Policy, Planning, and Research

WORKING PAPERS

Debt and International Finance

International Economics Department
The World Bank
January 1989
WPS 150

Improving the Currency Composition of External Debt:

Applications in Indonesia and Turkey

Ken Kroner and Stijn Claessens

The debt service ratio of many developing countries shot up when the dollar fell. The paper shows how developing countries can alter the currency composition of their external debt to minimize their vulnerability to fluctuations in international exchange rates.

Policy, Planning, and Research

WORKING PAPERS

Debt and International Finance

Changes in exchange rates affect both the structure and level of a country's external debt. Much of Indonesia's debt was denominated in yen, for example, so the depreciation of the dollar since 1985 has increased the level of Indonesia's debt and reduced the dollar-denominated portion of that debt.

Indonesia's debt service increased from 10 percent in 1980 to 37 percent in 1986, largely because of the depreciation of the U.S. dollar and the fall in oil prices. Other countries had similar experiences.

Developed countries can hedge against exchange rate and commodity price changes by purchasing currency futures or other hedging instruments, but most developing countries do not have access to futures markets. They can,

however, reduce their exposure by matching the currency composition of their external debt with the currency composition of the cash flows with which they service their debt.

Using advanced econometric techniques, the authors analyze what the currency exposures might have been in Indonesia and Turkey — and suggest borrowing portfolios that might be effective in hedging these countries' terms of trade against exchange rate fluctuations.

The results are promising for Indonesia, where the optimal currency portfolios might have resulted in a significant reduction in risk. The results are less satisfying for Turkey—although they do indicate possible research directions.

This paper is a product of the Debt and International Finance Division, International Economics Department. Copies are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Leah Chavarria, room \$7-033, extension 33730.

The PPR Working Paper Series disseminates the findings of work under way in the Bank's Policy, Planning, and Research Complex. An objective of the series is to get these findings out quickly, even if presentations are less than fully polished. The findings, interpretations, and conclusions in these papers do not necessarily represent official policy of the Bank.

Optimal Currency Composition of External Debt: Applications to Indonesia and Turkey

by Ken Kroner and Stijn Claessens*

Table of Contents

I.	Introduction	1
II.	Issues	4
III.	The Model	11
IV.	Data	15
V.	Estimation	20
	1. The Arch Model	20
	2. The Estimation Results	22
VI.	The Optimal Portfolios	26
VII.	Conclusion	34
	Appendix 1	35
	Appendix 2	37
	Appendix 3	44
	Bibliography	46

*University of Arizona and the World Bank respectively. We would like to thank Rob Engle, Amer Bhattacharya, Jeff Balkind, Sweder van Wijnbergen, Bela Balassa and participants of the Canadian Econometrics Study group conference in Bariff, Canada during October 1988 for comments, though we of course accept full responsibility for any errors ourselves.

I. Introduction

Over the last decade, Indonesia and Turkey have been affected by large changes in cross-currency exchange rates and in commodity prices. For example, the debt service ratio for Indonesia rose from 10% in 1980 to 37% in 1986, largely as a result of the depreciation of the US dollar and the fall in (About one-half of Indonesia's exports are petroleum.) exchange rate changes have affected the structure as well as the level of Indonesia's debt. Much of Indonesia's borrowing has been denominated in the Japanese yen, so the large depreciation of the dollar since 1985 has both increased the level of its debt as well as reduced the dollar-denominated portion of that debt. Assuming exchange rates remain constant, this will imply that the debt service level will continue to remain higher over the next couple of years than it would have been had exchanges rates remained at their 1985 level. In general, whether we are dealing with Indonesia, Turkey, or any other developing country, cross-currency exchange rate fluctuations and commodity price changes have affected both the structure and the level of the country's external obligations debt.

Developed countries are able to hedge against exchange rate changes by purchasing currency futures or other hedging instruments on organized or over-ths-counter markets. Most developing countries, however, do not have access to these markets, due to institutional constraints, credit constraints and other reasons. An alternative hedging instrument which can be used to reduce exchange risk exposure and is available to all countries, is the currency composition of external debt. Indonesia and Turkey could have minimized their exposures by matching in some fashion the currency composition of their debt with the currency composition of those cash flows with which

they are able to service their debt. Even though a perfect match might have been difficult to achieve for Turkey and Indonesia, as they might have been constrained in choosing and altering the currency composition of their new borrowings, such as those due to foreign governments, and might not have been able to use market techniques, such as currency swaps, to change the currency composition of their existing debt, marginal changes could still have led to some tangible benefits. Ex-post, one is able to determine what their optimal portfolios would have been, but this does not necessarily help in coming up with policy rules on how they can minimize their exposure to exchange risks in the future.

The purpose of this paper is to derive practical policy rules for developing countries in general on ways to use the currency composition of their external debt to minimize the exposure of their terms of trade (or other external account variables, e.g., exports) to exchange rate fluctuations. The model is subsequently estimated and applied to Indonesia and Turkey.

Section II presents a discussion of a number of alternative approaches and related issues regarding currency management of external debt. Section III presents the analytical model, which is an abbreviated version of the model given in Claessens (1988), and Section IV discusses the data used in the paper. The solution for the optimal portfolios involves estimating a covariance matrix of exchange rates movements, which is changing through time. An econometric technique (called Autoregressive Conditional Heteroskedasticity) which allows for this is described in Section V, along with some of the estimation results. The actual portfolios are presented in Section VI, and out of sample tests are conducted to see how effective these portfolios would be in hedging the terms of trade against exchange rate

exposure. The results are very promising for Indonesia and point out some further research directions for Turkey. Section VII concludes.

II. Issues

Large changes in cross-currency exchange rates and relative goods prices over the last decade have affected many developing countries. Changes in cross-currency exchange rates affect goods prices, as for instance has been observed in the relationship between the dollar exchange rate and the price of commodities before the most recent depreciation of the dollar. Cross-currency exchange rate changes impact not only export prices but import prices as well. Furthermore, Cross-currency exchange rate changes affect the relative competitiveness of countries with which the developing country competes. As a result, many developing countries' market shares and profit margins have been affected by movements in cross-currency exchange rates. In addition, commodity price movements have had a substantial impact on many developing countries because a large share of their export earnings have been derived from primary commodities.

The increased volatility of exchange rates and commodity prices (and interest rates) has thus led to increased fluctuations in the aggregate income and welfare levels of many developing countries through their impact on their external accounts. It has highlighted the importance for many countries of, first of all, developing a conceptual framework for managing external exposures, and secondly, for devising practical policy rules. We will discuss in this section the concepts and principles that have been proposed for external liability management, and discuss their strengths and weaknesses.

Several options are open to a country that wants to reduce its exposure to external factors. First of all, the country can try to engage in real diversification through the sourcing, producing and exporting of a mix of products which is close to optimal given the relationships between exchange

rates, interest rates, good prices, and other external factors. Of course, the composition of exports, and to a lesser extent of imports, can not be changed easily, but one can still expect some contribution in the long run. Secondly, a range of instruments is available to private firms in developed financial markets to manage short-lerm exposures and some of these instruments could be used to manage a country's external exposures. Transfer of certain risks to market participants more able to absorb them or to transfer risks further could substantially benefit a country in terms of reducing risks at reasonable costs. For example, well diversified financial institutions can transform a floating interest liability into a fixed interest liability at a cost which can be substantially below the opportunity costs for the liability holder. Thirdly, the country has a potential financial hedging instrument against unanticipated exchange rates movements in the form of the currency composition of its existing external liabilities. In addition, the country can influence the currency composition of new external capital flows. Fourthly, the composition of the external liabilities of the country can also be a useful tool to manage the exposure to commodity price movements to the extent that movements in exchange rates and goods prices are correlated and to the extent that the country has the flexibility to influence the composition.

It is easy to tell, with the benefit of hindsight, what a good external liability policy is and what the optimal currency composition of external liabilities would have been. Capital gains and losses due to exchange rate movements could have been avoided by matching in the right fashion the currency composition of external liabilities with the effective currency composition of cash flows, i.e. non-interest current account flows. However, such hindsight does not provide practical rules on how countries,

exchange rate movements. One needs to look at the right risk-return tradeoffs and incorporate the informational constraints that the nature of the problem suggests: hindsight does not help ex-ante.

A number of often intuitively appealing rules have been proposed for choosing the currency denomination of external liabilities and new borrowings. The most favored strategies are to base the currency composition of a country's debt on the pattern of trade, on the currency denomination of its export revenues or on the basis of the basket of currencies with respect to which the exchange rate is managed.

In the case of the pattern of trade rule, this would mean that a country would borrow in the currencies according to its trade pattern in the expectation that if the currency of an export market appreciates, the borrower's terms of trade are likely to improve and thus partially offsett the higher costs of servicing debt in that currency, and vice versa for imports. This relies on the appreciation of a currency of an export market to be accompanied by an increase in a country's ability to pay as its exports and terms of trade improve. However, it is not clear a priori that an appreciation of a currency of an export market has to mean an improvement in the country's terms of trade. One contradictory example would be of two different countries exporting the same good to the same country. The crossexchange rate between the two exporting countries would be more important for their relative competiveness and market shares in the importing country than the currency of the importing country itself. Of course, if Purchasing Power Parity (PPP) would hold, it would not matter in what currency exports were denominated and export shares would no be affected. For strong and conclusive

rejections of the PPP see Frenkel (1981) and Cumby and Obstfeld (1984).

The rule to base the currency composition of external debt on the currencies of invoice or denomination of exports (or imports or both) could equally be critized. There is sufficient evidence that the relationship between the nominally denomination and the real value of exports has at times been perverse. For example, the real price of commodities has in the past been inversely related to the real value of the dollar, even though most commodities are denominated and traded in terms of dollars. This meant that when the dollar went down, commodity prices went up and a commodity producing country's terms of trade improved too. Such a relationship would imply that the dollar would not necessarily have been the optimal currency to borrow as its value could have a perverse relationship with the country's ability to generate foreign exchange in the short run through exports. As a depreciation of the dollar implies an appreciation of other currencies, a reverse relationship might exist between the value of other currencies and commodity prices: when the dollar appreciates, commodity prices decline, but other currencies depreciate too, which might make non-dollar currencies good external liabilities. This shows that a currency composition based on a country's trade denomination pattern could still lead to large real risks. The currency basket rule could be equally critized as being based on nominal consepts.

In general, the nominal dimension of trade flows is not necessarily crucial in the long run for the currency choice of external debt. If one uses the nominal denomination of trade flows, the US dollar would be the predominant currency for most developing countries. However, when one realizes that prices in world commodity and manufactures markets depend on the

interaction between demanders and suppliers across the whole world, one realizes that the nominal currency denomination of a good or the nominal direction of trade do not have to reflect the real denomination of the price of a commodity or manufacture. In a world market for commodities and manufactures suppliers become more (or less) competitive depending not only on the changes in their own currency but also on the changes in other suppliers currencies. Similarly, demanders will consider goods more (or less) attractive depending on the movements of the exchange rates of multiple suppliers. As a result, the changes in the price of a particular grant price of a part commodity as a result of exchange rates changes will crucially depend on the type of market structure (perfect competitive, oligopolistic, etc.). Demand and supply elasticities will play a crucial role in distributing the effects of exchange rate in terms of (nominal) price and quantity changes over the different market participants. A not perfectly competive market structure can result in real effects of nominal exchange rate changes as the different suppliers and demanders are affected differently. For a further analysis of this issue see, for instance, Dornbusch (1987).

All of the rules mentioned so far can thus be critized as not being explicitly related to a specific goal or objective and not being based on an explicit definition and measurement of risk. It might then also be the case that these rules increase rather than decrease the real costs of borrowings and the fluctuations of these costs over time. A more integrated approach would to base the currency composition choice of external liabilities on some tradeoff of the level of real effectiv. costs of funds in a certain currency with the uncertainty of these costs, where the costs are related to, for instance, the ability of the country to generate foreign exchange as reflected

in the short run by its level of exports. In this approach, the real costs and riskiness of borrowing in a particular currency would depend not only on the nominal costs of borrowings and its uncertainty but also on whether the appreciation or depreciation of that particular currency is associated with an increase or decrease in the ability of the country to generate foreign exchange and pay its external liabilities. Commonly used indicators of the ability to service external debt are the level of exports and the country's terms of trade. For example, if the country exports only one commodity and if the price of that particular commodity is one to one related to a particular currency, say the Danish Krone, then borrowing in that currency would be the lowest risk strategy possible.

In this context, diversification of risk could suggest that a country should not only borrow in more than one currency, it could also suggest that a country determines the currency composition of external liabilities, or other aspects of its external liabilities, so as to reduce other forms of price risk (e.g., fluctuations in export or import prices, or the terms of trade).

As exchange rates (and interest rates) are very difficult to predict, it seems valid to use in the context of an optimal borrowing strategy the presumption that the objective of the country in choosing its currency composition is not to "beat" the market. The market expectations as a whole of the movements of cross-currency exchange rates are related to the nominal interest differentials between directent currencies. It can be expected that under perfect market conditions ex-ante deviations from uncovered interest rate parity are small and that expected borrowing costs will thus be similar in all currencies. The currency composition choice will then become predominantly a function of the correlation of each of the currencies with the

country's ability to pay external liabilities. The country will minimize the variability of its debt service obligation relative to its ability to pay and borrow a risk-minimizing hedge portfolio.

The conclusion of this section is that a solid theoretical framework is necessary and that only empirical work can clarify the optimal currency choice issue. We turn now to the theoretical model.

III. The Model

Consider a world which consists of a small open economy (the home country) and N developed countries. Let each of the N developed countries have an exchange rate e(i), i=1,...,N, which follows the diffusion process:

(1)
$$\frac{de(i)}{e(i)} = v_{e(i)}^{dt} + \sigma_{e(i)}^{dZ} e(i), \qquad i=1,...,N$$

Here e(i) is written in terms of the home countries' currency per unit of the foreign currency (e.g., Turkish liras per US dollar) and $dZ_{e(i)}$ is a Wiener process. So E(dZ)=0 and VAR(dZ)=dt. Thus, this differential equation says that the expected value of the depreciation of the i'th exchange rate during the time period dt is $v_{e(i)}$ and its standard deviation is $\sigma_{e(i)}$. We are thus assuming that the exchange rate depreciations are approximately normal for small intervals dt, and that the exchange rates themselves are lognormal.²

Suppose also that the means and standard deviations $v_{e(i)}$ and $\sigma_{e(i)}$ are all allowed to depend on both time and a vector of state variables (which will be defined later). So

$$v_{e(i)} = v_{e(i)}(s,t)$$
 and $\sigma_{e(i)} = \sigma_{e(i)}(s,t)$

where S is a (Sx1) vector of state variables which are assumed to follow Ito processes. Thus we have N foreign currencies in which the home country can invest its wealth and denominate its liabilities, which are assumed to follow the process

^{1.} This section is a brief summary of a restricted version of the model presented in Claessens (1988). See his paper for the general model and a discussion of the model and its assumptions.

^{2.} See Merton (1971) and Fischer (1975) for descriptions of the properties of Wiener processes and stochastic differential equations.

$$\frac{de(i)}{e(i)} = v_{e(i)}(s,t)dt + \sigma_{e(i)}(s,t)dZ_{e(i)}$$

Suppose that each country in the "world" has one nominal riskless (instantaneous) bond. Let $B^*(j)$ be the price in the j'th currency of country j's riskless bond, and B be the price in the home currency of the home country's riskless bond. The dynamics for $B^*(j)$ are given by

(3)
$$\frac{dB^{*}(j)}{B^{*}(j)} = R^{*}(j)dt, \qquad j=1,...,N$$

where $R^*(j)$ is the instantaneous nominal rate of return on the j'th bond in currency j. Also, let R be the instantaneous nominal return on the safe domestic bond. The interest rates R are assumed to be constant.

Define the excess return of the j'th foreign bond for a domestic investor, $dH(B^*(j))/H(B^*(j))$, as the return on one unit of domestic currency invested in the foreign bond, financed by borrowing at the interest rate R in the domestic country., i.e.

(4)
$$\frac{dH(B^{*}(j))}{H(B^{*}(j))} = R^{*}(j)dt + \frac{de(j)}{e(j)} - Rdt, \qquad j=1,...,N$$
$$= (R^{*}(j) + v_{e(j)} - R)dt + \sigma_{e(j)}dZ_{e(j)}$$

Notice that the foreign bonds are risk-free in their own country but exchange rate risks make them risky for investors from our "home country", and that their excess returns are perfectly correlated with the changes in the corresponding exchange rate.

Next, suppose there are K commodities consumed in the home country, whose domestic currency prices follow the differential equation

(5)
$$\frac{dP(i)}{P(i)} = v_{P(i)}dt + \sigma_{P(i)}dZ_{P(i)}, \qquad i=1,...,K$$

Again, $v_{P(i)}$ and $\sigma_{P(i)}$ are allowed to be functions of both time and a vector

of state variables. So the commodity price changes have a mean of $v_{p(i)}(s,t)$ and a standard deviation of $\sigma_{p(i)}(s,t)$ over short time intervals dt.³

The first K elements in the (Sxl) vector of state variables are assumed to be the changes in the logarithms of the commodity prices; the next N elements are assumed to be the changes in the logarithms of the exchange rates; and the remaining (S-K-N) elements are assumed to be other unspecified exogenous variables.

Finally, we assume that the domestic investor maximizes a time-additive von Neuman-Morgenstern lifetime expected utility function which depends only on the consumption of the K commodities and time, i.e. $E_t \left\{ t^{\infty} U\left[c_1(z), \ldots, c_k(z)\right] e^{-\delta z} dz \right\}$ where δ is the intertemporal rate of time preference and c_i is the consumption rate of good i. This assumption completes the model, and allows us to solve for the optimal investment portfolio.

Let b be the optimal holdings of foreign bonds, let \mathbf{v} be the (NxI) vector of excess returns, let \mathbf{V}_{aa} be the (NxN) covariance matrix of excess returns to the foreign bonds, and let \mathbf{V}_{as} be the (NxS) matrix of covariances between the excess returns and changes in the state variables. Notice that, because the excess returns are perfectly correlated with changes in the exchange rate changes, \mathbf{V}_{aa} is the same as the covariance matrix of exchange rate depreciations, and \mathbf{V}_{as} is the same as the matrix of covariances between the exchange rate depreciations and changes in the states variables, where the

^{3.} We do not assume that the law of one price holds necessarily exactly for all goods (nor that Purchasing Power Parity holds), i.e., $P(i) \neq P^*(i,j)e(j)$ necessarily for all i and j, where $P^*(i,j)$ is the price of the traded good i in terms of foreign currency j. Neither do we assume that changes in the terms of trade are perfectly correlated with the (weighted average of the) changes in the exchange rates.

first K state variables are the commodity prices. It can be shown (see Claessens, 1988; Stulz; 1981; or Breeden, 1979) that the optimal holdings of foreign bonds $b = \frac{1}{C_w} \begin{bmatrix} \frac{-U}{U_{cc}} & V_{aa}^{-1} \\ \frac{-U}{U_{cc}} & V_{aa}^{-1} \end{bmatrix} = V_{aa}^{-1} V_{aa} V_{as} C_s$, where C = C(W,S,t) is the consumption expenditure function of the investor, W is wealth, and subscripts refer to partial derivatives. Notice that this is a a linear combination of (s+1) column vectors, each of which (when appropriately scaled) can be interpreted as a mutual fund portfolio. The first portfolio is a mean-variance efficient portfolio (i.e., a speculative portfolio), given by $V_{aa}^{-1}v$, and the remaining s portfolios are hedging portfolios, given by the S columns of

$$\mathbf{v_{aa}^{-1}v_{as}}$$

The weights in the linear combination depend on the parameters of the utility function (such as degree of risk aversion and the consumption shares of the different goods) while the portfolios themselves do not. The weight on the speculative pc-tfolio is $\frac{-1}{C_w} - \frac{U_c}{U_{cc}}$ where $\frac{U_c}{U_{cc}}$ is just the inverse of the coefficient of risk aversion, and the weights on the hedging portfolios $\frac{-C_s}{C_w}$. For a country with a high degree of relative risk aversion, the hedging mutual funds will clearly be relatively more important in the overall optimal holding of foreign bonds than the speculative mutual fund. Assuming that most developing countries are relatively risk averse, and using the assumption that the expected costs of borrowings in different currencies after adjusting for exchange rate changes are all equal (i.e., v = 0), we can focus on the hedging portfolios for the rest of this paper.

The hedging portfolios are the portfolios which provide the maximum

correlation with the state variables 8, and hence they can be used to hedge against unanticipated changes in the state variables. This is because after a shock to a state variable, the hedging portfolio leaves the investor's wealth "as near as possible" to what it was originally was, where the "nearness" depends on the degree of correlation of that portfolio with the state variable.

The model says that the optimal way to hedge the K commodity prices (and thus consumer's welfare) against changes in the exchange rates is to borrow according to the first K elements of the matrix $V_{aa}^{-1}V_{as}$, because then a change in each currency leaves the investor's net welfare the least affected. The borrowing shares would apply to the country's <u>net</u> foreign liabilities, i.e., gross debt minus foreign exchange reserves.

In the next section of this paper we will assume that K is one, and that the first state variable will be the difference between the logarithm of the export prices and the logarithm of the import prices, i.e. this state variable is the terms of trade. 4

IV. Data

Weekly exchange rate data from April 1, 1977 to March 31, 1988 were collected (574 observations) for the Japanese Yen (JY), the Deutschemark (DM), the Swiss Franc (SWF), the Austrian Schiling (AUS), the Pound Sterling (PS), the French Franc (FF), the US Dollar (US), the Indonesian Rupiah (INDO), and

^{4.} The use of one price variable, terms of trade, instead of K can be justified if the utility function to be maximized exhibits constant consumption shares. The covariances of the terms of trade with the exchange rates can then be written as a function of the covariances of the individual prices with the exchange rates.

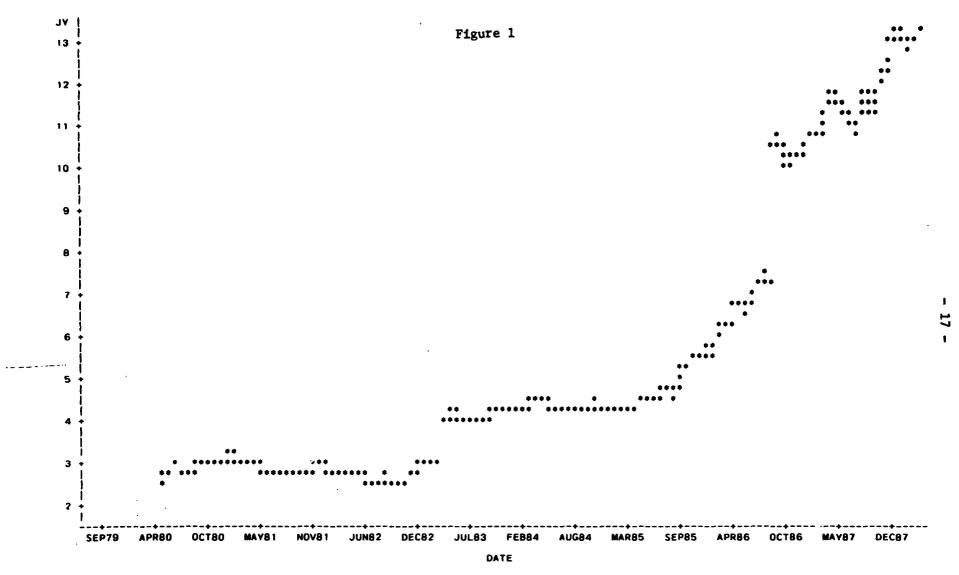
the Turkish Lira (TUR). From this data, the weekly exchange rates of the developed countries currencies were calculated in terms of number of Indonesian Rupiahs and Turkish Liras per unit of foreign currency (e.g., Liras per Pound Sterling), giving two sets of seven exchange rates.

For the first part of the sample period, both the Rupiah and the Lira were fixed with respect to the US Dollar. This would, of course, artificially affect our estimates of the variances and covariances of the exchange rates, so the sample size was truncated at April 30, 1980 for Indonesia (414 observations left) and at October 15, 1980 for Turkey (390 observations left). Since that time, both currencies have been under a managed float exchange rate system.

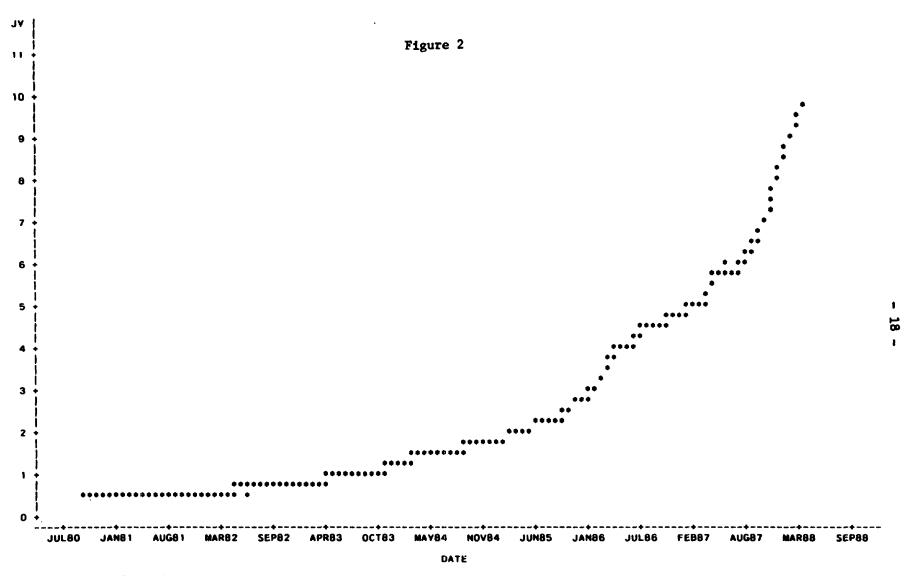
A few interesting facts become obvious when one examines the plots of the exchange rates. See Figures 1 and 2 for the plots of INDO/JY and TUR/JY. The most striking feature of these plots is the dramatic depreciation of both the Rupiah and the Lira with respect to JY and with respect to the other currencies as well. The Rupiah fell from 2.63 INDO/JY to 13.25 INDO/JY, and the Lira fell from 0.40 TUR/JY to 9.75 TUR/JY over our sample period. Another obvious feature is the two times that the Indonesian government devalued the Rupiah by large percentages - from 702 INDO/US to 970 INDO/US in March, 1983; and from 1134 INDO/US to 1643 INDO/US in September, 1986.

These two devaluations of the Rupiah would cause problems when estimating the covariance matrix of exchange rate changes, as the large

^{5.} The source for INDO and TUR was the International Monetary Fund database TIBMER, and the source for the other rates was the IMF database FTFROR. The Indonesian and Turkish rates are "representative" rates - i.e., they come from markets within Indonesia and Turkey respectively. The other rates are all London Noon Spot Quotations. Wednesdays were used whenever possible, but if a holiday fell on Wednesday then Thursday's quotation was used.



NOTE: 246 OBS HIDDEN



NOTE: 254 OBS HIDDEN

appreciations of all currencies at the same time would artificially increase covariance estimates. A dummy variable is therefore used throughout the ensuing analysis to capture each of these two depreciations (one dummy variable for each depreciation). It is interesting to note that with these dummies included, statistical tests (augmented Dickey-Fuller tests and examination of the correlograms) reveal that the Rupiah-based log exchange rates all are random walks with drift. All the Turkish Lira based exchange rates, however, fail these tests, perhaps suggesting that the Turkish government has some other, non-market-clearing motives in mind when it sets the value of the Lira.

As mentioned in the previous section, the analysis focusses on the behavior of the countries' terms of trade in relation to the behavior of the exchange rates. The unit value of exports (imports) are calculated by dividing the export (import) values by the export (import) volumes. The terms of trade are calculated as the unit value of exports divided by the unit value of imports.

The analysis which follows is based on the logarithms of the exchange rates and the logarithms of the terms of trade, multiplied by 100. This is in harmony with the literature on exchange rates, and gives us the additional benefit of being able to interpret differenced logs as percentage changes. Recall that the theoretical framework requires all the data to be differenced, so we will deal with percentage changes. It should be noted that the

^{6.} All the value data and the Indonesian export volume data come from the International Financial Statistics. The source for the Indonesian import volume data is the Indikator Ekonomi, a publication put out monthly by the Indonesian Bureau of Statistics, and the source for all the Turkish volume data is Aylik Istatistik Bulteni, a publication put out monthly by the Turkish Ministry of State.

percentage changes from month to month in the terms of trade are quite high compared to the monthly exchange rate changes, which are much less volatile and usually fluctuate between ±10%. The correlations between the exchange rate changes and the terms of trade changes are somewhat low, suggesting that the optimal portfolio we derive might be a less than perfect hedge against the changes in the terms of trade.

V. Estimation

1) The ARCH Model

We are interested in obtaining a portfolio of foreign assets which has maximum correlation with the changes in the terms of trade. This optimal hedging portfolio can be found by solving equation (6), where V_{as} is now the vector of covariances between the changes in the terms of trade and the changes in the exchange rates. But notice that $V_{aa}^{-1}V_{as}$ is just a simple OLS regression of the changes in the state variable on changes in the exchange rates. So one could calculate the optimal portfolio shares by running a simple OLS regression of the terms of trade changes on the exchange rate changes and using the parameter estimates for the slopes as shares. However, this procedure implicitly assumes that the variances and covariances of the exchange rate changes are constant through time, an assumption which has been proven false many times in the literature. It would seem appropriate, then, to use an estimation procedure which allows the covariance matrix to change with time.

Autoregressive Conditional Heteroskedasticity (ARCH) is an econometric technique developed by Engle (1982) to do just that. In the univariate version which he presents, the conditional variance of a time

manner. This means that during periods in which there are large unexpected shocks to the variable its estimated variance will increase, and during periods of relative stability its estimated variance will decrease. Bollerslev (1986) generalizes the ARCH model in much the same way that an Autoregressive model is generalized to an Autoregressive Moving Average (ARMA) model. His model, called Generalized ARCH, is the same as an ARMA model in squared residuals. So, just as the ARMA model allows the mean to change with time, the ARCH (and Generalized ARCH) model allows the variance to change with time.

The generalization of the univariate ARCH models to multivariate ARCH models involves allowing the whole covariance matrix to change with time, instead of allowing just the variance to change with time. The first multivariate ARCH model presented in the literature was the one presented by Kraft and Engle (1982), which allows the elements of the covariance matrix to be a function of all lagged squares and cross products of the residuals. So this generalization is the same as the generalization of a univariate ARMA process to a vector ARMA process. This model is very general, and does not always give positive definite covariance matrices, so Baba et.al. (1987) present a model which imposes positive definiteness on the covariance matrix. Bollerslev (1987) develops another multivariate ARCH model which, mile more restrictive, is simpler and much easier to estimate, while still allowing the covariance matrix to change with time. This model imposes the restriction that the correlation matrix is constant through time while the covariance matrix changes. Because of the large number of equations in our model, we will be using the constant correlations version of the multivariate ARCH model. See Appendix 1 for a description of the model.

2) The Estimation Results

If \mathbf{H}_{t} is the covariance matrix of exchange rate depreciations and \mathbf{y}_{t} is the exchange rate depreciation, then the model estimated for Indonesia is

$$y_t = c + d_1D_{1t} + d_2D_{2t} + e_t$$

$$H_t = V_tCV_t$$

where y_t , c, d_1 , d_2 , and e_t are all (7x1) vectors, V_t , C, and H_t are all (7x7) matrices (see Appendix 1), D_{1t} is a dummy variable for the first major depreciation in the rupiah and D_{2t} is a dummy variable for the second major depreciation. Restrictions are imposed which say that the effect of each of the two large depreciations was the same on all seven exchange rates (in terms of percentages). This amounts to imposing the restrictions that $d_1 = r_1i$ and $d_2 = r_2i$, where r_1 and r_2 are scalars and i is a (7x1) vector of ones. The model estimated for Turkey is the same as the model estimated for Indonesia, except without the dummy variables.

These models are estimated using a program, graciously supplied by Tim Bollerslev, which uses maximum likelihood estimation to estimate the values of the parameters in the model. The correlation matrix of weekly exchange rate depreciations for the Indonesian model is given in Table 1 (t-stats in parentheses)⁷, and the correlation matrix for Turkey is given in Table 2. Notice that in both models the correlation of the US dollar with all other exchange rates is much lower than the correlation between any of the

^{7.} It is interesting to note that without the dummy variables, all the correlations increased dramatically. For example, the correlations involving the US dollar increased from about -.40 to +.80.

other exchange rates. In the Indonesian model, the correlations are even negative. This is not surprising in light of the fact that the dollar often moved in a direction opposite to other developed countries' currencies over the sample period covered. Notice also that the European currencies (DM, SWF, AUS, and FF) form a block with high correlations. In fact, DM and AUS are almost perfectly correlated, which leads one to question how much additional information is added by including both in the analysis.⁸

Table 1
Correlation Matrix for Indonesian Based Exchange Rate Depreciations

JY	DM	SWF	AUS	PS	FF	us
1.00	.648	.701	.649	.460	.596	477
•	(22)	(27)	(21)	(11)	(15)	(-10)
	1.00	.919	.990	.649	.893	429
	-	(112)	(949)	(23)	(91)	(-9)
		1.00	.917	.627	.835	444
		-	(114)	(21)	(51)	(-10)
			1.00	.660	.901	422
			-	(24)	(106)	(-9)
				1.00	.660	~.335
				-	(21)	(-6)
					1.00	375
					-	(-7)
						1.00

^{8.} To keep the issues clear, the Austrian schiling is dropped in the ensuing analysis. This is done by simply deleting the row and column of the covariance matrix corresponding to AUS. It should be noted, however, that dropping AUS only causes a redistribution of the holdings of the European currencies in our optimal portfolio, leaving the portfolio weights of the non-European currencies (JY, PS, US) almost unaffected. This supports the conjecture that AUS is not adding much to the analysis.

See Appendix 2 for a presentation and a discussion of the other parameters in the model. A couple of observations which merit mention in the main text, however, are first that the parameters of the ARCH model are almost always significant, providing strong support for the hypothesis that the covariance matrix is changing with time; and second that the constants in the mean equations are significant, capturing the drift which is so obvious in the plots of the exchange rates.

Table 2
Correlation Matrix for Turkish Based Exchange Rate Depreciations

JY	DM	SWF	AUS	PS	PP	US
1.00	.602	.664	.606	.446	.548	.271
-	(17)	(21)	(17)	(11)	(15)	(6)
	1.00	.895	.988	.612	.862	.082
	-	(93)	(754)	(20)	(73)	(1.7)
		1.00	.892	.593	.793	.056
		-	(94)	(18)	(41)	(1.2)
			1.00	.628	.871	.086
			-	(21)	(79)	(1.8)
				1.00	.627	.253
				-	(17)	(5)
					1.00	.101
					-	(2)
						1.00

The terms of trade data are only available monthly, which means that weekly covariances between exchange rate depreciations and changes in the terms of trade cannot be calculated. Monthly covariances must then be used instead. A test is undertaken to see whether or not these covariances also follow an ARCH process. Under the null hypothesis that there is no ARCH in the monthly covariances between a currencies exchange rate depreciations and changes in the terms of trade, the test statistic is distributed χ^2_1 , with a 5% critical value of 3.84. See Table 3 for the computed test statistics. As the test statistics for the Turkish data are all less than 0.07, these values are all insignificant at any reasonable level of significance. We can

^{9.} See Appendix 3 for a theoretical derivation of the appropriate test statistic.

exchange rates are not changing with time, and subsequently the unconditional covariances can be used. The results for the Indonesian data set is similar, though the test statistic for the covariance between changes in the US dollar and the changes in the terms of trade is significant. Because all other test statistics are highly insignificant, we will treat the US dollar covariances as though they are also constant through time.

Table 3
Lagrange Multiplier Statistics for ARCH in the Covariances
between the Changes in the Terms of Trade and Currency Depreciations

	JY	DM	SWF	PS	pp	US
Turkey	0.0003	0.002	0.0007	0.070	0.02	0.0007
Indonesia	0.45	0.05	0.035	0.04	0.0055	10.41

VI. The Optimal Portfolios

In order to calculate a nation's optimal currency compostion of external debt over the next (say) three months, one must first obtain a forecast of the variance-covariance matrix of exchange rate depreciations for the next three months, then multiply the inverse of that by the forecasted covariance between exchange rate depreciations and changes in the terms of trade. The forecast of the covariance between the exchange rate depreciations and the changes in the terms of trade are easy to derive - they are just the unconditional covariances because they are not changing over time. The three month forecasts would be the monthly unconditional covariance multiplied by three.

The ARCH model can be used to forecast the variances and covariances of a series of exchange rate depreciations in exactly the same way that an ARMA model can be used to forecast the mean of a series. So we can just

forecast the weekly covariance matrix for the next thirteen weeks, and use these forecasts to get a forecast of the three month covariance matrix. Tables 4a and 4b give the forecasted covariance matrix for Indonesian based currencies for the first quarter of 1988 and the unconditional covariance matrix over the sample period (Apr/80 to Mar/88). Notice that they are quite different, and they suggest that the exchange rates in the first quarter of 1988 were expected to be relatively more stable than over the previous eight years combined.

Table 4a
Forecasted Covariance Matrix - Indonesia, 1988.1

Table 4b
Unconditional Covariance Matrix - Indonesia, 1980.2 to 1987.4

With this information, we are now able to calculate the optimal portfolio according to equation (6). The result for the shares for Indonesia for the first quarter of 1988, when scaled to sum to one, is

The most striking feature of this portfolio is the heavy weight in the US

expected, though, because Indonesia's exports are largely made up of petroleum and primary commodities, whose prices are evidently more related to the US dollar, and because Indonesia manages its exchange rate with respect to the US dollar. Borrowing a large fraction in US dollars provides then a hedge against changes in terms of trade and export values. The optimal portfolio for Indonesia for the first quarter of 1986 (i.e. based only on data up to the end of 1985, or the first 297 observations) was

This portfolio is not remarkably different from their optimal portfolio for the first quarter of 1988, but it differs substantially from their actual debt portfolio at the end of 1985. Indonesia's debt was actual outstanding in the following proportions:

It would seem, then, that the optimal portfolio could result in a large reduction in the variance of Indonesia's net position, as the optimal portfolio differs substantially from their actual portfolio.

In order to calculate how effective this portfolio strategy is in terms of dynamically hedging against changes in the terms of trade for Indonesia, we assume that the Indonesian authorities followed the optimal strategy every quarter since the end of 1985. I.e., they estimated the ARCH model, forecasted the covariance matrix of exchange rate depreciations, estimated the unconditional covariances of the exchange rate depreciations

with the changes in the terms of trade, and applied equation (6) to get the optimal hedging portfolio for each quarter since the end of 1985. then resulting portfolios, borrowed the the of ($\Delta \ln T + \beta \Delta \ln FX$) can be calculated for two years (1986 and 1987), where T = terms of trade, FX are the residuals from the exchange rate equations, and B are the portfolio weights. The variance of this sequence can be compared with the variance of the sequence which results when they use their 1985 portfolio throughout the two years, assuming borrowing at the absolute levels implied by the optimal portfolio strategies. Comparison of the variances of the two portfolios provides an indication of how well the optimal strategies are in hedging against changes in the terms of trade. Performing this exercise shows that the variance drops using the optimal strategy. Presumably, the movement in Indonesia's borrowing portfolio away from JY to US resulted in the increased stability.

The optimal quarterly portfolios which result from the above analysis, are shown in Table 5. The portfolios are scaled to sum to one. The relative shares of the currencies change quite a bit from quarter to quarter. However, it is interesting to note that the effective currency distribution of the portfolio does not change that much through time once one accounts for the high correlation between the European currencies. The sums of the shares of the European currencies (DM, SWF, PS and FF) are for each respective quarter (from the first quarter of 1986 through the first quarter of 1988): 10.9%, 13.8%, 7.4%, 18%, 34.4%, 19.7%, 25.8%, 11.1% and 6.1%. The combined European share is somewhat more stable than the individual shares, potentially a reflection of the high correlation among the European currencies. In addition to the changes in shares, the unscaled portfolios also change. The sum of the

unscaled portfolio weights ranges between about 5 and about 40, which suggests different absolute levels of borrowing.

Table 5
Optimal Portfolios - Indonesia

period	JY	DM	SWF	PS	FF	US
1986.1	-0.005	0.307	-0.055	0.007	-0.154	0.900
1986.2	-0.022	0.320	-0.028	0.028	-0.182	0.884
1986.3	-0.001	0.164	-0.012	0.021	-0.100	0.928
1986.4	-0.027	0.384	0.019	0.027	-0.252	0.849
1987.1	-0.009	0.801	0.026	0.150	-0.632	0.665
1987.2	0.006	0.462	0.015	0.075	-0.354	0.797
1987.3	-0.033	0.703	-0.017	0.050	-0.479	0.777
1987.4	0.044	0.323	0.001	0.029	-0.243	0.847
1988.1	0.031	0.191	-0.005	0.014	-0.139	0.907

A similar analysis can be conducted to find the portfolios that hedge against changes in export prices, export values, import prices or import values. These portfolios are very similar to the ones above and achieve equally remarkable reductions in variance. As expected, though, the import hedging portfolios are approximately the negative of the export hedging portfolios. The export hedging portfolios are approximately the same as the terms of trade portfolios.

The case for Turkey is analyzed similarly. Applying the strategy decribed above gives the optimal portfolio for the first quarter of 1988 as

This portfolio is much more diversified than Indonesia's optimal portfolio, which might not surprising because Turkey's trade is more diversified than Indonesia's and Turkey's exchange rate policy is less focussed on one currency. Turkey's optimal portfolio involves investing large amounts in SWF,

DM and US (the negative portfolio shares) while borrowing JY, PS and FF. The investing can be done through the central bank's foreign exchange rserves, which could be financed by the borrowings in the other currencies. Their actual portfolio (row 1) and their optimal portfolio (row 2) for the first quarter of 1986 are:

This large discrepancy between their actual debt portfolio and their optimal debt portfolio suggests that there is a lot of hedging that can be done by changing their debt portfolio. Unfortunately, this is not the case due to the large changes in portfolio shares from period to period. The optimal strategies for each quarter are given in Table 6, from which one notices the large changes in the optimal portfolio shares through time, unlike Indonesia where they were relatively stable.

Table 6
Optimal Portfolios - Turkey

	***	DV	CLIE	200	0.00	***
period	JY	DM	SWF	PS	P P	US
1986.1	0.911	-0.311	-0.893	0.255	1.131	-0.093
1986.2	0.335	-0.677	-0.589	0.531	1.265	0.135
1986.3	0.799	-0.479	-0.833	0.718	1.019	-0.225
1986.4	0.548	-0.365	-0.974	0.932	0.867	-0.007
1987.1	0.390	-0.363	-1.022	0.857	1.086	0.052
1987.2	0.362	-0.159	-0.863	0.618	1.190	-0.147
1987.3	0.237	-0.123	-1.234	0.483	1.783	-0.146
1987.4	0.524	-0.316	-1.155	0.052	1.728	-0.301
1988.1	0.115	-0.086	-1.842	0.498	2.511	-0.197

The sum of the unscaled portfolio weights ranges between about 0.9 and 2, suggesting, similar to the results for Indonesia, different absolute levels of borrowing. However, the variability in the suggested absolute level

of borrowing is not as large for Turkey as it was for Indonesia. The sums of the shares of European currencies are respectively for the nine quarters: 18.2%, 53%, 42.6%, 45.9%, 55.8%, 78.5%, 90.9%, 77.7% and 108%. The sums seems to suggest a relatively more stable weight for the European currencies as a whole compared to the individual European currency weights. Restricting the portfolio shares to be positive, i.e., not allowing any investing in foreign currencies, results for the nine quarters in the following portfolios:

Table 7
Optimal Portfolios, Shares Positive - Turkey

period	JY	DM	SWF	PS	FF	US	SUM
1986.2	0.075	0.000	0.000	0.337	0.349	0.239	0.554
1986.3	0.338	0.000	0.000	0.462	0.200	0.000	0.576
1986.4	0.117	0.000	0.000	0.677	0.067	0.138	0.705
1987.1	0.000	0.000	0.000	0.582	0.233	0.180	0.746
1987.2	0.020	0.000	0.000	0.436	0.544	0.000	0.759
1987.3	0.000	0.000	0.000	0.228	0.769	0.003	0.627
1987.4	0.037	0.000	0.000	0.251	0.712	0.000	0.663
1988.1	0.000	0.000	0.000	0.124	0.876	0.000	0.375

In general, restricting the weights of the currencies to be positive leads to less skewed and somewhat more stable portfolios. In addition, the sum of the unscaled portfolio amounts, the right hand column in the table 7, is more stable. This indicates that the relative amount to borrow each period to hedge against terms of trade movements is more constant when the weights are restricted to be positive. The sums of the shares of the European currencies for the eight quarters are respectively: 0.656, 0.662, 0.744, 0.815, 0.980, 0.997, 0.963 and 1.00. This sum is more stable than the individual currency weights, the PS and the FF (and more stable than without restrictions on the portfolio shares). The relatively high correlation between the PS and FF currencies makes these currencies in some sense "substitutes" in terms of

hedging against terms of trade movements. Small relative changes from period to period in the correlation between the PS and FF exchange rates and the terms of trade movements can then allow for large shifts within the European basket. The shifting shares within the European group need not be a reason for concern as long as Turkey sets its borrowing targets with respect to the groups of European currencies as a whole.

Overall the results for Turkey are not completely satisfying, most likely due to weaker data. It seems unlikely that using better econometric techniques and imposing more restrictions while solving for the portfolio weights will lead to more satisfying results unless the data used for Turkey are improved upon.

VII. Conclusion

This paper outlines a theoretical model of how to calculate the optimal debt portfolio for a nation which wants to hedge its terms of trade against exchange rate fluctuations. The paper applies the theoretical analyis to Indonesia and Turkey. The portfolio which we derive for Indonesia is shown to be an effective hedge, reducing the variance of the costs of borrowing reletative tot Indonesia terms of trade. So even though Indonesia might only have limited access to organized currency futures and other hedging markets, they could still manage their external exposure effectively if they could structure their external debt optimally. Historically the optimal portfolios did not change much from one quarter to the next, so the quarterly portfolio changes would mostly require fine-tuning their portfolio, which could make the borrowing strategy all the more beneficial to implement.

The results for Turkey are not completely satisfying. Part of it might be due to the data for Turkey, which were less than perfect. But it also points out some directions for further research. One is to reperform the analysis with some more currencies included. The Turkey model, for example, should probably have as one of its borrowing currencies a currency representing the Middle East - such as Saudia Arabia. Another direction would be to use an instrumental variable technique to forecast currency changes against the Turkish Lira and to obtain the deviations from the expected exchange rate changes. This might enhance the analysis, especially as the Turkish exhange rates do not seem to follow a random walk.

Appendix 1

The multivariate ARCH model used in this paper is the one first proposed by Bollerslev (1987). See his paper for a more detailed discussion of the model.

Let F_t (the information set) be the σ -field generated by past values of e_t , let y_t be a (nxl) vector and let H_t be a (nxn) matrix. Then the multivariate ARCH model is

$$y_t | F_{t-1} \sim N(X_t B, H_t)$$

$$H_t = V_t CV_t$$

where C is a (nxn) correlation matrix which is time-invariant, and V_t is a (nxn) diagonal matrix with the i'th diagonal element, $h_{i,t}^{\frac{1}{2}}$, being the standard deviation of the i'th variable, which is allowed to change with time. The variances are assumed to follow a univariate ARCH process; i.e. $h_{i,t} = w_i + a_i e_{i,t-1}^2 + b_i h_{i,t-1}$. This gives a covariance matrix, H_t , with constant correlations but time-changing variances and covariances.

Letting θ be the parameters of the model and T be the number of observations, the likelihood function is

L (0) =
$$-\frac{TN}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} (\log|H_t| + e_t'H_t^{-1}e_t)$$

which can be rewritten as

8

$$L (\theta) = -\frac{TN}{2} \log(2\pi) - \frac{1}{2}T\log|C| - \sum_{t=1}^{T} \log|D_t| - \frac{1}{2} \sum_{t=1}^{T} s_t C^{-1}s_t$$
 where s_t are the standardized residuals.

This version of the likelihood function highlights one of this model's major advantages (and the reason we chose to use this model instead of one of the other multivariate ARCH models) - evaluation of the likelihood

function requires inversion of only one (nxn) matrix instead of T. Because we use numeric methods to maximize the likelihood function and we have 50 parameters in the model with 400 observations, we reduce the number of matrix inversions from 20,000 to 50 for each iteration. Given that we are estimating about 20 different models with many iterations per model, the time and cost savings are enormous, while the results should not be qualitatively different than with another model.

Appendix 2

The full model estimated was

where y_t , c, and e_t are (7x1) vectors, H_t and C are (7x7) matrices, and V_t is a (7x7) diagonal matrix with diagonal elements $(h_{JY,t},h_{DM,t},\dots,h_{US,t})$ and each $h_{i,t}$ a univariate GARCH(1,1) model. For example, $h_{PS,t} = w_{PS} + a_{PS}e_{PS,t-1}^2 + b_{PS}h_{PS,t-1}$. The results for Indonesia are presented in Table A2.1 and the results for Turkey are presented in Table A2.2.

Table A2.1

ARCH Estimation Results - Indonesia, Data through Mar/1988

parameter	JY	DM	SWF	AUS	PS	PP	US
· c	0.327	0.199	0.237	0.204	0.091	0.104	0.029
t-stats	(4.6)	(2.6)	(2.9)	(2.7)	(1.2)	(1.2)	(5.3)
w	0.130	0.470	0.199	0.483	0.112	0.636	0.000
t-stats	(3.5)	(2.1)	(1.5)	(2.1)	(2.1)	(1.9)	(1.5)
4	0.072	0.058	0.035	0.056	0.047	0.098	0.292
t-stats	(5.7)	(2.4)	(3.6)	(2.2)	(2.9)	(4.4)	(5.8)
b	0.863	0.734	0.892	0.729	0.902	0.623	0.739
t-stats	(30.7)	(6.4)	(16.0)	(6.2)	(25.3)	(3.4)	(23.4)

dummy variables depreciation #1 32.26 (163)
depreciation #2 37.21
. (531)

Table A2.2

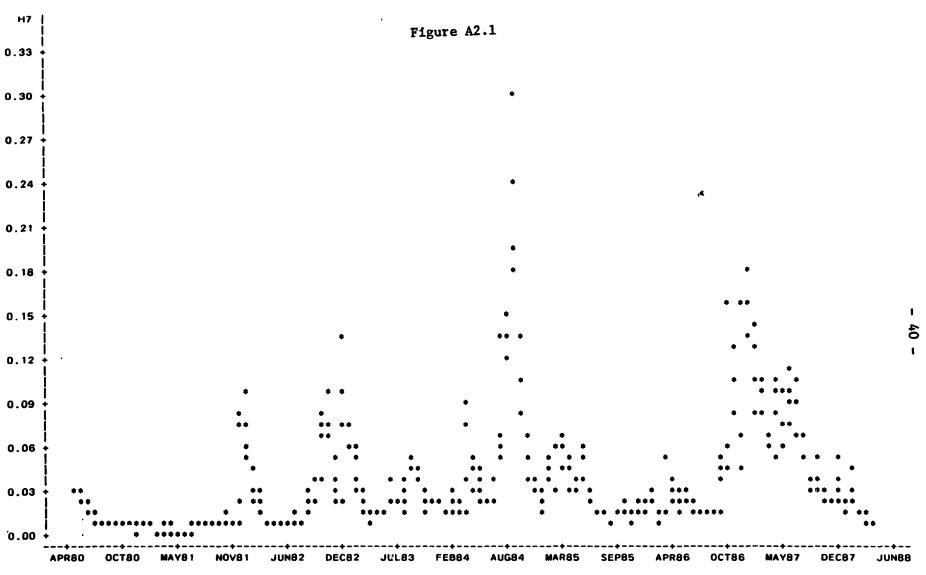
ARCH Estimation Results - Turkey, Data through Mar/88

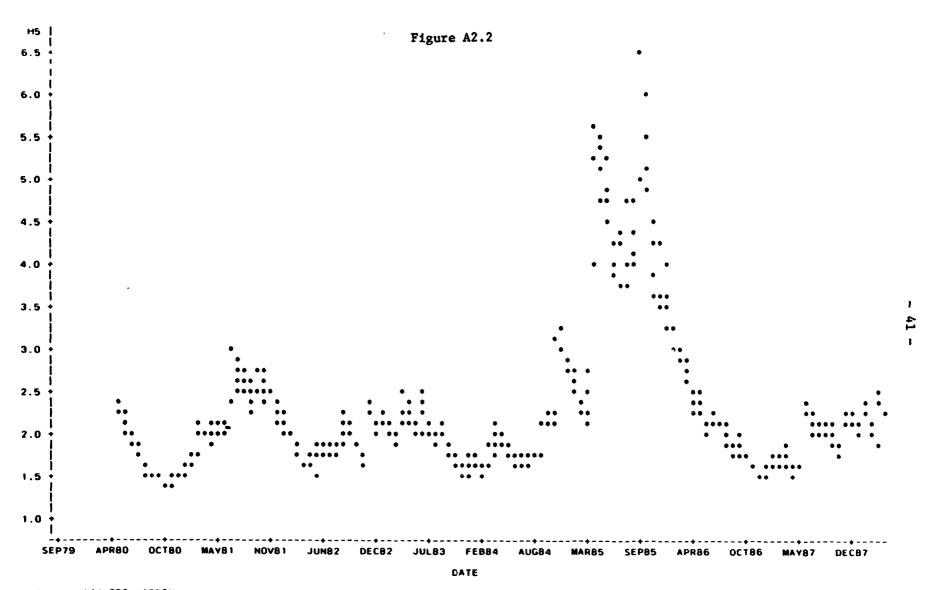
parameter	JY	DM	SWF	AUS	PS	pp	US
· c	0.786	0.691	0.717	0.697	0.609	0.609	0.676
t-stats	(9.9)	(10.1)	(9.2)	(10.3)	(7.2)	(8.6)	(10.8)
v	0.166	0.326	0.032	0.265	0.071	0.732	0.673
t-stats	(2.3)	(1.9)	(1.4)	(2.1)	(1.4)	(2.4)	(3.7)
a	0.095	0.016	0.046	0.044	0.025	0.116	0.234
t-stats	(3.5)	(2.6)	(2.2)	(2.5)	(2.5)	(4.8)	(3.0)
ь	0.823	0.763	0.968	0.797	0.943	0.462	0.156
t-stats	(16.9)	(7.0)	(60.7)	(9.2)	(32.6)	(2.4)	(0.9)

The first observation on these results is that the ARCH parameters a and b are almost always highly significant, which suggests that the variances and covariances are changing through time and the ARCH estimation procedure should give us better covariance estimates at any point in time than OLS. Another observation is that the constants in the mean equations are usually significantly positive, capturing the upward trend in the exchange rates. Notice that for Turkey (for example) the constants are around 0.70, which says that the average weekly depreciation of the Lira was about 0.70%. For Indonesia, the typical weekly depreciation was smaller, but the two dummy variables capture two large depreciations (which probably make up for the smaller typical weekly depreciation). The first depreciation was about 32% and the second was about 37%.

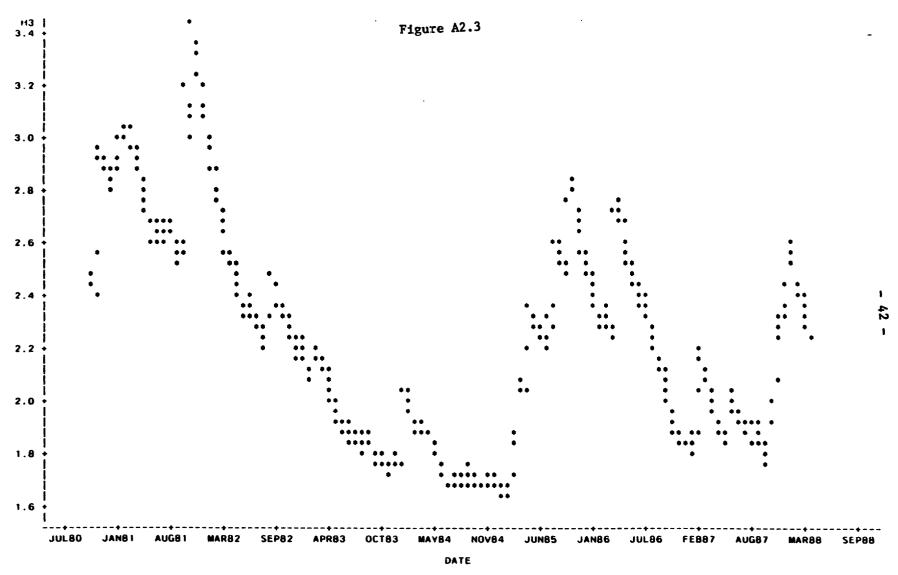
From these results, a series of conditional variances can be constructed which allows us to identify periods of stability and instability in each of the exchange rates. The conditional variances for INDO/US, INDO/PS, TUR/SWF and TUR/JY are plotted in Figures A2.1 to A2.4 respectively. These are just a representative sample of all of the possible plots; the conclusions derived from these plots are similar to those which

would be derived from other plots. Some interesting insights can be gained from these plots. First, the variances of the INDO/US series are much smaller than the other Indonesian-based series' variances (note the scale difference on the vertical axis). This is expected because the Rupiah is operating on a dirty float with respect to the dollar. Notice also that uncertainty in most exchange rates was at a peak in 1985; a time when uncertainty in the dollar was relatively small. One possible explanation for this is that this was when the dollar was falling. The market knew that the dollar was going to fall there was not much uncertainty there - but there was a lot of uncertainty in how the fall would be distributed. Into which currency would the dollar be converted? This leads to increased uncertainty in the other currencies.

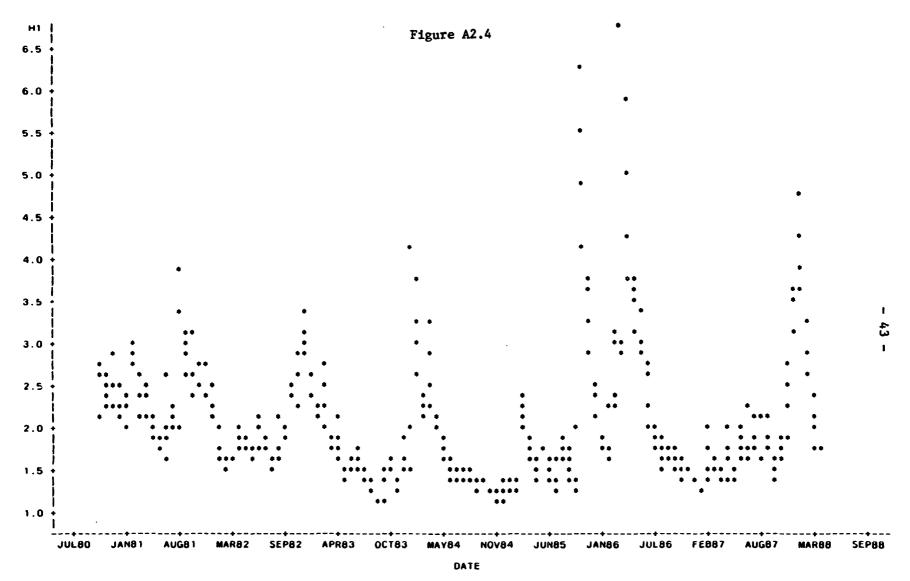




NOTE: 144 OBS HIDDEN



NOTE: 106 OBS HIDDLN



NOTE: 88 OBS HIDDEN

Appendix 3

When testing for restrictions in the ARCH models, the simplest test is the Lagrange Multiplier (LM) test. This is a test which examines whether the slope of the likelihood function evaluated at the parameters under the null hypothesis, is zero. So the LM test requires only the derivatives of the likelihood function with respect to all the parameters, evaluated under the null. If the null hypothesis is given by $H_0:0=0$ then the LM test statistic, which is distributed χ^2_q where q is the number of restrictions imposed by the null, is

$$\xi_{LM} = \left(\frac{\delta L}{\delta \Theta}\right)' \hat{\varsigma}^{-1} \left(\frac{\delta L}{\delta \Theta}\right)|_{\Theta = \Theta_{\Omega}}$$

where ¿ is the information matrix and L is the likelihood function.

Kroner (1987) shows that for ARCH models this test statistic always reduces to

$$\xi_{LM} = \frac{1}{2} (\frac{T}{\xi=1} v_t w_t^{-1} D_t) (\frac{T}{\xi=1} D_t w_t^{-1} D_t)^{-1} (\frac{T}{\xi=1} D_t w_t^{-1} v_t)$$

where

$$v_t = vec(e_te_t'-H_t)$$

$$W_t = H_t \cdot \Theta \cdot H_t$$
and
$$Q_t' = \underbrace{\delta vec}_{\Theta}Ht$$

In this paper, we are interested in testing whether or not the covariances between exchange rate depreciations and terms of trade changes are varying with time. This is done by setting up a bivariate seemingly unrelated regressions model, with the two variables being the appropriate exchange rate depreciations and the changes in the terms of trade. We then test for ARCH in

the covariances from this regression. So if the model is

$$H_{t} = \Omega + \begin{bmatrix} 0 & P \epsilon_{1t} - \hat{1} & 2t - 1 \\ p \epsilon_{1t} - 1 & 2t - 1 & 0 \end{bmatrix}$$

then the null hypothesis for no ARCH in the covariances is μ_0 : p=0, the parameters θ are θ = (vec Ω ,p) and Q_t is given by

$$\begin{bmatrix} I_4 & \varepsilon_{1t-1} & \sigma_{\varepsilon_{2t-1}} \\ \varepsilon_{1t-1} & \varepsilon_{2t-1} & \varepsilon_{2t-1} \end{bmatrix}_{4x5}$$

This test statistic is distributed as a χ^2_1 under the null of no ARCH in the covariances.

Bibliography

- Aylik Istatistik Bulteni, Republic of Turkey, Ministry of State, Institute of Statistics; various issues.
- Baba, Y., R.F. Engle, D.F. Kraft, and K.F. Kroner, (1987), "Multivariate Generalized Autoregressive Conditional Heteroskedasticity,"

 UC San Diego manuscript.
- Bollerslev, T., (1986), "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, 31, 307-327.
- Bollerslev, T., (1987), "Modelling the Coherence in Short Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model," Northwestern University manuscript.
- Breeden, D., (1979), "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," <u>Journal of Financial</u> Economics, 7, 265-296.
- Claessens, S., (1988), "The Optimal Currency Composition of External Debt,"
 Policy, Planning and Research Working Paper #14, Development
 Economics, World Bank.
- Cumby, R. and M. Obstfeld, (1984), "International Interest and Price Level
 Linkages under Flexible Exchange Rates: A Review of Recent
 Evidence", in John F.O. Bilson and Richard C. Marston, editors,
 Exchange Rate Theory and Practice, University of Chicago Press,
 1984
- Diebold, F.X. and M. Nerlove, (1985), "ARCH Models of Exchange Rate Fluctuations," University of Pensylvania manuscript.
- Diebold, F.X. and M. Nerlove, (1988), "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model,"

 Journal of Applied Econometrics, forthcoming.
- Dornbusch, R., (1987), "Exchange Rates and Prices," American Economic Review, 77, pp. 93-106
- Engle, R.F., (1982), "Autoregressive Conditional Heteroskedasticity with,"

 Estimates of the Variance of U.K. Inflation," Econometrica, 50, 987-1008.
- Fischer, S., (1975), "The Demand For Index Bonds," <u>Journal of Political</u> Economy, 83, pp. 509-534
- Frenkel, J. (1981), "Flexible Exchange Rates, Prices and the Role of 'News':

 Lessons from the 1970s", <u>Journal of Political Economy</u> 89, August
 1981.

- Gemmill, G., (1985), "Optimal Hedging on Futures Markets for Commodity-Exporting Nations," <u>European Economic Review</u>, 27, 243-261.
- Indikator Ekonomi, Biro Pusat Statistik, Jakarta, Indonesia; various issues.
- International Financial Statistics, International Monetary Fund Publication; various issues.
- Kraft, D.F. and R.F. Engle, (1982), "Autoregressive Conditional Heteroskedasticity in Multiple Time Series Models," UC San Diego Discussion Paper #82-23.
- Kroner, K.F., (1987) "Estimating and Testing for Factor ARCH," UC San Diego manuscript.
- Merton, R.C., (1971), "Optimal Consumption and Portfolio Rules in a Continuous-Time Model," <u>Journal of Economic Theory</u>, 3, pp. 373-413.
- Stulz, R. (1981), "A Model of International Pricing," <u>Journal of Financial</u> Economics, 9, 383-406.
- Svensson, L.E.O., (1987), "Optimal Foreign Debt Composition," manuscript, World Bank, December 1987.

PPR Working Paper Series

	Title	Author	e	Contact
WP\$130	Maternal Education and the Vicious Circle of High Fertility and Mal- nutrition: An Analytic Survey	Matthew Lockwood Paul Collier	December 1988	J. Klous 33745
:PS131	Implementing Direct Consumption Taxes in Developing Countries	George R. Zodrow Charles E. McLure, Jr	December 1988	A. Bhalla 60359
WPS132	Is the Discount on the Secondary Market A Case for LDC Debt Relief?	Daniel Cohen	November 1988	M. Luna 33729
WPS133	Lewis Through a Looking Glass: Public Sector Employment, Rent-Seeking and Economic Growth	Alan Gelb J.B. Knight R.H. Sabot	November 1988	A. Hodges 61268
WPS134	International Trade in Financial Services	Silvia B. Sagari	January 1989	W. Pitatatonakarn 60353
WP\$135	PPR Working Papers Catalog of Numbers 1 to 105	PPR Dissem. Center	November 1988	Ann Van Aken 31022
WPS136	Pricing Commodity Bonds Using Binomial Option Pricing	Raghuram Rajan	December 1988	J. Raulin 33715
WPS137	Trends in Nontariff Barriers of Developed Countries: 1966 to 1986	Sam Laird Alexander Yeats	December 1988	J. Epps 33710
WP\$138	Fiscal Adjustment and Deficit Financing During the Debt Crisis	William R. Easterly	January 1989	R. Luz 61760
WPS139	A Conceptual Framework for Adjustment Policies	Bela Balassa	January 1989	N. Campbell 33769
WPS140	Building Educational Evaluation Capacity in Developing Countries	John Middleton James Terry Deborah Bloch		

PPR Working Paper Series

	Title	Author	Date	Contact
WPS141	Payroll Taxes for Financing Training in Developing Countries	Adrian Ziderman	January 1989	C. Cristobal 33640
WPS142	Vocational Secondary Schooling in Israel: A Study of Labor Market Outcomes	Adrian Ziderman	January 1989	C. Cristobal 33640
WPS143	Decentralization in Education: An Economic Perspective	Donald R. Winkler		33040
WPS144	Product Differentiation and the Treatment of Foreign Trade in Computable General Equilibrium Models of Small Economies	Jaime de Meio Sherman Robinson		
WPS145	Revenue Raising Taxes: General Equilibrium Evaluation of Alternative Taxation in U.S. Petroleum Industries	Jaime de Melo Julie Stanton David Tarr		
₩PS146	Exchange Rate-Based Disinflation, Wage Rigidity, and Capital Inflows: Tradeoffs for Chile 1977-81	Timothy Condon Vittorio Corbo Jaime de Melo		
WPS147	The Private Sector's Response to Financial Liberalization in Turkey: 1980-82	lzak Atiyas	January 1989	W. Pitayatonakarn 60353
WPS148	Impact of the International Coffee Agreement's Export Quota System on the World Coffee Market	T. Akiyama P. Varangis		
WPS149	Reflections on Perestroyka and the Foreign Economic Ties of the USSR	Bela Baiassa	January 1989	N. Campbell 33769
WPS150	Improving the Currency Composition of External Debt: Applications in Indonesia and Turkey	Ken Kroner Stijn Claessens	January 1989	L. Chavarria 23730