New Results on Censored Regression with Applications to Transactions Costs, Household Decisions and Food Purchases

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Abstract

We generalize the Tobit censored regression to permit unique unobserved censoring thresholds conditioned by covariates and a set of common response coefficients. This situation, we argue, is one arising frequently in applications of censored regression and we provide three diverse examples to motivate the theory. We derive a robust estimation algorithm with three noteworthy features. First, by augmenting the observed-data likelihood with the censored observations, the estimation strategy is the same as Chib (1992) who derives Bayes estimates of the conventional censored regression. Second, by virtue of its generality, the model is applicable to a much broader set of circumstances than the conventional Tobit regression, which is nested as a special case of the more general framework. Third, despite its generality and wide applicability, the estimation algorithm is very simple, evidencing routine application of Markov chain Monte Carlo methods (MCMC)—Gibbs sampling in particular—and requiring only modest extensions of the basic algorithm in Chib (1992). The model and procedures are illustrated empirically in three applications that we use to motivate the theory, namely problems in transactions-costs economics, household decision-making and food-consumption (182 words).

Journal of Economic Literature classifications: O11, C34, O13.

Keywords: conditionally censored Tobit regression, Bayes inference.

Introduction

Because it is now routinized in applications, Gibbs sampling often de-emphasizes two of its most attractive features which are respectively breaking-down complex models into constituents yielding robust estimation algorithms and sequentially extending simplified structures into more meaningful probability models. These features are important in the context of Tobit regression which is one of many extensions of the normal linear model for which Gibbs sampling proves fruitful. Further application of Gibbs sampling to Tobit regression, sequentially extending its basic structure and breaking down its conditional components to yield robust estimation algorithms are the objectives of this paper. The start point is the normal linear model. In that environment it is well-known that the marginal distributions of interest are available in closed-form (Zellner, 1996) and Gibbs sampling is unnecessary. However, the error standard deviation and the regression coefficients can be estimated by sampling respectively from inverted-Gamma and Normal distributions (Gelfand and Smith, 1990), which is a two-step procedure. The introduction of censored data with a known censoring threshold—in other words, conventional Tobit regression (Tobin, 1958)—is easily handled by introducing latent data and appending a third step which is to sample from truncated-Normal distributions (Chib, 1992). And when the censoring threshold is random, but common across agents, the Tobit shares one feature of the ordered probit (Albert and Chib, 1993) and a fourth step—sampling from a uniform distribution—completes the algorithm (Holloway et al., 2004). Thus, Gibbs sampling has proved indispensable for conducting Bayes inference in the censored regression and has proved fruitful for furnishing extensions to the basic ideas in the seminal work of Tobin (1958). The spirit of the present inquiry is ostensibly the same and we seek to relax restrictions embedded in the censoring assumptions in Chib (1992) and in Holloway et al., (2004). In particular, we generalize the Tobit regression to permit unique unobserved censoring thresholds conditioned by covariates

and a set of common response coefficients. The model and procedures are illustrated empirically in application to the examples that we use to motivate the theory. In section two we present the model. In section three we present the motivating examples. In section four we present the estimation algorithm and in section five present the empirical applications. Conclusions and extensions are discussed in section six.

Statistical Model

The basic situation being considered is as follows. We observe a response y_i which is equal to a latent, response, z_i, if the unobserved response surpasses a threshold, v_i, which is also unobserved but is dependent on a set of observable covariates; if the response z_i fails to surpass the threshold we then observe that y_i is zero. In other words, data $\mathbf{y} \equiv (y_1, y_2, .., y_N)'$ are assumed to be generated from the model

$$(1) \qquad z_i = \mathbf{x}_i'b + \varepsilon_i, \qquad\qquad i = 1, 2, .. N,$$

$$(2) \qquad v_i = \mathbf{w}_i'd + \eta_i, \qquad\qquad i = 1, 2, .. N;$$

where $\mathbf{x}_i \equiv (x_{i1}, x_{i2}, .., x_{iK})'$ denotes a K-vector of factors affecting z_i; $\mathbf{w}_i \equiv (w_{i1}, w_{i2}, .., w_{iJ})'$ denotes a J-vector of factors affecting v_i; $b \equiv (\beta_1, \beta_2, .., \beta_K)'$ and $d \equiv (d_1, d_2, .., d_J)'$ denote corresponding coefficient vectors; $\varepsilon_i \sim f^N(\varepsilon_i|0,\sigma)$ and $\eta_i \sim f^N(\eta|0,\omega)$ denote unobserved random disturbances. The investigator observes $\mathbf{x}_i$, $\mathbf{w}_i$, and $y_i = z_i$ if $z_i > v_i$ and $y_i = 0$ otherwise. Thus at each observation there is random censoring at a conditionally dependent threshold and our objective is to use the observed $\{y_i, \mathbf{x}_i, \mathbf{w}_i\}_{i=1}^N$ to make inferences about the 2+K+J unobserved components of $q \equiv (\sigma, \omega, b', d')'$.

Examples

The model in (1) and (2) is considerably broader than its zero-censored counterpart and this breadth should appeal in a potentially wide set of circumstances. In this section we present

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diverse examples to emphasize the motivation.

<u>Example One: Transactions-Costs and Market Participation in the Ethiopian Highlands</u>

We apply the methodology to a sample of observations on milk supply from households in the Ethiopian highlands. There a significant impediment to market participation is a high, often prohibitive level of transactions costs. These costs relate to low levels of infrastructure, physical distances to market, perishability of fluid milk and a host of additional factors too numerous to itemize. These costs motivate a growing body of work (Goetz, 1992; Key, Sadoulet and de Janvry, 2000; Holloway <u>et al.</u>, 2001) aimed at devising strategies to circumvent the key impediment to development in sub-Saharan Africa, namely lack of density of market participation (Stiglitz, 1989). Additional background in the East African and Ethiopian contexts is presented in Staal, Delgado and Nicholson (1997) and Holloway <u>et al.</u> (2001). The reader is also referred to Holloway <u>et al.</u> (2004) for discussion of the data. Briefly, each of 68 households in the sample was visited 3 times in the 1998 production year. At each visit milk sales to the local milk cooperative were retrieved for the preceding 7 days, yielding a total of 1428 (= $68 \times 3 \times 7$) observations, of which some 85% are censored. Previous application (Holloway <u>et al.</u>, 2004, table 1, p. 102) suggests that a parsimonious choice of covariates works well in Tobit estimation and consists of five effects, namely the return-time to walk bucketed milk to the milk cooperative (<u>Distance</u>), the years of formal schooling of the household head (<u>Education</u>), the number of times in the preceding year that the household was visited my an extension agent discussing production and marketing techniques (<u>Extension</u>), the number of cross-breed milking cows (<u>Crossbreed</u>), the number of local-breed milking cows (<u>Localbreed</u>) and two site-specific dummy variables (<u>IluKura</u> and <u>Mirti</u>). Here we consider the value added to Holloway <u>et al.</u> (2004) of permitting conditionally censoring with unique thresholds for each of the households. In terms of the equations (1) and (2), $\mathbf{z} \equiv (z_1, z_2, .., z_N)'$ depicts supply of fluid milk to the milk cooperative

(litres per household per day); $\mathbf{v} \equiv (v_1, v_2, .., v_N)'$ depicts the unobserved censoring thresholds (liters of fluid milk per household per day); $\mathbf{x}_i \equiv (x_{i1}, x_{i2}, .., x_{iK})'$ depicts the covariates affecting milk supply; and $\mathbf{w}_i \equiv (w_{i1}, w_{i2}, .., w_{iK})'$ depicts the covariates affecting the thresholds. For illustrative purposes we include the variables IluKura and Mirti in both equations, group in $\mathbf{x}_i$ the variables Crossbreed and Localbreed and group in $\mathbf{w}_i$ the variables Distance, Education and Extension.

Example Two: Time Devoted to Extra-Marital Affairs

A second, planned application will apply the model and procedures developed to the data used by Fair (1978) in his application of Tobit regression examining the factors explaining allocation of time to extra-marital affairs. These data have been re-examined recently by Wells (2003).

Example Three: Eating-Away-From-Home Food Consumption Decisions

A third, planned application will apply the model and procedures to away-from-home food purchases using cross-sectional data from Italy.

Algorithm

The likelihood is based on recognition of two facts. First, in order for $y_i = z_i \neq 0$ to be observed, it must be that $v_i \leq y_i$. Second, in order for $y_i = 0$ to be observed, it must be that $z_i < v_i$. Consequently, with $\Phi(\times)$ the cumulative distribution function (cdf) corresponding to the standard Normal distribution, and noting that ε_i and η_i are independent, $\Phi((y_i-\mathbf{w}_i'd)/\omega)$ denotes the probability that $v_i \leq y_i$, $1-\Phi((z_i-\mathbf{w}_i'd)/\omega)$ denotes the probability that $v_i \geq z_i$ and $\Phi((v_i-\mathbf{x}_i'b)/\sigma)$ denotes the probability that $z_i \leq v_i$. Consequently, the observed-data likelihood,

$$(3) \qquad f(\mathbf{y}|q) = \prod_{i \in c}^{N} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Phi((v_i-\mathbf{x}_i'b)/\sigma) \, [1-\Phi((z_i-\mathbf{w}_i'd)/\omega)] \, dz_i \, dv_i$$

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$$\times \quad \prod_{i \notin c}^{N} f^{N}(y_i|\mathbf{x}_i'b,\sigma) \, \Phi((y_i-\mathbf{w}_i'd)/\omega),$$

is complicated by five integrals. There are three types. One type—the usual one encountered in Tobit regression—reflects the probability that the censored observations lie beneath the threshold. This probability is depicted by the term $\Phi((v_i-\mathbf{x}_i'b)/\sigma)$. A second type, signified by the terms $1-\Phi((z_i-\mathbf{w}_i'd)/\omega)$ and $\Phi((y_i-\mathbf{w}_i'd)/\omega)$, reflects the probability that the threshold lies above, respectively beneath, the non-censored data. The third type of integral corresponds to the explicit integrations depicted in the first component on the right-hand side corresponding to the censored observations. They arise because the limits of integration are latent. These five integrals make the observed-data likelihood quite intractable and we turn to consider the <u>complete-data likelihood</u> as in Chib (1992, equation (16), p. 88). The advantages of augmenting the likelihood with missing data is showcased in a number of influential papers including Lavine and West (1992), Albert and Chib (1993), Diebolt and Robert (1994) and Geweke, Keane and Runkle (1997). The conditionally censored Tobit regression is no exception.

Our strategy involves augmenting the likelihood with the missing $\mathbf{z} \equiv (z_1, z_2, .., z_N)'$ and the missing $\mathbf{v} \equiv (v_1, v_2, .., v_N)'$; working with the complete-data likelihood $f(\mathbf{y}|q,\mathbf{z},\mathbf{v})$, and a prior $\pi(q)$; and establishing the fully conditional distributions characterizing the joint posterior for the parameters augmented by the missing data, $\pi(q|\mathbf{y},\mathbf{z},\mathbf{v})$. This strategy lends itself to a Gibbs-sampling algorithm with desirable convergence properties and leads ultimately to robust estimates of the conditionally censored Tobit regression.

The essential observation Chib (1992, p. 88) relies on the fact that $f(\mathbf{y}|q,\mathbf{z},\mathbf{v})$ contains neither censoring (Chib's situation) nor integrations required by the fact that the model is conditioned by latent data (the present situation) and, although the latent data are obviously unavailable, estimates of them are efficiently simulated from standard distributions obeying

restrictions implicit in the model. Each of the component conditional distributions has a particularly simple form and it is thus straight-forward to form a Markov chain with desirable convergence properties. We outline the distributions before examining the convergence properties of the algorithm and following specification of the prior,

(4) $\pi(q) \propto f^{iG}(\sigma|\upsilon_{\sigma o},s_{\sigma o}{}^2)\, f^{mN}(b|b_o,\mathbf{C}_{bo})\, f^{iG}(\omega|\upsilon_{\omega o},s_{\omega o}{}^2)\, f^{mN}(d|d_o,\mathbf{C}_{do})$,

which is the conventional, natural-conjugate prior for Normal data. The prior is proper whenever $\upsilon_{\sigma o}$ and $\upsilon_{\omega o}$ exceed zero and $C_{\beta o}$ and $C_{\delta o}$ are finite and it reduces to the standard diffuse prior of Jeffrey's (1939) whenever $\upsilon_{\sigma o}$ and $\upsilon_{\omega o}$ equal zero. Although we present the fully conditional distributions in the presence of the proper prior, we are mostly interested in the case where the prior is diffuse. We will, of course, consider model comparisons in the presence of the proper prior.

The fully conditional distributions comprising $\pi(q|\mathbf{y},\mathbf{z},\mathbf{v})$ are six. Conditional on b and $\mathbf{z}$, σ is distributed

(6) $\pi(\sigma|\mathbf{y},b,\mathbf{z}) \propto f^{iG}(\sigma|\upsilon_\sigma,s_\sigma{}^2)$,

where $\upsilon_\sigma \equiv \upsilon_{\sigma o}+N$, $s_\sigma{}^2 \equiv \upsilon_{\sigma o}s_{\sigma o}{}^2+(\mathbf{z}-\mathbf{x}b)'(\mathbf{z}-\mathbf{x}b)$ and $\mathbf{x} \equiv (\mathbf{x}_1', \mathbf{x}_2', .., \mathbf{x}_N')'$. Conditional on σ and $\mathbf{z}$, b is distributed

(7) $\pi(b|\mathbf{y},\sigma,\mathbf{z}) \propto f^N(b|\hat{b}, \mathbf{C}_{\hat{b}})$,

where $\hat{b} \equiv \mathbf{C}_{\hat{b}}^{-1}(\mathbf{x}'\mathbf{y}+\mathbf{C}_{\hat{b}o}^{-1}b_o)$ and $\mathbf{C}_{\hat{b}} \equiv \sigma^2(\mathbf{x}'\mathbf{x})^{-1}+\mathbf{C}_{\hat{b}o}^{-1}$. Conditional on σ, b and $\mathbf{v}$, for each $i \in \mathbf{c}$, z_i is distributed

(8) $\pi(z_i|\mathbf{y},\sigma,b,v_i) \propto f^{tN}(z_i|\mathbf{x}_i'b,\sigma,v_i)$,

where truncation is from the right. Conditional on d and $\mathbf{v}$, ω is distributed

(9) $\pi(\omega|\mathbf{y},d,\mathbf{v}) \propto f^{iG}(\omega|\upsilon_\omega,s_\omega{}^2)$,

$\upsilon_\omega \equiv \upsilon_{\omega o}+N$, $s_\omega{}^2 \equiv \upsilon_{\omega o}s_{\omega o}{}^2+(\mathbf{v}-\mathbf{w}d)'(\mathbf{v}-\mathbf{w}d)$ and $\mathbf{w} \equiv (\mathbf{w}_1', \mathbf{w}_2', .., \mathbf{w}_N')'$. Conditional on ω and d $\mathbf{v}$, d is distributed

(10) $\pi(d|\mathbf{y},\omega,\mathbf{v}) \propto f^{N}(d|\hat{d},\mathbf{C}_{\hat{d}})$,

where $\hat{d} \equiv \mathbf{C}_{\hat{d}}^{-1}(\mathbf{w}'\mathbf{v}+\mathbf{C}_{do}^{-1}d_{o})$ and $\mathbf{C}_{\hat{d}} \equiv \omega^{2}(\mathbf{w}'\mathbf{w})^{-1}+\mathbf{C}_{do}^{-1}$. And conditional on d, ω and $\mathbf{z}$, each

v_i is distributed

(11) $\pi(v_{i}|\mathbf{y},\omega,d,z_{i}) \propto f^{tN}(v_{i}|\mathbf{w}_{i}'d,\omega,z_{i})$,

where if $i \in \mathbf{c}$ $f^{tN}(\times)$ is left truncated at z_i and if $i \notin \mathbf{c}$ $f^{tN}(\cdot)$ is right-truncated at z_i. Accordingly,

the Gibbs algorithm for estimating the conditionally censored Tobit regression (equations (1)

and (2)) consists of six steps.

A_1:

> Step 1: Draw σ from (5).
>
> Step 2: Draw b from (6).
>
> Step 3: Draw z_i from (7), truncated to the right.
>
> Step 4: Draw ω from (8).
>
> Step 5: Draw d from (9).
>
> Step 6: Draw v_i from (10), truncated to the left if $i \in \mathbf{c}$ and to the right if $i \notin \mathbf{c}$.

Several remarks are in order. First, steps 1-3 are essentially those of Chib (1992, equations

(18), p. 89). Consequently, the gains from estimating the generalized censored regression

model come at the cost of three additional steps, namely steps 4-6. Second, each of the fully

conditional distributions is easy to sample from. A draw from the inverted-Gamma

distribution is obtained by drawing a scaled-inverse chi-squared deviate (Gelman et al., p.

480). This approach is convenient because it relies only on transformations of standard

normal random deviates. The truncated draws in steps 3 and 6 are efficiently generated using

the probability integral transform (Mood et al., 1974, p. 202) as suggested by Geweke (1992)

and applied by Chib (1992, p. 89). Third, because the draws are easily simulated, it is

convenient to collect Gibbs samples of satisfactory sizes, say $G = 50{,}000$ following a <u>burnin</u> of sufficient length. Although experiments suggest that the algorithm converges after only a few iterations, each of the reports in this paper are obtained from a burnin of 50,000 followed by a collection sample of 50,000. This, of course is more than adequate to relieve the posterior inference of any dependence on starting values. However, we advocate using the starting values $\mathbf{z}^{(0)} = \mathbf{y}$, $\mathbf{v}^{(0)} = \mathbf{y}$, $b^{(0)} = (\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}'\mathbf{z}$, $d^{(0)} = (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{v}$ and $\omega^{(0)} = (\mathbf{z}-\mathbf{w}d)'(\mathbf{z}-\mathbf{w}d)/(N-J)$.

Empirical Applications

Table 1 presents results of derived from the Ethiopian data. The first column lists the covariates; the second column reports estimates of the traditional Tobit regression, under the zero censoring assumption; and the third and fourth columns report estimates of the conditionally censored Tobit regression—equations (1) and (2), respectively—using the approach of differencing the two equations in order to identify the threshold coefficients. The numerical values are the means of the posterior densities. Ninety-five percent highest posterior density (<u>hpd</u>) intervals are given in parentheses.

Three features of the results are especially noteworthy. First, differences between the traditional approach and the conditionally censored estimates are large. Second, the estimates derived from the conditionally censored model are markedly more precise. Third, the conditionally censored Tobit regression stems nuanced inference about the incremental effects of transactions costs impeding milk supply among the Ethiopian highlands producers.

We consider further differences between the traditional and the non-traditional approaches during application to the data on extra-marital affairs (Fair, 1978; Wells, 2003) and food purchases (currently incomplete).

Conclusions

Perhaps because it has become commonplace in Bayesian estimation, one important aspect of MCMC and Gibbs sampling, in particular, is now de-emphasized in the theoretical and applied literatures. This aspect is the breaking-down of complex problems into simpler, tractable components. With reference to the Tobit regression, if we begin by first ignoring the censoring, the Gibbs procedure is a two-step algorithm (draw from an inverted-Gamma distribution and draw from a multivariate-Normal distribution). With zero censoring we add an additional step (draw from a truncated-Normal distribution). With a common, random censoring point we add another step (draw from a uniform distribution). In this paper we show how this step-wise approach extends readily to the case of unique unobserved censoring thresholds conditioned by covariates and a set of common response coefficients. We show that this extension leads to important additional insights in three familiar settings, but requires only modest amendments to the basic ideas in Chib (1992). Because the algorithm evidences routine application of Gibbs sampling methodology, the model and procedures are available to a wide and broader set of circumstances than previously.

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Table 1. Tobit regressions: Ethiopian highlands milk supply.

	Tobit	Conditionally Censored Tobit		
	Milk Supply	**Milk Supply**	**Threshold**	**Elasticity**
Crossbreed	4.07	2.82		
	(3.53) (4.67)	(2.50) (3.17)		
Localbreed	1.81	0.99		
	(1.38) (2.28)	(0.75) (1.22)		
Distance	-0.06		0.04	0.53
	(-0.08) (-0.05)		(0.03) (0.05)	(0.40) (0.67)
Education	0.59		-0.36	-0.22
	(0.43) (0.76)		(-0.42) (-0.30)	(-0.25) (-0.18)
Extension	0.64		-0.23	-0.08
	(0.48) (0.80)		(-0.32) (-0.15)	(-0.11) (-0.05)
Ilukura	-9.33	-3.96	1.36	
	(-11.12) (-7.73)	(-4.90) (-3.11)	(-0.89) (1.81)	
Mirti	-14.56	-5.91	3.37	
	(16.80) (-12.54)	(-7.04) (4.92)	(2.79) (3.98)	
s	4.88	2.91		
	(4.40) (5.42)	(2.59) (3.27)		

Note: Reports are posterior means. Numbers in parentheses are 95% hpd intervals.