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The Measurement of Child Costs:
A Rothbarth-Type Method Consistent with Scale Economies and Parents' Bargaining

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Abstract: We propose a new methodology to estimate the share of household income accruing to children, i.e., the cost of children. The household behavior is represented according to the collective approach. That is, each household member is characterized by specific preferences. Following the principle of the Rothbarth approach, the identification of the children's share requires the observation of adult-specific goods. Our method differs from this traditional approach in that it is compatible with economies of scale as well as with parents' bargaining. In addition, it allows defining a new concept of child costs that takes into account economies of scale. We illustrate the method with an application on the French Household Budget Survey.

Key Words: Consumer Demand, Collective Model, Rothbarth Method, Cost of Children, Scale Economies, Equivalence Scales, Indifference Scales.

Classification JEL: D11, D12, D36, I31, J12.

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1 Introduction

Evaluating what parents spend on children is an essential prerequisite for inferring individual living standards from income data. Among the numerous methods suggested in the economic literature to measure the cost of children, the Rothbarth method is certainly one of the most theoretically sound. It consists in imputing the same level of aggregate consumption, whatever the demographic composition of the household in which they live, to adults that have the same level of consumption of some adult-specific goods, and deriving from this the fraction of household total expenditure devoted to children.¹ To fix ideas, let us illustrate this method with the simple specification proposed by Gronau (1988, 1991). The goods are supposed to be private (i.e., consumption is rival). First we denote the quantity of adult-specific goods purchased by the household by q_a , the total expenditure of the household by X, and the expenditure specifically devoted to children by Θ . The expenditure devoted to adults is thus equal to $X - \Theta$. Then we suppose that the demand for adult-specific goods in a household with children is represented by the following linear equation:

$$q_a = A + B(X - \Theta), \quad \text{so that} \quad \Theta = X + \frac{A - q_a}{B},$$
 (1)

where A and B are parameters. Thus the children in the household have a simple wealth effect on the demand for adult-specific goods that translates the resources available for the adults by Θ . Information on the level of total expenditure of the household and on the quantity of adult-specific goods purchased can be obtained from usual consumer expenditure surveys. The fundamental identifying idea of the Rothbarth-Gronau method is that the parameters A and B, which are crucial to recover the cost of children, are the same whatever the demographic composition of the household. In other words, the demand for adult-specific goods in a household without children is simply given by:

$$q_a = A + BX. (2)$$

The parameters of this equation can thus be estimated from a sample of childless adults, allowing one to identify the cost of children (1).

¹See Deaton, Ruiz-Castillo and Thomas (1989), Gronau (1991) and Lazear and Michael (1988) on the Rothbarth approach. See Browning (1992) and Lewbel (1997) for a survey of the various techniques used to measure the cost of children.

This method is remarkably simple. Needless to say, however, the identifying assumption, according to which the parameters A and B are independent of the demographic composition of the household, is questionable. In fact we can distinguish at least two serious problems that might invalidate the estimations obtained with this method. Firstly, the existence of economies of scale, due in particular to the possibility of joint consumption in multi-person households, may generate a wealth effect that will generally modify the structure of consumption.² Perhaps more importantly, scale economies may affect the consumption of adults' goods not only via a wealth effect but also via substitution effects. For instance, adult-specific goods which are typically private goods may appear as more costly in a multi-person household than other goods with a large public component (such as heating).³ Secondly, another important problem that may affect the validity of the Rothbarth method is concerned with the lack of individualistic foundations. The adults of the household are described by some constant parameters A and B (in the example above), the provenance of which is unknown. However, recent literature on collective models suggests that individuals in households, in particular, men and women, may differ in terms of objectives.⁴ Hence the decisions are often the result of a compromise – which may be affected by the presence of children – among household members. More generally, the notion of distribution factors, i.e., variables that affect the within-household bargaining without influencing preferences or the budget constraint (according to the traditional terminology of Bourguignon, Browning, and Chiappori, 2008), is potentially important to explain the level of the parents' expenditure devoted to children.⁵ Finally, to under-

²If adult-specific goods are necessary (luxury), the budget share for adult-specific goods will decrease (increase) with economies of scale so that the cost of children will be overstated (understated) by the econometrician. This mechanism is explained in greater detail in the core of the paper.

³Another traditional argument is that goods that are consumed by both adults and children become more expensive to the adult than goods that are only consumed by adults (Deaton and Muellbauer, 1986). To quote Deaton (1997): "on a visit to a restaurant, the father who prefers a soft drink and who would order it were he alone, finds that in the company of a child his soft drink is twice as expensive but that a beer costs the same, and so is encouraged to substitute towards the latter".

⁴See Chiappori and Donni (2010) and Donni (2008) for a survey of this literature.

⁵For instance, a shift of the bargaining power from the father to the mother (due, say, to an exogenous modification of their respective earnings) may change the expenditure devoted to children.

stand boy-girl discrimination (Deaton, 1989; Rose, 1999), it is necessary to be able to disentangle the mother's and the father's preferences in an equation such as (2).

In the present paper, we suggest a variation of the Rothbarth method which is consistent with economies of scale and with parental bargaining. Our approach is closely related to the most recent developments of the literature on collective models.⁶ In particular, Browning, Chiappori and Lewbel (2008) and Lewbel and Pendakur (2008) consider a model where each individual is characterized by a specific utility function and suggest the complete identification of (a) the sharing rule of household resources (which summarizes the bargaining process) and (b) the economies of scale, exploiting simultaneously data on couples and single-person households. Browning, Chiappori and Lewbel (2008) account for economies of scale using a (price) transformation à la Barten while Lewbel and Pendakur (2008) adopt an independence of base technology of production, i.e., they suppose that there exists a single function, which is independent of total expenditure, that scales the expenditure of each individual in the household and represents the economies from joint consumption. While these authors focus on childless couples, we extend the approach to families with children. To represent economies of scale, we follow Lewbel and Pendakur (2008) and make the independence of base assumption. This assumption allows us to recover the sharing of resources between wife, husband and children as well as the consumption technology without price variation, which makes the estimation much more tractable and is also very convenient when using data in which spatial or time variation in prices is limited. In line with the traditional Rothbarth method, we also suppose that the demand for some adult-specific goods is observed. Actually each adult in the household must exclusively consume at least one adult-specific good. This is slightly more demanding

To come back to our example, the parameters A and B for households with and without children have not to be the same.

⁶In the traditional literature on collective models, children and their implications for the intrahousehold allocation are generally ignored: empirical estimations are carried out using a sample of childless couples (Chiappori and Browning, 1998; Donni, 2009). We are aware of essentially two studies (Blundell, Chiappori and Meghir, 2005; Bourguignon, 1999) on collective models that explicitly deal with young children. Closely related are the papers of Menon and Perali (2007) and the test of Dauphin et al. (2008) on collective models with more than two deciders.

than in the traditional Rothbarth approach. From economies of scale and the sharing of resources, we can compute indifference scales, that is, the scalar by which household expenditure must be multiplied so that adults living in couple (with or without children) have the same level of welfare as adults living alone (Lewbel, 2003; Browning, Chiappori and Lewbel, 2008; Lewbel and Pendakur, 2008). We can also propose a new measure for the cost of children which takes into account economies of scale.⁷

Our theoretical results are implemented using the 2000 French Household Budget Survey (INSEE). We suppose that household expenditures on certain pieces of clothing can be seen as adult-specific and consider the case of couples with only one child. We first estimate the budget share equations for the two adult-specific goods in order to measure the cost of children and the economies of scale, then generalize our approach and estimate a system of ten budget share equations. Our evaluation of what parents spend for the child is comprised between 20% and 27% of the total expenditure of the household, which is much more conform to intuition than evaluations based on the traditional Rothbarth method. Once economies of scale are taken into account, it turns out that the cost is notably lower.

The paper is structured as follows. In Section 2, we describe the model and demonstrate how it can be identified. In Section 3, we present the functional form and the method of estimation. In Section 4, we present the data and report the results. In Section 5, we conclude. Further theoretical results are given in the Appendix.

2 The Model

2.1 Preferences, Technologies and the Decision Process

We consider three types of households, namely, a single individual (n = 1), a couple without children (n = 2) and a couple with one child (n = 3) that make decisions about consumption. Individuals are indexed by subscript i while superscript k = 1, ..., K

⁷The present paper must also be related to the recent contribution made, independently of ours, by Dunbar, Lewbel and Pendakur (2010). These authors suggest an alternative, interesting identification strategy of individual shares of total expenditure using only data on couples with children, but they do not propose a measure of child costs taking account of economies of scale.

denotes goods. By convention, we suppose that i = 1 is a male adult, i = 2 is a female adult and i = 3 is a child. The log total expenditure in a household is denoted by x and the vector of log prices by p.

In a single-person household (n = 1), individual utility is maximized with respect to a budget constraint. The indirect utility function of a single individual i endowed with log resources x is supposed to be well-behaved (monotonic, strictly quasi-convex, and twice-continuously differentiable) and is denoted by $v_i(x, \boldsymbol{p}, \boldsymbol{z}_i)$, where \boldsymbol{z}_i is a vector of individual characteristics for individual i (such as age, education, region of residence); hence, the budget share of individual i for good k is defined by

$$w_i^k(x, \boldsymbol{p}, \boldsymbol{z}_i) = -\frac{\partial v_i(x, \boldsymbol{p}, \boldsymbol{z}_i)/\partial p^k}{\partial v_i(x, \boldsymbol{p}, \boldsymbol{z}_i)/\partial x},$$
(3)

for i = 1, 2, 3 and k = 1, ..., K.

In a multi-person household (n > 1), however, budget share equations will change in a way that reflects (a) scale economies and (b) total expenditure sharing. More precisely, each individual in the household is characterized by a well-behaved utility function, the same as that of a similar single individual (that is, single and married persons have identical preferences over goods if they have the same individual characteristics). The relative allocation of household resources $\exp(x)$ among the household members is then defined according to some arbitrary rule, which may be seen as the outcome of an unspecified decision process.⁸ That is to say, individual i living in household of type n > 1 receives a share $\eta_{i,n}(x, \mathbf{p}, \mathbf{z})$ of total expenditure $\exp(x)$. The sharing functions $\eta_{i,n}(x, \mathbf{p}, \mathbf{z})$, with i = 1, ..., n and n = 2 and 3, are differentiable, comprised between zero and one, and sum up to unity, i.e., $\sum_{i=1}^{n} \eta_{i,n}(x, \mathbf{p}, \mathbf{z}) = 1$. They, in general, depend on prices and total expenditure.⁹ They also depend on a vector of household characteristics \mathbf{z} ; the latter includes individual characteristics \mathbf{z}_i with i = 1, ..., n as well as some specific variables $\bar{\mathbf{z}}$ that govern the intrahousehold allocation of resources (i.e., distribution factors). An interesting candidate for these variables is the ratio of

⁸In the collective framework, the existence of a first stage sharing of total expenditure can be justified by the sole efficiency assumption. The sharing may also be the result of parents' altruism.

⁹For instance, we can imagine that the resources accruing to the child vary with the price of child goods (such as child's clothing or toys); see also Bargain and Donni (2008) on this point.

spouses' exogenous incomes in as much as the household bargaining power of spouses depends on what they earn.¹⁰

To obtain our main results regarding identification, we also adopt the same assumption as Lewbel and Pendakur (2008) and Dunbar, Lewbel and Pendakur (2010), that is:

A.1. The shares of total expenditure are differentiable functions that do not depend on total expenditure x, that is, $\eta_{i,n}(x, \boldsymbol{p}, \boldsymbol{z}) = \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})$ for i = 1, 2, 3 and n = 2, 3.

This assumption is potentially strong but it is made essentially for the sake of simplicity. Indeed we show in the Appendix that the main identification results still hold, theoretically at least, when sharing functions depend on total expenditure. Yet, as explained, its implementation with real data may be difficult. Our objective here is to keep the empirical model simple and tractable at the expense of reasonable approximations. Moreover, this assumption is attractive as it implies, as explained below, that the scales we develop in this paper are independent of the base, a desirable property which is often imposed in the traditional equivalence scales literature. Finally, this assumption can be mitigated in empirical applications by including measures of household wealth other than total expenditure in income shares.

The publicness of goods, and hence economies of scale in the household, are represented by a particular technology of production. This technology must be sufficiently tractable so that the model can be estimated using cross-section data. The simplest, but not most convincing, framework to model economies of scale consists in using Engel scales. With A.1, the indirect utility function of individual i in household of type n then becomes: $v_i(\boldsymbol{p}, x + \log \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z}) - \log s_e, \boldsymbol{z}_i)$, where $s_e < 1$ is an Engel scale. So, the "value" of total expenditure is inflated by the presence of several persons in the household and economies of scale have a pure wealth effect. This is the case envisaged in the introduction. However, this approach is not satisfactory because, as it seems obvious, the level of joint consumption is not the same for all goods: some goods have a clear public component while other goods are completely private. Moreover, the pro-

¹⁰Numerous studies indeed show that the source of exogenous income influences the structure of consumption. For instance, Thomas (1991) note that unearned income in the hands of the mother has a bigger effect on the children's health.

¹¹In our previous example and with Engel scales, the demand for adult goods in a household with

portion of jointly consumed goods will generally not be the same for all the household members. To give the intuition, let us consider a couple with or without child and suppose that a constant proportion of all the goods, say ϑ , is consumed jointly within the household. In that case, the consumption of spouse i in household of type n > 1 is supplemented by a fraction of joint consumption of the other household members; it is equal to

$$\eta_{i,n}(oldsymbol{p},oldsymbol{z}) + artheta imes \left(1 - \eta_{i,n}(oldsymbol{p},oldsymbol{z})
ight) = rac{\eta_{1,n}(oldsymbol{p},oldsymbol{z})}{s_{i,n}^*(oldsymbol{p},oldsymbol{z})},$$

where

$$s_{i,n}^*(\boldsymbol{p}, \boldsymbol{z}) = \left(1 + \vartheta \times \frac{1 - \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})}{\eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})}\right)^{-1},$$
(4)

so that, even in this very simple case, the deflator representing economies of scale will depend on the vector of prices (at least if the sharing of total expenditure depends itself on the vector of prices). Therefore we decided to adopt a much more general approach than Engel scales and the scales such as (4), and to assume that economies of scale generated by joint consumption of certain goods in the household can be represented by a price-dependent deflator. We first introduce this assumption formally below and then discuss its implications.

A.2. (Independent of the Base) For each person i living in a household of type n > 1, we assume that there exists a scalar-valued, differentiable function $s_{i,n}(\mathbf{p}, \mathbf{z})$ such that the indifference curves of individual i satisfy the condition:

$$u_i = v_i(\boldsymbol{p}, x + \log \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z}) - \log s_{i,n}(\boldsymbol{p}, \boldsymbol{z}), \boldsymbol{z}_i)$$
(5)

for any level of log individual expenditure $x + \log \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})$.

children is given by:

$$q_a = s_e \times \left[A + B \left(\frac{X - \Theta}{s_e} \right) \right], \quad \text{so that} \quad \Theta = X + \frac{s_e A - q_a}{B}.$$

This expression underlines the distortions that may result from the omission of scale economies when using the Rothbarth method. Specifically, assuming that B > 0 and $s_e < 1$, the cost of children will be over-stated if the adult good is necessary (A > 0) and under-stated if it is luxury (A < 0).

The deflator measures the cost savings experienced by person i resulting from scale economies in the household. The Independent of the base (IB) assumption refers to the fact that these economies are assumed to be independent of the base expenditure (and hence utility) level at which they are evaluated. This assumption is similar to the IB restriction in the equivalence scale literature (Blackorby and Donaldson, 1993; Lewbel, 1989, 1991), but it concerns individual utility functions rather than aggregated household utility functions.¹² The scaling function $s_{i,n}(\mathbf{p}, \mathbf{z})$ can be interpreted by first discerning two polar cases: if $s_{i,n}(\mathbf{p}, \mathbf{z}) = 1$ for $i \leq n$, it is as if all the goods were purely private and if $s_{i,n}(\mathbf{p}, \mathbf{z}) = \eta_{i,n}(\mathbf{p}, \mathbf{z})$ for $i \leq n$, all the goods can be seen as purely public. Then a large range of intermediate situations can be obtained for other values of $s_{i,n}(\mathbf{p}, \mathbf{z})$.

The fact that the scaling function depends on prices makes the IB scale far more general than traditional Engel scales; in particular, the idea that some goods are consumed in common (and thereby largely affected by economies of scale) while other goods are not can be represented here, admittedly in a quite restrictive way, by the derivative of $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ with respect to prices. To take an intuitive example, let us suppose that good k has a large public component (like housing) so that it can potentially generate important economies of scale. Of course, the actual economies will depend on the quantity of good k purchased by the household. Then an increase in the price of good k that leads to a reduction of the purchased quantity of good k will have a positive effect on the scale $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ (i.e., a negative effect on economies of scale). Conversely, let us suppose that good k is purely private (like food). Then an increase in the price of good k will have a negative effect on the scale $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$. Moreover, economies of scale may differ between individuals within the same household, depending on how they value the good which is jointly consumed. In particular, if the consumption by member i of good k exerts a negative externality effect on the utility of the other members in the same household, and if member i internalizes this effect in his/her

¹²The scaling function $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ generally depends on all the individual characteristics of the persons living in the household, \boldsymbol{z} . Indeed, it cannot be excluded that the extent of joint consumption of one person in the household be related to the characteristics of his/her partner or his/her child. It seems logical, however, to suppose that distribution factors do not enter scale economies because they influence behavior only via the intra-household distribution of total expenditure. This is not important for our results, though.

utility function, then a decrease in the price of this good may be compensated by an increase of the scale $s_{i,n}(p, z)$. This flexibility of IB scales is particularly important. The apparition of a child in the household may indeed generate important externality effects; for example, the parents may decide to stop smoking and to change their leisure activities. Note finally that IB scales can be seen as an approximation of Barten scales (used by Browning, Chiappori and Lewbel, 2008) in the sense that indirect utility functions can be both IB and Barten scaled if at least one linear restriction exists on the log of Barten scales (Lewbel, 1991). For a more structural presentation of the model using Barten scales, the reader is referred to Lewbel and Pendakur (2008).

2.2 Economies of Scale, Indifference Scales and the Cost of Children

From the above discussion, it is clear that the level of the scale $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ cannot be interpreted directly: it must be compared to the level of the corresponding share $\eta_{i,n}(\boldsymbol{p},\boldsymbol{z})$. Fortunately, a normalized indicator of the 'individual' economies of scale for each member can be defined as

$$\sigma_{i,n}(\boldsymbol{p},\boldsymbol{z}) = 1 + \frac{\eta_{i,n}(\boldsymbol{p},\boldsymbol{z}) \left(1 - s_{i,n}(\boldsymbol{p},\boldsymbol{z})\right)}{s_{i,n}(\boldsymbol{p},\boldsymbol{z}) \left(1 - \eta_{i,n}(\boldsymbol{p},\boldsymbol{z})\right)},$$

for $n \geq 2$, which is equal to 1 in the purely private case and to 2 in the purely public case. If the scale is of the form (4), then $\sigma_{i,n}(\boldsymbol{p},\boldsymbol{z})$ is simply equal to $1 + \vartheta$.

Denote $\log I_{i,n}(\boldsymbol{p},\boldsymbol{z}) = \log s_{i,n}(\boldsymbol{p},\boldsymbol{z}) - \log \eta_{i,n}(\boldsymbol{p},\boldsymbol{z})$ so that equation (5) can be compactly written as:

$$u_i = v_i(\boldsymbol{p}, x - \log I_{i,n}(\boldsymbol{p}, \boldsymbol{z}), \boldsymbol{z}_i). \tag{6}$$

The term $I_{i,n}(\boldsymbol{p},\boldsymbol{z})$ is the indifference scale of member i as defined by Lewbel (2003), Lewbel and Pendakur (2009) and Browning, Chiappori and Lewbel (2008). It represents the income adjustment applied to person i when living in a multi-person household – consuming a share $\eta_{i,n}(\boldsymbol{p},\boldsymbol{z})$ of total resources and benefiting from scale economies represented by $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ – for her/him to reach the same indifference curve as when living alone.¹³ This concept differs from an ordinary equivalence scale, which

¹³The definition at stake here is slightly different from that found in the mentioned literature because the basis of reference is the single person and not the person living in a couple.

attempts to compare the welfare of an individual to that of a household, and hence suffers from the fundamental identification problem associated with interpersonal comparisons (Pollak and Wales, 1979, 1992). In contrast, indifference scales can be seen as comparing the same individual in two different situations: living alone and living with a partner (with or without children).¹⁴ Implicitly, the direct utility or disutility from living with others (such as love and companionship) is assumed to be separable from consumption goods and ignored.

The notion of indifference scale leads to a new measure for the cost of children. The scalar by which the total expenditure of a childless couple must be multiplied so that the level of utility of both spouses remain unaffected after the arrival of a first child is:

$$\lambda(oldsymbol{p},oldsymbol{z}) = \left[\sum
olimits_{i=1,2} \eta_{i,3}(oldsymbol{p},oldsymbol{z}) imes rac{s_{i,2}(oldsymbol{p},oldsymbol{z})}{s_{i,3}(oldsymbol{p},oldsymbol{z})}
ight]^{-1},$$

and the cost of the child as a fraction of total expenditure is:

$$c(\boldsymbol{p}, \boldsymbol{z}) = \lambda(\boldsymbol{p}, \boldsymbol{z}) - 1.$$

This measure recognizes the role of economies of scale when estimating the cost of children. It is the concept that is relevant for policy recommendations. For instance, let us suppose the government wants to compensate couples for the birth of their first child; it must give child benefits that are equal to $c(\mathbf{p}, \mathbf{z}) \times \exp x$ for some level x of log total expenditure. To distinguish this cost from more traditional measures of the cost of children and to underline the fact that it incorporates economies of scale, we shall refer to it as "the overall cost" in what follows. Note that this measure is proportionate to total expenditure. In fact, as it was anticipated, indifference scales $I_{i,n}(\mathbf{p},\mathbf{z})$, normalized economies of scale $\sigma_{i,n}(\mathbf{p},\mathbf{z})$, and the overall cost of the child $c(\mathbf{p},\mathbf{z})$ are independent of the base.

¹⁴It is fair to say that traditional equivalence scales are sometimes interpreted as comparing the utility of the sole adults in the household, and not the utility of the household as a whole (Nelson, 1993). However, this interpretation is not convincing in the unitary framework.

2.3 The Budget Shares of Total Expenditure

Denoting the log individual share as $x_{i,n} = x + \log \eta_{i,n}$ and applying Roy's identity to equation (5), individual i's budget share function for good k is defined as:

$$\omega_{i,n}^k(x, \boldsymbol{p}, \boldsymbol{z}) = -\left. \frac{\partial v_i(\boldsymbol{p}, x_{i,n} - \log s_{i,n}(\boldsymbol{p}, \boldsymbol{z}), \boldsymbol{z}_i) / \partial p^k}{\partial v_i(\boldsymbol{p}, x_{i,n} - \log s_{i,n}(\boldsymbol{p}, \boldsymbol{z}), \boldsymbol{z}_i) / \partial x_{i,n}} \right|_{x_{i,n} = x + \log \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})},$$

where the left-hand side of this expression is the fraction of member i's resource share, $\exp(x) \times \eta_{i,n}(\boldsymbol{p}, \boldsymbol{z})$, spent on good k. Developing the derivatives, it is easy to show that

$$\omega_{i,n}^k(\boldsymbol{p}, x, \boldsymbol{z}) = d_{i,n}^k(\boldsymbol{p}, \boldsymbol{z}) + w_i^k(\boldsymbol{p}, x - \log I_{i,n}(\boldsymbol{p}, \boldsymbol{z}), \boldsymbol{z}_i)$$
(7)

where

$$d_{i,n}^k(oldsymbol{p},oldsymbol{z}) = rac{\partial \log s_{i,n}(oldsymbol{p},oldsymbol{z})}{\partial p^k}$$

is the elasticity of $s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ with respect to the k-th price. The consequence of the IB assumption in the present context is that the budget share equations of person i when living in a household differ from when alone only in that they are translated over by $d_{i,n}^k(\boldsymbol{p},\boldsymbol{z})$ while log household expenditures x are translated over by $\log I_{i,n}(\boldsymbol{p},\boldsymbol{z})$. This property is referred to as "shape invariance" by Pendakur (1999). The translation function $d_i^k(\boldsymbol{p},\boldsymbol{z})$ is specific to good k and related to the differences that may exist between goods with respect to the possibility of joint consumption. Intuitively, economies of scale may have a wealth effect and a substitution effect. The former is represented by $\log s_{i,n}(\boldsymbol{p},\boldsymbol{z})$ and the latter by $d_{i,n}^k(\boldsymbol{p},\boldsymbol{z})$. The substitution effect is positive (negative) if good k is essentially public (private).

To unify our notation, we also use the following definitions.

N.1. For single men
$$(i = 1)$$
 or single women $(i = 2)$, we have: $\eta_{i,1}(\boldsymbol{p}, \boldsymbol{z}) = 1$, $d_{i,1}^k(\boldsymbol{p}, \boldsymbol{z}) = 0$, $s_{i,1}(\boldsymbol{p}, \boldsymbol{z}) = 1$ for any k .

This condition is also a normalization. It implicitly means that single individuals are used as the demographic structure of reference.

Now let us suppose that data are observed in a unique price regime, as provided in cross-sectional data, so that the vector of prices p is constant and can be taken out of equation (7). Formally, the implications of the IB assumption in a framework with no price variation are described in the following lemma:

Lemma 1. Assume A.1–A.2 and N.1. If prices are constant, the budget share of good k of person i living in household of type n is written:

$$\omega_{i,n}^{k}(x, \mathbf{z}) = d_{i,n}^{k}(\mathbf{z}) + w_{i}^{k}(x - \log I_{i,n}(\mathbf{z}), \mathbf{z}_{i}),$$
for $k = 1, ..., K, i = 1, ..., n, and $n = 1, 2, 3,$$

where $\log I_{i,n}(z) = \log s_{i,n}(z) - \log \eta_{i,n}(z)$ is the log deflator of total expenditure which combines the scaling $s_{i,n}$ and sharing $\eta_{i,n}$.

The left-hand side of (8) represents the 'reduced-form' budget share on good k of person i in household of type n as a function of (log) household resources x and household characteristics z. The right-hand side puts some structure on the budget share as a result of the IB restriction. The individual budget share function $w_i^k(\cdot, z_i)$ depends on person i's individual resources adjusted by the scaling $s_{i,n}(z)$ and on the individual characteristics z_i (but not on the characteristics of the other individuals in the household). This share is then translated by the elasticity $d_{i,n}^k(z)$.

For each good k, we can write household expenditure as the sum of individual expenditures on that good. Dividing this identity by total outlay $\exp(x)$, we obtain directly the household budget share function for good k:

$$W_n^k(x, \mathbf{z}) = \sum_{i=1}^n \eta_{i,n}(\mathbf{z}) \cdot \omega_{i,n}^k(x, \mathbf{z}), \tag{9}$$

for any n and any k, where $W_n^k(\cdot)$ is the share spent by the household of type n on good k. This is simply the sum of individual budget share equations over all household members, weighted by their individual resource shares. Using equation (8), the budget share equation can be written as:

$$W_n^k(x, \mathbf{z}) = \sum_{i=1}^n \eta_{i,n}(\mathbf{z}) \left[d_{i,n}^k(\mathbf{z}) + w_i^k \left(x - \log I_{i,n}(\mathbf{z}), \mathbf{z}_i \right) \right], \tag{10}$$

where individual budget shares are translated both in budget shares and log-expenditure.

2.4 Identification Strategy

Our goal here is to identify the important structural elements of the model, namely the sharing and scaling functions, from demand data. To account for unobserved factors,

we add error terms to the household budget shares previously defined:

$$\tilde{W}_{n}^{k}(x, \boldsymbol{z}) = W_{n}^{k}(x, \boldsymbol{z}) + \varepsilon_{n}^{k},$$
for $n = 1, 2, 3$ and $k = 1, \dots, K,$

where $\tilde{W}_n^k(\cdot)$ is the stochastic extension of $W_n^k(\cdot)$. The classical interpretation of the error term ε_n^k is that it represents optimization or measurement errors. This is the easiest way to understand this specification. Alternatively, the stochastic component could be interpreted as resulting from unobservable heterogeneity in the individual budget share equations (hence assuming random utilities), in the scales or in the resource shares. For instance, if the budget share equations are random, i.e., $w_i^k(\cdot) + \varepsilon_i^k$, then $\varepsilon_n^k = \sum_{i=1}^n \eta_{i,n}(z) \cdot \varepsilon_{i,n}^k$, where $\varepsilon_{i,n}^k$ is an idiosyncratic term for member i in the household. The discussion that follows is not modified provided that the terms $\varepsilon_{i,n}^k$ are independent of z. Moreover, in that case, the term ε_n^k will generally be heteroskedastic.

The equations (11) can be identified from well-known results in non-parametric econometrics provided the sample is sufficiently large and error terms satisfy normalization restrictions (see Matzkin, 2007, for instance). Identification can thus concentrate on how to retrieve the structural components $s_{i,n}(z)$, and $\eta_{i,n}(z)$, for i = 1, ..., n and n = 1, 2, 3, from the knowledge of the deterministic components $W_n^k(\cdot)$.

Identification exploits the following additional assumption:

A.3. There exists at least one adult-specific good for each adult in the household. More precisely, one good k_1 is consumed by men but not by women or children and one other good k_2 is consumed by women but not by men or children.

The concept of adult-specific goods plays a major role for applying the well-known Rothbarth method. Classic examples of such goods include certain pieces of clothing, tobacco and alcohol even if more inclusive definitions have also been used (as explained by Deaton, 1997). The assumption introduced here is a little more demanding as the good must be specific to the wife or the husband. We explain in the Appendix how this restriction could theoretically be relaxed. The extension to the case with a unique adult-specific good is not presented here because the data we use effectively contains a pair of goods that are specific to wives and husbands respectively. Moreover, we

believe that the identification of the structural components of the model with only one adult-specific good may be flimsy in practice.

The identification result that follows relies on a certain number of normalization conditions. First of all, the condition N.1 previously discussed is obviously necessary. Moreover, the terms that represent economies of scale in the budget share equations of children are actually meaningless in a world where young children are always living within the same family structure.¹⁵ Hence, without loss of generality, the following condition is also used.

N.2. For children (i = 3), we have: $d_{3,3}^k(z) = 0$, $s_{3,3}(z) = 0$ for any k.

The main result is then summarized in the following proposition.

Proposition 2. Assume A.1–A.3 and N.1–N.2. If prices are constant, and $\nabla_x w_i^{k_i} \neq 0$ and $\nabla_{xx} w_i^{k_i} \neq 0$ almost everywhere for i = 1, 2, then the sharing functions $\eta_{i,n}(\mathbf{z})$ and the scaling functions $s_{i,n}(\mathbf{z})$, for i = 1, 2, 3 and n = 1, 2, 3, can be identified from the estimation of the budget share equations $W_n^{k_i}(x, \mathbf{z})$ for the adult-specific goods.

The proof follows in three steps. We first discuss how to retrieve the "basic" budget share equations. We then consider identification in the case of couples without child and in the case of couples with one child.

Step 1. To retrieve the main structural components of the model, the basic idea is that differences between individual consumption as a single or in a multi-person household are assumed to be due to partially joint consumption, resource sharing and changes in total resources, but are not attributed to taste differences. Gronau (1988) argues that this assumption, as strong as it may be, is essential to make the comparison of individuals living in different households possible. Then, using N.1, we simply have:

$$W_1^k(x, \mathbf{z}) = w_i^k(x, \mathbf{z}_i),$$

for any k, with i=1,2, and identification of the functions $w_i^k\left(\cdot\right)$ can be obtained from a sample of single (male and female) individuals.

¹⁵It would be useful to account for children's economies of scale if we were considering more diversified family structures such as single-parent families or families with several children.

Step 2. We now consider the case of a childless couple, that is, n = 2. The household budget share equation for good k_i can be written as:

$$W_2^{k_i}(x, \mathbf{z}) = \eta_{i,2}(\mathbf{z}) \cdot \left[d_{i,2}^{k_i}(\mathbf{z}) + w_i^{k_i}(x - \log I_{i,2}(\mathbf{z}), \mathbf{z}_i) \right], \tag{12}$$

for i=1, 2, because this good is specific to only one person in the household. The following reasoning is, in fact, a new demonstration (in a slightly different context) of a result previously obtained by Lewbel and Pendakur (2008). The latter do not use individual-specific goods for their demonstration, though, but consider a system of budget share equations and suppose that the household total expenditure can be zero. To eliminate the function $d_{i,2}^{k_i}(z)$ from equation (12), we compute the first order derivative of this expression with respect to x and obtain:

$$\nabla_x W_2^{k_i}(x, \mathbf{z}) = \eta_{i,2}(\mathbf{z}) \nabla_x w_i^{k_i} \left(x - \log I_{i,2}(\mathbf{z}), \mathbf{z}_i \right), \tag{13}$$

where the left-hand side of this expression is identified. Differentiating again this expression with respect to x we obtain the second order derivative:

$$\nabla_{xx} W_2^{k_i}(x, \mathbf{z}) = \eta_{i,2}(\mathbf{z}) \nabla_{xx} w_{ii}^k \left(x - \log I_{i,2}(\mathbf{z}), \mathbf{z}_i \right). \tag{14}$$

Taking the ratio of (13) and (14), we have:

$$\frac{\nabla_x W_2^{k_i}(x, \boldsymbol{z})}{\nabla_{xx} W_2^{k_i}(x, \boldsymbol{z})} = \frac{\nabla_x w_i^{k_i} \left(x - \log I_{i,2}(\boldsymbol{z}), \boldsymbol{z}_i\right)}{\nabla_{xx} w_i^{k_i} \left(x - \log I_{i,2}(\boldsymbol{z}), \boldsymbol{z}_i\right)} = \Delta_i^{k_i} \left(x + \log I_{i,2}(\boldsymbol{z}), \boldsymbol{z}\right)$$

where the left-hand side of the first equality and the function $\Delta_i^{k_i}(\cdot, \mathbf{z})$ are known from step 1. This condition uniquely identifies the indifference scales $I_{i,2}(\mathbf{z})$ for i = 1, 2, provided the function $\Delta_i^{k_i}(\cdot)$ is not periodic in its first argument – a rather natural requirement. Then, for i = 1, 2, identification of sharing functions $\eta_{i,2}(\mathbf{z})$ follows from (13) and identification of translation functions $d_{i,2}^{k_i}(\mathbf{z})$ from (12). Finally, the scaling functions $s_{i,2}(\mathbf{z})$ can be computed for i = 1, 2 from the definition of $I_{i,2}(\mathbf{z})$.

Step 3. In the case of a couple with one child, the budget share equations for adult specific goods have exactly the same structure as above:

$$W_3^{k_i}(x, \mathbf{z}) = \eta_{i,3}(\mathbf{z}) \cdot \left[d_{i,3}^{k_i}(\mathbf{z}) + w_i^{k_i}(x - \log I_{i,3}(\mathbf{z}), \mathbf{z}_i) \right],$$

for i = 1, 2. Hence, identification of $\eta_{i,3}(\mathbf{z})$, $s_{i,3}(\mathbf{z})$ and $I_{i,3}(\mathbf{z})$ for i = 1, 2 is straightforward and does not deserve a detailed discussion. The share of total expenditure devoted to the child can then be obtained as:

$$\eta_{3,3}(z) = 1 - \sum_{i=1}^{2} \eta_{i,3}(z),$$

while the function $s_{3,3}(z)$ is given by N.2. This completes the proof. \square Several important comments are in order.

- (a) Identification necessitates that budget share equations for adult-specific goods be non-linear in log total expenditure, i.e., the second order derivative of the budget share equation must be different from zero. This is not necessarily a serious issue; as recognized by Banks, Blundell, and Lewbel (1997), budget share equations are generally non-linear. Nonetheless, the functional form must be sufficiently flexible to account for this nonlinearity. Moreover, the regularity conditions in Proposition 2 may be violated for some specific goods and must be checked in a preliminary step of the empirical analysis. If they are not convincingly satisfied in the data, modeling more budget share equations may be a solution as explained below.
- (b) It must be clear that modeling more budget share equations than those for the two adult goods will generate overidentification restrictions. In particular, any budget share equation in a childless couple can be written as:

$$W_2^k(x, \mathbf{z}) = D_2^k(\mathbf{z}) + \sum_{i=1}^2 \eta_{i,2}(\mathbf{z}) w_i^k(x - I_{i,2}(\mathbf{z}), \mathbf{z}_i),$$
(15)

with $k \neq k_1, k_2$, where

$$D_2^k(\mathbf{z}) = \sum_{i=1}^2 d_{i,2}^k(\mathbf{z}) \eta_{i,2}(\mathbf{z}). \tag{16}$$

The functions $w_i^k(\cdot, \mathbf{z}_i)$ can be identified from estimations made on a sample of singleperson households while the functions $\eta_{i,2}(\mathbf{z})$ and $I_{i,2}(\mathbf{z})$ are identified from estimations of the budget share equations for good k_1 and k_2 , as explained above. The only degree of freedom is then represented by the function $D_2^k(\mathbf{z})$; in particular, the derivative of the budget share equation with respect to log total expenditure for an arbitrary good k is completely determined by the knowledge of the behavior of single persons and the structural components recovered from adult-specific goods. Such overidentification can, naturally, be used to generate empirical tests. In particular, the slopes $\nabla_x w_i^k$ can be estimated for goods $k \neq k_1, k_2$ from the sample on childless couples, and these estimations can then be compared to those obtained from the sample on singles. Otherwise, overidentification helps improve the precision of the estimations. ¹⁶

(c) Many more structural components of the model can generally be identified, which is not made explicit in the proposition. In particular, if a complete system of budget share equations (instead of the sole budget share equations for the adult-specific goods) is estimated, the functions $D_2^k(z)$ can be retrieved as

$$D_2^k(z) = W_2^k(x, z) - \sum_{i=1}^2 \eta_{i,2}(z) w_i^k(x - I_{i,2}(z), z_i),$$
(17)

where the left-hand side is identified. Moreover, under some additional conditions, i.e., if there exists a distribution factor \bar{z}_1 (say) that enters the sharing functions as argument without entering the scaling functions, the functions $d_{1,2}^k(z)$ and $d_{2,2}^k(z)$, respectively, can be identified as well. Indeed,

$$abla_{ar{z}_1}D_2^k(m{z}) = \sum_{i=1}^2 d_{i,2}^k(m{z})
abla_{z_1} \eta_{1,2}(m{z}),$$

since $\nabla_{\bar{z}_1} d_{i,2}^k(z) = 0$ for i = 1, 2. This equation, together with equation (16), can generically be solved with respect to $d_{1,2}^k(z)$ and $d_{2,2}^k(z)$, which in turn allows recovering the effect of all the prices (computed at the current system of prices) on economies of scale. Finally, although the budget share equations of children cannot, in general, be retrieved, the derivatives of these equations with respect to log total expenditure can be identified. Indeed,

$$w_3^k \left(x - \eta_{3,3}(\boldsymbol{z}), \boldsymbol{z}_3 \right) = \frac{W_3^k(x, \boldsymbol{z})}{\eta_{3,3}(\boldsymbol{z})} - \frac{D_3^k(\boldsymbol{z})}{\eta_{3,3}(\boldsymbol{z})} - \sum_{i=1}^2 \frac{\eta_{i,3}(\boldsymbol{z})}{\eta_{3,3}(\boldsymbol{z})} w_i^k \left(x - I_{i,3}(\boldsymbol{z}), \boldsymbol{z}_i \right), (18)$$

¹⁶For the sake of simplicity, the discussion above is not complete. Firstly, the budget share functions for adult-specific goods, taken separately from the other budget share functions, are also overidentified. This is explained in the Appendix. Secondly, the budget share functions of a couple with one child also generate additional restrictions. This is explained below.

where

$$D_3^k(z) = \sum_{i=1}^3 d_{i,3}^k(z) \eta_{i,3}(z)$$

is an unknown function. Now differentiating expression (18) with respect to x shows that the derivative of the budget share equation of the child $\nabla_x w_3^k$ can be identified, allowing us to determine whether goods consumed by the child are luxury or necessary. Because the left-hand side depends only on a limited number of arguments, namely, $(x - \eta_{3,3}(z))$ and z_3 , the budget share equations for couples with child generate overidentifying restrictions (provided that z_3 is strictly included in z).

3 Empirical Implementation

3.1 Functional Form

In what follows, we shall discuss the empirical specification of the complete model which includes 10 equations. The model with only adult-specific goods, which will also be estimated, is simply a particular case. For the functional form, we suggest a parameterization that balances flexibility and empirical tractability. The first component, which appears in the specification of the different demographic groups, is the "basic" budget share equation. We adopt the following quadratic specification:

$$w_i^k(x_{i,n}, \mathbf{z}_i) = \bar{a}_i^k + \sum_j a_{i,j}^k z_j + b_i^k \left(x_{i,n} - \sum_j e_{i,j} z_j \right) + c_i^k \left(x_{i,n} - \sum_j e_{i,j} z_j \right)^2, \quad \text{for } i = 1, 2, 3 \text{ and } k = 1, ..., K,$$

where $x_{i,n}$ is defined as previously, and \bar{a}_i^k , $a_{i,j}^k$, b_i^k , c_i^k and $e_{i,j}$ are parameters. The parameters are specific to individual type (i.e., are indexed i=1 for men, i=2 for women, i=3 for children) but do not depend on the demographic type n since the "basic" budget share equations are the same for single women (men) and for women (men) living in a couple. The demographic variables enter the specification both as a translation of budget share equations and as a translation of log scaled expenditure. The characteristics entering $\sum_j e_{i,j} z_j$ for adults include dummies for age and education and those entering $\sum_j a_{i,j}^k z_j$ include the same variables plus dummies for car ownership,

house ownership, urban resident and Paris resident. For children, the characteristics include a dummy for gender and a dummy for age in both $\sum_{j} e_{i,j} z_{j}$ and $\sum_{j} a_{i,j}^{k} z_{j}$.

We now turn to the specification of the household budget share equations. For single male and female adults, they coincide with the "basic" budget share equations specified above plus an additive error term, that is,

$$\tilde{W}_1^k(x, \mathbf{z}) = w_i^k(x, \mathbf{z}_i) + \varepsilon_1^k. \tag{19}$$

For multi-person households $n \geq 2$, and for non-adult-specific goods, the household budget share equations,

$$\widetilde{W}_{n}^{k}(x, \boldsymbol{z}) = \sum_{i=1}^{n} \eta_{i,n}(\boldsymbol{z}) \left[d_{i,n}^{k}(\boldsymbol{z}) + w_{i}^{k} \left(x - \log I_{i,n}(\boldsymbol{z}), \boldsymbol{z}_{i} \right) \right] + \varepsilon_{n}^{k},$$
(20)

comprise the individual functions $w_i^k(\cdot, \mathbf{z}_i)$ as already specified and three other components that are defined as follows. Firstly, the *sharing functions* are specified using the logistic form:

$$\eta_{i,n}(z) = \frac{\exp(\bar{\beta}_{i,n} + \sum_{j} \beta_{i,j} z_{j})}{\sum_{i=1}^{n} \exp(\bar{\beta}_{i,n} + \sum_{j} \beta_{i,j} z_{j})}, \quad \text{for } i = 1, 2, 3 \text{ and } n = 2, 3,$$

where $\bar{\beta}_{i,n}$ and $\beta_{i,j}$ are parameters. To limit the number of parameters, variables in $\sum_{j} \beta_{i,j} z_{j}$ include the dummies for spouse i's age and education for i=1,2 or the dummies for gender and age for i=3 as well as a distribution factor – the wage ratio which is defined as the ratio of wife's over husband's labor earnings expressed in full-time equivalent – but it does not include individual characteristics of the partner. Almost all the parameters are the same whether a child is living or not in the household; only the constant differs so that it is possible to measure the effect of the child on the distribution of resources between parents. Secondly, the log scaling functions that translates expenditure within the basic budget shares can be written as:

$$\log s_{i,n}(\boldsymbol{z}) = \bar{\alpha}_{i,n} + \sum_{j} \alpha_{i,j} z_{j}, \qquad \text{for } i = 1, 2 \text{ and } n = 2, 3,$$

where $\bar{\alpha}_{i,n}$ and $\alpha_{i,j}$ are parameters. The scaling functions can, in principle, vary with all the variables entering preferences (i.e., z_i for i = 1, ..., n). In our specification,

¹⁷Normalization is obviously required. The variables entering exponentials corresponding to the wife are set to zero if they are also in the exponentials of the husband or the child.

however, it is restricted to depend only on variables regarding individual i. Moreover, to limit the number of parameters, only the constant is indexed by the type of family n. Concretely, variables in $\sum_j \alpha_{i,j} z_j$ include the dummies for age and education of spouse i if it concerns an adult and the dummies for gender and age if it concerns a child. Thirdly, the function that translates the basic budget shares $d_{i,n}^k(z)$ is a price elasticity. Measuring price effects is generally challenging and it is all the more difficult to capture their interaction with demographics in any plausible way. Therefore we restrict these terms to be constant:

$$d_{i,n}^k(z) = \overline{d}_{i,n}^k$$
, for $i = 1, 2, n = 2, 3, \text{ and } k = 1, \dots, K$.

3.2 Estimation Method

The complete model is estimated by the iterated SURE method. To account for the likely correlation between the error terms ε_n^k in each budget share function and the log total expenditure, each budget share equation is augmented with the 'Wu-Hausman' residuals \hat{v}_n^1 (and possibly \hat{v}_n^2) obtained from reduced-form estimations, specific to family type n, of x and x^2 respectively on all exogenous variables used in the model plus some excluded instruments (Banks, Blundell and Lewbel, 1997; Blundell and Robin, 1999, 2000; Smith and Blundell, 1986). For the latter, we choose the inverse of household disposable income and a fourth order polynomials in its logarithm. Since budget shares sum up to one, equation for good K is unnecessary. The household budget share equations for the K-1 goods and for the three demographic groups are estimated simultaneously. The error terms are supposed to be uncorrelated across households but correlated across goods within households. They are supposed to be homoskedastic for each family type n (and covariance matrices are supposed to be different for single male and female). Observations in the data are indexed by hand the number of singles, couples without children, and couples with children in the data is denoted by H_1 , H_2 , and H_3 , respectively. Let $\mathbf{W}_{n,h}$ be the (K-1) vector of observed budget shares for the first K-1 goods consumed by household h of type n and let $\mathbf{W}_{n,h}(\boldsymbol{\theta})$ be the corresponding (K-1) vector of predicted budget shares for some parameter vector $\boldsymbol{\theta}$. The vector of residuals is thus given by $\boldsymbol{\varepsilon}_{n,h}(\boldsymbol{\theta})$ $\mathbf{W}_{n,h} - \hat{\mathbf{W}}_{n,h}(\boldsymbol{\theta})$. If $\hat{\boldsymbol{\varepsilon}}_{n,h} = \boldsymbol{\varepsilon}_{n,h}(\hat{\boldsymbol{\theta}}_0)$, where $\hat{\boldsymbol{\theta}}_0$ is any initial consistent estimation of the vector of parameters, the estimated covariance matrix can be defined by

$$\hat{\mathbf{V}}_n = H_n^{-1} \times (\hat{\boldsymbol{\varepsilon}}_{n,h}) (\hat{\boldsymbol{\varepsilon}}_{n,h})'$$
.

The SURE criterion is then:

$$\min_{\boldsymbol{\theta}} \sum_{n=1}^{3} \sum_{h=1}^{H_n} \left(\boldsymbol{\varepsilon}_{n,h}(\boldsymbol{\theta}) \right)' (\hat{\mathbf{V}}_n)^{-1} \left(\boldsymbol{\varepsilon}_{n,h}(\boldsymbol{\theta}) \right),$$

which gives a new value $\hat{\boldsymbol{\theta}}_1$ for the estimates. The estimation procedure is then iterated with the new estimates until the covariance matrix converges.

4 Data and Empirical Results

4.1 Data and Sample Selection

Our sample is drawn from the 2000 French Household Budget Survey conducted by INSEE. This data gathers information on household expenditures, incomes and socio-demographics for 10, 350 representative households. It was collected over the year 2000 and only little price variation is witnessed over this period so that the sample can be treated as cross-sectional data. All household members who are at least 14 years of age are interviewed. Expenditures on clothing are recorded for the past two months, and consumption of daily services and goods are recorded in diaries over the 14 days of the study.

Our selection criterion is as follows. To begin with, we exclude households larger than the nucleus family (parents, children), with more than one child or where the child is aged 14 or more (and hence not differentiable from adults in terms of clothing expenditure in the data), which leaves out about 38% of the sample. We then select households where adults are aged 18-59, which further restricts the initial sample by 26% and we withdraw another 2% corresponding to households where adults are students, in the army or retired. Since leisure is not modeled here, but is likely endogenous to consumption (and savings) decisions, we finally restrict our sample to working adults and full-time working men. This excludes another 13% of the original sample, 7% of which is due to non-participating spouses in couples. The final sample is composed of 2, 153 observations and is described in Table 1.

In the estimation of the more general model, we use K = 10 non-durable commodities: food (in and out), "vices" (alcohol, tobacco and gambling), male, female and child clothing, transport, leisure, household operation, personal goods and services, and housing (the omitted good in the Engel curve system).¹⁸ Formally, one male-specific good and one female-specific good (and a residual good) are just what we need to identify the main components of the model. The first results we present are based on this simplified setup. However, we consider eight additional goods to improve the efficiency of the estimations. We also suppose that expenditures on vice goods are adult-specific while expenditures on child clothing are child-specific.

4.2 An Informal Look at the Data

The descriptive statistics in Table 1 provide a first overview of the problems we have to address. For one time, let us adopt the traditional Rothbarth way of thinking. If we consider adult-specific goods, we note that the presence of one child reduces the household budget shares devoted to parents' clothing. Expenditures in absolute terms also decrease. For instance, while the average yearly expenditure on male (female) clothing is $613 \in (766 \in)$ in childless couples, it drops to $570 \in (647 \in)$ in couples with one child. The Rothbarth intuition then suggests that, on average, the welfare the parents get out of consumption (at least) declines when the household becomes larger (in spite of a conjoined increase in household total expenditure). The decline in parents' welfare is due to the fraction of total expenditure the parents devote to children.

Yet, the story is not complete. In general, the budget share of all the typically private goods (i.e., food, total clothing and, to some extent, personal goods and services) increases with the size of the household while the budget share of typically public goods (i.e., housing) decreases. The decrease in the budget share devoted to housing when the household size increases is consistent with a reduction of the household living standard only if housing is a luxury good, which is certainly not the case. The simplest interpretation is that economies of scale are substantial, and that these economies of

¹⁸Traditionally, expenditures on housing are not modeled (because these expenditures may be difficult to evaluate for owners). Nonetheless, we believe that expenditure on housing cannot be ignored when economies of scale are considered. In doing so, we must mention that the size of the household may be endogenous in making housing decisions.

TABLE 1: DESCRIPTIVE STATISTICS OF THE SAMPLE

TABLE 1, DES	SCRIPTIVE STA	-11511C5 OF	THE SAMELE	-
	SINGLE WOMEN	SINGLE MEN	CHILDLESS COUPLES	COUPLES WITH ONE CHILD
	_	0.41	0.56	0.21
AGE (MALE) (1=LESS THAN 40)		(0.49)	(0.50)	(0.41)
	0.45	_	0.52	0.14
AGE (FEMALE) (1=LESS THAN 40)	(0.50		(0.50)	(0.35)
_ , , , , , , , , , , , , , , , , , , ,	_	0.37	0.29	0.30
EDUCATION (MALE) (1=TERTIARY)		(0.48)	(0.46)	(0.46)
_ , ,,,	0.46	_	0.34	0.40
EDUCATION (FEMALE) (1=TERTIARY)	(0.50)		(0.47)	(0.39)
	0.90	0.84	0.77	0.74
URBAN RESIDENT	(0.29)	(0.37)	(0.42)	(0.44)
_	0.19	0.20	0.15	0.16
PARIS RESIDENT	(0.39)	(0.40)	(0.36)	(0.36)
	0.78	0.81	0.96	0.97
CAR OWNER	(0.42)	(0.39)	(0.19)	(0.18)
	0.61	0.59	0.42	0.46
House owner	(0.49)	(0.49)	(0.49)	(0.50)
			0.84	0.88
WAGE RATIO	-	_	(0.68)	(1.16)
_	289	304	495	540
TOTAL EXPENDITURE (EUR/WEEK)	(126)	(160)	(255)	(262)
	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	(100)	,	0.49
CHILD'S SEX (1 =GIRL)	-	-	_	(0.50)
				0.47
CHILD'S AGE (1=LESS THAN 2)	-	_	_	(0.50)
BUDGET SHARES:				
_	0.20	0.21	0.23	0.25
FOOD	(0.09)	(0.10)	(0.09)	(0.09)
	0.03	0.05	0.04	0.03
Vices	(0.04)	(80.0)	(0.05)	(0.04)
_	0.11	0.14	0.13	0.13
TRANSPORT	(0.09)	(0.12)	(80.0)	(0.09)
_	0.09	0.10	0.10	0.10
LEISURE GOODS AND SERVICES	(0.07)	(0.10)	(80.0)	(0.07)
	0.05	0.04	0.07	0.07
Household operations	(0.06)	(0.07)	(80.0)	(80.0)
	0.05	0.02	0.05	0.05
PERSONAL GOODS AND SERVICES	(0.06)	(0.04)	(0.06)	(0.05)
	0.41	0.39	0.32	0.31
Housing	(0.14)	(0.14)	(0.11)	(0.11)
BUDGET SHARE (EXCLUSIVE GOODS):				
		0.044	0.023	0.019
MEN'S CLOTHING	-	(0.057)	(0.026)	(0.023)
	0.059	(5.557)	0.029	0.022
WOMEN'S CLOTHING	(0.059)	_	(0.030)	(0.025)
				0.022
CHILD'S CLOTHING	-	_	_	(0.022)
	0.059	0.044	0.052	0.063
TOTAL ON CLOTHING	(0.059)	(0.057)	(0.045)	(0.046)
PROPORTION OF POSTIVIE VALUES:	(0.033)	(0.007)	(0.045)	(0.040)
Men's clothing		0.74	0.74	0.76
Women's CLOTHING	0.85		0.82	0.81
CHILD'S CLOTHING	2.30	_	3.02	0.90
SAMPLE SIZE	 512	 497	 728	418
J LE VILL		-51	, 20	710

scale are not the same for all goods.¹⁹ That is, economies of scale generate a wealth effect that incites consumption of private goods (substituting away from public goods). This mechanism is similar to what is described by Deaton and Paxson (1998).

Some rough estimates of the distribution of total expenditure among household members can be obtained from the aggregate data on the budget shares devoted to clothing exhibited in Table 1. If (1) the adults' utility functions were identical, (2) the elasticity of clothing with respect to total expenditure was unitary (so that the budget shares devoted to clothing were independent of the level of scaled total expenditure) and (3) scale economies were independent of prices (so that the translation functions $d_{i,n}^k(z)$ were equal to zero), then the share received by each individual would be proportionate to the household budget share devoted to clothing for each individual. For instance, for the case of couples with children, the expenditure share of fathers would be equal to $0.30 \simeq 0.19/0.63$, that of mothers to $0.35 \simeq 0.22/0.63$ and that of children to $0.35 \simeq 0.22/0.63$ as well. The latter figure seems to be larger than any realistic measure of the cost of children. Such over-stating, however, may be partly explained by economies of scale in the household. Expenditure on children's clothing that are purely private cannot be compressed.

To check that budget share equations are nonlinear, we perform reduced-form estimations on subsamples for single-person households, two-person households, three-person households, respectively. The budget shares for male and female clothing are first regressed on the dummies for education, age, car ownership, house ownership, urban resident and Paris resident and the log total expenditure. The squared log total expenditure and the Wu-Hausman residuals are then sequentially added to the explanatory variables of the regression. The coefficient corresponding to the main variables, namely the log total expenditure, its squared value, and the Wu-Hausman residuals, are presented in Table 2. For all the subsamples, the coefficients of the linear model are positive, i.e., the budget share for male and female clothing increases when total expenditure increases (thereby implying that, on average, clothing is a luxury good). The coefficients of the quadratic model show that the effect of log total expenditure is decreasing. The same conclusion is obtained by Banks, Blundell and Lewbel (1997).

¹⁹The effect of the household size for the other goods, that are partially private and public, is more complicated to interpret and seems to be the result of opposite forces (and, possibly, externalities).

TABLE 2: ESTIMATED COEFFICENTS OF REDUCED-FORM REGRESSIONS

			LINEAR	QUADRATIC		
	Models	•		WITHOUT WH RESIDUALS	WITH WH RESIDUALS	
			0.136	2.196	2.556	
		LOG EXP	(0.055)	(1.297)	(1.319)	
			_	-1.070	-1.177	
	MALE CLOTHING	SQUARE OF LOG EXP		(0.674)	(0.677)	
		Wu-Hausman	_	_	-0.197	
SINGLE-		RESIDUAL			(0.136)	
PERSONS			0.200	1.122	1.128	
		LOG EXP	(0.070)	(1.269)	(1.273)	
			_	-0.490	-0.499	
	FEMALE CLOTHING	SQUARE OF LOG EXP		(0.675)	(0.685)	
		Wu-Hausman	_	_	0.012	
		RESIDUAL			(0.167)	
COUPLES WITHOUT CHILD			0.045	0.400	0.423	
	MALE CLOTHING	LOG EXP	(0.024)	(0.636)	(0.639)	
		SQUARE OF LOG EXP	-	-0.174	0.177	
				(0.313)	(0.313)	
		Wu-Hausman	_	_	-0.025	
		RESIDUAL			(0.054)	
			0.067	1.295	1.325	
	EELINE OLOTUNG	LOG EXP	(0.028)	(0.732)	(0.735)	
			_	-0.604	-0.609	
	FEMALE CLOTHING	SQUARE OF LOG EXP		(0.360)	(0.360)	
		Wu-Hausman	_	_	-0.028	
		RESIDUAL			(0.063)	
		LOC EVD	0.077	0.793	0.706	
		LOG EXP	(0.032)	(0.814)	(0.809)	
		COLLABE OF LOC EXP	_	-0.349	-0.233	
	MALE CLOTHING	SQUARE OF LOG EXP		(0.397)	(0.397)	
		Wu-Hausman	_	_	-0.196	
COUPLES WITH		RESIDUAL			(0.076)	
CHILD		LOC EVD	0.098	0.611	0.568	
		LOG EXP	(0.033)	(0.882)	(0.878)	
		COLLABE 07: 07 7:	_	-0.250	-0.169	
	FEMALE CLOTHING	SQUARE OF LOG EXP		(0.430)	(0.429)	
		Wu-Hausman	_	_	-0.167	
		RESIDUAL			(0.075)	

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

The results are consistent for all the subsamples which suggest that the budget share equations are indeed nonlinear. Nevertheless, the coefficients are not very precisely estimated. The introduction of Wu-Hausman residuals does not modify notably the estimates.

4.3 Estimations of the Simple Model

To begin with, we consider a three-equation model that consists in the budget share equations for the two adult-goods and the residual good (the latter being omitted from the estimations). In that case, the identification of the structural components of the model is based on a limited number of information so that efficiency may be diminished. The functional form in these primary estimations is thus simplified: all the parameters $\alpha_{i,j}$ and $e_{i,j}$ are set to zero. These simplifications turn out to be necessary, as shown below, to obtain significant results.

In a preliminary step, we want to perform a test of the endogeneity of log total expenditure. The technique consists in directly testing exogeneity through the significance of the 'Wu-Hausman' residuals in the regressions. It appears that the residuals for the square of log expenditure are not jointly significant; hence only the 'Wu-Hausman' residuals for log expenditure are introduced for the basic model.²⁰ The estimated coefficients of the budget share equations for male and female are presented in Table 3. Men and women are characterized by estimated coefficients of the same sign and the same order of magnitude. In particular, the coefficients of log scaled expenditure and its square are significantly different from zero, suggesting that the regularity conditions of Proposition 2 are satisfied; more precisely, the effect of log scaled expenditure on budget shares is positive but decreasing. These figures are compatible with reduced-form estimations reported in Table 2 for the sample of single persons (with lower standard deviations). Finally, as for socio-demographic variables, the coefficients are not precisely estimated; only the coefficient of the dummy variable for car owners is significantly negative at the 5% level.

More interesting for our purpose are the estimated coefficients of the sharing and scal-

 $^{^{20}}$ The residual for log expenditure does not turn to be essential. Only the coefficient in the male budget share equation is significant at the 10% level.

TABLE 3: ESTIMATED COEFFICIENTS OF THE THREE-EQUATION MODEL BUDGET SHARE EQUATIONS

		ARE FOR MALE THING	BUDGET SHARE FOR FEMALI CLOTHING				
CONSTANT	-1.099	(0.437)	-0.795	(0.320)			
ADULT'S AGE (1=LESS THAN 40)	-0.013	(0.010)	-0.014	(0.006)			
ADULT'S EDUCATION (1=TERTIARY)	0.004	(0.009)	0.002	(0.005)			
CAR OWNER	-0.030	(0.006)	-0.011	(0.005)			
House owner	0.005	(0.003)	0.003	(0.003)			
URBAN RESIDENT	-0.001	(0.003)	-0.002	(0.003)			
PARIS RESIDENT	0.006	(0.004)	0.006	(0.003)			
LOG SCALED EXP	2.120	(0.891)	1.679	(0.659)			
LOG SCALED EXP SQUARED	-0.934	(0.459)	-0.808	(0.343)			
DEMOGRAPHIC TRANSLATIONS							
ADULT'S AGE (1=LESS THAN 40)	-0.002	(0.003)	-0.045	(0.035)			
ADULT'S EDUCATION (1=TERTIARY)	0.010	(0.317)	-0.039	(0.036)			

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

ing functions that are shown in Table 4. To begin with, the coefficients of the sharing functions (in particular, those entering the child's exponential function) are not precisely estimated. Nonetheless, some results deserve attention. Firstly, the wage ratio seems to influence the distribution of resources among spouses in the household: an increase in the wife's wage relatively to the husband's entails a shift of the distribution of total expenditure from the husband to the wife. The effect of this variable on the share of total expenditure devoted to the child, on the other hand, is more ambiguous. These results, although intuitive, must be interpreted with caution because, as it will be shown below, their robustness is questionable. Secondly, the coefficient of the dummy variable for the child's sex is significantly different from zero at usual significance levels. More precisely, it turns out that girls receive, on average, a smaller fraction of total expenditure than boys. This result confirms the work of Rose (1999) – and Dunbar, Lewbel and Pendakur (2010) that use a technique similar to ours – showing that discrimination in favor of boys may be revealed by the structure of consumption. Our empirical results differ from these studies in that they are based on

²¹In contrast, Deaton (1989) does not observe any discrimination between boys and girls using data from Côte d'Ivoire and Thailand.

TABLE 4: ESTIMATED COEFFICIENTS OF THE THREE-EQUATION MODEL-SCALING AND SHARING FUNCTIONS

MALE ECONOMIES OF SCALE

FEMALE ECONOMIES OF SCALE

Translations of Budget Shares											
TRANSLATION OF LOG EXPENDITURE CONSTANT -0.583 -0.202 -0.599 -0.243) CONSTANT (IF CHILD) -0.449 -0.180) -0.656 -0.274) SHARES OF TOTAL EXPENDITURE VARIABLES ENTERING FEMALE EXPONENTIAL FUNCTION CONSTANT -0.000 - WOMAN'S AGE (1=LESS THAN 40) -0.048 -0.025 -0.000 -0.000 -0.000 -0.000 -0.000 -0.000 -0.000 -0.000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.0000 -0.00000 -0.00000 -0.00000 -0.000000 -0.00000000	TRANSLATIONS OF BUDGET SHARES										
CONSTANT	CONSTANT	0.004	(0.028)	0.001	(0.011)						
CONSTANT (IF CHILD)	TRANSLATION OF LOG EXPENDITURE										
VARIABLES ENTERING FEMALE EXPONENTIAL FUNCTION	CONSTANT	-0.583	(0.202)	-0.599	(0.243)						
VARIABLES ENTERING FEMALE EXPONENTIAL FUNCTION CONSTANT 0.000 _ CONSTANT (IF CHILD) 0.000 _ WOMAN'S AGE (1=LESS THAN 40) 0.048 (0.033) WOMAN'S EDUCATION 0.014 (0.025) (1=TERTIARY) 0.000 _ WAGE RATIO 0.000 _ CONSTANT -0.467 (0.393) CONSTANT (IF CHILD) -0.022 (0.398) MAN'S AGE (1=LESS THAN 40) -0.043 (0.036) MAN'S EDUCATION (1=TERTIARY) 0.044 (0.031) WAGE RATIO -0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	CONSTANT (IF CHILD)	-0.449	(0.180)	-0.656	(0.274)						
CONSTANT (IF CHILD) 0.000 WOMAN'S AGE (1=LESS THAN 40) -0.048 (0.033) WOMAN'S EDUCATION	SHARES OF TOTAL EXPENDITURE										
CONSTANT (IF CHILD) WOMAN'S AGE (1=LESS THAN 40)	VARIABLES ENTERING FEMALE EXPONENTIAL FUNCTION										
WOMAN'S AGE (1=LESS THAN 40)	CONSTANT	0.000	_								
WOMAN'S EDUCATION (1=TERTIARY) 0.014 (0.025) WAGE RATIO 0.000 _ VARIABLES ENTERING MALE EXPONENTIAL FUNCTION CONSTANT 0.467 (0.393) CONSTANT (IF CHILD) 0.022 (0.398) MAN'S AGE (1=LESS THAN 40) 0.043 (0.036) MAN'S EDUCATION (1=TERTIARY) 0.044 (0.031) WAGE RATIO 0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT 0.463 (0.455) CHILD'S SEX (1=GIRL) 0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	CONSTANT (IF CHILD)	0.000	_								
Constant Constant	Woman's Age (1=Less than 40)	-0.048	(0.033)								
VARIABLES ENTERING MALE EXPONENTIAL FUNCTION CONSTANT -0.467 (0.393) CONSTANT (IF CHILD) -0.022 (0.398) MAN'S AGE (1=LESS THAN 40) -0.043 (0.036) MAN'S EDUCATION (1=TERTIARY) 0.044 (0.031) WAGE RATIO -0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)		-0.014	(0.025)								
CONSTANT (IF CHILD) -0.022 (0.398) MAN'S AGE (1=LESS THAN 40) -0.043 (0.036) MAN'S EDUCATION (1=TERTIARY) 0.044 (0.031) WAGE RATIO -0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	WAGE RATIO	0.000	-								
CONSTANT (IF CHILD) -0.022 (0.398) MAN'S AGE (1=LESS THAN 40) -0.043 (0.036) MAN'S EDUCATION (1=TERTIARY) 0.044 (0.031) WAGE RATIO -0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	VARIABLES ENTERING MALE EXPONENT	TAL FUNCTION									
Man's age (1=less than 40) -0.043 (0.036) Man's education (1=tertiary) 0.044 (0.031) Wage ratio -0.026 (0.009) Variables entering child exponential function Constant -0.463 (0.455) Child's sex (1=girl) -0.197 (0.096) Child's age (1=less than 2) 0.100 (0.076)	CONSTANT	-0.467	(0.393)								
Man's education (1=tertiary) 0.044 (0.031) Wage ratio -0.026 (0.009) Variables entering child exponential function Constant -0.463 (0.455) Child's sex (1=girl) -0.197 (0.096) Child's age (1=less than 2) 0.100 (0.076)	CONSTANT (IF CHILD)	-0.022	(0.398)								
WAGE RATIO -0.026 (0.009) VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	Man's age (1=LESS THAN 40)	-0.043	(0.036)								
VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	Man's education (1=tertiary)	0.044	(0.031)								
CONSTANT -0.463 (0.455) CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	WAGE RATIO	-0.026	(0.009)								
CHILD'S SEX (1=GIRL) -0.197 (0.096) CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	VARIABLES ENTERING CHILD EXPONENT	TIAL FUNCTION									
CHILD'S AGE (1=LESS THAN 2) 0.100 (0.076)	CONSTANT	-0.463	(0.455)								
	CHILD'S SEX (1=GIRL)	-0.197	(0.096)								
WAGE RATIO -0.099 (0.071)	CHILD'S AGE (1=LESS THAN 2)	0.100	(0.076)								
	WAGE RATIO	-0.099	(0.071)								

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

data from a developed country.²² Needless to say, however, the larger proportion of household resources devoted to boys (by comparison with girls) does not necessarily mean that the utility of the former is greater. Indeed boys and girls do not generally benefit from the same level of joint consumption in the household. This result simply says that what the parents spend for a girl is lower than what they spend for a boy.

One last point to mention when examining Table 4 is that the parameters of the scaling functions are significantly different from one, underlining the existence of sizeable economies of scale in the household and invalidating the traditional Rothbarth approach.

To have a better understanding of these results, however, the estimated shares $\eta_{i,n}(z)$ for a representative household, the estimated (normalized) scales $\sigma_{i,n}(z)$, and the estimated overall cost of the child c(z), as well as their standard error and confidence interval, are reported in Table 5. The confidence intervals are useful because these functions are strongly nonlinear. A first suggestive point is that the wife's share of total expenditure is larger than the husband's (even if these differences are not significant because of large standard deviations). For a representative couple without children, the wife's share amounts to about 0.62 with a standard error of 0.09. To take a comparison point, the average wife's share estimated by Browning, Chiappori and Lewbel (2008), with Canadian data, is in excess of 0.60. Similarly, Bargain and Donni (2010), using data from Ireland, obtain estimations that are comprised between 0.51 and 0.63. Nonetheless, Lewbel and Pendakur (2008), using Canadian data too, obtain estimations that are notably smaller (depending on the model they consider the average wife's share varies between 0.36 and 0.46). The natural interpretation – ignoring for a while that the equal sharing hypothesis cannot be statistically rejected - is that women have the leading voice in the household. Note that each household budget share is the weighted average of the individual budget shares, with weights being equal to individual shares of total expenditure. If the wife's share is greater than a half, then the behavior of couples resembles more that of single women than that of single men. It may be the result of self-selection at the time of marriage – the men

²²Evidence from developed countries is rare and inconclusive. For instance, Lundberg and Rose (2004) estimate Engel curves on U.S. data and do not discern a clear phenomenon of discrimination between boys and girls.

TABLE 5: ESTIMATED ECONOMIES OF SCALE AND SHARES OF TOTAL EXPENDITURE FOR A REPRESENTATIVE HOUSEHOLD OBTAINED WITH THE THREE EQUATION MODEL

	EXPECTED	STANDARD	95%-CONFIDENCE INTERVAL			
	VALUE	DEVIATION	LOWER BOUND	UPPER BOUND		
WIFE'S SHARE OF TOTAL EXPENDITURE (NO CHILD)	0.616	0.090	0.461	0.758		
WIFE'S SHARE OF TOTAL EXPENDITURE (ONE BOY)	0.387	0.073	0.271	0.512		
WIFE'S SHARE OF TOTAL EXPENDITURE (ONE GIRL)	0.402	0.075	0.281	0.531		
HUSBAND'S SHARE OF TOTAL EXPENDITURE (NO CHILD)	0.383	0.090	0.241	0.538		
HUSBAND'S SHARE OF TOTAL EXPENDITURE (ONE BOY)	0.377	0.099	0.220	0.547		
HUSBAND'S SHARE OF TOTAL EXPENDITURE (ONE GIRL)	0.391	0.101	0.228	0.564		
BOY'S SHARE OF TOTAL EXPENDITURE	0.235	0.092	0.105	0.406		
GIRL'S SHARE OF TOTAL EXPENDITURE	0.205	0.093	0.081	0.382		
BOY'S OVERALL COST	0.036	0.052	-0.032	0.131		
GIRL'S OVERALL COST	-0.002	0.055	-0.072	0.100		
WIFE'S NORMALIZED ECONOMIES OF SCALE (NO CHILD)	1.649	0.186	1.313	1.871		
WIFE'S NORMALIZED ECONOMIES OF SCALE (ONE BOY)	1.792	0.096	1.623	1.936		
WIFE'S NORMALIZED ECONOMIES OF SCALE (ONE GIRL)	1.845	0.105	1.660	2.003		
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (NO CHILD)	1.977	0.130	1.776	2.196		
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (ONE BOY)	1.830	0.123	1.636	2.020		
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (ONE GIRL)	1.885	0.139	1.667	2.102		

NOTE: THE REPRESENTATIVE HOUSEHOLD IS COMPOSED OF ADULTS AGED UNDER 40 WITHOUT TERTIARY EDUCATION. IF THEY HAVE A CHILD, IT IS A BOY ABOVE 2. WAGE RATIO IS EQUAL TO ONE. STANDARD DEVIATIONS ARE COMPUTED BY BOOTSTRAP.

that decide to marry have preferences more comparable to that of unmarried women – or changes in tastes after the marriage. One last point which is really interesting in the results of Table 5 is that, for a representative couple with one child, the wife's and the husband's shares are approximately the same. In other words, the mother seems to bear the largest fraction of child expenditures in the household.

Now let us consider the share of total expenditure devoted to the child. For a representative household, it amounts to about 23% of total expenditure for a boy and to 20% for a girl. Studies based on more traditional Rothbarth approaches obtain estimations of expenditures for children that are usually lower: about 15% of household total expenditure in Gronau (1991), using US data; between 11% and 18% in Deaton, Ruiz-Castillo and Thomas (1989) with Spanish data; and between 9% and 13% in Tsakloglou (1991) with Greek data. Our estimations are not very indicative, though, because confidence intervals are large. Moreover, the "overall cost" of a child, which is also presented in Table 5, turns out to be rather small. For instance, for a boy, it is equal to 0.036, with an upper bound for the 95% confidence interval at 0.131. That is to say, the supplement of income necessary to maintain the level of welfare of parents after the birth of a boy is equal at most to 13% of total expenditure; and it is probably lower. These small overall costs may be explained by important economies of scale in the household, as we shall see.

To show this, the scales $s_{i,n}(z)$ (not reported in tables) can be computed. If these scales are to be interpreted as reflecting joint consumption, they should, in principle, lie between $\eta_{i,n}(z)$ (complete jointness of consumption) and 1.00 (purely private consumption) for a childless couple. Also it turns out that the estimates of scales $s_{i,n}(z)$ for childless couples are reasonable in magnitude, but small. To take an example, the women's scale for a representative childless couple is equal to 0.70; so the cost of living for a woman with a man is 70% of the cost she would experience should she live alone. One naturally expects that economies of scales increase (i.e., deflators decrease) in families with one child compared to childless couples. Nevertheless, the magnitude of the deflators is difficult to interpret as household members consume only a fraction of total expenditure. That is why the normalized measures of scale economies $\sigma_{i,n}(z)$ are presented in the lower panel of Table 5. They amount to 1.98 (1.65) for a man (woman) living in a couple without. They are of the same order for households with

children, that is, 1.83 (1.79) when the child is a boy and 1.89 (1.84) when this is a girl. Overall, these values are remarkably large. Indeed, let us recall that, in the limit case where $\sigma_{i,n}(z) = 2$ all the goods consumed by spouses can be assimilated to purely public goods. Hence joint consumption among households is certainly important.²³ As a consequence, it can be shown that indifference scales for spouses (not reported here) are close to one. For instance, the household income must be multiplied by no more than 1.15 for a woman to obtain the same level of welfare in a couple with a boy than when alone. Such woman, if living alone, would need $0.87 \approx 1/1.15$ of the couple's income to reach the same indifference curve as when in couple. This is clearly larger than a half because single persons would not benefit from the important scale economies.

4.4 Estimations of the Complete Model

The estimates obtained with the simple model, although based on quite restrictive functional forms, are not sufficiently precise. Therefore we consider here a more complete model including ten budget share equations and a completely general specification: all the parameters of the functional form discussed in Section 3.1 are now free.

Since each additional equation generates overidentifying restrictions, the structural components of the model are expected to be more precisely estimated in the complete model. The Hausman-Wu residuals for log total expenditure and its square are introduced in each budget share equation (except that for male and female clothing which includes only one residual). The estimated coefficients of these residuals are not reported here but it turns out that the majority of them are significantly different from zero. Exogeneity of log total expenditure is clearly rejected by the data.

²³By comparison, Browning, Chiappori and Lewbel (2008) obtain economies of scale (aggregated over the household using a measure different from ours) comprised between 1.27 and 1.41. Bargain and Donni (2010) obtain a confirmation of the present measures of scale economies when using data for Ireland. Using US data, Nelson (1989) estimates the economies of scale in the household for each good (including housing). Her estimations are very large. In particular, economies of scale for housing seem larger than what they would be in the case of pure joint consumption. She explains it by increasing returns in household production

TABLE 6: ESTIMATED COEFFICIENTS OF THE COMPLETE MODEL - BUDGET SHARE EQUATIONS OF ADULTS

	Fo	Ю	Vı	CE	CLOT	THING		LEISURE GOODS AND TRANSPORT SERVICES					USEHOLD ERATIONS	
	MAN	Woman	Man	Woman	Man	Woman	MAN	Woman	Man	Woman	Man	Woman	MAN	Woman
CONCTANT	-1.185	-4.560	1.011	3.630	-0.381	-0.822	-0.622	0.750	0.399	-2.557	0.164	1.502	3.294	3.231
CONSTANT	(1.241)	(1.391)	(0.656)	(0.688)	(0.241)	(0.365)	(0.948)	(1.030)	(1.053)	(1.310)	(0.549)	(0.899)	(1.228)	(0.994)
A	0.024	0.032	0.005	-0.005	0.004	0.010	0.002	-0.016	-0.030	-0.011	0.000	-0.002	0.017	0.000
ADULT'S AGE	(0.011)	(800.0)	(0.006)	(0.003)	(0.004)	(0.003)	(0.010)	(0.006)	(0.011)	(0.007)	(0.004)	(0.007)	(0.010)	(0.006)
ADULT'S	0.000	0.021	-0.017	-0.009	0.005	0.006	0.021	0.000	0.003	0.004	0.000	-0.019	0.010	0.002
EDUCATION	(800.0)	(0.009)	(0.005)	(0.004)	(0.003)	(0.003)	(800.0)	(0.007)	(800.0)	(0.009)	(0.003)	(800.0)	(800.0)	(0.006)
CAROWNER	-0.030	-0.021	-0.014	0.002	-0.027	-0.013	-0.011	0.005	0.121	0.073	-0.007	0.004	-0.020	-0.009
CAR OWNER	(0.011)	(0.010)	(800.0)	(0.005)	(0.005)	(0.005)	(0.010)	(0.007)	(0.012)	(0.010)	(0.004)	(0.007)	(800.0)	(0.007)
House owner	0.005	0.002	0.018	0.007	0.002	0.004	0.011	0.003	0.016	0.025	0.001	0.019	-0.007	-0.008
House owner	(0.009)	(800.0)	(0.005)	(0.004)	(0.003)	(0.003)	(800.0)	(0.006)	(0.009)	(800.0)	(0.004)	(0.005)	(0.006)	(0.006)
Linner present	-0.001	0.003	0.000	0.003	-0.001	-0.002	0.011	0.006	-0.008	-0.024	0.002	-0.001	0.016	0.002
URBAN RESIDENT	(0.011)	(0.010)	(0.007)	(0.005)	(0.003)	(0.003)	(0.010)	(800.0)	(0.011)	(0.010)	(0.004)	(0.006)	(0.007)	(0.007)
Dunio provincia	0.011	0.008	-0.018	-0.009	0.007	0.006	0.006	0.005	0.003	0.001	0.005	-0.018	-0.014	-0.009
PARIS RESIDENT	(0.010)	(0.010)	(0.005)	(0.004)	(0.004)	(0.004)	(0.009)	(0.007)	(0.010)	(0.009)	(0.004)	(0.006)	(0.007)	(0.007)
LOG SCALED	3.124	10.376	-2.063	7.661	0.825	1.676	1.199	-1.743	0.488	5.675	0.374	-3.756	6.876	-6.933
EXPENDITURE	(2.665)	(2.989)	(1.442)	(1.487)	(0.554)	(0.756)	(2.062)	((2.206)	(2.288)	(2.801)	(1.181)	(1.921)	(2.581)	(2.141)
SQUARE OF LOG	-1.725	-5.649	1.118	4.068	.370	-0.775	-0.469	1.093	0.123	-3.061	-0.188	2.343	3.496	3.780
SCALE EXPENDITURE	(1.435)	(1.609)	(0.797)	(0.804)	(0.311)	(0.392)	(1.127)	(1.183)	(1.249)	(1.499)	(0.636)	(1.028)	(1.358)	(1.154)
DEMOGRAPHIC TRAI	NSLATION													
HUSBAND'S AGE (1=LESS THAN 40) IN ALL MEN'S EQUATIONS:		0.044				HUSBAND'S EDUCATION (1=TERTIARY) IN ALL MEN'S				0.026				
			(0.019)			EQUATIONS:					(0.015)			
WIFE'S AGE (1=LES	s than 40) i	N ALL WOME	n's	0.004					Winnia no					-0.001
EQUATIONS:				(0.006)		Wife's EDUCATION (1=TERTIARY) IN ALL WOMEN'S EQUATIONS:			(0.008)					

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

TABLE 7: ESTIMATED COEFFICIENTS OF THE COMPLETE MODEL - BUDGET SHARE EQUATIONS OF CHILDREN

	FOOD	CLOTHING	LEISURE GOODS AND SERVICES	TRANSPORT	PERSONAL GOODS AND SERVICES	HOUSEHOLD OPERATIONS
CONSTANT	0.662	0.030	0.030	-0.224	-0.321	0.704
	(0.333)	(0.067)	(0.283)	(0.348)	(0.317)	(0.386)
CHILD'S AGE	-0.267	-0.058	-0.142	-0.110	-0.078	0.285
(1=LESS THAN 2)	(0.298)	(0.101)	(0.257)	((0.359)	(0.341)	(0.459)
CHILD'S SEX (1=GIRL)	0.344	0.318	0.309	0.765	0.852	-1.232
	(2.002)	(0.101)	(0.257)	(0.359)	(0.341)	(0.459)
LOG SCALED EXPENDITURE	-0.110	-0.160	-0.252	-0.477	-0.588	0.694
	(0.854)	(0.213)	(0.884)	(0.689)	(0.554)	(0.734)
SQUARE OF LOG SCALE EXPENDITURE	-0.067	-0.024	0.098	-0.001	0.050	0.004
	(0.421)	(0.149)	(0.610)	(0.055)	(0.315)	(0.052)
DEMOGRAPHIC TRANSLATION						
CHILD'S AGE (1=LESS THAN 2) IN ALL CHILD'S EQUATIONS:		0.413		CHILD'S SEX (1	=GIRL) IN ALL	-1.845
		(0.526)		CHILD'S EQUATIONS:		(10.247)

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

One advantage of the general model is that the hypothesis according to which the parameters for singles and couples are the same can be tested. To do that, we construct a more general model where the parameters b_i^k and c_i^k of the budget shares (others than for male and female clothing) may be different for singles and for persons living in couple. We then make a NR-squared test (accounting for the heteroskedasticity of error terms across goods). The number of restrictions is equal to 24 (i.e., four restrictions per equation). The R^2 of the auxiliary regression amounts to 0.0025 and the total number of observations to 16,600 (i.e., the number of households in the sample multiplied by the number of goods). The NR-squared statistic, which follows a Chi-squared distribution under the null hypothesis, is then equal to 41.50 with 24 degrees of freedom. The null hypothesis is rejected at the 5% level, but not at the 1% level. In view of the large number of observations, supposing that the parameters for single persons and for persons living in couple are the same seems to be a reasonable approximation. This preliminary step allows us to go further in the analysis.

The general specification has, all in all, 251 parameters (out of which 98 are signifi-

cantly different from zero at the 10% level). The estimated parameters of the male and female budget share equations are reported in Table 6. Some comments are in order. Firstly, the estimated parameters of the budget share equations for male and female clothing are of the same order as those obtained with the simple model (reported in Table 3), but standard deviations are generally lower. Going one step further, it turns out that, for all the budget share equations, the estimated parameters are similar to those obtained from the sample of single-person households.²⁴ Secondly, the effect of the socio-demographic variables for men and women are consistent between them. In particular, several dummies have the same significant effect on budget share for both men and women: the dummy for age has a positive effect on the food budget shares; the dummy for education has a negative effect on the vice budget shares; the dummy for car owners has a negative effect on the food budget shares, on the male and female clothing budget shares, and a positive effect on the transport budget shares; the dummy for Paris resident has a negative effect on the vice budget shares; the dummy for house owner has a positive effect on the transport budget shares and on the vice budget shares. The estimated parameters of the child's budget share equations are presented in Table 7 but are unfortunately not precisely estimated. The slopes of the child's budget shares with respect to log total expenditure do not allow inferring the nature of goods (luxury or necessary) even though this information is identifiable, as explained in the theoretical section.

The estimates of the coefficients of the sharing and scaling functions are reported in Table 8. Regarding the distribution of resources between adults, the first stable result is that living with an older partner reduces the share of total expenditure that a person receives. It seems also that the level of education of the wife has a negative effect on her share, but this effect is not very significant. The distribution factor, i.e., the wage ratio, does not significantly influence the intrahousehold distribution of resources, contrary to what was observed with the simple model. The sign of the estimated coefficient in both models is, however, the same.²⁵ One possible explanation is that the significant

²⁴To save on space, the estimates obtained with the sample of single-persons are not reported here.

²⁵Whether she works or not may be the margin that matters in this respect, more than differences in productivities. As explained before, we focus here on two-earner couples and do not have variation in female labor market participation; see Zamora (2008) on this issue.

Table 8: Estimated coefficients of the complete model - Scaling and sharing functions

,	MALE ECONOMIES OF SCALE		FEMALE ECONOMIES OF SCALE			
TRANSLATIONS OF BUDGET SHARES (CONSTANTS)						
FOOD	-0.554	(0.365)	0.517	(0.297)		
VICE	0.038	(0.049)	0.026	(0.040)		
CLOTHING	0.032	(0.014)	0.001	(0.007)		
LEISURE GOODS AND SERVICES	-0.078	(0.267)	0.125	(0.228)		
TRANSPORT	0.319	(0.299)	-0.294	(0.254)		
PERSONAL GOODS AND SERVICES	0.291	(0.227)	-0.202	(0.185)		
HOUSEHOLD OPERATIONS	-0.765	(0.324)	0.728	(0.237)		
TRANSLATION OF LOG EXPENDITU	<u>RE</u>					
CONSTANT	-0.528	(0.120)	-0.633	(0.144)		
CONSTANT (IF CHILD)	-0.940	(0.149)	-0.725	(0.197)		
ADULT'S AGE	-0.005	(0.012)	-0.021	(0.016)		
ADULT'S EDUCATION	-0.029	(0.016)	0.014	(0.013)		
	SHARES OF TO	TAL EXPENDITURE				
VARIABLES ENTERING FEMALE EX	PONENTIAL FUN	CTION				
CONSTANT	0.000	-				
CONSTANT (IF CHILD)	0.000	_				
WOMAN'S AGE(1=LESS THAN	-0.048	(0.019)				
40) Woman's Education	0.006	(0.013)				
(1=TERTIARY)	0.000	(0.013)				
WAGE RATIO	0.000	_				
VARIABLES ENTERING MALE EXPONENTIAL FUNCTION						
CONSTANT	0.217	(0.261)				
CONSTANT (IF CHILD)	0.047	0.285				
Man's age (1 =LESS THAN 40)	-0.066	(0.024)				
Man's Education (1=TERTIARY)	-0.057	(0.023)				
WAGE RATIO	-0.004	(0.005)				
VARIABLES ENTERING CHILD EXPONENTIAL FUNCTION						
CONSTANT	-0.354	(0.280)				
CHILD'S SEX (1=GIRL)	-0.200	(0.073)				
CHILD'S AGE (1=LESS THAN 2)	0.040	(0.053)				
WAGE RATIO	0.006	(0.007)				

NOTE: STANDARD DEVIATIONS ARE IN PARENTHESES.

TABLE 9: ESTIMATED ECONOMIES OF SCALE AND SHARES OF TOTAL EXPENDITURE OBTAINED WITH THE COMPLETE MODEL

	EXPECTED STANDARD		95%-CONFIDENCE INTERVAL	
	VALUE	DEVIATION	Lower Bound	UPPER BOUND
WIFE'S SHARE OF TOTAL EXPENDITURE (NO CHILD)	0.554	0.063	0.447	0.657
HUSBAND'S SHARE OF TOTAL EXPENDITURE (NO CHILD)	0.445	0.063	0.342	0.552
WIFE'S SHARE OF TOTAL EXPENDITURE (ONE BOY)	0.358	0.050	0.277	0.443
HUSBAND'S SHARE OF TOTAL EXPENDITURE (ONE BOY)	0.375	0.065	0.271	0.487
BOY'S SHARE OF TOTAL EXPENDITURE	0.265	0.053	0.183	0.360
WIFE'S SHARE OF TOTAL EXPENDITURE (ONE GIRL)	0.375	0.052	0.291	0.464
HUSBAND'S SHARE OF TOTAL EXPENDITURE (ONE GIRL)	0.394	0.069	0.283	0.511
GIRL'S SHARE OF TOTAL EXPENDITURE	0.230	0.056	0.146	0.330
Boy's overall cost	0.053	0.027	0.012	0.100
GIRL'S OVERALL COST	0.004	0.026	-0.034	0.051
WIFE'S NORMALIZED ECONOMIES OF SCALE (NO CHILD)	1.847	0.060	1.739	1.925
WIFE'S NORMALIZED ECONOMIES OF SCALE (ONE BOY)	1.854	0.047	1.770	1.925
WIFE'S NORMALIZED ECONOMIES OF SCALE (ONE GIRL)	1.921	0.051	1.832	1.997
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (NO CHILD)	1.693	0.089	1.545	1.837
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (ONE BOY)	1.619	0.103	1.440	1.775
HUSBAND'S NORMALIZED ECONOMIES OF SCALE (ONE GIRL)	1.669	0.108	1.482	1.831

NOTE: THE REPRESENTATIVE HOUSEHOLD IS COMPOSED OF ADULTS AGED UNDER 40 WITHOUT TERTIARY EDUCATION. IF THEY HAVE A CHILD, IT IS A BOY ABOVE 2. WAGE RATIO IS EQUAL TO ONE. STANDARD DEVIATIONS ARE COMPUTED BY BOOTSTRAP.

effect observed in the budget share equations for clothing is due to the endogeneity of wages. Indeed, if higher-paid jobs require more expensive work clothing, then the incomes of the wife and husband will enter the budget share equations even if we condition on individual shares. Finally, concerning the expenditure devoted to the child, it appears that boys are favored over girls. This confirms the conclusion drawn with the simple model.

The estimated shares of total expenditure for a representative household, the estimated (normalized) scales, and the estimated overall cost of the child, as well as their standard error and their confidence interval, are reported in Table 9. Overall, the results obtained with the simple model are confirmed, but the standard deviations are lower. First, the estimations of the shares of total expenditure are comparable to those previously obtained. In particular, the average share devoted to the child amounts to 0.27 for a boy and 0.23 for a girl. Second, the overall cost of a boy is around 5% of household total expenditure while the overall cost of a girl is close to zero. Again these values seem to be very small. Third, the estimation of the normalized measures of scale economies confirm that joint consumption is important. For a man and a woman living in a childless couple, the normalized measures are 1.694 and 1.848 respectively. They are of 1.619 and 1.854 if the husband and the wife have one boy and of 1.669 and 1.921 if they have a girl. To summarize, the estimations of the main structural components are similar to those obtained with the simple model in spite of the fact that these two models are based on quite different sets of maintained assumptions.

5 Conclusion

In this paper, we have suggested a new method to estimate the cost of children that generalizes the more conventional Rothbarth method. This approach is consistent with the existence of economies of scale and parental bargaining. Our empirical results on French data indicate that the parents' expenditures made for children living in the household are relatively important. They amount to about 23 - 27% of household total expenditure. Nevertheless, the economies of scale in multi-person households turn out to be very large as well, so that the income necessary to compensate parents after the birth of a first child is after all very modest. In fact the estimations of

this alternative measure of the cost of a child which takes into account economies of scale are unexpectedly small – around 5% of household total expenditure. This result certainly needs more scrutiny in future research. Another important empirical contribution of this paper is that expenditures made by parents for boys seem to be larger than for girls, suggesting the existence of some discrimination within the household (even if other explanations for this result can be envisaged). This is one of the rare contributions that underlines this phenomenon in a developed country.

Future research should generalize the approach suggested in this paper. In particular, the cost of children is measured only for families with a single child. It is necessary to consider more diversified demographic structures in order to measure how the overall cost of children changes when the size of the household increases. Moreover, the time devoted by parents to child care certainly represents a significant fraction of non-market time. It should be incorporated in our model. In particular, the mothers' part-time participation in the labor market, which is generally associated with the provision of child care, should be modeled to define a more complete concept of the cost of children.

Appendix: Further Identification Results

The model is largely overidentified as it will be clear below. Overidentification could, in principle, be used to relax some of the controversial postulate upon which the model is based. One of the most restrictive of them is the assumption that the sharing functions are independent of log total expenditure x. To show how we can get it without such a constraint, we have first to understand why the model is overidentified. Let us take the expenditure share equation for one adult specific good k_i for the case of childless couples and suppose that socio-demographic variables are constant $z = \bar{z}$, that is,

$$W_2^{k_i}(x,\bar{z}) = \eta_{i,2}(\bar{z}) \cdot \left[d_{i,2}^{k_i}(\bar{z}) + w_i^{k_i} \left(x + \eta_{i,2}(\bar{z}) - \log s_{i,2}(\bar{z}), \bar{z}_i \right) \right], \tag{21}$$

where i=1 or 2, $d_{i,2}^{k_i}(\bar{z})$, $\eta_{i,2}(\bar{z})$ and $s_{i,2}(\bar{z})$ are constants and $W_2^{k_i}(\cdot,\bar{z})$ and $w_i^{k_i}(\cdot,\bar{z}_i)$ are one-variable functions. The latter functions are supposed to be observed (i.e., estimated from data) as is explained in the main text. Therefore, when x varies within its domain, expression (21) can be seen as a continuum of equations in $d_{i,2}^k(\bar{z})$,

 $\eta_{i,2}(\bar{z})$ and $s_{i,2}(\bar{z})$ for any value of \bar{z} . To be more concrete, let us consider three arbitrary values of log total expenditure, i.e., $\{x_1, x_2, x_3\}$. This provides a system of three equations in three unknowns:

$$W_2^k(x,\bar{z}) = \eta_{i,2}(\bar{z}) \cdot \left(d_{i,2}^k(\bar{z}) + w_i^k \left(x_T + \eta_{i,2}(\bar{z}) - \log s_{i,2}(\bar{z}), \bar{z}_i \right) \right),$$

where T = 1, 2, 3, that can, in general, be solved. Hence, the functions $d_{i,2}^k(\bar{z})$, $\eta_{i,2}(z)$ and $s_{i,2}(z)$ are generically identified for any value of the vector \bar{z} . The same reasoning applies in the case of couples with children, thereby showing that children's cost is identified. Note that this result is only generic in the sense that it is 'almost always' satisfied in the traditional mathematical sense. However it may be violated for particular forms of preferences. For instance, it is clear that the structural components are not identifiable if the budget share equation for good k is linear in its first argument. This explains the regularity conditions that are used in Proposition 2. Finally, since only three values $\{x_1, x_2, x_3\}$ of log total expenditure are, in principle, sufficient for identifying the main structural components, the model is largely over-identified.

From the previous reasoning, one can straightforwardly conclude that the structural components of the model are still identified when there is only one adult-specific good, that is, a good which is not specific to the wife or the husband. Indeed, the budget share equation for the adult-specific good in a household of type n can be written as:

$$W_n^k(x,\bar{z}) = D_n^k(\bar{z}) + \sum_{i=1}^2 \eta_{i,n}(\bar{z}) \cdot \left[w_i^k \left(x + \eta_{i,n}(\bar{z}) - \log s_{i,n}(\bar{z}), \bar{z}_i \right) \right], \tag{22}$$

where $D_n^k(\bar{z}) = \sum_{i=1}^2 \eta_{i,n}(\bar{z}) \cdot d_{i,n}^k(\bar{z})$. This represents a continuum of equations in $D_{i,n}^k(\bar{z})$, $\eta_{1,n}(\bar{z})$, $\eta_{2,n}(\bar{z})$, $s_{1,n}(\bar{z})$ and $s_{2,n}(\bar{z})$ for any value of \bar{z} . Nonetheless, even if identification is theoretically possible, it may be very difficult to estimate these constants with any precision from real data.

Let us come back to the initial case of two adult-specific goods and consider a generalization of the model where $\eta_{i,n} = \eta_{i,n}(x, \bar{z})$. In that case, the budget share equations for adult-specific goods become:

$$W_n^{k_i}(x,\bar{z}) = \eta_{i,n}(x,\bar{z}) \cdot \left[d_{i,n}^{k_i}(\bar{z}) + w_i^{k_i}(x + \eta_{i,n}(x,\bar{z}) - \log s_{i,n}(\bar{z}), \bar{z}_i) \right],$$

with i=1 or 2. Then inverting this equation with respect to $\eta_{i,n}(x,\bar{z})$ (under the assumption that such an inversion is possible) gives:

$$\eta_{i,n}(x,\bar{z}) = \Phi_{i,n}^{k_i}\left(x, W_n^{k_i}(x,\bar{z}), d_{i,n}^{k_i}(\bar{z}), s_{i,n}(\bar{z}), \bar{z}_i\right), \tag{23}$$

where $\Phi_{i,n}^{k_i}(\cdot)$ is a known function. That is to say, each sharing function $\eta_{i,n}(x,\bar{z})$ is identified up to two constants $d_{i,n}^{k_i}(\bar{z})$ and $s_{i,n}(\bar{z})$, with i=1,2. To obtain a complete identification, additional information is necessary. For instance, let us suppose that we have at our disposal an additional adult-specific good k_0 , that is,

$$W_n^{k_0}(x,\bar{z}) = \sum_{i=1}^2 \eta_{i,n}(x,\bar{z}) \cdot \left[d_{i,n}^{k_0}(\bar{z}) + w_i^{k_0} \left(x + \eta_{i,n}(x,\bar{z}) - \log s_{i,n}(\bar{z}), \bar{z}_i \right) \right]. \tag{24}$$

Incorporating (23) in (24), we obtain a continuum of equations in $d_{1,n}^{k_0}(\bar{z})$, $d_{2,n}^{k_0}(\bar{z})$, $d_{1,n}^{k_0}(\bar{z})$, $d_{2,n}^{k_0}(\bar{z})$, $s_{1,n}(\bar{z})$ and $s_{2,n}(\bar{z})$ for any value of \bar{z} . Again, if this continuum of equations is solved for any value of \bar{z} , the functions $\eta_{1,n}(x,z)$ and $\eta_{2,n}(x,z)$ can be generically identified.

Finally, using the same reasoning, it would be possible to show that, with a sufficiently large system of budget share equations and with adult-specific goods, the structural components of the model are generically identified in the more general case where the scaling functions can be written as $s_{i,n} = s_{i,n}(x, \mathbf{z})$, provided that the elasticities $d_{i,n}^k(\mathbf{z})$, for k = 1, ..., K, are independent of x.

References

- Banks, Jeffrey, Richard W. Blundell, Arthur Lewbel (1997), "Quadratic Engel Curves and Consumer Demand", Review of Economics and Statistics, vol. 79, No. 4, pp. 527–539.
- [2] Bargain, Olivier and Olivier Donni (2008), "Indirect Taxation, Targeting and Child Poverty", Working Paper, University College of Dublin.
- [3] Bargain, Olivier, Olivier Donni and Monnet Gbakou (2010), "The Measurement of Child Costs: Evidence from Ireland", *Economic and Social Review*.

- [4] Blackorby, Charles and David Donaldson. (1993), "Adult-equivalence scales and the economic implementation of interpersonal comparisons of well-being", Social Choice and Welfare, vol. 10, pp. 335-361.
- [5] Blundell, Richard W. and Arthur Lewbel (1991), "The Information Content of Equivalence Scales", *Journal of Econometrics*, vol. 50, pp. 49-68.
- [6] Blundell, Richard W. and Jean-Marc Robin (1999), "Estimation in Large and Disaggregated Demand Systems: An Estimator for Conditionally Linear Systems", Journal of Applied Econometrics, vol. 14, pp. 209–232
- [7] Blundell, Richard W. and Jean-Marc Robin (2000), "Latent Separability: Grouping Goods Without Weak Separability", *Econometrica*, vol. 68, No. 1, pp. 53-84.
- [8] Blundell, Richard W., Pierre-André Chiappori and Costas Meghir (2005), "Collective labor supply with children", Journal of Political Economy, vol. 113, No. 6, pp. 1277–1306
- [9] Bourguignon, François (1999), "The cost of children: May the collective approach to household behaviour help?" Journal of Population Economics, vol. 12, No. 4, pp. 503–522.
- [10] Bourguignon, François, Martin Browning and Pierre-André (2008), "Efficient Intra-Household Allocations and Distribution Factors: Implications and Identification", Review of Economics Studies, vol. 76, pp. 503-528.
- [11] Browning, Martin (1992), "Children and Household Economic Behavior", *Journal of Economic Literature*, vol. 30, pp. 1434-1475.
- [12] Browning, Martin and Pierre-André Chiappori (1998), "Efficient Intrahousehold Allocations: A General Characterization and Empirical Tests", Econometrica, vol. 56, pp. 1241–1278.
- [13] Browning, Martin, Pierre-André Chiappori and Arthur Lewbel (2008), "Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power", Working Paper, Boston College.
- [14] Chiappori, Pierre-André and Olivier Donni (2010), "Non-Unitary Models of

- Household Behavior: A Survey of the Literature". In: A. Molina (eds), *Household Economic Behaviors*, Berlin: Springer.
- [15] Dauphin, A., A-R. El Lahga, B. Fortin and G. Lacroix (2008): "Are Children Decision-Makers Within the Household?", IZA working paper, 3728.
- [16] Deaton, Angus (1997), The Analysis of Household Surveys: A Microeconometric Approach to Development Policy, Baltimore: The John Hopkins University Press.
- [17] Deaton, Angus, Javier Ruiz-Castillo, Duncan Thomas (1989), "The Influence of Household Composition on Household Expenditure Patterns: Theory and Spanish Evidence", Journal of Political Economy, vol. 97, pp.179-200.
- [18] Deaton, Angus and Christina Paxson (1998), "Economies of Scale, Household Size, and the Demand for Food", Journal of Political Economy, vol. 106, pp. 97-930.
- [19] Deaton, Angus and John Muellbauer (1986), "On Measuring Child Costs: With Applications to Poor Countries", Journal of Political Economy, vol. 94, No. 4, pp. 720-744.
- [20] Dunbar, Geoffrey, Arthur Lewbel and Krishna Pendakur (2010), "Children's Resources in Collective Households: Identification, Estimation and an Application to Child Poverty in Malawi", Workin Paper, Simon Fraser University & Boston College.
- [21] Donni, Olivier (2008), "Collective Models of Household Behavior".In: L. Blume et S. Durlauf (eds). The New Palgrave Dictionary of Economics, 2nd Edition. Londres: Palgrave McMillan.
- [22] Donni, Olivier (2009), "A Simple Approach to Investigate Intrahousehold Allocation of Private and Public Goods", The Review of Economics and Statistics, vol. 91, No. 3, pp. 617–628.
- [23] Gronau, Reuben (1988), "Consumption Technology and the Intrafamily Distribution of Resources: Adult Equivalence Scales Reexamined", Journal of Political Economy, vol. 96, No. 6, pp. 1183-1205.

- [24] Gronau, Reuben (1991), "The Intrafamily Allocation of Goods How to Separate the Adult from the Child", Journal of Labor Economics, vol. 9, No. 3, pp. 207– 235.
- [25] Lazear, Edward P. and Robert T. Michael (1988), Allocation of Income within the Household, Chicago: University of Chicago Press.
- [26] Lewbel, Arthur (1989), "Household Equivalence Scales and Welfare Comparisons", *Journal of Public Economics*, vol. 39, pp. 377-391.
- [27] Lewbel, Arthur (1991), "Cost of Characteristics Indices and Household Equivalence Scales", European Economic Review, vol. 35, pp.1277-1293.
- [28] Lewbel, Arthur (1997), "Consumer Demand Systems and Household Equivalence Scales". In: M.H. Pesaran and P. Schmidt (ed), Handbook of Applied Econometrics, Volume II: Microeconomics, Oxford: Blackwell Publishers Ltd.
- [29] Lewbel, Arthur, (2003), "Calculating Compensation in Cases of Wrongful Death," Journal of Econometrics, vol. 113, pp. 115-128.
- [30] Lewbel, Arthur and Krishna Pendakur (2008), "Estimation of Collective House-hold Models with Engel Curves", Journal of Econometrics, vol. 148, pp. 350-358.
- [31] Lunberg, Shelly and Elaina Rose (2004), "Investments in Sons and Daughters: Evidence from the Consumer Expenditure Survey" in: Ariel Kalil and Thomas DeLeire (eds), Family Investments in Children's Potential: Resources and Parenting Behaviors That Promote Success, Mahwah, N.J.: Lawrence Erlbaum Associates Inc.
- [32] Matzkin, Rosa L. (2008), "Nonparametric Identification", in: J. Heckman and E. Leamer (eds), *Handbook of Econometrics*, vol. VI, pp. Elsevier.
- [33] Menon, M. and F. Perali (2007): "The cost of rearing children, child welfare and child poverty within the collective household model", mimeo
- [34] Nelson, Julie A. (1988), "Household Economies of Scale in Consumption: Theory and Evidence", *Econometrica*, vol. 56, pp. 1301-1314.
- [35] Nelson, Julie A. (1993), "Household Equivalence Scales: Theory vs. Policy?", Journal of Labor Economics, vol. 11, pp. 471-493.

- [36] Pendakur, Krishna (1999), "Estimates and Tests of Base-Independent Equivalence Scales", *Journal of Econometrics*, vol. 88, pp. 1-40
- [37] Pollak, Robert and Terence J. Wales (1979), "Welfare Comparisons and Equivalence Scales", American Economic Review, vol. 69, pp. 216–221.
- [38] Pollak, Robert and Terence J. Wales (1992), Demand System Specification and Estimation, New York: Oxford University Press.
- [39] Rose, Elaina (1999), "Consumption Smoothing and Excess Female Mortality in Rural India," *Review of Economics and Statistics*, vol. 81, pp. 41-49.
- [40] Smith, Richard J. and Richard W. Blundell (1986), "An Exogeneity Test for a Simultaneous Equation Tobit Model with an Application to Labor Supply", *Econometrica*, vol. 54, No. 3, pp. 679–685.
- [41] Thomas, Duncan (1991), "Intre-household Resource Allocation: An Inferential Approach", *Journal of Human Resources*, vol. 25, pp. 635-664.
- [42] Tsakloglou, Panos (1991), "Estimation and Comparison of Two Simple Models of Equivalence Scales for the Cost of Children", The Economic Journal, vol. 101, No. 405, pp. 343-357.
- [43] Zamora, Bernarda (2010): "Does female participation affect the sharing rule?", Journal of Population Economics, forthcoming.



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