Strategic Defense and Attack for Series and Parallel Reliability Systems: Reply on Comment

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Abstract
Kovenock and Roberson's (2010) paper has the potential to advance the research frontier, but has deficiencies. This paper suggests how Kovenock and Roberson's (2010) paper can be developed into a more substantial paper. Kovenock and Roberson's (2010) paper consists of three sections. The first section is an introduction which is OK but has no results. The second section, titled "Model and Main Result", provides no contribution beyond Hausken (2008a). It consists of equations (1)-(10) which are equivalent to equations developed by Hausken (2008a), and equation (11) which is equivalent to the requirement $u \geq 0$ and $U \geq 0$ provided after equation (17) in Hausken (2008a). The third section quotes Hausken (2008a) once in one sentence which means that section 3 does not belong as a comment on the paper written by Hausken (2008a). The authors are encouraged to develop a new paper based on many interesting ideas in this note. The new paper should develop further the idea of mixed strategies presented in section 3. The new paper may be titled: "Strategic Defense and Attack for Series and Parallel Reliability Systems when Allowing Mixed Strategies".

1 Introduction

Kovenock and Roberson (2010) have written a paper which comments on a paper written by Hausken (2008a). Kovenock and Roberson's (2010) paper is a first step towards supplementing Hausken's (2008a) work by allowing mixed strategies. The objectives of this paper are, first, to illustrate how Kovenock and Roberson's (2010) approach can be extended to advance the research frontier. Second, we clarify a few misunderstandings in Kovenock and Roberson's (2010) paper and show how there are no technical errors in Hausken's (2008a) paper. Section 2 provides a systematic analysis referring to page numbers P and line numbers L in Kovenock and Roberson's (2010) paper. Section 3 concludes.

2 Systematic analysis

1. P5,L40-44: The authors state: "Furthermore, it is incorrect to assert, as does Hausken (2008a), that if a player's expected payoff given in (9) or (10) is negative, then there exists an equilibrium in which that player allocates….."

That statement is incorrect and should be removed. The term "equilibrium" is not present in Hausken's (2008a) paper. After (17) Hausken (2008a) writes:

"When (17) gives negative utilities, a corner solution emerges with zero utility and zero investment for that agent (either the defender or the attacker) which according to (17) would otherwise get negative utility. Using (12) or (13), the other agent gets a utility equal to the value of the system by investing arbitrarily small but positive amounts into defending or attacking the components."

As the authors correctly point out, the solution when (17) gives negative utilities is not a Nash equilibrium. The reason is that the defender can unilaterally and preferably deviate by choosing a strategy of investing more than the attacker. (The reason is not, as the authors state, that "the defender's best response is to outbid the attacker," since this is not a sequential game where the players can inch up on each other in sequential bidding. If the players were to outbid each other in sequential bidding, they would find no final resting point since no Nash equilibrium exists.) From a gametheoretic point of view, the issue boils down to specifying what happens in a simultaneous move game when no Nash equilibrium exists in pure strategies. This author thinks that the literature will certainly benefit from specifying the many solutions that may arise when a Nash equilibrium does not exist in pure strategies. Hausken's (2008a) solution is one of many possible assumptions about what players may do when a Nash equilibrium does not exist in pure strategies. Another possible solution is that the attacker chooses an investment which makes the defender indifferent between choosing no investment and choosing a positive investment, both of which give zero utility to the defender. That solution is proposed by Hausken (2010b).

For the event that the attacker moves first, that paper does not specify that the attacker chooses an arbitrarily small investment, since the defender will then not respond by choosing zero investment, but will respond by choosing a positive investment. In contrast, in that paper, the attacker, moving first, chooses a strictly positive investment which makes the defender, moving second, indifferent between choosing no investment and choosing a positive investment, both of which give zero utility to the defender. A third possible assumption about what players may do when a Nash equilibrium does not exist in pure strategies, is to use ideas from the gametheoretic bargaining literature in the 1980s about "off-the-equilibrium-path conjectures". As we know, infinitely many assumptions and solutions may be proposed for the event that no Nash equilibrium exists in pure strategies. See for example Rapoport and Guyer (1966) for non-existence of Nash equilibrium in pure strategies in 2 x 2 games:

But, as we know, a Nash equilibrium always exists in mixed strategies. Hence a fourth possible assumption about what players may do when a Nash equilibrium does not exist in pure strategies, is to assume that the players may choose mixed strategies. I think the current work initiated by the authors on mixed strategies can provide a useful addition to the literature. I encourage the authors to develop a new paper with such a focus. The paper can be titled: "Strategic Defense and Attack for Series and Parallel Reliability Systems when Allowing Mixed Strategies".

2. P1,L24-28, abstract: The authors state: "This note identifies a technical error that invalidates Hausken's characterization of Nash equilibrium for a substantial portion of the parameter space that he examines.…."

That statement is incorrect and should be removed. There is no technical error in Hausken's (2008a) paper. Hausken (2008a) provides a correct solution which is a Nash equilibrium in pure strategies when both players' utilities are positive. For the event of negative utilities Hausken (2008a) provides one possible solution based on one of many possible assumptions about what players may do when a Nash equilibrium does not exist in pure strategies. The authors are advised to revise the statement as follows: "This paper supplements Hausken's characterization of Nash equilibrium with a mixed strategy Nash equilibrium analysis for all parameter values, including that portion of the parameter space where no Nash equilibrium exists in pure strategies."

3. P1,L26: The authors state: "…substantial portion of the parameter space".

That statement is inappropriate and should be removed unless the authors can somehow quantify "substantial portion" and supplement with further reasoning about which parameter values are realistic. Empirical support is needed here, which neither Hausken (2008a) nor the authors provide. Assume that either expert judgment or empirical support or other tools is/are used to assess lower and upper limits for all parameters. Having established such ranges for all parameters, one can calculate which portion of these ranges corresponds to $u \geq 0$ and $U \geq 0$. That portion is a number between 0 and 1. With "substantial portion" the authors possibly mean that this number is above 0.5. If so, this author would like to see another argument than armchair reasoning for why this number is above 0.5. But, more importantly, regardless of whether or not this number is above or below 0.5, a crucial question is which parameter values are realistic. If only 10% of the possible combinations of parameter values is/are realistic, the remaining 90% constitutes a substantial portion which is unrealistic. Let us illustrate with an example. Consider a network defended by a defender and attacked by an attacker. That no pure-strategy Nash equilibrium exists, which means that either $u<0$ or $U<0$, is quite a disastrous situation. For example, $u<0$ means that the parameter values are such that the defender cannot earn positive utility as no Nash equilibrium exists. From a common sense point of view that may be considered as an unrealistic situation, regardless of whether $m_i = 1$ or $m_i \neq 1$. Quite the contrary, it may be argued that realistic situations are those where $u>0$ so that the defender has an incentive to defend the network. Let us further reflect on the case $U<0$. Many networks exist which are not attacked by an attacker, and hence $U<0$ may be realistic. This author considers it valuable that the authors analyze mixed strategy equilibria for the case that either $u<0$ or $U<0$ (and for the case $u \geq 0$ and $U \geq 0$).

4. P1,L24-29, abstract: The authors state: "….and provides necessary conditions for his solution to form a pure-strategy Nash equilibrium."

That statement, which constitutes equation (11) in the authors' Proposition 1, is correct, but is equivalent to requiring $u \geq 0$ and $U \geq 0$ in equation (17) in Hausken (2008a). Hence the insight has already been provided by Hausken (2008a). The case $\{u \geq 0$ and $U \geq 0\}$ has to be distinguished from the case that either $u<0$ or $U<0$ as explained after equation (17) in Hausken (2008a).

5. P1,L29-32, abstract: The authors state: "….Many of the existing results in the contest-theoretic literature on the attack and defense of networks of targets rely upon Hausken's (2008a) characterization and require corresponding parameter restrictions."

That statement should be reformulated since it is already known from the existing results in the contest-theoretic literature which parameter restrictions apply for the various solutions. The statement can be reformulated as follows: "This paper also supplements with a mixed strategy Nash equilibrium analysis for all parameter values the many results in the contest-theoretic literature on the attack and defense of networks that rely upon Hausken's (2008a) characterization."

6. The last sentence in the authors' abstract is correct and can be kept unchanged.

7. P2,L37-41: The authors state: "In this comment we provide necessary conditions for the solution in Hausken (2008a) to form a pure-strategy Nash equilibrium point and show that these conditions are quite restrictive."

That statement is misleading and should be removed. The conditions for a pure-strategy Nash equilibrium has already been provided by Hausken (2008a) requiring $u \geq 0$ and $U \geq 0$ after equation (17). Hausken's (2008a) requirement $u \geq 0$ and $U \geq 0$ is equivalent to the authors' equation (11) in the authors' Proposition 1. The case $\{u \geq 0$ and $U \geq 0\}$ has to be distinguished from the case that either $u<0$ or $U<0$ as explained after equation (17) in Hausken (2008a). When either $u<0$ or $U<0$, no Nash equilibrium exists in pure strategies. Regarding the restrictiveness of the conditions, please see elsewhere in this report.

8. P2,L42-45: The authors state: "Section 2 reviews the characterization of equilibrium given by Hausken (2008a) and provides necessary conditions for the solution in Hausken (2008a) to form a pure-strategy Nash equilibrium point."

That statement should be removed since the necessary conditions have already been provided by Hausken (2008a) as $u \geq 0$ and $U \geq 0$, as stated after equation (17) in Hausken (2008a). These conditions are equivalent to the authors' equation (11) and Proposition 1.

9. P3,L19: The two words "Main Result" in this heading should be removed since there are no main results in this section 2. Equation (11) and Proposition 1 are equivalent to $u \geq 0$ and $U \geq 0$ provided by Hausken (2008a).

10. P3-6: The authors' equations (1)-(10) are equivalent to equations developed by Hausken (2008a). The authors' equation (11) is equivalent to the requirement $u \geq 0$ and $U \geq 0$ provided after equation (17) in Hausken (2008a).

11. P6,L31-33: The authors state: "Therefore solving the system of first-order conditions, as does Hausken (2008a), does not guarantee equilibrium."

That statement is correct, but this is already known from Hausken (2008a) in the sense of distinguishing between $u \geq 0$ and $U \geq 0$ on the one hand and $u<0$ or $U<0$ on the other hand. The first-order conditions are solved only when $u \geq 0$ and $U \geq 0$, consistent with the statement after equation (17) in Hausken (2008a). When either $u<0$ or $U<0$, no Nash equilibrium exists in pure strategies, consistent with the statement after equation (17) in Hausken (2008a).

12. P6,L44-55 and P7,L1-5: The authors discuss how restrictive the conditions $u \geq 0$ and $U \geq 0$ are. For example, "pure-strategy equilibria may fail to exist even in the popular case of $m_i = 1$ for all i." I advise the authors to be careful with such armchair reasoning. Empirical support is needed to assess restrictiveness. Neither Hausken (2008a) nor the authors provide empirical support. That $m_i = 1$ is popular is no argument one way or the other. But, we can make the following observations. Consider a network defended by a defender and attacked by an attacker. That no pure-strategy Nash equilibrium exists, which means that either $u<0$ or $U<0$, is quite a disastrous situation. For example, $u<0$ means that the parameter values are such that the defender cannot earn positive utility based on pure strategies as no Nash equilibrium exists in pure strategies. From a common sense point of view that may be considered as an unrealistic situation, regardless of whether $m_i = 1$ or $m_i \neq 1$. Quite the contrary, it may be argued that realistic situations are those where $u>0$ so that the defender has an incentive to defend the network. Let us further reflect on the case $U<0$. Many networks exist which are not attacked by an attacker, and hence $U<0$ may be realistic. I consider it valuable and encourage the authors to analyze mixed strategy equilibria also for the case that either $u<0$ or $U<0$.

13. P7,L6-16: The authors state: "Because the solution in Hausken (2008a) forms a pure-strategy Nash equilibrium point only for a restrictive set of parameters, statements that are based on this solution — such as all of the propositions in Hausken (2008a) — fail to hold for all combinations of m_i, v_A, v_D, $c_{A,i}$ and $c_{D,i}$ that violate either of the two conditions in Proposition 1."

That statement should be removed since this is already known from Hausken (2008a). Of course all the propositions in Hausken (2008a) hold only when $u \geq 0$ and $U \geq 0$, consistent with the statement after equation (17) in Hausken (2008a). Yes, combinations of m_i, v_A, v_D, $c_{A,i}$ and $c_{D,i}$ that violate $u \geq 0$ and $U \geq 0$ cause all the propositions in Hausken (2008a) to be invalid, consistent with what is already known from Hausken (2008a).

14. P7,L17-19: The authors state: "There are a number of articles that analyze variations of the Hausken (2008a) framework that are also erroneous without the imposition of more restrictive parameter assumptions."

That statement is incorrect and should be removed. Hausken (2008a) has already provided the parameter assumptions as $u \geq 0$ and $U \geq 0$ after equation (17). Hence nothing is erroneous.

15. P7,L20-23: The authors state: "First, both Hausken (2008b) and Hausken (2010a) contain closely related technical errors that invalidate those characterizations of Nash equilibrium for a substantial portion of the parameter space.

That statement is incorrect and should be removed. There are no technical errors in Hausken (2008b) and Hausken (2010a). It is clear from those papers when the results are valid, for example when $u \geq 0$ and $U \geq 0$ or when the players earn higher utilities than their security values.

16. P7,L23-24: The authors state: "…substantial portion of the parameter space".

That statement is inappropriate and should be removed as discussed in point 3 above.

17. P7,L24-32: The authors state: "In particular, in both of those games each of the players has a secure utility that can be assured regardless of the action of the other player. But in the solutions that are given Hausken (2008b) and Hausken (2010a) there exist large portions of the parameter space in which one or both of the players obtain a level of utility that is below their secure utility level, and hence these solutions do not form pure-strategy equilibria."

That's correct but this is already known from Hausken (2008b) and Hausken (2010a). When the players earn utilities that are below their secure utilities, these solutions do not form pure-strategy equilibria.

18. P7,L33-54: The authors correctly quote a part of Hausken's (2008a) Proposition and add: "Unfortunately, there are a number of papers that, relying on Hausken (2008a), incorrectly assume that when $c_{A,i} = c_A$, $c_{D,i} = c_D$, and $m_i = m$ for all components i =1, . . . , n, it is optimal for each player to allocate forces evenly across components."

That statement is incorrect and should be removed. Hausken's (2008a) Proposition 1 is correct and applies when $u \geq 0$ and $U \geq 0$ which specify the parameter restrictions needed for a pure strategy Nash equilibrium to exist. Consequently, when $u \geq 0$ and $U \geq 0$, the result $t_i c_i / r = T_i C_i / R$ follows as an implication. That result is correct and independent of m. It is certainly not correct to state that $c_{A,i} = c_A$, $c_{D,i} = c_D$, and $m_i = m$ for all components i =1, . . . , n imply $t_i c_i / r = T_i C_i / R$. This author is not aware of any papers that, relying on Hausken (2008a), claim the implication in the previous sentence. Applying a mixed strategy equilibrium analysis, I hope the authors can develop expressions for the case that either u<0 or U<0.

19. P8,L11-12: The authors state: "It is beyond the scope of this comment to provide necessary and sufficient conditions for the existence of a pure-strategy equilibrium with an even allocation of forces that is assumed in all of these articles. However, it is straightforward to modify the necessary conditions given in Proposition 1 above for each of those particular models."

That statement should be modified since conditions such as u≥0 and U≥0 are already clear from those papers. But, in future work, I hope the authors can determine mixed strategy equilibria for each of these models. Yes, this author considers "the completion of this equilibrium characterization as an important area for future research."

20. P8-10: Much, but not all, of section 3 "Mixed-Strategy Equilibria" consists of quotes of earlier research. Section 3 contains many interesting ideas that can be developed into one or several separate future papers. Section 3 quotes Hausken (2008a) once in one sentence which means that section 3 does not belong as a comment on the paper written by Hausken (2008a).

21. P8,L40-43: The authors state: "assume that for each player the unit costs are symmetric across targets and players and are normalized to one (i.e., $c_{A,i} = c_{D,i} = 1$ for all i) and that the level of noise at each target is the same (i.e., $m_i = m$ for all i)."

I recommend that the authors write a future paper where those restrictive assumptions are generalized maximally.

22. P10,L4-8: The authors state: "However, there is good reason to believe that cases involving finite but high m are qualitatively closer to this benchmark than to the pure-strategy profiles examined in Hausken (2008a)."

This is the only quote of Hausken (2008a) in section 3. This author is not convinced by the authors' armchair reasoning about how pure-strategy profiles examined by Hausken (2008a) and others are close or not close to one or the other benchmark. Empirical support is needed. Pure-strategy profiles are indeed common. A government seeking to protect a network, and a terrorist seeking to attack a network, can be expected first to search for pure-strategy profiles, and thereafter to search for mixed-strategy profiles. Hausken and Levitin (2008) and Hirshleifer (1995) assess the issue of low versus high decisiveness parameter or contest intensity m.

23. P10,L4-8: The authors state: "A complete treatment of simultaneous move games of attack and defense in which the conditions in equation (11) are violated is still an open question."

I hope the authors will write a future paper where this open question is fully and exhaustively answered. That the conditions in equation (11) are violated means that we no longer have $u \geq 0$ and $U \geq 0$ in Hausken's (2008a) equation (17). Hausken (2008a) provides one possible suggestion for how the players may handle the event that either $u<0$ or $U<0$. That suggestion can well be argued to be plausible (the player earning negative utility simply withdraws leaving everything else to the other player), but this suggested solution is not a pure-strategy Nash equilibrium since no pure-strategy Nash equilibrium exists when either $u<0$ or $U<0$. An exhaustive analysis of the event that either $u<0$ or $U<0$ would be a valuable addition to the literature. For example, networks are often defended but not always attacked. This may suggest that the event $U<0$ is common. Understanding that event exhaustively will certainly be valuable for e.g. a defending government which then gets insight into how to deter attackers.

24. P10,L11-12: The authors state: "However, we do know that for a single contest with linear costs, (the famous Tullock rent-seeking model), a pure-strategy equilibrium exists only for m less than or equal to 2."

To supplement this insight, assuming unitary actors, Hirshleifer (1995:33) derives two conditions under which anarchy breaks down. The first is that "an excessively large decisiveness parameter m (i.e. $m > 1$) leads to dynamic instability, that is, movement toward a corner solution" where one actor gets the entire product, and the other gets nothing. "

Hirshleifer, J. (1995) Anarchy and Its Breakdown. *Journal of Political Economy* **103**(1) 26-52.

In contrast, accounting for the collective action problem within each group, Hausken (2006) develops the opposite result: fighting/production is stable even for large decisiveness parameters (above one) and strict income requirements for each agent.

Hausken, K. (2006), "The Stability of Anarchy and Breakdown of Production," Defence and Peace Economics 17, 6, 589-603.

25. P10,L22-25: The authors state: "Unfortunately, the nonexistence of pure-strategy equilibrium and characterization of mixed-strategy equilibria has been almost entirely overlooked in the contest-theoretic literature on attack and defense of networks of targets."

This author agrees and hopes that the authors will write a future paper to remedy this deficiency.

26. P10,L25-28: The authors state: "In fact, as shown in Proposition 1, the pure-strategy equilibria applied extensively in the literature exist only for a very restrictive set of parameters."

The statement about restrictiveness is misleading and should be rewritten in accordance with points about restrictiveness made elsewhere in this report.

27. P10,L25-28: The authors state: "This characterization remains an important area for future research."

Yes, this author agrees and hopes the authors will conduct future research on this characterization.

28. The possibility that either u<0 or U<0 may occur, which implies that no Nash equilibrium exists in pure strategies, is one reason that many of the papers the authors list in the reference list do not solve for a Nash equilibrium, but analyze a two period minmax game where the defender moves in the first period minimizing for example the vulnerability of a system, while the attacker moves in the second period maximizing the vulnerability.

3 Conclusion

Kovenock and Roberson's (2010) paper has the potential to advance the research frontier, but has deficiencies. This paper suggests how Kovenock and Roberson's (2010) paper can be developed into a more substantial paper. Kovenock and Roberson's (2010) paper consists of three sections. The first section is an introduction which is OK but has no results. The second section, titled "Model and Main Result", provides no contribution beyond Hausken (2008a). It consists of equations (1)-(10) which are equivalent to equations developed by Hausken (2008a), and equation (11) which is equivalent to the requirement $u \geq 0$ and $U \geq 0$ provided after equation (17) in Hausken (2008a). The third section quotes Hausken (2008a) once in one sentence which means that section 3 does not belong as a comment on the paper written by Hausken (2008a). The authors are encouraged to develop a new paper based on many interesting ideas in this note. The new paper should develop further the idea of mixed strategies presented in section 3. The new paper may be titled: "Strategic Defense and Attack for Series and Parallel Reliability Systems when Allowing Mixed Strategies".

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