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Halkos, George and Kevork, Ilias
University of Thessaly, Department of Economics

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George E. Halkos and Ilias S. Kevork

Department of Economics, University of Thessaly

ABSTRACT

In this paper we show that the random walk model with drift behaves like an ARIMA (0,2,1) when its parameter θ is greater but close to –1. Using the random walk for predicting future values of an ARIMA (0,2,1) process, we find out that when θ is not so close to –1, the performance of the prediction interval for the ℓ period forecast is not satisfactory. Particularly, for ℓ large, the achieved coverage, namely, the probability the prediction interval to include the future value is quite low. Even in the cases of large samples and small ℓ, although the random walk coverage approaches that of the ARIMA, the random walk produces wider prediction intervals. This picture changes when we forecast ARIMA (0,2,1) values for θ close to –1. The random walk should be preferred as it produces on average narrower confidence intervals, and its coverage is almost the same with the nominal coverage of the ARIMA (0,2,1).

Keywords: ARIMA, Random Walk, Monte Carlo Simulations

1. INTRODUCTION

Several studies have showed that many economic time series appear to behave like random walk models or seem at least to have random walk components. The acceptance that certain economic variables follow random walk models is important as regressing one variable against the others can lead to spurious results. This is because a relationship between economic variables is concluded when in fact such a relationship does not exist. At the same time, the effects of a temporary shock will not dissipate after several years but instead will be permanent (Pindyck and Rubinfeld, 1998).

Equation (1) gives the random walk model with drift, where ε_t's are independent variables normally distributed with constant variance σ_ε^2:

$$y_t = \mu + y_{t-1} + \varepsilon_t \qquad (1)$$

Taking first differences in model (1) gives a stationary random normal process, where data will fluctuate around a horizontal level located at μ, with constant over time variance equals to σ_ε^2. Over-differencing the random walk model, namely, taking second order differences, we would expect again a stationary process with mean zero and constant variance $2\sigma_\varepsilon^2$. Empirical results based on Monte Carlo Simulations support the outcome of first differences. However, autocorrelation and partial autocorrelation plots for the second differences do not display the typical pattern for a white noise process. Figure 1 displays the representative pattern for the autocorrelation and partial autocorrelation plots met in 200 replications of size 150 generated from the population model

$$y_t = 6.5 + y_{t-1} + \varepsilon_t \qquad (2)$$

where y_o = 10000, and σ_ε = 160. Details about the testing of the adopted random number generator can be found in Kevork(1990) and Halkos & Kevork(2003)[1]. For first differences in y_t, autocorrelations and partial autocorrelations lie within the two borderlines, confirming that Δy_t follows a white noise process. On the contrary, the autocorrelation and partial autocorrelation

[1]Model (2) was extracted after analysing certain type of environmental data from Australia. The data refer to sulfur emissions (A.S.L. and Associates, 1997; Lefohn *et al.,* 1999) which includes sulfur emissions from various fuels as well as sulfur emissions from mining and smelting activities for most of the countries from 1850 to 1990.

plots of second differences indicate a typical MA(1) pattern (see Makridakis et al., 1998), as there is only one non-zero autocorrelation, and partial autocorrelations decay exponentially to zero.

Furthermore, fitting an ARIMA (0,2,1) to the 200 replications from model (2), we identified that a model without a constant term is appropriate, and the p-value of the ARIMA parameter θ was almost zero in every replication. Additionally to the previous remarks, two other important issues should be raised based on the frequency distribution of the estimated values for θ, which is presented in table 1. First, the majority of the estimates lie very close to –1, and second the variability of the estimates decreases by increasing the sample size. This discussion, therefore, leads us to impose the following questions. First, is it legitimate to consider the random walk model as a special case of an ARIMA (0,2,1)? And if this is true, how powerful are the Dickey-Fuller (DF) and the augmented Dickey-Fuller (ADF) tests under the alternative hypothesis that the population model is an ARIMA (0,2,1)? And finally how valid is for an ARIMA (0,2,1) with θ close to –1 to produce forecasts using the minimum mean-square-error prediction equation and the corresponding confidence intervals proposed by the theory of random walks?

<u>Figure 1:</u> Representative autocorrelation and partial autocorrelation plots of first and second differences of realisations generated from the model $y_t = 6.5 + y_{t-1} + \varepsilon_t$

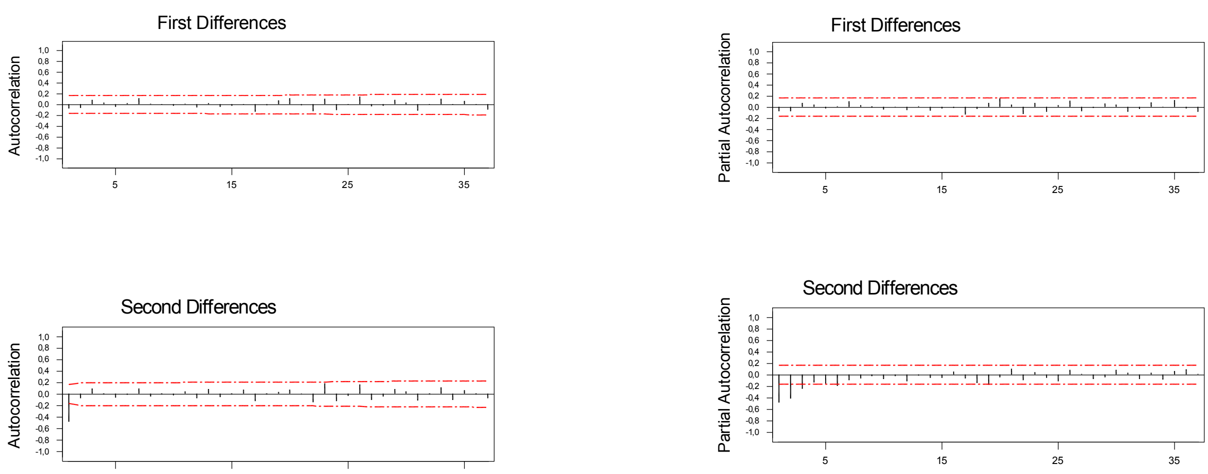


Table 1: Frequency distribution for the estimates of θ, after fitting an ARIMA (0,2,1) to replications from the random walk model $y_t = 6.5 + y_{t-1} + \varepsilon_t$

	Sample Size	
Estimates of θ	n = 50	n = 150
< -1	15	7
-1 , -0.99	2	12
-0.99, -0.97	91	174
-0.97, -0.95	65	5
-0.95, -0.90	20	2
-0.90, -0.85	2	
-0.85, -0.80	4	
-0.80, -0.75	1	

The answers to the above questions are given in the current paper. More specifically, we show that the random walk model with drift behaves like an ARIMA (0,2,1) when θ is greater but very close to –1, as first differences of the ARIMA (0,2,1) indicate a white noise process. Ignoring, therefore, second differences, and considering only first differences, we could wrongly support that the population model is a random walk instead of an ARIMA (0,2,1). At this stage, the DF and the ADF unit root tests cannot help at all, as their power is too low when the alternative hypothesis model is the ARIMA (0,2,1) with θ close to –1. Under this situation, there are two available prediction equations for forecasting future values of the process; the first one is based on the true ARIMA (0,2,1) model, and the second one on the mathematical properties of the random walk model with drift. These two methods are compared according to two criteria; the first one is the coverage that the confidence interval for the ℓ - period forecast achieves; the second one is the precision expressed in terms of the half-width of the prediction interval for the ℓ - period forecast. For the two models, that is the random walk with drift and the ARIMA, both criteria are estimated through Monte-Carlo simulations. The results of the comparisons are rather unexpected for θ very close to –1 (e.g. θ = -0.99). Although the coverage of both methods is almost the same, the random walk method produces narrower prediction intervals than ARIMA. On the other hand, for θ not close to –1 (e.g. θ = -0.90) the performance of the random walk method is very poor, from both the coverage and the accuracy point of view, especially when ℓ takes larger and larger values.

The structure of the paper is as follows. In section 1 we review the existing relative literature. In the next section, we show that the random walk with drift behaves like an ARIMA

(0,2,1) when θ approaches -1^{+}, and report the estimated power of the Dickey-Fuller and the Augmented Dickey-Fuller unit root tests. The unit root tests are evaluated under different sample sizes and different values of θ, close to –1. In section 3, we state the prediction equations and the error variance of the ℓ- period forecast for both models under consideration. Additionally, we compare the performance of the ARIMA and Random Walk methods using as criteria the coverage and the average half-length of the prediction intervals for the ℓ- period forecast. The comparison takes place for different sample sizes, and different values of θ and ℓ.

Figure 2a: Representative plots of first differences of realizations generated from the model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$

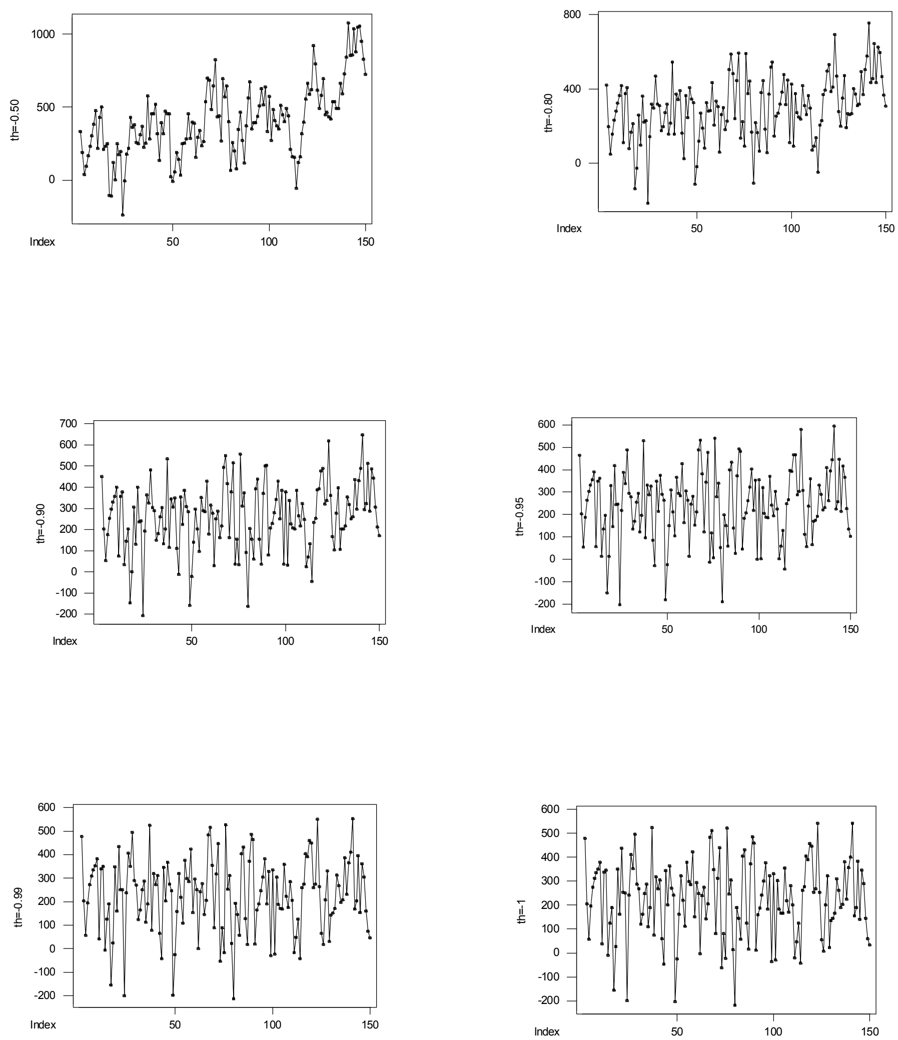
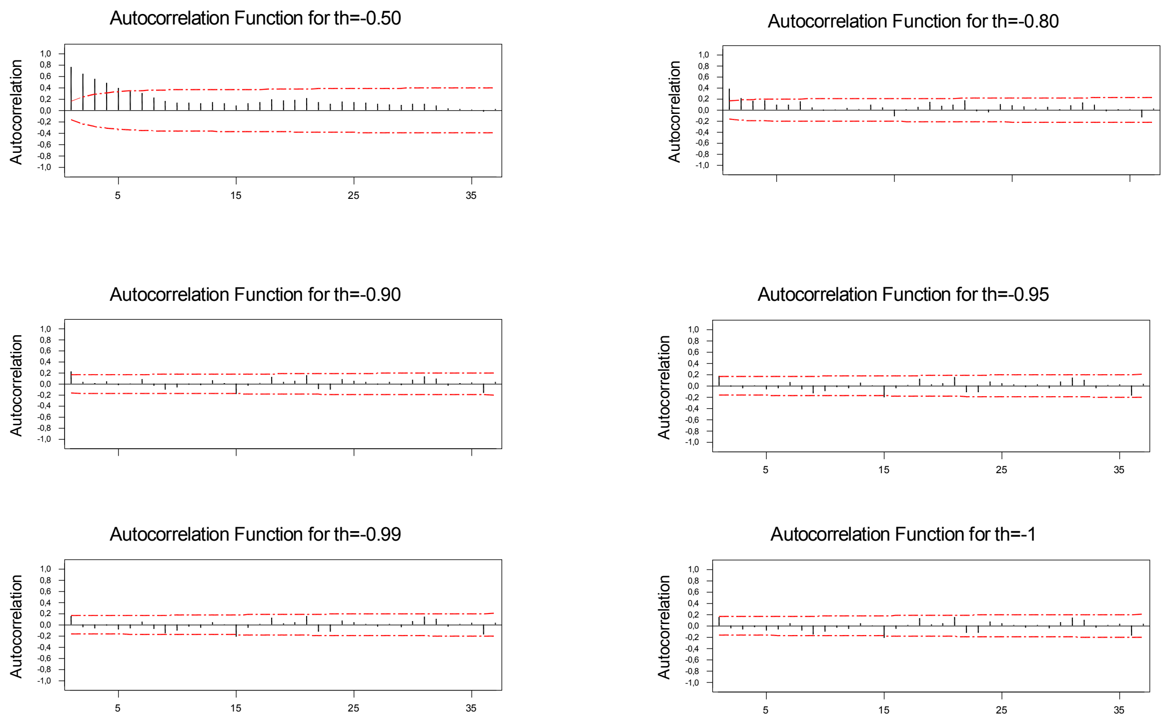


Figure 2b: Representative plots of autocorrelations for the first differences of realizations generated from the model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$



2. LITERATURE REVIEW

As there is no previous work on over-differencing a random walk, the literature review focuses on three basic issues: previous studies on forecasting an ARIMA model, over-differencing an empirical non-stationary series, and validity of Dickey Fuller and Augmented Dickey Fuller tests. Specifically, in empirical research applications of Box-Jenkins ARIMA (p,d,q) models for making valid predictions, we have to identify correctly the proper ARIMA model, which governs the behavior of the empirical time series (hereafter TS). For a non-stationary time series before identifying the parameter p and q we must identify the times the series should be differenced.

The number of times that the TS under consideration must be differenced is determined intuitively by using the autocorrelation or/and partial autocorrelation functions of the differenced series. Model identification is complicated especially if the TS under consideration is seasonal or periodic. For non-seasonal TS, manual identification may be achieved by using the autocorrelation or/and partial autocorrelation functions, the extended autocorrelation function and the smallest canonical correlation table (Tsay and Tiao, 1984, 1985; Box and Jenkins, 1970, 1994; Pankratz, 1991). The above methods seem to be ineffective in seasonal TS. In this case the identification may be performed using a filtering method (Liu, 1989; Liu and Hudak, 1992; Liu, 1999). This method is effective for automatic identification of ARIMA models for both seasonal and non-seasonal TS.

Reilly (1980) and Reynolds *et al.* (1995) developed automatic methods for identifying ARIMA models for TS. The method developed by Reynolds *et al.* (1995) employs a neural network approach and is restricted to non-seasonal TS, while the method developed by Reilly (1980) works properly for non-seasonal TS but it is less effective in the case of seasonal TS. Analytical neural network techniques have been extensively used for prediction (Chiu *et al.*, 1995; Cook and Chiu, 1997; Gao *et al.*, 1997; Saad *et al.*, 1998).

The above-mentioned methods require the existence of long TS, which are used for model development and validation before we proceed to parameter estimates and predictions. The ARIMA approach for TS predictive model development is justified in both theoretical and statistical grounds. But Makridakis *et al.* (1983) claim the complexity of these models has been an obstacle for their adoption as a forecasting tool in organizations. The one-step ahead forecast for an ARIMA (0,1,1) model is equivalent to forecasting using an exponential smoothing method when the smoothing constant leads to minimum mean square error forecast (Abraham and Ledolter, 1983). A unit root in the moving average polynomial can be interpreted in various ways depending on the modeling application. Testing for a unit root in the moving average polynomial is equivalent to testing that the series is over-differenced (Brockwell and Davis, 2002).

At the same time, the development of neural network models requires extensive network design, training and testing for model development as well as regular monitoring to assure that the model continues to realistically represent the process. Alternatively, we may use models based on fractional integration, which allow the difference parameter d to take any real value. These types of models have received attention since the seminal papers of Granger and Joyeux (1980) and Hosking (1981). This increasing application of the models is due to their ability to capture the persistent temporal dependence, which is a component in many financial and microeconomic TS as well as to their advantage to include the unit root hypothesis as a special case.

A large literature has been recently developed for analyzing TS regression with difference stationary processes. Dickey (1976) and Dickey and Fuller (1976, 1981) in their seminal papers examined the OLS estimation when the innovations in the unit root process were i.i.d. Phillips (1987) extended these results to a more general setting for the innovation process in such a way as to allow both time dependence and heterogeneity. Phillips and Perron (1988) explored data generating mechanisms with drift and trend. Phillips (1990) and Chan and Tran (1989) have explored the estimation of the autoregressive parameter and tested for a unit root when the random walk process has errors, which obey to a stable law. Phillips (1990) generalizes this case using a semi-parametric modification of the usual t-ratio.

Leybourne and Newbold (1999) using simple theoretical calculations, confirmed simulation evidences that probabilities of rejecting the null hypothesis of the Dickey Fuller and the Phillips-Perron tests differ substantially when the true generating process is the stationary second order autoregression. On the contrary, Halkos and Kevork (2003) evaluated simple versions of the Dickey-Fuller test under the null hypothesis of a random walk model or an alternative non-stationary mean reverted process. Through Monte Carlo simulations they show that, apart from few cases, testing the existence of a unit root, using both McKinnon critical values and an F test, recommended by Pindyck and Rubinfeld, they obtain actual type I error and power very close to their nominal levels.

Ahn *et al.* (2001) analyze both asymptotically and in finite sample the properties of some unit root test when the errors obey to a stable law. They consider a number of test statistics (such as the Dickey Fuller and the Lagrange Multiplier) when the data generating process is a driftless random walk and the regression model matches exactly the data generation process. Gallegari *et al.* (2003) in a similar analysis, characterize as limited both the behavior of OLS estimators of regression coefficients and the DF tests under the data generating processes usually encountered in the unit root literature (random walk with and without drift and the associated regression models with constant term, without deterministic component and with constant and time trend terms). They also investigate the consequences of the 'local to finite' variance analysis assessing that the size distortion of the DF test as the departure from the standard finite variance set up tends to decrease as the sample size tends to infinite.

Sanchez (2003) analyzes the relationship between the prediction errors of a predictor, which assumes the presence of a unit root as well as the efficient detection of such a root. Dickey and Fuller (1979) based their analysis on the asymptotic properties of the OLS estimator. Important variations of the DF tests are their extensions to other estimation methods such as Maximum Likelihood (Shin and Lee, 2000; Skin and Fuller, 1998), the generalized least squares detrending under a fixed local alternative (Elliot *et al.*, 1996; Xiao and Phillips, 1998; Hwang and Schmidt, 1996) and the weighted symmetric estimator (Park and Fuller, 1995; Fuller, 1996). Hassler and Wolters (1994) claim that the Augmented Dickey Fuller (hereafter ADF) compared to fractional alternatives loses considerable power when augmented terms are added. On the other hand, Krämer (1998) finds that ADF is consistent if the order of autoregression does not tend to infinity too fast. Bisaglia and Procidano (2002), using Monte Carlo simulations, clarify this contradiction and find that the ADF bootstrap works in general better than the ADF even if the power of the test is quite low, especially if the data generating process is a non-stationary fractional integrated one.

Finally, a number of researchers have developed tests for a single structural break with unknown break points in various dynamic models (Andrews, 1993; Perron and Vogelsang, 1992). In most cases, these tests were either designed to test for a structural change in regression coefficients with stationary series or for a unit root against a stationary alternative with an unknown single break point. The applications of these tests were extremely successful in analyzing breaking points in variables like real exchange rates, real GNP and other integrated processes (Banerjee *et al.*, 1992; Perron and Vogelsang, 1992; Zivot and Andrews, 1992).

3. THE POWER OF UNIT ROOT TESTS UNDER THE ALTERNATIVE OF AN ARIMA (0,2,1)

To examine the behaviour of the ARIMA (0,2,1) for values of θ greater but close to –1, we generated 200 replications of size 150 observations from the population model

$$y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1} \qquad (3)$$

under different values of θ. The analysis of certain type of environmental data from Australia leads us to fix the initial values in each replication at y_{-1}=45 and y_o=40. The size of the error standard deviation was set at 160. Figures (2a) and (2b) display, for different values of θ and for n=150, representative plots for the first differences in y_t and the corresponding autocorrelation functions. It is obvious that as θ approaches –1, the plot of Δy_t from an obvious non-stationary pattern is changed gradually to a stationary one. The autocorrelation plots support the previous argument. For θ quite far away from –1, the autocorrelation function has the representative pattern of a non-stationary process. On the contrary, for θ close to –1, the autocorrelation plots indicate obviously a white noise process. For all the previous cases, the autocorrelation and partial autocorrelation plots for the second differences present a typical MA(1), where there is only one significant autocorrelation at lag 1, and the partial autocorrelations decay exponentially to zero.

The previous analysis shows that when θ is negative and quite far away from –1, the plot of first differences, which will indicate a non-stationary process, combined with the autocorrelation and partial autocorrelation plots can reveal the ARIMA (0,2,1). However, for θ close to –1, examining only the plot of first differences leads to a white noise process, which itself indicate a random walk model. For such values of θ it is necessary to examine at what extent the established Dickey-Fuller (DF) and Augmented Dickey Fuller (ADF) unit root tests are able to reject the random walk null hypothesis.

Assuming that the movement of first differences in y_t are described by the following equation

$$\Delta y_t = \mu + \beta t + \gamma y_{t-1} + \varepsilon_t \tag{4}$$

and providing that the ε_t's are uncorrelated, the random walk null hypothesis H_o: γ=0 is rejected when the t-value of the Ordinary Least Squares (OLS) estimate of γ is less than the corresponding Mac-Kinnon (1991) critical value. This method is known as the simple version of the Dickey-Fuller test (1979). In the case where there is a serial correlation in ε_t, the augmented Dickey-Fuller test (ADF) should be used instead, according to which lagged difference terms of y_t should be added in (5). The parameters of the new model

$$\Delta y_t = \mu + \beta t + \gamma y_{t-1} + \lambda_1 \Delta y_{t-1} + \lambda_2 \Delta y_{t-2} + \ldots + \lambda_p \Delta y_{t-p} + \varepsilon_t \tag{5}$$

should be estimated using OLS, and the null hypothesis H_o: γ=0 is rejected again when the t value of the estimated γ is less than the Mac-Kinnon critical value.

Another method for testing whether a time series follows the random walk was offered by Dickey and Fuller (1981) and recommended by Pindyck & Rubinfeld (1998). This second alternative is based on an F test for the random walk null hypothesis Ho: β = 0, γ = 0. The test is applied by estimating first the unrestricted regression model (5), or (6) in case where ε_t's are correlated, and then the corresponding restricted ones under H_o. The null hypothesis is rejected when the calculated F-ratio

$$F = \frac{(n-k)(ESS_R - ESS_{UR})}{2 \cdot ESS_{UR}} \tag{6}$$

is greater than the critical values of a non-standard distribution tabulated by Dickey and Fuller. ESS_{UR} and ESS_R are the sum of squared residuals in the unrestricted and restricted regressions respectively, whereas k represents the number of estimated parameters in the unrestricted model.

The two versions of Dickey-Fuller test, namely, the first one based on Mac-Kinnon critical values (denoted as URT_1) and the second one based on the F-ratio (denoted as URT_2) were applied to the 200 replications from the ARIMA model (3) for n = 50, 100,150, and for θ = -0.90, -0.95, -0.99. In each combination of n and θ we fitted to the data model (5), and using Durbin-Watson statistic (DW), we tested at α=1% if the errors were uncorrelated. For each case where DW rejected the null hypothesis of no first order autocorrelation, we applied the ADF test. As it seems from table 2, in every combination of n and θ, the percentage of replications where the simple version of DF should be applied is close to1. Besides, by taking a larger sample, this percentage decreases when θ is not so close to –1, whereas, given the sample size, this percentage increases as θ is approaching –1. Finally, for the cases where the ADF test was applied, there was no need to consider in model (5) values of p greater than 1, as the estimated model with p=1 produced errors that were uncorrelated.

<u>Table 2:</u> Percentage of replications from the ARIMA model $y_t = 2y_{t-1} - y_{t-2} + \varepsilon_t + \theta\varepsilon_{t-1}$ where the DF or ADF test should be applied

Sample	θ = -0.90		θ = -0.95		θ = -0.99	
Size	DF	ADF, p=1	DF	ADF, p=1	DF	ADF, p=1
50	0.950	0.050	0.945	0.055	0.965	0.035
100	0.940	0.060	0.980	0.020	0.985	0.015
150	0.885	0.115	0.985	0.015	0.985	0.015

Let $1-\hat{\beta}_A$ be the estimated power for each combination of n and θ defined as the percentage of rejections of the random walk null hypothesis. Table 3 presents the estimated power of the two tests at nominal level of significance α_N = 1%, 5%, and 10%. For both tests, the highest power is attained at nominal level of significance 10%. Regarding URT_1, its power is very low for every combination of n and θ under consideration. Moreover, for θ not close to –1, there is a significant probability to receive a positive γ (something that contradicts the alternative hypothesis), which probability increases by taking a larger sample. On the other hand, URT_2 attains a rather acceptable power only when θ is not close to –1, and the sample size is sufficiently large. For all the other cases, the power of URT_2 is also very low, and especially for θ too close to –1, it reduces by increasing the sample size.

Table 3: Estimated power of URT_1 and URT_2 over 200 replications from an ARIMA (0,2,1) with σ_ε=160

θ	n	$Pr(\gamma>0)$	α_N = 1%		α_N = 5%		α_N = 10%	
			URT_1	URT_2	URT_1	URT_2	URT_1	URT_2
-0.90	50	0.035	0.000	0.030	0.060	0.130	0.090	0.195
	100	0.225	0.015	0.280	0.035	0.405	0.050	0.475
	150	0.290	0.025	0.480	0.040	0.555	0.055	0.610
-0.95	50	0.005	0.015	0.010	0.070	0.075	0.145	0.130
	100	0.085	0.010	0.050	0.025	0.135	0.045	0.190
	150	0.125	0.000	0.165	0.015	0.275	0.025	0.335
-0.99	50	0.000	0.020	0.015	0.080	0.055	0.120	0.115
	100	0.005	0.020	0.015	0.080	0.055	0.120	0.115
	150	0.000	0.010	0.015	0.035	0.045	0.065	0.075

4. FORECASTING USING THE RANDOM WALK MODEL

The previous analysis showed that when the parameter θ of an ARIMA (0,2,1) is close to –1, due to the low powers of the DF and ADF unit root tests, it is very likely to accept that the process of generating the data is the random walk with drift. Aiming therefore to forecast future values of the series, instead of using the real ARIMA (0,2,1) model, we shall use the prediction equation and the error variance of the ℓ - period forecast given by the random walk model. The consequences of such a decision are investigated in the current section by comparing the validity of predictions, which are produced fitting both an ARIMA (0,2,1) and a random walk model to realisations from the ARIMA (0,2,1) under different values of θ close to –1. The performance of both models is explored using two criteria. The first one is the actual coverage of the confidence interval for the ℓ - period forecast, that is, the probability the interval to include the real future value of the variable. The second criterion is the precision of the forecast, measured in terms of the half-length of the prediction interval.

Assuming that the process generating the data is the random walk model with drift, the ℓ - period forecast and its error variance are given (Pindyck and Rubinfeld, 1998) respectively by the following relationships:

$$\hat{y}_T(\ell) = E(y_{T+\ell} / y_T, y_{T-1},\dots,y_1) =$$
$$= E(y_T + \ell\mu + \varepsilon_{T+\ell} + \varepsilon_{T+\ell-1} + \varepsilon_{T+\ell-2} + \dots + \varepsilon_{T+1}) = y_T + \ell\mu \quad (7)$$
$$V[y_T(\ell) - \hat{y}_T(\ell)]_{RW} = \ell \cdot \sigma_v^2 \quad (8)$$

Estimates for μ and σ_v are obtained by fitting the model $\Delta y_t = \mu + v_t$ to the available sample using ordinary least squares.

For the ARIMA (0,2,1) model, the minimum mean-square-error forecast function is given by the set of the following three equations (Box and Jenkins, 1970):

$$\hat{y}_T(1) = 2y_T - y_{T-1} + \theta\varepsilon_T \quad (9a)$$
$$\hat{y}_T(2) = 2\hat{y}_T(1) - y_T \quad (9b)$$
$$\hat{y}_T(\ell) = 2\hat{y}_T(\ell-1) - \hat{y}_T(\ell-2) \quad (9c)$$

It is known that when the errors ε_t's of the ARIMA model are normal and given the information up to time t, the conditional distribution of a future value $y_T(\ell)$ of the process will be normal with mean $\hat{y}_T(\ell)$ and variance

$$V[y_T(\ell)] = \sigma_\varepsilon^2\left(1 + \sum_{j=1}^{\ell-1}\psi_j^2\right) \quad (10)$$

Equating the coefficients of B in

$$(1-B)^2(1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \dots) = 1 + \theta B$$

the ψ weights are computed recursively from the following equation

$$\psi_j = j(1+\theta) + 1, \quad j \ge 1 \quad (11)$$

Substituting (11) to (10), the ℓ-period forecast error variance for the ARIMA (0,2,1) would be given by

$$V[y_T(\ell) - \hat{y}_T(\ell)]_{AM} = \sigma_\varepsilon^2\left\{1 + \sum_{j=1}^{\ell-1}[j(1+\theta)+1]^2\right\} = \sigma_\varepsilon^2\left\{1 + (1+\theta)^2\sum_{j=1}^{\ell-1}j^2 + 2(1+\theta)\sum_{j=1}^{\ell-1}j + (\ell-1)\right\} =$$

$$= \ell\sigma_\varepsilon^2\left\{1+(1+\theta)^2\frac{(\ell-1)(2\ell-1)}{6}+(\ell-1)(1+\theta)\right\} \tag{12}$$

Replacing σ_ε and θ by their estimates, we take for the ARIMA (0,2,1) the corresponding estimated error variance for the ℓ - period forecast.

Equations (7), (8), (9) and (12) were used to estimate for each model the coverage that the prediction interval achieves, as well as its precision stated in terms of the average half-length. In particular, we fitted both an ARIMA (0,2,1) and a random walk to three different sample sizes n = 50, 100, 150 in each of the 200 realisations generated from model (4) setting θ = -0.90, -0.95 and –0.99. In each realisation and for each sample size, after obtaining the predicted value, using (7) for the random walk and (9) for the ARIMA, we constructed the prediction interval

$$\hat{y}_T(\ell) \pm 2\cdot\sqrt{S[\hat{e}(\ell)]}$$

where $S[\hat{e}(\ell)]$ is the estimated error variance of the corresponding model. Then the coverage of each model was estimated as the percentage of replications where the prediction interval included the real future value of the series. Additionally, taking the mean of the prediction intervals half-length over the 200 realisations, we obtained the average half-length, which, as it was mentioned, measures the precision of the forecast for each model. Tables 4 and 5 provide the estimated coverage and the average prediction interval half-length, which each model gives for different values of ℓ.

For θ not close to –1 (e.g. θ = -0.90, -0.95), the coverage that the random walk model achieves for large ℓ is quite low, even in large samples. On the contrary, for small ℓ and large samples, the coverage of the random walk model approaches that of ARIMA. However, whenever this happens, the random walk model produces on average wider prediction intervals.

The picture changes when we consider values for θ very close to –1 (e.g. θ = -0.99). For any sample size and any value of ℓ the coverage that the prediction interval of the random walk model achieves does not differ significantly from the coverage of ARIMA. Furthermore, although the real process of generating the data is the ARIMA (0,2,1), the random walk provides more precise forecasts as the prediction intervals have on average smaller half-length.

5. CONCLUSIONS

In this paper we examined the consequences of using a random walk model with drift for predicting future values of an ARIMA (0,2,1) with parameter θ close to -1. We reached this problem as we found out that, when the time series is generated by an ARIMA (0,2,1) with θ close to -1 it is very likely to accept that this series follows a random walk model with drift. The reasons for making this wrong decision are the following:

a) First differences of an ARIMA (0,2,1) with θ close to -1 indicate a white noise process
b) Second differences of a random walk model with drift display a typical MA(1) autocorrelation structure
c) The power of the Dickey-Fuller and the Augmented Dickey Fuller unit root tests under the alternative of an ARIMA (0,2,1) with θ close to -1 is very low.

After generating 200 realizations of size 150 observations from an ARIMA (0,2,1) under different values of θ close to -1, we estimated the probability the prediction intervals, constructed using both the random walk model and the ARIMA (0,2,1), to include the actual ℓ-period future value of the series. We called this probability "coverage". Apart from the coverage, we also estimated the average half-length of the prediction intervals, which was used as a measure of the precision of forecasts produced by each one of the two models. The experimentation took place using three different sample sizes, n = 50, 100, 150, and different values of ℓ.

For θ not so close to -1, the performance of the random walk model is not satisfactory. For ℓ large, the achieved coverage is quite low, while in the case of large samples and small ℓ, although the random walk coverage approaches that of the ARIMA, the random walk produces wider prediction intervals. The results were unexpected for values of θ close to -1. In such cases, the random walk model should be preferred at the stage of predicting future values of an ARIMA (0,2,1). The reason is that both models achieve the expected coverage, but the forecasts of the random walk are more precise as its prediction interval has on average smaller half-length.

It is evident therefore that at the stage of testing whether a time series follows a random walk model, it is not enough to apply only the established unit root tests. We should also examine the plots of first and second differences of the series, as well as, their autocorrelation structure using the autocorrelation and partial autocorrelation plots. The analysis in this paper showed that when first differences of the series behave like a white noise, whereas second differences display a typical MA(1) autocorrelation structure, the series is very likely to be generated by an ARIMA

(0,2,1) with parameter θ close to –1. Ignoring therefore the behavior of the second differences, and applying only the unit root tests to the levels of the series, it is very likely to reach the wrong conclusion that the series follows a random walk with drift. Whenever therefore we meet the previous patterns in first and second differences, we recommend to fit to the available sample an ARIMA (0,2,1) model without a constant term and to accept the model providing that the p-value of the parameter θ is almost 0. Whenever the estimated value of θ is very close to –1, the random walk model should be preferred in making prediction for future values of the series. On the other hand, if θ is not so close to –1, no matter where the unit root tests results in, the minimum mean-square- error prediction equation of the ARIMA (0,2,1) should be chosen instead.

Table 4: Coverage of the prediction interval of each model

n = 50	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	0.965	0.955	0.96	0.96	0.96	0.96
2	0.93	0.92	0.95	0.945	0.955	0.94
3	0.91	0.875	0.92	0.915	0.93	0.935
4	0.885	0.835	0.915	0.88	0.93	0.9
5	0.895	0.79	0.92	0.89	0.935	0.91

n = 100	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	0.94	0.92	0.94	0.935	0.945	0.945
2	0.915	0.855	0.935	0.9	0.955	0.935
3	0.955	0.81	0.965	0.935	0.97	0.97
4	0.925	0.785	0.935	0.9	0.95	0.945
5	0.925	0.77	0.935	0.89	0.935	0.94
6	0.915	0.745	0.935	0.855	0.94	0.945
7	0.925	0.73	0.94	0.88	0.945	0.945

n = 150	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	0.935	0.92	0.94	0.935	0.95	0.945
2	0.945	0.87	0.95	0.935	0.95	0.95
3	0.97	0.83	0.97	0.94	0.96	0.975
4	0.95	0.765	0.955	0.93	0.965	0.965
5	0.945	0.735	0.945	0.9	0.96	0.955
6	0.925	0.7	0.91	0.855	0.95	0.945
7	0.935	0.675	0.93	0.85	0.955	0.94
8	0.955	0.67	0.95	0.825	0.97	0.96
9	0.96	0.65	0.94	0.83	0.95	0.955
10	0.955	0.625	0.94	0.805	0.955	0.955

Table 5: Average half-length for the prediction interval of each model

n = 50	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	313.7514	313.1339	315.9355	312.2321	320.2546	315.8258
2	456.401	442.8381	454.9673	441.5628	459.7463	446.6451
3	574.7234	542.3637	567.3281	540.8018	571.5211	547.0263
4	682.0152	626.2677	666.8737	624.4641	669.7706	631.6515
5	783.2532	700.1886	758.8628	698.1721	759.9027	706.2079

n = 100	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	317.2237	329.886	317.3541	317.7403	319.8011	317.3386
2	467.5096	466.5293	457.2126	449.3526	456.9693	448.7846
3	596.0202	571.3794	570.3564	550.3423	565.4617	549.6466
4	715.5881	659.772	670.6798	635.4806	659.6595	634.6772
5	830.9254	737.6476	763.453	710.4889	745.071	709.5907
6	944.316	808.0524	851.323	778.3016	824.4907	777.3177
7	1057.036	872.7964	935.8293	840.6618	899.5608	839.5991

n = 150	θ= -0.90		θ= -0.95		θ= -0.99	
ℓ	ARIMA	Random Walk	ARIMA	Random Walk	ARIMA	Random Walk
1	318.2309	343.8388	318.1701	321.8922	319.6191	317.9393
2	469.8063	486.2615	458.7939	455.2243	455.334	449.6341
3	599.9061	595.5463	572.8106	557.5336	561.7538	550.687
4	721.3239	687.6776	674.1076	643.7844	653.3912	635.8786
5	838.7476	768.8469	767.9476	719.7728	735.8234	710.9339
6	954.4444	842.2296	856.9718	788.4716	811.8889	778.7891
7	1069.677	909.712	942.7148	851.6467	883.2615	841.1883
8	1185.214	972.523	1026.154	910.4486	951.0236	899.2681
9	1301.547	1031.516	1107.952	965.6766	1015.921	953.8179
10	1419.007	1087.314	1188.579	1017.912	1078.495	1005.412

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