNAD (2001) for Brazil, IES(2000) for South Africa, and LIS(2000) for the U.S.

[6] The racial groups used in our analysis are "White", "Black", "Pardo", and "other" in Brazil, "White", "African", and "other" (combining Coloreds, Asians/Indians, and others) in South Africa, and "White", "Black", "Hispanic", "Asian", and "American Indian" in the U.S.

[7] The observed differences in between-group inequality may also depend on the number of groups under consideration, making the specific definition of groups a non-negligible issue. For example, the share of between-group inequality attributable to caste in India when one groups people simply into "high", 'medium", or "low" caste groupings, can be quite different from that which emerges when the partitions are *finer*, i.e. when one makes distinctions between castes within each broad category.

population constituted a separate group. Thus, the conventional practice is equivalent to comparing observed between-group inequality (across a few groups) against a benchmark (across perhaps millions of groups) that is quite extreme. It is not surprising that one rarely observes a high share of between-group inequality.[8] In this paper, we propose an alternative measure to assess between-group inequality. Specifically, we suggest replacing total inequality in the denominator of the conventional ratio with the maximum between-group inequality that could be obtained if the number of groups and their sizes were restricted to be the same as for the numerator. Because our proposed measure normalizes by the number of groups and their relative sizes in a country, decompositions can be better compared across settings where the number of groups (or the population shares for those groups) is very different.

We also argue that our measure is better suited to capture the salience of a specific population breakdown to the assessment of inequality of opportunities. As indicated earlier, inequality of opportunity is concerned with systematic differences among groups who differ only in skin color, caste, gender, etc. – predetermined characteristics that are arguably "morally irrelevant". Consider the following example (illustrated in figure 1). Imagine a country with two population groups of equal size: serfs and landlords. Mean income of the serfs is low while landlords enjoy a high mean income. In period 1, there is no variation in the incomes of individuals who belong to the same group, i.e. one's social group determines his or her income perfectly. In period 2, some random noise, ε_i, is added to each income. One can think of ε_i as pure luck or, alternatively, measurement error. Hence, in the second period, there is income inequality within each group, although the respective means are still the same as in period 1.

[8] See Anand (1983), Cowell and Jenkins (1995), Elbers et al. (2004).

Suppose that the resulting two distributions do not overlap. Has inequality of opportunity changed from period 1 to period 2?[9]

It is not clear why the presence of some random variation around these group means should be indicative of any change in underlying opportunities. After all, even the luckiest serf is still far poorer than the poorest landlord. However, if the question above were to be assessed on the basis of the between-group share that derives from traditional inequality decomposition, one would conclude that inequality of opportunity had fallen from period 1 to period 2. This is because within-group inequality in period 2 increased while between-group inequality remained unchanged (as mean incomes and population shares for the two groups were unchanged), implying that the share of between-group inequality fell between periods 1 and 2. As will be elaborated further in section 2, our proposed measure would remain unchanged – suggesting that inequality of opportunity did not fall between periods 1 and 2.[10]

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[9] Some readers will note that this example is somewhat similar to Example 1 in Esteban and Ray (2004), where their Figure 1b is analogous to period 1 in our example, while Figure 1a is akin to period 2. However, there is one important difference: the groups in Esteban and Ray are defined by incomes. Whether blacks and whites, or serfs and landlords fill the income distribution is of no consequence in their example. We are interested in income differences between groups defined by another characteristic, such as race, class, gender, parent's education, etc. In general, in an example such as this, the Esteban and Ray polarization index would register a decline in polarization as we move from period 1 to period 2. However, it is interesting to note that in the specific case of the two groups having identical sizes, the Esteban and Ray polarization index would in fact register an *increase* in polarization. For our purposes, the point to emphasize is that the polarization index, like between-group inequality, would not remain unchanged in moving from period 1 to period 2.

[10] Our alternative measure would also start declining in period 2 if the two income distributions started to overlap, but at a rate much slower than the traditional between-group inequality share (see section 2).

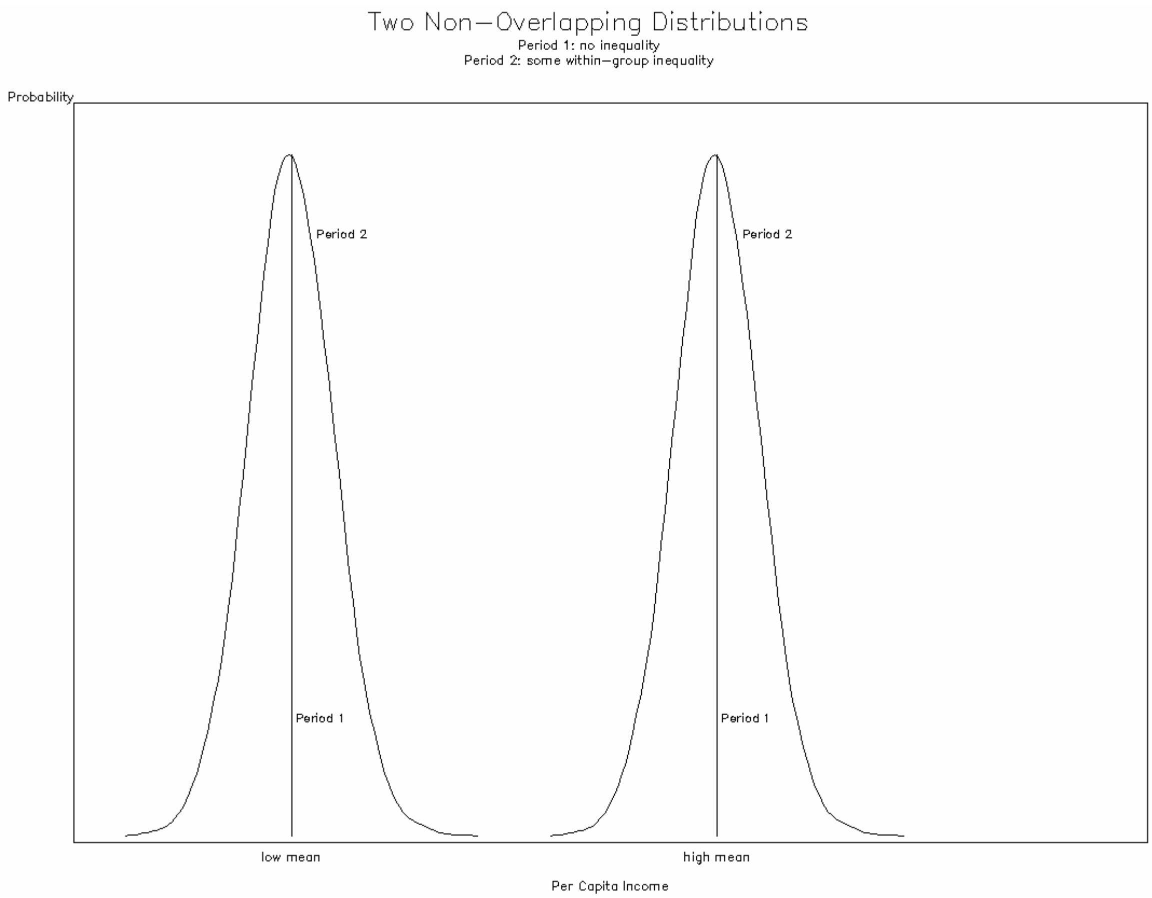


Figure 1: Inequality Between Serfs and Landlords

The rest of the paper is organized as follows. Section 2 briefly reviews the theoretical inequality decomposition literature and introduces our alternative approach. Section 3 draws on a newly compiled database of inequality and sub-group contributions for just under 100 developed and developing countries to demonstrate that qualitative assessments of the importance of between-group differences can, but need not, be markedly higher when based on our alternative approach. This section also discusses a thought-provoking finding of a strong positive correlation across countries between overall inequality and our proposed measure of inequality between groups. Section 4 concludes.

2. Methodology

The standard approach to decomposing inequality by population sub-groups breaks overall inequality into a between-group and a within-group component. The first component indicates how much of overall inequality would remain if incomes were equalized within each population group, i.e. each member of a particular group being given the group's average per capita income. The within-group component captures the amount of inequality that would remain if differences between groups in terms of their average incomes were eliminated and only within-group differences remained.

Not all summary measures of inequality can be neatly decomposed into these two components. The most commonly decomposed measures in the literature come from the General Entropy class. These take the following form:

$$GE = \frac{1}{c(c-1)} \sum_i f_i \left[\left(\frac{y_i}{\mu} \right)^c - 1 \right] \qquad \text{for } c \neq 0,1$$

$$= -\sum_i f_i \log\left(\frac{y_i}{\mu} \right) \qquad \text{for } c=0$$

$$= \sum_i f_i \left(\frac{y_i}{\mu} \right) \log\left(\frac{y_i}{\mu} \right) \qquad \text{for } c=1$$

where f_i is the population share of household i, y_i is per capita consumption of household i, μ is average per capita consumption, and c is a parameter that is to be selected by the user.[11] This class of inequality measures can be decomposed into a between and within-group component as follows:

[11] Lower values of *c* are associated with greater sensitivity to inequality amongst the poor, and higher values of c place more weight to inequality among the rich. A *c* value of 1 yields the well known Theil entropy measure, a value of 0 provides the Theil L or mean log deviation, and a value of 2 is ordinally equivalent to the squared coefficient of variation.

$$GE = \frac{1}{c(c-1)}\left[1 - \sum_j g_j \left(\frac{\mu_j}{\mu}\right)^c\right] + \sum_j GE_j g_j \left(\frac{\mu_j}{\mu}\right)^c \qquad \text{for } c \neq 0,1$$

$$= \left[\sum_j g_j \log\left(\frac{\mu}{\mu_j}\right)\right] + \sum_j GE_j g_j \qquad \text{for } c=0$$

$$= \left[\sum_j g_j \left(\frac{\mu_j}{\mu}\right) \log\left(\frac{\mu_j}{\mu}\right)\right] + \sum_j GE_j g_j \left(\frac{\mu_j}{\mu}\right) \qquad \text{for } c=1$$

where j refers to the sub-group, g_j refers to the population share of group j and GE_j refers to inequality in group j. The between-group component of inequality is captured by the first term: the level of inequality if everyone within each group j had consumption level μ_j. The second term gives within-group inequality.

Table 1 decomposes inequality on the basis of "social group" in eight countries. Social group is defined differently across countries, but refers loosely to the racial, ethnic, or caste breakdown that is relevant to each country. For example, the breakdown for the United States corresponds to five racial groups: Whites, Blacks, American Indians, Asians and Hispanics. In India the three groups comprise Scheduled Caste households, Scheduled Tribes, and Others. The number of groups and their respective sizes are clearly not the same in all countries. Inequality is estimated on the basis of per-capita consumption for each country and we have chosen to measure it using a General Entropy Class measure with parameter value zero. This is often referred to as the Theil L measure or the mean log deviation, and compared with other General Entropy Class measures places a good deal more weight on inequalities amongst the poor.

Based on the standard approach to decomposing inequality, as described above, between group inequality in each country in our list is rather low (Table 1, column III).

Only South Africa stands out with a between-race share of 38%. Even here, however, it is striking to note that nearly two-thirds of total inequality in South Africa can be attributed to differences *within* racial groups as opposed to differences across groups.

The generally low between-group shares we observe in Table 1 are typical in the literature, even for other population breakdowns commonly encountered.[12] But how should this finding of a low between-group share be interpreted? Does it mean that the population breakdown along social dimensions is not terribly relevant to thinking about inequality in these countries?

In what follows, we propose an alternative perspective on the between-group share of inequality. While South Africa's between race inequality share is "only" 38%, we show that observed between race inequality accounts for more than 50% of the 'maximum possible' between-race inequality in South Africa given its current income distribution, the number of racial groups, their sizes, and their ranking in terms of average income (see Table 1, column IV). A similar point can be made when comparing Brazil and Panama. Based on the standard decomposition by race/ethnic group, the between-group share of inequality in both countries is only about 16%. This calculation would conventionally be interpreted as suggesting that race or ethnicity is of limited relevance to an understanding of inequality in these two countries. However, in Panama, observed between-race inequality accounts for well over a third of 'maximum possible' between-race inequality, while in Brazil the conclusion based on our measure is only slightly different from that which obtained from the standard calculation (see Table 1).

[12] For example, Elbers et al. (2004) demonstrate that the share of inequality attributable to differences between the 1248 communities into which Madagascar can be subdivided is approximately 25%.

2.1 Maximum Between-Group Inequality

In studies of inequality one often encounters statements of the following type: "between-group income inequality accounts for only 20% of total inequality". Such statements, however, should not be taken to mean that 100% of total inequality would have been a realistic possibility. A between-group share of 100% would be possible only under two, rather unlikely, scenarios. First, if each household constitutes a separate "group" then total inequality is clearly also equal to between-group inequality. Second, a between group share of 100% would occur if there were fewer groups than households, but somehow all the households within each of these groups genuinely happened to have identical per capita incomes. Rather than having a bell-shape, the density function of income in this latter case would consist of a series of spikes occurring at the average per capita income level for different groups (as in the first period in figure 1 above). It is difficult to imagine a realistic setting in which this would occur: for virtually any empirically relevant income distribution and a limited number of groups, the share of maximum between-group inequality that can be attained is strictly below 1.

Hence, not all possible groupings of the population are equally relevant in assessing the salience of inequality between certain groups. But, what groupings are most relevant? In this section, we propose a measure that evaluates observed between-group inequality (BGI) against a benchmark of maximum between-group inequality that can be attained when the number and relative sizes of groups under examination are fixed:[13]

[13] A different approach could be based on statistical testing. One could ask how (un)likely it is that a particular value of between-group inequality is the result of pure chance given the number of groups and their relative sizes, i.e., test the null hypothesis that it is the result of a random allocation of incomes over households in society (keeping the overall income distribution constant). However, in practice, this exercise always leads to the rejection of that null hypothesis, i.e. observed

$$Rb' = \frac{BGI}{\text{maximum } BGI} = Rb \frac{total\ inequality}{\text{maximum } BGI}$$

Since BGI can never exceed total inequality, it follows that *Rb′* cannot be smaller than *Rb*.[14] It is also clear from the formula above that if maximum BGI attainable is close to total inequality, then *Rb* and *Rb′* will also be close to each other. Put differently, if there is a way of reordering the population into a given number of groups with fixed sizes such that the inequality between the resulting groups is almost equal to total inequality, then *Rb′* will not differ significantly from *Rb.* This is true, for example, in the case of inequality between social groups in Brazil, where our alternative measure is only slightly higher at 20% than the conventional between-group share of 16%, but not in Panama, where the analogous figures are 36% and 17%, respectively (see Table 1).

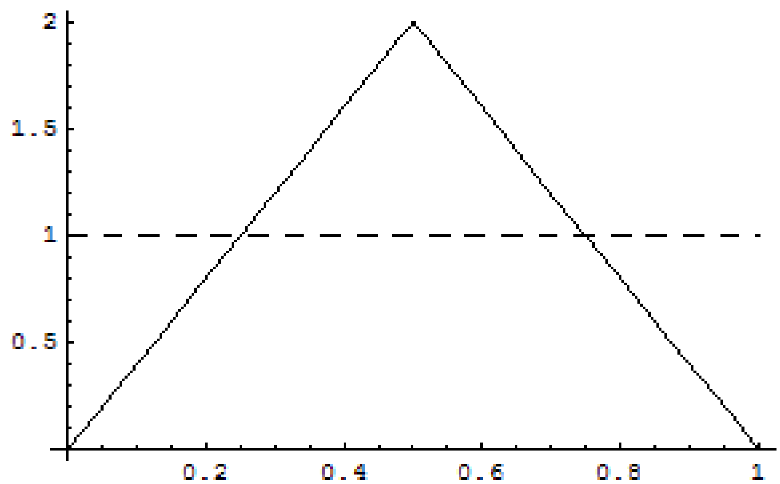
To see how the maximum attainable BGI can differ from context to context, take the rectangular and triangular distributions depicted in figure 2 below. In both cases, assume that there are two groups, each containing half the population. A necessary condition for BGI to be at its maximum is that these two groups occupy non-overlapping partitions of the distribution of income: if $\{y\}$ is an income distribution for which BGI is maximized, and g and h are different groups then either *all* incomes in g are higher than *all* incomes in h, or *vice versa*. (See Shorrocks and Wan, 2004, section 3.). Hence, for each of the distributions in figure 2, the maximum BGI is attained when one group

between-group inequalities are almost always above zero in a statistically significant manner. While we could use these significance levels to rank levels of between-group inequality, it is not so clear how to interpret a difference between significance levels of, say, 0.999 and 0.9995.

[14] We follow the notation in Cowell and Jenkins (1995).

occupies the bottom half of the income distribution, and the other the top.[15] In this particular example, it can be readily verified that maximum BGI as measured by GE(0) is 0.14 in the uniform case and 0.06 for the triangular distribution. In this hypothetical society, an observed between-group inequality of, say, 0.05 is arguably much more extreme (or salient) in the case of the triangular income distribution than the uniform one.

Figure 2: Uniform and triangular densities



In order to calculate *Rb′* we need to know BGI, which can be calculated in the usual way, and maximum BGI, which is slightly more difficult to compute. A "brute-force" approach to calculating maximum BGI uses the property (mentioned above) that under a BGI-maximizing distribution, group incomes occupy non-overlapping intervals. In the case of n groups, the following approach can be followed: take a particular permutation of groups $\{g_{(1)},\ldots, g_{(n)}\}$, allocate the lowest incomes to group $g_{(1)}$, then to $g_{(2)}$, etc., and calculate the corresponding BGI. Repeat this for all possible $n!$ permutations. The highest resulting BGI is the maximum sought. This approach is obviously easier when the number of groups under examination is small. In the

[15] Because the sizes of the groups are identical, it does not matter which group occupies which part of the distribution.

appendix, we describe some alternative approaches to solving the problem of maximizing between-group inequality for the Gini coefficient.[16] The same appendix also shows that without restrictions on the income distribution, no group order can be a priori excluded as a candidate for the BGI maximizing one.

In general, maximum BGI need not increase if the number of groups grows.[17] However, BGI cannot decline if more groups are obtained via proper sub-divisions of the existing groups.[18] In the limit, when every individual constitutes her own group, maximum BGI equals total inequality, and consequently $Rb=Rb'$.

A possibly more appealing benchmark against which to evaluate between-group inequality can be obtained by introducing one more restriction. In addition to fixing the number of groups and their relative sizes, we can also arrange the groups under examination according to their observed mean incomes, keeping their 'pecking order' unchanged.[19] In many cases, there is a well-understood hierarchy of population groups in terms of their mean incomes. Comparing actual between-group inequality to a counterfactual maximum BGI which preserves the actual, observed, rank ordering of groups is conceivably of greater interest than a counterfactual which allows for random

[16] The choice of Gini is for computational convenience and is not uncommon in the literature that precedes us, such as Davies and Shorrocks (1989).

[17] In the case of the Gini coefficient, the difference between total inequality and maximum BGI can be crudely bounded and the bounds are functions of the population share of the largest group, and not by the number of groups (see the appendix for a proof). Maximum BGI will stay bounded away from total inequality unless the maximum group size becomes sufficiently small. This implies that although the expected value of between group inequality might increase as the number of groups increases (Shorrocks and Wan (2004), proposition 3), this value may be well below total inequality if one of the groups remains very large. For example, with a lognormal (0,1) distribution and one group occupying 70% of the population while every other individual constitutes a separate group (implying effectively an infinite number of groups), the maximum possible between group inequality (measured by GE(0)) would be 0.373 – well below the inequality level of 0.5 in this distribution. For related results, see Davies and Shorrocks (1989) and Shorrocks and Wan (2004).

[18] Starting from a BGI-maximizing group order, split-up a group in such a way that mean incomes in the new groups are equal to the old parent group. BGI among these groups is equal to the old BGI. Obviously, if the incomes of the two new groups are non-overlapping, then BGI will increase. Hence a refinement of a particular grouping will generally lead to an increase in BGI. See also Shorrocks and Wan (2004), proposition 2.

[19] Ordering population groups by their mean incomes using, say, a household survey would introduce a possible difficulty due to sampling variability. In other words, our ability to order groups by mean income (or consumption) could be limited by the fact that some of the group means are statistically indistinguishable from each other. For the time being, we ignore the standard errors associated with the observed group means.

re-ordering of groups. For example, when decomposing inequality by race in Brazil, South Africa, or the U.S. (see the example in Section 1), the ordering of racial groups in terms of mean incomes is well-documented, and it is not obvious to what extent a counterfactual of say, average income of blacks exceeding that of whites would be realistic and of any inherent interest. This approach is also appealing for practical reasons as it involves just one, rather than n! calculations of BGI.

Obtaining the maximum possible BGI given the current income distribution, relative group sizes, and their rankings by mean incomes is trivial: allocate the lowest incomes to members of the group with lowest mean income, the lowest remaining incomes to the group with the 2nd lowest mean income, etc. In the rest of this paper, *Rb′* will refer to our index of BGI normalized by the maximum possible BGI given the current income distribution, relative group sizes, *and* their "pecking order."

Salience of grouping for income inequality

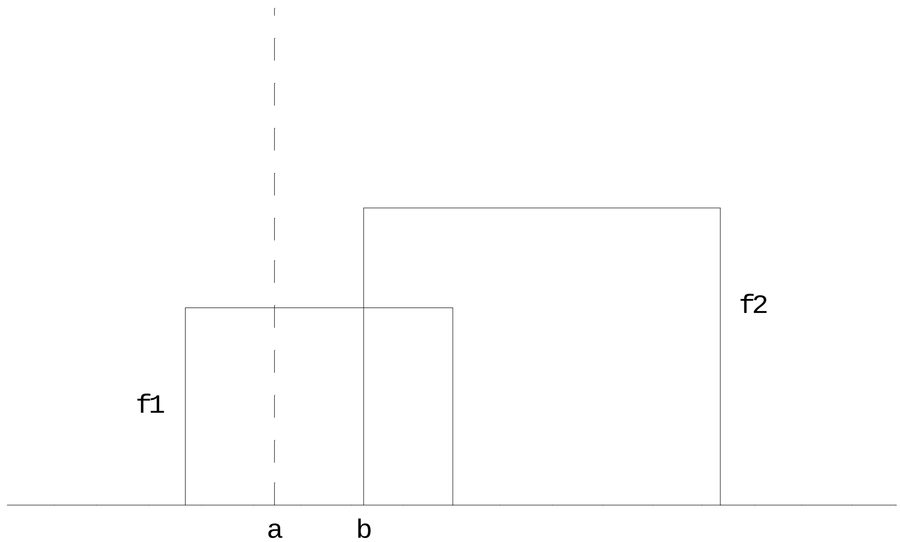
In the preceding sections we have introduced *Rb′* in an effort to assess whether group sub-divisions that should not be relevant in a moral sense, in fact display significant between group inequality. We can compare one way of sub-dividing the population to another. We could have asked an alternative question: how well does a person's income predict his or her group membership in one kind of population grouping relative to another? It is clear that in the case where an income distribution is divided into non-overlapping groups, a person's group can be perfectly predicted once her income is known. In general, if income is a good predictor of group membership then it would seem reasonable to regard that particular grouping as "salient" to the analysis of inequality, especially inequality of opportunity. Thus we are led to ask if Rb' indicates salience in the above sense. We are able to show that, compared to *Rb*, it is indeed more sensitive to overlap in the support of the groups' income distributions and is less sensitive to inequality within those groups.[20] An illustration may clarify.

Consider again a population consisting of two groups. Each group's (weighted) density of the income distribution is graphed in figure 3. Suppose we introduce a series of progressive transfers amongst the population represented by density *f1*. Specifically, we transfer incomes from those individuals with income below *b* but above *a*, to those individuals with income below *a*. We continue with these transfers until all individuals with incomes below *b* have the same income *a*. Clearly, redistributing incomes in this specific way has not affected our ability predict membership of either of the two groups based on knowledge only of observed incomes. Because group means, BGI, and maximum BGI are unaffected, Rb' is also unchanged. However, within-group inequality and total inequality decrease (because of the progressive transfers within group 1) and so *Rb* will go up. This change in *Rb* reflects a drop in total inequality that is not correlated with one's success in the 'salience game' described above.[21]

[20] Whether Rb' can be interpreted as an average success rate of guessing a person's group membership on the basis of income information alone, is a question we leave for future research.

[21] Pyatt (1976) also invokes the concept of a "game" when exploring the feasibility of sub-group decomposition of the Gini coefficient. He highlights the significance of the degree of "overlap" between groups in this procedure.

Figure 3



The perspective on 'salience' of group definitions offered by *Rb´* may be of interest also in settings other than the analysis of inequality of opportunity. While the latter exercise starts from the position that certain predetermined group definitions are judged to be "morally irrelevant" (Roemer, 1998) and then seeks to ascertain to what extent these groups are relevant to an understanding of inequality, there may also be situations where group definition does not precede the analysis, but is determined ex-post. For example, a politician may be interested in tailoring his economic policies and messages to specific groups in the population and would like to know which group definitions are most 'salient' in the sense we are considering here.[22] In this case, he (or she) might consider performing a search across different group definitions until the most 'salient' definition is identified. Basing this "search" on *Rb´* would appear to be

[22] Kanbur (2005) gives a similar example while discussing the policy implications of using conventional inequality decompositions.

particularly appealing as it is more readily comparable across different group definitions as a result of the normalization by maximum BGI.[23]

3. Evidence

Expressing observed between-group inequality as a fraction of the maximum possible BGI can provide additional insight in the analysis of inequality. Figures 4-6 decompose inequality on the basis of three different sub-group definitions for a number of developed and developing countries. Our data come from nationally representative household surveys from each of the countries and all refer to a year during the 1990s. We consider three ways of breaking down the population in each country: by social group membership, rural-urban location of residence, and education of household head (not all countries' data permit a breakdown along all three dimensions, however). Of these groups, only social group membership (which loosely refers to racial, ethnic, or caste breakdown relevant to each country) can be truly considered as a 'circumstance' or a pre-determined characteristic, and as such consistent with measuring inequality of opportunity. In the rest of our empirical work, we would have ideally used *place of birth* instead of *rural-urban residence*, and *parents' education* in place of *education of household head*. However, many of the household surveys in our database do not permit us to break the population down by these circumstance variables, and hence the variables we use instead can be viewed at best as crude proxies for an individual's circumstances.[24] For each country and for each sub-group definition, the between-group share is calculated

[23] We are grateful to Sam Bowles for suggesting to us this interpretation of Rb'.
[24] Such data on individual circumstances, of course, exists for an increasing number of countries. For example, drawing on the distinction between 'circumstance' and 'effort' variables in Roemer's work on equality of opportunity, Bourguignon, Ferreira, and Menéndez (2003) uses a different method to decompose earnings inequality in Brazil into a component due to unequal opportunities and a residual term.

in both the conventional manner, with total inequality as denominator, as well as on the basis of the Rb' calculation outlined above.

The data are not strictly comparable as inequality is typically measured differently across countries – based sometimes on a consumption measure of welfare and sometimes on an income measure. Even where the welfare indicators are based on the same concept, the precise definition is almost never the same across countries. We have explored the sensitivity of decomposition analyses to alternative welfare indicators for a sub-set of 14 countries in which we have both income and consumption data. We have found that while overall measured inequality typically varies markedly across welfare indicators (with measured inequality based on an per-capita income measure usually being higher than inequality based on a per-capita consumption measure), decomposition results tend to vary only slightly. This finding that inequality "profiles" are less sensitive to different underlying welfare definitions than direct inequality comparisons echoes a similar finding in the poverty literature that poverty profiles are often quite robust to varying underlying welfare definitions and poverty lines (see Lanjouw and Lanjouw, 2001). Thus, while the data examined here are far from comparable in terms of overall measured inequality we contend that comparisons of decomposition results are much less problematic.

Moreover, as we have emphasized above, one of the attractions of the Rb' measure is that it normalizes by the observed number and relative size of observed groups, within each distribution of income that is being considered. We have already described above how, for example, decomposition by social group involves quite different group definitions and sizes in different countries. Working with *Rb′* rather than *Rb* is thus less subject to comparability concerns from this perspective as well.

Figure 4 decomposes inequality on the basis of a rural/urban breakdown in 85 countries. Countries are grouped by region and ranked within each region by conventionally calculated *Rb*. In each country both *Rb* and *Rb′* are reported. Several observations can be offered. First, in most countries the conventionally calculated between-group share is generally well below the *Rb'* calculation. Indeed, in Senegal, Guinea, Burundi, Kenya, Guatemala, Panama and Bolivia, the between-group contribution based on *Rb′* rises above 40%, suggesting that in these countries inequality is strongly colored by these spatial issues. In only two out of 85 countries is conventionally calculated *Rb* as high as 30%, but the number increases eight-fold to 16 out of 85 on the basis of *Rb′*. The between-sector inequality contribution is generally lowest for the most developed countries in our sample, as well as for a number of the Eastern Europe and Central Asian countries, irrespective of the manner in which this contribution is calculated (see further below).

Figure 5 returns to the breakdown of inequality by social groupings described in Table 1, now for a total of 35 countries. Again, the definition of social group (and number of groups) differs across countries, but is generally based on some criterion related to ethnicity, races, or religion. The evidence in Figure 5 suggests that social grouping is a particularly important dimension of the inequality profile in South Africa, Paraguay, Guatemala and Panama. In these countries, *Rb′* is above 30%, and indeed in South Africa it reaches to nearly 60%.

While *Rb'* is always higher than conventionally measured *Rb*, the degree to which these two statistics differ varies considerably across countries. In Nepal and Madagascar, for example, one's assessment of the salience of social groups does not much vary across the two approaches, while in South Africa, Paraguay, Vietnam, France, Panama, and Peru, they yield very different conclusions. It is interesting to note the rank reversals between U.S., Germany, and France when our alternative approach is employed.

Figure 6 decomposes inequality on the basis of roughly five education groups in each of 91 countries.[25] Education level of household head is a particularly salient dimension of inequality in many Latin American countries, as well as in several African countries and Thailand. In general, although *Rb'* is naturally higher than *Rb*, the difference between the two statistics is not as large for decompositions by education as in the previous two population breakdowns. Indeed, the ranking of countries on the basis of *Rb'* is not much different from that on the basis of *Rb*.

Overall, we can see from these illustrative calculations that employment of the *Rb'* calculation has the general effect of significantly raising one's assessment of the importance of group differences in an examination of inequality. To the extent that this approach is viewed to contribute a meaningful perspective on the importance of group differences, the qualitative conclusions that have tended to be drawn in the conventional literature may merit reconsideration.

Correlating Total Inequality and Between-Group Inequality

[25] The five broad education categories correspond to levels achieved by the household head, and refer to: no education, up to primary only, above primary but below secondary completion, secondary completion, post-secondary education. This definition of education groupings could not be applied in an exactly identical manner in all countries and is therefore only broadly comparable across countries.

As mentioned in Section 1, Kanbur (2000) has cautioned against concluding that simply because (conventionally calculated) between-group contributions to inequality are generally low, this should be taken to imply that between group differences are of only limited importance to an overall assessment of inequality. In the spirit of probing further this concern we ask here whether, across our set of countries, there is any statistical relationship between overall inequality and the percentage contribution that is attributable to between-group differences. We regress overall inequality in each country separately on the between-group contribution (based on *Rb′*) attributable to four population breakdowns: rural-urban location of residence, social group, occupation of household head, and education of household head. As we have noted, our data are far from comparable in terms of overall measured inequality due to different definitions of welfare being employed in different countries. To accommodate this concern, albeit only partially, we include in our regression a set of regional dummy variables as well as a dummy indicating whether a particular country's inequality is measured on the basis of per-capita consumption or income. Regression results have also been screened for the influence of outliers and influential observations.[26]

Figure 7 presents our results. There is strong evidence of a positive correlation between overall inequality and the between-group contribution, irrespective of the specific group definition. It is important to realize that there is nothing inherent in the mechanics of the decomposition calculation that ensures that there should be a positive relationship between the overall level of inequality and the percentage contribution that

[26] We do not report regressions results based on a model of overall inequality on *Rb*. Our qualitative findings are similar, but as described in the text there are grounds for doubting the comparability across countries of these measures of between-group inequality contribution.

can be attributed to between group differences.[27] In Figure 7, we can see that in all cases considered here there is great sensitivity of overall inequality to between-group differences and this is strongly significant for all group decompositions.

These correlations are suggestive but, of course, far from conclusive. Nevertheless they are consistent with an argument that has been articulated most recently in the World Bank's 2006 World Development Report, namely that overall inequality in the developing world tends to be high and to persist over long periods of time in those countries in which inequalities of opportunity across population groups are accentuated.[28] The Report argues that the level and persistence of such inequalities of opportunity act as a brake on economic growth and dampen prospects for rapid poverty reduction. For this reason policy makers have an important instrumental reason for concentrating on reducing group differences alongside the more conventionally acknowledged intrinsic objections to inequality.

4. Concluding Remarks

In this paper, we propose a modification to the conventional approach of decomposing income inequality by population sub-groups. We note that the conventional practice of calculating the share of between-group inequality is equivalent to comparing observed between-group inequality (across a few groups) against a benchmark (across perhaps millions of groups) that is quite extreme. Specifically, we propose a measure that evaluates observed between-group inequality against a benchmark of maximum

[27] Indeed, if there were concerns about noise in the data, high inequality countries would likely be countries in which there was more noise. Pure noise would result in smaller between-group shares (because of greater overlap across groups). As a result, if anything one might expect a *negative* relationship.
[28] As mentioned earlier in this section, only differences between 'social groups' in these countries can strictly be interpreted as inequality of opportunity in the Roemer sense. The income/consumption differences between other groups, such as rural-urban, education, and occupation are likely due, at least in part, to choices people have made.

between-group inequality that can be attained when the number and relative sizes of groups under examination are fixed. As our measure normalizes between group inequality by the number and relative size of groups under examination, it is also less subject to problems of comparability across different settings. We argue that our modification can provide a complementary perspective on the question of whether a particular population breakdown is salient to an assessment of inequality in a country.

It is important to note that our measure is not the result of a statistical decomposition of any inequality measure of a certain class. Rb' is concerned with evaluating between-group inequality against a proper benchmark and as such places less emphasis on inequality within groups. It is our contention that if one is interested in assessing inequality of opportunity between certain groups, using the traditional contribution of between-group inequality to overall inequality may unduly color that assessment.

Our measure is simple to calculate, particularly when we preserve the "pecking order" of the groups under examination. We find that for a large set of countries our assessment of the importance of group differences typically increases substantially on the basis of this alternative approach. The ranking of countries (or different population groups) can also differ from that obtained using traditional decomposition methods. Finally, we observe an interesting pattern of higher levels of overall inequality in countries where our measure finds higher shares of between-group contributions.

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Appendix: Maximum Between Group Inequality: Analysis and Programming

In this appendix we will be mainly concerned with the Gini coefficient for measuring inequality. For reasons of exposition we will assume that the income distribution is absolutely continuous with density $f(y)$, CDF $F(y)$ and Lorenz curve $L(p)$. We define between-group inequality as the total inequality one would obtain if all incomes within groups were equal. As mentioned in Foster and Shneyerov (2000), between-group inequality can be defined in several ways that lead to different outcomes for a between-group Gini if groups have overlapping income ranges. However, the same ambiguity does not exist for *maximum* between-group inequality, since the maximum implies non-overlapping income ranges.

Figure A.1 depicts the Lorenz curve: $L(p), p \in [0,1]$, as well as a Lorenz curve based on the same distribution, but for groups $j=1,\ldots,n$, with non-overlapping incomes, and equal incomes within groups: $L_g(p)$. The share of each group in the population is w_j. The following are easily verified:

1. If the groups have non-overlapping income they can be mapped along the horizontal axis of the graph according to increasing per capita income, as adjacent intervals of width w_j.
2. $L(p) = L_g(p)$ at boundery points p of the intervals.
3. L_g is a piecewise-linear approximation to L. The approximation is better, the smaller are the population shares w_j.
4. $L(p) \le L_g(p)$. Hence the Gini (and all Lorenz-consistent inequality indices) of L_g is smaller than that of L.

The between-group Gini for groups with non-overlapping income ranges is unambiguously given by the Gini coefficient corresponding to $L_g(p)$. Since non-overlapping income ranges are a necessary condition for achieving maximum between-group inequality we have proved

Theorem

Under the conditions of this section, maximizing the between-group Gini is equivalent to ordering the groups $j=1\ldots,n$ along the horizontal axis in such a way that the area under the resulting Lorenz curve L_g is minimal.

In principle, one can solve the max-BGI problem by trying out all $n!$ group orders and calculate the BGI index (Gini or other) for the resulting piecewise-linear Lorenz curves. Obviously, this is not a viable strategy if the number of groups is big.[29]

The max-BGI problem can be formalized as an integer programming problem as follows. Define integer variables a_{ij} to equal 1 if group i has strictly lower expenditure than group j, and zero otherwise. (So group i comes before j in the income distribution.) The a_{ij} must satisfy the following conditions:

$$a_{ij} \in \{0,1\}$$
$$a_{ii}=0$$
$$i \neq j \Rightarrow a_{ij}+a_{ji}=1$$
$$\forall i,j,k: a_{ik} \geq a_{ij}+a_{jk}-1$$

[29]One may verify that if the Lorenz curve is quadratic, the between-group Gini does not depend on the order of groups. Further, it should be clear that one cannot exclude a particular group order *a priori*. Take any group order and give equal incomes to members from the same group (and incomes increasing with group rank). Then that particular order maximizes BGI for the resulting income distribution.

The last condition is in fact a 'linear version' of the transitivity condition $\forall i, j, k : a_{ik} \geq a_{ij} a_{jk}$ which states that if i is poorer than j and j is poorer than k, then i is poorer than k. The location of group j's interval on the horizontal axis can now be expressed as $(\ell_j, u_j]$, where

$$\ell_j = \sum_i w_i a_{ij}$$
$$u_j = \ell_j + w_j,$$

while group j's income share is $L(u_j)$-$L(\ell_j)$. Obviously, $u_j = \ell_{j+1}$. Geometrically, from the total size of all groups poorer than j, i.e. ℓ_j, and group j's size w_j, the location of the group's chord on the Lorenz curve $L(p)$ can be determined. See again Figure A.1. Group j's contribution S_j to the area under the linearized Lorenz curve is

$$S_j = w_j \frac{L(\ell_j) + L(u_j)}{2}$$

and so the programming problem becomes

$$\min \sum_{j=1}^{N} w_j \left(L(\ell_j)+L(u_j)\right) \qquad \text{with respect to } a_{ij},$$

subject to

$$a_{ij} \in \{0,1\}$$
$$a_{ii}=0$$
$$i \neq j \Rightarrow a_{ij}+a_{ji}=1$$
$$a_{ik} \geq a_{ij}+a_{jk}-1$$
$$\ell_j = \sum_i w_i a_{ij}$$
$$u_j = \ell_j + w_j,$$

where $i,j,k=1,\ldots,N$.

For other inequality measures the max-BGI problem can be formulated using the same constraints but a different objective function. Many inequality measures (such as all those in the GE class and the Atkinson inequality measures) are defined as functions of the groups' relative mean incomes μ_j/μ, where μ denotes overall average income and μ_j average income in group j. Such inequality measures can be expressed in terms of the above notation by noting that with non-overlapping income ranges

$$\mu_j/\mu = (L(u_j)\text{-}L(\ell_j))/w_j.$$

Although all the constraints in the program are linear, the objective function is not, and the problem turns out to be a highly 'non-convex' integer program. Easily available solvers for this type of problem do not work.[30] Fortunately, some further analytical results can be obtained for the Gini. Moreover, as mentioned in the text, a maximum BGI that preserves the income ranking of groups represents a practical, if not entirely exact, alternative approach.

The Gini coefficient of BGI: further analysis and computation

In this sub-section we present some further analytical results on the problem of calculating maximum BGI. These results are for the Gini coefficient and as such of limited value for the analysis of between-group inequality. On the other hand, a between-group Gini maximizing group order is very likely to have a close-to-maximum between-group inequality index for other measures as well.

[30]We tried the GAMS solvers DICOPT and SBB.

We use the following extension to Jensen's inequality.

Lemma

Let $g(x)$ be a (not necessarily strictly) convex function on a convex set X. Then for $t>0$ and $(x-t,x+t)\subset X$:

$$\frac{g(x+t)+g(x-t)}{2} \geq \frac{1}{2t}\int_{-t}^{t} g(x+u)du \geq g(x).$$

Proof

Obvious. The theorem essentially states that the effect of a mean-preserving spread gets stronger as probability mass is shifted outward, the mean being equal to x here. Further below we apply the lemma with the first derivative of the Lorenz curve (*i.e.* $F^{-1}(p)$) in the role of $g(x)$.

Corollary

Let G the an anti-derivative of g defined by $G(x+t)=\int_{x}^{x+t} g(s)ds$, then

$$H(t)=\frac{G(x+t)-G(x-t)}{2t} \text{ is increasing in } |t|.$$

This can be seen by taking the derivative of the above expression and noting that for $t>0$

$$H'(t)=\frac{1}{t}\frac{g(x+t)+g(x-t)}{2}-\frac{1}{2t^2}\left[G(x+t)-G(x-t)\right]$$

$$=\frac{1}{t}\left(\frac{g(x+t)+g(x-t)}{2}-\frac{1}{2t}\int_{-t}^{t} g(x+u)du\right)$$

from which $H'(t) \geq 0$ follows by applying the lemma. The corollary can be used to proof the following

Theorem

Let groups A and B be adjacent in a group order that maximizes the between-group Gini coefficient, with A having the lower incomes. Let $F^{-1}(p)$ be convex on the interval $(\ell_A, u_B]$. Then $w_A \geq w_B$.

Remark

Since $F(y)$ is nondecreasing, it is concave where $F^{-1}(p)$ is convex and *vice versa*. This corresponds to an income distribution which has monotonically decreasing density.[31] According to the theorem a BGI (Gini) maximizing group order must have decreasing group sizes if the density is decreasing over the relevant income range.

Proof

Let μ be average income, then the Lorenz curve satisfies

$$\mu L(p) = \int_0^p F^{-1}(s)ds.$$

Recall that the contribution of groups A and B to the between-group Gini is minus the surface S_{AB} under the chords connecting the points $(\ell_A, L(\ell_A))$, $(u_A, L(u_A)) = (\ell_B, L(\ell_B))$ and $(u_B, L(u_B))$. If the group order A,B is part of a Gini-maximizing order, the surface under the chords must not be higher than with groups A and B in reverse order.

[31] Convex functions are differentiable almost everywhere.

Let A and B be located between percentages $s = \ell_A$ and $s + w_A + w_B = u_B$. If A is before B the contribution of the two groups is

$$2S_{AB} = w_A(L(s) + L(s + w_A)) + w_B(L(s + w_A) + L(s + w_A + w_B)).$$

Alternatively, if B is before A the contribution is

$$2S_{BA} = w_B(L(s) + L(s + w_B)) + w_A(L(s + w_B) + L(s + w_A + w_B)).$$

The difference can be written as

$$S_{AB} - S_{BA} = \frac{1}{2}(w_B - w_A)(w_A + w_B)\left(\frac{L(s + w_A + w_B) - L(s)}{w_A + w_B} - \frac{L(s + w_B) - L(s + w_A)}{w_A - w_B}\right).$$

The difference can be negative only if $w_A > w_B$. To see this set $x = s + (w_A + w_B)/2$, $t_{AB}=(w_A+w_B)/2$ and $t_{BA} = (w_B - w_A)/2$. Since $t_{AB} > t_{BA}$ it follows from the lemma's corollary that

$$\frac{L(s + w_A + w_B) - L(s)}{w_A + w_B} = \frac{L(x + t_{AB}) - L(x - t_{AB})}{2t_{AB}} \geq$$

$$\frac{L(x + t_{BA}) - L(x - t_{BA})}{2t_{BA}} = \frac{L(s + w_B) - L(s + w_A)}{w_A - w_B}.$$

QED

Note that the same method of proof can be used *mutatis mutandis* to show that groups must be in *in*creasing size order if the income distribution has increasing density. Thus we have the following corollary.

Corollary

If the income distribution is unimodal, then a between-group Gini maximizing group order has groups arranged in increasing size where the density is increasing and in decreasing size where the density is decreasing.[32]

This last result is useful for practical computation. It implies that for the common case of a unimodal distribution the largest group is located near the mode of the distribution with increasing group sizes for incomes lower than the mode, and decreasing group sizes for higher incomes. When the fraction of the population with incomes below the mode is of the same order of magnitude of (or smaller than) group sizes, only n or n^2 of the $n!$ group orders will have to be checked to find the order maximizing BGI. Note further that if a particular group order maximizes the between-group Gini, it is likely also to lead to values for other inequality measures, close to the maximum.

Bounds on the maximum between-group Gini

The difference between the maximum between-group Gini and the overall Gini originates from the fact that L_g is an imperfect approximation to L. As mentioned above, the difference is smaller the smaller are the group sizes. In fact, the difference can be crudely bounded by maximum group size $w^M = \max_c(\{w_c\})$. First note that $u_j = \ell_{j+1}$ and define $\ell_{n+1} = u_n = 1$. Then the difference in Ginis is

[32]See also Davies and Shorrocks (1989) and Shorrocks and Wan (2004).

$$
\begin{aligned}
&2\int_0^1 L_g(p)dp - 2\int_0^1 L(p)dp \le \\
&2\sum_{j=1}^n w_j \frac{1}{2}[L(\ell_{j+1}) + L(\ell_j)] - 2\sum_{j=1}^n w_j L(\ell_j) = \\
&\sum_{j=1}^n w_j [L(\ell_{j+1}) - L(\ell_j)] \le \\
&w^M \sum_{j=1}^n [L(\ell_{j+1}) - L(\ell_j)] = \\
&w^M .
\end{aligned}
$$

The difference is bounded by a weighted average of group sizes, the weights being Lorenz-curve increments. If the maximum group size is less than 10% of the population, the BGI for an arbitrary group order is at most 10% (points) below the overall Gini and the difference will be even smaller for the BGI maximizing group order. With group sizes smaller than 10% the gain of using maximum BGI as a benchmark instead of total inequality is therefore limited.

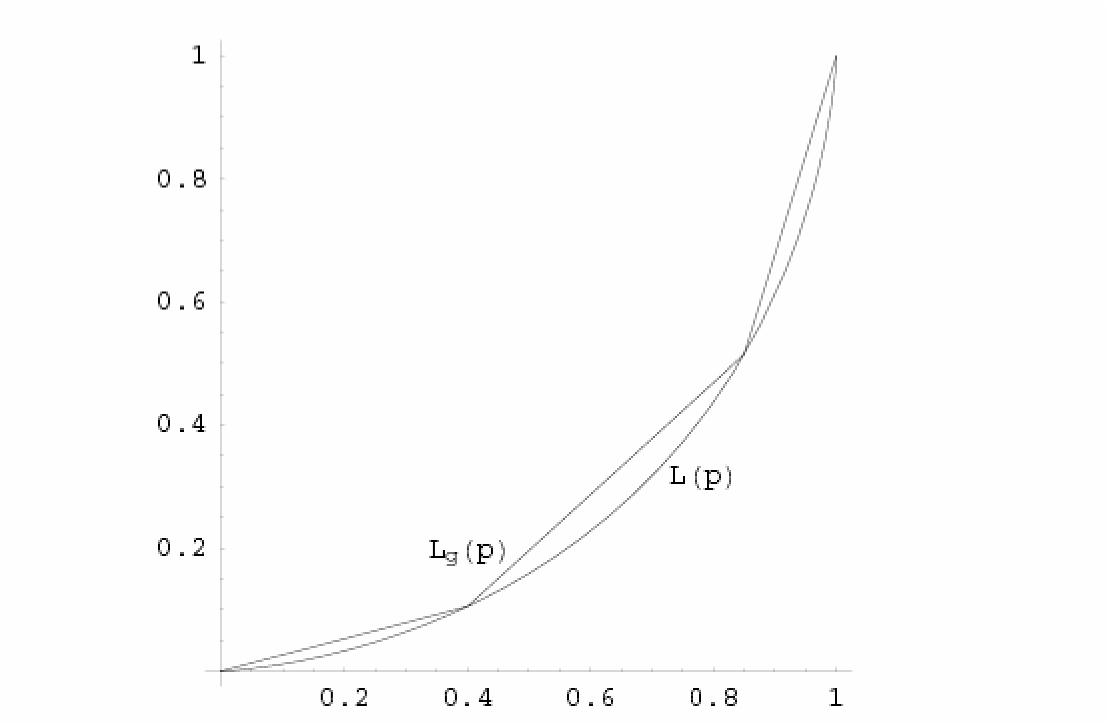
To derive a lower bound for the difference, note that the between-group Gini does not decrease (and will typically increase) if groups are refined into smaller groups. It follows that for $w \le w^M$ we have

$$
\begin{aligned}
&\min_{L_g}\left(2\int_0^1 L_g(p)dp - 2\int_0^1 L(p)dp\right) \ge \\
&\min_s\left(w(L(s) + L(s+w)) - 2\int_s^{s+w} L(p)dp\right) \ge \\
&\min_s\Big(w(L(s) + L(s+w)) + \\
&-2[\frac{1}{4}w(L(s) + L(s+\frac{1}{2}w)) + \frac{1}{4}w(L(s+\frac{1}{2}w) + L(s+w))]\Big) = \\
&w \min_s\left(\frac{L(s) + L(s+w)}{2} - L(s+\frac{1}{2}w)\right).
\end{aligned}
$$

Putting $w = w^M$ and $a(w) = \min_s [(L(s) + L(s+w))/2 - L(s+w/2)]$, it follows[33] that

$$\min_{L_g}\left(2\int_0^1 L_g(p)dp - 2\int_0^1 L(p)dp\right) \geq a(w^M)\, w^M .$$

Figure A.1: Lorenz curve L(p) and piecewise linear approximation Lg(p). There are three groups, with w1 = 0.4,w2 = 0.45,w3 = 0.15.



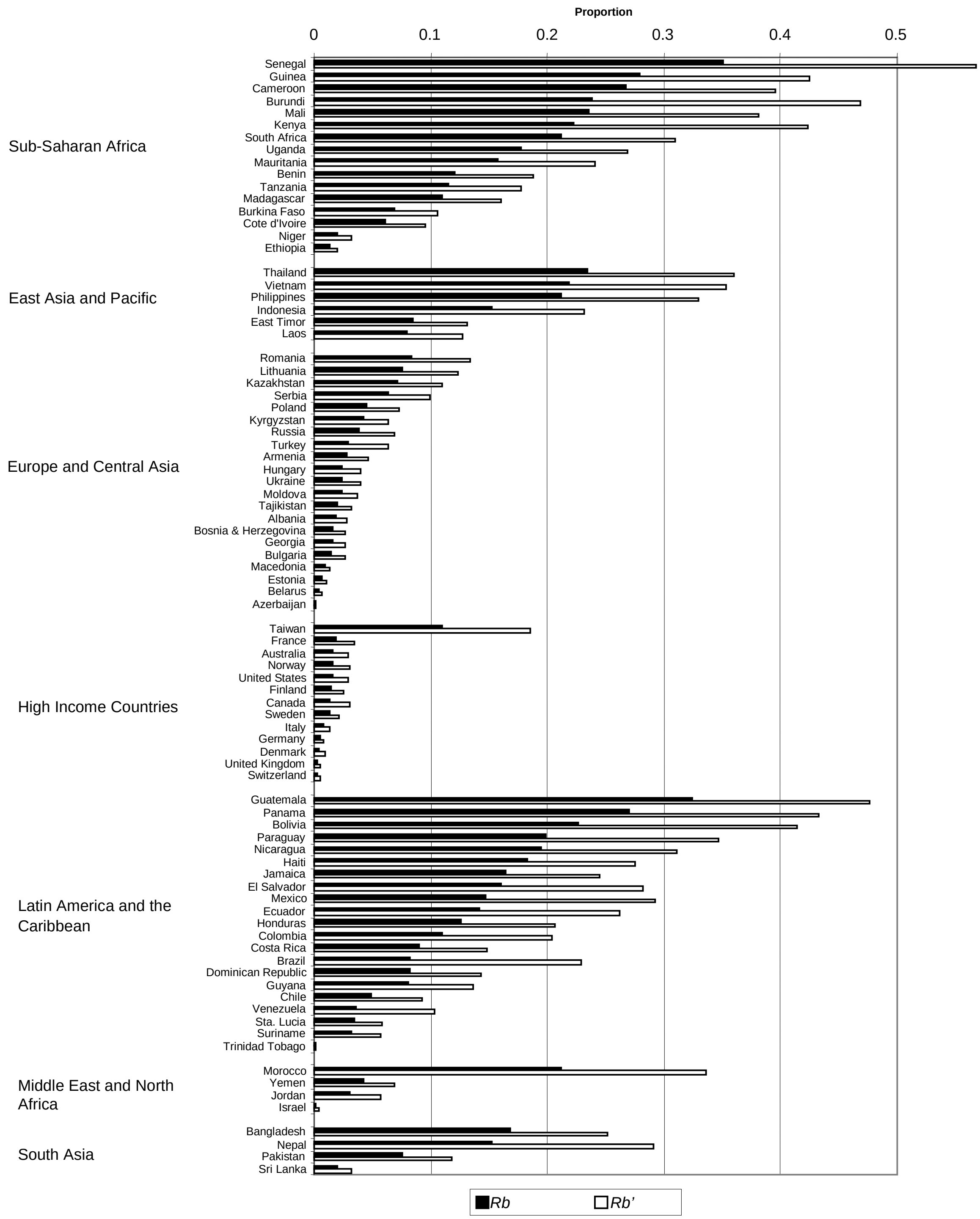
[33] $g(w)$ is a measure of the convexity of $L(p)$ over an interval of width w.

Table 1: Decomposing Inequality by "Social" Group in 8 countries.

Country	*No of "social" groups*	*GE(0)*	*Between-Group Contribution (%)*	*Rb' (%)*
India	3	0.136	5.1	10.1
Bangladesh	4	0.181	20.3	28.7
Kazakhstan	3	0.217	9.0	14.7
Nepal	10	0.220	23.3	23.7
United States	5	0.295	8.4	14.7
Panama	10	0.423	16.7	36.4
Brazil	4	0.442	16.2	20.0
South Africa	3	0.563	38.0	55.0

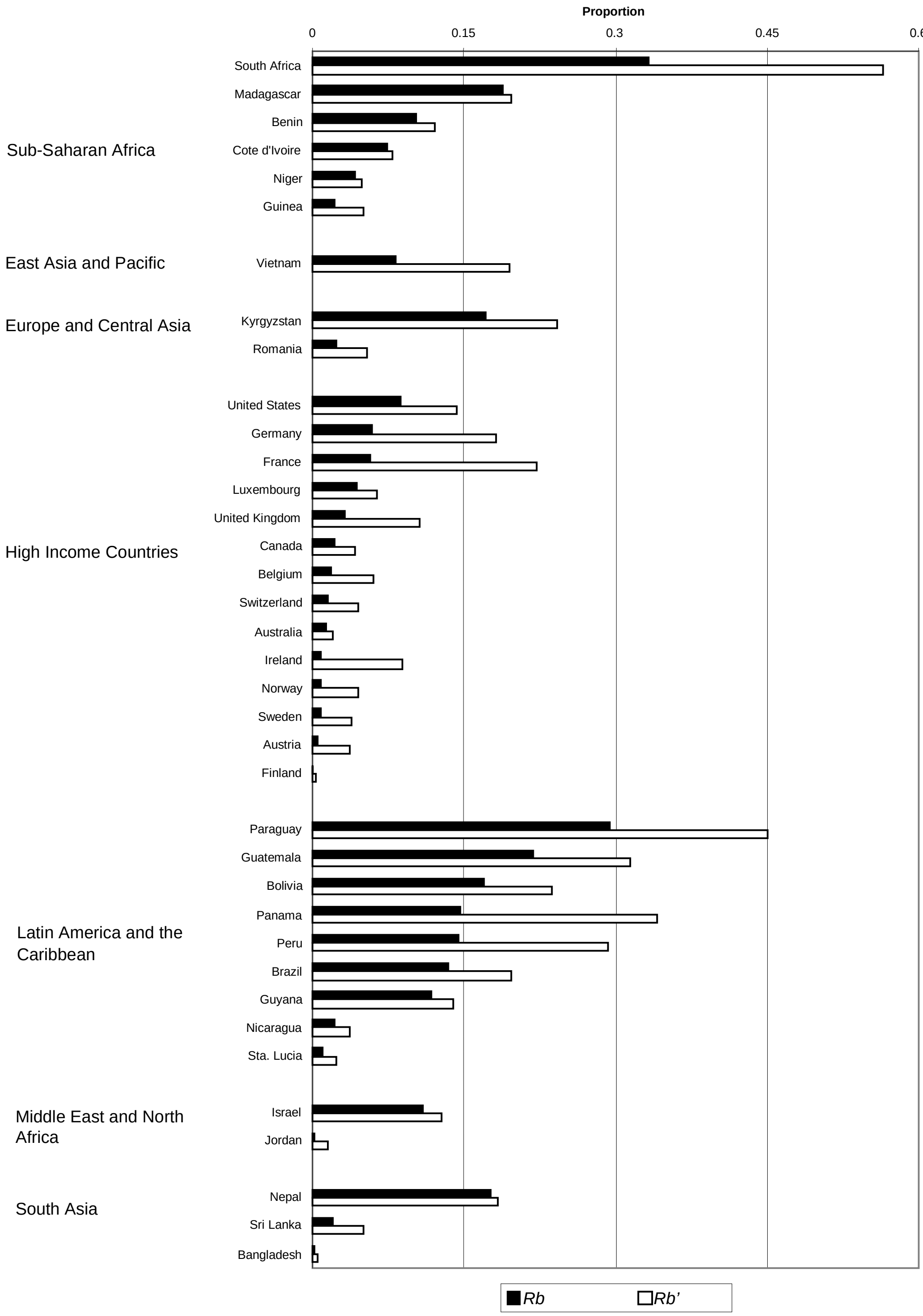
Note: data for India refer to rural areas only.

Figure 4: Between-group inequality decompositions: urban-rural



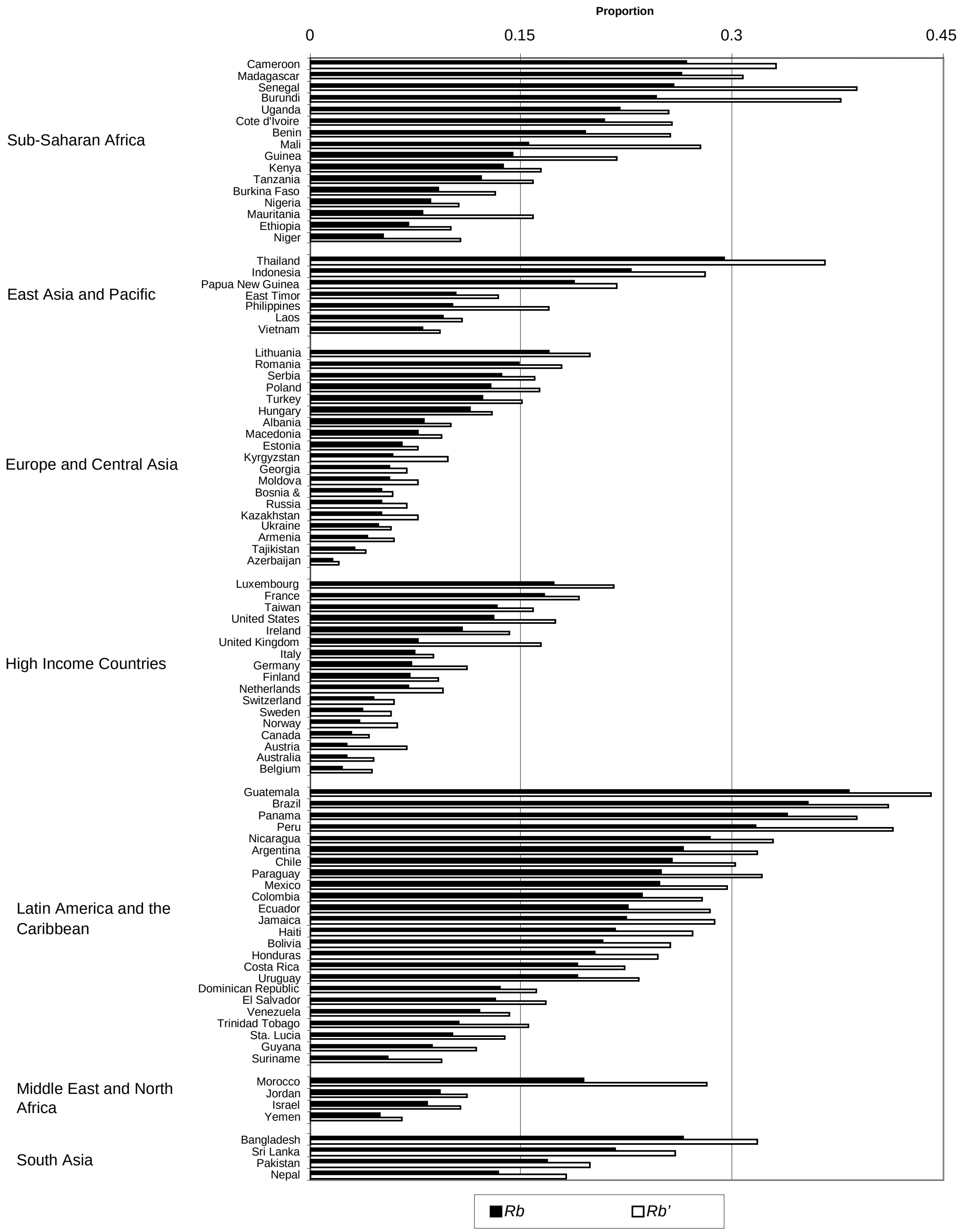
Source: Authors' calculations from household survey data.

Figure 5: Between-group inequality decompositions: social group of the household head



Source: **Authors' calculations from household survey data.**

Figure 6: Between-group inequality decompositions: education of the household head



Source: Authors' calculations from household survey data.

Figure 7:
Regressions of total inequality on shares of between-group inequality of different household characteristics
(based on *Rb'*)