WPS 1279

POLICY RESEARCH WORKING PAPER 1279

Optimal Hedging Strategy Revisited

Acknowledging the Existence of Nonstationary Economic Time Series

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The optimal portfolio model for hedging commodity price and exchange rate risks is extended to nonstationary economic time series data. The new approach corrects the problem of unstable solutions often found with earlier models using economic time series that are nonstationary.

The World Bank
International Economics Department
International Trade Division
March 1994

Summary findings

Recognizing that a country's commodity prices, foreign exchange rates, and export earnings are related, earlier studies developed an optimal portfolio model based on an integrated approach. But the estimates were inefficient because they assumed that the time series data used in the model were stationary. As a result, the model produced unstable solutions that were sensitive to exogenous changes.

Many economic time series — including aggregate consumption, national income, exchange rates, interest rates, commodity prices, and volume of trade — are nonstationary (drift over time). A shock to the nonstationary series has a permanent effect. Problems of nonsense regression or spurious regression can arise when performing regression with nonstationary series.

To correct the problem, Qian and Duncan used Engle and Granger's (1987) vector error correction (VEC) specification in the optimal portfolio estimation process. The VEC approach expands the application of the optimal portfolio model to nonstationary economic time series data.

They apply the new approach to data for Papua New Guinea in an analysis of optimal hedging of commodity price and exchange rate risks using commodity-linked bonds and varying the mix of foreign-currency-dominated borrowings.

They find the time series of commodity prices and foreign exchange rates to be nonstationary. When the VEC approach is applied, the results are comparable to those from the earlier study where the nonstationary was ignored.

The optimal portfolio of commodity-linked bonds and foreign currency borrowings derived from the new model shows more significant risk reduction (measured by ex-ante risk reduction) and less sensitivity to changes in assumption about the real interest rate.

In addition, establishing the cointegration relationships among the commodity prices and foreign exchange rates makes it easier to develop economic intuition in explaining the composition of the optimal portfolio.

The VEC's most significant advantage, however, is the stability achieved in the optimal portfolio solutions to changes in assumptions because of the superior long-run properties of the cointegration and error-correction representation.

This paper — a product of the International Trade Division, International Economics Department — is part of a larger effort in the department to investigate the feasibility and benefits of using risk management instruments by primary commodity producers and exporters in developing countries. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Sarah Lipscomb, room S7-062, extension 33718 (40 pages). March 1994.

Optimal Hedging Strategy Re-visited:

Acknowledging the Existence of

Non-stationary Economic Time Series

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and
Ronald Duncan

Summary

The management of commodity price risks and foreign exchange risks is urgently needed in many developing countries. Recognizing that a country's commodity prices, foreign exchange rates and export earnings are inter-related, an optimal portfolio model, based on an integrated approach, was developed in earlier studies (Myers and Thompson (1989), Claessens and Qian (1991) and Coleman and Qian (1991)) aimed at providing an appropriate mix of borrowings consisting of commodity-linked bonds and foreign currency denominated debts to minimize commodity price risks and foreign exchange risks. However, the estimation process was inefficient because it assumed that the time series data used in the model were stationary. As the result the model produced unstable solutions which were sensitive to exogenous changes.

Empirical studies have shown that many economic time series are non-stationary, including aggregate consumption, national income, volume of trade, exchange rates, interest rates and commodity prices. Compared with stationary data, non-stationary series drifts over time. A shock to the non-stationary series has a permanent effect. Problems of nonsense regression (Yule 1926), or spurious regression (Granger and Newbold 1974) can arise when performing regression with non-stationary series.

In order to correct for the problem, Qian and Duncan decided to use Engle and Granger's (1987) vector error correction (VEC) specification in the optimal portfolio estimation process. The VEC approach expands the applicability of the optimal portfolio model to non-stationary economic time series data and the derivation of cointegration relationships between the variables carried out as part of the estimation process helps to develop economic intuition about the

composition of the optimal portfolio. The VEC's most significant advantage, however, is the stability achieved in the optimal portfolio solutions to changes in assumptions because of the superior long-run properties of the cointegration and error correction representation.

The new approach is applied to data for Papua New Guinea (PNG) in an analysis of optimal hedging of commodity price and exchange rate risk using commodity-linked bonds and varying the mix of foreign currency-denominated borrowings. The time series of commodity prices and foreign exchange rates were found to be non-stationary. When the VEC approach is applied the results are found to be comparable to those from the earlier study using the VAR approach where non-stationarity was ignored. However, the optimal portfolio of commodity-linked bonds and foreign currency borrowings derived from the new model shows more significant risk reduction (measured by *ex-ante* risk reduction) and less sensitivity to changes in assumption about the real interest rate. In addition, establishing the cointegration relationships among the commodity prices and foreign exchange rates makes it easier to develop economic intuition in explaining the composition of the optimal portfolio.

I. Introduction

The uncertainties in primary commodity prices and cross-currency exchange rates impose large risks on primary commodity producing countries. With an appropriate mix of borrowings in commodity-linked bonds and foreign currency denominated debts, the risks can be reduced through linking the debt service to the movements of primary commodity prices and cross-currency exchange rates. An optimal portfolio model of commodity bonds and foreign currencies was developed earlier[1] to minimize commodity price risks and foreign exchange risks. However, a handicap of the model is that it uses potentially non-stationary time series data and results spurious regressions and unstable solutions which were sensitive to exogenous changes, particularly when the sample size is small.

Although much conventional theory for least squares estimation assumes stationarity of the explanatory variables, empirical studies have shown that many economic time series are non-stationary, including aggregate consumption, national income, volume of trade, exchange rates, interest rates and commodity prices. When performing regression with non-stationary series, nonsense regression (Yule 1926), or spurious regression (Granger and Newbold 1974) can result a significant relationship between two completely unrelated but non-stationary series. The earlier studies assumed that data were stationary, but unstable solutions of the optimal portfolio resulted, largely, it is guessed, because some or all of the time series were non-stationary, causing instability in the model estimates. Moreover, if the model is used on non-stationary data

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1 See Myers and Thompson (1989), Claessens and Qian (1991), and Coleman and Qian (1991).

then the true model can hardly be distinguished from spurious regressions and makes it difficult to explain composition of the optimal portfolio.

Two techniques have been recommended in the literature to handle the spurious regression problem with non-stationary data. First, Engle and Granger (1987) developed a vector error correction (VEC) model whereby differenced data and error correct terms (if there are cointegrated vectors) are used to induce stationarity and preserve long-run information lost through differencing. Second, Phillips and Durlauf (1986), Sims *et al* (1990), Stock (1987) and West (1988) developed the asymptotic properties of non-stationary and cointegrated data and showed that estimates from straightforward VAR with non-stationary data that are cointegrated are consistent as long as the sample size is sufficiently large.

Fanchon and Wendel (1992) compared the forecast performance of a VEC specification and a straightforward VAR on cattle prices and concluded that results were comparable. Shoesmith (1992) compared forecasts of state retail sales and personal incomes and concluded that VEC significantly improved medium-term forecasting over the use of straightforward VAR specification.

Dumas and Jorion (1993) addressed the non-stationarity problem in their search for a long-term hedging portfolio by performing a modified cointegration regression. The portfolio they found, however, may not be truly optimal because there is no mechanism in their approach to minimize the residual variance from the cointegration regression. Additionally, their approach requires a long time series to search for the cointegration relationship.

Because sample sizes are usually small and variables are often not cointegrated, we decided to apply Engle and Granger's (1987) approach whereby the Granger Representation Theorem established a link between the VAR on non-stationary variables and the VEC. The theorem says that the VAR of multivariate non-stationary variables is valid if cointegration is present among variables in the system. By estimating the VEC representation, consistent estimates of the VAR coefficient matrix and the residual covariance matrix can be obtained. The model was applied to the data for Papua new Guinea used in an earlier study, with beneficial results.

The immediate advantage of utilizing the VEC representation is that we can expand the applicability of the optimal portfolio model to non-stationary economic time series data while the model derivations and explanations in the previous studies will still hold. Another advantage is the economic intuition gained from the cointegration relationships. Because of the associated "structural" flavor, the cointegration expressions are much more easily understood in describing an economic relationship than a VAR representation. However, the VEC's most significant advantage is the consistency achieved in the optimal portfolio solutions — despite changes in various assumptions — because of the superior long-run properties of the cointegration and error correction representation.

The paper follows the following format. Part II considers the properties and implications of the previous model which is based on VAR representation, and discusses the Granger representation theorem and its application in converting VAR to VEC. Part III elaborates the policy implications of risk management achieved through optimizing use of commodity-linked bonds and foreign currency (other than US dollar) denominated debts. Part IV presents estimates from the VEC representation using data from Papua New Guinea, the subject of an earlier study reported in Coleman and Qian (1991), to see if the new model gives improved results. Part V concludes.

II. Theoretical Consideration

As referenced earlier, a simple rational expectations/VAR model was developed to select the optimal portfolio of foreign currency and commodity-linked bonds for a small, primary commodity exporting country vulnerable to the external financial risks of fluctuations in commodity prices and cross-currency exchange rates.[2] The model was constructed in the framework of a single agent which maximizes lifetime utility under rational expectations. The permanent income hypothesis and assumptions such as uncovered interest rate parity and parity among the expected costs of borrowing from different financial instruments were utilized.

The model has a few distinctive features as compared to conventional mean-variance portfolio selection models.[3] In applying the conventional mean-variance model to financial analysis, the mean term is referred to as the speculation component because it represents the expected cost of borrowing within a certain portfolio. The variance term is referred to as the hedging component because it represents the volatility or the riskiness of the portfolio. So the portfolio selection procedure based on the mean-variance model involves analyzing the tradeoff between the expected return (represented by the mean) and risks (represented by the variance). Also, mean-variance analysis typically focuses on the nominal cash flow in a single time period.

The rational expectations/VAR representation sees the real effects of commodity-price and

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[2] Model description is in Annex A.

[3] See Newbery and Stiglitz (1981) pp. 85-7. The mean-variance analysis is based on the maximization of the expression $y = \bar{y} - \frac{1}{2}av^2$, where y can be the level of utility, income or cash flow, and is assumed to be normally distributed with mean $\bar{y}$ and variance v^2. Parameter a is the measure of relative risk aversion.

exchange rate risks in terms of a country's real imports (i.e., it is assumed that all imports are consumed and that imports are the only source of consumption). It incorporates risk-offsetting effects, and includes only the non-diversifiable (within the country itself) aggregate risk in the portfolio selection. Optimality in risk management is achieved only if the country's lifetime utility is maximized; thus the (expected) long-run effect of the portfolio is fundamental. Under the assumptions of rational expectations, uncovered interest rate parity, and uncovered parities among a variety of financial instruments, the speculation component of the portfolio disappears. Lastly, the model derives the *ex ante* reduction of variance of consumption for the next period, conditioned on the current period, as a measure of the effectiveness of the risk management.

Generally, the estimation of the model can be carried out in two steps: (i) estimating an unrestricted VAR representation:

$$A(L)y_t=\epsilon_t \tag{1}$$

where A(L) is a matrix polynomial in lag operators, y is a set of variables including export revenue, commodity prices and foreign exchange rates, and ϵ_t is a zero mean, serially uncorrelated error vector with covariance matrix Ω; and (ii) retrieving the cross-equation covariance matrix Ω from the VAR residual vectors ϵ_t and the multiplier γ from the coefficient matrix A and computing the optimal portfolio vector using the following equation:

$$b_t=\Omega_{\pi\pi}^{-1}\Omega_{\pi y}\gamma \tag{2}$$

where $\Omega_{\pi\pi}$ and $\Omega_{\pi y}$ are covariance matrices of export revenue, commodity prices and foreign exchange rates.[4]

[4] See Annex A for exact definition of $\Omega_{\pi\pi}$, $\Omega_{\pi y}$ and γ.

However, in the presence of non-stationary data and relatively small sample sizes, the direct VAR estimation on the levels of variables can easily result in spurious regressions. The VAR on differenced data (done in order to overcome the non-stationarity) is not equivalent to the VAR in levels because any long-run properties are lost through differencing.

Engle and Granger (1987), Johansen and Juselius (1990) and Alogoskoufis and Smith (1991) have all contributed to the re-parameterization technique, which links the VAR of non-stationary variables and the VEC representation. In essence, the Granger representation theorem says that:[5] (i) the VAR representation, as in equation (1), estimated for multivariate non-stationary variables is valid if there exist cointegration relationships among the variables, and (ii) by estimating the VEC representation, the consistent estimates of A(L) can be obtained. Thus, we can see that the VAR and VEC are closely related. If the data are non-stationary, in the presence of cointegrating variables in the system the simple VAR in first differences is mis-specified because it omits the level term which appears in the error correction representation. Consistent VAR estimates can be obtained from the estimation of the VEC representation. Nevertheless, if there is no cointegration relationship among the variables, the error correction term in the VEC would disappear and become the VAR on the first differences.

The estimation for the optimal portfolio model in the presence of non-stationary data thus involves three steps: (i) searching for all cointegrating relationships among the variables (estimating static cointegration equations); (ii) estimating the VEC representation; and (iii) retrieving the cross-equation covariance matrix Ω, composing the coefficient matrix A from the VEC, and computing the optimal portfolio as of equation (2).

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[5] The theorem is given in Annex B.

III. Integrated Framework for Financial Risk Management Analysis

Because it is not likely that many commodity-dependent developing countries will quickly reduce their reliance on primary commodity exports, improved commodity price risk management is urgently needed. In addition to their external financial risk exposure due to commodity price fluctuations, these developing countries have significant exchange rate risks. In general, the volatility in nominal and real cross-currency exchange rates was very high during the 1980s, significantly above levels experienced in earlier periods.[6] The debt service changes due to changes in cross-currency exchange rates affect developing countries adversely if the changes are not matched by changes in the value of net hard currency earnings which provides the capacity to service these debts. It has been observed that, in general, commodity prices measured in real terms tend to move inversely with the real value of the dollar (Dornbusch, 1987). When the dollar appreciates value, commodity prices tend to decline and vice-versa. For a primary commodity exporting country whose external debt is mostly denominated in US dollars while the sources of its imports of manufacturing goods are well diversified, a higher dollar would impose a higher service cost for the external debt in terms of the country's real imports (i.e., more non-US dollar currencies would be needed to service the US dollar denominated debt, thus imports of manufacturing goods would be forgone). At the same time, real export earnings would fall due to the decline in primary commodity prices in real terms.

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6 If purchasing power parity (PPP) holds perfectly, currency fluctuations would have no real effects on export earnings and costs of imports. However, strong rejections of the PPP hypothesis, at least in the short term, can be found in many studies.

The external uncertainties in primary commodity prices and cross-currency exchange rates can impose large negative welfare effects on developing countries. These uncertainties can result in highly variable income and consumption streams, complicate planning and investment activities of the government and private sectors, and lower investment returns and long-run output levels.

Optimality in financial risk management should be achieved in an integrated and *ex ante* fashion because of the risk-offsetting effect[7] and the deadweight losses in *ex post* risk sharing. First, the notion of risk-offsetting has been found to be important, i.e., the risk of holding an individual asset needs to be defined with respect to a measure of aggregate risk (e.g., the risk of holding a diversified portfolio of assets). Risks which are diversifiable do not receive any higher expected return and therefore should not be carried. Only non-diversifiable risks require a risk premium. This suggests the following for developing countries: (i) risks from commodity price and exchange rate movements need to be defined in an integrated fashion and relative to their aggregate economic activities; and (ii) risks that are diversifiable in world capital markets should not be carried. Thus, financial portfolio decisions should be made in an integrated fashion.

Second, the optimal risk-sharing structure between creditors and debtors is based on an *ex ante* point of view. As the experience of the 1980s has shown, *ex post* the impact of a financial crises is shared between creditors and debtors in the form of re-scheduling, debt write-offs, and internal and external adjustments. This *ex post* risk sharing involved considerable deadweight losses which could have been avoided through a better *ex ante* structuring of the external debt.

Over the past decade, significant innovations have been made in risk management

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7 The total exposure to all risks may not equal the sum of individual risks, because certain risks can cancel out.

techniques in the international financial markets in response to the increased need to manage uncertainties in currency exchange rates, interest rates, and commodity prices. These risk-management instruments include forwards, futures, options and swaps. Commodity price-linked financing instruments, which combine risk management and finance, have recently been added to the tool kit. With an appropriate mix of commodity-linked instruments, primary commodity exporters can reduce the threat of external shocks and separate financial price risks from production risks. Reduced price risks can make investment planning easier and economic growth more even. Commodity-linked instruments can also provide better financing opportunities and access to international financial markets that would otherwise not be available. In the long run, the improved ability to service debts can lead to improved creditworthiness because a significant amount of creditworthiness risk such as adverse price shocks, which threaten the ability of the country to service its external obligations, can be eliminated.

Most financial risk management instruments can be applied to cross-currency exchange rate risks as well, in the form of currency and interest rate swaps, futures and forwards. These financial risk management tools are being used by many firms/entities in developed countries. In principle, they could be used by developing countries too. However, only a limited number of developing countries have made even occasional use of these instruments. Institutional and credit constraints prevent most developing countries from having the necessary access to these markets Under such circumstances, an alternative hedging instrument to reduce cross-currency exchange rate risk is the management of the currency composition of the external debt.[8]

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[8] For some developing countries, an optimal currency composition of external debt might be difficult to attain because of constraints in altering the currency composition (e.g., in the case of bilateral loans). But knowing what the optimal currency composition should be and what changes are required to achieve such a composition would still lead to substantial potential benefits (e.g.,

Thus, developing countries can minimize their vulnerability to the combined effects of currency and commodity price risks by optimizing the composition of their external debt in foreign currency and by utilizing commodity price-linked bonds. In essence, compared to conventional debt with a fixed interest rate, both commodity-linked bonds and foreign currency composition as external debt management schemes have similar debt servicing profiles; that is, their debt service (interest and/or principal) is not at a predetermined fixed rate. In the case of commodity price-linked bonds, the debt service is linked to commodity prices in the next period. In the case of the currency composition of external debt, the debt service depends on the exchange rate in the next period.

The selection of the optimal portfolio of such financial instruments for a developing country, using the VEC representation outlined above, is the subject of the next part of the paper.

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in selecting and negotiating bilateral and multilateral loans).

IV. Model Estimations

Papua New Guinea has been selected for study because of the availability of the data from an earlier paper (Coleman and Qian, 1991) in which the above-mentioned rational expectations/VAR model was used to calculate the optimal external debt portfolio. In that study it was assumed that all economic time series data were stationary. Therefore, in this study we can compare two sets of results and assess the usefulness of the VEC representation.

The data selected include commodity prices and foreign exchange rates for PNG during the period January 1971 to June 1991. The commodity price series include cocoa, coffee, copper, gold, logs and crude oil. These commodities have accounted for over 80% of PNG's total exports in recent years. Exchange rates are US dollar exchange rates against the British pound, the Deutsche mark, Japanese yen and the Swiss franc. Exports per capita is used as a proxy for the income stream. All value variables are deflated using PNG's domestic consumer price index (in US$ terms), and monthly seasonalities have been removed by regressing each variable on 12-monthly dummies. The four foreign exchange rates deserve attention because of the imbalance between currencies earned from exports and the composition of PNG's external debt. For example, during the sample period, an average of 60% of PNG's exports went to Germany and Japan but the share of debt denominated in Deutsche mark and Japanese yen was only 20%.

Before searching for all possible cointegration relationships among these variables, all need to be established as I(1). Table 1 presents the results of integration tests in levels. Dickey-Fuller (DF) statistics are presented for both regressions with and without a time trend on the right-hand side (RHS). The column "Low Tail Area" indicates the significance of the null-hypothesis of non-stationarity. For example, the DF statistic for cocoa price is -1.08 and the low tail area is 0.96, indicating that the probability of the cocoa price being non-stationary is 0.956. Thus, one cannot reject the null-hypothesis that the cocoa price is non-stationary. Judging from the test results, all variables selected in the model are non-stationary in levels except for the exchange rate of the British pound, for which the DF statistic without the time trend has a low tail area of 0.004. However, the DF statistic for the British pound with the time trend is not significant; thus, it is accepted as non-stationary.

Table 1: Integration Tests
(Dickey-Fuller Tests on Levels)

Variables	Statistics w/o Trend	Low Tail Area	Statistics w/ Trend	Low Tail Area
Export/Capita	-1.450	0.912	-1.496	0.893
Cocoa	-1.079	0.956	-1.698	0.817
Coffee	-1.503	0.880	-1.778	0.784
Copper	-2.124	0.599	-2.324	0.472
Gold	-2.105	0.611	-1.813	0.768
Logs	-2.451	0.393	-2.917	0.164
Crude Oil	-1.827	0.762	-1.498	0.882
British Pound	-4.278	0.004	-2.031	0.655
Deutsche Mark	-1.647	0.835	-1.752	0.795
Japanese Yen	-2.179	0.565	-1.470	0.889
Swiss Franc	-1.647	0.835	-1.752	0.795

Table 2 shows the results of DF tests on the first differences of the same set of variables. Obviously, all DF statistics are highly significant, which indicates that the variables are stationary in their first differences. Combining the findings from Table 1 and 2, we conclude that all variables are I(1), and we should proceed to search for possible cointegration relationships before specifying the VEC.

Table 2: Integration Tests (Dickey-Fuller Tests on First Differences)

Variables	Statistics w/o Trend	Low Tail Area	Statistics w/ Trend	Low Tail Area
Export/Capita	-11.505	0.000	-11.509	0.000
Cocoa	-11.923	0.000	-11.986	0.000
Coffee	-10.029	0.000	-10.046	0.000
Copper	-11.516	0.000	-11.521	0.000
Gold	-11.981	0.000	-12.025	0.000
Logs	-11.613	0.000	-11.613	0.000
Crude Oil	-11.451	0.000	-11.519	0.000
British Pound	-12.822	0.000	-13.472	0.000
Deutsche Mark	-11.301	0.000	-11.303	0.000
Japanese Yen	-11.373	0.000	-11.514	0.000
Swiss Franc	-11.301	0.000	-11.303	0.000

Table 3 shows the results of the cointegration regressions. Among the 11 variables, three pairs are found to be cointegrated. They are exports per capita with the copper price; the cocoa price with the coffee price; and the crude oil price with the Japanese yen exchange rate. Two versions of cointegration regression are presented (with and without the time trend on the right-hand side). Variables across the columns in the table are left-hand side (LHS) variables, and variables down the rows are right-hand side variables (RHS). Numbers in the parenthesis are t-statistics. R^2 indicates the goodness of fit. The DF-Statistics and the Low-Tail Area demonstrate the significance of the cointegration relationships between LHS and RHS variables. The remaining variables not included in the table did not reveal any cointegration relationships.

If we set the significance level at 90% (i.e., Low-Tail Area < 10%), the cointegration relationships between exports per capita and the copper price, and between the crude oil price

Table 3: Cointegration Tests (with and without time trend)

w/o time	Export	Cocoa	Coffee	Copper	Oil	Yen
Constant	13.78	20.86	60.74	1247.75	55.75	668.89
	(8.38)	(2.19)	(5.17)	(11.50)	(37.15)	(83.09)
Export				23.270		
				(4.94)		
Cocoa			1.107			
			(23.47)			
Coffee		0.656				
		(23.47)				
Copper	0.005					
	(5.25)					
Oil						-9.796
						(-24.64)
Yen					-0.076	
					(-24.64)	
R^2	0.117	0.727	0.727	0.106	0.745	0.745
DF-Stat.	-4.950	-3.116	-3.342	-5.217	-3.668	-3.679
Low-Tail	0.002	0.240	0.150	0.001	0.072	0.071
w/ time						
Constant	16.96	119.59	-9.78	1560.87	56.08	673.35
	(7.97)	(9.13)	(-0.416)	(11.78)	(37.54)	(65.91)
Export				18.820		
				(4.01)		
Cocoa			1.241			
			(20.57)			
Coffee		0.542				
		(20.57)				
Copper	0.004					
	(4.22)					
Oil						-9.860
						(-24.17)
Yen					-0.075	
					(-24.17)	
Time	-0.018	-0.605	0.387	-2.047	-0.008	-0.031
	(-2.32)	(-9.48)	(3.43)	(-3.88)	(-2.24)	(-0.71)
R^2	0.139	0.809	0.742	0.117	0.751	0.745
DF-Stat.	-5.294	-3.306	-3.466	-4.701	-3.719	-3.695
Low-Tail	0.001	0.160	0.110	0.004	0.064	0.068

and the Japanese yen exchange rate are fairly robust. The without-time-trend relationship between prices of cocoa and coffee is insignificant in the cointegration. However, with the time trend, the relationship is marginally significant. Because it is conceivable that prices of cocoa and coffee are linked together in the long run, we assume that their are cointegrated.

Thus far, we have finished the first step (searching for cointegration relationships) in estimating the optimal portfolio model with non-stationary economic time series, as laid out in Part III above. The second step is the estimation of the VEC representation which spells out the short-term adjustment process and the long-term cointegration relationships. For the six variables included in Table 3, a semi-unrestricted VEC is specified (certain lags are omitted because of degrees of freedom considerations). For example, the specification for the cocoa price equation is:

$$\begin{aligned}\Delta cc = {} & \Delta cc_{-1}+\Delta cc_{-2}+\Delta cc_{-3}+\Delta cc_{-4}+\Delta cc_{-5}+\Delta cc_{-6}+\Delta cc_{-12}+\Delta cc_{-24}+\Delta cc_{-36} \\ & +\Delta cf_{-1}+\Delta cf_{-2}+\Delta cf_{-3}+\Delta cf_{-4}+\Delta cf_{-5}+\Delta cf_{-6}+\Delta cf_{-12}+\Delta cf_{-24}+\Delta cf_{-36} \\ & + cc + cf + constant\end{aligned} \tag{3}$$

where cc is the price of cocoa, cf is the price of coffee, and Δ indicates the first difference. Up to 36 lags are used so that the 1st through 6th lags pick up the dynamic process of the instantaneous adjustments, and the 12th, 24th and 36th lags uncover the longer time span adjustments. The main feature is the inclusion of level variables such as the prices of cocoa and coffee in (3). We choose the unrestricted form on level variables.

For those variables where no cointegration relationship was found, a regular semi-unrestricted VAR on first differences is specified. For example, the equation for the Deutsche mark/US Dollar exchange rate is:

$$\begin{aligned}\Delta dm = {} & \Delta bp_{-1}+\Delta bp_{-2}+\Delta bp_{-3}+\Delta bp_{-4}+\Delta bp_{-5}+\Delta bp_{-6}+\Delta bp_{-12}+\Delta bp_{-24}+\Delta bp_{-36} \\ & +\Delta dm_{-1}+\Delta dm_{-2}+\Delta dm_{-3}+\Delta dm_{-4}+\Delta dm_{-5}+\Delta dm_{-6}+\Delta dm_{-12}+\Delta dm_{-24}+\Delta dm_{-36} \\ & +\Delta jy_{-1}+\Delta jy_{-2}+\Delta jy_{-3}+\Delta jy_{-4}+\Delta jy_{-5}+\Delta jy_{-6}+\Delta jy_{-12}+\Delta jy_{-24}+\Delta jy_{-36} \\ & +\Delta sf_{-1}+\Delta sf_{-2}+\Delta sf_{-3}+\Delta sf_{-4}+\Delta sf_{-5}+\Delta sf_{-6}+\Delta sf_{-12}+\Delta sf_{-24}+\Delta sf_{-36} + constant\end{aligned} \quad (4)$$

where dm is the Deutsche mark/US Dollar exchange rate, bp, jy and sf are exchange rates of the British pound, Japanese yen and Swiss franc.

Table 4 summarizes the structure of the VEC assembled for the 11 variables. The VEC representation for the rest of the cointegrated variables in Table 3 follows (3) for cocoa. The VAR on first differences for the remainder of the variables follows the Deutsche mark exchange rate as in (4), and the selection of explanatory variables is based on economic intuition, statistical significance of certain groups of variables (F test) and the overall goodness of fit.

The third step in estimating the optimal portfolio model is to retrieve the cross-equation covariance matrix Ω_{yy}, which is presented in Table 5[9]. This matrix represents the inter-relationships of unexpected shocks of commodity prices and foreign exchanges out of the "specified" rational expectation process. The portfolio, which consists of commodity bonds and foreign currency debt for PNG, would be optimal if it utilizes these inter-relationships to neutralize (cancel out) the risk exposure of the export earnings from various sources of unexpected shocks of commodity prices and/or foreign exchange rates.

The optimal portfolio model is solved on a monthly basis for four different sample periods in order to assess its sensitivity to exogenous shocks. Tables 6 through 9 display four sets of solutions for four sample periods ending in June 1991; June 1989; June 1987 and June

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[9] For ease of observation, the corresponding correlation matrix is presented. XT is for export; CC is for the price of cocoa; CF for coffee; CP for copper; GD for gold; LG for logs; OL for oil; BP for British Pound, DM for Deutsche Mark, JY for Japanese Yen and SF for Swiss Franc.

Table 4: The System of VAR/Error Correction Representations

Error Correction Representations:

Exports Per Capita:	Copper Price
Cocoa:	Coffee Price
Coffee:	Cocoa Price
Copper:	Exports Per Capita
Crude Oil:	Japanese Yen/US Dollar Exchange Rate
Japanese Yen:	Crude Oil Price

VARs:

Gold:	Copper Price
Logs:	Copper Price
British Pound:	Crude Oil Price, Deutsche Mark/US Dollar Exchange Rate, Swiss Franc/US Dollar Exchange Rate
Deutsche Mark:	British Pound/US Dollar Exchange Rate, Japanese Yen/US Dollar Exchange Rate, Swiss Franc/US Dollar Exchange Rate
Swiss Franc:	Gold Price, Crude Oil Price, Deutsche Mark/US Dollar Exchange Rate

Table 5: Cross Equation Correlation Matrix (1977.1-1991.6)

	XT	CC	CF	CP	GD	LG	OL	BP	DM	JY	SF
XT	1.00										
CC	0.36	1.00									
CF	0.34	0.87	1.00								
CP	0.23	0.09	0.04	1.00							
GD	-0.01	-0.26	-0.24	0.08	1.00						
LG	0.23	-0.23	-0.16	0.50	0.43	1.00					
OL	-0.08	0.12	0.07	-0.26	0.49	-0.11	1.00				
BP	0.35	0.82	0.68	0.33	0.01	-0.03	0.17	1.00			
DM	0.38	0.68	0.54	0.49	-0.16	0.13	-0.30	0.83	1.00		
JY	0.21	0.05	0.01	0.29	-0.40	0.18	-0.87	0.02	0.49	1.00	
SF	0.33	0.52	0.35	0.44	-0.12	0.20	-0.39	0.69	0.94	0.61	1.00

1985. All samples start in January 1977. Within each table, six different annual real interest rates are assumed, and solutions are in constant 1985 US dollars.

Table 6 shows the optimal portfolio for the sample period ending in June 1991[10]; and the results are consistent with the earlier paper (Coleman and Qian, 1991). The portfolio is on a per capita basis and is in terms of a unit of constant 1985 US dollars. The item "Variance" shows the variance of expected imports without the optimal portfolio. "Variance Reduction" can be interpreted as the contribution of the optimal portfolio in reducing the "Variance".[11] "Risk Reduction" is calculated as the percentage decline of the standard deviation before and after the application of the optimal portfolio.

According to these results, PNG should hold assets in Deutsche marks and Swiss francs and borrow extensively in commodity-linked bonds and British pound and Japanese yen denominated currencies. However, unlike the earlier model where the solution was sensitive to real interest rate assumptions, the portfolio solution here is fairly stable to different real interest rate assumptions ranging from 1% to 16%. One reason for the stability is that the model is estimated on the monthly data; the annual real interest rate has been transformed to a monthly rate, thereby de-emphasizing the impact of higher rates. Nevertheless, because lags of variables covering up to a three year span are included in the VEC, the high real interest rates should have a significant cumulative effect on a monthly basis.

Results from Table 6 cannot be compared with results from the earlier paper which was based on annual data and solved for year 1988 in constant 1980 US dollars. However, since

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10 The positive sign indicates that PNG should borrow using that instrument up to the stated amount per capita, and the negative sign indicates that PNG should lend.

11 These two items are from equation (A29) in Annex A. "Variance" is the first part of the equation and "Variance Reduction" is the second part.

Table 6: Optimal Portfolio of External Debt Per Capita (Constant 1985 Dollar, 1977.1-1991.6)

Interest Rate	0.01	0.04	0.07	0.10	0.13	0.16
Cocoa	0.40	0.40	0.40	0.40	0.41	0.41
Coffee	0.17	0.17	0.17	0.17	0.17	0.17
Copper	18.23	17.64	17.10	16.61	16.17	15.77
Gold	0.11	0.12	0.12	0.12	0.12	0.12
Logs	3.04	3.05	3.07	3.09	3.11	3.12
Crude Oil	0.63	0.63	0.64	0.64	0.64	0.65
British Pound	1.46	1.47	1.48	1.49	1.49	1.50
Deutsche Mark	-0.56	-0.56	-0.57	-0.57	-0.57	-0.58
Japanese Yen	5.81	5.85	5.88	5.91	5.95	5.98
Swiss Franc	-2.69	-2.71	-2.72	-2.74	-2.75	-2.77
Commodity Bond	22.58	22.01	21.49	21.03	20.61	20.24
Currency	4.02	4.05	4.07	4.09	4.12	4.14
Variance	18.94	18.13	17.42	16.81	16.28	15.81
Variance Reduction	16.01	15.16	14.43	13.78	13.21	12.71
Risk Reduction	60.7%	59.5%	58.6%	57.5%	56.6%	55.7%

prices (in US dollar terms) were virtually the same in PNG in 1980 and in 1985, and the monthly data in the current model have been de-seasonalized, it seems to be acceptable to transform them to annually-based numbers by multiplying the monthly figures by 12. If we do this transformation, we find that the results from the monthly VEC are very comparable to the previous annual VAR results in terms of the commodity bond and foreign exchange aggregates. Under a mid-range real interest rate assumption (r=4%), the annualized total commodity bond from the VEC is $264, and the total foreign currency debt is $48.6. The previous model gave a solution of $285 for total commodity bonds and $81 for foreign currency debt.

As can also be seen from Table 6, the new model is more effective in terms of risk reduction than the previous model. The dominating instrument is found to be the copper-linked bond. This is understandable because PNG's export earnings have been heavily dependent on copper. A major difference between optimal portfolio from the new model and that from previous one is that the new one uses all instruments, whereas in earlier solution, only cocoa and oil bonds were found to be positive. As a result, the risk reduction effect of the new portfolio is much greater. The new portfolio reduces the risk exposure by 60% when real interest rate is 4%, whereas the previous one reduced risk by 34%. In addition, risk reduction is not sensitive to real interest rate assumptions. The risk reduction of the previous model declined from 39% to 21% as the real interest rate increased from 1% to 8%. In the new model, the risk reduction only declines marginally from 60.7% to 55.7% as the real interest rate increases from 1% to 16%.

Note that the variance derived from the new model for the portfolio is comparable to portfolio variance from the pervious model if the correct transformation is done. The transformation is to take the square root of the monthly variance, multiply it by 12, and then compute the square. In that case, the annualized variance (under r=7%) is 2508, where the previous model gave 2355.

The optimal portfolio model is further tested using three other sample sets. Table 7 shows solutions based on the sample period January 1971 - June 1989. Comparing results in Table 7 and 6, we see that the overall structure of the portfolio is very much alike in that: (i) the solution is insensitive to real interest rate assumptions; (ii) the risk reduction is large (around 70%); and (iii) commodity-linked bonds dominate the portfolio, especially the copper-linked bond. However, there are some differences in Table 7, most notably they are: (i) the level of total borrowing in commodity-linked bonds is 70% higher and the level of copper-linked bond is almost doubled; (ii) British pound borrowing becomes negative while Deutsche mark borrowing is positive; and (iii) the level of variance is more than twice as high as in Table 6.

Table 7: Optimal Portfolio of External Debt Per Capita (Constant 1985 Dollar, 1977.1-1989.6)

Interest Rate	0.01	0.04	0.07	0.10	0.13	0.16
Cocoa	0.25	0.25	0.26	0.26	0.26	0.26
Coffee	0.29	0.29	0.29	0.29	0.29	0.29
Copper	35.47	33.91	32.54	31.33	30.25	29.29
Gold	-0.44	-0.44	-0.44	-0.44	-0.45	-0.45
Logs	3.03	3.05	3.07	3.09	3.11	3.13
Crude Oil	0.66	0.66	0.67	0.67	0.68	0.68
British Pound	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
Deutsche Mark	2.49	2.51	2.52	2.54	2.56	2.57
Japanese Yen	3.96	3.98	4.01	4.03	4.06	4.08
Swiss Franc	-3.02	-3.03	-3.05	-3.07	-3.09	-3.11
Commodity Bond	39.26	37.73	36.38	35.19	34.14	33.20
Currency	3.38	3.40	3.43	3.45	3.47	3.49
Variance	48.73	45.29	42.41	39.95	37.85	36.03
Variance Reduction	45.26	41.78	38.85	36.35	34.20	32.34
Risk Reduction	73.3%	72.2%	71.0%	70.0%	68.9%	68.0%

The significance of the copper-linked bond in the optimal portfolio can be explained by copper-price shocks in real terms during the sample period. The copper price experienced two major shocks from its relatively stable level in the early 1980s. The first came in late 1987 and the second in late 1989. Both shocks were of a magnitude of more than a 40% increase in the space of two months. In both cases, the after-shock price level in real terms returned to the stable level of the first half of the 1980s. Although the copper price continued to exhibit similar (if somewhat smaller scale) shocks two or three more times after 1989, to a rational expectationist in 1989, those two earlier shocks would be viewed as very much unexpected, and imposed huge risks to PNG's export earnings. Thus, we can see why the variance in Table 7 is larger than variance in Table 6, and the amount of the copper-linked bond is twice as large as in Table 6.

Unfortunately, unlike the case of copper-linked bond, it is difficult to pinpoint the reason why British pound and Deutsche mark borrowings change sign in Tables 6 and 7, since the results are derived from complicated simultaneous processes involving all the inter-relationships between commodity prices, foreign exchanges and export earnings.

Table 8 presents the solution based on the sample period January 1977 - June 1987. The optimal portfolio in Table 8 demonstrates the similar behavior to that in Tables 6 and 7. These are: (i) insensitivity with respect to the real interest rate assumption; (ii) high level of risk reduction; and (iii) relatively strong position of commodity-linked bonds in the portfolio. However, some differences exist. For example: (i) the variance is low compared to Table 7 and is similar to Table 6; (ii) the amount of copper-linked bond is reduced rather significantly because the copper price was relatively stable during that period; and (iii) the Swiss franc changes its sign from negative in Tables 6 and 7 to positive. Another interesting feature is that the crude oil-linked bond becomes important when the sample period ends earlier. For example, the oil-linked bond in the optimal portfolio presented in Table 6 is $0.63 (under r=0.04) per capita, in Table 7 it is $0.66 per capita, while in Table 8 it increases to $1.67 per capita.

Table 9 shows the optimal portfolio calculated for the sample period January 1977 - June 1985. Comparing these result with Tables 6, 7 and 8, we see great similarities in the property of the portfolio, such as real interest rate insensitivity, significant risk reduction and a large share of commodity-linked bonds. One major difference with to the other tables is the significance of the crude oil-linked bond. Table 9 suggests that the oil-linked bond should be around $7.50 per capita in the optimal portfolio. The reason for this is that in 1985, the rational expectationists might still be experiencing repercussions from the second oil shock of 1980 and sensing large

Table 8: Optimal Portfolio of External Debt Per Capita
(Constant 1985 Dollar, 1977.1-1987.6)

Interest Rate	0.01	0.04	0.07	0.10	0.13	0.16
Cocoa	1.45	1.46	1.46	1.47	1.48	1.48
Coffee	0.74	0.74	0.74	0.75	0.75	0.75
Copper	8.04	7.73	7.44	7.17	6.93	6.70
Gold	1.06	1.06	1.07	1.07	1.08	1.08
Logs	3.58	3.60	3.61	3.63	3.65	3.67
Crude Oil	1.66	1.67	1.68	1.69	1.70	1.70
British Pound	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Deutsche Mark	0.32	0.32	0.32	0.33	0.33	0.33
Japanese Yen	4.67	4.69	4.72	4.74	4.76	4.78
Swiss Franc	2.72	2.73	2.74	2.76	2.77	2.78
Commodity Bond	16.53	16.25	16.00	15.78	15.57	15.39
Currency	7.71	7.74	7.78	7.82	7.85	7.89
Variance	18.37	17.84	17.39	16.99	16.63	16.32
Variance Reduction	14.21	13.65	13.15	12.71	12.32	11.97
Risk Reduction	52.4%	51.5%	50.6%	49.8%	49.0%	48.3%

risks arising from oil price movements. Thus, they assigned a relatively large value to variance as shown in Table 9 (compared to Table 8), and picked a higher value for the oil-linked bond in order to hedge the oil risk.

Table 9: Optimal Portfolio of External Debt Per Capita
(Constant 1985 Dollar, 1977.1-1985.6)

Interest Rate	0.01	0.04	0.07	0.10	0.13	0.16
Cocoa	2.06	2.07	2.08	2.09	2.10	2.11
Coffee	0.24	0.24	0.24	0.25	0.25	0.25
Copper	7.88	7.50	7.15	6.83	6.52	6.24
Gold	0.91	0.92	0.92	0.93	0.93	0.94
Logs	4.09	4.11	4.13	4.15	4.17	4.19
Crude Oil	7.83	7.86	7.90	7.94	7.98	8.02
British Pound	-2.58	-2.60	-2.61	-2.62	-2.63	-2.65
Deutsche Mark	6.44	6.47	6.50	6.53	6.56	6.59
Japanese Yen	5.35	5.38	5.40	5.43	5.45	5.48
Swiss Franc	0.14	0.15	0.15	0.15	0.15	0.15
Commodity Bond	23.01	22.70	22.42	22.17	21.94	21.73
Currency	9.35	9.39	9.44	9.48	9.53	9.57
Variance	23.02	22.39	21.84	21.35	20.91	20.52
Variance Reduction	18.14	17.46	16.86	16.32	15.84	15.40
Risk Reduction	54.0%	53.1%	52.2%	51.5%	50.8%	50.0%

VI. Conclusion

Commodity price risks and foreign exchange risks have a marked impact on the economic performance of primary commodity producing countries and effective risk management programs are urgently needed there. This paper proposed an integrated approach, recognizing that a country's commodity prices, foreign exchange rates and export earnings are inter-related. A rational expectation model aimed at providing the optimal portfolio consisting of commodity-linked bonds and foreign exchanges is analyzed. It is suggested that the earlier VAR representation in estimating the rational expectations model of the optimal portfolio was less efficient because the assumption of stationarity of the economic time series was not met, and as the result it produced relatively unstable solutions which were sensitive to exogenous changes.

The theory of re-parameterization of the VAR on non-stationary time series proposed by Engle and Granger (1987) provides us with a tool to re-specify and transform the model into a VEC representation. This transformation ensures efficient econometric estimation when using non-stationary time series.

The empirical application of the VEC yields results comparable to those of the earlier model where in all economic time series data were assumed to be stationary. The new model confirms the usefulness of commodity-linked bond in the optimal portfolio combined with foreign currency, in providing significant risk reduction (measured by the *ex-ante* risk reduction). However, compared to the earlier model, the VEC is superior in several aspects. First, it is applicable to non-stationary economic time series data, which makes the model flexible and realistic. This is because most economic time series data, including commodity prices, are found to be non-stationary. Second, it yields relatively insensitive solutions of the optimal portfolio to changes in exogenous variables. Third, the new model unveils the structural behavior among economic time series from cointegration relationships, thus making it easier to develop economic intuition about the composition of the optimal portfolio.

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Annex. An Optimal Portfolio Model

Mathematically, the model can be presented as follows. In a small open economy, assume all external debt is issued by a single agent. Debt can be issued as conventional debt (i.e., with a fixed real interest rate) or as debt with a variable debt service scheme. At time t, the agent produces exports x_t and consumes imports m_t, with utility function:

$$u_t(m_t)=\bar{m}m_t-\frac{1}{2}m_t^2 \qquad \textbf{(A1)}$$

where m_t is the level of imports, and $\bar{m}$ is the bliss level of imports. The agent can also borrow from various sources under different debt service schemes; thus the budget constraint the agent faces is:

$$m_t+rd_{t-1}+\pi_t'b_{t-1}\leq\theta_t x_t+(d_t-d_{t-1})+\omega_t'b_t \qquad \textbf{(A2)}$$

where x_t is the level of exports, θ_t is the terms of trade, and d_t is a conventional loan at time t, with a fixed rate of interest r. Vector $b_t = (b_{1t}, b_{2t}, \ldots, b_{nt})'$ represents borrowing whose elements are in the variable debt service scheme,[12] $\omega_t = (\omega_{1t}, \omega_{2t}, \ldots, \omega_{nt})'$ is the vector for converting b_t to the unit of real imports at the time of borrowing, and $\pi_t = (\pi_{1t}, \pi_{2t}, \ldots, \pi_{nt})'$ is the converting vector of b_t at debt service time. Thus $\omega_t'b_t$ is the total value borrowed in units of real imports at time t and $\pi_t'b_{t-1}$ is the debt service.

The agent also face a transversality condition:

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12 Elements of b_t are debt denominated in foreign currencies and/or commodity-linked bonds. All vectors are denoted as column vectors. The sign ' indicates the "transform".

$$\lim_{t\to\infty}(1+r)^{-t}d_t=\lim_{t\to\infty}(1+r)^{-t}\omega_t' b_t=0 \qquad \text{(A3)}$$

The agent's problem is to choose a portfolio of d_t and b_t so as to maximize the expected lifetime utility function:

$$E_0\sum_{t=0}^{\infty}\beta^t u(m_t) \qquad \text{(A4)}$$

subject to (A2) and (A3).

The associated Euler equations are:

$$u'(m_t)-\beta(1+r)E_t u'(m_{t+1})=0 \qquad \text{(A5)}$$

$$u'(m_t)\omega_t-\beta E_t(u'(m_{t+1})\pi_{t+1})=0 \qquad \text{(A6)}$$

The optimal import path can be derived following the permanent income hypothesis of consumption. Rewrite the binding budget constraint (A2):

$$m_t=\theta_t x_t-(1+r)d_{t-1}-\pi_t' b_{t-1}+d_t+\omega_t' b_t \qquad \text{(A7)}$$

Lead (A7) one period, multiply both sides by $1/(1+r)$ and take the expectation:

$$\frac{1}{1+r}E_t m_{t+1}=\frac{1}{1+r}(E_t\theta_{t+1}x_{t+1}-(1+r)d_t-E_t\pi_{t+1}' b_t+E_t d_{t+1}+E_t\omega_{t+1}' b_{t+1}) \qquad \text{(A8)}$$

Assume the following:

$$E_t[\omega_{t+i}-\frac{\pi_{t+i+1}}{1+r}]=0, \quad i=0,1,\ldots,\infty \qquad \text{(A9)}$$

Add (A7) and (A8):

$$m_t+\frac{1}{1+r}E_tm_{t+1}=\theta_tx_t+\frac{1}{1+r}E_t\theta_{t+1}x_{t+1}-(1+r)d_{t-1}-\pi_t'b_{t-1}+\frac{1}{1+r}E_td_{t+1}+\frac{1}{1+r}E_t\omega_{t+1}'b_{t+1} \quad \text{(A10)}$$

Lead (A7) two periods, multiply both sides by $1/(1+r)^2$, take the expectation and add to (A10):

$$\begin{aligned} m_t+\frac{1}{1+r}E_tm_{t+1}+\frac{1}{(1+r)^2}E_tm_{t+2}=\theta_tx_t+\frac{1}{1+r}E_t\theta_{t+1}x_{t+1}+\frac{1}{(1+r)^2}E_t\theta_{t+2}x_{t+2} \\ -(1+r)d_{t-1}-\pi_tb_{t-1}+\frac{1}{(1+r)^2}E_td_{t+2}+\frac{1}{(1+r)^2}E_t\omega_{t+2}b_{t+2} \end{aligned} \quad \text{(A11)}$$

Repeat above operation an infinite number of times, and assume the transversality condition:

$$\sum_{i=0}^{\infty}(1+r)^{-i}E_tm_{t+i}=\sum_{i=0}^{\infty}(1+r)^{-i}E_t\theta_{t+i}x_{t+i}-(1+r)d_{t-1}-\pi_t'b_{t-1} \quad \text{(A12)}$$

Assume that the permanent income hypothesis holds:

$$m_t=E_tm_{t+1}=E_tm_{t+2}=...=E_tm_{t+i}=E_tm_{t+i+1}=...,\quad i=0,1,...,\infty \quad \text{(A13)}$$

thus the optimal import path can be found as:

$$m_t=\frac{r}{1+r}\left[\sum_{i=0}^{\infty}(1+r)^{-i}E_t(\theta_{t+i}x_{t+i})-\pi_t'b_{t-1}-(1+r)d_{t-1}\right] \quad \text{(A14)}$$

Define $y_t = (\theta_tx_t, \pi'_t, s_t)'$, where s_t is a set of other state variables, and assume y_t follows the autoregressive process:

$$A(L)y_t=\epsilon_t \quad \text{(A15)}$$

where A(L) is a matrix polynomial in lag operators[13] and ϵ_t is a zero mean, serially uncorrelated error vector with covariance matrix Ω. Given (A14), the optimal projection of the future income

[13] Specifically $A(L)=I-A_1L-A_2L^2-...-A_qL^q$ where I is the identity matrix and A_q is the coefficient matrix of q-lagged variables.

stream (in units of imports) derived from exports can be found as:[14]

$$\sum_{i=0}^{\infty}(1+r)^{-i}E_t(\theta_{t+i}x_{t+i})=\gamma' y_t+B(L)y_{t-1}$$
$$where\ \gamma'=\phi A\left(\frac{1}{1+r}\right)^{-1},\ B(L)=\phi A\left(\frac{1}{1+r}\right)^{-1}\left[\sum_{j=1}^{q-1}\left[\sum_{k=j+1}^{q}(1+r)^{j-k}A_k\right]L^{j-1}\right] \tag{A16}$$

where ϕ is a row vector with a one in the first column and zeros elsewhere.[15] Substituting (A16) into (A14) gives the operational decision rule:

$$m_t=\frac{r}{1+r}\left[\gamma' y_t+B(L)y_{t-1}-\pi_t' b_{t-1}-(1+r)d_{t-1}\right] \tag{A17}$$

Rearranging the Euler equations (A5) and (A6) gives:

$$E_t\left[u'(m_{t+1})\left(\omega_t-\frac{\pi_{t+1}}{1+r}\right)\right]=0 \tag{A18}$$

Assume that the expected real rate of return on holding b_t is equal to the real interest rate on the conventional loan d_t:

$$E_t\left(\omega_t-\frac{\pi_{t+1}}{1+r}\right)=0 \tag{A19}$$

Then, by (A18) and (A19):

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[14] See Hansen and Sargent (1980).

[15] $A(\frac{1}{1+r})^{-1}=(I-\frac{1}{1+r}A_1-(\frac{1}{1+r})^2A_2-\ldots-(\frac{1}{1+r})^qA_q)^{-1}$

$$E_t(u'(m_{t+1}))E_t(\pi_{t+1})=E_t(u'(m_{t+1})\pi_{t+1}) \qquad \textbf{(A20)}$$

which implies that the conditional covariance at time t between marginal utility and prices π_{t+1} at time t+1 is zero:

$$COV_t(u'(m_{t+1}),\pi_{t+1})=0 \qquad \textbf{(A21)}$$

From (A1), the first derivative of $u(m_{t+1})$ with respect to m_{t+1} is:

$$u'(m_{t+1})=\overline{m}-m_{t+1} \qquad \textbf{(A22)}$$

So (A21) is equivalent to:

$$COV_t(m_{t+1},\pi_{t+1})=0 \qquad \textbf{(A23)}$$

Leading (A17) one period and substituting into (A23):

$$COV_t\left(\left(\frac{r}{1+r}(\gamma' y_{t+1}+B(L)y_t-\pi'_{t+1}b_t-(1+r)d_t)\right),\pi_{t+1}\right)=0 \qquad \textbf{(A24)}$$

Rearranging (A24)[16] and recognizing that all variables at time t are known and thus can be dropped out of the covariance expression, leads to:

$$\Omega_{\pi y}\gamma-\Omega_{\pi\pi}b_t=0 \qquad \textbf{(A25)}$$

where $\Omega_{\pi y}$ is the covariance operation between vector π and y; $\Omega_{\pi\pi}$ is the covariance matrix of

[16] For simplicity, subscript t+1 has been dropped.

vector π.[17] Solving for b gives:

$$b_t = \Omega_{\pi\pi}^{-1}\Omega_{\pi y}\gamma \tag{A26}$$

In estimating the optimal portfolio, it is important to be able to determine whether the variance of real import levels is reduced via external financing through commodity-linked bonds and the composition of the foreign currency borrowing in order to evaluate the hedging effectiveness of the portfolio. Leading (A17) one period, the conditional variance of m_{t+1} at time t is equal to:

[17] In the estimation process, $\Omega_{\pi y}$ and $\Omega_{\pi\pi}$ are obtained from the cross-equation covariance matrix Ω_{yy} of the VAR system specified by equation (A15). Specifically, if we define the residual vector of the VAR system $y_R = (\theta x, \pi_1, \pi_2, \ldots, \pi_n)$, then Ω_{yy}, $\Omega_{\pi y}$ and $\Omega_{\pi\pi}$ can be shown as follows:

$$\Omega_{yy} = \begin{pmatrix} (\theta x)^2 & (\theta x)\pi_1 & (\theta x)\pi_2 & \cdots & (\theta x)\pi_n \\ (\theta x)\pi_1 & \pi_1^2 & \pi_1\pi_2 & \cdots & \pi_1\pi_n \\ (\theta x)\pi_2 & \pi_2\pi_1 & \pi_2^2 & \cdots & \pi_2\pi_n \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ (\theta x)\pi_n & \pi_n\pi_1 & \pi_n\pi_2 & \cdots & \pi_n^2 \end{pmatrix}$$

$$\Omega_{\pi y} = \begin{pmatrix} (\theta x)\pi_1 & \pi_1^2 & \pi_1\pi_2 & \cdots & \pi_1\pi_n \\ (\theta x)\pi_2 & \pi_2\pi_1 & \pi_2^2 & \cdots & \pi_2\pi_n \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ (\theta x)\pi_n & \pi_n\pi_1 & \pi_n\pi_2 & \cdots & \pi_n^2 \end{pmatrix} \qquad \Omega_{\pi\pi} = \begin{pmatrix} \pi_1^2 & \pi_1\pi_2 & \cdots & \pi_1\pi_n \\ \pi_2\pi_1 & \pi_2^2 & \cdots & \pi_2\pi_n \\ \cdot & \cdot & \cdots & \cdot \\ \pi_n\pi_1 & \pi_n\pi_2 & \cdots & \pi_n^2 \end{pmatrix}$$

$$VAR_t(m_{t+1}) = \left(\frac{r}{1+r}\right)^2 (VAR(\gamma' y_{t+1}) + VAR(\pi'_{t+1} b_t) - 2COV(\gamma' y_{t+1}, \pi'_{t+1} b_t))$$
$$= \left(\frac{r}{1+r}\right)^2 (\gamma' \Omega_{yy} \gamma + b_t' \Omega_{\pi\pi} b_t - 2b_t' \Omega_{\pi y} \gamma) \qquad \text{(A27)}$$

Rearrange (A25):

$$\Omega_{\pi y} \gamma = \Omega_{\pi\pi} b_t$$
$$b_t' \Omega_{\pi y} \gamma = b_t' \Omega_{\pi\pi} b_t \qquad \text{(A28)}$$

Substituting (A28) into (A27):

$$VAR_t(m_{t+1}) = \left(\frac{r}{1+r}\right)^2 (\gamma' \Omega_{yy} \gamma + b_t' \Omega_{\pi\pi} b_t - 2b_t' \Omega_{\pi y} \gamma)$$
$$= \left(\frac{r}{1+r}\right)^2 (\gamma' \Omega_{yy} \gamma - b_t' \Omega_{\pi\pi} b_t) \qquad \text{(A29)}$$

It is apparent that two items in (A29) are in the quadratic form and that they are, by definition, non-negative. If external financing through non-conventional loans (e.g., commodity-linked bonds and foreign currency composition) is not available (i.e., b_t is zero), then the variance of m_{t+1} is simply the first item in the expression.[18] If the optimal portfolio of commodity-linked bonds and foreign currency debt can be obtained (i.e., b_t is determined by equation (A26) and not all elements in b_t are zero), then the second item in (A28) is always positive. Thus the conditional variance of m_{t+1} with non-zero b_t is always smaller than the conditional variance of m_{t+1} without b_t.[19]

[18] This variance can be considered as a measure of the non-diversifiable risks under the assumptions of rational expectations and the permanent income hypothesis when no borrowing opportunities on b_t are available.

[19] Empirically speaking, by introducing commodity-linked bonds and optimizing the composition of foreign currency debt, a developing country's non-diversifiable risks can be reduced.

The solution formula for b_t as of equation (A26) is difficult to comprehend in its present mathematical terms. However, after certain steps of decomposition, it can more easily be seen that the optimal portfolio b_t is a hedging portfolio which directly relates to the effects of unexpected changes of π_{t+1} on export earnings of $\theta_{t+1}x_{t+1}$.

Redefine $y_t = (\theta_t x_t, \pi_t')$; thus the set of other state variables s_t is null. Rewrite (A26) as follows:

$$b_t=\Omega_{\pi\pi}^{-1}\Omega_{\pi y}\gamma=\Omega_{\pi\pi}^{-1}[V_{\pi x},\Omega_{\pi\pi}]\gamma=[\Omega_{\pi\pi}^{-1}V_{\pi x},I]\gamma=[\beta,I]\gamma \qquad \text{(A30)}$$

where $V_{\pi x}$ is the column vector of covariance between the unexpected export income θx and the unexpected vector of π,[20] I is the identity matrix with diagonal elements equal to one and others zero, and β can be considered as the OLS regression coefficients of the residual of θx on the residuals of π' from the VAR representation given as:

$$(\theta x)_R=\beta'\pi_R=\beta_1\pi_{R1}+\beta_2\pi_{R2}+\ldots+\beta_n\pi_{Rn} \qquad \text{(A31)}$$

where $(\theta x)_R$ and $\pi_R=(\pi_{R1}, \pi_{R2}, \ldots\pi_{Rn})$ are the residuals from the VAR as of equation (A15).[21] Each element in coefficient vector β indicates the amount of instantaneous hedge in each element of b_t in response to the respective unexpected shocks in π. Because an unexpected move in price π_i in the amount of π_{Ri} of the i^{th} borrowing instrument will result in a $\beta_i\pi_{Ri}$ change in the unexpected export earnings $(\theta x)_R$, then export earnings can be used to hedge the risk exposure of the β_i amount of debt that is linked to π_i (which experiences the unexpected shock).

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20 The terms "unexpected" refers to the residuals of the variables of interest in the VAR system defined in equation (A15).

21 In empirical studies, $(\theta x)_R$ and π_R can be explained as the unexpected shocks in export earnings, commodity prices, and foreign exchange rates.

However, as suggested by equation (A30), in order to derive the final composition of debt, β has to be adjusted by the vector of γ, which is defined in equation (A16). Leading (A16) one period:

$$R_t\left(\sum_{i=1}^{\infty}(1+r)^{-i}E_t(\theta_{t+i}x_{t+i})\right)=R_t(\gamma' y_{t+1}+B(L)y_t)=R_t(\gamma' y_{t+1}) \qquad (32)$$

where R_t is the "unexpected at time t" operator. $R_t(\gamma' y_{t+1})$ can be further written as:[22]

$$R_t(\gamma' y_{t+1})=\gamma' y_{Rt}=\gamma_{\theta x}(\theta x)_R+\gamma_{\pi 1}\pi_{R1}+\ldots+\gamma_{\pi n}\pi_{Rn} \qquad (33)$$

where $y_{Rt+1} = ((\theta x)_{Rt+1}, \pi_{Rt+1}')'$, and $\gamma=(\gamma_{\theta x}, \gamma_{\pi 1}, \gamma_{\pi 2}, \ldots, \gamma_{\pi n})'$ which can be thought of as the vector of the multiplier of unexpected shocks $(\theta x)_R$ and π_R to the permanent level of export income. Thus, as given by equation (A30), the borrowing b_{it} which is linked to price π_{it+1} is the following:

$$b_{it}=\gamma_{\theta x}\beta_i+\gamma_{\pi_i} \qquad i=1,\ldots,n \qquad (34)$$

Variable b_{it} has an interesting interpretation. Reasoning from equation (A31), the unexpected shock π_{Ri} on price π_i can be hedged instantaneously by the amount of π_i-linked debt β_i because of the unexpected export income $\beta_i\pi_{Ri}$. However, shocks π_{Ri} and $\beta_i\pi_{Ri}$ continue to spill over into the permanent level of export income through vector γ. Thus, according to equation (A33), the permanent level of export income is changed by the level of $\gamma_{\theta x}\beta_i\pi_{Ri}+\gamma_{\pi i}\pi_{Ri}$. So the amount of price π_i-linked debt which can be optimally hedged against the unexpected price shock π_{Ri} is $\gamma_{\theta x}\beta_i+\gamma_{\pi i}$.

Some implications can also be derived through such decomposition. Firstly, the

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22 For simplicity, subscript t has been dropped in the last part of the expression.

decomposition helps to explain unstable results for the optimal portfolios derived in empirical studies, and possible solutions to this problem. In particular, equation (A30) establishes the relationship between the OLS regression coefficients on the VAR residuals and the final result for the optimal portfolio. If multicollinearity exists among price residuals π_{R1}, π_{R2}, ... , π_{Rn}, which is most likely among groups of primary commodity prices or foreign exchange rates, the efficiency of estimating instantaneous amount of debts β_1, β_2, ... , β_n would be undermined, and the result for each individual β_i would be unstable. As a result, the optimal composition of the debt portfolio may vary greatly if conditions change only slightly. A simple solution to this problem would be to drop some price series from the instrument list in order to avoid multicollinearity, and the efficiency of estimating the remaining instantaneous debt could be improved without reducing the effectiveness of hedging.

Secondly, the presence of the vector multiplier γ indicates that the effect on permanent export income of unexpected shocks in π determines the optimal hedging strategy, not the instantaneous one. Intuitively speaking, if π_i refers to the price of an exported commodity, a positive shock in π_i will most likely bring about a positive instantaneous rise in unexpected export income. But its effect on permanent export income is not clear, due to the quantity offsetting effect, or "Dutch disease", etc. It is possible for a positive shock in π_i to induce a decline in permanent export income. Thus the π_i-linked debt has no effectiveness in hedging from the permanent income point of view.

Annex B. The Granger Representation Theorem

The Granger representation theorem states the following: if y_t is a $N \times 1$ vector, and each component of y_t is I(1), there will always exist a multivariate Wold representation:

$$(1-L)y_t = C(L)\epsilon_t \tag{B1}$$

where L is the back-shift operator, $C(0) = I_N$, and ϵ_t are zero mean white noise vectors. If y_t is co-integrated and with co-integrating rank k, then:

1. C(1) is of rank N-k.
2. There exists a vector ARMA representation

$$A(L)y_t = d(L)\epsilon_t \tag{B2}$$

   and A(1) has rank k and d(L) is a scaler lag polynomial with d(1) finite, and $A(0)=I_N$. When $d(L)=1$, this is a vector autoregression.
3. There exist $N \times k$ matrices, α, λ, of rank k such that $\alpha' C(1)=0$, $C(1)\lambda=0$, and $A(1)=\lambda\alpha'$.
4. There exists an vector error correction (VEC) representation with $z_t = \alpha' y_t$, a $k \times 1$ vector of stationary random variables:

$$A^*(L)(1-L)y_t = -\lambda z_{t-1} + d(L)\epsilon_t \tag{B3}$$

with $A^*(0)=I_N$.

5. The vector z_t is given by

$$z_t = K(L)\epsilon_t$$
$$(1-L)z_t = -\alpha'\lambda z_{t-1} + J(L)\epsilon_t \tag{B4}$$

where K(L) is a k×N matrix of lag polynomials given by $\alpha' C^*(L)$ with all elements of K(1) finite with rank k, and $\det(\alpha'\lambda) > 0$. $C^*(L)$ is defined by the identity: $C(L)=C(1)+(1-L)C^*(L)$.

6. If a finite vector autoregressive representation is possible, it will have the form given by (B2) and (B3) above with $d(L)=1$ and both A(L) and $A^*(L)$ as matrices of finite polynomials.

By comparing equations (B2) and (B3), it can be seen that:

$$A(L) = A^*(L)(1-L) + \lambda\alpha' \tag{B5}$$

where, according to the theorem, α' is the matrix of cointegrating vectors in the VAR estimated in levels, and λ is the coefficient matrix of error correction terms in the VEC representation defined as in equation (B3). Equation (B5) can be seen as the link between VEC coefficient matrices estimated on the first differences and VAR coefficient matrices in levels.

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