WPS3577

Testing Genuine Saving

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Abstract. The World Bank has been publishing estimates of adjusted net or 'genuine' saving since 1999. This measure of saving treats depletion of natural resources as a type of economic depreciation. Recent theoretical results relating growth in saving to growth in future consumption are used to provide a test of genuine saving using historical data – did measured genuine saving in 1976, for example, 'predict' the observed changes in consumption over subsequent decades? Four alternative measures of saving are tested econometrically. The worst measure, in terms of explained variation, is traditional net saving. Genuine saving adjusted to reflect population growth exhibits the worst fit with theory. Both gross saving and genuine saving perform better, with good concordance with theory, while genuine saving exhibits a moderate advantage in terms of goodness of fit.

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[1] Environment Department, World Bank. The comments of Susana Ferreira are acknowledged with thanks.

Introduction

Intuition suggests that saving today should have an effect on future economic performance, and indeed the large body of work on cross-country analysis of economic growth supports this (see, for example, Sala-i-Martin (1997)). This intuition was made formal in Hamilton and Clemens (1999), where it is shown that current net or genuine saving is precisely equal to the change in the present value of future utility along the optimal development path for an economy. The work has been extended by Dasgupta and Mäler (2000) and Asheim and Weitzman (2001). This theory can provide a basic framework for testing, using historical data, whether current saving does in fact predict future changes in welfare. Recent papers by Ferreira and Vincent (forthcoming) and Ferreira *et al.* (2003) have explored the question in detail. This paper provides an alternative framework and empirical test.

A key motivation for an alternative test lies in the restrictiveness of the assumptions underlying other frameworks. For example, the very general model of Weitzman (1976) requires (i) that the economy be on the optimal path which maximizes the present value of consumption and (ii) that the interest rate be constant. These are both strong assumptions. The model of Ferreira *et al.* (2003) presents other problems for estimation, as the following section illustrates.

Alternative models

Assume a Dasgupta-Heal type economy with a finite stock of resource S which is extracted at rate R, and where production depends on the capital stock and flow of resources, i.e. $F = F(K, R)$. We assume constant returns to scale. The basic accounting identities for the economy are:

$$\dot{K} = F(K, R) - C$$

$$S = \int_t^\infty R(s)\,ds$$

$$\dot{S} = -R$$

Assuming profit maximization, the price of the resource is given by F_R, which must satisfy the usual arbitrage relationship (the Hotelling rule),

$$\dot{F}_R / F_R = F_K,$$

while F_K is the interest rate for the economy. Total wealth is defined as,

$$W = K + \int_t^\infty F_R(s)R(s) \cdot \exp\left(-\int_t^s F_K(\tau)\,d\tau\right)ds = K + F_R S.$$

These basic relationships plus constant returns to scale lead to the following derivation:

$$
\begin{aligned}
C &= F - \dot{K} + \dot{F}_R S - \dot{F}_R S \\
&= F_K K + F_R R - \dot{K} + F_K F_R S - \dot{F}_R S \\
&= F_K (K + F_R S) - \dot{K} - F_R \dot{S} - \dot{F}_R S \\
&= F_K W - \dot{W}
\end{aligned}
\tag{1}
$$

This differential equation has particular solution,

$$
W = \int_t^\infty C(s) \cdot \exp\left(-\int_t^s F_K(\tau) d\tau\right) ds ,
$$

so wealth is just the present value of consumption along the profit-maximizing path.

Since genuine saving for this economy is given by $G \equiv \dot{K} - F_R R$, expression (1) can be re-written as,

$$
F_K \int_t^\infty C(s) \cdot \exp\left(-\int_t^s F_K(\tau) d\tau\right) ds - C = G + \dot{F}_R S .
\tag{2}
$$

This is the expression tested by Ferreira *et al.* (2003) – genuine saving plus capital gains at time *t* should equal the difference between a particular average of future consumption and current consumption. This approach to the problem entails two restrictive assumptions – profit maximization and constant returns to scale – and encounters one considerable practical problem, the measurement of capital gains. Although expression (2) shows only capital gains on the exhaustible resource, a more general model would suggest that all capital gains should be included – however, cross-country time series data on capital gains are lacking.

An alternative approach to testing genuine saving may be derived from Hamilton and Hartwick (2005). For the same profit-maximizing model,

$$
\begin{aligned}
\dot{C} &= \dot{F} - \ddot{K} - \dot{F}_R R + \dot{F}_R R \\
&= F_K \dot{K} - F_K F_R R - \ddot{K} + F_R \dot{R} + \dot{F}_R R . \\
&= F_K G - \dot{G}
\end{aligned}
$$

This has a particular solution,

$$
\int_t^\infty \dot{C}(s) \cdot \exp\left(-\int_t^s F_K(\tau) d\tau\right) ds = G ,
\tag{3}
$$

which provides the basic test of saving employed below: current genuine saving should equal the present value of future changes in consumption. This is a more parsimonious model, requiring only profit maximization. Hamilton and Withagen (2004) show that this result holds for a much more general model provided that the economy is competitive (households maximize utility

while producers maximize profits) and that any externalities are internalized through Pigouvian taxes.

The econometric test of genuine saving

As in Ferreira *et al.* (2003), the econometric test of saving which we wish to apply is,

$$PVC_i = \alpha + \beta \cdot G_i + \varepsilon_i, \tag{4}$$

where G_i is one of several alternative measures of saving, while PVC_i is the present value of changes in future consumption for country *i,* as suggested by expression (3). If the data fit the theory, then we would expect $\alpha = 0$ and $\beta = 1$.

We need to account for population growth when measuring saving[2]. We therefore assume population N to be growing at exogenous rate g; GDP is denoted Y. The key variables to be subjected to econometric analysis are therefore calculated in the base period ($t = 0$) as,

$$PVC_0 = \left(\frac{Y_0}{N_0}\right)^{-1} \sum_{j=1}^{T} \left(\frac{C_j/N_j - C_{j-1}/N_{j-1}}{(1+r)^j}\right)$$

$$G_0 = \left(\frac{Y_0}{N_0}\right)^{-1} \left(\frac{(Y_0 - C_0) - \delta K_0 + \sum p_{i0}\dot{X}_{i0}}{N_0} - g \cdot \frac{\left(K_0 + \sum p_{i0}X_{i0}\right)}{N_0}\right).$$

Here δK_0 is depreciation of produced assets, while p_{i0} is the shadow price of the *i*-th asset. Both expressions are normalized to current GDP per capita for expositional purposes. The four alternative measures of saving which we test are:

1. Gross saving $= \left(\frac{Y_0}{N_0}\right)^{-1} \left(\frac{(Y_0 - C_0)}{N_0}\right).$

2. Net saving $= \left(\frac{Y_0}{N_0}\right)^{-1} \left(\frac{(Y_0 - C_0) - \delta K_0}{N_0}\right).$

3. Genuine saving $= \left(\frac{Y_0}{N_0}\right)^{-1} \left(\frac{(Y_0 - C_0) - \delta K_0 + \sum p_{i0}\dot{X}_{i0}}{N_0}\right).$

4. Malthusian saving $= \left(\frac{Y_0}{N_0}\right)^{-1} \left(\frac{(Y_0 - C_0) - \delta K_0 + \sum p_{i0}\dot{X}_{i0}}{N_0} - g \cdot \frac{\left(K_0 + \sum p_{i0}X_{i0}\right)}{N_0}\right).$

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[2] See Hamilton (2003) for details.

We term the fourth measure 'Malthusian' saving owing to the final term reflecting the immiserating effects of population growth.

Data

All data for the analysis – GDP, gross saving, consumption of fixed capital[3], and depletion of natural resources (energy, minerals and net forest depletion) – are taken directly from the *World Development Indicators* (World Bank 2002). Total wealth, employed in the Malthusian saving calculation, is derived using a Perpetual Inventory Model for produced capital stock estimates, present values of mineral and energy rents, and present values of forestry, fishing and agricultural rents, all measured in constant 1995 dollars, providing the basic estimates – these are the same total wealth data employed in Ferreira *et al.* (2003).

As in Ferreira and Vincent (forthcoming) and Ferreira *et al.* (2003), we exclude public expenditures on education from the saving measures – these were shown to perform exceedingly badly in the earlier work. There are a number of plausible reasons for the poor performance: (i) these are gross, rather than net, investment estimates; (ii) private expenditures are excluded; and (iii) expenditures may be a particularly poor proxy for human capital formation, particularly in developing countries – see Pritchett (1996).

We also exclude damages from CO2 emissions. This is partly because the bulk of the damages occur in the longer term, but mostly because damages to other countries (the major effect of emitting CO2) should have no effect on future consumption in the emitting country in the absence of a binding agreement to pay compensation.

Methodology of estimation

One of the key choices to be made in estimating expression (4) is the choice of period over which to calculate changes in consumption. The underlying theory, as expressed in (3), suggests that there is in principle an infinite time horizon. As a practical matter, however, the WDI data on genuine savings are limited to the period 1970-2000, with data for the early 1970s being particularly sparse.

A reasonable choice of time horizon would be the mean lifetime of produced capital stocks, roughly 20 years (machinery and equipment lifetimes are typically shorter, 10 years or so, but buildings and infrastructure have lifetimes of several decades). Choosing 20 years would be saying, in effect, that the effects of savings will be felt over the lifetime of the produced capital in which they are presumed to be invested. This is the assumption used below, and testing the estimation for a 10 year time horizon produced less robust estimates overall (in terms of

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[3] Ferreira *et al.* (2003) use estimated figures for consumption of fixed capital derived from the Perpetual Inventory Model used to estimate total stocks of produced capital. Inspection of these figures reveals a fairly large number of anomalous estimates.

explained variation, probability of rejecting a linear relationship between dependent and independent variables, and significance of the coefficients on saving).

The other decision required for estimation concerns the discount rate. The underlying theory (see Ch. 3 and Ferreira *et al.* (2003)) suggests that the rate should be the marginal product of capital less depreciation rates for produced capital, less population growth rates, which argues for a low value. We use a uniform rate of 5%, and tests of alternatives suggest that the estimates are fairly insensitive to small changes in the discount rate.

Allowing for the sparse early-1970s savings data[4], therefore, expression (4) was estimated using OLS for consecutive 20 year periods from 1976-1980. These results, as well as more informal methods, are reported below.

Empirical results

To give a feel for the data, we first scatter the present value of changes in consumption against the four different savings measures for 1980 in Figures 1-4. The broad picture which emerges is that there is no monotonic improvement in the fit with theory as more stringent measures of saving are applied. The coefficient on saving actually drops going from gross saving to net saving, and the explained variation drops considerably. For genuine saving the coefficient on saving is very near 1 and the explained variation is the highest of the four saving measures. Finally, for Malthusian saving the coefficient on saving drops to the lowest level of the four measures, while explained variation reaches its highest value.

Figure 5 presents the same scatter for high income countries only. As seen in Ferreira and Vincent (forthcoming) and Ferreira *et al.* (2003), the model fit is particularly poor for these countries – further tests show the coefficient on saving to be insignificant, while the explained variation is very low.

Table 1 presents the results of the individual OLS estimates of the model for each of the five years and four measures of saving. This table reports the coefficient values with t-statistics, r-squared, degrees of freedom, the probability of rejecting a linear relationship (from the F statistic) and a simple two-sided t-test of whether the coefficient on saving is equal to 1 (values greater than 2.00 imply the coefficient is significantly different from 1 at the 5% confidence level). While there is some heterogeneity in the results, the following broad conclusions hold:

- The results for 1977 are the weakest of the five years, with low r-squared, higher probabilities of rejecting a linear relationship than other years and two saving coefficient estimates that are significantly different from 1 (although the coefficient for net saving is not itself significant). This suggests some systematic shock being picked up by the data for this year.

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[4] From 1970 to 1975 there are fewer than 40 countries with the necessary data, and these are primarily developed countries.

- Results for net saving are generally the weakest of the four saving measures tested, with insignificant coefficients on saving at the 5% level in 1976 and 1977, and generally low r-squared and higher probability of rejecting a linear relationship than other measures.

- Malthusian saving exhibits the worst fit with theory, with the coefficients on saving being the lowest of the four saving measures, and significantly different from 1 in four out of the five years tested.

- The results for gross and genuine saving have similarities, with the coefficients on saving being significant and not significantly different from 1 in all years. Genuine saving explains much more of the total variation in four out of five years, and exhibits lower probability of rejecting a linear relationship in the same four years, suggesting a more robust fit with theory.

Quantitative analysis suggests a moderate advantage to using genuine saving as a 'predictor' of future welfare, in the sense of a one percentage change in saving translating into a one percent change in the present value of changes in future consumption. Figures 1 and 3 suggest a more qualitative test. In Figure 1 it can clearly be seen that gross saving provides many 'false positives' in the form of positive base year savings translating into negative welfare outcomes – these are the scatter points lying in the lower right quadrant. Similarly, the upper left quadrant points in Figure 3 represent 'false negatives' – countries where negative base year genuine savings were associated with increases in welfare.

Table 2 assembles the proportions of false positives and false negatives[5] for all saving measures for all years, along with an average for each saving measure weighted by the number of countries with positive or negative savings observed. A few observations:

- Malthusian saving has the lowest proportion of false positives, but in fact the vast majority of the countries with positive Malthusian saving are developed countries – the result is therefore unsurprising. This saving measure also has the highest proportion of false negatives, which is consistent with the results of the quantitative analysis.

- Gross and net saving have relatively low proportions of false negatives, but this represents very few countries (only one in the case of gross saving) across all years. There are simply very few countries with negative gross or net saving.

- Genuine saving has lower proportions of false positives than either gross or net saving, but this is balanced by a much higher proportion of false negatives.

Conclusions

Growth theory provides the basis for a stringent test of whether saving does in fact translate into future welfare. This paper confronts the theory with 'real world' data, with positive results at least for measures of gross and genuine saving. Even without appealing to theoretical

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[5] This is clearly a rather *ad hoc* test, but one that policy makers may care about.

models, it may be asked when a dollar is saved how it could *not* show up in future production and consumption. Many answers to this question are possible: (i) saving may be measured very badly; (ii) funds appropriated for public investments may not in fact be invested, owing to problems of governance, and (iii) investments, particularly by the public sector, may not be productive.

It is important to note the many caveats pertaining to this analysis. First, measurement error may be significant, particularly for consumption of fixed capital (where government estimates may be incorrect), depletion of natural resources (where World Bank resource rent estimates depend on rather sparse cost of extraction data, and where the methodology probably inflates the value of depletion for countries with large resource deposits), and total wealth estimates (especially produced capital in developing countries, where public investments may be particularly inefficient – see Pritchett (2000)).

Missing variable bias may also be an issue. Although human capital is excluded from the analysis for the reasons outlined above, in principle net investment in human capital should be an important contributor to future welfare – however, the negative effects of including education spending in the analysis of saving and future welfare in Ferreira and Vincent (forthcoming) and Ferreira *et al.* (2003) may simply be another manifestation of the small or negative growth impact of public education spending in developing countries analyzed by Pritchett (1996). In addition, for some countries the exclusion of natural resources such as diamonds and fish may be a significant omission.

Exogenous shocks may present problems for testing the theory of saving and social welfare. The period under analysis in this paper includes, in the early and least heavily discounted stages, the second oil shock in 1979 and a steep worldwide recession in 1981. However, Ferreira *et al.* (2003) do not find any significant effects of exchange rate shocks in their analysis of the theory.

Turning to the results of the analysis, we find that the various saving measures are poor at signaling future changes in welfare in developed countries, similar to what Ferreira and Vincent and Ferreira *et al.* find. This probably reflects factors other than capital accumulation being key for the growth performance of these economies, in particular technological innovation. For all countries combined, we find that both net and Malthusian saving fit the theory poorly. The significantly low coefficients on Malthusian saving suggest that this measure overstates the effects of population growth on wealth accumulation per capita. Gross and genuine saving perform well, with estimated coefficients not being significantly different from the predicted values and with lower probabilities of rejecting a linear relationship between dependent and independent variables than for other measures. Genuine saving performs marginally better than gross saving in terms of goodness of fit.

In terms of the more qualitative question of false positives and negatives, genuine saving provides on average a lower false positive ratio than gross saving (22% of countries with positive genuine saving at a point in time actually experienced welfare declines, compared with 29% of countries with positive gross saving). Conversely, negative genuine saving falsely signaled future welfare decreases in 38% of cases on average.

It should be noted that the theory being tested is particularly stringent, since it implies that measuring positive or negative saving *at a point in time* leads to future welfare being higher or lower than current welfare over some interval of time. In the real non-optimal world a positive exogenous shock (such as an improvement in the terms of trade) in the year immediately following the time when saving turned negative could easily swamp the effect of negative saving, and conversely for positive saving and negative shocks.

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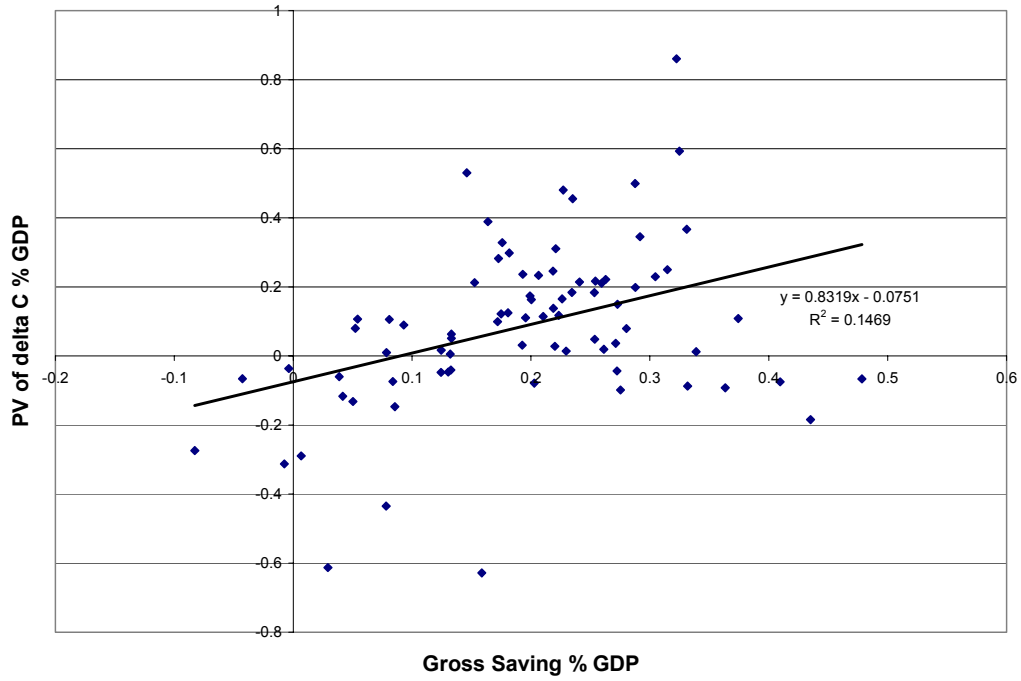World Bank, 2002. *World Development Indicators 2002*. Washington: The World Bank.

Figure 1. PV of change in consumption vs. gross saving, 1980

Figure 2. PV of change in consumption vs. net saving, 1980
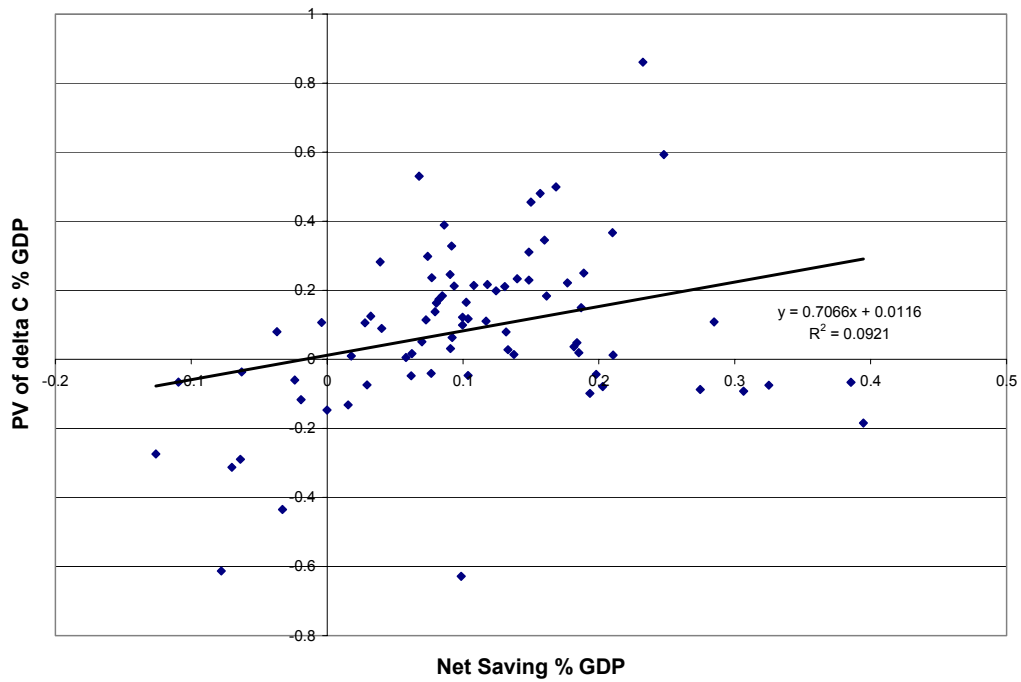
Figure 3. PV of change in consumption vs. genuine saving, 1980

$y = 0.9882x + 0.0568$
$R^2 = 0.2391$

Genuine Saving % GDP

PV of delta C % GDP
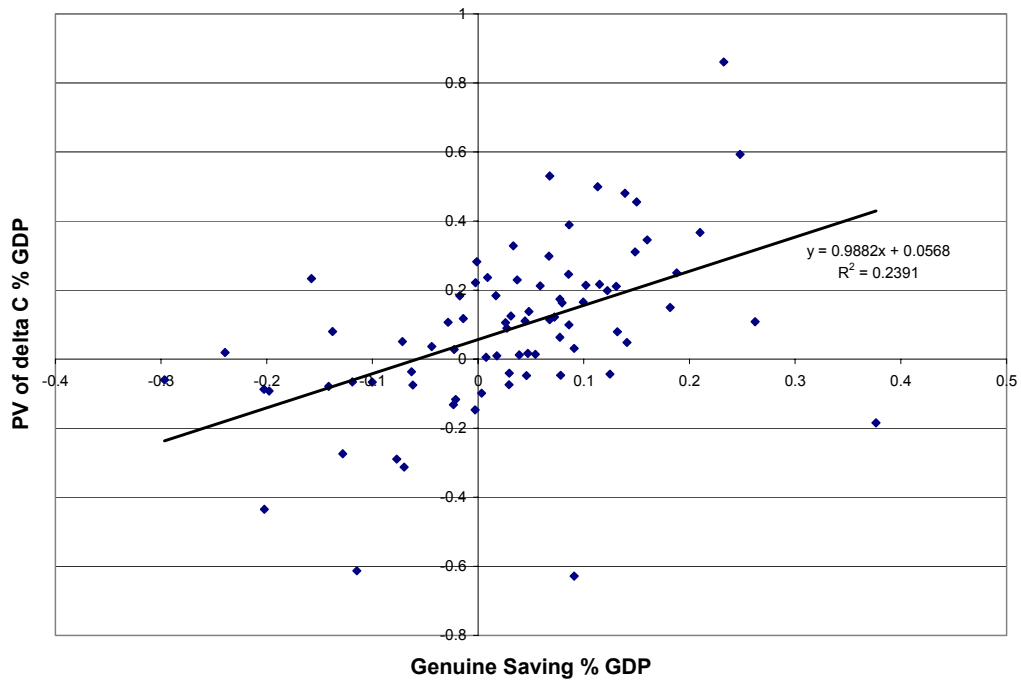
Figure 4. PV of change in consumption vs. Malthusian saving, 1980

$y = 0.5221x + 0.1249$
$R^2 = 0.3194$

Malthusian Saving % GDP

PV of delta C % GDP
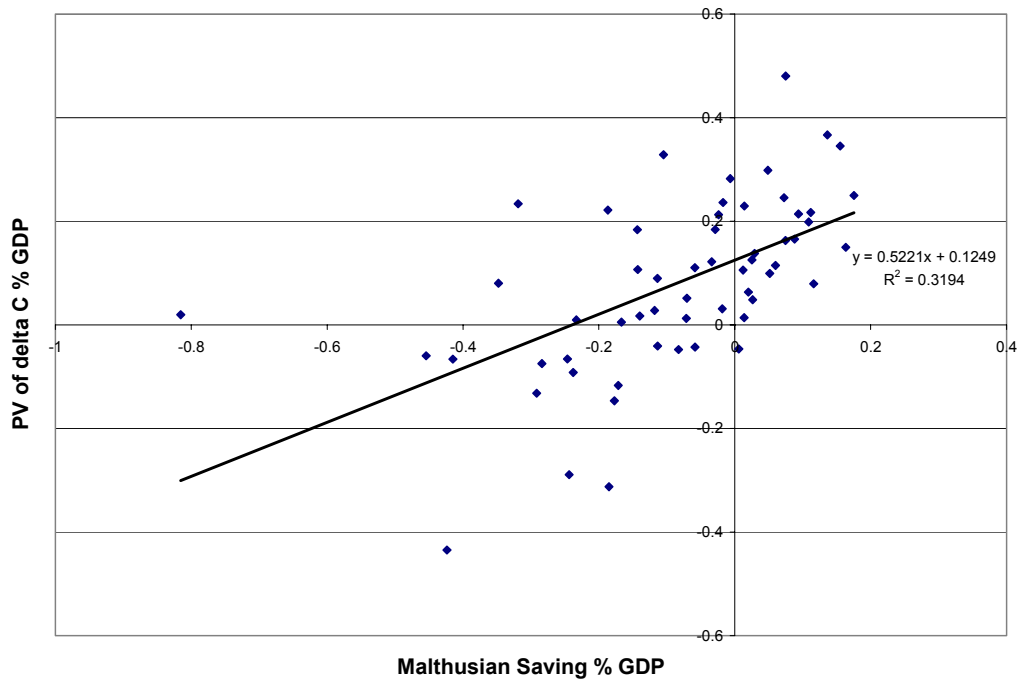
Figure 5. PV of change in consumption vs. genuine saving, high income countries, 1980

$y = 0.1507x + 0.18$
$R^2 = 0.0156$
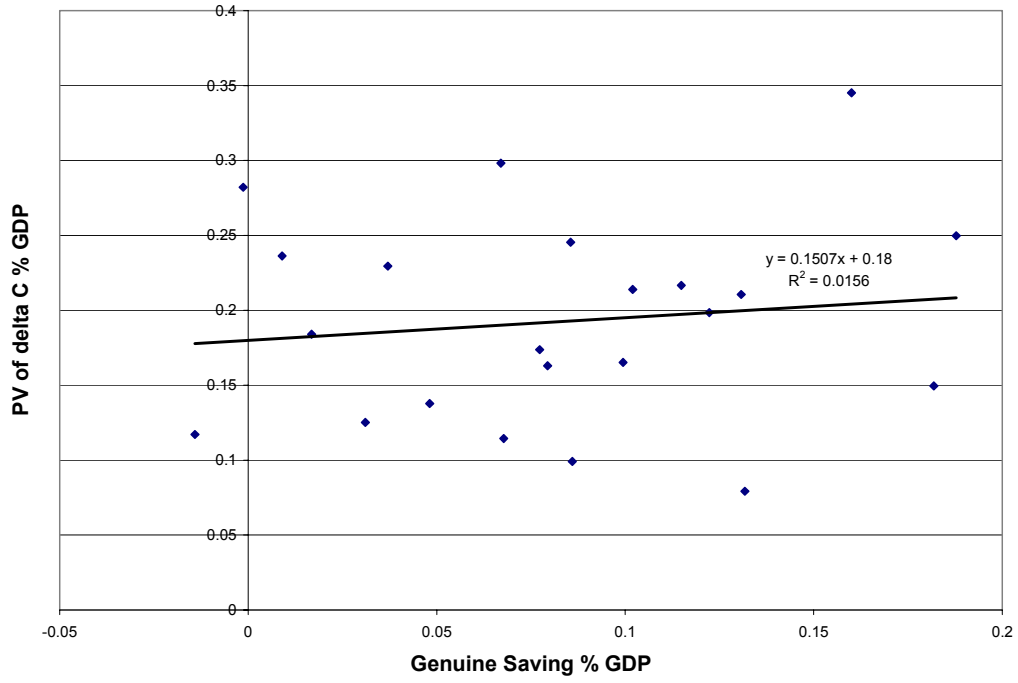
Table 1 Regression results for PVC = alpha + beta * Saving

	1976		1977		1978		1979		1980	
	beta	alpha	beta	alpha	beta	alpha	beta	alpha	beta	alpha
Gross saving										
Coeff.	1.0152	-0.0737	0.7596	-0.0338	1.0484	-0.1212	1.2325	-0.1743	0.8319	-0.0751
tstat	3.0335	-0.9511	2.4358	-0.4628	3.7257	-1.8992	4.7372	-2.8601	3.6416	-1.4656
Rsq	0.1479		0.0803		0.1598		0.2351		0.1469	
Df	53		68		73		73		77	
Pr > F	0.0037		0.0175		0.0004		0.0000		0.0005	
beta = 1	0.0445		-0.7595		0.1697		0.8814		-0.7264	
Net saving										
Coeff.	0.6634	0.0606	0.2161	0.1047	0.6485	0.0209	0.9835	-0.0293	0.7066	0.0116
tstat	1.7723	1.0787	0.6471	2.0414	1.9740	0.4433	3.2791	-0.6574	2.7943	0.3102
Rsq	0.0560		0.0061		0.0507		0.1284		0.0921	
Df	53		68		73		73		77	
Pr > F	0.0821		0.5198		0.0522		0.0016		0.0066	
beta = 1	-0.8823		-2.3125		-1.0555		-0.0542		-1.1451	
Genuine saving										
Coeff.	1.2803	0.0483	0.8532	0.0677	1.2553	0.0131	0.7815	0.0580	0.9882	0.0568
tstat	4.5524	1.4442	3.4246	2.1915	4.9943	0.4654	4.2716	2.3469	4.9187	2.3175
Rsq	0.2811		0.1471		0.2547		0.2000		0.2391	
Df	53		68		73		73		77	
Pr > F	0.0000		0.0010		0.0000		0.0001		0.0000	
beta = 1	0.9780		-0.5808		1.0019		-1.1781		-0.0578	
Malthusian saving										
Coeff.	0.7757	0.1337	0.5741	0.1200	0.4663	0.1061	0.3599	0.1117	0.5221	0.1249
tstat	3.8801	5.1418	3.2489	5.0664	4.0371	5.0553	3.7425	5.2683	5.1265	6.1294
Rsq	0.2785		0.1772		0.2352		0.2030		0.3194	
Df	39		49		53		55		56	
Pr > F	0.0004		0.0021		0.0002		0.0004		0.0000	
beta = 1	-1.0937		-2.3613		-4.5343		-6.5358		-4.6100	

Table 2 False signals regarding future changes in consumption (ratios)

	1976	1977	1978	1979	1980	Wt. Avg.
Gross saving						
False positive	0.241	0.246	0.320	0.360	0.267	0.294
False negative	1.000	0.000	0.000	0.000	0.000	0.167
Net saving						
False positive	0.226	0.250	0.275	0.338	0.209	0.266
False negative	0.500	0.500	0.167	0.250	0.167	0.231
Genuine saving						
False positive	0.188	0.200	0.226	0.293	0.154	0.218
False negative	0.429	0.400	0.231	0.412	0.407	0.378
Malthusian saving						
False positive	0.043	0.080	0.037	0.077	0.043	0.056
False negative	0.611	0.615	0.464	0.452	0.600	0.543