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Government Bias in Education, Schooling Attainment and Growth

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Abstract

A surprising cross country stylized fact is that there is little positive correlation between growth and public spending on education. The empirical relationship suggests that a higher public spending on education tends to lower the long run per capita growth rate and schooling returns. This goes contrary to the conventional wisdom that education is a major driver of growth. In this paper, we revisit this issue and try to understand these puzzling facts in terms of an endogenous growth model. Our cross country calibration of the growth model predicts that countries with greater government involvement in education experience lower schooling efforts and lower growth.

1 Introduction

The effect of public expenditure on educational attainment and growth is an unresolved issue. A recent flow of literature questions the effect of gov-

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ernment involvement in education on the educational attainment of pupils. Based on US and international data Hanushek (2003) persuasively argues that a resource based policy of the government has little effects on pupils' educational attainment. In majority of the cases, the active involvement of the government in the education sector is deemed to be a failure. Blankenau and Camera (2009) show that in a strategic environment when a government spends more on education sector, students may underinvest in efforts. If the government involvement in education has such a questionable effect on pupils' educational attainment, the spillover effect of this on economic growth also becomes debatable.[1]

The objective of this paper is to ask specifically in the context of an endogenously growing economy why a resource based public education policy can be counterproductive. There is a growing literature that explores the link between public expenditure on education and growth (Glomm and Ravikumar, 1992, 1997, 1998). Our paper is closely related to Blankenau and Simpson (2004) and Blankenau et al. (2007) who address the relationship between growth and public education spending. Our model, however, differs from both these papers in several important ways. First, besides using a bigger sample of countries than Blankenau et al. (2007), we focus on household's time allocation between schooling and work as an important determinant of long run growth and public education spending. Blankenau et al. (2007), on the other hand, do not model time allocation to schooling. Second, unlike Blankenau and Simpson (2004) and Blankenau et al.(2007), we focus on the proactive role of the government as a factor determining the public education spending behavior. Third, we also analyze the relationship between public education spending and schooling returns while Blankenau and Simpson (2004) and Blankenau et al.(2007) do not explore any such relationship. Finally, Blankenau et al. use an overlapping generations model

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[1] Pritchett (2001) shows significant skepticism about the positive effect of education spending on growth. Sylwester (2000) demonstrates that the contemporaneous education expenditure has a negative effect on growth. Temple (2001) revisits the empirical evidence and shows with alternative statistical procedures that the link between education expenditure and growth is tenuous. Blankenau et al. (2007) argue that government spending on education has insignificant effect on growth for low and middle income countries while it has a positive effect on rich countries.

while we use an infinite horizon model making the model more naturally amenable to calibration using the cross country data.

The principal insight from our endogenous growth model is that when the government spends more on teachers and pupils, two opposing effects are at work. First is a positive complementarity effect because of the government provision of intermediate inputs in the form of teacher salary and other school aids. Second, there is a negative distortionary effect due to the tax on the non-education sector (goods sector) to finance education spending. The latter discourages physical capital investment and it lowers the capital stock in the goods sector. Due to diminishing returns, the return on physical capital thus rises. Households respond to this by reallocating more time to the goods sector and less time to schooling to rebalance the return on physical capital and human capital. This crowding out effect on schooling time is the key factor that lowers the long run growth rate in the economy. The relative strengths of these two opposing forces depends critically on the proactive role of the government in the education sector which we call *government bias* in education.

Our approach is novel because we highlight the role of government bias in education as a central determinant of the cross country relationship between growth and public spending on education. This government bias is an institutional feature which has so far been overlooked in the literarture. We formulate this government bias in terms of a simple neoclassical schooling technology where public spending appears as an intermediate input together with private schooling efforts in education for creation of human capital. The relative importance of public and private inputs in the production of human capital is treated as a technology parameter, which we characterize as the government bias in education. Our central question is then to understand how this government bias in education influences public education spending propensity, pupils' incentive to learn, schooling returns and ultimately the economic growth of a nation.

Using our calibrated growth model, we estimate this government bias for a wide range of countries. Our model predicts that countries with a greater government bias in education experience a greater public education share in GDP, lower private schooling effort and a lower long run growth. Thus

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even though there is a technological complementarity between private and public inputs in the human capital production, a greater government bias in the education sector crowds out private schooling efforts for majority of countries in our sample.

Our model also provides lessons for optimal public spending policy in education. A benevolent government that aims to promote societal welfare should spend more on education in an economy where the government involvement in education is already present in terms of a rich public educational infrastructure. Thus contrary to conventional wisdom, a blanket increase in government spending on education may not necessarily promote growth and welfare in all countries.

The paper is organized as follows. The following section presents some key development facts to motivate our growth model. Section 3 lays out an endogenous growth model and characterizes the balanced growth properties of model variables. Section 4 reports the quantitative implications of the model based on cross country calibration. Section 5 discusses welfare and policy implications. Section 6 concludes. Construction of data and details on derivation of equilibrium conditions and solution procedure of the model are given in the Appendix.

2   Some Development Facts

Figure 1 plots the average per capita growth rate (1970-2005) against the public education spending ratio for 166 countries. Given the well known cross country volatility of growth rate, a negative relationship holds. The correlation coefficient is -0.15 which is statistically significant at the 5% level.

A clearer relationship emerges if countries are broadly grouped. Figure 2 plots the cross country per capita growth rate and education spending ratio averaged over the period 1970-2005 for 18 groups of countries sorted by per capita income. The correlation coefficient is -0.38. Higher spending ratio generally lowers growth except for countries at the very top education spending ratio. At this very top end, a higher spending ratio tends to raise the per capita growth. The relationship between education spending ratio

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resembles nonlinearity as pointed out by Blankenau et al. (2007).

Figure 3 plots the rate of return on education against the education spending ratio for 48 countries for which the rate of return data are available.[2] The correlation coefficient is -0.15 and statistically signficant at the 5% level.

The essence of these cross country stylized facts can be summarized as follows. Growth rate and return on education are lower in countries where the government spends more on education. The following section presents a growth model to understand these stylized facts.

3   The Model

The model is an adaptation of the Lucas-Uzawa (Lucas, 1988) model. There are two sectors, goods and education. A fixed time (normalized at unity) is allocated between schooling and goods production. Time l_{Ht} allocated to schooling at date t creates effective labour or human capital (h_{t+1}) in the following period. The productivity of schooling effort which is the same as the quality of schooling depends on **pupils'** learning ability parameter (A_H) which is assumed to be a country-specific parameter, and the public spending on education (g_t).

The schooling technology thus follows the Cobb-Douglas form as in Glomm and Ravikumar (1997) and Blankenau et al. (2007):

$$h_{t+1} = (1 - \delta_h)h_t + A_H g_t^{\eta}(l_{Ht}h_t)^{1-\eta} \tag{1}$$

The schooling technology (1) is actually a production function for new human capital. In a similar vein as in Barro (1990), the inputs in the production function are public spending one education g_t and private spending $l_{Ht}h_t$. The latter is the imputed (opportunity) cost of diverting time from goods production to human capital production.[3]

The schooling technology parameter η is of central interest in this paper.

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[2]Details of all the data sources are discussed in the appendix.

[3]If an explicit labour market is in place, this opportunity cost will be measured by the foregone wages due to spending time at school.

η is simply the elasticity of the flow of knowledge with respect to government spending on intermediate input to human capital production (e.g. teachers). For example, a one percent increase in teacher's salary creates a higher percent increase in pupils' attainment in an economy with a better educational infrastructure (e.g. school library, internet facility). Viewed from this perspective, η can be interpreted as the infrastructural role of the government in the education sector. In other words, η is higher in countries where the government had already taken a proactive role in the past by investing a lot of resources to build such a rich infrastructure. Rather than explicitly modelling the government's infrastructural investment in education, we treat this infrastructural role of the government as a country-specific technology parameter. Absent such a government role in the education (η equals zero), the schooling technology reverts to the Lucas (1988) form. Given this interpretation, hereafter we label the schooling technology parameter (η) as the *government bias in the education sector*. We find that this government bias parameter is quite fundamental in determining the cross-country relationship between public spending on education and growth.

Final goods (y_t) are produced with the help of human and physical capital via the Cobb-Douglas production technology:

$$y_t = A_G k_t{}^\alpha (l_{Gt} h_t)^{1-\alpha} \tag{2}$$

where l_{Gt} (that equals $1 - l_{Ht}$) is the remaining time allocated to the production of goods and A_G is a constant total factor productivity (TFP) in the goods sector.[4]

The investment goods technology is specified as follows:

$$k_{t+1} = (1 - \delta_k)k_t + i_t^k \tag{3}$$

where δ_k is a fixed rate of depreciation of physical capital.

The government finances the education spending (g_t) by levying a proportional tax (τ_t) on goods sector output, y_t . In other words, the government budget constraint is:

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[4]We assume that leisure time is fixed.

$$g_t = \tau_t y_t \tag{4}$$

The representative household takes the sequence of tax rates $\{\tau_t\}$ as given and chooses the sequences $\{c_t\}, \{i_t\}, \{l_{Ht}\}$, that maximize

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to the resource constraint:

$$c_t + i_t = (1 - \tau_t)y_t \tag{5}$$

and the schooling technology (1).

Given that the private sector behaves optimally, the government sets the tax rates $\{\tau_t\}$ to maximize societal welfare.

3.1 Balanced Growth Properties

Since the central goal of the paper is to understand the role of public spending in education in determining the dispersion in cross country long run growth rates, we assume that each country has already embarked on a balanced growth path. We thus focus on the balanced growth property of our proposed economy. We have the following proposition.

Proposition 1 *Along the balanced growth path, the optimal share of public spending in GDP is given by:*

$$\tau = \frac{\frac{1-\alpha}{1-\eta} \cdot \frac{\eta l_H}{l_G}}{1 + \frac{1-\alpha}{1-\eta} \cdot \frac{\eta l_H}{l_G}} \tag{6}$$

Proof. Appendix. ∎

Along the balanced growth path, the time allocations to goods and schooling sectors are stationary which we denote as l_H and l_G dropping the

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time subscripts. The ratios of output to capital (y_t/k_t) and the physical to human capital (k_t/h_t) are also constants. Proposition 1 establishes that the share of education in GDP is also constant. In other words, the steady state government spending share in GDP is given by:

$$\frac{g_t}{y_t} = \tau \tag{7}$$

Define the gross balanced growth rate as γ. There are three key balanced growth equations. Based on the first order condition for the physical capital stock we get:

$$\gamma = \beta \left[(1 - \tau)(\alpha y_t/k_t) + 1 - \delta_k \right] \tag{8}$$

Based on the first order condition for the human capital stock, one gets:

$$\gamma = \beta[1 - \delta_h + A_H(1 - \eta)\tau^\eta l_H^{-\eta}(y_t/h_t)^\eta] \tag{9}$$

Finally, using the human capital technology (1), we get a third balanced growth equation:

$$\gamma = 1 - \delta_h + A_H\tau^\eta l_H^{1-\eta} A_G^\eta l_G^{(1-\alpha)\eta}(k_t/h_t)^{\alpha\eta} \tag{10}$$

Given the production function (2), these three equations solve for three unknowns, namely k/h, l_H and γ. The appendix provides the details of the derivation.

3.2   Implications for Return to Schooling

The model establishes a tight link between private schooling efforts, **growth rates** and returns to schooling. Along the balanced growth path, the return to human capital (R^h) is given by:

$$R^h = 1 - \delta_h + MPH^E \tag{11}$$

The private agents allocate time between goods production and schooling to equate the marginal returns to physical and human capital. In other words, we have the fundamental arbitrage condition that the return on human

capital must balance the after tax return on physical capital.:

$$R^h = (1 - \tau)(\alpha y/k) + 1 - \delta_k \qquad (12)$$

The appendix shows the details of the derivation of (11) and (12).

4 Cross country calibration of government bias in education

In this section, we report the results of a cross country calibration experiment. Our basic premise is that all countries share the same structural parameters except the government bias parameter η and the level of cognitive skill, A_H. This premise is based on the evidence that the education spending ratio differs a lot across countries (see Fig 1). The difference in learning ability across countries is also well documented by Hanushek and Woessman (2008). Other structural parameters are fixed at some baseline levels. The capital share parameter α and the rate of depreciation of physical capital, δ_k are fixed at the conventional levels 0.36 and 0.1 (Prescott, 1986). The remaining parameters are fixed at $\beta = 0.94$, $A_G = 3.9$ and $\delta_h = .05$ with a goal to arrive at reasonable cross-country steady state distribution for average time allocation between work and schooling (l_H), the average level of cognitive skill, A_H and the average government bias in the education η. Table 1 summarizes the baseline parameter values.

Table 1: Baseline estimates of structural parameters common to all countries

α	β	δ_k	δ_h	A_G
0.36	0.94	0.1	0.05	3.9

In order to get cross country estimates of the two crucial educational technology parameters, η and A_H, we follow a method of reverse engineering. We focus on 4 key steady state equations, (6), (8), (9) and (10). We fix

$\alpha, \beta, A_G, \delta_k, \delta_H$ at the baseline levels. We assume that all 166 countries in our sample have embarked on various steady states. This means that for each country γ and τ are equal to the historical average growth rate of GDP and education share in GDP. Given that all countries share the same baseline estimates of the structural parameters as shown in Table 1, it means that for each country in our sample, we have four equations in four unknowns, namely η, A_H, l_H and k/y. These four unknowns can be thus backed out from the model. Doing so we make our growth model perfectly match the cross country growth rates and education shares. We then back out the two crucial education technology parameters, η, A_H as well as the time to schooling l_H and the capital-output ratio (k/y) for each of the countries in our sample.[5]

Table 2 reports the mean and standard deviation of the cross country distribution of the four key steady state variables. The average time to schooling l_H is 0.47 which is similar to the estimate of Gomme and Rupert (2007). A cross country average A_H of 0.15 is in the vicinity of the value calibrated by Basu et al. (2010) and an average $\eta = 0.07$ is close to the cross country average share of public spending on education in GDP (which is 0.05) for our sample. The cross country average capital:output ratio of 1.91 is in line with the estimate of capital:output ratio for the US based on a Solow growth model (Mankiw, 2003). The cross country dispersion is highest for the capital-output ratio which is not surprising given the enormous disparity in the per capita output across countries.

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[5]Although some cross country data are available for time to schooling (e.g. Barro and Lee, 1994), we use our model to generate cross country estimates of l_H. The reason is that the variable l_H in our model more accurately represents schooling efforts which cannot be fully reflected by the cross-country data on schooling time. For example, parents might spend a significant amount of time in tutoring their children which means a lot of schooling efforts. It is hard to find cross country data for this kind of efforts. In a similar vein, Blankenau and Camera (2009) argue that schooling attendance may be the same across countries but efforts may differ. We also rely on the model to generate cross country series for capital/output ratio (k/y) and pupil's learning ability A_H because reliable cross-country series for capital stock are hard to get. About learning ability, the closest series available are Hanushek and Weissman (2006). These estimates are, however, based on standardized test scores which are the end results of learning ability and even debatable proxy for learning ability.

Table 2: Cross country steady state distribution of the education technology

	l_H	A_H	η	k/y
Mean	0.47	0.15	0.07	1.91
Std Deviation	0.07	0.02	0.03	0.21

Table 3 reports the summary description of η for broad groups of countries classified in regions. The range of regional variation of η is from 0.036 to 0.096 which is substantial. The government bias is the highest in the North American region where Canada provides the lead (0.12). Next to North America are OECD countries. The government bias is the lowest in the South Asian region. The bottom row of the table reports the historical average growth rate of GDP for each region. A sharp negative regional relationship emerges between the government bias in education and the long run average growth rates. (see Figure 4).

Table 3: Regional Features of the Government Bias in Education

	Asia	Europe	Latin America and Caribbean	Middle East and North Africa	OECD	North America	South Asia	Afric
η	0.057	0.078	0.068	0.063	0.08	0.096	0.036	0.077
g(%)	2.830	2.111	1.478	1.518	2.259	1.756	3.811	0.731

Table 4 summarizes the cross correlations of the key macroeconomic variables of interest based on our calibrated growth model for the full sample of 166 countries. The correlation between growth and government bias is -0.46 which further confirms the negative relation reported in Figure 4. Since schooling return is proportional to growth rate (see equation (A.13)) and growth rate negatively varies with η, the immediate implication is that returns to schooling also covary negatively with η. The model thus reflects the stylized facts reported in section 2 that there is a negative cross country correlation between growth rate and spending ratio as well as schooling return and spending ratio.

Table 4: Cross country correlations of the key macroeconomic varaibles

	l_H	A_H	η	k/y	τ	γ	R^h
l_H	1						
A_H	0.92	1					
η	-0.64	-0.39	1				
k/y	-0.94	-0.96	0.35	1			
τ	-0.14	0.12	0.81	-0.19	1		
γ	0.93	0.99	-0.46	-0.95	0.01	1	
R^h	0.93	0.99	-0.46	-0.95	0.01	1	1

4.1 Government bias in education and schooling efforts

The cross country correlation between η and l_H is -0.64 that is found to be significant at the 5% level. Figure 4 plots l_H against η for all 166 countries in the sample. The scatter plot also confirms the strong negative relationship between η and l_H. Countries with a greater government bias in education experience crowding out of private schooling efforts.

Not surprisingly, the model predicts that the cross country correlation between the education share τ and government bias, η is strongly positive (correlation coefficient is 0.84). Figure 5 plots the government bias in the education sector against the education share. Governments in countries with a greater government bias in education spend more on education as a fraction of GDP.

These two cross-country plots shed light on the finding of Hanushek (2003) that greater public resources in education does not help promote pupils' learning incentive. In fact, a greater government involvement in education crowds out private schooling efforts. Although this crowding out effect is apparently counterintuitive, a closer examination reveals that a fundamental arbitrage condition (12) is at work in explaining this. Everything else equal, an increase in η raises the optimal tax rate τ to finance education spending. Since the goods sector is more capital intensive than the education sector, this increase in tax lowers the ratio of physical to human capital in the economy which raises the marginal product of physical capital. If agents can alter the

time allocation, they will allocate more time to work and less to schooling to preserve the arbitrage condition (12).

For 100% depreciation of human capital ($\delta_h = 1$), an analytical expression for l_H exists and it confirms this intuition. Use (A.13) and (10) to get:

$$l_H = \beta(1 - \eta) \qquad (13)$$

which upon substitution in (6) yields

$$\tau = \frac{\beta(1 - \alpha)}{(1 - \beta)\eta^{-1} + \beta + (1 - \alpha)\beta} \qquad (14)$$

It is straightforward to verify from (13) and (14) that countries with a greater government bias in education (higher η) experience a crowding out of private schooling efforts and also a greater share of GDP in education.

5   Welfare Implications

Does an increase in public spending on education necessarily make the society worse off in the long run? In the appendix, we have shown that the education spending share τ that maximizes growth also maximizes societal welfare. Thus any cross country variation of τ that lowers growth rate would also lower societal welfare. Growth can be thus a sufficient statistic of a measure of country's welfare in our representative agent growth model.

Given this connection between growth and welfare, the first question that we ask is: Do countries with a greater government bias in education (η) necessarily experience lower welfare? Our calibrated model based on cross country evidence suggests that this is indeed the case. A higher η lowers long run per capita growth rate of a country and thus lowers welfare. One has to be careful though to generalize this result. An increase in η has two opposing effects on growth, (i) crowding out effect on schooling effort, (ii) complementarity effect. The former lowers growth while the latter promotes growth. In our cross country calibration, we find that (i) is stronger than (ii). However, if one allows a large variation of η, it is possible that for countries with a very high η, greater government involvement might be beneficial for

growth. Figure 10 plots growth against education share when η is allowed to vary from 0.05 to 0.4. The relationship is U shaped. For very high education spenders, an increase in government involvement may be beneficial for growth. This nonlinear relationship between growth and education spending is also consistent with Blankenau and Simpson (2007) who find that higher education spending promotes growth particularly in rich countries.

6   Conclusion

The effect of public education spending on growth is an empirically unsettled issue. A plethora of studies document that public education spending does not help promote growth. Our cross country stylized facts also support this finding. Growth and schooling returns are in fact lower in countries with a higher ratio of public spending to GDP except for very high education spenders. In this paper, we reopen this issue and investigate this within an endogenous growth framework. Public spending on education appears directly in the human capital technology. The relative intensity of public and private education spending, which we call government bias in education, appears to be a fundamental determinant of cross country dispersion in long run growth and schooling returns. A higher government bias has conflicting effects on growth. On the one hand, it lowers growth by crowding out private schooling efforts. On the other hand, it promotes growth through the complementarity channel. The latter effect is stronger in countries which have historically a greater government bias in education. Based on our growth model, we estimate this government bias parameter for a wide range of countries and find that the government bias in education is generally higher in rich countries.

The policy implications of our analysis is that an increase in public spending on education without an adequate infrastructural support may not necessarily be beneficial for the society. Currently 80 percent of education spending goes to teacher salaries[6] which is likely to be spent more on consumption than in investment. For the complementarity effect of public spending to

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[6]Based on UNECSCO database on education available at http://www.uis.unesco.org/.

dominate, a nation may need a greater educational infrastructure. This infrastructural role of the government in education is an area worth exploring in future research.

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A   Appendix

A.1   Data Sources

Database for PPP adjusted per capita income and education spending ratios were taken from the World Development Indicators

(available at http://www.esds.ac.uk/international/).

Our sample period generally ranges from 1970 to 2008. For some countries there are paucity of data from 1970 in which case we changed the sample

period from 1980 onward. The time average of the annual growth rate of per capita income, share of public spending in GDP were constructed for each country in our sample of 166 countries.

Rate of return on education are not available for all countries in our sample. Psacharopoulos and Patrinos (2004) and Pritchetts (2001) compiled rate of return on education series for 48 countries based on Mincer (1974) type analysis. These series are available on the World Bank web site (go.worldbank.org/W0WKLRECX0). We could use their rates of return on schooling data for a sample of 48 countries and for 18 groups in WDI as shown in Figures **3** and **2** respectively.

A.2 First order conditions

Let λ_t, μ_t, be the Lagrangian multipliers associated with the flow budget constraint (5), human capital technology (1) respectively.

The Lagrange is:

$$L = \sum_{t=0}^{\infty} \beta^t U(c_t) + \sum_{t=0}^{\infty} \lambda_t [A_G(1 - \tau_t)k_t^\alpha (l_{Gt}h_t)^{1-\alpha} + (1 - \delta_k)k_t - c_t - k_{t+1}]$$
$$+ \sum_{t=0}^{\infty} \mu_t [(1 - \delta_h)h_t + A_H g_t^\eta (l_{Ht}h_t)^{1-\eta} - h_{t+1}]$$

First order conditions are:

$$c_t : \beta^t U'(c_t) = \lambda_t \tag{A.1}$$

$$k_{t+1} : -\lambda_t + \lambda_{t+1}[(1 - \tau_{t+1})\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_k] = 0 \tag{A.2}$$

$$h_{t+1} : \quad \mu_t = \mu_{t+1}[1 - \delta_h + A_H g_{t+1}^\eta (1 - \eta)h_{t+1}^{-\eta} l_{Ht+1}^{1-\eta}] \tag{A.3}$$
$$+ \lambda_{t+1}[A_G(1 - \tau_{t+1})(1 - \alpha)k_{t+1}^\alpha h_{t+1}^{-\alpha} l_{Gt+1}^{1-\alpha}]$$

$$l_{Gt} : \lambda_t (1-\alpha)(1-\tau_t) A_G l_{Gt}^{-\alpha} k_t^{\alpha} h_t^{1-\alpha} - \mu_t(1-\eta) g_t^{\eta} A_H h_t^{1-\eta} l_{Ht}^{-\eta} = 0 \quad \text{(A.4)}$$

$$\tau_t : \lambda_t y_t = \mu_t A_H \eta \tau_t^{\eta-1} (h_t l_{Ht})^{1-\eta} y_t^{\eta} \quad \text{(A.5)}$$

A.3 Proof of Proposition 1

The expression for the optimal tax rate in proposition 1 immediately follows after substituting out λ_t/μ_t from (A.4) and (A.5). One gets the optimal tax rate:

$$\tau_t = \frac{\frac{1-\alpha}{1-\eta} \cdot \frac{\eta l_{Ht}}{l_{Gt}}}{1 + \frac{1-\alpha}{1-\eta} \cdot \frac{\eta l_{Ht}}{l_{Gt}}}$$

Next, we exploit the fact that along the balanced growth path, the time allocations to goods and schooling (l_{Gt} and l_{Ht}) are constants. Unless the time allocations are constant, a constant balanced growth rate does not exist because the marginal product of capital will be time varying (see (A.2)). Since l_{Gt} is a constant, this means that the optimal tax rate τ_t is also stationary.

A.4 Derivation of the Balanced Growth Equations

Hereafter, we drop time subscripts for variables which are stationary along the balanced growth path. To prove (8), use (A.1) and (A.2).

To get (9), rewrite (A.3) as:

$$\begin{aligned}
\frac{\mu_t}{\lambda_t} &= \frac{\mu_{t+1}}{\lambda_{t+1}} \cdot \frac{\lambda_{t+1}}{\lambda_t} [1 - \delta_h + A_H g_{t+1}^{\eta}(1-\eta)(1 - l_{Gt+1})^{1-\eta} h_{t+1}^{-\eta}] \quad \text{(A.6)} \\
&\quad + \frac{\lambda_{t+1}}{\lambda_t} \{A_G(1-\alpha)(1-\tau_{t+1}) k_{t+1}^{\alpha} h_{t+1}^{-\alpha} l_{Gt+1}^{1-\alpha}\}
\end{aligned}$$

Using (A.1), check that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{\beta c_t}{c_{t+1}}$. Use (A.5) to substitute out $\frac{\mu_t}{\lambda_t}$ and also use the balanced growth condition $\frac{\lambda_{t+1}}{\lambda_t} = \beta/(1+g)$ which upon substitution in (A.6) yields:

$$\gamma = \beta[1 - \delta_h + A_H(1-\eta)\tau^\eta l_H^{-\eta}(y_t/h_t)^\eta] \tag{A.7}$$

To get (10) use (1), (2) and (4).

A.5   Derivation of (11) and (12)

Think of human capital as a Lucas (1978) tree with valuation q_t^h which is akin to Tobin's q of physical capital. This valuation is driven by the return and opportunity cost of going to school. The value of human capital is the same as the ratio of the shadow price of consumption to that of investment in schooling. In other words,

$$q_t^h = \frac{\mu_t}{\lambda_t} \tag{A.8}$$

where μ_t and λ_t are the Lagrange multipliers associated with the schooling technology (1) and the flow resource constraint (see 5).

Using the Euler equation for human capital (see (A.6)), one gets the following valuation equation for the human capital:

$$q_t^h = m_{t+1}[\{q_{t+1}^h\{1-\delta_h+A_H g_{t+1}^\eta(1-\eta)(1-l_{Gt+1})^{1-\eta}h_{t+1}^{-\eta}\}+\{A_G(1-\tau_{t+1})(1-\alpha)k_{t+1}^\alpha h_{t+1}^{-\alpha}l_{Gt+1}^{1-\alpha}\}] \tag{A.9}$$

where m_{t+1} is the intertemporal marginal rate of substitution in consumption given by λ_{t+1}/λ_t.

Next verify from (A.4) in the appendix that

$$q_t^h = \frac{(1-\tau_t)MPH_t^G}{MPH_t^E} \tag{A.10}$$

where MPH_t^G and MPH_t^E are the marginal products of effective labour in the goods and education sectors respectively.

Rewrite (A.9) as

$$q_t^h = m_{t+1} \left[q_{t+1}^h (1 - \delta_h + l_{Ht+1} MPH_{t+1}^E) + l_{Gt+1}(1 - \tau_{t+1}) MPH_{t+1}^G \right]$$
(A.11)

The valuation equation for human capital looks similar to a Lucas (1978) tree valuation equation. The value of this tree at date t is the discounted next period marginal product of human capital in the goods sector, $l_{Gt+1} MPH_{t+1}^G$ and the imputed next period value of unused portion of the tree $(1 - \delta_h) q_{t+1}^h$ plus the replenishment of it, $l_{Ht+1} MPH_{t+1}^E$ due to new education.

The return to schooling (R_{t+1}^h) is thus given by:

$$R_{t+1}^h = \frac{\left[q_{t+1}^h (1 - \delta_h + l_{Ht+1} MPH_{t+1}^E) + l_{Gt+1}(1 - \tau_{t+1}) MPH_{t+1}^G \right]}{q_t^h}$$
(A.12)

Along the balanced growth path, q_t^h and τ_t are stationary. Using (A.10) one obtains (11).

Using (11) one can rewrite the balanced growth equation (9) as follows:

$$1 + g = \beta R^h$$
(A.13)

Comparison of (8) with (A.13) one obtains the arbitrage condition (12).

A.6   Proof of Proposition 1

Proposition 2 *The tax rate that maximizes growth also maximizes the long run welfare.*

Proof. The steady state welfare can be written as:

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$$W_t \;=\; \sum_{j=0}^{\infty} \beta^j \ln c_{t+j} \qquad (A.14)$$

$$= \frac{\ln c_t}{1-\beta} + \frac{\beta}{(1-\beta)^2}\ln\gamma$$

$$= \frac{\ln k_t}{1-\beta} + \frac{\ln(c_t/k_t)}{1-\beta} + \frac{\beta}{(1-\beta)^2}\ln\gamma$$

Use the resource constraint (5) and the balanced growth condition to verify that

$$\frac{c_t}{k_t} = \frac{(1-\tau)y_t}{k_t} + (1-\delta_k) - \gamma \qquad (A.15)$$

Next plug (8) into (A.15) to find

$$\frac{c_t}{k_t} = \frac{1-\alpha\beta}{\alpha\beta}\gamma - \frac{(1-\delta)(1-\alpha\beta)}{\alpha\beta} \qquad (A.16)$$

which upon substitution in (A.14) yields

$$W_t = \frac{\ln k_t}{1-\beta} + \ln(\gamma - (1-\delta)) + \frac{\beta}{(1-\beta)^2}\ln\gamma + \ln\left(\frac{1-\alpha\beta}{\alpha\beta}\right) \qquad (A.17)$$

This shows that the steady state welfare is positively related to growth rate.

Thus the growth maximizer tax rate is also a welfare maximizer. ∎