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Alternative Instruments for Smoothing the Consumption of Primary Commodity Exporters

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To reduce price risk caused by the instability of primary commodity markets, countries that depend for export earnings on a single primary commodity can find substantial long-run protection by rolling over one-period futures. The practical benefits of a substantially longer hedging horizon may often be small.

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This paper — a product of the International Trade Division, International Economics Department — is part of a larger effort in PRE to improve developing countries' management of the substantial commodity price risk which most face. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Julie Carroll, room S7-069, extension 33715 (69 pages).

Countries that depend on a single primary export for their foreign earnings are likely to experience sharp fluctuations in export earnings and their underlying wealth, because of the instability of all primary commodity markets. As part of structural adjustment, several countries have liberalized their trade regimes, so domestic producers are no longer insulated from international price fluctuations.

Kletzer, Newbery, and Wright review the costs of export price instability and consider the role of conventional instruments (loans, price stabilization measures, future contracts, and futures rollovers for longer-term price protection), as well as instruments loosely called "commodity bonds." They weigh the implications of the risk of borrower default when the borrower's aim is smoothing consumption. They conclude:

• In principle, consumption-smoothing contracts might be valuable to countries dependent on an export commodity subject to price risk. Futures coverage could help if longer maturities were available. They conclude that substantial long-term protection is possible by rolling over one-period futures. The marginal net benefits of lengthening the horizon beyond the one production period (roughly observed in practice) depend upon transaction costs, the degree of serial correlation, and the discount factor. In practice, the extra benefits of a substantially longer hedging horizon may often be small.

• If production responds to incentives with a one-period lag, the rollover strategy does not provide perfect protection at the time the hedge is made — even if the production response to inputs is nonstochastic, as opposed to the case of one-period hedging.

• When a sovereign exporter can offer no collateral and is short of liquid resources, the use of futures is precluded by the need to furnish the margins that guard against default. The disadvantage of standard loans and buffer funds is that they will probably reach crisis states in which the resolution of the crisis is ill-defined. The lenders' recognition of this will dampen their enthusiasm.

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* This is a revised version of a paper presented at the World Bank, Commodity Economics Division, June 1990.

SMOOTHING THE CONSUMPTION OF PRIMARY COMMODITY EXPORTERS: AN ASSESSMENT OF SOME ALTERNATIVE INSTRUMENTS

by

Kenneth M. Kletzer, David M. Newbery, and Brian D. Wright

The value of oil in Norway was estimated at 71 percent of total wealth for 1980, at a price of oil of $16.40 ($1980) per barrel. At various times since 1980 the forecast price of oil has been twice this level and is now probably half this estimate. Given that Norwegian oil is costly to extract, the effect of doubling or halving the price of oil would be to more than double or halve the value of oil output, indicating that fluctuations in the price of oil will cause wild swings in Norway's net income, and permanent price shifts will have a profound effect on the estimated value of Norway's assets, and hence on her estimated sustainable consumption level.

Similar calculations could be performed for Mexico, Nigeria, Zambia, the OPEC nations and a range of other countries which depend on a single primary export for most of their foreign earnings. Given the substantial instability in all primary commodity markets, such countries are likely to experience sharp fluctuations in export earnings and their underlying wealth. To the extent that these fluctuations will affect consumption they will be costly, and we would expect the countries to seek ways of managing these risks and reducing their costs.

As part of the package of structural adjustments several countries have liberalized their trade regimes, with the consequence that domestic producers are no longer insulated from international price fluctuations. Indeed, it may well be better for the individual producers to manage their risk directly, rather than the government intervening to stabilize domestic prices and absorbing the resulting risk. This paper does not inquire into this important issue, and instead concentrates on the management of country-level consumption risk, and considers actions which the government might undertake to reduce the cost of that risk. In many countries the nature of the resource endowment and its comparative advantage rule out production diversification as a significant near-term strategy, and we assume it away here. In addition, we rule out diversification via exchange of equity investments with foreigners.

The paper is organized as follows. First we review the costs of export price instability, with some reference to the empirical magnitudes. Then in section 2 we consider the role of conventional instruments, including loans, price stabilization measures, and futures contracts. Particular attention is paid to the potential use of futures rollovers for longer-term price protection, and the effect of production response on that protection.

The above instruments encounter difficulties in the presence of sovereign risk and/or capital shortages. In section 3 we discuss instruments that are come under the loose heading "commodity bonds," and consider the implications of borrower default risk in the presence of a consumption-smoothing motivation on the part of the borrower. (In this paper the lenders are assumed always to honor their contracts.) Dynamic consumption smoothing paths, with and without a borrower default constraint, are addressed in section 4, using the constrained optimal fully state contingent contract as a benchmark. Conclusions follow in section 5.

1. The Costs of Income Variability

Consider a country that has economically unresponsive production ("zero supply elasticity") and seeks to maximize the expected utility of its representative consumer

$$V_t = E\sum_{t=0}^{\infty}(1+\delta)^{-t}u(c_t) \tag{1}$$

where E is the expectations operator, c_t is consumption in period t, and U is felicity, concave in consumption. The rate at which utility is discounted, that is, the rate of pure time preference, is δ. There is no storage. Output and price are each subject to one discrete i.i.d. random disturbance per period.

To dramatize the issues, assume that exports from a single commodity account for 33% of GNP on average, and suppose that the coefficient of variation (CV) of output and price of the commodity are both 30%, and that the correlation between output and price can be ignored. Suppose also that all other income is nonstochastic and that the country optimally shares risks internally. There is, however, no saving or borrowing or other intertemporal income smoothing. Using the standard formulas[1] for the cost of risk, if the coefficient of relative risk aversion is R (defined for one-period variations in consumption), and if the CV of consumption is s, then the annual cost of risk, ρ, is defined implicitly by $u(\bar{c}-\rho)=Eu(c_t)$, where a bar over a variable indicates its expected value, and the relative cost, $\rho/\bar{c}$, is approximately (exactly if utility is quadratic in income per period) $Rs^2/2$. If consumption must be equal to income each year, then $s = 0.33e$ where e is the CV of export revenue (and 0.33 is the average share of exports to GNP). If output and price are independently normally distributed, then $e^2 = 0.19$ (and this will hold approximately even if output and price are not normal). In this case, if R has the not unreasonable value of 2, the cost of risk is approximately 2% of average income, the amount representative consumers would be willing to forego each year in return for a stabilized consumption stream of $\bar{c}$.

Now, this figure of 2% of income locks high—if it were discounted at 10% real, then it would amount to 20% of GNP, which is a sizeable amount of wealth to give up. At this point, one should draw attention to a potentially serious and currently unresolved problem in interpreting attitudes to risk. Briefly, the problem is this. Risk aversion as originally defined, referred to static problems in which income, wealth and consumption were all equal and there was no saving (as there was no future). In this context, the coefficient of relative risk aversion, R_W, is defined as -WU"(W)/U'(W), where W is

[1]If consumption c is a random variable with coefficient of variation s, $u(E(c)-\rho)=Eu(c)$. Expand both sides in a Taylor series: $u(E(c))-\rho u'(E(c)) \approx u(E(c))+0.5s^2E(c)u''(E(c))$ or $\rho/E(c) \approx 0.5s^2R$.

wealth. In practice, income and consumption differ, and attitudes to risk are typically defined over the variations in annual consumption. The coefficient of partial risk aversion, Rp, is defined as $-cU''(c)/U'(c)$ $= (c/W)R$. Normally, c/W is small, if one thinks of W as lifetime wealth, which is equivalent to saying that small variations in current income make little difference to lifetime wealth, and so should not lead to marked fluctuations in consumption and should not therefore be very costly, in present value terms. In fact, the assumption that consumers behave as though they viewed fluctuations in current income in terms of its impact on lifetime wealth appears empirically untrue, as Binswanger's (1981) experiments suggest. (See also, for a related set of "anomalies", Thaler, 1990.)

The use and estimated value of R given above strictly speaking refers to partial risk aversion, Rp, which appears to describe behaviour, though it is hard to believe that it accurately measures the true cost of the risk. Put it another way, if, as we do, we observe agents violating the behavioral predictions of the theory, then we should be wary of using that theory to measure the costs of risk. It is hard to know how to respond to this difficulty, but one obvious conclusion is that it may be seriously misleading to compute the present discounted value of the apparent annual costs of risk measured by the amount of consumption the consumer would be willing to forego in order to avoid that risk. This would turn the cost of risk into a wealth measure, which is inappropriate given that risk aversion in the example referred to fluctuations in income, not in wealth. It therefore remains somewhat unclear whether a risk cost of 2% of annual consumption should be viewed as "small" or "large".

The example given above was over-dramatic in its estimate of the amount of risk reduction which might be possible. The next step is to inquire into the likely sizes of possible risk reduction which might come from reducing the effects of commodity price instability.

1.1 The magnitude of the problem

Newbery and Stiglitz (1981) estimated the price variability of six agricultural primary commodities of importance to LDCs over the period 1951-75. They estimated the coefficient of variation (CV) of prices in a number of different ways. Three measures are of particular relevance. The first is the CV of price changes from one year to the next, or, to be precise, the standard deviation of $2(p_t - p_{t-1})/(p_t + p_{t-1})$. This is a crude measure of the year to year variability. A rather better measure of the unpredictability of commodity price movements is the CV of the price forecast errors. Newbery and Stiglitz used a simple first order autoregressive formula to predict prices, but in principle more sophisticated time series methods could be used. Finally, they gave the CV of deviations from 5-year centered moving averages, which measures the potential reduction in price instability which might be achieved with some time-averaging smoothing resulting perhaps from buffering prices, or some other system of price smoothing.

They found that the 5-year moving average gave the lowest measures of instability, slightly lower than the forecast errors, which in turn were slightly lower than the CV of price changes, though the differences were slight. Newbery (1990) recently updated estimates of the variability of 7 "soft" commodities, and two of the measures are given in Table 1.

The costs of price instability increase as the *square* of the CV, which means that if we take the measures of 5-year MA for the period 1970-86, coffee price instability (CV = 24%) is four times as costly as that of jute (CV = 12%), not twice as costly, as the figures might otherwise suggest. Table 2 gives the squared CVs for these two measures of price instability for the two periods.

It is desirable to reduce price instability for the world as a whole, provided that it is not too expensive to do so, as it generates arbitrage benefits. Indeed the price series reflect the fact that commodities are shifted (via storage) from low-price low-value dates to higher price higher value dates. However, the exporting countries are not directly concerned with the worldwide benefits of stabilization, and are instead interested in what happens to their average export revenue and its variability. International buffer stock schemes normally affect the average revenues received by exporting countries, and Newbery and Stiglitz (1981) argued that exporters might be adversely affected in the absence of supply restrictions. If, on the other hand, countries individually act to reduce the costs of risk, then they may have little effect on average revenue. Suppose we consider the effect of stabilizing the price received by these countries on the variability of their export revenue. Newbery and Stiglitz (1981, chapter 20) estimated the impact of completely stabilizing the price on the variability of crop export revenues by country. For typical countries price stabilization had a rather small effect on the crop export revenue instability, as this was often largely determined by variations in quantities. In some cases the reduction in variability was appreciable, and well worth having.

At this point it is worth asking an important question: How should the cost of country-level risk be measured? If the primary product producers face world prices, and if these are stabilized in some way, then the relevant measure depends on the variability of income from that commodity. If, on the other hand, the government already buffers domestic incomes, then the risk costs are borne by the government and the relevant question is what happens to the variability of total export revenue.

In the present paper we are primarily concerned with country level instability, assuming that the government is successful in shifting risk from producers to the government. It would therefore be useful to update these earlier estimates, broadening the question to ask what effect price stabilization might have on overall export revenue variability, rather than just on that part of export variability arising from the commodity exports. Table 3 shows the results of such an exercise for copper price stabilization for four countries for which copper was the main export, and for coffee price stabilization in Brazil and Colombia.

The table shows that stabilizing the price of copper and coffee has a rather small effect on the variability of export revenues for these commodities, and a smaller effect still on the variability of total export revenue. Figure 1 shows the effect of stabilizing the price of copper for Zambia, which is the extreme example of a country heavily dependent on one primary commodity. The vertical logarithmic scale gives the current US $ value of exports. Total exports, as can be seen, are almost synonymous with copper exports (shown dotted), while total exports when copper prices are stabilized are shown as the heavier line. The 5-year moving average of this stabilized export revenue is shown dashed, and is the reference level against which to compare the CV of variations.

The figures 3 through 6 show the effect of stabilizing the price of coffee for Brazil and Colombia. The vertical logarithmic scale gives the deflated U.S. $ value of exports. Figures 3 and 5 give deflated export values. Figures 4 and 6 give the percentage deviations from the 5 year Moving Average. In the second case, the graphs of deviations of commodity exports and total exports are displaced vertically to make it easier to see what is going on—coffee deviations are shown on the left-hand scale and total exports are on the right-hand scale. In both cases stabilizing the price of the main primary export appears to have little effect either on the variability of real income from the commodity or of total export revenue. It might be argued that supply variability would be reduce with reductions in price variability, in which case price stabilization would be more favourable than the graphs and table suggests.

2. Buffering the Export Price Instability Using Conventional Instruments

2.1 Loans and Savings

Countries have a variety of alternatives when confronted with instability in the price of their key export. If commodity prices are uncorrelated from year to year, and fluctuate around a known trend, and if the country can lend and borrow freely on international financial markets, then consumption can be smoothed out by lending and borrowing. The fluctuations in consumption will thus be substantially reduced and the cost of the risk correspondingly reduced.

Even if the country cannot borrow, it can accumulate savings in buffer funds which can be drawn down in adverse times with a substantial reduction in the costs of risk. Papua New Guinea has followed a conservative policy of averaging export receipts from copper (and other less critical export crops) over a lengthy time horizon, with a buffer fund held in convertible currency. While it is rare for a developing country to resist the temptation to invest such funds in its own capital stock (thereby losing the international liquidity needed to buffer trade fluctuations), both the International Monetary Fund, through its Compensating Financial Facility, and the Lomé countries, through STABEX, offer a similar buffering facility, being prepared to lend to countries at favorable rates when their export earnings drop sufficiently below trend.

Can a country optimally smooth consumption by borrowing and lending from overseas sources? Let us consider the most favorable and simplest case. Suppose that export revenue fluctuates from period to period in a serially uncorrelated way with no trend, and that the utility function is quadratic, and that the rate of pure time preference, δ, is equal to the rate of interest, r, abroad, then the country would have no motive for saving or borrowing other that to smooth consumption. We make this assumption to focus on the consumption smoothing aspect of international borrowing, and to show the kinds of problems that arise even in this simple case, when serial correlation is ignored.

If there are no constraints on lending and borrowing, then the optimally smoothed consumption of a borrower committed to borrowing and lending only for smoothing and to meeting his interest payment obligations is (Newbery and Stiglitz, 1981, pp. 201-3) $c_t = E_t(c_{t+1}) = \bar{y}_t - rL_t$. Under this scheme accumulated debt, L_t follows a discrete random walk with increment equal to the difference between income, y_t, and its mean value, $\bar{y}$. If this scheme is to continue to work, there must be no limit on L. But in finite time, L will pass the value at which reputation becomes more attractive than continued interest payments, even if lending and borrowing opportunities are then cut off. Knowing this, competitive lenders would not make unlimited loans. Any feasible loans would offer at best only suboptimal and/or impermanent smoothing.

Before addressing this particular income smoothing problem, it makes sense to look at other complications which affect income smoothing by lending and borrowing, and also to examine alternative instruments available. The first qualification is that, either because of poverty, or, more plausibly, because of the shortage of capital, the rate at which the country discounts future income (i.e. the return to investment) is higher than the world rate of interest. This would imply that in the absence of uncertainty the country should borrow as much as is prudent (i..e. as much as it would be willing to repay).

This will affect the potential for income smoothing, for two reasons. First, there is a temptation to approach the prudential borrowing limit, which reduces the ability to buffer shortfalls in income by further borrowing. This would not be so important if current investment could be readily scaled back at an intertemporal opportunity cost close to the marginal return to investment.

The next complication is that public sector expenditures may be hard to adjust swiftly. At each moment there may be high adjustment costs to departing form the original plan, and the fraction of the budget which is available for reallocation may be initially small, and growing over time. Certainly countries experiencing major debt crises and an urgent need for structural adjustment have found it very hard to make more than gradual adjustments to current public expenditures. If so, then recent work on hysteresis and adjustment costs by Dixit suggests that governments should embark on such expenditure programs only when their rate of return exceeds some test rate of discount by an adequate margin to cover the possibility of incurring adjustment costs from premature abandonment.

A problem with consumption smoothing is that it requires the country to know the trend level of earnings (and hence commodity price) about which to smooth. If prices follow a random walk, then the best estimate of the future price is the current price, and the more quickly consumption is adjusted to the revised level of export earnings, the better. Put another way, the variability in underlying wealth (or permanent income) is a small fraction (equal to the rate of interest) of the variability in current earnings if these fluctuate around a constant level, but is equal to the variability in current earnings if prices follow a random walk. The costs of risk in the latter case are thus much greater. It is therefore important to see which of these two extremes—serially uncorrelated prices or prices which follow a random walk—is nearer the truth.

2.2 Consumption smoothing with serial correlation

Many of the major traded primary commodities exhibit significant annual serial correlation which dramatically affects the costs and the benefits, and the feasibility of price stabilization. Cuddington and Urzúa (1987) have attempted to identify the extent to which price changes of primary commodities persist, using annual data on 24 commodity prices for the period 1900-1987, deflated by an index of manufacturing unit values. They regress the change in the log of the real commodity price on a constant plus error, e(t), which is in turn expressed at A(L)u(t), where u(t) is white noise. Their measure of persistence is then a_i, where a_i are the coefficients of A(L), and is a measure of the extent to which the price change will persist.

Table 4, reproduced from Cuddington and Urzúa (1987), gives the persistence measures of three groups of commodities, each group ranked in increasing order of persistence. (It also gives the highest order significant lag for the more parsimonious lag specification). Thus if one looks at cocoa, 65 percent of a price change is expected to persist, and the remaining 35 percent can be accounted by short term fluctuations, with a maximum (statistically significant) lag of two years. In each group the average persistence is over 50 percent, and for many commodities prices seem to follow a random walk with persistence of 100 percent. It was impossible to reject the null hypothesis that all commodity price series followed a random walk using the (rather weak) statistical tests available. One must interpret this rather carefully, for even if it is hard to reject the hypothesis that commodity prices follow random walks, there are no plausible theories which suggest that these prices should follow random walks, and rather good arguments why eventually they should return to an equilibrium determined by demand and supply.

Deaton and Laroque (1989) have also studied this problem, using more sophisticated methods, but the same commodity price data. Their measures of persistence are the sum of all autocorrelation coefficients (whether significant or not), with the sums being linearly declining weighted averages over the window widths of 20 or 40 years. Their results are reported in Table 5. Where the same commodity appears in both studies the measures of persistence from both studies are given in Table 4. In some cases the agreement is close--thus either 45 or 59 percent of the price shock in bananas is persistent, similarly for rice and palm oil. For others the differences are considerable, with Deaton and Laroque's (more reliable) measures tending to be lower than those of Cuddington and Urzúa, which Deaton and Laroque argue are likely on statistical grounds to be biassed upwards.

If one takes the Cuddington and Urzúa evidence then perhaps one-half of price shocks are persistent for many of the important export crops of developed countries. If one takes Deaton and Laroque's estimates then about one-quarter of price shocks are permanent. Even in this case, though, Table 5 shows the high first-order autocorrelations, so three-quarters or more of the price shock will persist for at least a year, and even after two years typically 60 percent of the price shock will persist. The evidence suggests, therefore, that serial correlation is prevalent for the world prices of primary commodities, and this fact should be taken into account in designing methods for consumption smoothing.

If income is serially correlated because prices are serially correlated, a fall in current income signals lower than anticipated income next year, and hence lower y_{t+1} and lower c_{t+1} (other things being equal). This will raise $u'(c_{t+1})$ and hence lower current consumption. If the autoregression coefficient is near unity, consumption may be depressed almost as much as current income and little smoothing will take place.

2.3 Price stabilization

If it is unattractive or infeasible to stabilize consumption by lending and borrowing, then perhaps it is possible to partially stabilize income and hence reduce the need to smooth consumption relative to income. If part of the reason for the export revenue fluctuations lies in commodity price instability, then perhaps price stabilization would achieve this goal. For the moment we ignore the fact that feasible market stabilization generally changes mean price. (See for example Newbery and Stiglitz 1981, or Williams and Wright forthcoming.) If we also ignore the distinction between income and consumption, then the risk benefits of reducing the CV of income as a proportion of initial income are approximately $1/2R\Delta[\sigma_y^2]$, where $\Delta[x]$ means the change in x and σ_y is the CV of income, and R is again the coefficient of (partial) relative risk aversion. If price and output are uncorrelated and income is taken as price times output, then the change in the squared CV of income will be the same as the change in the squared CV of price, and the risk benefit will be $1/2R(1-\alpha^2)\sigma_p^2$. On the earlier assumption that the coefficient of partial risk aversion is about 2, if $\alpha = 1/2$, and the share of exports to GDP is again one-third, (so that if risks are spread over the whole of GDP then the value of σ_p must be divided by 3), the risk benefits are then three fourths of the total cost of the income (consumption) instability.

It is not sufficient in this case to stabilize domestic income, for that merely shifts the risk from the producer to the government. International price stabilization may be able to achieve these gains, though, as noted above, with consequent problems of lowering the average price and revenue to the exporting country. Moreover, few commodities are at any moment subject to successful international price stabilization schemes, and so this option is typically not available (Gardner 1986; Gilbert 1986). What is needed is some way for the country to secure stable or partially stabilized prices, and the obvious answer is to use futures or forward markets, in the absence of international price stabilization.

2.4 Futures contracts

If prices were serially uncorrelated (which in turn means that storage from year to year is negligible) and if the futures price at the start of the crop year were an unbiased predictor of the post-harvest spot price, then there would be no reason for the opening futures price to vary from year to year, and the producer could lock in the same, stable price each year. Potatoes in the US provide a good example of such a commodity, for carryovers are costly and unusual.

But, as we have seen, serial correlation is prevalent, and the best start-of-year predictor of next year's post harvest price (i.e. the opening futures price) is unlikely to be the same as last year's start-of-year's forecast, or the opening futures price last year. It is now no longer so easy to stabilize income from year to year by hedging on futures markets at the start of the year.

To see what role futures markets may play, let us define notation:

p_t	Spot price at harvest in year t
$F_{t,j}$	Futures price for delivery after harvest in year t at date j - t.
$b_t = p_t - F_{t,t}$	Contemporaneous basis
$f_t \equiv F_{t,t-1}$	futures price at start of year t
$f_{t+1} - f_t$	Intertemporal basis

Trading in futures markets exposes producers to two different kinds of risk, both confusingly called basis risk. Contemporaneous basis risk arises because the producer who has sold futures to hedge output typically liquidates this by buying them back in the terminal month, and selling his output. If the terminal futures price were equal to the spot price there would be no risk, but in general this is not true, so ex ante the producer faces the risk that the two prices will not be the same—that is, he faces basis risk at the point of sale. While this basis is the stuff on which futures markets survive or perish, the risk involved is small compared to the risk of not hedging for most producers (i.e. those for which the futures market offers an appropriate contract). Of countries exporting primary commodities, many are selling on forward contracts which are linked to terminal futures prices, for which there is clearly no such basis risk (though, for example, transport costs to that point might be another source of risk in practice). We shall therefore ignore this type of risk, and assume in what follows that $b_t = 0$, or $F_{t,t} = p_t$. We shall also adopt the convention that $Ex_t \equiv E_{t-1}x_t$, and assume that the futures market is unbiased, so that $f_t = E_{t-1}P_t$.

Suppose prices follow the simple autoregressive scheme:

$$(2) \qquad \tilde{p}_t = \alpha p_{t-1} + (1-\alpha)\bar{p} + \tilde{u}_t,$$

where $\tilde{u}_t$ is i.i.d. with zero mean, and is the forecast error. This can also be written as

$$\tilde{p}_t = f_t + \tilde{u}_t, \quad f_t = E_{t-1}\tilde{p}_t = \alpha p_{t-1} + (1-\alpha)\bar{p}.$$

Again, f_t is the expected price, equal to the futures price at the start of period t in an unbiased market. The intertemporal basis, $f_{t+1} - f_t = \alpha(p_t - p_{t-1})$, will now fluctuate from year to year, possibly substantially. Hedging from year to year on futures markets will not provide insurance against this basis risk, but in the Appendix we show how to construct a sequence of rollover hedges in the futures market that provides considerable risk reduction and insurance against this basis risk. Even if futures markets only extend one year ahead, it is possible to roll over hedges to provide additional income smoothing to that achievable within the crop year.

The way the roll-over works is to sell more futures initially than needed for one-period hedging, and then use the surplus futures sales to finance the next year's futures transactions. This is not perfect, for the amount of hedging required next year will depend on production, and that will depend on the futures price prevailing next year, which is not yet known. Consequently, despite the absence of production risk, future output cannot be perfectly hedged, and there remains some residual risk (as there would be if there were output risk). Nevertheless, because the costs of risk increase with the square of the deviation, reducing the risk by a given fraction reduces the cost of risk by more than that fraction and can be worthwhile.

The appendix shows how to construct a rolling n-period hedge for the special case of no output risk, but supply responsive to futures prices. The model has a linear supply schedule (linear in the futures price, which is the action certainty equivalent price in the absence of output risk). In year t, production q_t is planned, and at the start of the year $q_t[1+\alpha\beta+...+(\alpha\beta)^{n-1}]$ hedges are sold on the futures market. Hedging for longer periods reduces risk, but requires additional purchases of hedges, which of course involve additional transactions costs. The Appendix derives a formula for the value of the additional risk benefit derived per extra present value of hedge, as increasing the current number of rollovers involves a stream of future transaction costs as well as a flow of future risk benefits. The formula for the marginal benefit/cost ratio from increasing the period of hedging from n-1 to n (and the number of hedges by $(\alpha\beta)^{n-1}$) when each extra futures contract costs μ is

$$\frac{(\alpha\beta)^{n+1}}{1+\beta}\frac{R\sigma^2}{\mu}$$

Clearly, as the time horizon of the hedge increases, the marginal benefit also falls. Figure 2 graphs the ratio of risk benefits to additional costs for four different values of the serial correlation coefficient, when the coefficient of relative risk aversion is 1, the discount rate is 5 per cent real, the CV of forecast error is 15 percent, and the transaction costs as a fraction of the value of the hedge is 0.3 of 1% — a figure taken from Gardner (1989).

Thus the graph shows that if α = 0.8, then it would be worth setting n = 4, and at α = 0.9, n should be 8. But it is clear that the value of such hedging (on the favorable assumption of no output risk) is quite low, as transaction costs are low and the benefit-cost ratio is in terms of these transaction costs. Higher transactions costs would shorten the horizon over which hedging was cost-effective.

The other point to make is that the number of hedges rises with the horizon, which would increase the risk of performance default if the contracts did not require payment of margin calls as the futures price changes, to cover any change in the value of the contract. The transaction costs calculated by Gardner include the foregone interest rate differential on the money left on deposit to cover margin calls, and this can be thought of as ensuring contract performance. A country could follow this hedging strategy and avoid performance risk, if it were unconstrained in credit markets. But this is exactly the situation in which lending and borrowing would also be a viable consumption smoothing strategy.

If the exporter is credit constrained, then performance risk must be a serious issue, and the hedging strategy may not be available. Instead it may be more logical to consider how commodity bonds might be used for consumption smoothing even in the presence of default risk. Let us begin with a brief review of commodity bonds as they have been used in practice.

3. Commodity Bonds

Commodity bonds are bonds whose terminal value (and perhaps dividend payments) are denominated in units of physical commodity (or the terminal value of some appropriate futures contract). Thus, a country might issue a bond paying 10 ounces of gold in 10 years' time with a current face value of $3,000 or a bond paying one lot of 10 tonnes of December maturing U.S. futures in cocoa for 10 years with a terminal payment of 25 contracts for a current face value of $350,000. Typically, the buyer has an option to receive the face value or the commodity bundle. That is, the bond usually comes with a call option for the buyer.

Before the second (1979) oil shock awakened the corporate interest in commodity bonds, governments were already using these instruments for various purposes. In 1863 the Confederate States of America issued bonds payable in bales of cotton (O'Hara). The French government used an electricity-indexed bond to compensate for 1945 nationalization of its utilities; and in 1973 "Le Giscard," a $1.5 billion issue with an untimely gold-guaranteed redemption value, was designed to persuade French gold hoarders to deposit their hidden treasure with the government (*New York Times*). The type of internationally-oriented government financing considered here was initiated later in the decade when a Mexican government agency made several bond issues in local currency backed by barrels of crude oil.

Recently, corporations have issued bonds with returns (principal and/or interest) payable in silver (Sunshine Mining); gold (Peggold); oil (Standard Oil Company); coal (Semirara Coal Corporation of the Philippines); and, for small investors requiring guaranteed liquidity of another sort, wine from the French Dordogne (Henry Ryman of the United Kingdom) or port wine (Dourosa Investments, United Kingdom).[2]

3.1 Smoothing with Commodity Bonds under Full Commitment

If the borrower can be fully committed to honor her contracts, commodity bonds are a powerful means of smoothing price variation.To simplify the exposition, assume that the package under discussion is a zero-coupon bond issued by the borrower with repayment upon maturity consisting only of a completely specified commodity bundle. We assume the purchaser (lender) is competitive and market risk-neutral with respect to this bond (see O'Hara for analysis of the demand side of the market for c-bonds under other assumptions). As above, assume initially that all contracts are always honored.

[2]The port contract is a pure zero coupon commodity bond; other contracts contain options to redeem at monetary face value.

Under these assumptions, if the country issues c-bonds (which in this model need only be one-period bonds) and if these can be issued (and indefinitely re-issued) at the present value of the expected price for next period, then their risk-reducing properties in the steady state are exactly the same as those of an optimal forward or futures hedge at the same price. Newbery and Stiglitz (p. 186) show that, in the case of stationary, uncorrelated output and price disturbances, the ratio of income variance with and without optimal forward hedging, is roughly $1/(1 + k^2)$, where k is the ratio of the CVs of price and output. In our numerical example in Section 1 above, k equals 1. If there is no other means of consumption smoothing by lending and borrowing, then c-bonds will halve the steady state costs of the risk—to 1% of GNP in our example. If the CV of income were the same, but only price were stochastic, then c-bonds eliminate risk, worth 2% of GNP.

Assume, henceforth, that no other borrowing is possible and that all income variation is due to price. Then with credible commitment, complete smoothing is achieved by selling c-bonds for the whole (deterministic) output. The borrower then has constant income and consumption and delivers all output of random value to the lender.

But what makes the commitment to deliver credible? Is this simple commodity bond contract subgame perfect? Note first that the lender's obligation within this contract is fulfilled at the start of the deal, by making the loan. Only the borrower has an unfulfilled obligation after the initial loan.

So within the contract period, only the sovereign borrower has any unfulfilled obligation, so she alone has an incentive to default. The incentive for her to default is state-dependent. This case with pure price uncertainty is illustrated in Figure 7, in which the world spot price P_t is on the horizontal axis and the exporter's contract payment per unit committed are shown on the vertical axis. If all sales are spot, then payment per unit and P_t are related by the 45^o line OA.

The simple (non-contingent) commodity bond can be considered as a combination of a one-period loan and a forward contract of the same duration. Under a *forward* contract, the borrower's incentive to default is the difference between the spot price at maturity, P_t, and the forward price to be paid on delivery. The latter equals the expected price $\bar{P}$ as of the signing of the contract, under the assumptions of risk neutrality, competitive buyers, and credible seller commitment to deliver. The short-run temptation to default (to be weighed against any effects on future smoothing opportunities) is $P_t - \bar{P}$; the higher the spot price, the greater the temptation. The short-run default incentive of the buyer of the contract (the "long" side) is, symmetrically, $\bar{P} - P_t$.

In a commodity bond contract, the borrower incurs at the outset a repayment obligation of $\bar{P}$ per unit of exports (from a loan of $\bar{P}/(1 + r)$ per unit in the previous period) in addition to the delivery obligation. This adds the amount of the loan repayment under compliance, $\bar{P}$, to the short-run incentive to default. The temptation to default is thus P_t.

This default temptation at time t must be balanced against the opportunity cost of defaulting. For full commitment the latter must dominate. For conventional domestic lending the loan is backed by collateral that may be seized by the lender in the event of default. Assuming the lender has sufficient collateral, default does not occur.

In the finance literature, studies of the pricing of commodity bonds (Schwartz; Carr) do not distinguish bonds issued by foreign governments from private corporate bond issues—though the recent literature on foreign borrowing recognizes that the distinction is crucial for ordinary bonds. It is also crucial for commodity bonds.

3.2 Sovereign borrowing and default prevention

The main distinction between corporate and sovereign borrowing, described in masterly fashion by Keynes and incorporated in the seminal work of Eaton and Gersovitz, is that collateral is generally unavailable to creditors of a sovereign borrower since the assets of the latter are located within its borders. Only in exceptional cases can they be attached by lenders in the event of default. In the absence af attachable collateral, some substitute must be found if any lending is to occur in equilibrium. Three such substitutes, recognized in the literature, are witholding of future access to loans, direct intervention backed by military power, and interference with trade (Eaton and Gersovitz 1981, Bulow and Rogoff 1989).

The absence of a final distribution of assets to creditors as seen in domestic bankruptcy also changes the nature of default on conventional loans. It arises in the context of a sequence of strategic moves by creditors and the sovereign debtor who retains (and, in fact, cannot credibly foreswear) the power to make subsequent decisions that affect the interests of creditors. The cost and uncertainty of these renegotiations is widely recognized as a serious problem for lenders, borrowers and intermediaries.

Here we focus on income-smoothing financial transactions between investors in developed countries (DCs) and a less-developed country (LDC) heavily dependent on a single commodity subject to substantial revenue fluctuations. The default penalty is enforcement of debt seniority clauses in the courts of all potential financial partners of lender nations so that a defaulter's foreign investments or servicing of new debt would be subject to seizure. Default means permanent elimination of foreign borrowing or lending opportunities. We assume throughout that the lenders in developed countries must always honor their commitments.[3]

The cost of default is the loss of expected future consumption smoothing that could be had given no default at time t. The borrower's motivation to fulfill her part of the contract depends on her expectations of continued lending (beyond current contractual horizons) conditional on her current behavior. If her behavior complies with equilibrium expectations of the lender, then she can expect the competitive lenders to be willing to conform to the equilibrium in the future as they have proved to currently.

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[3]For an anlysis under the alternate assumption, see Kletzer and Wright (1990).

If the price distribution is such that *P* is always less than the minimum level that makes default profitable, there is no default problem, and borrower commitment to repay in all states is credible. If not, then potential lenders, foreseeing the possibility of default, realize the above contract will yield an expected loss, and do not buy the bond.

The credibility of a no-default commitment depends upon the parameters of the model. Henceforth we concentrate on the case of pure price uncertainty with income $y = p\bar{q}$,where $\bar{q}$ is fixed output. Consider the simple example in which $\bar{y}$ and $\bar{P}$ normalized at unity, and $y_t = \bar{y}(1 + u_t)$. The probability density for the multiplicative disturbance u is i.i.d. with u=+/-v, each with probability of one half, so that the coefficient of variation of price, and of income is v. The annual current cost of risk in this case is $Rv^2/2$ with present value $Rv^2/2r$, where r is the discount rate.

Consider the stochastic steady state in which a fraction α of output, $0 < \alpha < 1$, is covered each period by commodity bonds. Each period a fraction α of output is delivered in payment of the previous loan and a new loan of $\alpha/(1 + r)$ is received. Given a price draw of $(1 + v_t)$, consumption is $[(1 - \alpha)(1 + v_t) + \alpha/(1 + r)]$ if the old contract is fulfilled. The contract is rationally honored if the current temptation to default, $v - \alpha v$, is less than the present value of the extra risk cost incurred, $Rv^2(1-\alpha^2)/2$, that is, if $r < Rv(1+\alpha)/2$; coverage of at least some portion of output is feasible if $r < Rv/2$.

4. Overview of Dynamic Smoothing Strategies

4.1 Default Constraint Nonbinding

From an initial uncovered situation, the availability of commodity bonds adds to the short-run resources represented by initial income y_1. Assume that the sovereign starts with no savings, but that she can save overseas in the countries that host the international lenders. Assume also that these lender countries collectively enforce financial contracts within their borders. In particular, they cooperatively enforce claims by foreigners on domestic assets, and senior claims of domestic lenders on sovereign borrowers are enforced with respect to all inflows from sovereign borrowers, including savings deposits as well as loan repayments.[4] If so, one description of the optimal infinite horizon smoothing plan for implementation in period 1, given current income, y_1 (assumed for this exposition to be entirely from export of one commodity at price p), and the discount rate equal to the interest rate is as follows: Invest βy_1, where $\beta \equiv 1/(1 + r)$, overseas for a certain periodic rate of return of r, issue a simple c-bond payable in units of the commodity to cover all output, with current sale price $\beta \bar{y}$, and consume $r\beta y_1 + \beta \bar{y}$ in each period 1, 2, 3,.... Full consumption smoothing is immediately achieved forever: consumption is the same for all periods and states.

The opportunities for legally protected overseas investment at the (certain) market interest rate and for sale of c-bonds at unbiased prices are all the financial facilities needed for this plan. Furthermore, note that, if the initial income, y_1, is invested where it

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[4]Both types of enforcement together support the dynamic smoothing contracts that follow. Bulow and Rogoff (1989) show that if the former type alone is effective, the smoothing strategies formulated below do not work.

can be collateralized for the c-bond loan (for example in the lending country), the default constraint is relaxed relative to the comparative static analysis above that assumed all income was from sales of c-bonds and none of the current income in the period in which c-bonds were introduced was saved. So, even if full c-bond coverage seemed infeasible in that analysis, the above strategy may work.

If one ignores transactions costs, as we do here, a number of different combinations of contracts could replicate the above arrangement, given the assumption of a nonbinding default constraint. One example is a short forward contract plus a loan on the anticipated proceeds of the contract. Several commentators have suggested that a combination of a futures contract and a loan would also be equivalent. If the futures contract were continuously marked to market as price varied over the time between commitment and maturity, the incentive to default would be removed because losses from adhering to the contract are paid out as margin calls as they accrue. (For example, when the futures price of a commodity rises by a dollar, the short side pays a dollar per unit hedged to the clearing house.) And this leads to additional uncertain increases or decreases in credit requirements on the part of the hedger. In practice this can result in serious complications, especially if trading is obstructed by price move limits for significant periods, and/or interest rates move substantially and are not themselves hedged.

If the default constraint binds on hedging with commodity bonds or forward contracts, the full smoothing described above is infeasible. The alternative of using futures markets is precluded because the variation margin requirements that make default unattractive cannot be met by a liquidity-starved borrower. Nor will the margin calls be loaned by a third party lender because of the induced incentive of the borrower to default on those loans.

4.2 Default Constraint Binding

If the default constraint binds, preventing full coverage via a simple commodity bond, the immediate transition to permanent full consumption smoothing is precluded. What kinds of consumption smoothing contracts are feasible in such cases?

The common type of commodity bond, (as reviewed above), with a call option for the buyer, is clearly inappropriate for this type of smoothing. True, the premium associated with the option would increase the lower consumption levels if the contract were feasible. But by selling the call to the lender, the borrower places herself under great temptation of default when price is high, and gets very inefficient low-end protection.

A more promising strategy is to limit the maximum temptation for the borrower by giving her some share in the marginal gain from increases in high prices, while limiting her maximum losses. An optimal state-contingent loan contract would be ideal.

4.2.1 The optimal state-contingent, no-default contract

Before presenting the commodity bond package, we consider as a standard of comparison the optimal consumption plan for a risk-averse sovereign commodity exporter, "the borrower." Assume that the risk-neutral lender must achieve non-negative expected profits in a long-term contract and that there is free entry. The borrower can repudiate the long-term relationship, and will do so whenever permanent autarky is superior to continuation of the consumption-smoothing relationship.

The problem is to maximize

$$u(c_1)+E\sum_{t=2}^{\infty}\beta^{t-1}u(c_t), \tag{3}$$

with respect to state-contingent consumption plans $\{c_t\}_{t=1}^{\infty}$, where $c_t=c_t(y_1,\dots,y_t)$,

subject to the no-default constraint for the borrower

$$u(c_t)+E\sum_{i=1}^{\infty}\beta^i u(c_{t+i})\geq u(y_t)+E\sum_{i=1}^{\infty}\beta^i u(y_{t+i})=u(y_t)+\left(\frac{\beta}{1-\beta}\right)E\,u(y) \tag{4}$$

for every $t=1,2,\dots$, and the profitability constraint for the lender,

$$(y_1-c_1)+E\sum_{t=1}^{\infty}\beta^{t-1}(y_t-c_t)\geq 0. \tag{5}$$

This is similar to the problem of finding an optimal implicit long-term wage contract, as in Holmstrom (1983). We can rewrite it as a dynamic programming problem. Define the history of states as w_t, where $w_t\equiv(y_0,\dots,y_t)$ and $w_{t+1}\equiv(w_t,y_{t+1})$. Let $V_1(V_2^t)$ represent the maximal surplus that the exporter gets at time t from the consumption smoothing plan over permanent autarky when the risk-neutral "lender" receives profit V_2^t. At time t the lender's profit V_2^t.

will be a function of w_t, that is $V_2^t \equiv V_2(w_t)$. The function, $V_1(\cdot)$, is given by the optimality equation:

$$(6) \qquad V_1(V_2^t) = \max\left\{u(c_t) - u(y_t) + \beta E\, V_1(V_2(w_{t+1}))\right\},$$

with respect to c_t and $\left\{V_2(w_t, y_{t+1})\right\}_{y_{t+1}=y^1}^{y^S}$

subject to

$$(7) \qquad V_2^t \leq (y_t - c_t) + \beta E\, V_2(w_t, y_{t+1}),$$

and

$$(8) \qquad 0 \leq V_1(V_2(w_t, y_{t+1})), \forall y_{t+1} \varepsilon \{y^1, \ldots, y^S\}$$

and

$$0 = V_2(y_0).$$

That is, solving (3) - (5) for the state-contingent infinite horizon consumption plan is equivalent to solving problem (6) at each date t, in each event w_t, for c_t and the (promised) profit to the lender in each state of nature, y_{t+1}, for the next period, by Bellman's principal. A sufficient condition for a solution to (6) to exist is that the global endowment is bounded in each period (so that c_t is bounded).

Because u(c) is strictly concave and continuously differentiable, the function $V_1(V_2^t)$ can be shown to be strictly concave and continuously differentiable as well. (The constraints define a convex choice set; with bounded global endowment, it is also compact.) We form a Langrangean for problem (6) and assigning the multiplier $\lambda(w_t)$ to constraint (7) and multipliers $\phi(w_t, y^1), \ldots \phi(w_t, y^S)$ to the constraints (8), we obtain from the first order conditions and envelope condition:

(9) $$u'(c_t) = \left(1+\phi\left(w_t, y_{t+1}\right)\right) u'\left(c_{t+1}\right),$$

(10) $$\phi\left(w_t, y_{t+1}\right) \geq 0 \text{ and } \phi_{t+1} \cdot V_1\left(V_2\left(w_{t+1}\right)\right) = 0,$$

for every $y_{t+1} = y^1, \ldots, y^S$.

Therefore, consumption is monotonically increasing over time and whenever the default constraint is not binding, in equilibrium, $(\phi_{t+1} = 0)$, $c_t = c_{t+1}$. Consumption in period $t+1$ will be smoothed across states for which the default constraint is not binding.

To finish characterizing the optimum subject to potential repudiation, we define the minimum consumption the debtor will accept in each state in the optimum. Define $\underline{c}^i$ by

$$V_1\left(\overline{V}_2^i\right) = u\left(\underline{c}^j\right) - u\left(y^j\right) + \beta \, E V_1\left(V_2\left(w_{t+1}\right)\right) = 0,$$

where $\overline{V}_2^i = y^j - \underline{c}^j + \beta \, E V_2\left(w_{t+1}\right)$ and $V_1\left(V_2^i\right)$ solves problem (6).

We can prove the following:

$$\underline{c}^1 < \underline{c}^2 < \underline{c}^3 < \ldots < \underline{c}^S \text{, and } \underline{c}^1 = y^1.$$

Next, we can calculate $\underline{c}^j$ using (7) and (8):

$$\begin{aligned} 0 &= u\left(\underline{c}^S\right) - u\left(y^S\right) + \beta \, E V_1^{t+1} \\ &= u\left(\underline{c}^S\right) - u\left(y^S\right) + \frac{\beta}{1-\beta} E\left(u\left(\underline{c}^S\right) - u(y)\right), \end{aligned}$$

because $\underline{c}^S > \underline{c}^J$, for all $j < S$; therefore, by (9) $c_{t+1} = \underline{c}^S$ for every state y_{t+1}.

Thus,

$$u(\underline{c}^S) = (1-\beta)u(y^S) + \beta\,E(u(y)).$$

The solution for $\underline{c}^j$ can be found to be given by:

$$u(\underline{c}^j) = (1-\beta)u(y^j) + \beta\left[\sum_{i=1}^{j-1}\pi_i\,u(y^i) + u(y^j)\sum_{i=j}^{S}\pi_i\right]$$

where π_i is the probability that state i occurs.

Under free entry, initial consumption will satisfy

$$(y_1 - c_1) + \beta\,E\,V_2^1 = 0.$$

we have the following result:

There exists a state N (which depends on all of the parameters of the problem) such that if $y_1 \geq y^N$, then consumption is fully smoothed for all dates and states. That is for $y_1 \geq y^N$, we can find c_1 to satisfy

$$c_1 = (1-\beta)y_1 + \beta\,E\,y \qquad \text{(zero profit)}$$

and

$$c_1 \geq \underline{c}^S$$

For $y_1 < y^N$ (if $n > 1$), these cannot both be satisfied, so that we have a state j such that

$$c_1 \geq \underline{c}^j \text{ and } c_1 < \underline{c}^{j+1},$$

for for $1 \leq j < S$. Consumption is the same in period 2 as in period 1 for all y in period 2 less than or equal to y^j. If $y_2 = y^i > y^j$, then by (9) and (10), $c_2 = \underline{c}^i$. Therefore, $c_t = c_1$, as long as a state exceeding j does not occur. Once a state greater than y^j occurs, consumption permanently rises. That is, $c_t = c_1$, until some state $y > y^j$ occurs; thereafter, $c_{t+1} = \underline{c}^i$, where $y^i = \max\{y_1, \dots, y_t\}$, for $y_{t+1} \varepsilon \{y^1, \dots, y^i\}$ for $y_{t+1} = y^l$ s.st. l>i. Whenever $c_1 < \underline{c}^s$ the long run consumption is given by $\underline{c}^s$— in the steady state consumption is fully smoothed. Before the steady state is achieved, consumption is smoothed for each date (t+1) over the set of states $\{y^1, \dots, y^i\}$ where y^i is the highest state realized before t+1. Consumption is monotonically increasing over time (weakly).

4.2.2 Implementation with one-period state-contingent loan contracts

The optimum can be achieved using one-period loan contracts with state-contingent repayment schedules under free entry by competitive, risk-neutral lenders. Suppose the loan contract specifies an amount l_t and repayment schedule $R_t(y_{t+1})$ for period t+1.

The zero expected profit condition is

$$-l_t + \beta\, E\, R_t(y_{t+1}) = 0$$

Returning to problem (6):

$$V_1(V_2^t) = \max\left\{u(c_t) - u(y_t) + \beta\, E\, V_1(V_2^{t+1})\right\}$$

$$\text{(11)} \qquad \text{s.t.} \quad y_t - c_t + \beta\, E\, V_2^{t+1} = V_2^t$$

<u>Define</u>

$$R_{t-1}(y_t) = V_2^t$$
$$R_t(y_{t+1}) = V_2^{t+1} \equiv V_2(w_t, y_{t+1})$$
$$\text{and } l_t = c_t + V_2^t - y_t .$$

Then the constraint (11) satisfied by the solution to (6) assures that

$$-l_t + \beta E(R_t(y_{t+1})) = 0,$$

and $$V_2(y_0) = 0 \Rightarrow -l_1 + \beta E(R_1(y_2)) = 0,$$

for the initial loan. Therefore, a sequence of one-period state-contingent contracts suffice to attain the optimal plan, as Worrall (1989) has shown.

Next, we note that because consumption is completely smoothed for all states, y, less than or equal to the historical maximum or state j (defined above) whichever is greater, we have c_{t+1} constant for all i s.t. $y^i \leq \max\{y^j, y_1, \ldots, y_t\} \equiv y_t^*$. This implies that for all $y_{t+1}^i \leq y_t$, the optimum state-contingent loan contract (1-period) has repayments $R_t(y_{t+1})$ s.t. $y_{t+1} - R_t(y_{t+1}) =$ constant. For states $y_{t+1} > \hat{y}_t$, $(y_{t+1} - R_t(y_{t+1}))$ will, in general, vary with y_{t+1}. The new loan contract, $(l_{t+1}, R_{t+1}(y_{t+2}))$, will remain the same as last period for all $y_{t+1} \leq \hat{y}_t$, but will change for $y_{t+1} > \hat{y}_t$.

4.2.3 Smoothing with commodity bonds

We now show that a commodity bond constructed as a package of a one-period loan and a put option for the exporter can achieve feasible smoothing. Indeed it results in a pattern of smoothing similar to that seen in the more complex optimal state-contingent loan contract discussed above. The commodity bond contract never dominates the fully state contingent contract in our model. But it has the offsetting advantage of simplicity, and the associated potential for more liquid trading as a relatively standardized instrument.

The 1-period put option on the commodity has strike value y_t^*. The exporter exercises the put if $y_t \leq y_t^*$, receiving income y_t^*. The option premium, z_t^*, is given by the zero profit condition:

$$z_t^* = E\left(\max\{y_t^* - y, 0\}\right).$$

This must be paid in every state. We let it be paid at date t, the same date that the put is exercised or expires. Therefore, the exporter gets from this option the net income:

$$\max\{y_t, y_t^*\} - z_t^*.$$

If the put is exercised $\left(\text{i.e. } y_{t+1} \leq y_t^*\right)$, then assume that the same put option is contracted for the next period. If $y_{t+1} > y_t^*$ then a new put is chosen with higher strike value y_{t+1}^*.

Now, let the exporter also have access to a loan market with standard non-state-contingent bonds (one-period). The penalty for non-repayment is removal of all opportunities for smoothing in the future, whether by borrowing or via options. In period t, she chooses an option contract with strike value y_{t+1} for the next period and a loan l_t (positive or negative). The repayment due at t+1 is l_t / β, so that her consumption at t+1 is

$$c_{t+1} = [\mathrm{Max}\{y_{t+1}, y_{t+1}^*\} - z_t^* - l_t / \beta] + l_{t+1}$$

To find the equilibrium path in c-bonds, define the state variable:

$$B_t = \text{Max}\{y_t^*, y_t\} - z_t^* - l_{t-1}/\beta.$$

At time t, the exporter chooses l_t and y_t^*.

The equilibrium problem is the same as the dynamic programming problem:

$V_q(B_t) = \text{Max}\{u(B_t+l_t)-u(y_t)+\beta E_k V_k(B_{t+1})\}$ with respect to (l_t, y_{t+1}^*) such that:

$$B_{t+1} = \text{Max}\{y_{t+1}, y_t^*\} - z_t^* - l_t/\beta$$

and

$$V_k(B_{t+1}) \geq 0, \quad \forall k,$$

where subscripts q and k indicate realized states y^q and y^k.

The first order conditions imply

$$u'(c_t) = E[V_k'(B_{t+1}).(1+\phi_k)]$$

and

$$E[V_k'(B_{t+1}).(1+\phi_k)] = E[V_k'(B_{t+1}).(1+\phi_k) \mid y_{t+1} \leq y_{t+1}^*]$$

where ϕ_k is the multiplier for the constraint $V_k(B_{t+1}) \leq 0$,

$$\phi_k . V_k(B) = 0, \quad \phi_k \geq 0.$$

If $y_{t+1} \le y_t^*$ then B_{t+1} is constant. For each state $y_{t+1} \le y_t^*$, the borrower chooses the same consumption by choosing the same l_{t+1}. Therefore, for states less then or equal to y_t^* the state-contingent pattern of net income is similar to that of the optimal one-period state-contingent loan contract.

However, for $y_{t+1} > y_t^*$, "net income" B is $[y_{t+1} - z_t^* - l_t / \beta]$, so that B increases with y_{t+1} one for one, and

$$V_k'(B_{t+1}) = u'(B_{t+1} + l_{t+1}) = u'(c_{t+1}(k)).$$

This is not necessarily true for the optimal one-period state-contingent loan contracts; net income for them is given by

$$[y_{t+1} - R_t(y_{t+1})].$$

From the first order conditions,

$$u'(c_t) \ge u'(c_{t+1}) \quad \text{for } y_{t+1} \le y_{t+1}^*$$

and $\phi_k = 0$ for k<n where

$$n = \text{Max}\{k: \; y^k \le y_{t+1}^*\}.$$

For $y^k < y^n$, consumption c_{t+1} is monotone increasing in y_{t+1}^*.

How does the put option cum non-contingent bond scheme work? Start off with y_1; the exporter takes an initial loan l_1 and consumes $c_1 = y_1 + l_1$ at t=1 and contracts for a put option with strike income y_1^* and premium

$$z_1^* = E[\mathrm{Max}\{y_1^* - y, 0\}].$$

Next period, if $y_2 \le y_1^*$,

$$c_2 = (y_1^* - z_1^* - l_1 / \beta) + l_2.$$

y_1^* is chosen such that in equilibrium the exporter does not default in any states at t=2, given l_1, and $c_2 = c_1$ if $y^2 \le y_1^*$. If $y_1^* < y^s$, then there is some state such that if y_2 equals this state the exporter is just indifferent between being able to continue smoothing her consumption and permanent autarky. Choosing a higher strike income than y_1^* to obtain more insurance for period 2 to insure compliance by the exporter lowers first period consumption. There is a trade-off between smoothing across states of nature at t=2 and between dates one and two.

If $y_2 \le y_1^*$, the exporter just repeats her choice of contract, choosing the same put option and non-contingent loan repayment (l_1 / β) for period 3. But in the first period t in which y_t exceeds y_1^*, her choice of strike income rises to a level sufficient to smooth her consumption in the following period for all $y_{t+1} \le y_t$. Her choice of loan also changes. The new put is exercised for all $y_{t+1} \le y_t$, and the borrower chooses a new loan to make c_t as large as possible without causing her to choose to default in any state in the next period, t+1.

The strike income rises each time a new historical high occurs. Once the highest state y^S occurs, consumption is smoothed across all states and remains constant thereafter. The steady state consumption is given by

$$c^* = Ey + (1 - 1/\beta)l^*$$

since

$$z^S = E\, Max(y^S - y, 0) = y^S - Ey.$$

To avoid default in the steady state, c* must be at least as great as $\underline{c}^s$. This implies an upper bound on l*. For some sets of parameters of the model, $\underline{c}^s > \varepsilon y$ so that l* must be negative. The exporter will have to invest some of her current resources externally and use the interest to augment the feasible smoothed consumption level c^* so that $c^* \geq \underline{c}^s$. (Since domestic investment is ruled out by assumption here, the investment must be external. We assume it is subject to seizure by creditors in the event of default.)

$$c_1 = y_1 + l^* < y_1,$$

so if y_1 is low, say $y_1 = y^1$, then c_1 is strictly less than c^*.

If the initial state, y_1, is low, full insurance might not be possible immediately. To achieve a high enough level of smoothed income in the next period to preclude repudiation in a high state, the initial loan must satisfy $\varepsilon y + l(1 - 1/B) = \underline{c}^s$. Therefore first period consumption is $c_1 = y_1 + l$ which can be much lower than c*. (For example let $y_1 = y^1$.) Thus full smoothing at a certain, constant level of c for all future periods cannot be achieved immediately in all cases. Furthermore, feasible full smoothing after a one period delay is less desirable for the exporter than a program with a higher initial loan and initial consumption, but no guarantee of full smoothing next period in all states.

In general, there is a dynamic tradeoff between the current consumption level and the amount of insurance coverage for the next period. The consumption floor ratchets upward whenever a new historically highest state is first visited, until full smoothing is achieved.

Thus this scheme, using only 1-period bonds and put options, exhibits the type of monotonically rising consumption floor seen in the pattern of smoothing generated by the first-best long-term contract. Note that $\underline{c}^s$ is the same in the first-best as here, but $\underline{c}^j$, $\forall j < S$, is in general different from the first best. Steady state consumption is $c^* \geq \underline{c}^s$.

The constrained first-best path and the put-cum-bond path are the same if initial income is high enough to allow immediate complete smoothing at $c^* \geq \underline{c}^s$, or if there is only one realization of y, y^S, that is above the minimum strike income chosen when $y_1 = y^1$. The paths differ in other cases because the put-cum-bond scheme is not fully state-contingent for realizations of y above the strike income. The only way to alter the differential between the consumptions associated with two such states in t+1 is by varying l_{t+1}, but that affects the relation between c_{t+1} and c_{t+2}. There are too few instruments to replicate the constrained first-best path in such cases.

The non-contingent bond and the option premium are repaid in the same amount in every state. For states below the strike income this is not a problem as all income variation between such states is removed by the option. If there are multiple higher states the contracts are incomplete in the sense that they do not allow for state-contingent repayments as in the more complex first-best contractual design, when initial income is low. Though the smoothing behavior under the two types of contracts is qualitatively similar in such cases, with a minimum consumption level that ratchets up till it achieves a constant level, there is a difference in the efficiency of the smoothing achieved.

5. Conclusions

Consumption smoothing contracts might in principle be quite valuable to many countries heavily dependent on an export commodity subject to price risk. It is frequently said that futures coverage could be useful in this role if they had longer maturities. In this paper we have shown that substantial long-term protection can be achieved by rolling over one-period futures. The marginal net benefits of lengthening the horizon beyond one production period (roughly what is observed in practice) depend upon transactions cost, the degree of serial correlation, and the discount factor. In practice, the extra benefits of a substantially longer hedging horizon may often be rather small.

If production responds to incentives with a one-period lag, the rollover strategy does not provide perfect protection at the time the hedge is made. This is true even if production response to inputs is non-stochastic, in contrast to the case of one-period hedging.

In cases where a sovereign exporter can offer no collateral, and is short of liquid resources, the use of futures is precluded by the need to furnish the margins that guard against default. Standard loans and buffer funds have the disadvantage that they will with probability one reach crisis states in which the resolution of the crisis is ill-defined; recognition of this by lenders no doubt dampens their enthusiasm somewhat.

In this context commodity bonds with a put for the seller (borrower-exporter) offer at least part of the benefits of using fully state-contingent contracts constrained only by sovereign immunity. In fact when initial conditions are sufficiently good the two are identical. A straight commodity bond suffices for fully smoothing consumption. When the initial state is bad, commodity bonds combining a put and a loan for the exporter can achieve some degree of consumption smoothing in the face of random export prices for commodity-dependent countries that cannot offer credible collateral for foreign loans. Consumption is nondecreasing over time and becomes fully smoothed if and when the highest income state is visited.

Though put-cum-loan bond contracts do not in general achieve a constrained efficient consumption path, they have the significant practical advantage of comprising two similar and simple instruments, a conventional loan and a put option. Any additional state contingencies needed for a constrained efficient contract will be country-specific. The put options, on the other hand, could in principle be used by multiple countries as appropriate, and therefore are more likely to have a liquid market and therefore lower transaction costs. Likewise, in equilibrium the bonds are always repaid in full; there is no prospect of costly loan renegotiations. This commodity bond, constructed as a conventional loan with fixed repayment obligation and an attached put for the seller, contrasts with commonly observed commodity bond contracts, which generally attach a call option for the buyer to the loan. The consumption-smoothing achieved reduces downside exposure of the seller, while leaving her a sufficiently large share of high realizations that she is not tempted to default.

Though we have shown this only in the case of pure price uncertainty with i.i.d. disturbances (and, hence, no interperiod storage), availability of a constant risk-free rate of return and market risk neutrality of lenders, our results suggest further investigation of the smoothing possibilities of these instruments in more general circumstances. Drawing on our results for futures rollovers, we infer that in the presence of serial correlation of of the disturbance, commodity bond "rollovers" will not in practice be used to eliminate all consumption variations over multiple periods.

Table 1 Commodity price instability, 1950-1986

Commodity	Coefficient of variation, percentages			
	1950-69		1970-86	
	5-yr MA	price change	5-yr MA	price change
Cocoa	21	25	22	28
Coffee	12	16	24	35
Tea	7	11	19	23
Sugar	35	39	38	47
Cotton	6	13	13	19
Jute	20	22	12	18
Rubber	16	24	18	23

Source: Newbery (1990, Table 5.1)

Notes: Price change is the standard deviation of $2(p_t - p_{t-1})/(p_t + p_{t-1})$.
5-year MA is the CV of deviations from the 5-year moving average.

Table 2 Squared coefficient of variation of prices, 1950-1986

percentages

Commodity	1950-69		1970-86	
	5-yr MA	price change	5-yr MA	price change
Cocoa	4.4	6.3	4.4	7.8
Coffee	1.4	2.6	5.8	12.3
Tea	.05	1.2	3.6	5.3
Sugar	12.3	15.2	14.4	22.1
Cotton	0.4	1.7	1.7	3.6
Jute	4.0	4.8	1.4	3.2
Rubber	2.6	5.8	3.2	5.3

Source: Table 1

Table 3 Effects of stabilizing copper and coffee prices. 1961-1986

percentages

Country	Average export share (1)	Revenue unstabilized (2)	Revenue stabilized (3)	Exports unstabilized (4)	Exports stabilized (5)
	Coefficients of variation				
Copper					
Chile	50	18	17	12	8
Zaire	39	16	16	31	24
Zambia	88	18	16	18	12
Papua New Guinea	20	33	13	14	13
Coffee					
Brazil	22	22	22	17	16
Colombia	43	19	15	10	10

Source: World Bank data

Notes:
(1) is average share of exports in total export revenue
(2) is the CV of deviations from 5-yr MA export revenue
(3) is the CV of deviations from 5-yr export revenue valuing the exports at prices stabilized at their 5-yr MA level
(4) is the CV of deviations from 5-yr MA total export revenue
(5) is the CV of deviations from 5-yr MA total export revenue valuing the exports at prices stabilized at their 5-yr MA level

All export revenues were deflated by the Index of Manufacturing Unit Value (MUV).

Table 4 Persistence of price shocks, 1900-1987

Commodity	Persistence		Longest Lag years
	Autocorr measure	Deaton PER20	
Rice	0.11	0.18	9
Palm oil	0.13	0.13	5
Coffee	0.38	0.17	11
Bananas	0.45	0.59	10
Wheat	0.46	0.24	10
Sugar	0.52	0.11	6
Cocoa	0.65	0.29	2
Tea	0.72	0.37	2
Beef	1.0		0
Maize	1.1	0.19	10
Lamb	1.30		4
Average	0.61		6.28
Timber	0.1		8
Wool	0.35		2
Jute	0.4	0.19	5
Hides	0.43		2
Cotton	0.67	0.39	3
Tobacco	0.73		4
Rubber	1.0		0
Average	0.51		3.43
Oil	0.51		11
Silver	0.65		8
Tin	0.65	0.43	5
Lead	0.73		3
Aluminium	0.93		5
Zinc	1.0		0
Copper	1.0	0.31	0
Coal	1.0		0
Average	0.81		4.0

Source: Cuddington and Urzúa (1987), Deaton and Laroque (1989, Table 2)

Notes: Annual data. The first measure is the sum of the statistically significant autocorrelation coefficients, as calculated by Cuddington and Urzúa and explained in the text. Deaton and Laroque's measure of persistence is PER20, given in from Table 5 below, and explained therein. The longest lag is the highest order statistically significant lag.

Table 5 Variability and persistence of annual commodity prices, 1900-1987

Commodity	CV	AR1	AR2	PER20	PER40
Bananas	0.17	0.91	0.82	0.59	0.52
Cocoa	0.54	0.83	0.66	0.29	0.24
Coffee	0.45	0.80	0.62	0.17	0.11
Copper	0.38	0.84	0.64	0.31	0.22
Cotton	0.35	0.88	0.68	0.39	0.13
Jute	0.33	0.71	0.45	0.19	0.09
Maize	0.38	0.76	0.53	0.19	0.10
Palm oil	0.48	0.73	0.48	0.13	0.05
Rice	0.36	0.83	0.61	0.18	0.08
Sugar	0.60	0.62	0.39	0.11	0.06
Tea	0.26	0.78	0.59	0.37	0.28
Tin	0.42	0.90	0.76	0.43	0.18
Wheat	0.38	0.86	0.68	0.24	0.11

Source: Deaton and Laroque (1989, Table 2).

Notes: CV is the coefficient of variation. AR1 and AR2 are the first and second order autocorrelation coefficients of the deflated series of prices. PER20 and PER40 are the Campbell/Mankiw-Cochrane measures of persistence with window widths of 20 and 40 years.

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APPENDIX
DERIVATION OF RISK FORMULAS

The cost of income risk

If $\bar{c}$ is consumption, a random variable with mean $\bar{c}$, $CV\sigma_c$, and U(c), is utility, then the cost of risk, is defined by the equation $U(\bar{c}-p)=EU(\bar{c})$. Expand both sides in a second-order Taylor series:

$$U(\bar{c})-pU'(\bar{c}) \approx U(\bar{c})+\frac{1}{2}\,Var(c)U''(\bar{c}),$$

or $q/\bar{c} \approx \frac{1}{2}R\sigma_c^2$, where $R \equiv -\bar{c}\,U''(\bar{c})/U'(\bar{c})$.

Hedging with risky prices

Producers choose inputs at the start of the year, to produce output, q. There is no uncertainty about output, but prices are risky and not yet known when inputs are chosen. The futures price is, however, known, and the producer can choose a hedge at the same time as the choice of inputs, so that the futures price is the *action certainty equivalent price*, i.e. the price which would induce the same actions, in this case the choice of inputs, as a perfectly certain output price of the same level. The price at harvest is $\tilde{p}=f+\tilde{u}$, where u is a mean-zero random variable, and f is the expected value of p (and also the futures price if there is an unbiased futures market). The producer's income (in the absence of a futures market) is

$$\tilde{y}=\tilde{p}q-\frac{1}{2}cq^2, \tag{1}$$

and the producer has a constant absolute risk aversion utility function. Assuming p is normally distributed (or under weaker assumptions, given in Newbery, 1988, to a high degree of approximation), he chooses q to maximize

(2) $$U = E\,\tilde{y} - \frac{1}{2} A \operatorname{Var}(\tilde{y}).$$

This can be written

(3) $$U = \bar{p}q - \frac{1}{2}\left(cq^2 - A\,\bar{p}^2 q^2 \sigma_p^2\right),$$

where A is the coefficient of absolute risk aversion, and σ_p is the CV of price, $SD(\tilde{u})/\bar{p}$. The optimal choice of q is given by

(4) $$q = \frac{\bar{p}}{c\phi}, \quad \phi = 1 + 2R\sigma_p^2; \quad R = \frac{Ap}{2c}.$$

Here *R* is the (dimensionless) coefficient of relative risk aversion, evaluated at the risk-free level of income, $\bar{y} = \frac{1}{2}\bar{p}^2/c$. The certainty equivalent level of income is then $U = \frac{1}{2}\bar{p}^2/(c\phi)$.

Perfect price stabilization

The effect of perfectly stabilizing price is to set $\sigma_p = 0$, and to change the value of ϕ to 1. The cash value of stabilization to the farmer is measured by the change in certainty equivalent income, ΔU:

(5) $$\frac{\Delta U}{U} = \phi - 1 = 2R\sigma_p^2.$$

If the farmer has access to an unbiased futures market with price f, then income must be augmented by the term $z(f-\bar{p})$, where z is the volume of futures sold. Substituting this term in (2) and choosing z optimally (see e.g Newbery and Stiglitz, 1981, p184), shows that z = q, and the effect on (3) is to eliminate the term with A. The benefit of introducing the futures market will be the same as (5).

Price stabilization when prices are autocorrelated

Suppose prices follow the following simple autoregressive scheme:

$$\tilde{p}_t = \alpha p_{t-1} + (1-\alpha)\bar{p} + \tilde{u}_t, \tag{6}$$

where $\bar{u}_t$ is i.i.d. with zero mean. This can also be written as

$$\tilde{p}_t = f_t + \tilde{u}_t, \;\; f_t = E_{t-1}\,\tilde{p}_t = \alpha p_{t-1} + (1-\alpha)\bar{p}.$$

Again, f_t is the expected price, equal to the futures price at the start of period t in an unbiased market. The same arguments as before imply that in the absence of a futures market:

$$q = \frac{f}{c\phi}, \;\; U = \frac{f^2}{2c\phi}, \;\; \phi = \left(1 + 2R\sigma_p^2\right), \tag{7}$$

where the time subscript has been suppressed. Now consider the problem facing a farmer with a two-year horizon, with the initial expected price $f_t = \bar{p}$, its long-run average level. This in turn implies that $p_t = \bar{p}(1+\tilde{\varepsilon}_t)$, where $\tilde{\varepsilon}_t = \tilde{u}_t / \bar{p}$, and $\tilde{f}_{t+1} = \bar{p}(1+\alpha\tilde{\varepsilon}_t)$. Consider the present value of the two certainty equivalent incomes at the end of period t:

$$(8) \qquad W=\frac{f_t^2}{2c\phi}+\frac{\beta \tilde{f}_{t+1}^2}{2c\phi}=\frac{\bar{p}^2}{2c\phi}[1+\beta\left(1+\alpha\tilde{\varepsilon}_t^2\right)].$$

where β is the discount factor, and $\phi=1+2R\sigma_p^2$. Each term in (8) is the certainty equivalent income for that period, but at the start of period t, the certainty equivalent income in period $t+1$ is uncertain. The certainty equivalent present value can be found by replacing y by W in (2) and is

$$(9) \qquad V=\frac{\bar{p}^2}{2c\phi}\left\{1+\beta\left(1+\alpha^2\sigma_p^2\right)-R\beta^2\alpha^2\sigma_p^2\left(2+\alpha^2\sigma_p^2\right)/\phi\right\}.$$

The effect of having an unbiased futures market and allowing one-period ahead hedging is to set $\phi=1$ in equations (8) and (9). If $\alpha>0$, there is no simple analytical expression for the proportional gain in certainty equivalent income in introducing a futures market, but suppose $\beta=0.9$, $\sigma_p^2=0.1$, $R=1$, then the benefits of a futures market fall with α and are only 85% as large at $\alpha=1$ as $\alpha=0$.

Even if the futures market only extends one period ahead, it is possible to increase the degree of hedging by rolling over contracts as they mature. Consider the strategy of selling $(\bar{p}/c)(1+\beta z)$ futures at the start of period t, and find the optimal value of z. The present value at the end of period t is:

$$(10) \qquad W=\frac{\bar{p}^2}{2c}\left\{1+\beta\left(1+\alpha\varepsilon_t^2\right)+2\beta z\left(\frac{f_t-\bar{p}_t}{\bar{p}}\right)\right\}.$$

The choice of z is that which minimizes the certainty equivalent value of W, which in this case amounts to minimizing the variance of W. The solution is $z=\alpha$, and this gives, ignoring terms is σ_p higher than σ_p^2:

$$(11) \qquad V=\frac{\bar{p}^2}{2c}\left\{1+\beta\left(1+\alpha^2\sigma_p^2\right)\right\}.$$

It is immediate that this strategy of sequential futures trading reduces risk further than futures trading confined to each year.

Extensions to n-year rolling hedge

Choose units so that $\bar{p}=1=c$, and suppose that $f_o=1$. (This last assumption is not innocuous, but can be relaxed, and does not alter the thrust of the argument.) The level of output when the expected price is at its long-run average level of 1 is also 1, and suppose that a single futures contract is also for this amount. Then

$$(12) \qquad f_i=1+\alpha^i\varepsilon_0+..\alpha\varepsilon_{i-1}, \quad i\geq 1, \quad f_i-p_i=-\varepsilon_i.$$

Consider the risk-minimizing n-period hedge constructed by selling a number of hedges n_i at the start of period i:

$$(13) \qquad n_i=f_i\left[1+\alpha\beta+\ldots(\alpha\beta)^{n-i+1}\right].$$

This can be thought of as follows. The first term hedges planned output in period i, which, given the normalizations, is equal to the certainty equivalent price, f_i. The remaining terms cover the costs of rolling forward the hedge next period. At the end of each period, current income will be

$$\frac{1}{2} f_i^2 - (n_i - 1)\,\varepsilon_i,$$

where the first term is the profit from the perfectly hedged current output, and the second term is the net income from liquidating the remaining $(n_i - 1)$ hedges. Now consider the NPV of income discounted to the end of period 0 (i.e to the date of settling period-0 hedges):

$$\frac{1}{2}\left(f_0^2 + \beta f_1^2 + .. \beta^{n-1} f_{n-1}^2 + \ldots\right)$$
$$-f_0\,\varepsilon_0\left[\alpha b + ..(\alpha b)^{n-1}\right] - \beta f_1 \varepsilon_1 (n_1 - 1) - ..$$

where the tildes over the fs are a reminder that future futures prices are uncertain. If we are looking for the variance-minimizing strategy, and if we ignore terms smaller than σ_p^2, and if we examine the i-th term in this expression, then we need to evaluate typical terms such as

$$\frac{1}{2}\beta^i f_i^2 - \beta^i \varepsilon_i f_i\left(\alpha\beta + .. + (\alpha\beta)^{n-i-1}\right).$$

The variance will involve taking the expected values of squared terms, bearing in mind that as the εs are i.i.d. that $E\,\varepsilon_i \varepsilon_j = 0$ for $i \neq j$. Ignoring higher order terms, this will involve terms such as

$$\beta^i\left(\alpha^i \varepsilon_0 + .. + \alpha\,\varepsilon_{i-1}\right) - \beta^i (n_i - 1)$$

Collecting terms together and substituting for f_i gives

(14) $$\sum_{i=0}^{\infty}\beta^i\left\{\sum_{j=1}^{i-1}\alpha^j\varepsilon_{i-j} - \varepsilon_i\sum_{k=1}^{n-1}(\alpha\beta)^k\right\},$$

which shows that if the world came to an end after n periods, then an n-period rollover hedge would eliminate risk. In the general case the terms left uncancelled are

(15) $$\sum_{i=n}^{\infty}\beta^i\left\{\sum_{j=1}^{i-1}\alpha^j\varepsilon_{i-j}\right\}$$

and the practical question is by how much an n-period rollover reduces risk compared to an (n-1)-period hedge, and what this risk reduction is worth to the producer, and what it might cost in terms of additional numbers of hedges to finance. The reduction in variance in increasing the period of hedging by 1 (and the number of hedges by $(\alpha\beta^{n-1})$) is

(16) $$\beta^{2n-2}\sum_{j=1}^{n-2}\alpha^{2j}\sigma^2 = \beta^{2n-2}\left\{\frac{1-\alpha^{2n-2}}{1-\alpha^2}\right\}\sigma^2$$

The value of this reduction in risk is R times this reduction in variance (which, by normalization, is also the CV squared), and if each extra futures contract costs μ, the marginal benefit per unit cost is

$$\left(\frac{\beta}{\alpha}\right)^{n-1}\frac{1-\alpha^{2n-2}}{1-\alpha^2}\frac{R\sigma^2}{\mu}.$$

Clearly, as the time horizon of the hedge increases, the marginal benefit also falls.

Figure 1

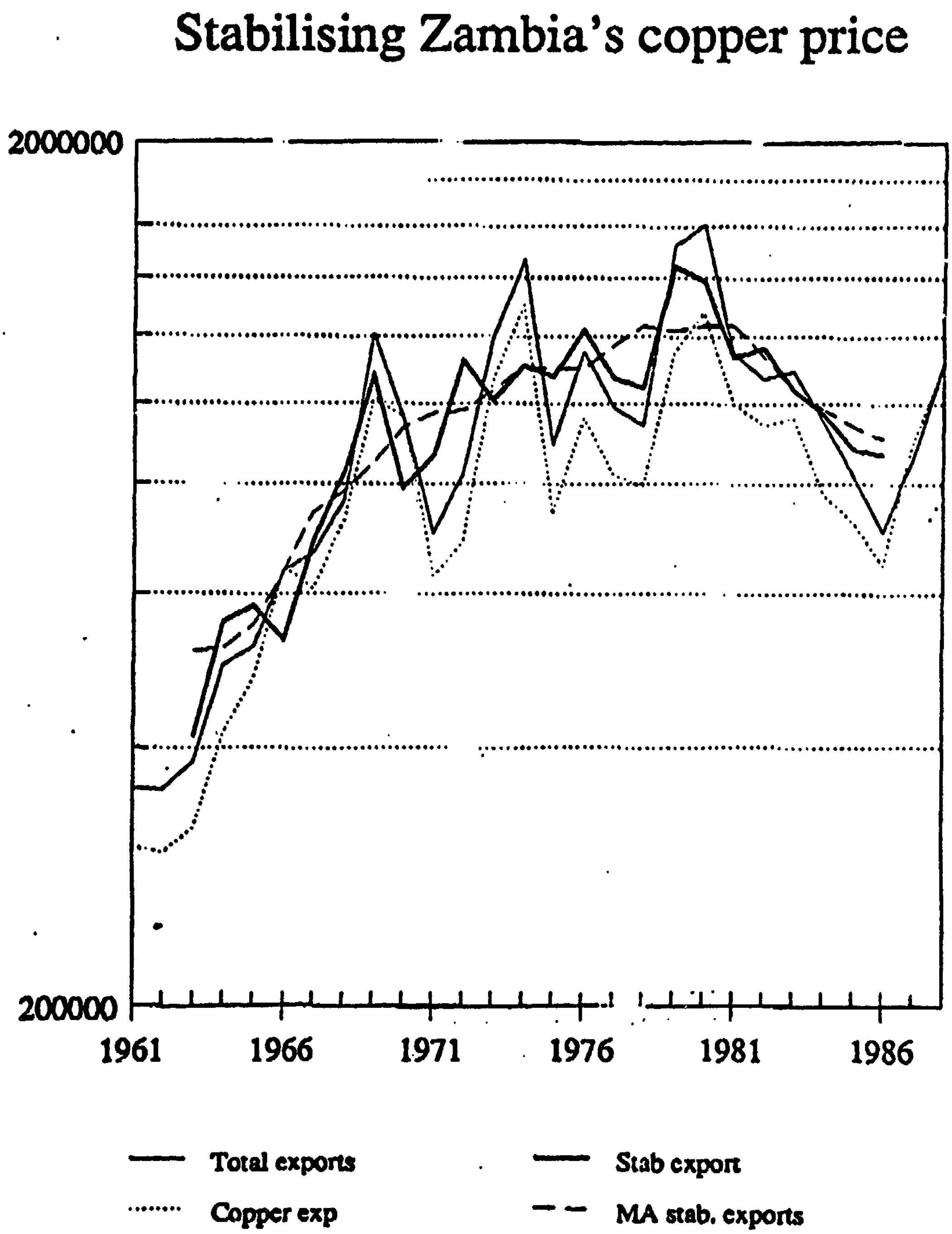
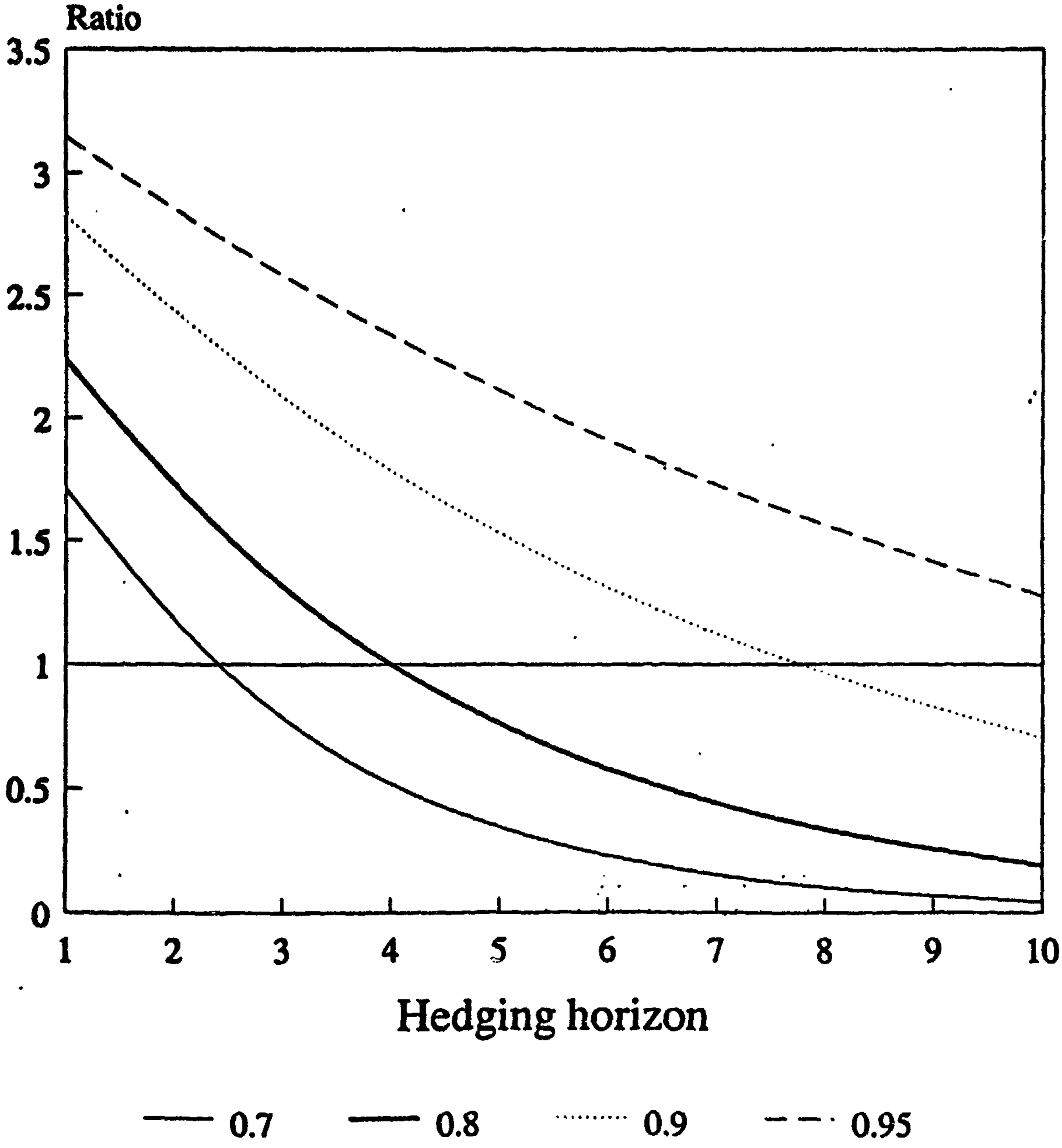

Stabilising Zambia's copper price
2000000
200000
1961
1966
1971
1976
1981
1986
Total exports
Stab export
Copper exp
MA stab. exports

Figure 2

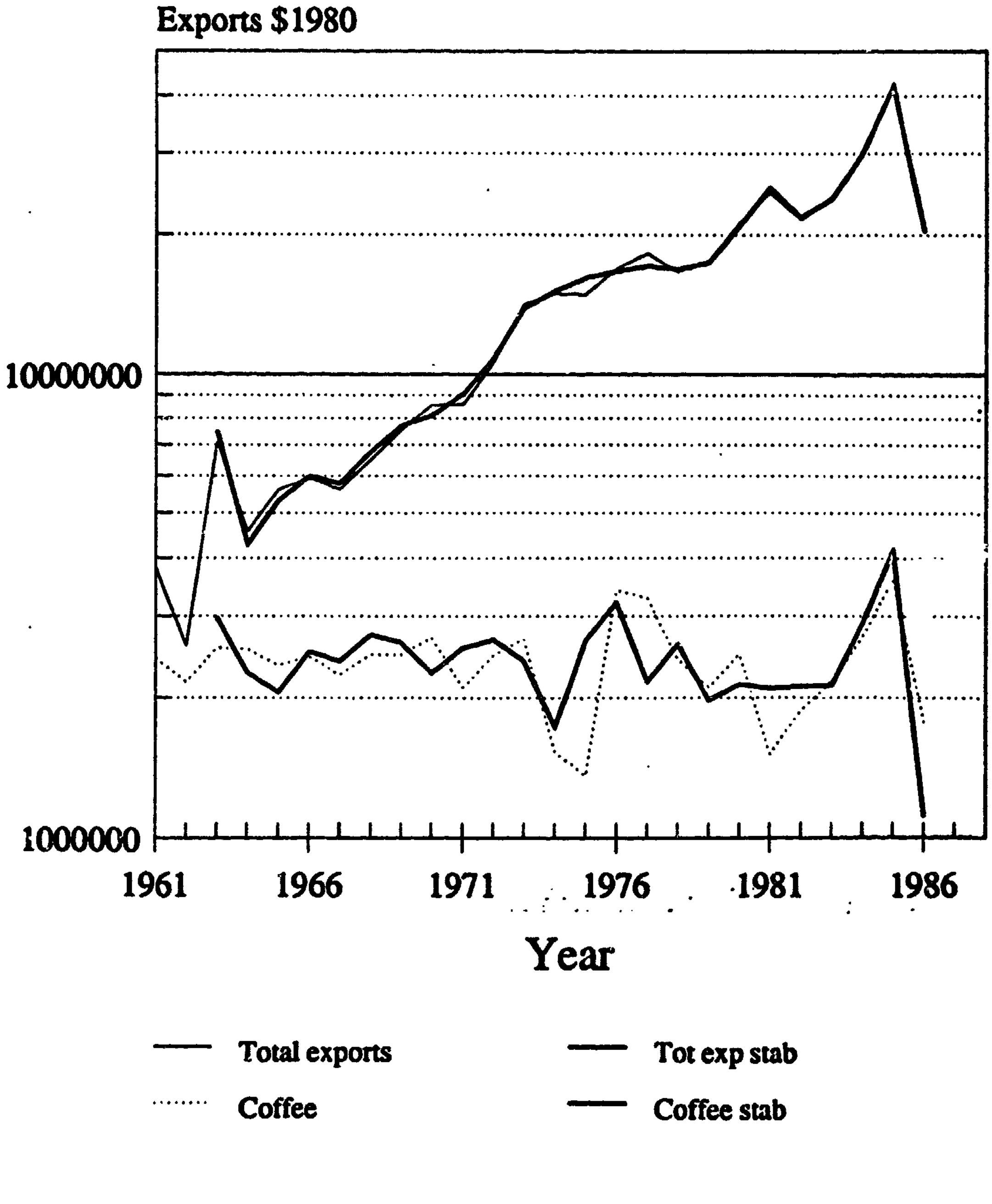
Benefit cost ratio
for rolling hedges



Autocorrelations

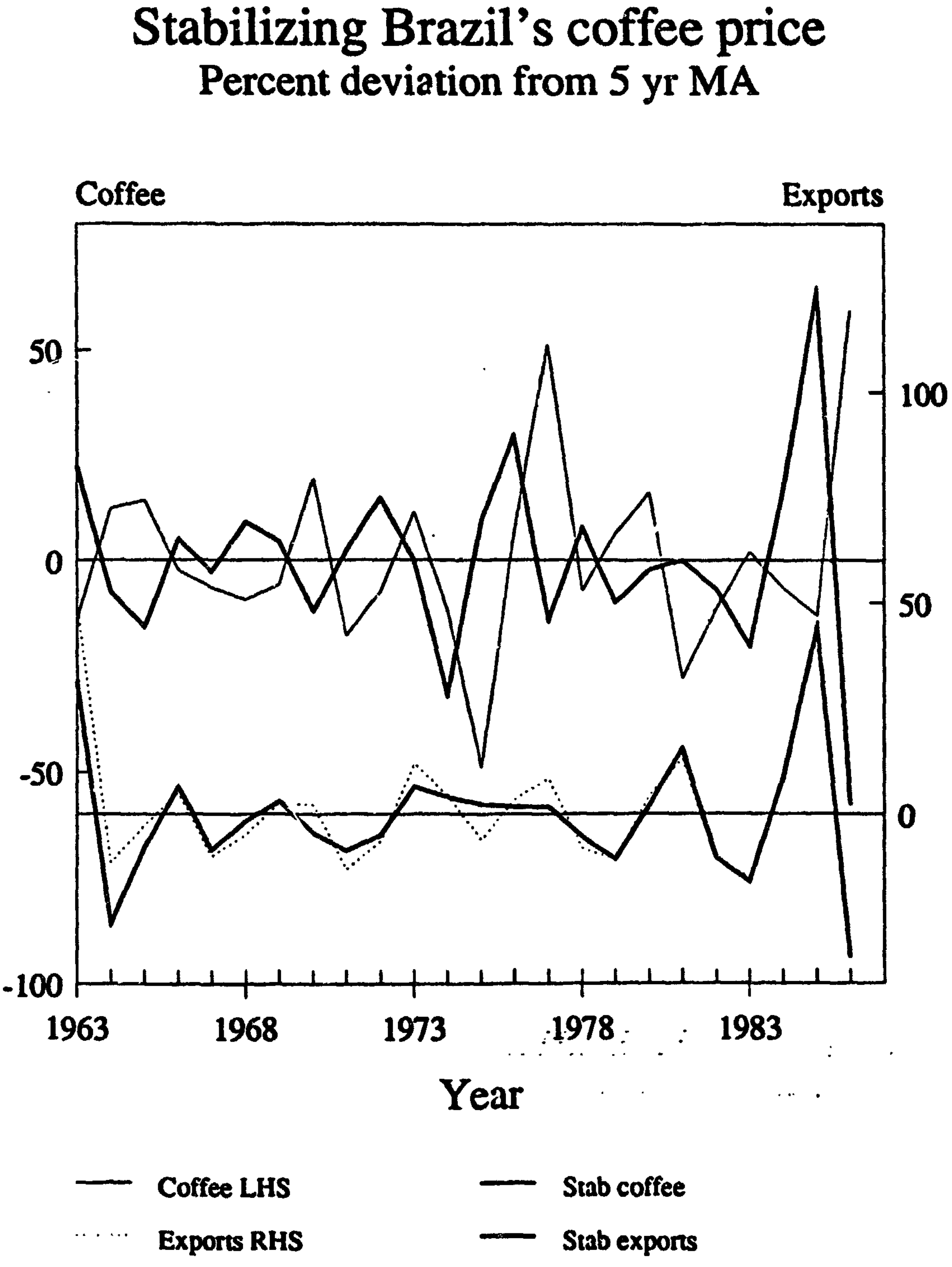
Figure 3

Stabilizing Brazil's coffee price exports at constant prices



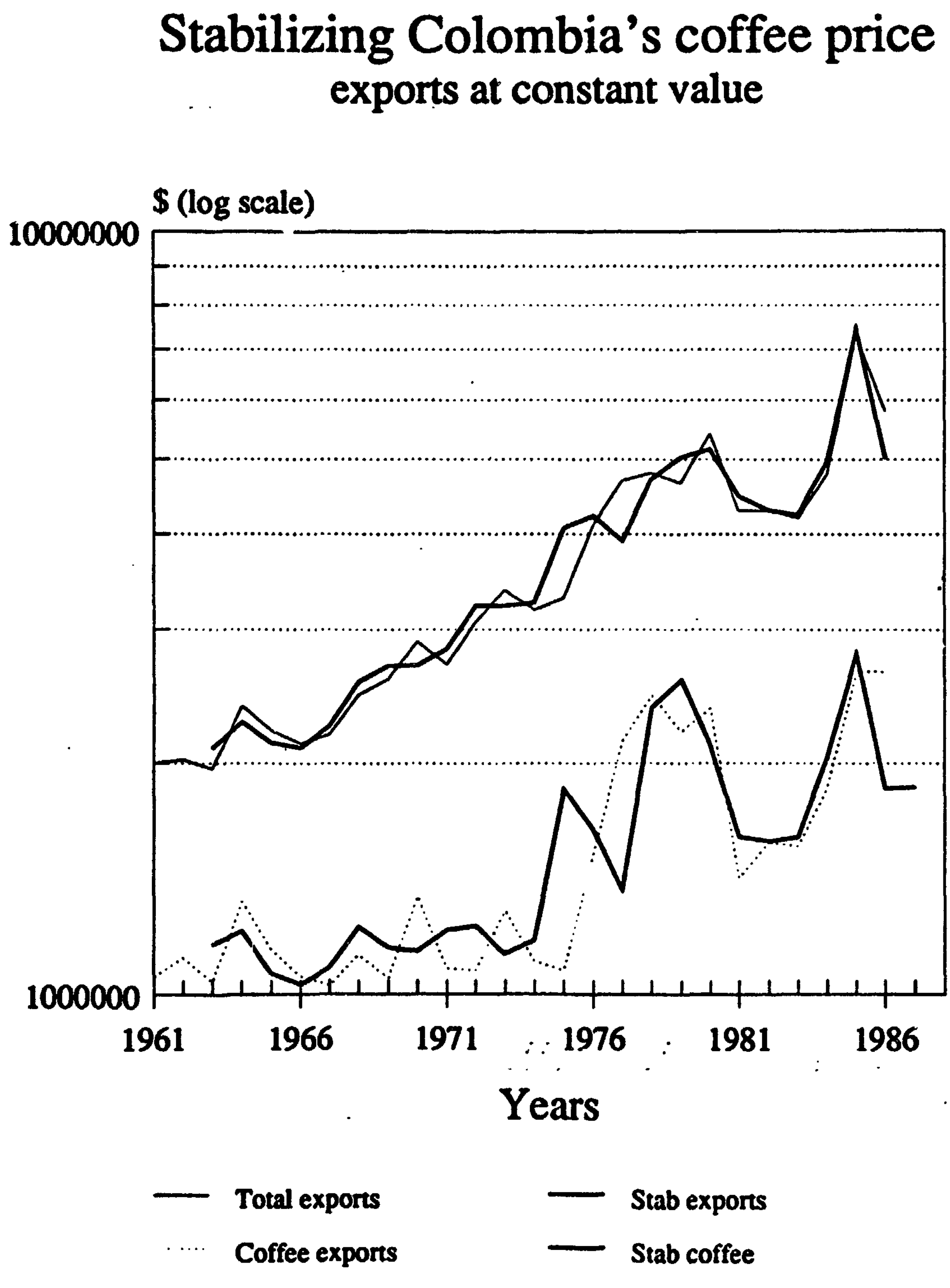
Deflated by MUV

Figure 4



Deflated by MUV

Figure 5



Deflated by MUV

Figure 6

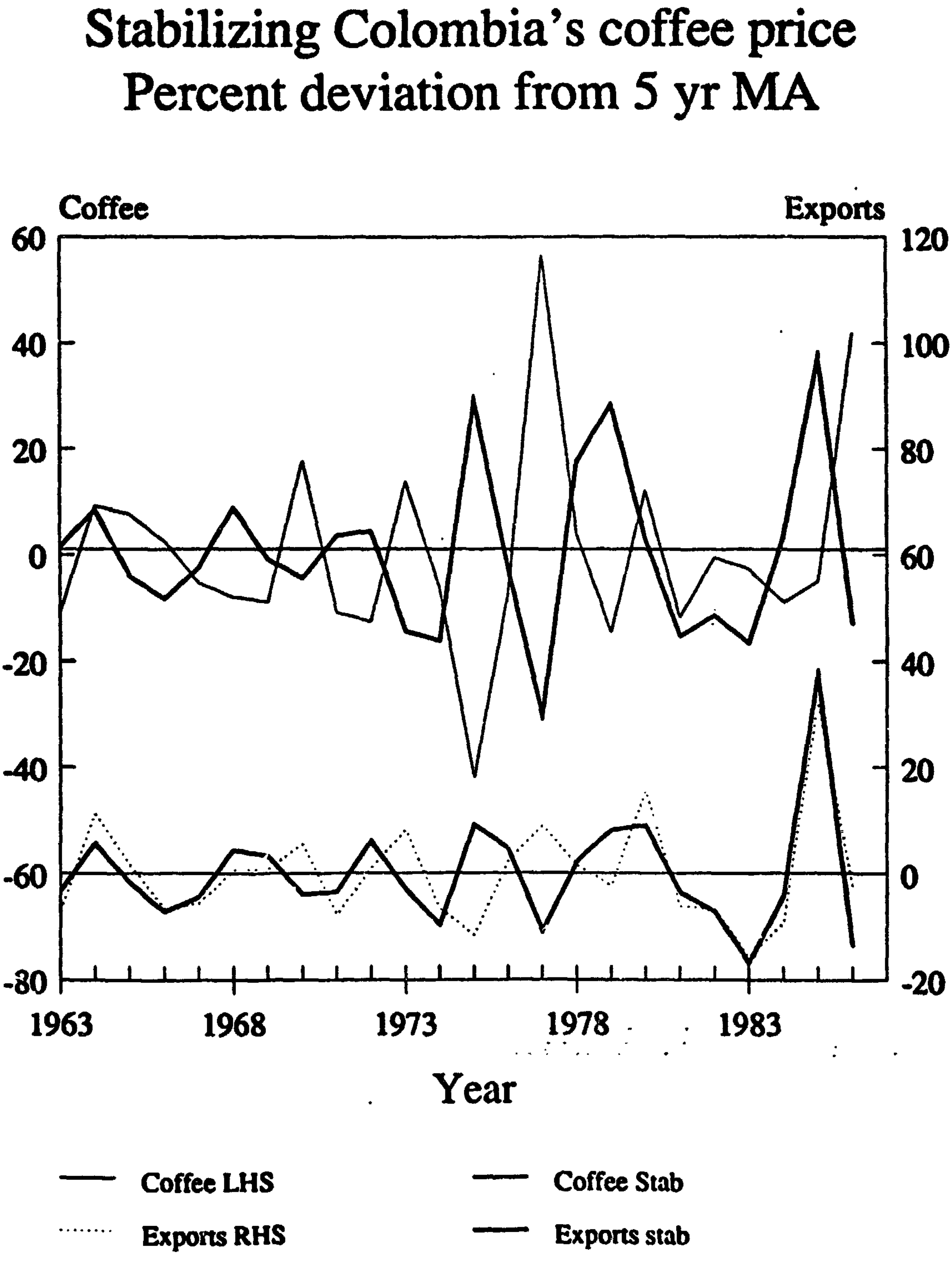


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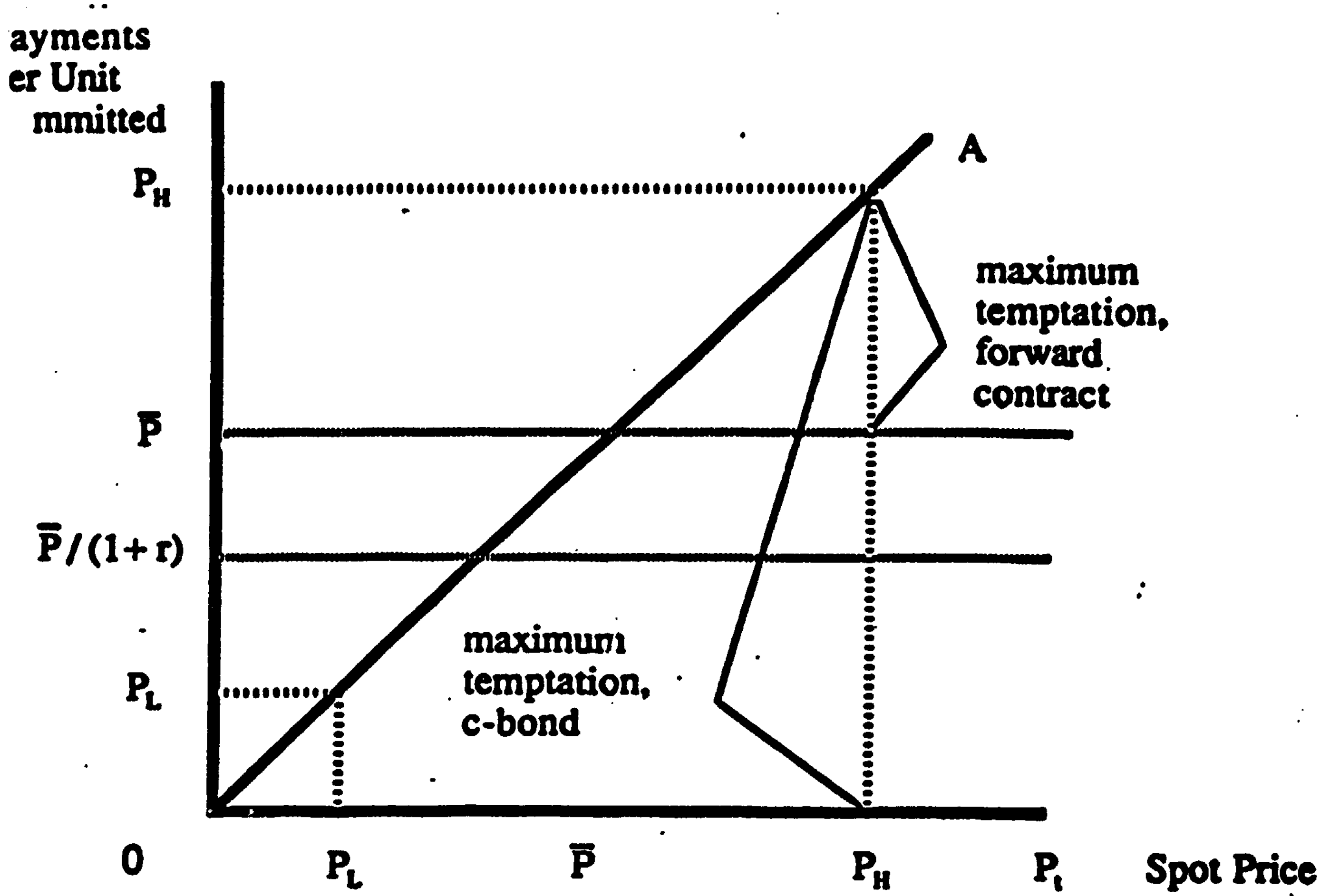
Figure 7



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