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Chakrabarty, Debajyoti

Working Paper

Poverty traps and growth in a model of endogenous time preference

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Debajyoti Chakrabarty

Poverty Traps and Growth in a Model of Endogenous Time Preference

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Working Paper

Poverty traps and Growth in a model of Endogenous Time Preference*

Debajyoti Chakrabarty[†]

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Abstract

We study the effect of endogenous time preference in a simple neo-classical model of growth. The variation of time preference causes the economy to have multiple steady states, some of which are similar to poverty traps. The stability properties of these steady states are analyzed. The results are interpreted in light of the growth experiences of developing economies. The model can explain why two economies that have identical production technologies and identical preferences may converge to different levels of income depending on initial conditions.

Keywords: Intertemporal choice, Saving, Growth, Local stability, Poverty traps
JEL classification: D91, E21, C62, O40

1 Introduction

The consumption-saving choices of economic agents are driven by intertemporal utility trade-offs between current and future consumption. Any inter-temporal decision making process can be broken down into basically two components: First, the rate of time preference, i.e., how much importance agents give to the future, and second, the rate of return on savings, i.e., the reward for foregoing current consumption. Economic theory predicts that individuals who give more importance to future would defer consumption of a larger portion their current income to future. The consumption foregone can be utilized for investment in a productive activity which would increase consumption possibilities in the future.

Solow's ([12]) neo-classical model of growth, which is also the benchmark model for analyzing growth related issues focuses on the second aspect of this economic decision making process. According to the neo-classical model, when an economy is poor, that is, it has very little capital to work with, the rate of return on investment is very high. This induces people to save and invest more, and the economy embarks on the process of growth and development. As societies accumulate more capital the return on investment declines and hence people become less thrifty. The process of accumulation of capital goes on until the society reaches its desired level of capital and output.

An important implication of the neo-classical model is that poorer economies would grow at a more rapid rate than richer economies provided they have access to the same technology. In long run, all economies converge to the same level of capital and output. The initial conditions of an economy do not play any role in determining the long-run level of affluence of an economy. This implication of the neo-classical model is commonly referred to as the convergence hypothesis. Subsequent empirical studies have failed to vindicate the convergence hypothesis, however, there is evidence of convergence among the OECD countries. This phenomenon called club convergence implies that rates of growth rates of economies with similar levels of affluence will tend to converge.

One of the common features of most models of growth is that all individuals give the same amount of importance to the future and that weight is assumed to be an exogenously given parameter called subjective discount factor. Usually it is assumed that the discount factor is less than one, implying that people give less importance to future

consumption in comparison to current consumption. One possible economic rationale behind this assumption may be that there is a chance that an agent may not survive to the future period to reap the benefits of his current savings. However, assuming the discount factor to be same for all agents requires some careful analysis. Treating the discount factor as an exogenously given parameter, makes the preferences of agents seperable over time and hence greatly simplifies the analysis of their optimal consumption-savings decisions. This technical simplicity comes at a cost of economic plausibility. Uzawa([13]) and Koopmans([7]) first introduced the possibility of discount factors to be determined endogenously by allowing the preferences of agents to be recursive. Their approach was extended and studied in greater detail by Epstein([4]), Iwai([6]) and Obstfeld([10]). Both Epstein and Obstfeld assumed the discount factor of agents to be decreasing in the level of consumption. This behavioral assumption regarding the discount factors basically implies that agents become more impatient as they grow richer and their level of consumption rises. While they acknowledge that there may be equally compelling reasons to believe that people become more patient as they grow richer, they work with this behavior of discount rates as it ensures a unique steady state which is also stable. They conclude that the long-run level of consumption and output of an economy is unique and independent of the initial conditions, a result very similar to the neo-classical model.

In this paper, we interpret the discount rate as the probability of an agent surviving to the next period. We allow this probability of survival to be determined endogenously. The probability of survival is increasing as the agents current level of consumption increases. Hence, the importance given to the future is influenced by his endowment of wealth and other productive factors. We draw this relationship from the vast literature in development economics which have recorded the effect of malnutrition and undernourishment on the economic behavior of individuals. Individuals who cannot afford certain subsistence level of consumption are trapped in poverty and save very little. On the other hand, there is another section of population called the "middle class" which have recorded significant improvements in their real income in the development experience of countries like India. We attempt to explain these by studying the effect of initial endowment of wealth on the saving investment choice of individuals when the rate of time preference is allowed to vary.

At this point we would like to point out that the basic difference of our paper from

rest of the literature. In our model, an individual's probability of surviving to a future period is increasing with his current level of consumption. This has the effect of decreasing the rate of time preference as current consumption of an individual rises. This assumption is not only has a certain economic rationale, it also has been found to be true in empirical studies. Ogaki and Atkeson([11]) in their study of panel data on three village districts in India, find that the inter-temporal elasticity of substitution rises with the level of wealth. The rate of time preference is not allowed to differ according to their model. Lawrance([8]) in her study on inter-temporal preferences based on U.S. panel data finds that subjective rate of time preference is about three to five percentage points higher for households with lower incomes than those with higher incomes. Controlling for race and education widens this difference even more. These results suggest one possible explanation for the observed heterogeneity in savings behavior across socioeconomic classes within a society as well as across different societies with different levels of affluence. This kind of behavior also has significant policy implications. Higher rates of time preference may reduce investment in education and thereby induce a negative relation between time preference and long-run income. Also, poor households will give less importance to the future and have a higher marginal propensity to consume which would adversely affect their savings-investment behavior.

A similar theoretical approach as ours was taken by Mantel([9]), where he studies the impact of decreasing rate of time preference on the optimal growth path of an economy. However, Mantel's primary focus was to study the monotonocity properties of optimal consumption and investment paths.

The paper is organized as follows. The next section describes our model and presents our basic results. Results concerning the steady states are presented in section 3. In section 4 we study the stability properties of the steady states. Section 5 provides a discussion of the results and we conclude in section 6 with some possible research questions.

2   The Model

Consider a closed economy in a one-good world. The good can be used for either consumption or investment. The production of this good requires two kinds of inputs, labor(N)

and capital(K). There are a large number of competetive firms having the same constant returns to scale production technology. The aggregate production function of the economy in every period is described by

$$Y_t = F(K_t, N) \tag{1}$$

Our economy consists of a representative agent who seeks to maximize his lifetime welfare. The agent derives his income from selling productive factors in every period. The agent's endowment of labor is assumed to be constant in every period. However, the capital can change over time depending on the savings decision of the agent. The agent, given the initial endowment of capital, has to decide his consumption and savings.

At any period t there is a chance that the agent will not survive to the next period $t+1$. The agent's probability of surviving to the next period depends on his current period consumption. If the agent's current period consumption is below a certain subsistence level of consumption($\underline{C}$), then the agent's probability of survival is extremely low($\underline{\beta}$). As the consumption increases from this subsistence level the probability of survival also increases until consumption rises to a level of basic comfort($\bar{C}$), where the agents probability of survival reaches a maximum($\overline{\beta}$) and becomes insensitive to the changes in consumption. Thus, the probability of survival($\rho_{t,t+1}$) from any period t to period $t+1$ is a continuous function of consumption at time t in the following way:

$$\rho_{t,t+1} = \begin{cases} \underline{\beta} < 1 & \text{if } C_t < \underline{C} \\ \beta(C_t) & \text{if } C_t \in [\underline{C}, \bar{C}] \\ \overline{\beta} < 1 & \text{if } C_t > \bar{C} \end{cases} \tag{2}$$

where $\beta'(C_t) > 0$, if $C_t \in [\underline{C}, \bar{C}]$ and $\underline{\beta} < \overline{\beta}$. Suppose the agent has to choose a path of consumption and savings at period 0 to maximize his lifetime welfare. The weight given to future consumption will depend on the agent's probability of surviving to that future date. Let the probability of surviving to period t be denoted by $\rho_{0,t}$. These probabilities are the discount factors of the agent in his intertemporal maximization problem. Let us take note of some properties of the discount factors $\rho_{0,t}$ which will help in simplifying our analysis in future.

(P1) $\rho_{0,0} = 1$, $\rho_{0,t} = \beta(C_0)\beta(C_1).........\beta(C_{t-1})$

(P2) $\rho_{0,t+1} = \rho_{0,t} \, \beta(C_t)$

(P3) $\left(\dfrac{\rho_{0,s}}{\rho_{0,t}}\right) = \rho_{t,s}.$

The maximization problem faced by the agent is

$$\max \sum_{t=0}^{\infty} \rho_{0,t} \, U(C_t) \ ,$$

subject to

$$K_{t+1} = F(K_t, N) + (1-\delta)K_t - C_t \ ,$$

and a transversality condition

$$\lim_{t \longrightarrow \infty} \rho_{0,t} K_t \geq 0 \ . \tag{TC}$$

The agent's period utility function is $U(C_t)$ and δ is the depreciation rate of capital where, $0 < \delta < 1$. At this point we make some assumptions concerning the functions $U(.)$ and $\beta(.)$ to ensure that the necessary conditions for maximum are also sufficient.

(A1) $U(C_t) > 0$, $U'(C_t) > 0$, $U''(C_t) < 0$.

(A2) $0 < \beta(C_t) < 1$, $\beta'(C_t) > 0$ and $\beta''(C_t) < 0$ for $C_t \in [\underline{C}, \bar{C}]$.

(A3) $F(0,0) = 0$, $F_K(.) > 0$, $F_{KK}(.) < 0$, $\lim_{K \to 0} F_K(.) = \infty$

The Lagrangian for the agent's problem can be written as

$$\pounds = \sum_{t=0}^{\infty} \{\rho_{0,t} \, U(C_t) + \tilde{\lambda}_t[F(K_t, N) + (1-\delta)K_t - C_t - K_{t+1}]\} \ .$$

The first-order conditions for maximum are

$$\rho_{0,t} U'(C_t) + \sum_{s=t+1}^{\infty} \frac{\partial \rho_{0,s}}{\partial C_t} U(C_s) = \tilde{\lambda}_t \ ,$$

$$\tilde{\lambda}_t = \tilde{\lambda}_{t+1}[F_K(K_{t+1}, N) + 1 - \delta] \ .$$

and the transversality condition holding with equality. Let $\left(\dfrac{\tilde{\lambda}_t}{\rho_{0,t}}\right) = \lambda_t$ and $\sum\limits_{s=t+1}^{\infty} \rho_{t+1,s} U(C_s) = \phi_{t+1}$, where ϕ_{t+1} is the present discounted value of future consumption from period $t+1$ onwards. The first order conditions can now be re-written as

$$U'(C_t) + \beta'(C_t) \, \phi_{t+1} = \lambda_t \,, \tag{3}$$

$$\lambda_t = \lambda_{t+1}\beta(C_t) \, [F_K(K_{t+1}, N) + 1 - \delta] \,, \tag{4}$$

and

$$K_{t+1} = F(K_t, N) + (1 - \delta)K_t - C_t \,. \tag{5}$$

Note that $U''(C_t) + \beta''(C_t) \, \phi_{t+1} < 0$ since the functions $U(.)$ and $\beta(.)$ are concave. Thus the second order condition for maximum is also satisfied. Substituting (3) in (4) we get,

$$\frac{U'(C_t) + \beta'(C_t) \, \phi_{t+1}}{\beta(C_t)[U'(C_{t+1}) + \beta'(C_{t+1}) \, \phi_{t+2}]} = [F_K(K_{t+1}, N) + 1 - \delta] \,. \tag{6}$$

Notice that the variable ϕ_t, the present discounted value of utilities from period t onwards, evolves in the following fashion:

$$\phi_t = U(C_t) \, + \beta(C_t) \, \phi_{t+1} \qquad \text{for all } t \geq 1 \,. \tag{7}$$

Definition 1 *A perfect foresight equilibrium(PFE) of this econmomy are sequences $\{C_t\}_{t=0}^{\infty}$, $\{K_{t+1}\}_{t=0}^{\infty}$, $\{\phi_t\}_{t=1}^{\infty}$ such that (5), (6), (7) and TC hold for a given K_0.*

Equation (5) is the intertemporal budget constraint of the agent. Equation (6) which is derived from (3) and (4) tells us that the loss in welfare due to foregoing consumption in period t has to equal the discounted value of gain in welfare from period $t+1$ onwards. This condition is commonly referred to as the Fisher equation.

3 Steady-state Equilibria

Let us first study the steady state solutions to the difference equations (5), (6), (7) . In a steady state $C_t = C_{t+1} = C$, $K_t = K_{t+1} = K$ and $\phi = \frac{U(C)}{1-\beta(C)}$ for all t. In a steady state, equations (5) and (6) reduce to

$$C = F(K, N) - \delta K \,, \tag{BC}$$

and

$$\beta(C) \left[F_K(K, N) + 1 - \delta\right] = 1 . \tag{RR}$$

Equation (RR) is the steady state counterpart of the Fisher's intertemporal optimum. Equation (BC) gives us the locus of points along which the agent's consumption and capital stock are constant and satisfy the budget constraint. We are going to restrict our attention to only the positively sloped part of the BC curve[1]. The slope of RR curve in consumption-capital plane is given by $\frac{-F_{KK}(K,N)\beta^2}{\beta'(C)}$ which is always positive from our assumptions. We will soon derive the conditions for the existence of a steady state equilibrium. However, assuming those conditions hold, plotting BC and RR curves on consumption-capital plane shows that there could be multiple steady state solutions to the agent's problem(see **Figure 1**). We characterize the steady states as two types. A steady state is of type "H" (or SS_H) if the slope of RR curve is greater than BC curve and of type "L" (or SS_L) otherwise. The existence of various kinds of steady states is characterized in a series of lemmas and propositions. The first lemma provides us with a bound on the steady state levels of capital.

Lemma 1 *In any steady state solution to the agent's problem the steady state level of capital stock $K \in [\underline{K}, \overline{K}]$, where $\underline{K}$ satisfies $[F_K(., N) + (1 - \delta)] \underline{\beta} = 1$ and $\overline{K}$ satisfies $[F_K(., N) + (1 - \delta)] \overline{\beta} = 1$.*

Proof: See the appendix. ∎

The result follows from the steady state equation (RR) and says that the possible steady state capital levels are bounded within an interval which is determined by the minimum and maximum value of the discount factors. However between these bounds there may be more than one steady state equilibrium. The next proposition characterizes the number and type of such equilibria.

Proposition 1 *If the subsistence level of consumption($\underline{C}$) is equal to zero, then the steady state level of capital is strictly greater than $\underline{K}$. There exists at least one steady state of*

---

[1]The slope of BC curve is $F_K(K, N) - \delta$. This slope is positive for low values of capital and becomes negative after capital increases beyond the point where $F_K(K, N) = \delta$. According to RR in a steady state $F_K(K, N) + 1 - \delta = [\beta(C)]^{-1}$. If $F_K(K, N) < \delta$ then $F_K(K, N) + 1 - \delta < 1$, which is a contradiction since $[\beta(C)]^{-1}$ is always greater than one.

type "H". If there exists "n" steady states of type "H", then there must be "(n-1)" steady states of type "L".

Proof: See the appendix. ∎

The following lemma establishes a sufficient condition for the existence of at least one steady state of type "L".

Lemma 2 *Let $\underline{C} = 0$ and $F_K(\overline{K}, N) > \delta$ and K^1 denote the lowest steady state level of capital. If there exists a $K' > K^1$ such that $\beta^{-1}(\frac{1}{F_K(K',N)+1-\delta}) < F(K',N) - \delta K'$; then there exists at least one steady state of type "L" and two steady states of type "H".*

Proof: See the appendix. ∎

In the subsequent analysis we are going to assume the requirement of lemma 2 is satisfied and the economy has two steady states of type "H" with an intermediate steady state of type "L". All the intresting qualitative properties of the model can be studied within such a setup. We first study the local stability properties of the steady states.

4 Local equlibria

The stability of the steady states will depend on the behavior of the C, K and ϕ around steady state. Log-linearization of the first order conditions around steady state yields a system of difference equations in $\widehat{C}_t$, $\widehat{\phi}_t$ and $\widehat{K}_t$ where '^' denotes percentage deviation of the variable from its steady state value. The dynamical system can be expressed as

$$
\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = M \begin{bmatrix} \widehat{C}_t \\ \widehat{\phi}_t \\ \widehat{K}_t \end{bmatrix} ,
\tag{8}
$$

where

$$
M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ -\beta^{-1}\Delta_2 & \beta^{-1} & 0 \\ s_1 & 0 & \beta^{-1} \end{bmatrix} ,
$$

and Δ_2 is a weighted average of the elasticity of the period utility function and the discount factor at the steady state. Details of the derivation of equation (8) and the components of matrix M are provided in the appendix. The roots of the polynomial $\det[M - \mu I]$ are going to determine the local stability properties of any steady state equlibrium. In this context we have the following proposition.

Proposition 2 *Any steady state of type "H" is a saddle path and locally unique. Any steady state of type "L" is locally unstable.*

Proof: See the appendix. ■

If an economy is endowed with a level of capital stock close to a steady state capital of type "H" then the economy will converge to that steady state level of capital. Howver the steady states of type "L" are unstable. So far we have asumed that the agent is prefectly informed about the effect of his current consumption on his probability of survival. However, the results will not change he fails to internalize this effect.

Proposition 3 *If the agent fails to internalize the effect of consumption on the probability of survival, the possible steady state equilibria to the agent's optimization problem and their local stability properties are unaffected.*

Proof: See the appendix. ■

5   Long run output and Poverty traps

Depending on the initial level of capital, the economy might have three possible steady states. The pair (K_p, C_p) denotes a situation like poverty trap. One the other hand the pair (K_h, C_h) is the high level equilibrium. Both these steady states are locally stable and unique. The pair (K_u, C_u) is the unstable steady state(see **Figure 2**). This kind of behavior is very similar to Becker, Murphy and Tamura([3]) model with endogenous fertility. In the first part of their paper, human capital was the only accumulable factor. If the initial level of human capital was too low, it resulted in investment in education of children being too expensive as opposed to having more children. If the initial capital stock is sufficiently high then individuals have less children and invest on their education. There was an intermediate unstable steady state. The long run fertility behavior of the economy was dependent on the initial capital stock. In the latter half of their paper

they introduced another accumulable factor in form of physical capital. It is shown that depending on initial level of human capital the economy may have two different growth rates. If the initial endowment of human capital is very low the economy exhibits an under-development equilibrium, in which there is no investment in human capital and there is no growth. If the initial endowment of human capital is above a critical level the economy invests in both the productive assets and shows a steady long run growth. The introduction of human capital accumulation of the kind assumed in Becker, Murphy and Tamura([3]) would result in the possibility of two distinct growth rates in our model also. A detailed discussion of that scenario falls beyond the scope of the present essay.

In our model the causes of an economy falling into a poverty trap are slightly different. If an individual is extremely poor, then he gives less weight to future and hence his saving and investment behavior is adversely effected. The economy may be stuck at a low-level equilibrium as a result of perfectly rational intertemporal decision process. This kind of behavior is called investment in patience by Becker and Mulligan([2]).

The population and hence the labor force is assumed to be constant in our economy. It means that each agent is replaced with another on his death. An alternative interpretation of our set up is that every individual lives for one period and at the end of that period he leaves behind some bequest. This bequest depends on the importance given to the welfare of the future generations. For individuals who are poor this altruistic motive is weaker, and future generations of their dynasty are hence trapped in poverty. This was the motivation behind the Galor and Zierra ([5]) paper. In their model certain individuals are not able to invest in human capital and hence get trapped in poverty because their previous generations didn't leave any bequest. At this point we should point out that their qualitative results are very similar to what we derive.

So far we have assumed that the subsistence level of consumption ($\underline{C}$) is zero. However, if this level of consumption becomes too high then the steady state level of capital will be at its lowest possible level. The following proposition formalizes impact of subsistence and basic comfort level of consumptions on the steady state equilibria.

Proposition 4 *If the subsistence level of consumption($\underline{C}$) is greater than the $F(\underline{K}, N) - \delta\underline{K}$ then there exists a steady state equilibrium where $K = \underline{K}$ and $C = F(\underline{K}, N) - \delta\underline{K}$. If the level of basic comfort($\overline{C}$) is less than $F(\overline{K}, N) - \delta\overline{K}$ then there exists a steady state*

equilibrium where $K = \overline{K}$ and $C = F(\overline{K}, N) - \delta\overline{K}$. Moreover both the steady states are saddle paths and locally unique.

Proof: See the appendix. ∎

If the subsistence level of consumption is very high compared to the initial capital stock of the economy there is little incentive to save and accumulate capital. The economy will find itself stuck at a low level equilibrium with the lowest possible level of capital and output. On the other hand once the probability of survival reaches its maximum possible level the additional incentive to save goes away and the economy is content to be at its highest desirable capital stock $\overline{K}$.

If this economy was a small open economy which could borrow any amount of funds at a given world intersest rate, then there would be a unique steady state. The arbitrage opportunities would imply that the world interest rate equals domestic rate return on capital. However, if the economy faces credit rationing then possibility of multiple steady states would arise.

6   Conclusion

The traditional theories of development predict that as an economy embarks on growth path the size of its backward sector is going to decline. However experiences in Latin American, African and some Asian countries have shown that the presence of the backward sector has been very persistent. The neo-classical growth model predicts improvement in welfare as the economy accumulates more capital. Those models assume exogenous time preference and hence the possibility of poverty traps completely escape their purview. We have provided an alternative motivation behind an economy getting trapped at a low-level equilibrium.

We show that it may be possible that a economy with very little capital and output shows no growth at all while a richer economy exhibits a positive rate of growth. This occurs because the poorer economy may be trapped in a low level equilibrium while the richer economy shows positive growth in its endeavour to attain a higher level of steady state output. Unlike the neo-classical model history of an economy has an imprtant role in the process of development.

An interesting avenue for future research would be to allow the economy to be comprised of heterogeneous individuals in terms of their endowment of capital and study growth related issues in that framework. The endogeneity of time preference might cause the distribution of income to change as an economy embarks on its growth path.

Appendix

Proof of Lemma 1:

In order for the RR equation to be satisfied the marginal product of capital can never fall below $\underline{\beta}^{-1} + \delta - 1$. This in turn implies that in any steady state equilibria the level of capital has to exceed $\underline{K}$ from the strict concavity of $F(., N)$. A similar argument applies for $\overline{K}$. ∎

Proof of Proposition 1:

If $\underline{C} = 0$, then along the RR curve as $K \to \underline{K}$, $C \to 0$. From the assumptions made about the functions $F(K, N)$ and $\beta(C)$ we know that along the RR curve as $K \to \overline{K}$, $C \to \infty$. Hence there would exist at least one steady state and that steady state is of type "H". From the nature of the RR curve it follows that there must be one steady state of type "L" between two steady states of type "H". ∎

Proof of Lemma 2:

The existence of a type "H" steady state capital like K^1 follows from Proposition 1. If $\beta^{-1}(\frac{1}{F_K(K',N)+1-\delta}) < F(K', N) - \delta K'$ then there is some value of capital for which the RR curve is below the BC curve. Thus there is a steady state of type "L". The assumption of $F_K(\overline{K}, N) > \delta$ then guarantees the existence of the second steady state of type "H". ∎

Derivation of Equation 8:

Log-linearization of (5) around a steady state yields

$$\widehat{K}_{t+1} = \beta^{-1}\widehat{K}_t + s_1 \, \widehat{C}_t \, , \tag{I}$$

where '^' denotes percentage deviation of the variable from its steady state value and $s_1 = -C/K$ at steady state. From (3), we have

$$[s_2\sigma(C) + s_3\eta_\beta(C)\phi] \, \widehat{C}_t + s_3 \, \widehat{\phi}_{t+1} = \widehat{\lambda}_t \, ,$$

where $\sigma(C) = \left(\dfrac{U''(C)\,C}{U'(C)}\right) < 0$ and $\eta_\beta(C) = \left(\dfrac{\beta''(C)\,C}{\beta'(C)}\right) < 0$. $\quad s_2 = \left(\dfrac{U'(C)}{\lambda}\right)$ and $s_3 = 1 - s_2$. We write the above equation more compactly as

$$\Delta_1 \, \widehat{C}_t + \, s_3 \, \widehat{\phi}_{t+1} = \widehat{\lambda}_t \, , \tag{II}$$

where $\Delta_1 = [s_2\sigma(C) + s_3\eta_\beta(C)\phi] < 0$, from our previous assumptions.

From (4) we have

$$\widehat{\lambda}_t - \, \widehat{\lambda}_{t+1} = \epsilon_\beta(C) \, \widehat{C}_t + \, \tau_K \, \eta_F(K, N) \, \widehat{K}_{t+1}, \tag{III}$$

where $\epsilon_\beta(C) = \left(\dfrac{\beta'(C)\,C}{\beta(C)}\right) > 0, \eta_F(K, N) = \left(\dfrac{F_{KK}(K, N)\,K}{F_K(K, N)}\right) < 0$, and $\tau_K = F_K(K, N)\beta(C) > 0$.

From (7) we get

$$\widehat{\phi}_t = [(1 - \beta)\epsilon_U(C) + \beta\epsilon_\beta(C)] \, \widehat{C}_t + \, \beta \, \widehat{\phi}_{t+1} \, , \tag{IV}$$

where $\epsilon_U(C) = \left(\dfrac{U'(C)\,C}{U(C)}\right) > 0$. We rewrite the above equation as

$$\widehat{\phi}_{t+2} - \widehat{\phi}_{t+1} = -\beta^{-1}\Delta_2 \, \widehat{C}_{t+1} + (\, \beta^{-1} - 1) \, \widehat{\phi}_{t+1} \, , \tag{V}$$

where $\Delta_2 = (1 - \beta)\epsilon_U(C) + \beta\epsilon_\beta(C) > 0$. We can now use equations (I)-(V) to write a system of difference equations in $\widehat{C}_t$, $\widehat{\phi}_t$ and $\widehat{K}_t$ where the dynamical system can be expressed as

$$\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = M \begin{bmatrix} \widehat{C}_t \\ \widehat{\phi}_t \\ \widehat{K}_t \end{bmatrix} , \tag{9}$$

where

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ -\beta^{-1}\Delta_2 & \beta^{-1} & 0 \\ s_1 & 0 & \beta^{-1} \end{bmatrix} ,$$

$$M_{11} = \left(\frac{\epsilon_\beta(2 - \beta^{-1}) + s_1\tau_K \ \eta_F - \Delta_1}{s_3\beta^{-1}\Delta_2 - \Delta_1} \right), M_{12} = \left(\frac{s_3(\beta^{-1} - 1) \ \beta^{-1}}{s_3\beta^{-1}\Delta_2 - \Delta_1} \right), \text{ and } M_{13} = \left(\frac{\tau_K \ \eta_F \ \beta^{-1}}{s_3\beta^{-1}\Delta_2 - \Delta_1} \right).$$

■

Proof of Proposition 2:

The roots of the polynomial $\det[M - \mu I] = 0$, will determine the behavior of the above system.

$$
\begin{aligned}
\det[M - \mu I] &= (\beta^{-1} - \mu)[(M_{11} - \mu)(\beta^{-1} - \mu) + M_{12}\beta^{-1}\Delta_2] - s_1 M_{13}(\beta^{-1} - \mu) \\
&= (\beta^{-1} - \mu)[(M_{11} - \mu)(\beta^{-1} - \mu) - s_1 M_{13}] \ .
\end{aligned}
$$

Therefore $\mu_1 = \beta^{-1}$ is one of the roots of the polynomial. The other two roots of $\det[M - \mu I]$ are the roots of the polynomial,

$$P(\mu) = \mu^2 - (M_{11} + \beta^{-1})\mu + (\beta^{-1}M_{11} + M_{12}\beta^{-1}\Delta_2 - s_1 M_{13}).$$

Now let us consider s_3. From our definition

$$s_3 = \frac{\beta'(C) \ \phi}{U'(C) + \beta'(C) \ \phi} = \frac{\beta'(C)}{U'(C)/\phi + \beta'(C)} \ .$$

Now using the fact that at steady state

$$
\begin{aligned}
\phi &= \frac{U(C)}{1 - \beta(C)} \\
\implies s_3 &= \frac{\beta\epsilon_\beta}{(1 - \beta)\epsilon_U(C) + \beta\epsilon_\beta(C)} \\
&= \frac{\beta\epsilon_\beta}{\Delta_2} \ .
\end{aligned}
$$

Therefore, we can write $M_{11} = \left(\frac{\epsilon_\beta + \epsilon_\beta(1 - \beta^{-1}) + s_1\tau_K \ \eta_F - \Delta_1}{\epsilon_\beta - \Delta_1} \right), M_{12} = \left(\frac{\epsilon_\beta(\beta^{-1} - 1)\Delta_2^{-1}}{\epsilon_\beta - \Delta_1} \right),$

and $M_{13} = \left(\frac{\tau_K \ \eta_F \ \beta^{-1}}{\epsilon_\beta - \Delta_1} \right).$ Now consider the one kind of steady state SS_H. In SS_H the slope of the RR curve is greater than the slope of the BC curve. That is

$$\frac{-F_{KK}(.)\beta(C)^2}{\beta'(C)} > (\beta^{-1} - 1) \implies \frac{s_1\tau_K \ \eta_F}{\epsilon_\beta} > (\beta^{-1} - 1)$$

or $s_1\tau_K \ \eta_F + \epsilon_\beta(1 - \beta^{-1}) > 0$. Conversely in the other kind of steady state SS_L, $s_1\tau_K \ \eta_F + \epsilon_\beta(1 - \beta^{-1}) < 0$. Therefore for SS_H $M_{11} > 1$ and $P(0) = \beta^{-1}M_{11} + M_{12}\beta^{-1}\Delta_2 - s_1 M_{13} =$

$\beta^{-1} > 0$, if real roots of the polynomial $P(\mu)$ exist then they must be positive. $P(1) = 1 - \beta^{-1} - M_{11} + \beta^{-1} = 1 - M_{11} < 0$. Therefore around SS_H there exists one eigenroot μ_3 which is less than one in absolute value. It is easy to show that the eigenroot μ_2 will be greater than one. Hence, SS_H is a saddle path and the system is locally unique around the steady state. For the second part of the lemma, we split up the analysis of the roots into various cases. Firstly, we note that

$$M_{11} + \beta^{-1} = \frac{\epsilon_\beta(\beta^{-1} - 1) + \epsilon_\beta(1 - \beta^{-1}) + s_1 \tau_K \, \eta_F - \Delta_1(1 + \beta^{-1})}{\epsilon_\beta - \Delta_1} > 0 \; .$$

Case 1(Imaginary roots). The polynomial $P(\mu)$ will have imaginary roots if $(M_{11} + \beta^{-1})^2 < 4\beta^{-1}$. The modulus of the imaginary roots will be $\sqrt{\beta^{-1}} > 1$.

Case 2. (Real and distinct roots) The polynomial $P(\mu)$ will have real and distinct roots if $(M_{11} + \beta^{-1})^2 > 4\beta^{-1}$. Around SS_L, $s_1 \tau_K \, \eta_F + \epsilon_\beta(1 - \beta^{-1}) < 0$, hence, $M_{11} < 1$. Therefore, $P(0) = \beta^{-1} > 0$, $P(1) = 1 - M_{11} > 0$. The polynomial $P(\mu)$ attains a minimum at $\frac{M_{11} + \beta^{-1}}{2}$. Hence the roots will be greater than one in absolute value.

Case 3. (Real and repeated roots) The polynomial $P(\mu)$ will have real and repeated roots if $(M_{11} + \beta^{-1})^2 = 4\beta^{-1}$. This implies that $M_{11} + \beta^{-1} > 2$. Hence the roots will be greater than 1.

So SS_L is a source and the system is locally unstable. ■

Proof of Proposition 3:

If the agent does not take into account the effect of current consumption on the probability of survival then the first order conditions for an optimum would have to satisfy

$$U'(C_t) = \lambda_t \; , \tag{3'}$$

(4) and (5). The steady state solutions to the agent's problem would have to satisfy equations (BC) and (RR). Hence the existence of possible steady states is not affected. Log-linearizing the first order conditions around steady state yields the following dynamical system

$$\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & M_{13} \\ -\beta^{-1}(1-\beta)\epsilon_U & \beta^{-1} & 0 \\ s_1 & 0 & \beta^{-1} \end{bmatrix} \begin{bmatrix} \widehat{C}_t \\ \widehat{\phi}_t \\ \widehat{K}_t \end{bmatrix} , \tag{10}$$

where $M_{11} = \frac{\sigma(C) - \tau_K \eta_F s_1 - \epsilon_\beta}{\sigma(C)} > 1$ and $M_{13} = -\frac{\tau_K \eta_F \beta^{-1}}{\sigma(C)} < 0$. The roots of the polynomial $\det[M - \mu I] = 0$, will determine the behavior of the above system where

$$\det[M - \mu I] = (\beta^{-1} - \mu)[(M_{11} - \mu)(\beta^{-1} - \mu) - s_1 M_{13}].$$

Therefore $\mu_1 = \beta^{-1}$ is one of the roots of the polynomial. The other two roots of $\det[M - \mu I]$ are the roots of the polynomial,

$$P(\mu) = \mu^2 - (M_{11} + \beta^{-1})\mu + \beta^{-1}(1 - \epsilon_\beta/\sigma(C)).$$

Note that $1 - \epsilon_\beta/\sigma(C) < \frac{\sigma(C) - \tau_K \eta_F s_1 - \epsilon_\beta}{\sigma(C)} = M_{11}$. The discriminant of the above quadratic equation is $D = (M_{11} + \beta^{-1})^2 - 4\beta^{-1}(1 - \epsilon_\beta/\sigma(C)) > (M_{11} + \beta^{-1})^2 - 4\beta^{-1}M_{11} = (M_{11} - \beta^{-1})^2 \geq 0$. Therefore real roots exist. One of the roots of the polynmial $P(\mu)$, $\mu_2 = \frac{M_{11} + \beta^{-1} + \sqrt{D}}{2} > 1$. In order to show that the third root μ_3 is less than one, note that $P(0) = \beta^{-1}(1 - \epsilon_\beta/\sigma(C)) > 0$. $P(1) = 1 - \beta^{-1} - M_{11} + \beta^{-1}(1 - \epsilon_\beta/\sigma(C)) < 0$ if and only if $\tau_K \eta_F s_1 > (\beta^{-1} - 1)\epsilon_\beta$. So steady states of type "H" are locally stable and steady states of type "L" are unstable. ∎

Proof of Proposition 4:

From Lemma 1, we know that the steady state capital stock can never be less than $\underline{K}$. The (RR) curve is a vertical line until the subsistence level of consumption is reached. If $\underline{C} > F(\underline{K}, N) - \delta\underline{K}$, it imples that the (RR) curve will intersect the (BC) curve at $C = F(\underline{K}, N) - \delta\underline{K}$ and hence $K = \underline{K}$, $C = F(\underline{K}, N) - \delta\underline{K}$ will be a steady state equilibrium. Log-linearization of the first order conditions around steady state yields the following dynamical system

$$\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & M_{13} \\ -\underline{\beta}^{-1}(1-\beta)\epsilon_U & \beta^{-1} & 0 \\ s_1 & 0 & \underline{\beta}^{-1} \end{bmatrix} \begin{bmatrix} \widehat{C}_t \\ \widehat{\phi}_t \\ \widehat{K}_t \end{bmatrix}, \qquad (11)$$

where $M_{11} = \frac{\sigma(C) - \tau_K \eta_F s_1}{\sigma(C)} > 1$ and $M_{13} = -\frac{\tau_K \eta_F^{-1}\beta}{\sigma(C)} < 0$. It is easy to check that that two roots of the polynomial $\det[M - \mu I] = 0$ are strictly greater than 1 and one root is stricly less than 1. A similar argument applies for the steady state equilibrium $K = \overline{K}$, $C = F(\overline{K}, N) - \delta\overline{K}$. ∎

References

[1] Azariadis, C and Drazen, A., (1990) Threshold Externalities in Economic Development, *Quarterly Journal of Economics* 105 (2), 501-26.

[2] Becker, G. and Mulligan, C.B.,(1997) The Endogenous Determination of Time Preference, *Quarterly Journal of Economics* 112, 729-758.

[3] Becker, G., Murphy, K. and Tamura, R.(1990) Human Capital, Fertility and Economic Growth, *Journal of Political Economy* 98(5), S12-S37.

[4] Epstein, L.G. (1983) Stationary Cardinal Utility and Optimal Growth under Uncertainty, *Journal of Economic Theory* 31, 133-152.

[5] Galor, O. and Zeira, J.(1993) Income Distribution and Macroeconomics, *Review of Economic Studies* 60, 35-52.

[6] Iwai, K. (1972) Optimal Economic Growth and Stationary Ordinal Utility-A Fisherian Approach, *Journal of Economic Theory* 5, 121-151.

[7] Koopmans, T. C. (1963) Stationary Ordinal Utility and Impatience, *Econometrica* 28, 287-309.

[8] Lawrance, E.C. (1991) Poverty and the Rate of Time preference: Evidence from the panel data, *Journal of Political Economy* 99 (February), 54-77.

[9] Mantel, R. R. (1998) Optimal economic growth with recursive preferences: Decreasing rate of time preference, *Estudios de Economia* 25(2), 161-178.

[10] Obstfeld, M. (1990) Intertemporal dependence, impatience and dynamics, *Journal of Monetary Economics* 26, 45-75.

[11] Ogaki, M. and Atkeson, A. (1997) Rate of Time preference, Intertemporal elasticity of substitution and Level of Wealth, *Review of Economics and Statistics* 79 (November), 564-72.

[12] Solow, R.M. (1956) A contribution to the theory of Economic Growth, *Quarterly Journal of Economics* 70 (February), 65-94.

[13] Uzawa, H. (1968) Time preference, the consumption function, and optimum asset holdings, chapter 21 in J. N. Wolfe, ed., *Value, Capital, and Growth.* Papers in honor of Sir John Hicks. Edinburgh: University Press, 485-504.

Figure 1
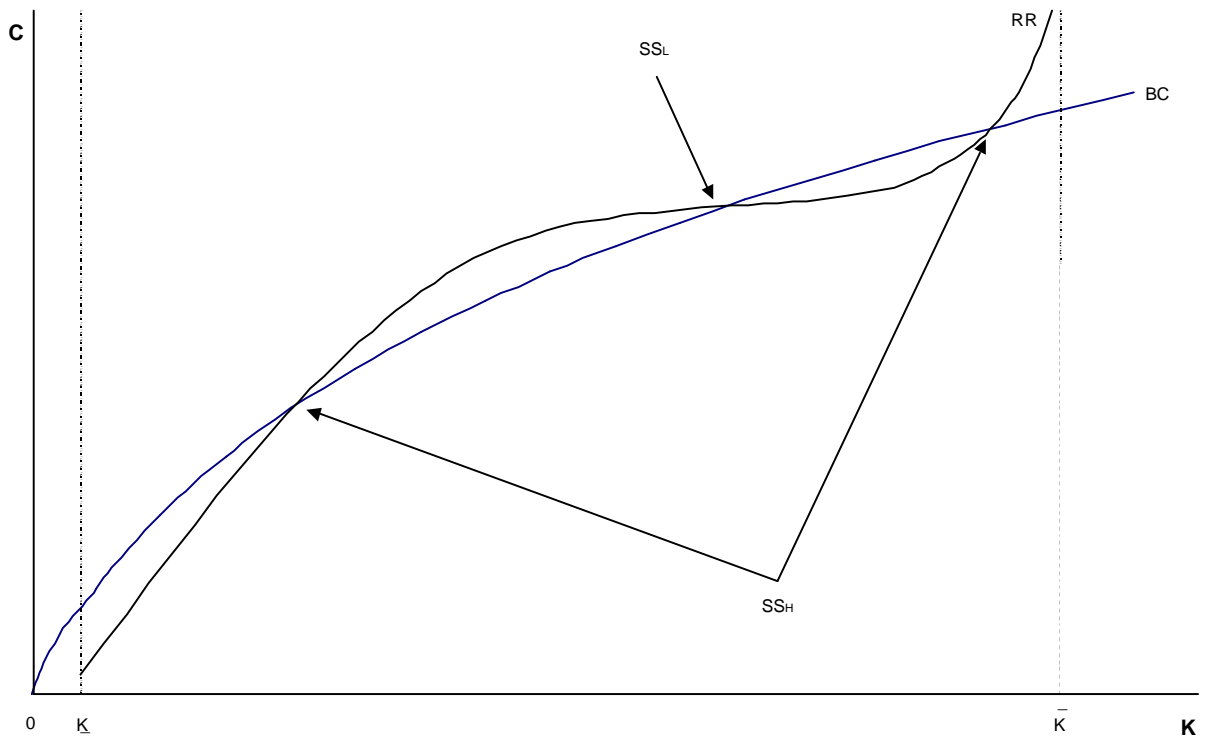
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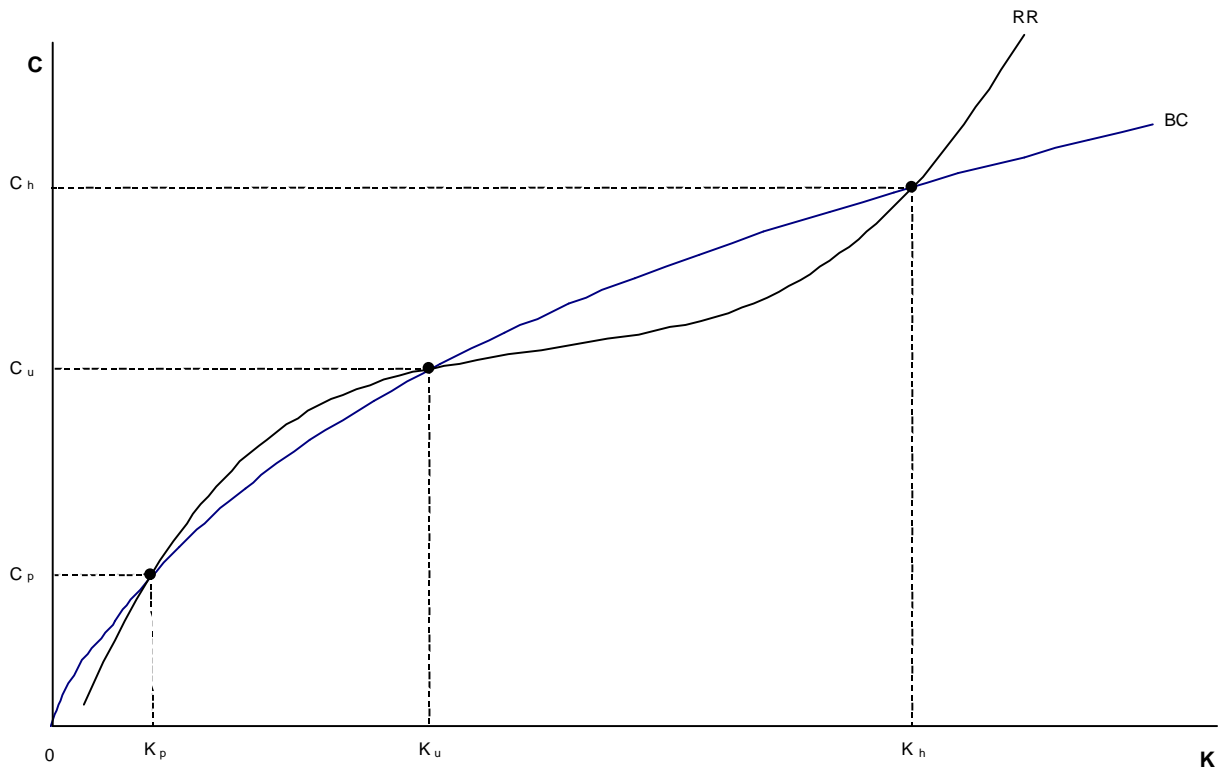
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