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USEFULNESS OF IMPERFECT MODELS FOR THE FORMULATION OF STABILIZATION POLICIES

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This article describes a method for evaluating the performance of the optimal policy derived from an econometric model. The theoretical framework is applied to determine the usefulness of two simplified models for stabilization policy. Here, even though one of the models differs from the other in terms of reduced forms and multipliers, it can still be used effectively as a guide to policy even if the world is accurately described by the other model.

I. INTRODUCTION

Econometric models are widely used to forecast the national economy. Are they accurate enough to be used by the government authorities for the formulation of macroeconomic policies? What kind of accuracy is required for them to be useful as a guide to policy? This paper provides a theoretical framework to answer this accuracy equation, and applies it to ascertain the usefulness of two simplified models in the determination of stabilization policies.

In a review article on the comparative forecasting abilities and the multiplier effects of the major U.S. econometric models currently in use, Carl Christ writes [5, p. 54], "though the models forecast well over horizons of four to six quarters, they disagree so strongly about the effects of important monetary and fiscal policies that they cannot be considered reliable guides to such policy effects, until it can be determined which of them are wrong in this respect and which (if any) are right." The method of this paper can be applied to decide whether two models disagree significantly in terms of their policy recommendations. The existing models which imply different multiplier effects do "forecast well over horizons of four to six quarters." They do contain useful information, however imperfect, which can be exploited to make forecasts. Since sound economic policy is based on good economic forecasts made under the assumption of alternative policy proposals, one cannot automatically assume that the same information is useless for the formulation of economice policy. Furthermore, just as two structures having different multiplier effects may produce forecasts closer to each other than to a naive forecast, they may also produce policy recommendations which are closer to each other than to a passive policy.

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To show that two different models may yield the same or similar policy recommendations, consider the univariate difference equation

$$y_t = ay_{t-1} + cx_t + u_t \tag{1.1}$$

where y_t is a dependent variable, x_t is a policy instrument or control variable and u_t is a serially independent random disturbance with mean zero and variance v. If the objective is to minimize the expectation Ey_t^2, then the optimal feedback policy is to set $ay_{t-1} + cx_t$ equal to zero, so that Ey_t^2 achieves its minimum $Eu_t^2 = v$. The policy is therefore

$$x_t = (-c^{-1}a)y_{t-1}. \tag{1.2}$$

Another model, which has coefficients $\tilde{a}$ and $\tilde{c}$ instead of a and c, will yield the same policy provided that the ratio $\tilde{a}/\tilde{c}$ is the same as a/c. The multipliers $a^k c$ of x_{t-k} in the final form of model (1.1) could certainly be very different from those of the alternative model, as illustrated by $a = .9$, $c = 1$, $\tilde{a} = .09$ and $\tilde{c} = .1$. Thus, an imperfect model with coefficients .09 and .1 may yield a policy close to being optimal, if the true coefficients are .9 and 1 respectively.

An interesting question concerning the usefulness of imperfect models is whether they will yield policies which are superior to an inactive policy allowing for no feedback. In the above example, an inactive policy is to set $x_t = 0$. Under this policy and assuming (1.1) to be the true model with $|a| < 1$, we can easily find the variance to approach $v/(1 - a^2)$ as t increases. If the government authority uses the inaccurate coefficients $\tilde{a}$ and $\tilde{c}$ and the resulting feedback policy, the system (1) will become

$$y_t = [a + c(-\tilde{c}^{-1}\tilde{a})]y_{t-1} + u_t, \tag{1.3}$$

which has the steady-state variance $v/\{1 - [a + c(-\tilde{c}^{-1}\tilde{a})]^2\}$. This variance is smaller than the variance prevailing under the inactive policy provided merely that $[a + c(-\tilde{c}^{-1}\tilde{a})]^2$ is smaller than a^2. Given a and c, a wide range of values for $\tilde{a}$ and $\tilde{c}$ will produce this required result. Hence using imperfect models can still be better than using a passive policy without feedback for the determination of macroeconomic policy.

We will generalize the above discussion in section 2 to treat dynamic econometric systems involving many variables and higher-order lags. Section 3 provides two illustrative models to be used for stabilization policy. Section 4 applies the method of section 2 to evaluate the usefulness of one of the models of section 3, assuming that the other model is the correct one. It illustrates how an imperfect model performs for the determination of policy as compared with using no feedback at all. Section 5 contains some concluding remarks.

2. Evaluation of Imperfect Models for Policy Analysis

Let the economy be governed by a time-varying linear system

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \tag{2.1}$$

where y_t is a vector of p endogenous variables, x_t is a vector of q policy or control variables with $q < p$, and u_t is a random vector independently distributed through time, having mean zero and covariance matrix V. The true parameters A_t, C_t, b_t and V are of course unknown to the policy maker. We will assume that the policy maker has available an imperfect model explaining a subset of the endogenous variables y_t. Written in the form (2.1), with appropriate zeros added, this imperfect model has coefficients $\tilde{A}_t$, $\check{C}_t$ and $\check{b}_t$. The question is how well a policy based on these inaccurate parameters would work, as compared with a policy of using no feedback, for certain hypothetical values of A_t, C_t and b_t. High-order lags in both the endogenous and policy variables are subsumed under the notation of (2.1) by suitable definitions, as illustrated by (3.2) in section 3 below. Nonlinear systems can be approximated by time-varying linear systems of the form (2.1) for our analysis, as will be explained later in this section.

The performance of the economy is measured by the expectation of the loss function

$$\sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) \tag{2.2}$$

where a_t are the targets and K_t are diagonal matrices giving the relative penalties of the squared deviations of the different variables from their targets. If the behavior of the policy variables also matters, they will be included in the vector y_t by appropriate definitions. We will be interested in comparing the performance of three policies. Policy I is the optimal policy assuming perfect knowledge of the true model (2.1). Policy II is obtained by minimizing the expectation of (2.2) under the assumption of an imperfect model, with coefficients $\tilde{A}_t$, $\check{C}_t$ and $\check{b}_t$. Policy III specifies a smooth time path for the policy variables which will not be altered by future observations of the economy.

As shown in Chow [1, Chapter 7], the optimal policy I is given by a set of linear feedback control equations

$$x_t = G_t y_{t-1} + g_t \tag{2.3}$$

The coefficients G_t and g_t can be calculated from the model parameters A_t, C_t and b_t, and the parameters a_t and K_t of the loss function. The economy under policy I will follow (2.1) and (2.3) which combine to yield

$$y_t = R_t y_{t-1} + r_t + u_t \tag{2.4}$$

where

$$R_t = (A_t + C_t G_t); \qquad r_t = b_t + C_t g_t. \tag{2.5}$$

The mean path of the economy as of the beginning of the planning horizon will follow

$$\bar{y}_t = R_t \bar{y}_{t-1} + r_t. \tag{2.6}$$

By subtracting (2.6) from (2.4) and defining the deviation from the mean path as $y_t^* = y_t - \bar{y}_t$, we have

$$y_t^* = R_t y_{t-1}^* + u_t \tag{2.7}$$

The covariance matrix of the system will therefore be

$$Ey_t^* y_t^{*\prime} = R_t(Ey_{t-1}^* y_{t-1}^{*\prime})R_t' + V \quad (t = 1,2,\ldots,T) \tag{2.8}$$

with initial condition $Ey_0^* y_0^* = 0$ since y_0 is constant and $y_0^* = 0$.

By considering the deviation $y_t - a_t$ as the sum of y_t^* and $\bar{y}_t - a_t$, we will decompose the expectation of the loss function (2.2) into two parts,

$$tr \sum_{t=1}^{T} K_t E y_t^* y_t^{*\prime} + \sum_{t=1}^{T} (\bar{y}_t - a_t)' K_t (\bar{y}_t - a_t). \tag{2.9}$$

One part is a weighted sum of the variances of y_t, to be calculated by using the covariance matrix (2.8). The other is a weighted sum of the squared deviations of the means $\bar{y}_t$ from the targets a_t. This decomposition will be used to study the expected losses of policies II and III as well.

Policy II is obtained by minimizing the expectation of (2.2) subject to a model of the form (2.1) with coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$. This policy is given by a feedback control equation of the form (2.3), with coefficient $\tilde{G}_t$ and $\tilde{g}_t$ which are computed by using the coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$ instead. The economy under policy II will be governed by (2.1) and this feedback control equation, namely

$$y_t = \tilde{R}_t y_{t-1} + \tilde{r}_t + u_t \tag{2.10}$$

where

$$\tilde{R}_t = (A_t + C_t \tilde{G}_t); \qquad \tilde{r}_t = b_t + C_t \tilde{g}_t. \tag{2.11}$$

The mean path and the covariance matrix of this system will be given respectively by (2.6) and (2.8) with $\tilde{R}_t$ and $\tilde{r}_t$ replacing R_t and r_t. The expected loss under this regime can be similarly decomposed as in (2.9).

Policy III allows for no feedback. If one refuses to use econometric models for the formulation of macroeconomic policy, what alternatives are available? One alternative is still to adjust the policy instruments according to the current state of the economy by some ad hoc rules which are not derived systematically from an econometric model. Such rules, once stated explicitly in the form of feedback control equations, can and should be evaluated by the method here proposed. Skeptics of the use of econometric models are under the obligation to show that their alternatives are no worse. The second alternative, which we will further examine, is not to use any feedback. It can always be written as $x_t = g_t^0$ for some fixed path g_t^0 to be specified without regard to the state of the economy. Under such a rule, which implies $G_t = 0$ in our notation, the mean and covariance matrix of the economic variables will be given by (2.6) and (2.8) respectively, with $R_t = A_t$ and $r_t = b_t + C_t g_t^0$. The two components of the expected loss can be computed by (2.9).

If the true model is nonlinear and consists of random disturbances, one cannot obtain analytically an optimal policy which would minimize the expectation of (2.2) under the assumption of perfect knowledge of the model parameters. However, for our analysis, policy I will be replaced by the following nearly optimal policy, which is described more fully in Chow [1, Chapter 12] and [4]. First, ignoring the random disturbances in the model, one finds an optimal path to minimize (2.2) using the resulting deterministic model. One then linearizes the model about this path, producing a system of the form (2.1) with time-varying coefficients. The analysis suggested above can be carried out in exactly the same way. The feedback control coefficients $\tilde{G}_t$ and $\tilde{g}_t$ for policy II are obtained by employing an imperfect nonlinear model which is similarly linearized to yield the coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$ needed to compute them. Policy III remains to be $x_t = g_t^0$. The two components of the expected loss resulting from each policy can be calculated as before. As a generalization of the discussion of section 1, an imperfect model yielding the feedback coefficients $\tilde{G}_t$ can be used to stabilize the economy better than using no feedback provided that $R_t = (A_t + C_t\tilde{G}_t)$ entering equation (2.8) will produce smaller variances than $R_t = A_t$.

In this section, we have suggested some analytical methods to evaluate policy recommendations derived from imperfect models. Without them, one would have to perform very expensive stochastic simulations to obtain sample paths of the economy under the assumptions of a hypothetically true model and alternative policy rules. The analytical methods can be used to deduce the means and covariance matrices of the sample paths without resort to the perhaps prohibitive computer simulations.

3. Fitting Two Illustrative Models

To illustrate the method of section 2, we will employ two hypothetical linear models. These models are derived from the multipliers reported in Christ [5] for the Michigan quarterly model and the Wharton Mark III model. Given the multipliers of the final form of an econometric model, the following procedure is applied to construct an approximate reduced form for policy analysis.

The procedure is based on the well-known relation between the reduced form and the final form. Let the reduced form be

$$(3.1)\quad y_t = B_1 y_{t-1} + B_2 y_{t-2} + B_3 x_t + B_4 x_{t-1} + B_5 x_{t-2} + b_0 + v_t.$$

We will convert it to first order and eliminate the lagged control variables by writing

$$(3.2)\quad \begin{bmatrix} y_t \\ y_{t-1} \\ x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & B_4 & B_5 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} B_3 \\ 0 \\ I \\ 0 \end{bmatrix} x_t + \begin{bmatrix} b_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which will be rewritten simply as

$$(3.3)\quad y_t = Ay_{t-1} + Cx_t + b + u_t.$$

Note that the new vector y_t of dependent variables includes the original dependent variables and control variables as subvectors. The matrices A and C and the vector b in (3.3) are defined by (3.2). By repeated elimination of lagged y's using (3.3), we obtain the final form

$$\begin{aligned}(3.4)\quad y_t = {} & Cx_t + ACx_{t-1} + A^2Cx_{t-2} + \cdots + A^{t-1}Cx_1 \\ & + A^t y_0 + b + Ab + A^2 b + \cdots + A^{t-1}b \\ & + u_t + Au_{t-1} + A^2 u_{t-2} + \cdots + A^{t-1}u_1.\end{aligned}$$

To construct a reduced form from the given final-form multipliers, we first make a tentative decision on the number of lagged y's and the number of lagged x's required as the reduced form was originally written in the form of equation (3.1). The coefficients B_i in (3.1) are related to A and C in (3.3) by definitions similar to those given in (3.2). The matrix C of impact multipliers are known. Denote the delayed multipliers AC, $A^2C, \ldots, A^kC$, respectively by $M_1, M_2, \ldots, M_k$ which are also known. We will use the relations

$$(3.5)\quad AC = M_1; \qquad AM_1 = M_2; \qquad AM_2 = M_3; \ldots; \qquad AM_{k-1} = M_k,$$

or

$$A[C M_1 M_2 \ldots M_{k-1}] = [M_1 M_2 M_3 \ldots M_k].$$

Each row a_i' of unknown elements in A will be chosen to minimize the sum of squares of the deviations of $a_i'[CM_1 \ldots M_{k-1}]$ from the i-th row m_i' of $[M_1 M_2 \ldots M_k]$. By the method of least squares,

$$(3.6) \qquad a_i = [(CM_1 \ldots M_{k-1})(CM_1 \ldots M_{k-1})']^{-1}(CM_1 \ldots M_{k-1})m_i.$$

If the fit is poor, as judged by the sizes of the above deviations, we will increase the numbers of lagged y's and/or lagged x's in the reduced form (3.1).

For illustrative purpose, we have chosen two dependent variables, nominal and real GNP, and two instruments, Federal government nondefense purchases and unborrowed reserves. The multiplier effects of a \$1 billion increase in nominal government purchases on nominal and real GNP (in billions of 1958 dollars) are given in Table 3 of Christ [5, pp. 66-67], lines 3 and 11 showing the effects for the Michigan Model and lines 5 and 13 for the Wharton Model. Similarly, the effects of a \$1 billion increase in unborrowed reserves (or a cut of 50 basis point in the Treasury bill rate) are given in Table 4 of Christ [5, pp. 68-69], lines 2 and 10 for the Michigan Model and lines 4 and 12 for the Wharton Model. The multipliers from the Michigan Model are based on simulations for the 40

TABLE 1

A. FINAL FORM MULTIPLIERS FOR MODEL M

Lag of x_1	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i	Lag of x_2	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i
0	.700	.700	.800	.800	0	.100	.100	.100	.100
1	.556	.556	.528	.528	1	.300	.300	300	.300
2	.425	.425	.302	.302	2	.500	.500	.500	.500
4	.217	.217	.115	.115	4	1.552	1.552	1.326	1.326
6	.045	.045	-.029	-.029	6	1.716	1.716	1.630	1.630
8	-.059	-.059	-.137	-.137	8	1.437	1.437	1.050	1.050
12	-.096	-.137	-.144	-.185	12	-.264	-.250	-.584	-.573
16	-.053	-.081	-.075	-.130	16	-.314	-.352	-.554	-.621
20	-.018	.067	-.024	.075	20	-.146	-.168	-.274	-.278
24	-.002	.112	-.002	.081	24	-.014	.069	-.075	-.020
28	.003	.059	.004	.034	28	-.044	.101	.012	.030
32	.003	.021	.004	.009	32	.057	.068	.033	.001
36	.002	.004	.002	.002	36	.051	.024	.028	.000

B. FINAL FORM MULTIPLIERS FOR MODEL W

Lag of x_1	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i	Lag of x_2	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i
0	1.300	1.300	1.300	1.300	0	1.300	1.300	1.400	1.400
1	.258	.258	.983	.983	1	1.240	1.240	1.330	1.330
2	.205	.205	.750	.750	2	1.180	1.180	1.260	1.260
4	.132	.132	.351	.351	4	1.030	1.103	1.070	1.070
6	.084	.084	.100	.100	6	.800	.800	.817	.817
8	.046	.054	-.019	-.022	8	.389	.350	.385	.400
10	.012	.035	-.061	-.092	10	.068	.075	-.016	-.049
12	-.015	.020	-.068	-.128	12	-.106	-.090	-.200	-.181
14	-.029	.009	-.057	-.123	14	-.155	-.187	-.216	-.213
16	-.029	.000	-.038	-.083	16	-.131	-.220	-.145	- 171
18	-.022	-.002	-.018	-.040	18	-.077	-.157	-.058	-.086
20	-.011	-.003	-.002	.004	20	-.025	.048	.008	-.027
23	.002	.000	.010	.000	23	.020	.000	.045	.000

quarters from 58.1 to 67.4. From the Wharton Model, they are based on simulations for the 16 quarters from 62.1 to 65.4. The results reported are the cumulative effects of a sustained increase in the instruments. In the notation of (3.5), they are the partial sums $M_1 + M_2 + \cdots + M_i$ for different i. The 2×2 matrices M_i have been obtained from these cumulative effects by differencing. Since the cumulative effects were given in Christ [5] only for selected i, curde graphic interpolations have been employed to obtain the multipliers M_i for each quarter as given by the figures under the columns M_t in Tables 1A and 1B.[2]

After some experimentation with different numbers of lagged dependent variables and lagged instruments, it was decided that a reduced form having dependent variables lagged 3 quarters and instruments lagged 9 quarters would fit the interpolated multipliers from the Michigan Model reasonably well; and that dependent variables lagged 3 quarters and instruments lagged 6 quarters would suffice to approximate the multipliers from the Wharton Model. Because of our crude graphic interpolation of the multipliers, our linearization of the models, our assumption that the parameters in the linear models are time-invariant, and our somewhat arbitrary truncation of the number of lagged variables in the reduced forms, the resulting models, to be called M and W respectively, may behave quite differently from the original Michigan and Wharton models, but they serve to illustrate the possible value of the policy recommendations from imperfect models. Note the differences between the multipliers in Tables 1A and 1B. For model M, the effects of government purchases on GNP become negative from period 7 on and are fairly large in absolute value; not so for model W. The multipliers of the monetary instrument increase in the first six quarters for model M while they decrease for model W. The reduced form coefficients obtained by our fitting procedure are given in Table 2; they are also fairly different for the two models. The final-form coefficients A^iC of x_{t-i} deduced from the reduced form are given in Table 1; they resemble the observed coefficients M_i.

The intercepts of the reduced forms for M and W are assumed to be linear functions of time t, which takes the value 1 for 1966.1. Using the historical data[3] from 1966.1 to 1969.4 and the coefficients of Table 2, we

[2]A referee has pointed out the inaccuracies of our graphic interpretation of Christ's Tables, especially for the multipliers M_i in column 5 of Table 1B measuring the effects of government purchases on real GNP according to the Wharton model. Since a main point of our paper is to show that models having different multipliers may imply similar optimal policy responses, the illustrative models constructed from the multipliers of Tables 1A and 1B will serve our purpose well.

[3]The time series used are quarterly data on nominal GNP, GNP in 1958 dollars, Federal government non-defense purchases of goods and services (all in billions of dollars at seasonally adjusted annual rates, from the *Survey of Current Business*), and nonborrowed member bank reserves in billions of dollars, (seasonally adjusted, from the *Federal Reserve Bulletin*).

TABLE 2

Reduced Form Coefficients for Models *M* and *W*

Model	$y_{1,t-1}$	$y_{2,t-1}$	$y_{1,t-2}$	$y_{2,t-2}$	$y_{1,t-3}$	$y_{2,t-3}$	x_{1t}	x_{2t}		$x_{1,t-1}$	$x_{2,t-1}$
M	.777	.592	−.023	−.354	.207	−.224	.700	.100		−.462	.163
	−.556	1.935	.430	−.788	.121	−.185	.800	.100		−.631	.162
W	1.545	−.392	−.604	.568	−.111	−.177	1.300	1.300		−1.241	−.219
	.801	1.468	−1.259	−.556	.493	−.011	1.300	1.400		−1.967	−1.767
	$x_{1,t-2}$	$x_{2,t-2}$	$x_{1,t-3}$	$x_{2,t-3}$	$x_{1,t-4}$	$x_{2,t-4}$	$x_{1,t-5}$	$x_{2,t-5}$		$x_{1,t-6}$	$x_{2,t-6}$
M	−.021	.127	.051	.130	−.035	.787	−.067	−.044		−.040	.014
	−.081	.122	.093	.124	−.005	.559	−.045	.128		−.039	−.014
W	.238	−.224	.110	.176	−.009	.132	−.007	.118		−.010	.090
	1.461	.730	−.490	.062	.005	.031	−.014	.073		.006	.003
	$x_{1,t-7}$	$x_{2,t-7}$	$x_{1,t-8}$	$x_{2,t-8}$	$x_{1,t-9}$	$x_{2,t-9}$	t	1	Model	t	1
M	−.027	−.080	−.028	−.149	−.020	−.745	.717	21.296	*W*	1.863	87.162
	−.032	−.095	−.019	−.392	−.012	−.508	.282	29.176		−.472	22.373

have estimated the trend terms by least squares, as given in the lower right corner of Table 2. The sample residuals of these reduced-form equations have covariance matrices given by

$$(3.7) \qquad V_M = \begin{bmatrix} 16.605 & 13.170 \\ 13.170 & 11.569 \end{bmatrix}; \qquad V_w = \begin{bmatrix} 22.012 & 16.914 \\ 16.914 & 40.524 \end{bmatrix}$$

The GNP figures are in billions of current or 1958 dollars. The standard deviations of the residuals are between 3.4 and 6.4 billions. The covariance matrices (3.7) will be used as the population values when the corresponding models are regarded as the true models in future analysis.

4. Illustrative Evaluation of Two Imperfect Models

Before applying any stabilization policy, be it derived from an imperfect econometric model or from some ad hoc reasoning, the government authorities should examine how it would perform under reasonable assumptions about the dynamic structure of the economy. Although the structure is unknown, it is necessary to assume hypothetical structures to test the performance of any policy being seriously considered for adoption. In this section, we use one of the models of section 3 as the hypothetical structure and evaluate the policy recommendations derived from using the other model. The planning horizon T is 32 quarters, with initial conditions given by historical data up to the last quarter of 1965. The target growth rates for nominal and real GNP are assumed to be .018 and .008 per quarter respectively; these are their average historical rates from 1966.1 to 1969.4. The diagonal elements of the K matrix are 1 and 1 for these target variables, and .2 and .2 for the instruments which are assigned growth rates of .011 and .013, their average historical rates from 1966 to 1969. This assignment is to inhibit excessive variations in the instruments.

The inactive policy provides constant growth rates for the two instruments. The growth rates chosen in our experiment are respectively .011 and .013, the average historical growth rates. In practice, a nondiscretionary policy of maintaining constant growth rates for the instruments is hard to design partly because one does not know what growth rates are consistent with price stability and full employment. We have partly bypassed the problem by using the average historical growth rates of nominal and real GNP as our target rates, and the historical growth rates of the instruments to define the inactive policy. Since both models M and W fit the historical data fairly well, applying the average historical growth rates to the instruments insures that the dependent variables will also follow the historical or target rates, on the average. A more realistic evaluation of a nondiscretionary policy would utilize the growth rates proposed by its advocate. Our analysis tends to favor the inactive policy.

TABLE 3

A. COMPONENTS OF WELFARE LOSS ASSUMING MODEL M TO BE TRUE

	Sum of Variances of GNP$ and GNP58			Sum of Squared Deviations of Means from Targets		
Period	Policy I	Policy II	Policy III	Policy I	Policy II	Policy III
1	28.2	28.2	28.2	2.6	8.7	53.8
2	31.1	34.1	74.5	8.3	32.7	156.9
3	32.7	41.5	126.7	18.0	17.3	255.5
4	34.1	43.4	180.3	27.4	63.3	322.8
5	35.5	46.3	232.3	43.2	22.5	338.3
6	37.1	55.9	279.7	42.7	61.5	317.3
7	38.5	64.0	320.6	36.1	29.1	272.7
8	39.5	73.1	354.8	29.2	68.2	214.6
9	40.2	74.9	382.6	30.7	15.8	146.5
10	41.0	77.7	404.9	22.1	59.9	83.1
11	41.6	107.3	422.5	16.5	12.6	34.2
12	42.1	178.0	436.6	13.6	40.8	6.3
Sum	441.6	824.4	3,243.7	290.4	432.4	2.202.0
B. COMPONENTS OF WELFARE LOSS ASSUMING MODEL W TO BE TRUE						
1	62.5	62.5	62.5	30.1	96.0	121.6
2	119.8	326.9	242.0	14.9	50.2	279.2
3	135.6	540.1	420.6	4.4	129.1	344.3
4	138.5	917.6	544.3	1.3	139.6	359.2
5	139.2	1.349.7	627.1	0.5	73.8	340.9
6	139.4	1.693.8	680.9	0.3	24.9	294.6
7	139.5	1.909.9	714.2	.1	111.3	236.1
8	139.5	2.108.5	736.2	.0	121.6	176.8
9	139.5	2.438.9	753.3	.0	303.8	121.3
10	139.5	3.332.8	767.4	.1	303.3	73.9
11	139.5	4.577.6	778.7	.2	349.6	39.9
12	139.5	6.313.0	787.0	.3	464.7	25.2
Sum	1,572.0	25.571.3	7.114.2	52.2	2,167.9	2.413.0

Table 3A and 3B give the main results of our illustrative calculations. For Table 3A. Model M is assumed to be true. Policy I is the optimal policy derived from using Model M. Policy II is the optimal policy for model W. Policy III uses the average historical rates of change for the two instruments. For each policy and each period. we show separately the loss due to the variances of the variables and to the deviations of their means from the targets, as indicated by expression (2.9). Table 3B gives analogous results, assuming W to be the true model, with policy II being the optimal policy derived from model M. Without the stochastic control theory of section 2, one would have to solve an optimal control problem for 32 periods using the true model or the imperfect model as the case may be, and obtain the optimum values for the instruments in period 1; apply these values. together with a random drawing of the residuals u_1 in period 1 from the true model, to generate a set of dependent variables y_1 for period 1: using y_1 as the initial condition, solve a second optimal control problem for 31 periods, and obtain the optimum values of the instruments in period 2: apply these values to generate y_2 stochastically and so forth. This tedious process only provides one observation, covering 32 periods, of the stochastic time path for a hypothetically true model and a given strategy. The process has to be repeated many times in order to estimate the mean vector and the covariance matrix of the multivariate stochastic time series describing the economy under control. The analytical method of section 2 was used to calculate the means and variances for Tables 3A and 3B in lieu of such stochastic simulations and countless optimal control calculations.

Because the end of the time horizon is fixed, the policy recommendations for the later periods are subject to the well-known limitations of being myopic, and should therefore not be taken seriously. Furthermore, to evaluate the policy recommendations from an imperfect model realistically, one ought to allow for possible revisions of model parameters through time. For these two reasons, we consider the dynamic behavior of the economy described by Tables 3A and 3B only for the first 12 periods. The sum for each component of the loss function over the first 12 periods is given at the bottom of Tables 3A and 3B.

For each combination of the true world and the policy, the total expected loss due to both the variances and the squared deviations of means from targets is given in the following payoff matrix (negative sign omitted).

	True Model	
	M	*W*
Optimal Policy Derived from *M*	731.0	27,739.2
Optimal Policy Derived from *W*	1,256.8	1,624.2
Inactive Policy	5,445.7	9,527.2

Thus, the policy based on model *W* would be much better than the inactive policy even if the true world were model *M*, and in spite of the apparent differences in the multipliers and the reduced form equations for the two models. However, the policy derived from Model *M* would be much worse than the inactive policy if the true world were model *W*. If the policy maker were to face only these two possible states of the world, he should formulate his policy according to model *W* rather than following an inactive policy, since the latter policy is dominated by the former according to the payoff matrix. Of course, if the true state of the world were very different from both models *M* and *W*, one may do very poorly by following the optimal policy based on model *W*.

The calculations of this section are merely illustrative of the method of section 2. The results are *not* intended to apply to the original Michigan and Wharton models for obvious reasons. The method, however, applies to nonlinear models as pointed out in section 2, by using the (nearly) optimal feedback control equations of Chow [1, Chapter 12] and [4] for nonlinear models.

5. Concluding Remarks

In this paper, we have described a method to evaluate the performance of the optimal policy derived from an econometric model, and illustrate it with two simplified models. Although model W differs a great deal from model M in terms of the reduced forms and the multipliers, it can still be used effectively as a guide to policy even if the world is accurately described by model M. We propose to calculate the expected loss associated with an optimal policy derived from an imperfect model under different assumptions about the true state of the world. Certainly, from an imperfect econometric model, other rules can be derived than the optimal rule given by section 2 above. For example, uncertainty in the parameters can be allowed for as indicated in Chow [1, Chapter 10], [2] and [3]. Such a policy may perform better under the assumption that a different model is true. One may also devise a rule by somehow combining the parameter values from two different models so that it will behave reasonably well under both worlds. These matters are subjects for further research. Hopefully, the method outlined in this paper will facilitate the evaluations of alternative policy recommendations and econometric models.

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