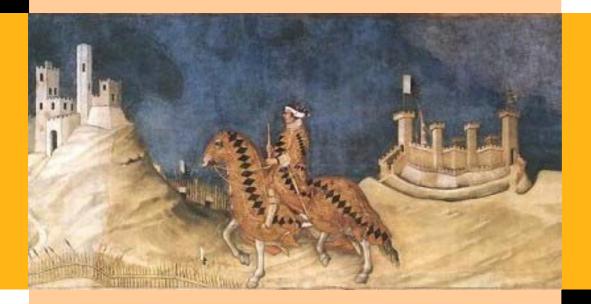# UNIVERSITÀ DEGLI STUDI DI SIENA

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Design-Based Treatment of Unit Nonresponse
by the Calibration Approach

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Abstract - The use of nonresponse calibration weighting is considered in a complete design-based frameworkto account for the cases in which nonresponse is a fixed characteristic of the units, just like the interest variable. Approximate expressions of design-based bias and variance of the calibration estimator are derived and some estimators of the sampling variance are proposed. The choice of auxiliary variables is discussed from theoretical and practical point of view. The results of an extensive simulation study demonstrate how the reliability of the procedure is mainly determined by the capability of selecting auxiliary variables in such a way that their relationship with the interest variable is similar for both the respondent and nonrespondent sub-populations.

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1. Introduction

Unit nonresponse is often a problem in sample surveys, arising when the values of the survey variable cannot be recorded for some sampled units.

Widely applied methods to account for unit nonresponse, recently referred to as *nonresponse propensity weighting* (*e.g*. Haziza *et al*., 2010), view the respondent set as the result of a two-phase sampling: in the first phase a sample is selected from the population by means of the established sampling scheme while, in the second phase, the respondent set is realized as a subset of the first-phase sample, assuming the existence of a response mechanism for which every population unit has its own (invariably positive) response probability. Then, a realistic model is formulated in which the unknown response probabilities depend on some auxiliary variables and as such they are subsequently estimated on the basis of the auxiliary information available at sample or at population level (the so called Info S and Info U, in the parlance of Särndal and Lundström, 2005). The estimation theory built around the idea that units are equipped with design-based inclusion probabilities and model-based response probabilities has been termed *quasi-randomization theory* by Oh and Scheuren (1983). As pointed out by Kott (1994), the procedure requires that all the response probabilities were strictly positive, while this requirement is often unrealistic because most populations contain units that do not respond under any circumstances. Moreover, Särndal and Lundström (2005, p.52) point out that the knowledge about response behaviour is usually limited, so it is difficult to defend any proposed model adopted to estimate the response probabilities as being more realistic than any alternative.

Apart from these two (relevant) drawbacks, in some surveys unit responses cannot be viewed as outcomes of dichotomous experiments with unknown probabilities (just as tosses of unfair coins). Indeed, there exist situations in which the response pattern is fixed, in the sense that the population is strictly partitioned into respondent and nonrespondent units and responses are fixed characteristics of the units, just like the values of the interest variable. In these cases, it seems

1

natural to perform estimation in a complete design-based framework. It is worth noting that the non random nature of responses is quite common in environmental surveys such as forest inventories, when a population of sites scattered over the study area is sampled. In some circumstances, it may occur that some selected sites are located in difficult terrains and cannot be reached. Thus, the values of the interest variable corresponding to such sites are missed. Obviously, in this situation no random experiment can be claimed, since the sites can be reached or not.

When nonresponse is a fixed characteristic, the quasi-randomization approach cannot be adopted and *imputation* or *nonresponse calibration weighting* (Haziza *et al*., 2010) should be used. Imputation is a procedure in which missing values are replaced by substitutes and estimation is performed on the completed data, thus achieving the so called *imputed estimator*. As pointed out by Särndal and Lundström (2005, p.52), imputed values are artificial and as such affected by errors. Accordingly, imputation errors may be treated as measurement errors, as when an erroneous value is recorded for a sampled unit. Commonly used techniques of imputation are *regression imputation*, *nearest neighbour imputation*, *hot deck imputation* and *multiple imputation* (for a review see *e.g.* Little and Rubin, 2002 and Durrant, 2005). Without entering on these techniques, it should be once again pointed out that, since knowledge about response behaviour is usually limited, it is difficult to defend any proposed method/model of imputation as being more realistic than others.

As an alternative to imputation, the nonresponse calibration weighting (henceforth NCW) may be adopted: in order to compensate the reduction in the estimate value due to nonresponse, the weights originally attached to each respondent observation are changed into new weights able to estimate the total of a set of auxiliary variables without error (in the case of Info U) or as if the complete sample were available (in the case of Info S). The rationale behind the approach is quite obvious: if the calibrated weights guess the total of the auxiliary variables or their complete sample estimates without errors, then they should be suitable also for estimating the total of the interest variable, providing that a close relationship exists between the interest and the auxiliary variables.

2

Even if no superiority of NCW with respect to imputation can be generally claimed, NCW seems to be more convincing than imputation because even at its best, *i.e.* when all the imputed values are guessed without errors, imputation cannot improve upon the performance of the complete-sample estimator. On the other hand, in the case of Info U and if a perfect linear relationship exists between interest and auxiliary variables, the NCW approach estimates the total without error (Särndal and Lundström, 2005, p.61). Accordingly, NCW is likely to perform well for suitable choices of auxiliary variables, accomplishing both the goal of reducing nonresponse bias and increasing the accuracy of estimates. Moreover, the NCW approach does not refer explicitly to any model, allowing for a straightforward design-based treatment.

The present paper deals with the application of the NCW approach in a complete design-based framework, *i.e.* viewing population values and nonresponse as fixed characteristics. To this purpose, in section 2 some preliminaries and notations are given, while in section 3 the approximate expressions of the design-based bias and variance of the calibrated Horvitz-Tompson (HT) estimator are considered and some variance estimators are proposed. Subsequently, in section 4, the choice of the auxiliary variables adopted to perform calibration is discussed from both theoretical and practical point of view. In section 5, the performance of the calibrated HT estimator and of some variance estimators are checked by means of a simulation study. Concluding remarks are contained in section 6.

2. Preliminaries and notations

Let U be a population of N units, let y_j be the value of a positive survey variable Y for the j-th unit and suppose that the population total, say $T_y = \sum_{j \in \mathsf{U}} y_j$ be the interest quantity to be estimated on the basis of a sample $\mathsf{S} \subset \mathsf{U}$ of size n. Suppose S be selected from the population by means of a fixed-size scheme inducing first- and second-order inclusion probabilities π_j and π_{jh} ($h > j = 1, \ldots, N$).

Suppose also that $\pi_{jh} > 0$ for any $h > j = 1, \ldots, N$. If the y_js are recorded for each $j \in \mathsf{S}$ (complete response), then the HT estimator $\hat{T}_{HT} = \sum\limits_{j \in \mathsf{S}} \dfrac{y_j}{\pi_j}$ is design-unbiased with design-based variance which can be unbiasedly estimated by means of the well-known Sen-Yates-Grundy (SYG) or HT variance estimators. On the other hand, if the y_js are recorded only for $j \in \mathsf{R} \subset \mathsf{S}$ (partial response), then the R-based HT estimator

$$\hat{T}_R = \sum_{j \in \mathsf{R}} \frac{y_j}{\pi_j} \tag{1}$$

turns out to be invariably smaller than $\hat{T}_{HT}$ and hence negatively biased. In order to compensate the decrease of $\hat{T}_R$ due to nonresponse, the weights $1/\pi_j$ attached to each y_j should be suitably modified. As already mentioned, consider situations in which the response pattern is fixed, in the sense that the population is partitioned into two strata: the respondent stratum, say U_R of size N_R and the nonrespondent stratum $\mathsf{U} - \mathsf{U}_R$ of size $N - N_R$.

In order to perform the NCW approach, auxiliary information is necessary. Denote by x_{jk} the value of an auxiliary variable X_k for the j-th unit in the population. Now, suppose that the values of L auxiliary variables, say $X_1^*, \ldots, X_L^*$, are known for each unit of the population (info U), *i.e.* the L-vectors $\mathbf{x}_j^* = \left[x_{j1}^*, \ldots, x_{jL}^* \right]^{\mathsf{T}}$ are known for each $j = 1, \ldots, N$. Accordingly, the vector of population totals for the L variables, say $\mathbf{T}^* = \sum\limits_{j \in \mathsf{U}} \mathbf{x}_j^*$ is also known. Moreover, suppose that the values of M auxiliary variables, say $X_1^\circ, \ldots, X_M^\circ$, are known for each unit in the sample (info S), *i.e.* the M-vectors $\mathbf{x}_j^\circ = \left[x_{j1}^\circ, \ldots, x_{jM}^\circ \right]^{\mathsf{T}}$ are known for each $j \in \mathsf{S}$. In this case, the vector of the HT estimates of the population totals computed on S, say $\hat{\mathbf{T}}^\circ = \sum\limits_{j \in \mathsf{S}} \dfrac{\mathbf{x}_j^\circ}{\pi_j}$ is known. Obviously $\hat{\mathbf{T}}^\circ$ constitutes an

unbiased estimator of the vector of population totals of the M variables, say $\mathbf{T}^{\circ} = \sum_{j \in \mathsf{U}} \mathbf{x}_j^{\circ}$. Now, for simplicity of notation, the two sets of variables can be joined into a unique set of $K = L + M$ variables simply by using the K-vector $\mathbf{x}_j = \begin{bmatrix} \mathbf{x}_j^* \\ \mathbf{x}_j^{\circ} \end{bmatrix}$ as well as the K-vector $\hat{\mathbf{T}} = \begin{bmatrix} \mathbf{T}^* \\ \hat{\mathbf{T}}^{\circ} \end{bmatrix}$. In the parlance of Särndal and Lundström (2005, Table 6.1), the joined information is referred to as the InfoUS while the vector $\hat{\mathbf{T}}$ is referred to as the *information input* owing to its basic role in the subsequent calibration. It is worth noting that while the first L components of $\hat{\mathbf{T}}$ are known constants, the remaining M components are random variables depending on S. Obviously the design-based expectation of $\hat{\mathbf{T}}$ turns out to be $\mathrm{E}(\hat{\mathbf{T}}) = \mathbf{T}$ where $\mathbf{T} = \begin{bmatrix} \mathbf{T}^* \\ \mathbf{T}^{\circ} \end{bmatrix}$.

In order to compensate the reduction of $\hat{T}_R$ due to non response, the weights $1/\pi_j$ in (1) are changed into new weights, say w_j, in such a way to satisfy the so called *calibration equation* $\sum_{j \in \mathsf{R}} w_j \mathbf{x}_j = \hat{\mathbf{T}}$. Rewriting the w_js as modifications of the HT weights, *i.e.* $w_j = v_j / \pi_j$, and from the fact that (even if it is not mandatory for all the js in R) the w_js should constitute enlargements of the π_js, a suitable structure for the v_js as enlargement factors (*i.e.* $v_j > 1$) may be of type $v_j = 1 + \mathbf{c}^{\mathrm{T}} \mathbf{x}_j$ (Särndal and Lundström, 2005, Chapter 6). Then, solving the calibration equation with respect to $\mathbf{c}$, the resulting estimator, henceforth referred to as the *calibration estimator*, turns out to be

$$\hat{T}_{CAL} = \hat{\mathbf{b}}_R^{\mathrm{T}} \hat{\mathbf{T}}, \tag{2}$$

where $\hat{\mathbf{b}}_R = \left(\sum_{j \in \mathsf{R}} \frac{\mathbf{x}_j \mathbf{x}_j^{\mathrm{T}}}{\pi_j} \right)^{-1} \sum_{j \in \mathsf{R}} \frac{y_j \mathbf{x}_j}{\pi_j}$ providing that the matrix to be inverted is positive definite and that an auxiliary variable (say the first without loss of generality) is invariably equal to 1,

i.e. $\mathbf{x}_j = \left[1, x_{j2}, \ldots, x_{jK}\right]^{\mathrm{T}}$ for any $j = 1, \ldots, N$ (Särndal and Lundström, 2005, section 6.8).

Henceforth, the auxiliary variable invariably equal to 1 will be tacitly included in the set of the K auxiliary variables, in such a way that the calibration weights reproduce the population size without error, *i.e.* $\sum_{j \in \mathsf{R}} w_j = N$. As previously emphasized, from (2) it can be easily proven that under Info U and if $y_j = \mathbf{b}^{\mathrm{T}} \mathbf{x}_j$ for all $j = 1, \ldots, N$, then $\hat{T}_{CAL} = T_y$.

3 Design-based bias and variance.

In order to treat nonresponse as a fixed characteristic a dummy variable, say R, is considered such that $r_j = 1$ for $j \in \mathsf{U}_R$ while $r_j = 0$ for $j \in \mathsf{U} - \mathsf{U}_R$. Hence, the vector $\hat{\mathbf{b}}_R$ in (2) can be rewritten as

$$\hat{\mathbf{b}}_R = \left(\sum_{j \in \mathsf{S}} \frac{r_j \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}}}{\pi_j} \right)^{-1} \sum_{j \in \mathsf{S}} \frac{r_j y_j \mathbf{x}_j}{\pi_j}$$ in such a way that $\hat{T}_{CAL}$ depends on the sole sample S while

nonresponse is accounted for in the r_js ($j \in \mathsf{S}$). Now, denote by $\hat{\mathbf{A}}_R = \sum_{j \in \mathsf{S}} \frac{r_j \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}}}{\pi_j}$ and

$\hat{\mathbf{a}}_R = \sum_{j \in \mathsf{S}} \frac{r_j y_j \mathbf{x}_j}{\pi_j}$ the two HT-like estimators in $\hat{\mathbf{b}}_R$. Obviously,

$\mathrm{E}(\hat{\mathbf{A}}_R) = \sum_{j \in \mathsf{U}} r_j \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}} = \sum_{j \in \mathsf{U}_R} \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}} = \mathbf{A}_R$ and $\mathrm{E}(\hat{\mathbf{a}}_R) = \sum_{j \in \mathsf{U}} r_j y_j \mathbf{x}_j = \sum_{j \in \mathsf{U}_R} y_j \mathbf{x}_j = \mathbf{a}_R$. Thus, keeping in mind

that $\hat{\mathbf{T}}$ may be viewed as the HT estimator of T and as such $\mathrm{E}(\hat{\mathbf{T}}) = \mathbf{T}$, $\hat{T}_{CAL}$ can be rewritten as a function of the three HT estimators $\hat{\mathbf{A}}_R, \hat{\mathbf{a}}_R$ and $\hat{\mathbf{T}}$. Accordingly, the first-order Taylor series approximation of $\hat{T}_{CAL}$ gives rise to

$$\hat{T}_{CAL} \approx \hat{\mathbf{a}}_R^T \mathbf{A}_R^{-1} \mathbf{T} - \mathbf{b}_R^{\mathrm{T}} \hat{\mathbf{A}}_R \mathbf{A}_R^{-1} \mathbf{T} + \mathbf{b}_R^{\mathrm{T}} \hat{\mathbf{T}}, \tag{3}$$

where $\mathbf{b}_R = \mathbf{A}_R^{-1}\mathbf{a}_R$ is the coefficient vector of the least-square hyperplane fitted from the respondent population scatter $\{(\mathbf{x}_j, y_j), j \in \mathsf{U}_R\}$. In this sense, $\hat{\mathbf{b}}_R$ may be viewed as an approximately unbiased estimator of $\mathbf{b}_R$, obtained from the respondent sample R.

3.1 Design-based nonresponse bias

Särndal and Lundström (2005, equation 9.14) derive a general expression for the approximate bias of $\hat{T}_{CAL}$ under the so called *nonresponse model approach* (NMA), from which inference is made with respect to the joint distribution induced by the sampling design and the nonresponse mechanism, if a response probability θ_j is supposed for each unit and it is also supposed that units respond independently of one another. If the θ_js are set equal to 1 for $j \in \mathsf{U}_R$ and 0 otherwise, then the Särndal-Lundström approximate bias expression under NMA reduces to

$$\mathrm{AB}(\hat{T}_{CAL}) = \sum_{j \in \mathsf{U}-\mathsf{U}_R}\mathbf{b}_R^{\mathrm{T}}\mathbf{x}_j - \sum_{j \in \mathsf{U}-\mathsf{U}_R} y_j \qquad (4)$$

which constitutes the approximate design-based bias of $\hat{T}_{CAL}$. As pointed out by Särndal and Lundström (2005, p.99) the approximate bias does not depend from the design.

In order to obtain more insights, expression (4) can be suitably rewritten as

$$\mathrm{AB}(\hat{T}_{CAL}) = \sum_{j \in \mathsf{U}-\mathsf{U}_R}\mathbf{b}_R^{\mathrm{T}}\mathbf{x}_j - \sum_{j \in \mathsf{U}-\mathsf{U}_R}(\mathbf{b}_{NR}^{\mathrm{T}}\mathbf{x}_j + e_{NRj}) = \sum_{j \in \mathsf{U}-\mathsf{U}_R}(\mathbf{b}_R - \mathbf{b}_{NR})^{\mathrm{T}}\mathbf{x}_j , \qquad (5)$$

where the e_{NRj} denotes the 0-sum residuals from the regression performed on the nonrespondent population scatter $\{(\mathbf{x}_j, y_j), j \in \mathsf{U}-\mathsf{U}_R\}$, *i.e.* $e_{NRj} = y_j - \mathbf{b}_{NR}^T\mathbf{x}_j$ for $j \in \mathsf{U}-\mathsf{U}_R$ and

$\mathbf{b}_{NR} = \left(\sum_{j \in \mathsf{U}-\mathsf{U}_R}\mathbf{x}_j\mathbf{x}_j^{\mathrm{T}}\right)^{-1}\sum_{j \in \mathsf{U}-\mathsf{U}_R}y_j\mathbf{x}_j$ is the coefficient vector of the least-square hyperplane fitted from the nonrespondent population.

It is at once apparent from (5) that the design-based approximate bias of $\hat{T}_{CAL}$ strictly depends on the difference between the least-squares hyperplanes fitted from the respondent and nonrespondent population scatters. Approximate unbiasedness is achieved when the two hyperplanes are identical, *i.e.* the linear relationship among interest and auxiliary variables is similar for respondent and nonrespondent units.

3.2 Design-based nonresponse variance

In order to derive an approximate expression for the design-based variance of $\hat{T}_{CAL}$ and the corresponding estimators, it does not seem convenient to quote from the general results achieved under the NMA (Särndal and Lundström, Chapter 11, Haziza *et al.*, 2010). Indeed in the NMA the complexity of the problem is inflated by the fact that nonresponse, working like a sort of second-phase selection, provides an additional variance component to be estimated. Such a component is obviously absent when nonresponse is a fixed characteristic. In this case the approximate variance can be straightforwardly achieved using the standard linearization approach (*e.g.* Sarndal *et al.*, 1992 , section 5.5) and the resulting expression is straightforwardly interpretable.

To this purpose, it should be noticed that an additive constant is present in the first-order approximation (3) owing to the fact that the first L components of $\hat{\mathbf{T}}$ are the true population totals of $X_1^*, \ldots, X_L^*$ (info U). Accordingly, (3) can be more suitably expressed as

$$\hat{T}_{CAL} \approx \hat{\mathbf{a}}_R^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T} - \mathbf{a}_R^{\mathrm{T}} \mathbf{A}_R^{-1} \hat{\mathbf{A}}_R \mathbf{A}_R^{-1} \mathbf{T} + \mathbf{b}_R^{*\mathrm{T}} \mathbf{T}^* + \mathbf{b}_R^{\circ\mathrm{T}} \hat{\mathbf{T}}^{\circ} =$$

$$= \left(\sum_{j \in \mathsf{S}} \frac{r_j y_j \mathbf{x}_j}{\pi_j} \right)^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T} - \mathbf{a}_R^{\mathrm{T}} \mathbf{A}_R^{-1} \left(\sum_{j \in \mathsf{S}} \frac{r_j \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}}}{\pi_j} \right) \mathbf{A}_R^{-1} \mathbf{T} + \mathbf{b}_R^{\circ\mathrm{T}} \left(\sum_{j \in \mathsf{S}} \frac{\mathbf{x}_j^{\circ}}{\pi_j} \right) + const =$$

$$= \sum_{j \in \mathsf{S}} \frac{r_j (y_j \mathbf{x}_j^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T} - \mathbf{a}_R^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T}) + \mathbf{b}_R^{\circ\mathrm{T}} \mathbf{x}_j^{\circ}}{\pi_j} + const =$$

$$= \sum_{j \in \mathsf{S}} \frac{r_j e_{Rj} \mathbf{x}_j^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T} + y_j - e_{Rj}^{\circ}}{\pi_j} + const = \sum_{j \in \mathsf{S}} \frac{u_j}{\pi_j} + const \ , \tag{6}$$

where for any $j \in \mathsf{U}$, $u_j = r_j e_{Rj} \mathbf{x}_j^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{T} + y_j - e_{Rj}^{\circ}$ are the so called *influence values* (*e.g.* Davison and Hinkley, 1997), e_{Rj} denotes the 0-sum residuals from the least-square fitting performed on the respondent population scatter, *i.e.* $e_{Rj} = y_j - \mathbf{b}_R^T \mathbf{x}_j$ for $j \in \mathsf{U}_R$ and $e_{Rj} = 0$ otherwise, $\mathbf{b}_R^{\circ}$ denotes the last M components of $\mathbf{b}_R$ and $e_{Rj}^{\circ} = y_j - \mathbf{b}_R^{\circ \mathrm{T}} \mathbf{x}_j^{\circ}$ are the non-0-sum residuals obtained neglecting the Info-U-variable coefficients of $\mathbf{b}_R$.

Up to a constant term, the approximation (6) to $\hat{T}_{CAL}$ may be viewed as the HT estimator of the total of the u_js over U. Accordingly, the approximate variance of $\hat{T}_{CAL}$ turns out to be

$$\mathrm{AV}(\hat{T}_{CAL}) = \sum_{h > j \in \mathsf{U}} (\pi_j \pi_h - \pi_{jh}) \left(\frac{u_j}{\pi_j} - \frac{u_h}{\pi_h} \right)^2 \ . \tag{7}$$

From the previous expressions it readily follows that the design-based approximate variability of $\hat{T}_{CAL}$ jointly depends on: *i*) the ability of the whole set of K auxiliary variables to predict the interest variable in the respondent population; *ii*) the ability of the M Info-S variables to predict the interest variable in the whole population neglecting the contribution of the L Info-U variables; *iii*) the estimation of the total of the interest variable.

3.3 Variance estimation.

Standard variance estimator can be straightforwardly achieved from expression (7). The Sen-Yates-Grundy (SYG) variance estimator is given by

$$V_{SYG}^2 = \sum_{h > j \in \mathsf{S}} \frac{\pi_j \pi_h - \pi_{jh}}{\pi_{jh}} \left(\frac{\hat{u}_j}{\pi_j} - \frac{\hat{u}_h}{\pi_h} \right)^2 \ , \tag{8}$$

9

where for each $j \in S$, $\hat{u}_j = r_j \hat{e}_{Rj} \mathbf{x}_j^T \hat{\mathbf{A}}_R^{-1} \hat{\mathbf{T}} + \hat{\mathbf{b}}_R^{\circ T} \mathbf{x}_j^{\circ}$ are the empirical influence values, $\hat{e}_{Rj}$ are the residual achieved from the least-square fitting performed on the respondent point scatter $\{(\mathbf{x}_j, y_j), j \in R\}$, i.e. $\hat{e}_{Rj} = y_j - \hat{\mathbf{b}}_R^T \mathbf{x}_j$ for $j \in R$ and $\hat{e}_{Rj} = 0$ if $j \in S\text{-}R$ and $\hat{\mathbf{b}}_R^{\circ}$ denotes the last M components of $\hat{\mathbf{b}}_R$.

Alternatively, the HT variance estimator is given by

$$V_{HT}^2 = \sum_{j \in S} \frac{1-\pi_j}{\pi_j^2} \hat{u}_j^2 + 2 \sum_{h>j \in S} \frac{\pi_{jh} - \pi_j \pi_h}{\pi_j \pi_h \pi_{jh}} \hat{u}_j \hat{u}_h \ . \tag{9}$$

Finally, the jackknife variance estimator by Berger and Skinner (2005) can be used. The jackknife estimator is analogous to (9) but with the empirical influence values which are numerically approximated instead of being obtained by analytic differentiation. Quoting from Berger and Skinner (2005), denote by $v_{(j)} = \left(1 - \dfrac{1}{\hat{N}\pi_j}\right)\left\{\hat{T}_{CAL} - \hat{T}_{CAL(j)}\right\}$ where $\hat{N} = \sum_{j \in S} \dfrac{1}{\pi_j}$, $\hat{T}_{CAL(j)} = \hat{\mathbf{b}}_{R(j)}^T \hat{\mathbf{T}}_{(j)}$,

$$\hat{\mathbf{b}}_{R(j)} = \left(\sum_{h \in S_{-j}} \frac{r_h \mathbf{x}_h \mathbf{x}_h^T}{\pi_h}\right)^{-1} \sum_{h \in S_{-j}} \frac{r_h y_h \mathbf{x}_h}{\pi_h}, \qquad \hat{\mathbf{T}}_{(j)} = \left[T_1^*, \ldots, T_L^*, \hat{T}_{1(j)}^{\circ}, \ldots, \hat{T}_{M(j)}^{\circ}\right]^T, \qquad \hat{T}_{k(j)}^{\circ} = \frac{N}{\hat{N}} \sum_{h \in S_{-j}} \frac{x_{h,k}^{\circ}}{\pi_h} \quad \text{for}$$

$k = 1, \ldots, M$ and finally S_{-j} consists of the sample S with the j-th unit deleted. Accordingly, the jackknife estimator for the variance of $\hat{T}_{CAL}$ turns out to be

$$V_{jack}^2 = \sum_{j \in S} (1 - \pi_j) v_{(j)}^2 + 2 \sum_{h>j \in S} \frac{\pi_{jh} - \pi_j \pi_h}{\pi_{jh}} v_{(j)} v_{(h)} \ . \tag{10}$$

4. Selecting effective auxiliary information

Särndal and Lundström (2005, p. 98) point out as the bias of any nonresponse-adjusted estimator should be the main concern. The authors emphasize that variance is of minor importance since "*if an estimator is greatly biased, it is poor consolation that its variance is low*".

At least in the case of Info U, $\hat{T}_{CAL}$ estimates T_y without error in presence of a perfect linear relationship between interest and auxiliary variables. Thus, the search for auxiliary information should be guided by the following criterion, referred to as *Principle 2* in the parlance of Särndal and Lundström (2005, p. 110): *the auxiliary vector should explain the interest variable*. In this framework, a good indicator of the capacity of the $\mathbf{x}_j$s to predict the y_js should be obviously given by the fraction of the *Y*-variance explained by the selected variables $X_1,\ldots,X_K$, *i.e.*

$$\eta^2 = 1 - \frac{\sum_{j \in \mathsf{U}} (y_j - \mathbf{b}^{\mathrm{T}} \mathbf{x}_j)^2}{\sum_{j \in \mathsf{U}} (y_j - \bar{Y})^2} , \tag{11}$$

where $\mathbf{b} = \left(\sum_{j \in \mathsf{U}} \mathbf{x}_j \mathbf{x}_j^{\mathrm{T}} \right)^{-1} \sum_{j \in \mathsf{U}} y_j \mathbf{x}_j$ is now the coefficient vector of the least-square hyperplane fitted from the whole population scatter $\{(\mathbf{x}_j, y_j), j \in \mathsf{U}\}$ and $\bar{Y} = T_y / N$ is the population mean. Unfortunately, Principle 2 does not seem a suitable solution, at least in a complete design-based setting. Indeed, η^2 is unknown and consequently we are forced to estimate it by means of

$$\hat{\eta}_R^2 = 1 - \frac{\sum_{j \in \mathsf{R}} \frac{1}{\pi_j} (y_j - \hat{\mathbf{b}}_R^{\mathrm{T}} \mathbf{x}_j)^2}{\sum_{j \in \mathsf{R}} \frac{1}{\pi_j} (y_j - \hat{\bar{Y}}_R)^2} , \tag{12}$$

where $\hat{\bar{Y}}_R = \hat{T}_R / \hat{N}_R$ and $\hat{N}_R = \sum_{j \in \mathsf{R}} \frac{1}{\pi_j}$. In order to derive the design-based properties of (12) as an estimator of (11), it is convenient to rewrite (12) in a more suitable form. After trivial algebra we have

$$\hat{\eta}_R^2 = 1 - \frac{\hat{Q}_R - \hat{\mathbf{a}}_R^{\mathrm{T}} \hat{\mathbf{A}}_R^{-1} \hat{\mathbf{a}}_R}{\hat{Q}_R - \frac{\hat{T}_R^2}{\hat{N}_R}} , \tag{13}$$

where $\hat{Q}_R = \sum_{j \in \mathsf{S}} \dfrac{r_j y_j^2}{\pi_j}$ and $\hat{T}_R$ and $\hat{N}_R$ are suitably expressed in term of S as $\hat{T}_R = \sum_{j \in \mathsf{S}} \dfrac{r_j y_j}{\pi_j}$ and

$\hat{N}_R = \sum_{j \in \mathsf{S}} \dfrac{r_j}{\pi_j}$. Obviously, it is at once apparent that $\mathrm{E}(\hat{Q}_R) = \sum_{j \in \mathsf{U}} r_j y_j^2 = \sum_{j \in \mathsf{U}_R} y_j^2 = Q_R$ as well as

$\mathrm{E}(\hat{T}_R) = \sum_{j \in \mathsf{U}} r_j y_j = \sum_{j \in \mathsf{U}_R} y_j = T_R$ and $\mathrm{E}(\hat{N}_R) = \sum_{j \in \mathsf{U}} r_j = N_R$. Thus, $\hat{\eta}_R^2$ is a function of the five HT

estimators $\hat{Q}_R, \hat{\mathbf{a}}_R, \hat{\mathbf{A}}_R, \hat{T}_R$ and $\hat{N}_R$. Then, from the first-order Taylor series approximation of $\hat{\eta}_R^2$, it

follows that

$$\mathrm{E}(\hat{\eta}_R^2) \approx 1 - \frac{Q_R - \mathbf{a}_R^{\mathrm{T}} \mathbf{A}_R^{-1} \mathbf{a}_R}{Q_R - \dfrac{T_R^2}{N_R}} = 1 - \frac{\displaystyle\sum_{j \in \mathsf{U}_R} (y_j - \mathbf{b}_R^{\mathrm{T}} \mathbf{x}_j)^2}{\displaystyle\sum_{j \in \mathsf{U}_R} (y_j - \bar{Y}_R)^2} = \eta_R^2$$

where $\bar{Y}_R = T_R / N_R$. Practically speaking $\hat{\eta}_R^2$ provides an approximately unbiased estimator of the Y-variance portion explained by the selected variables, not in the whole population U but only in the respondent population U_R. Paradoxically, the procedure based on $\hat{\eta}_R^2$ may provide reliable choices of the auxiliary variables only if the linear relationship among interest and auxiliary variables is similar for respondent and nonrespondent units, a situation which alone ensure approximate unbiasedness.

In order to search for auxiliary variables which behave similarly for respondent and nonrespondent units, a promising, even if trivial, procedure should be based on the comparison of ranges in respondent and nonrespondent populations (Info U) or samples (Info S). Indeed, if the values of an auxiliary variable in the respondent population (sample) tend to be much greater or lower than the values of the same variable in the nonresponse counterpart, then it is quite difficult that the same linear relationship may hold for both cases. As a very simple example, consider the slope of terrain as an auxiliary variable adopted to predict the timber volume in forest inventories. Slopes (in degrees) in sites/plots which can be easily surveyed by foresters (respondent population) usually

range from 0 to 40 degrees while they range from about 40 to 60 degrees (with some values reaching 80 degrees) for those plots which cannot be surveyed (nonrespondent population). Thus, it is quite unlike that the same linear relationship might be valid to predict timber volume as linear function of slope in the whole range 0-80.

From these considerations, it seems that the choice of auxiliary information in design-based NCW approach should be guided by practical considerations about the nature of the variables and their relationship with the interest variable rather than by rigid quantitative indicators which, being necessarily computed from the respondent sample, can reflect the actual situation only for the respondent population.

Finally, even if obvious, it is also worth noting that the selection of highly correlated auxiliary variables should be avoided and only one of them should be used. Indeed, the use of highly correlated variables deteriorates the estimation of the regression coefficients without providing relevant additional information.

5. Simulation study.

Empirical investigations were used to throw light in: a) the capability of the approximate expressions for the bias and the variance to guess the actual values; b) the design-based accuracy of the calibration estimator in terms of amount of nonresponse, effectiveness of the auxiliary variables to predict the interest variable, differences in the behaviour of the auxiliary variable between respondent and nonrespondent units and multicollinearity among auxiliary variables; c) the capability of variance estimators to evaluate the accuracy of the calibration estimator and to give confidence interval with coverage near to the nominal level.

To this purpose a population of $N = 1,000$ individuals was considered, partitioned into respondent and nonrespondent stratum. The size of respondent stratum was presumed to be $N_R = 300, 600, 900$ corresponding to respondent percentages (say RP) of $30\%, 60\%, 90\%$. Then, two auxiliary variables

X_1^* and X_2^* were supposed to be known for each population unit (Info U). For each unit $j \in U$, the values x_{j1}^* and x_{j2}^* were generated from a bivariate normal distribution with expectations $\mu_1 = \mu_2 = 1$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Moreover, in order to take into account different degrees of multicollinearity (MC), a correlation of 0, 0.5 and 0.9 was presumed between X_1^* and X_2^*. Then, for each unit of the respondent stratum the interest variable Y was achieved from the relation

$$y_j = 1 + 0.5x_{j1}^* + 0.5x_{j2}^* + \varepsilon_j, \tag{14}$$

where ε_j was an error term generated from a centred normal distribution. On the other hand, as to nonrespondent stratum, three similarity levels (SL) with relation (14) were considered: a first situation (say SL1) in which the y_js were generated by the same relation adopted in respondent stratum, a second situation (say SL2) in which the coefficients attached to x_{j1}^* and x_{j2}^* were two times those adopted in respondent stratum, a third situation (say SL3) in which the coefficients were four times those adopted in respondent stratum. Finally, the variances of the error terms in respondent and nonrespondent stratum, say δ_R^2 and δ_{NR}^2, were chosen in such a way to achieve a fraction of explained variance (say FEV) equal to 0.3, 0.6 and 0.9 for both the respondent and nonrespondent population point scatters. Then, from the possible combinations of RP, SL, FEV and MC, a final set of 81 populations was achieved.

In each population, 10,000 samples of size $n = 50$ (corresponding to sampling fraction of 5%) were selected by means of simple random sampling without replacement (SRSWOR). For each selected sample the following quantities were computed: $\hat{T}_{CAL}$, V_{SYG}^2 and V_{jack}^2 (note that V_{SYG}^2 and V_{HT}^2 coincide under SRSWR). From the variance estimates the corresponding estimates of the relative standard error $RSE_{SYG} = V_{SYG}/\hat{T}_{CAL}$ and $RSE_{jack} = V_{jack}/\hat{T}_{CAL}$ were also computed together with the confidence intervals $\hat{T}_{CAL} \pm 1.96V_{SYG}$ and $\hat{T}_{CAL} \pm 1.96V_{jack}$. Then, from the resulting Monte

14

Carlo distributions, the relative bias, coefficient of variation and relative root mean squared error of $\hat{T}_{CAL}$, say RB-CAL, CV-CAL and RRMSE-CAL were empirically evaluated together with the expectations of RSE_{SYG} and RSE_{jack}, say ERSE-SYG and ERSE-JACK and the coverage of the confidence intervals, say CVRG-SYG and CVRG-JACK, achieved as the percentage of times the intervals included the true total. Moreover, for each population the approximate bias and variance of $\hat{T}_{CAL}$, say ARB-CAL and ACV-CAL were analytically computed by means of equations (4) and (7), together with the coefficient of variation which would be achieved by the HT estimator in the case of complete response, say CV-HT. This quantity was included as a bench-mark with which the accuracy of $\hat{T}_{CAL}$ can be compared.

For each population, Tables 1-5 reports the percent values of ARB-CAL, RB-CAL, ACV-CAL, CV-CAL, RRMSE-CAL, CV-HT, ERSE-SYG, ERSE-JACK, CVRG-SYG and CVRG-JACK. The simulation results motivates the following comments:

- when the relationship among interest and auxiliary variables is similar in respondent and nonrespondent sub-populations (SL1), underestimation due to nonresponse turns out to be negligible but it markedly increases as differences in the relationships are present (SL2 and SL3); downward bias also increases with the amount of nonresponse but it seems to be poorly influenced by FEV and MC factors (see Table 1);

- the approximate bias expression (4) turns out to be quite accurate; for RP equal to 30%, the approximate relative bias shows differences with the actual relative bias always smaller than 5 percentage points except for FEV equal to 0.3 and MC equal to 0.9, when the differences are of about 10 percentage points; the accuracy of the approximation quickly increases as RP increases with differences which become negligible when RP reaches 90% (see Table 1);

- even if the approximate variance expression (7) invariably provides underestimation of the actual variance, it turns out to be satisfactory: the differences between the approximate and actual

coefficient of variations are always smaller then 3 percentage points for RP equal to 30% and become negligible as RP increases (see Table 2);

- when the relationship among interest and auxiliary variables is similar in respondent and nonrespondent sub-populations (SL1), calibration estimation is worse than the complete-sample HT estimator only in presence of a massive amount of nonresponse (RP=30%), while for smaller amounts the calibration procedure even provides improvement with respect to the complete-sample performance; on the other hand, when the difference between the relationships in respondent and nonrespondent sub-populations become marked, the presence of substantial bias deteriorates the performance of calibration estimator with relative errors 3-5 times greater that those achieved with complete samples (see Table 3);

- the SYG/HT variance estimator always provides underestimation of the relative standard error as opposite to jackknife estimator which proves to be invariably conservative; both downward and upward bias tend to reduce as RP and FEV increase: for RP equal to 90% and FEV equal to 0.9 both the estimators are approximately unbiased (see Table 4);

- the coverage of confidence intervals well approximate the nominal level only when the relationship among interest and auxiliary variable is similar in respondent and nonrespondent sub-populations (SL1) and for respondent percentages of 60 and 90%; in the other cases (SL2 and SL3), the presence of bias skews the confidence intervals entailing disastrous coverage losses; intervals achieved using the jackknife variance estimator invariably perform better than those achieved using SYG/HT estimator (see Table 5).

6 . Final remarks

The design-properties of the calibration estimation are approximated considering the unit nonresponse as a fixed characteristic, just like the values of interest and auxiliary variables, a situation which is likely to occur in enviromnental surveys. On the basis of the approximate

expression of the variance of calibration estimator, three variance estimators were attempted using the SYG, HT and jackknife criteria. Obviously all these estimators are likely to provide reliable accuracy evaluations and confidence intervals only when nonresponse bias is small. The results of simulation study largely confirm these considerations. In presence of a large bias, which is mainly generated when different relationships among interest and auxiliary variables hold in respondent and nonrespondent sub-populations, any inference (estimation, estimation of accuracy and confidence interval construction) turns out to be completely unreliable. Thus, attention should be paid in the selection of auxiliary variables which should be chosen not on the basis of their capability to explain the interest variable (which can only be checked on the respondent population) but rather on the basis of the stability of their relationship with the interest variable in respondent and non-respondent sub-populations. In this framework, the choice of auxiliary variables should then be mainly guided by practical considerations and previous experiences. Under small bias and small amount of nonresponse, simulation results prove the effectiveness of NCW: the calibration estimator compares well with (and in some situations even improves over) the complete-sample HT estimator while the jackknife variance estimator is moderately conservative providing confidence intervals with good coverage. Then, conditional to small biases, NCW approach seems to be especially appealing in environmental surveys, where nonresponse percentages are usually be smaller than 5%.

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Table 1. Percentage values of approximate relative bias of calibration estimator compared with the actual values for samples of size $n = 50$ selected from 81 populations of 1,000 obtained from each combination of RP, SL, FEV and MC.

SL	FEV	MC	RP = 30%		RP = 60%		RP = 90%	
			ARB-CAL	RB-CAL	ARB-CAL	RB-CAL	ARB-CAL	RB-CAL
SL1	0.3	0	0	-2	0	3	0	0
		0.5	0	3	0	4	0	1
		0.9	0	-7	0	1	0	1
	0.6	0	0	-2	0	1	0	0
		0.5	0	-2	0	-1	0	0
		0.9	0	1	0	1	0	0
	0.9	0	0	1	0	0	0	0
		0.5	0	0	0	0	0	0
		0.9	0	1	0	0	0	0
SL2	0.3	0	-26	-26	-17	-19	-5	-4
		0.5	-24	-30	-17	-20	-4	-2
		0.9	-25	-28	-16	-17	-5	-3
	0.6	0	-27	-27	-16	-15	-4	-5
		0.5	-27	-26	-16	-17	-5	-4
		0.9	-26	-27	-16	-16	-6	-5
	0.9	0	-26	-26	-16	-17	-5	-5
		0.5	-24	-25	-17	-17	-5	-5
		0.9	-24	-24	-17	-17	-4	-5
SL3	0.3	0	-49	-50	-36	-39	-14	-14
		0.5	-50	-50	-37	-37	-14	-12
		0.9	-57	-46	-37	-41	-14	-14
	0.6	0	-50	-51	-37	-38	-12	-12
		0.5	-51	-50	-37	-37	-12	-14
		0.9	-50	-55	-41	-39	-13	-15
	0.9	0	-51	-50	-38	-39	-13	-13
		0.5	-51	-51	-37	-37	-12	-12
		0.9	-51	-52	-40	-39	-13	-13

Table 2. Percentage values of approximate coefficient of variation of calibration estimator compared with the actual values for samples of size $n = 50$ selected from 81 populations of 1,000 obtained from each combination of RP, SL, FEV and MC.

SL	FEV	MC	RP = 30%		RP = 60%		RP = 90%	
			ACV-CAL	CV-CAL	ACV-CAL	CV-CAL	ACV-CAL	CV-CAL
SL1	0.3	0	12	14	10	10	8	8
		0.5	16	19	12	13	9	10
		0.9	18	21	13	14	10	11
	0.6	0	7	8	5	5	4	4
		0.5	9	10	6	7	5	5
		0.9	10	12	7	8	6	6
	0.9	0	3	3	2	2	2	2
		0.5	3	4	3	3	2	2
		0.9	4	5	3	3	2	2
SL2	0.3	0	10	11	8	8	7	8
		0.5	13	15	10	11	10	10
		0.9	14	15	11	11	9	10
	0.6	0	5	6	5	5	4	4
		0.5	7	8	5	6	5	5
		0.9	7	8	6	6	5	5
	0.9	0	2	2	2	2	2	2
		0.5	3	3	2	2	2	2
		0.9	3	4	2	3	2	2
SL3	0.3	0	6	7	6	6	6	7
		0.5	7	8	7	7	9	9
		0.9	10	12	8	8	9	9
	0.6	0	4	4	3	3	4	4
		0.5	5	5	4	4	5	5
		0.9	5	5	4	5	5	5
	0.9	0	2	2	1	1	2	2
		0.5	2	2	2	2	2	2
		0.9	2	2	2	2	2	2

Table 3. Percentage values of relative root mean squared error of calibration estimator compared with the coefficient of variation of the complete-sample Horvitz-Thompson estimator for samples of size $n = 50$ selected from 81 populations of 1,000 obtained from each combination of RP, SL, FEV and MC.

SL	FEV	MC	RP = 30%		RP = 60%		RP = 90%	
			RRMSE-CAL	CV-HT	RRMSE-CAL	CV-HT	RRMSE-CAL	CV-HT
SL1	0.3	0	14	9	10	9	8	9
		0.5	19	11	14	11	10	11
		0.9	22	13	14	12	11	12
	0.6	0	8	6	5	6	4	6
		0.5	10	8	7	8	5	8
		0.9	12	9	8	9	6	9
	0.9	0	3	5	2	5	2	5
		0.5	4	6	3	6	2	6
		0.9	5	7	3	7	2	7
SL2	0.3	0	28	11	21	11	9	10
		0.5	34	14	23	14	10	13
		0.9	32	16	21	15	10	12
	0.6	0	27	9	16	8	7	7
		0.5	27	10	18	10	7	9
		0.9	28	12	17	12	7	10
	0.9	0	26	7	17	7	5	6
		0.5	25	9	18	8	5	7
		0.9	24	10	17	10	5	8
SL3	0.3	0	51	15	40	16	16	13
		0.5	51	18	38	18	15	16
		0.9	47	22	42	21	17	18
	0.6	0	51	12	38	12	13	9
		0.5	51	13	37	14	15	11
		0.9	56	14	39	16	16	13
	0.9	0	50	10	39	11	13	9
		0.5	51	12	37	13	12	10
		0.9	52	13	39	13	13	11

Table 4. Percentage values of the expectations of Sen-Yates-Grundy and jackknife estimator of relative standard error of calibration estimator compared with the actual values of relative root mean squared error (in parenthesis) for samples of size $n = 50$ selected from 81 populations of 1,000 obtained from each combination of RP, SL, FEV and MC.

SL	FEV	MC	RP = 30%			RP = 60%			RP = 90%		
			ERSE-SYG		ERSE-JACK	ERSE-SYG		ERSE-JACK	ERSE-SYG		ERSE-JACK
SL1	0.3	0	12	(14)	16	9	(10)	10	8	(8)	9
		0.5	16	(19)	20	12	(13)	13	9	(10)	10
		0.9	20	(21)	26	13	(14)	14	10	(11)	11
	0.6	0	7	(8)	9	5	(5)	5	4	(4)	4
		0.5	9	(10)	12	6	(7)	7	5	(5)	5
		0.9	10	(12)	13	7	(8)	8	6	(6)	6
	0.9	0	3	(3)	4	2	(2)	2	2	(2)	2
		0.5	3	(4)	4	3	(3)	3	2	(2)	2
		0.9	4	(5)	5	3	(3)	3	2	(2)	2
SL2	0.3	0	13	(11)	17	10	(8)	11	8	(8)	8
		0.5	18	(15)	24	13	(11)	14	10	(10)	10
		0.9	18	(15)	24	13	(11)	14	10	(10)	10
	0.6	0	7	(6)	9	5	(5)	6	4	(4)	4
		0.5	9	(8)	11	6	(6)	7	5	(5)	6
		0.9	9	(8)	12	7	(6)	8	5	(5)	6
	0.9	0	3	(2)	4	2	(2)	2	2	(2)	2
		0.5	3	(3)	4	3	(2)	3	2	(2)	2
		0.9	4	(4)	5	3	(3)	3	2	(2)	2
SL3	0.3	0	12	(7)	16	10	(6)	11	8	(7)	8
		0.5	15	(8)	20	11	(7)	12	10	(9)	10
		0.9	19	(12)	24	13	(8)	14	11	(9)	11
	0.6	0	7	(4)	9	5	(3)	6	4	(4)	4
		0.5	9	(5)	12	6	(4)	7	5	(5)	6
		0.9	10	(5)	13	7	(5)	8	6	(5)	6
	0.9	0	3	(2)	4	2	(1)	2	2	(2)	2
		0.5	4	(2)	5	3	(2)	3	2	(2)	2
		0.9	4	(2)	5	3	(2)	3	2	(2)	2

Table 5. Percentage values of the actual coverage of Sen-Yates-Grundy and jackknife confidence intervals with nominal coverage of 95% for samples of size $n = 50$ selected from 81 populations of 1,000 obtained from each combination of RP, SL, FEV and MC.

SL	FEV	MC	RP = 30%		RP = 60%		RP = 90%	
			CVRG-SYG	CVRG-JACK	CVRG-SYG	CVRG-JACK	CVRG-SYG	CVRG-JACK
SL1	0.3	0	88	95	92	94	93	94
		0.5	88	94	92	94	93	95
		0.9	86	93	93	95	94	95
	0.6	0	88	94	92	94	93	95
		0.5	88	94	92	94	94	95
		0.9	88	94	92	94	94	95
	0.9	0	89	95	93	95	93	95
		0.5	88	94	92	94	94	95
		0.9	87	94	93	95	94	95
SL2	0.3	0	25	39	34	41	90	92
		0.5	33	51	47	55	93	95
		0.9	40	56	62	68	92	93
	0.6	0	2	6	9	13	72	76
		0.5	4	11	11	15	85	87
		0.9	6	15	25	32	82	84
	0.9	0	0	0	0	0	18	21
		0.5	0	0	0	0	31	35
		0.9	0	0	0	0	40	44
SL3	0.3	0	0	0	0	0	39	43
		0.5	0	1	0	0	69	72
		0.9	2	7	0	0	65	69
	0.6	0	0	0	0	0	8	10
		0.5	0	0	0	0	14	16
		0.9	0	0	0	0	16	18
	0.9	0	0	0	0	0	0	0
		0.5	0	0	0	0	0	0
		0.9	0	0	0	0	0	0