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Equivalence Scales Declining with Expenditure: Evidence and Implications for Income Distribution

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Abstract - We estimate expenditure dependent equivalence scales for Italian couples with and without children. Following Donaldson and Pendakur (2006) we incorporate the generalized absolute equivalence-scale exactness (GAESE) restrictions into a translated quadratic almost ideal (TQAI) demand system. We obtain declining with expenditure scales, a pattern that tends to strengthen when the number of children increases. This implies that scale economies in current consumption are lower for families with poor expenditure capacities. We also show that families living in the South and the Islands suffer a substantial additional cost to achieve, *ceteris paribus*, the same well-being of those living in the North. Finally, we find that ignoring the declining with expenditure pattern results in a relevant understatement of measured inequality.

JEL codes: D11; D12
Key Words: Equivalent Expenditure Functions; Equivalence Scales

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1. INTRODUCTION

Equivalence scales are indexes that convert the expenditure of a generic consumption unit into the expenditure of a specific reference unit, called equivalent expenditure, maintaining the same well-being of the former. A generic unit may be any household, but the reference unit is a household with specified demographic characteristics, as a childless couple or a single. Equivalence scales measure the extent to which consumption units share goods internally: higher values mean lower economies of scale. For example, given a childless couple as the reference unit, if a couple with a child spends 1,200 euros per month and the corresponding index is 1.2, this household enjoys the same level of well-being as a childless couple spending 1,000 euros (=1,200/1.2). With the scale at 1.5, this household is equivalent to a childless couple spending 800 euros (=1,200/1.5).

Equivalence scales play an essential role in a widening range of empirical applications where welfare comparisons among individual consumption units are to be made. Indexing schemes for social benefits payments or exemptions are usually based on this device. Studies typically based on equivalence scales, such as poverty analysis or welfare distribution analysis, often represent essential pieces of information from which important policy decisions are made. Along with these well-established applications, new fields have been proposed in recent years, for example in dealing with legal problems such as alimony and wrongful death calculations (Allen, 2007; Lewbel, 2003).

The large number of applications has been encouraged by long-standing theoretical research, closely related to the analysis of consumer demand. In fact, "modern equivalence scales measure well-being in terms of utility, using cost (expenditure) functions estimated from consumer demand data via revealed preference theory" (Lewbel and Pendakur, 2006**,** p. 3). But scale computation from estimated cost functions poses a crucial identification problem that can only be solved introducing additional restrictions on preferences. Keeping up with the fundamental progress in empirical demand systems specification occurred in past decades, increasingly general scale identification restrictions have been proposed.

The case in which an equivalence scale is independent of utility, but depending on the other variables entering a cost function, i.e. prices and demographic characteristics of consumption units, was analysed by Lewbel (1989) and by Blackorby and Donaldson (1993). They called this condition "independence of base" (IB) and "equivalence scale exactness" (ESE), respectively. The IB/ESE is a rather general condition representing a benchmark when scales are to be estimated. However, it involves strong restrictions on empirical cost functions and the utility-independence requirement may be unacceptable if departures from it are large.

Additional fixed costs connected with specific household characteristics, such as the presence of children or disabled people, should result, *ceteris paribus*, in an equivalence scale declining with expenditure. Also, it seems intuitive that the household members' ability to share goods depends on the household's expenditure capacity. For example, necessity may encourage sharing. But scarcity and sharing may be conflicting circumstances, as well; when this is the case, necessity would discourage sharing. The resulting effect is *a priori* unpredictable.

Two cases of expenditure dependent equivalence scales have been considered by Donaldson and Pendakur (2004, 2006). The condition called generalized equivalence-scale exactness (GESE, 2004) is a generalization of IB/ESE where equivalence expenditures are isoelastic with respect to expenditure. The condition called generalized absolute equivalence-scale exactness (GAESE, 2006) is a different generalization of IB/ESE not implying isoelasticity. The latter, incorporated into the rank-4 translated quadratic almost ideal (TQAI) demand system recently proposed by Lewbel (2004), is the one preferred by these authors.

Empirical estimations of these models using Canadian data give the rather general and clear result of equivalence scales declining with expenditure (Donaldson and Pendakur, 2004, 2006). General, because scales of all household types show the same pattern, even though for childless couples the GESE scale declines very slowly. Clear, because scale' reductions, measured between the top and the bottom vigintiles of the expenditure distributions, are substantial, even though not very large, especially those given by GAESE.

Other applications giving similar results have been developed using a survey method by Koulovatianos *et al*. (2005a) for Germany and France, and by Koulovatianos *et al*. (2005b) for Cyprus. Majumder and Chakrabarty (2008) have estimated, through Engel's curves analysis, an equivalence scale declining with expenditure for India.

In our opinion, further empirical evidence is needed to explore the equivalence scales' expenditure dependence. The value of this information is best expressed by Donaldson and Pendakur (2004, pag. 200). "Because we find declining equivalence scales for households with children, this suggests that the use of equivalence scales from the middle of the distribution of well-being understates poverty rates. In the case of inequality measurement, equivalence scales that decrease with expenditure for some household types imply more inequality for those types than expenditure-independent equivalence scales do. In addition, equivalence scales that decline with expenditure increase the optimal amount of progressivity because rich households with children face lower (relative) costs of children than poor households do".

In Italy, the long-standing and rather inconclusive debate on taxation seems now close to end in a fiscal reform strongly impacting on the whole economic and administrative system. All factors involved should therefore be carefully considered; the existence and the pattern of equivalence scale dependence on expenditure is one of such elements.

When new fields of application are considered, the availability of proper equivalence scale estimates is equally important. For example, alimony is a critical element in regulating many long-term relationships and inaccurate calculations may have important social consequences even when the degree of inaccuracy is small.

In this paper we calculate GAESE scales using Italian data finding that the strength and the direction of the scales' dependence on expenditure varies with the presence and number of children, and with the working condition of the household members. When the number of children increases, the decreasing pattern tends to be very strong.

The remainder of this paper is organised as follows: equivalence scales and their properties are defined in section 2; the empirical model is specified in section 3; the data, econometric strategy and results are shown in sections 4 and 5; section 6 discusses some implications for income distribution; section 7 concludes.

2. EQUIVALENCE SCALES: DEFINITION AND PROPERTIES

A cost function depending on a vector of demographic characteristics, *z*, in addition to the utility level, *u*, and the price vector, *p*, is called a conditional cost function (Pollak, 1989), $C(p,u,z)$. Since only households' total current expenditure, *x*, is considered, static demand theory as a modelling tool is fully adequate, i.e. $x = C(p,u,z)$. Hereafter, we assume the existence of a cost and an indirect utility function satisfying all usual regularity conditions implied by the theory. In what follows we summarize the discussion proposed in Donaldson and Pendakur (2006); the interested reader may refer to this work for further details.

Modern equivalence scales, based on the equalization of utility levels between consumption units, households in this context, may be defined through conditional cost functions, as follows:

$$s_R = C(p,u,z)/C(p,u,\bar{z}) \quad (1)$$

$$s_A = C(p,u,z) - C(p,u,\bar{z}) \quad (2)$$

where $\bar{z}$ is the vector of characteristics of a reference household, for example a childless couple living in the centre of Italy, s_R is a relative equivalence scale (the standard notion of equivalence scale when no further specification is done), and s_A is an absolute equivalence scale.

Once the reference household has been specified, $\bar{z}$ is a constant vector, therefore both the cost function and the corresponding indirect utility of this household may be written as:

$$C(p,u,\bar{z}) = C^r(p,u) \quad (3)$$

$$V(p,x,\bar{z}) = V^r(p,x) \tag{4}$$

Given an household total current expenditure x, where $x = C(p,u,z)$, the term $C(p,u,\bar{z})$ may be expressed in terms of *x*, *p* and *z*, by replacing *u* with $V(p,x,z)$:

$$x^e = X(p,x,z) = C^r(p,V(p,x,z)) \tag{5}$$

$X(p,x,z)$ is the equivalent expenditure, i.e. the expenditure x^e giving the reference family the same level of satisfaction the expenditure *x* gives to the family with demographic characters *z*. Given the properties of *C* and *V*, $X(p,x,z)$ is homogeneous of degree one in (p,z), is increasing in *x* and, for all (p,z), $X(p,x,\bar{z}) = x$. Moreover, the following expression holds:

$$V(p,x,z) = V^r(p,x^e) \tag{6}$$

Since $X(p,x,z) = x / s_R$ (or $X(p,x,z) = x - s_A$ if absolute scales are considered), equivalence scales convert an economy of heterogeneous families into an economy of identical families, i.e the reference household.

Expression (1) and (2) may be rewritten in terms of equivalent expenditure as:

$$S_R(p,x,z) = x / X(p,x,z) = s_R(p,u,z) \tag{7}$$

$$S_A(p,x,z) = x - X(p,x,z) = s_A(p,u,z) \tag{8}$$

From (7) and (8) an equivalence scale depends, in general, on the price vector, the demographic characters and the expenditure level. However, when a scale has to be estimated empirically through observable data, and a cost (or, equivalently, an indirect utility) function has to be specified, a fundamental identification problem arise. The source of the problem is the ordinal nature of the notion of utility defined in static demand theory, implying $C(p,u,z) = C(p,\phi(u,z),z)$ for any function $\phi(u,z)$ strictly monotonically increasing in *u*. This introduces an unsolvable indeterminacy when S_R (or S_A) is to be estimated from behavior (cf. Lewbel and Pendakur, 2006).

A way to solve this problem is imposing specific structures on the cost function. A class of equivalence scales that are independent of utility, and therefore are independent of expenditure levels, has been investigated by Lewbel (1989) and Blackorby and Donaldson (1993, 1994).

A relative scale is independent of *u*, i.e. $S_R(p,x,z) = \bar{S}_R(p,z)$, if and only if the cost function is multiplicatively decomposable, i.e. $E(p,u,z) = \bar{S}_R(p,z)E^R(p,u)$. This condition has been called ‘independence of base’ (IB) and ‘equivalence-scale exactness’ (ESE), respectively, by Lewbel (1989) and Blackorby and Donaldson (1993). An absolute scale is independent of *u*, i.e. $S_A(p,x,z) = \bar{S}_A(p,z)$, if and only if the cost function is additively decomposable, i.e. $E(p,u,z) = \bar{S}_A(p,z) + E^R(p,u)$. This condition has been called 'absolute equivalence-scale exactness' (AESE) by Blackorby and Donaldson (1994). However, ESE and AESE are not sufficient, if accepted as maintained hypothesis, to ensure the identification of, respectively, relative and absolute scales from demand data if additional restrictions upon the reference cost function are not imposed. Such restrictions have been introduced by Blackorby and Donaldson (1993, 1994; see Donaldson and Pendakur, 2006, for a review).

Donaldson and Pendakur (2006) have recently proposed a generalization of the ESE/AESE conditions. The starting point of their discussion is a common property shared by ESE and AESE conditions: the response of equivalent expenditure to a marginal change in expenditure is independent of expenditure. Consistently with this property, they assume the existence of a real function ρ such that:

$$\frac{\partial X(p,x,z)}{\partial x} = \rho(p,z) \tag{9}$$

Integrating with respect to *x*, an alternative equivalent expenditure function is obtained:

$$x^e = X(p,x,z) = \rho(p,z)x + \alpha(p,z) = \frac{x - A(p,z)}{R(p,z)} \tag{10}$$

where α is a generic function on (p,z), $R(p,z) = 1/\rho(p,z)$ and $A(p,z) = -\alpha(p,z)/\rho(p,z)$.

From the properties of X, ρ and α are homogeneous functions in *p*, of degree zero and one, respectively. Moreover, for all *p*, $\rho(p,\bar{z}) = 1$ and $\alpha(p,\bar{z}) = 0$; therefore, for all *p*, $R(p,\bar{z}) = 1$ and $A(p,\bar{z}) = 0$ also hold.

Combining expressions (6) and (10), the following indirect utility function is obtained:

$$V(p,x,z) = V^r\left(\frac{x - A(p,z)}{R(p,z)}\right) \tag{11}$$

Moreover, combining expressions (5) and (10), the following cost function results:

$$C(p,u,z) = R(p,z)C^r(p,u) + A(p,z) \tag{12}$$

Donaldson and Pendakur (2006) call conditions (10)-(12) generalized absolute equivalence-scale exactness (GAESE). When GAESE is the maintained hypothesis, relative and absolute equivalence scales become:

$$S_R(p,x,z) = \frac{R(p,z)x}{x - A(p,z)} \tag{13}$$

$$S_A(p,x,z) = \frac{(R(p,z) - 1)x + A(p,z)}{R(p,z)} \tag{14}$$

GAESE generalizes both ESE and AESE. This means that ESE and AESE are special cases of GAESE: (i) ESE holds if, and only if, $A(p,z)=0$ (or $\alpha(p,z)=0$) for all (p,z); (ii) AESE holds if, and only if, $R(p,z)=1$ (or $\rho(p,z)=0$) for all (p,z). This also implies that more general patterns are possible: (i) S_R is increasing (decreasing) in x if and only if $A(p,z)<0$ ($A(p,z)>0$); S_A is increasing (decreasing) in x if and only if $R(p,z)>1$ ($R(p,z)<1$). Moreover, for goods not consumed by the reference household (such as a child good in case of a childless reference family), ESE and AESE constrain expenditure elasticities to be one and zero, respectively, but GAESE does not.

When GAESE is the maintained hypothesis, equivalent expenditure functions are not identified without additional restrictions. Donaldson and Pendakur prove that equivalent expenditure functions are uniquely identified by demand data if GAESE holds and the reference cost function is neither affine nor log-affine, provided the vector *z* has at least two continuous components and V is a continuous function of them. Such additional conditions, considered separately, are similar to those ensuring identification when AESE and ESE are the maintained hypothesis (Blackorby and Donaldson, 1993 and 1994).

3. A TRANSLATED QUADRATIC ALMOST IDEAL DEMAND SYSTEM INCORPORATING GAESE

Lewbel (2003) has proposed a rank four demand system consistent with utility maximization which nests some lower rank commonly used demand systems as special cases. Since the maximum rank of an exactly aggregable demand system is three (Gorman, 1981), this model, called translated quadratic almost ideal demand system (TQAI), may only be applied to single consuming units, such as households. The class of indirect utility functions from which TQAI comes is the following:

$$V(p,x,z)=\left(\left(\frac{\log\left[(x-d(p,z))/a(p,z)\right]}{b(p,z)}\right)^{-1}-q(p,z)\right)^{-1}, \qquad (15)$$

where none of the terms *a*, *b*, *q* and *d* can be written as a function of the other three. Since *V* is homogeneous of degree zero in (p,x), *b* and *q* are homogeneous of degree zero in *p*, and exp(*a*) and *d* are homogeneous of degree one in *p*; since V is increasing in x, then $a(p,z)>0$ for every (p,z).

When $d(p,z)=0$ equation (15) reduces to the class of indirect utilities embodying the rank three quadratic almost ideal (QAI) demand system of Banks, Blundell and Lewbel (1997);

when $q(p,z)=0$ and $d(p,z)=0$, equation (15) reduces to the class of indirect utilities embodying the rank two almost ideal (AI) demand system of Deaton and Muellbauer (1980).

Assuming GAESE and the existence of an household with TQAI preferences, from equation (11) such preferences may be written as follows:

$$V(p,x,z)=\left(\left(\frac{\log\left[\left(\frac{x-A(p,z)}{R(p,z)}-d^r(p)\right)\Big/a^r(p)\right]}{b^r(p)}\right)^{-1}-q^r(p)\right)^{-1}$$

$$=\left(\left(\frac{\log\left[\frac{x-(A(p,z)+R(p,z)d^r(p))}{R(p,z)a^r(p)}\right]}{b^r(p)}\right)^{-1}-q^r(p)\right)^{-1} \tag{16}$$

Imposing GAESE into TQAI preferences still gives TQAI preferences, and, comparing (15) term by term with the last member of (16), the following relations hold:

$$d(p,z)=R(p,z)d^r(p)+A(p,z) \tag{17}$$

$$a(p,z)=R(p,z)a^r(p) \tag{18}$$

$$b(p,z)=b^r(p) \tag{19}$$

$$q(p,z)=q^r(p) \tag{20}$$

Since (11) is a general relation, so is (16). It follows that, assuming some households having TQAI preferences and given GAESE as a maintained hypothesis, all households have TQAI preferences. Any other possibility would violate (16).

Donaldson and Pendakur (2006) use results (17)-(20) to define the analytical framework essential to calculate GAESE equivalence scales and to test some of the implied hypotheses. In particular, (19) and (20) restrict the range of suitable TQAI preferences and may be used to test GAESE against more general specifications, while (17) and (18) allow the calculation of terms *A* and *R* which determine the equivalent expenditure function:

$$R(p,z)=a(p,z)\big/a^r(p) \tag{21}$$

$$A(p,z)=d(p,z)-R(p,z)d^r(p) \tag{22}$$

Relation (21) also allows testing AESE against GAESE, since testing whether $R(p,z)=1$ is equivalent to test $a(p,z)=a^r(p)$. Relation (22) also allows testing ESE against GAESE, because testing whether $A(p,z)=0$ is equivalent to test $d(p,z)=d^r(p)=0$. Since $a(p,z)>0$ then $R(p,z)>0$ for every (p,z), but $A(p,z)$ may have any sign.

To complete the indirect utility function specification, the terms *a*, *b*, *q* and *d* have to be defined. Donaldson and Pendakur (2006) have proposed the following:

$$d(p,z)=\sum_{k=1}^{m} d_k(z)p_k \ , \tag{23}$$

$$\log a(p,z)=a_0(z)+\sum_{k=1}^{m} a_k(z)\log p_k + \frac{1}{2}\sum_{k=1}^{m}\sum_{j=1}^{m} a_{kj}\log p_k \log p_j \ , \tag{24}$$

where $\sum_{k=1}^{m} a_k(z)=1$, $\sum_{l=1}^{m} a_{kl}(z)=0$ for every *k*, $a_{kl}(z)=a_{lk}(z)$ for every *k* and *l*,

$$\log b(p,z)=\sum_{k=1}^{m} b_k(z)\log p_k \ , \tag{25}$$

where $\sum_{k=1}^{m} b_k(z)=0$, and

$$q(p,z)=\sum_{k=1}^{m} q_k(z)\log p_k \ , \tag{26}$$

where $\sum_{k=1}^{m} q_k(z)=0$, k=1, …, m; m is the number of commodities.

Expressions (23)-(26) coincide with those of Lewbel (2003) except for (23) defining $d(p,z)$[1], and for the fact that Lewbel do not explicit the terms *a*, *b*, *q* and *d* as functions of demographic characters.

We adopt the Donaldson and Pendakur's specification except for the specification of parameters d_k, a_k, b_k and q_k as functions of the demographic variables vector *z*.

First, we do not consider the terms b_k and q_k as varying with *z*. In practice, we renounce to test GAESE against an unrestricted TQAI alternative. Donaldson and Pendakur (2006) actually do not reject this alternative, nevertheless they then assume GAESE as the maintained hypothesis. When empirical analysis is done using microdata, we conjecture that any additional variable tends to explain a significant part of an extremely large variability, since the occurrence of spurious correlations becomes very likely.

Second, for the parameters a_k and d_k specified as functions of *z*, according to GAESE, we choose the less sophisticated and less parsimonious formulation adopted by Balli and Tiezzi (2009). Donaldson and Pendakur demographics specification may handle a wide range of

[1] The term *d(p,z)* of Donaldson and Pendakur (2006) corresponds to the term *a(p)* of Lewbel (2003). The correct expression of Lewbel's *a(p)*, which has the same structure of the term *b(p,z)* defined above, is reported in the corrected version of the mentioned Lewbel's article (see references).

household situations by imposing specific functional structures. However, since only few household types are considered here, the chosen specification seems to be more general, not imposing any functional structure, and takes into account the geographic location and working condition of the family.

Third, we introduce a logarithmic annual trend, only modifying the log linear component of the translog price index. Then, parameters $a_k(z)$ are to be written as $a_k(z,t)$ (but $a_0(z)$ remains unchanged), and, in turn, the terms $a(p,z)$ and $a^r(p)$, defined by equation (24), are to be written as $a(p,t,z)$ and $a^r(p,t)$. This specification is the most parsimonious allowing a trend and maintaining nestability with the AI system estimated by Balli and Tiezzi (2009).

These considerations bring to the following:

$$d_k(z) = \bar{d}_k + d_k^{c1}c1 + d_k^{c2}c2 + d_k^{c3}c3 + d_k^{nw}nw + d_k^{ne}ne + d_k^{si}si + d_k^{w2}e2 + d_k^{e1}q1 + d_k^{e2}q2\,, \qquad (27)$$

$$a_0(z) = \bar{a}_0 + a_0^{c1}c1 + a_0^{c2}c2 + a_0^{c3}c3 + a_0^{nw}nw + a_0^{ne}ne + a_0^{si}si + a_0^{w2}e2 + a_0^{e1}q1 + a_0^{e2}q2\,, \qquad (28)$$

$$\begin{aligned} a_k(z,t) &= \bar{a}_k + a_k^{c1}c1 + a_k^{c2}c2 + a_k^{c3}c3 + a_k^{nw}nw + a_k^{ne}ne + a_k^{si}si \\ &+ a_k^{w2}e2 + a_k^{e1}q1 + a_k^{e2}q2 + a_k^{t}\log t \end{aligned} \qquad (29)$$

$$b_k(z) = \bar{b}_k \quad \text{[implying that } \log b(p,z) = \log b(p) \text{]}, \qquad (30)$$

$$q_k(z) = \bar{q}_k \quad \text{[implying that } q(p,z) = q(p) \text{]}. \qquad (31)$$

where $\bar{d}_k$, $\bar{a}_k$, $\bar{b}_k$ and $\bar{d}_k$ are associated to the reference household (a childless couple living in a central region of Italy with only one employed adult); *c1*, *c2* and *c3* are dummies for the presence of 1, 2 or 3 children, respectively; *nw*, *ne* and *si* are dummies indicating the family is living in a North-West, North-Est or South and Islands region, respectively; *w2* is a dummy for the presence of a second employed adult; *q1* and *q2* are continuous variables indicating the qualification level of the first two family members; t = 1, …, T (number of years considered); k = 1, …, m.

Applying Roy's identity to the indirect utility defined by (15), and incorporating (23)-(26), the expenditure share functions $w(p,x,z)$ are generated. The i-th commodity's share is:

$$w_i(p,x,z) = \frac{x - d(p,z)}{x}\left(a_i(z) + \sum_{k=1}^{m} a_{ik}\log p_k + b_i\hat{x} + \frac{q_i}{b(p)}\hat{x}^2\right) + \frac{p_j d_j(z)}{x} \qquad (32)$$

where i = 1, …, m, and

$$\hat{x} = \log\left[(x - d(p,z))/a(p,z)\right] \qquad (33)$$

4. DATA, ESTIMATION AND RESULTS

Data consist of monthly cross-sections, from January 1997 to December 2004, of individual Italian households' current expenditures collected by the Istituto Nazionale di Statistica (ISTAT) through a specific and routinely repeated survey. A sample of 43,701 observations has been selected taking families formed by couples of adults (aged between 25 and 64), at least one of them employed, childless or with a number of children (aged 14 or less) between one and three. Ten commodities are considered: (i) food; (ii) alcohol and tobacco; (iii) clothing; (iv) housing (excluding rent); (v) household equipment (including child care); (vi) health; (vii) transport; (viii) communications; (ix) recreation and culture; (x) other goods and services (that also includes education and hotels/restaurants for which ISTAT supplies separate values). Monthly and regional[2] consumption price indexes matching such goods are also available from ISTAT. Summary statistics are shown in table 1.

Two other data sets have been generated by partitioning the above sample according to the reference person (RP)' age[3]. Given the 43,701 original observations, a subsample of 22,873 is generated selecting households whose RP is aged less than 40, and another subsample of 20,828 is generated selecting households whose RP is aged 40 or more. There are not relevant differences in the summary statistics between the three sample, shown in table 1, apart from the proportion of childless and one-child couples. Childless couples represent 40% of households with the older RP, and 25% of those with the younger RP; such values are roughly inverted when one-child couples are considered. Moreover, slight differences in the mean total monthly expenditures occur, with families with the younger (older) RP spending around 60 euros less (more) than the overall average.

We apply the same empirical model, defined by (28), and the same econometric strategy discussed below, to the three distinct data sets. Results will be referred to as the full sample, the younger RP (or RP<40) sample and the older RP (or RP>40) sample, according to the data set used to generate them. The expenditure share corresponding to the tenth (residual) commodity is the left-out equation. As in Donaldson and Pendakur, we set rather than estimate the parameter $\bar{a}_0$, which is difficult to identify, at the average log total expenditure of the reference household (childless couple living in the centre, with one member employed). We obtain $\bar{a}_0 = 7.07$, $\bar{a}_0 = 7.02$ and $\bar{a}_0 = 7.09$ for full, younger RP and older RP sample models, respectively.

[2] ISTAT actually delivers monthly indexes for a more disaggregated geographical level than the region, that of provinces. To match with the expenditure data, where only the household's region is known, we take the prices of the province corresponding to the regional capital as representative of the whole region.
[3] The RP is the holder of the file recorded into the households' register handled by each Italian municipality.

Table 1. The data[a]. Summary statistics.

Variable	Mean	St. d.	Min.	Max.
Current expenditures (euro/month):				
Total Expenditure	1613.3	945.7	250.5	8972.0
1. Food From Stores	425.7	227.8	0.0	2946.9
2. Alcohol and Tobacco	40.1	50.0	0.0	624.0
3. Clothing	200.4	267.7	0.0	4824.0
4. Housing	133.2	100.7	0.0	1173.6
5. Household Equipment	75.5	118.4	0.0	3268.6
6. Health	93.0	243.2	0.0	6615.6
7. Transports	209.9	233.3	0.0	5508.5
8. Communication	44.4	36.8	0.0	623.9
9. Recreation	118.1	137.9	0.0	2520.4
10. Other Goods and Services	273.2	396.8	0.0	5603.4
Price indexes (P_{1995}=1):				
P_1	1.11	0.07	0.98	1.34
P_2	1.24	0.11	1.06	1.56
P_3	1.16	0.07	1.02	1.40
P_4	1.19	0.09	1.02	1.39
P_5	1.12	0.05	1.02	1.28
P_6	1.15	0.07	1.01	1.31
P_7	1.15	0.07	1.01	1.36
P_8	0.97	0.06	0.75	1.07
P_9	1.10	0.05	0.98	1.26
P_{10}	1.15	0.08	1.01	1.35
Other exogenous variables that distinguish households:				
Childless Couple (C0)	0.32	0.47	0.00	1.00
One child couple (C1)	0.31	0.46	0.00	1.00
Two children couple (C2)	0.32	0.47	0.00	1.00
Three children couple (C3)	0.05	0.21	0.00	1.00
North-West (NW)	0.25	0.43	0.00	1.00
North-Est (NE)	0.22	0.42	0.00	1.00
Centre (CE)	0.17	0.38	0.00	1.00
South and Island (SI)	0.36	0.48	0.00	1.00
One employed adult (E1)	0.48	0.50	0.00	1.00
Two employed adults (E2)	0.52	0.50	0.00	1.00
Education of the RP (Q1)	4.88	1.46	1.00	8.00
Education of the 2nd adult (Q2)	4.78	1.43	1.00	8.00

[a] The total number of households is 43,701.

Consistent estimates are obtained using the two-step procedure proposed by Shonkweiler and Yen (1999) to handle zero individual expenditures[4]. Standard errors of parameters' functions, such as elasticities and equivalence scales, are calculated using bootstrapping (with 500 replications); symmetric confidence interval, at the 95% level, are also generated.

Assuming GAESE, some restricted alternatives have been tested using the log-likelihood ratio. First, the two conditions ensuring identification of GAESE equivalence scales from demand data are considered. Non-identification requires the reference expenditure function to be either affine or log-affine and these conditions are equivalent to set $\log b(p,z) = q(p) = 0$ or $d(p,z) = q(p) = 0$, respectively. Second, AESE and ESE alternatives are considered, the latter implying that TQAI reduces to QAI. In particular, AESE and ESE are equivalent to set $a(p,z) = a^r(p)$ and $d(p,z) = 0$, respectively.

Table 2. Tests' results.

Data set	Hypothesis: Model / Restriction	Model df	Parametric restrictions	Model log-likelihood	Likelihood ratio test statistic
Full sample	H_0: TQAI/GAESE	280	none	553,938	
	H_A: Affine/GAESE	262	$\log b(p,z) = q(p) = 0$	552,256	3,364
	H_A: Log-affine/ESE	171	$d(p,z) = q(p) = 0$	553,024	1,828
	H_A: TQAI/AESE	190	$a(p,t,z) = a^r(p,t)$	552,936	2,004
	H_A: QAI/ESE	180	$d(p,z) = 0$	553,441	994
RP<40 sample	H_0: TQAI/GAESE	280	none	291,238	
	H_A: Affine/GAESE	262	$\log b(p,z) = q(p) = 0$	290,339	1,798
	H_A: Log-affine/ESE	171	$d(p,z) = q(p) = 0$	290,790	896
	H_A: TQAI/AESE	190	$a(p,t,z) = a^r(p,t)$	290,781	914
	H_A: QAI/ESE	180	$d(p,z) = 0$	290,999	478
RP>40 sample	H_0: TQAI/GAESE	280	none	263,704	
	H_A: Affine/GAESE	262	$\log b(p,z) = q(p) = 0$	261,525	4,358
	H_A: Log-affine/ESE	171	$d(p,z) = q(p) = 0$	263,131	1,146
	H_A: TQAI/AESE	190	$a(p,t,z) = a^r(p,t)$	263,185	1,038
	H_A: QAI/ESE	180	$d(p,z) = 0$	263,413	582

Selected 99-th percentiles of the chi-squared distribution, with the specified d.f., $\chi^2_{df;0.01}$:

$\chi^2_{18} = 34.805$ $\chi^2_{90} = 124.116$ $\chi^2_{100} = 135.807$ $\chi^2_{109} = 146.257$

[4] This number includes 9 parameters (one per estimated equation) introduced to implement the two-step estimation procedure. All parameters' estimates will be made available by the authors on request.

Results are summarized in table 2. Alternative hypotheses are always rejected at conventional levels of significance. Like Donaldson and Pendakur, we conclude that GAESE equivalent expenditure functions may be estimated from consumption data and that a demand system incorporating GAESE fits the data, for each of the three sets considered here, better than a system only incorporating AESE or ESE. This implies that the behavior of Italian couples is significantly consistent with expenditure-dependent equivalent-expenditure functions.

In table 3, estimations of $\log a(p,z)$ and $d(p,z)$ are reported, each calculated at distinct values of the binary demographic variables, and at the mean values of the continuous variables, i.e. prices, *q1* and *q2*.

The values of the translation terms $d(p,z)$ are all negative, and almost all are significant. The non-significant ones are those referred to younger RP sample estimates, and, within the latter, to three-child couples with only one employed adult, in all geographic areas. Estimates associated to the younger RP sample are always higher than those referred to the older RP sample, and full sample estimates always lie into the range defined by the subsample estimates.

There are no theoretical indications about the sign of the translation terms $d(p,z)$. However, positive values seem to have a more straightforward interpretation, corresponding to a consumption level ensuring only subsistence, below which no utility is achieved. Negative values seem to indicate the presence of utility even when there is no expenditure, and this would raise a question about the source of such utility. A possible and attractive explanation is to interpret the negative terms as a proof of the so called Italian way to welfare, i.e. the occurrence of inter-familiar transfers from aged parents towards children' families. In our opinion, this point would deserve careful consideration and further research. For example, if this hypothesis is true negative values would become insignificant (or turn to positive) if only households formed by aged people (older than 64) were investigated.

The values of $\log a(p,z)$, i.e. the translog price indexes, are all positive, as prescribed by economic theory, and significant. All but one values associated to full sample estimates lie outside the range defined by the corresponding estimates of the two subsamples (they are usually, though not always, greater). Moreover, in most cases, the 95% confidence intervals of full sample $\log a(p,z)$ estimates do not overlap with any of the two corresponding intervals from subsample estimates.

Table 3. Estimated values of $\log a(p,z)$ and $d(p,z)$, calculated at specific values of dicotomous demographic variables, and at mean values of *p*, *q1* and *q2*.

All dummy demographic variables are zero, except:	estimated $\log a(p,z)$ [a]			estimated $d(p,z)$ [a]		
	RP<40 s.	full s.*	RP≥40 s.	RP<40 s.	full s.*	RP≥40 s.
nw = 1	**6.72**	**7.35***	**7.00**	**-201.2**	**-401.4**	**-508.8**
nw = 1; *c1* = 1	**6.87**	**8.49***	**7.10**	**-151.2**	**-316.6**	**-429.2**
nw = 1; *c2* = 1	**6.97**	**8.53***	**7.15**	**-164.9**	**-288.4**	**-380.4**
nw = 1; *c3* = 1	**6.83**	**8.75***	**7.43**	*-67.6*	**-219.2**	**-397.0**
ne = 1	**6.77**	**7.52***	**6.91**	**-256.4**	**-405.3**	**-426.2**
ne = 1; *c1* = 1	**6.91**	**8.66***	**7.01**	**-206.4**	**-320.5**	**-346.6**
ne = 1; *c2* = 1	**7.02**	**8.70***	**7.05**	**-220.1**	**-292.3**	**-297.8**
ne = 1; *c3* = 1	**6.88**	**8.92***	**7.34**	*-122.7*	**-223.1**	**-314.4**
(reference household)	**6.92**	**7.37***	**7.00**	**-226.4**	**-361.8**	**-456.9**
c1 = 1	**7.07**	**8.51***	**7.10**	**-176.4**	**-277.0**	**-377.3**
c2 = 1	**7.17**	**8.55***	**7.14**	**-190.1**	**-248.8**	**-328.4**
c3 = 1	**7.03**	**8.78***	**7.42**	*-92.8*	**-179.6**	**-345.1**
si = 1	**6.93**	**7.57***	**7.17**	**-205.2**	**-409.4**	**-600.4**
si = 1; *c1* = 1	**7.08**	**8.71***	**7.27**	**-155.3**	**-324.7**	**-520.8**
si = 1; *c2* = 1	**7.18**	**8.75***	**7.31**	**-168.9**	**-296.4**	**-471.9**
si = 1; *c3* = 1	**7.04**	**8.98***	**7.59**	*-71.6*	**-227.2**	**-488.6**
nw = 1; *e2*=1	**6.75**	**6.64**	**7.06**	**-285.9**	**-463.9**	**-635.1**
nw = 1; *c1* = 1; *e2* = 1	**6.90**	**7.79***	**7.16**	**-236.0**	**-379.2**	**-555.5**
nw = 1; *c2* = 1; *e2* = 1	**7.01**	**7.83***	**7.20**	**-249.6**	**-351.0**	**-506.7**
nw = 1; *c3* = 1; *e2* = 1	**6.86**	**8.05***	**7.49**	**-152.3**	**-281.7**	**-523.3**
ne = 1; *e2* = 1	**6.80**	**6.81**	**6.97**	**-341.1**	**-467.8**	**-552.5**
ne = 1; *c1* = 1; *e2* = 1	**6.95**	**7.95***	**7.07**	**-291.2**	**-383.1**	**-472.9**
ne = 1; *c2* = 1; *e2* = 1	**7.05**	**7.99***	**7.11**	**-304.8**	**-354.8**	**-424.1**
ne = 1; *c3* = 1; *e2* = 1	**6.91**	**8.22***	**7.39**	**-207.5**	**-285.6**	**-440.7**
e2 = 1	**6.95**	**6.67**	**7.05**	**-311.1**	**-424.3**	**-583.2**
c1 = 1; *e2* = 1	**7.10**	**7.81***	**7.15**	**-261.2**	**-339.6**	**-503.6**
c2 = 1; *e2* = 1	**7.21**	**7.85***	**7.20**	**-274.8**	**-311.4**	**-454.7**
c3 = 1; *e2* = 1	**7.06**	**8.08***	**7.48**	**-177.5**	**-242.1**	**-471.4**
si = 1; *e2* = 1	**6.96**	**6.87**	**7.23**	**-290.0**	**-472.0**	**-726.7**
si = 1; *c1* = 1; *e2* = 1	**7.11**	**8.01***	**7.33**	**-240.0**	**-387.2**	**-647.1**
si = 1; *c2* = 1; *e2* = 1	**7.22**	**8.05***	**7.37**	**-253.7**	**-359.0**	**-598.2**
si = 1; *c3* = 1; *e2* = 1	**7.07**	**8.27***	**7.65**	**-156.4**	**-289.8**	**-614.9**

[a] The total number of households is 43,701. Significant (at 95% level) values are in bold, insignificant values are in italic.

* Indicate the 95% confidence intervals do not overlap with any of the two corresponding intervals constructed around both the left (RP<40) and the rigth hand (RP<40) estimates.

In table 4 expenditure and compensated (Hicksian) own price elasticities are shown. For each commodity, nine expenditure elasticities are reported: for each model, elasticities are calculated at the mean value of all variables, and by replacing the average expenditure with the first and the third quartile. Correspondingly, nine compensated own price elasticities are also calculated.

Table 4. Expenditure and compensated (Hicksian) own price elasticities, calculated at the sample mean of all variables (for expenditure, also at the first and third quartile).

Commodity	Expend-iture level	Expenditure elasticities[a]			Compensated own price elasticities[a]		
		RP<40 s.	full s.	RP≥40 s.	RP<40 s.	full s.	RP≥40 s.
(1) food	3rd q.	**0,64**	**0,62**	**0,63**	**-0,31**	**-0,38**	**-0,34**
	mean	**0,65**	**0,63**	**0,63**	**-0,33**	**-0,41**	**-0,38**
	1st q.	**0,67**	**0,66**	**0,65**	**-0,39**	**-0,49**	**-0,51**
(2) alcohol and tobacco	3rd q.	**0,49**	**0,45**	**0,31**	**-0,25**	**-0,30**	*-0,14*
	mean	**0,56**	**0,52**	**0,40**	**-0,36**	**-0,44**	**-0,38**
	1st q.	**0,75**	**0,76**	**0,71**	**-0,71**	**-0,92**	**-1,21**
(3) clothing	3rd q.	**1,42**	**1,22**	**1,08**	**-0,85**	**-0,48**	**-0,40**
	mean	**1,47**	**1,24**	**1,10**	**-0,77**	**-0,51**	**-0,45**
	1st q.	**1,54**	**1,32**	**1,20**	**-0,47**	**-0,63**	**-0,63**
(4) housing (excluding rent)	3rd q.	**0,52**	**0,53**	**0,78**	**-1,59**	**-1,42**	**-1,40**
	mean	**0,52**	**0,53**	**0,73**	**-1,60**	**-1,48**	**-1,45**
	1st q.	**0,48**	**0,50**	**0,57**	**-1,66**	**-1,68**	**-1,63**
(5) household equipment	3rd q.	**0,94**	**0,84**	**0,95**	**-2,66**	**-2,02**	**-1,87**
	mean	**1,00**	**0,88**	**0,99**	**-2,67**	**-2,12**	**-1,88**
	1st q.	**1,10**	**0,96**	**1,05**	**-2,71**	**-2,48**	**-1,91**
(6) health	3rd q.	**1,76**	**2,08**	**2,48**	**-1,22**	**-1,80**	**-1,88**
	mean	**1,65**	**1,96**	**2,31**	**-1,21**	**-1,60**	**-1,69**
	1st q.	**1,40**	**1,64**	**1,92**	**-1,20**	**-0,87**	**-0,97**
(7) transport	3rd q.	**0,95**	**1,15**	**1,20**	**-0,59**	**-0,72**	**-0,76**
	mean	**0,95**	**1,18**	**1,24**	**-0,69**	**-0,77**	**-0,81**
	1st q.	**1,01**	**1,29**	**1,35**	**-0,98**	**-0,95**	**-0,98**
(8) communica-tions	3rd q.	**0,53**	**0,55**	**0,52**	**-0,59**	**-0,66**	**-0,62**
	mean	**0,49**	**0,52**	**0,49**	**-0,61**	**-0,66**	**-0,65**
	1st q.	**0,36**	**0,40**	**0,34**	**-0,68**	**-0,67**	**-0,74**
(9) recreation and culture	3rd q.	**1,16**	**0,86**	**1,08**	**-0,27**	**-0,23**	**-1,02**
	mean	**1,25**	**0,90**	**1,15**	*-0,18*	**-0,28**	**-0,93**
	1st q.	**1,39**	**1,00**	**1,26**	*0,16*	**-0,46**	**-0,57**
(10) other goods and services	3rd q.	**1,61**	**1,74**	**1,40**	**-1,20**	**-1,19**	**-0,86**
	mean	**1,53**	**1,67**	**1,38**	**-1,25**	**-1,21**	**-0,91**
	1st q.	**1,38**	**1,50**	**1,32**	**-1,41**	**-1,26**	**-1,11**

[a] Significant (at 95% level) values are in bold, insignificant values are in italic.

All expenditure elasticities are significantly positive. Results generally confirm (but in some cases correct) findings in Balli and Tiezzi (2009). Necessities are food, alcohol and tobacco, housing, and communications; luxuries are clothing, health, and recreation. Household operations and equipment have an expenditure elasticity very close to 1. The same for transport, but only for families with younger RP; otherwise (families with older RP) transport is a luxury.

Nearly all compensated own price elasticities are significantly negative. There is a positive but insignificant value for recreation, when household with younger RP are considered and expenditures fixed at the first quartile. As for expenditure elasticities, results generally confirm Balli and Tiezzi values: housing, household equipment and health are elastic. However, the rank of the model along with the data set partitioning allow for the emergence of more articulated patterns. For example, alcohol and tobacco become elastic, and health tends to be relatively inelastic, when older RP families with lower income are considered. Also, recreation and culture tend to be relatively elastic when older RP families with a higher expenditure are considered. In general, all necessities tend to be more elastic when income is low.

Table 5. Compensated cross price elasticities, calculated at the sample mean of all variables (range of values)[a].

Elasticity range	RP<40 s.	full s.	RP≥40 s.	
EXY > 1	E510, E54, E93, E83, E210	E54, E83, E510, E93	E54, E83, E67	
0.5 < EXY ≤ 1	E53, E45, E41, E39, E810, E71	E210, E67, E53, E45, E61, E71, E41, E39, E27, E107, E101	E93, E61, E71, E210, E76, E45, E94, E39, E41	
0 < EXY ≤ 0.5	E67, E27, E105, E61, E87, E107, E710, E101, E57, E35, E38, E64, E110, E94, E102, E76, E17, E14, E49, E46, E108, E72, E75, E610, E78, E104, E410	E710, E94, E76, E810, E49, E59, E105, E64, E110, E38, E89, E35, E17, E56, E82, E84, E87, E28, E102, E95, E57, E14, E46, E65, E72, E98, E16, E48, E108, E104, E75, E78, E410	E27, E59, E107, E510, E710, E89, E84, E49, E38, E82, E95, E17, E28, E64, E810, E105, E101, E102, E98, E14, E16, E110, E72, E48, E410, E108, E104	Substitutability ↑
-0.5 ≤ EXY < 0	E62, E12, E109, E19, E26, E18, E15, E74, E910, E37, E73, E68, E91, E47, E79	E12, E109, E19, E18, E92, E58, E910, E96, E15, E85, E29, E68, E74, E79, E37, E73, E91, E21, E47	E12, E32, E18, E15, E58, E68, E79, E85, E23, E52, E37, E69, E73, E74, E25, E96, E97, E47	Complementarity ↓
-1 ≤ EXY < -0.5	E21, E97, E86, E52, E25	E52, E97, E86, E25	E21, E86, E51, E81	
< -1	E51, E81	E81, E51		

[a] Only significant (at 95% level) values are reported. The meaning of EXY terms is straightforward, being both X and Y goods which are numbered in the same order as in the list of table 4.

Table 5 shows compensated cross price elasticities (only those computed at the mean values of all variables). The stronger substitution relationships detected are those between household equipments and housing, recreation and clothing, communication and clothing, housing and food, transport and food, health and transport, and transport, household equipments and clothing. On the other hand, the stronger complementarities are those between household equipments and food, communications and food, communications and health, recreation and transport, alcohol/tobacco and household equipments, alcohol/tobacco and food. These results substantially give the same picture as in Balli and Tiezzi (2009).

Responses from the three estimated models seem qualitatively rather similar. In several cases full sample estimates lie into the range defined by the two corresponding subsamples estimates. In other cases this does not happen, as for expenditure elasticities of household equipments. For the same good, full sample compensated own price elasticities lie into the ranges defined by the corresponding subsamples elasticities, but show a pattern, the elasticity decreasing with expenditure, which is inexistent when the two separate samples are considered. Indeed, the full sample 95% confidence intervals of compensated own price elasticities of household equipments at the first and third total expenditure quartiles are not overlapping. As to cross price compensated elasticities, in 25 cases over 90 the value from the full sample model does not lie into the specified range.

5. EQUIVALENCE SCALES

The full sample model generally fails to generate an equivalence scale lying into a plausible range, i.e. upperly bounded by the household size. For example, at the mean value of continuous variables (the total expenditure y, the price vector p and the demographics $q1$ and $q2$), the relative scale values for couples with one and two children, living in the Centre and with only one member employed, are 6.65 and 7.68 respectively. Moreover, as pointed out in the previous section, the full sample model also reveals problems in producing an intermediate picture, in terms of price and expenditure elasticities, with respect to the two subsample estimates. However, consumers' behavior depicted from this model is consistent with the theory, in line with previous findings, and, apart from the problems just mentioned, there are no other substantial differences with the other two models.

The failure of the full sample model to generate a plausible GAESE scale therefore seems to reveal a specific sensitivity of such indexes to the model specification and/or the estimation strategy. Even though this is a purely qualitative and provisional statement, we consider it as a result of this work. Further research about the presence and the pattern of such a sensitivity may give important indications for the empirical application of GAESE scales.

In table 6 estimated relative and absolute equivalence scales, defined by equations (21) and (22), are reported for the older RP model. All distinct demographic effects are shown, i.e. the effects of single demographic features from those considered through the dummy components of the vector *z*: number of children, geographic location and working condition. Since such features are introduced into the model additively, without interactions, all other combined effects may be approximately derived from the distinct ones. Some of the combined effects are also shown in table 6 for the sake of exposition.

All values are economically plausible. When the family total current expenditure is close to the sample mean, and only one adult member is employed, the presence of a child increases such value by 20% with respect to the reference household (childless couple); a second child increases household expenses by 9% (i.e. the per cent ratio between 1.31 and 1.20, the scale values for two-children and one-child couples, respectively); a third child involves a 48% (per cent ratio between 1.94 and 1.31) additional increase. The corresponding *A(p,z)* terms are positive and significant, implying the scale to be decreasing with total expenditure. Moreover, this pattern tends to be stronger when the number of children rises: differences between the values calculated at the top and at the bottom deciles of the monthly total current expenditure distribution are -0.19, -0.36 and -1.31 for families with 1, 2 and 3 children, respectively. Consequent percentage decreases are -14%, -23% and -43%. For example, considering households with two children, the scale is 1.60 at the bottom decile of the expenditure distribution (about 710 euro), and 1.26 at the top decile (about 2910 euro), with an absolute decrease of 0.36, about 23% of the first value.

On the other hand, for a childless couple with only one member employed (table 6, lines with *nw* = *ne* = *si* = 1), *A(p,z)* terms are negative but insignificant. The corresponding scale may therefore be considered as flat with respect to the total current expenditure. However, a second employed adult has no welfare impact if the scale is computed at the expenditure's mean value (as shown in table 6, line with *e2* = 1), but the corresponding *A(p,z)* term is negative and significant, so that the scale is increasing with expenditure. When expenditure is less than average, having a second employed member involves a gain which increases as expenditure decreases: at the bottom decile, the same level of well-being is obtained with a 7% saving. The opposite is true when expenditure is above average: the additional cost at the top decile is about 3%.

When children are present and both adults are employed, two conflicting forces are working at the same time. The *A(p,z)* terms are insignificant, however negative signs prevail and a pattern decreasing with expenditure remains, even though a weakened one (as shown in table 6, lines with *c1*/*c2*/*c3* = 1 and *e2* = 1).

Table 6. RP>40 model: estimated *R(p,z)*, *A(p,z)*, relative (S_r) and absolute (S_a) scales, at specific values of *z* and *y*, and at mean values of *p*, *q1* and *q2*.

All dummy demographic variables are zero, except:	$R(p,z)^a$	$A(p,z)^b$	1st decile of *x*		1st quartile of *x*	
			x_{d1} = 709,4		x_{q1} = 997,6	
			S_r	S_a	S_r	S_a
ref. household	1.00	0.0	1.00	0.0	1.00	0.0
c1 = 1	1.11	**127.9**	1.35	183.5	1.27	211.0
c2 = 1	**1.15**	**198.9**	1.60	267.0	1.44	305.5
c3 = 1	**1.53**	**353.9**	3.05	477.0	2.37	576.8
e2 = 1	1.06	**-98.5**	0.93	-52.3	0.97	-35.8
c1 = 1; *e2* = 1	**1.17**	32.3	1.23	132.1	1.21	174.6
c2 = 1; *e2* = 1	**1.22**	104.6	1.44	215.4	1.37	268.2
c3 = 1; *e2* = 1	**1.62**	270.0	2.62	438.7	2.23	549.3
nw = 1	1.01	-48.8	0.94	-43.6	0.96	-41.6
ne = 1	0.92	-6.8	0.91	-70.7	0.91	-96.4
si = 1	**1.19**	-57.7	1.10	63.6	1.12	109.2

Table 6 (continued).

All dummy demographic variables are zero, except:	sample mean of *x*		3rd quartile of *x*		9th decile of *x*	
	x_{mean} = 1.678,1		x_{q3} = 2.073,9		x_{d9} = 2.912,4	
	S_r(a)	S_a(b)	S_r	S_a	S_r	S_a
ref. household	1.00	0.0	1.00	0.0	1.00	0.0
c1 = 1	**1.20**	**276.1**	1.18	313.9	1.16	394.1
c2 = 1	**1.31**	**396.4**	1.28	449.2	1.24	561.2
c3 = 1	**1.94**	**812.5**	1.84	949.6	1.74	1.239.9
e2 = 1	1.00	3.2	1.01	25.9	1.03	73.9
c1 = 1; *e2* = 1	**1.20**	**274.9**	1.19	333.2	1.19	456.8
c2 = 1; *e2* = 1	**1.31**	**392.8**	1.29	465.3	1.27	618.8
c3 = 1; *e2* = 1	**1.93**	**810.4**	1.87	962.3	1.79	1.284.1
nw = 1	0.98	-36.9	0.98	-34.2	0.99	-28.5
ne = 1	0.91	-157.1	0.92	-192.4	0.92	-267.2
si = 1	1.15	216.8	1.16	279.4	1.16	412.0

[a] Significance is with respect to the value 1; significant (at 95% level) values are in bold.
[b] Significant (at 95% level) values are in bold.

Figure 1 displays relative scales depending on expenditure.

Menon and Perali (2009) obtained a very similar equivalence scale, at the sample mean of expenditure, for Italian couples with one child[5].

[5] These authors computed distinct scales for couples with a child in distinct age ranges (the childless couple is the reference). They obtained the following values: 1.19 for a child between 0 and 5 years old; 1.16 for a child between 6 and 13; 1.18 for a child between 14 and 18.

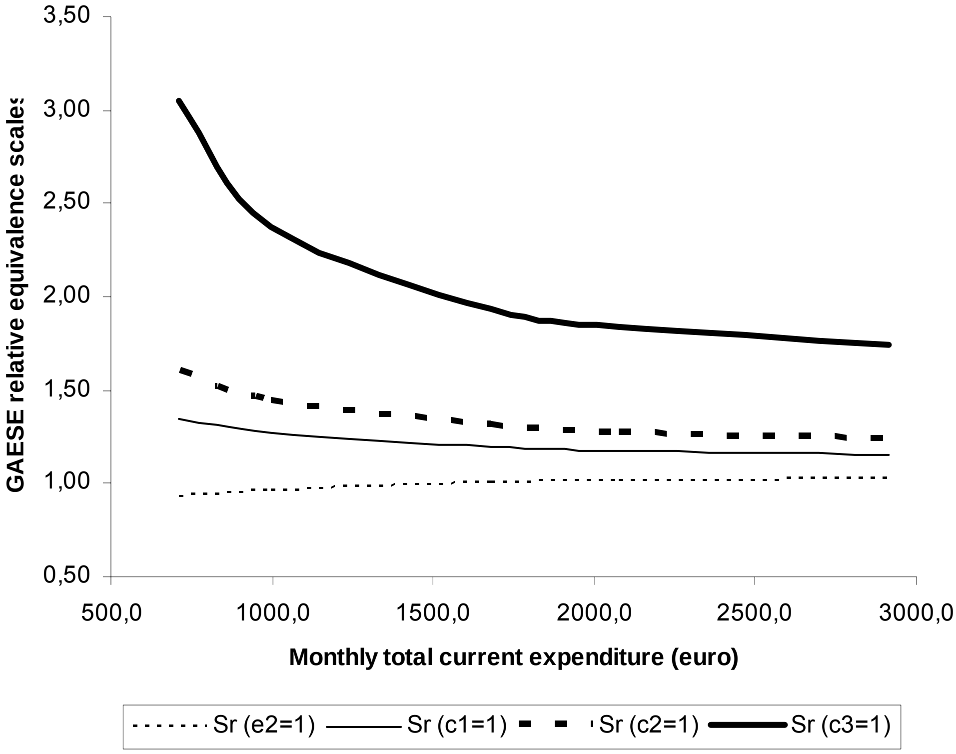

Figure 1. Estimated GAESE equivalence scales as functions of total monthly current expenditure.

GAESE scales estimated by Donaldson and Pendakur (2006) are all decreasing[6] with expenditure; for couples this pattern tends to strengthen when the number of children increases. GESE scales estimated by Donaldson and Pendakur (2004) are also declining with expenditure, but for childless couples this pattern is hardly perceptible and statistically insignificant. Our results are different in some respect, partly because here the number of working adults is an explanatory variable. The decreasing pattern does not occur when children are not present and only one adult member is employed: the term *A(p,z)* is not significant; if both members are employed, the pattern is actually reversed. When children are present and only one adult is employed, the decreasing pattern is significant and tends to strengthen when the number of children rises; this effect is stronger than that found by Donaldson and Pendakur. With two employed members these patterns still emerge but with a lower intensity.

In table 7 estimated relative and absolute equivalence scales are reported for the younger RP model. The structure is exactly the same as table 6.

[6] Differences between the values calculated by these authors at the top and at the bottom vigintiles of the annual current expenditure distribution are -0.10, -0.13 and -0.15 for households with 0, 1 and 3 children, respectively. They consider vigintiles from conditional distributions; while in this work deciles are taken from the joint distribution; effects on comparison between the two set of results are not relevant.

Table 7. RP<40 model: estimated *R(p,z)*, *A(p,z)*, relative (S_r) and absolute (S_a) scales, at specific values of *z* and *x*, and at mean values of *p*, *q1* and *q2*.

All dummy demographic variables are zero, except:	$R(p,z)^a$	$A(p,z)^b$	1st decile of *x* x_{d1} = 695,9 S_r	S_a	1st quartile of *x* x_{q1} = 966,2 S_r	S_a
ref. household	1.00	0.0	1.00	0.0	1.00	0.0
c1 = 1	**1.16**	**86.4**	1.33	171.1	1.28	208.6
c2 = 1	**1.29**	101.7	1.51	234.8	1.44	295.4
c3 = 1	1.12	**160.0**	1.45	216.0	1.34	244.2
e2 = 1	1.04	**-76.8**	0.93	-50.5	0.96	-41.3
c1 = 1; *e2* = 1	**1.20**	11.0	1.22	126.1	1.22	171.6
c2 = 1; *e2* = 1	**1.33**	27.2	1.39	194.7	1.37	262.4
c3 = 1; *e2* = 1	**1.16**	84.2	1.32	166.7	1.27	203.2
nw = 1	**0.82**	-15.5	0.80	-171.6	0.81	-230.9
ne = 1	**0.86**	-61.9	0.79	-186.2	0.81	-230.6
si = 1	1.01	23.6	1.05	30.7	1.04	33.6

Table 7 (continued).

All dummy demographic variables are zero, except:	sample mean of *x* y_{mean} = 1.554,3 S_r([a])	S_a([b])	3rd quartile of *x* y_{q3} = 1.905,6 S_r	S_a	9th decile of *x* y_{d9} = 2.623,0 S_r	S_a
ref. household	1.00	0.0	1.00	0.0	1.00	0.0
c1 = 1	**1.23**	**290.2**	1.22	339.0	1.20	438.6
c2 = 1	**1.38**	**427.1**	1.36	505.9	1.34	667.0
c3 = 1	1.24	305.6	1.22	342.3	1.19	417.2
e2 = 1	0.99	-21.2	1.00	-9.3	1.01	15.1
c1 = 1; *e2* = 1	**1.21**	**270.5**	1.21	329.6	1.21	450.2
c2 = 1; *e2* = 1	**1.36**	**409.7**	1.35	497.7	1.35	677.3
c3 = 1; *e2* = 1	1.22	282.5	1.21	329.9	1.19	426.7
nw = 1	**0.81**	**-359.9**	0.81	-437.0	0.82	-594.4
ne = 1	**0.83**	**-327.0**	0.83	-384.7	0.84	-502.4
si = 1	1.03	39.9	1.02	43.7	1.02	51.3

[a] Significance is with respect to the value 1; significant (at 95% level) values are in bold.
[b] Significant (at 95% level) values are in bold.

Reported values are all economically plausible, but those referred to couples with three children (table 7, line *c3*=1) are lower than the corresponding ones referred to two-children couples.

Couples with three-children are 4.20% of all observations in the younger RP subsample. They are concentrated in the South and Islands (58,5%); those in the other areas are rather concentrated in the North-East and not evenly distributed over time (there are less than 10 observations from the Centre in 5 years over 8). When the older RP sample is considered, figures are not dissimilar, but all changes are in the right direction: the category is more represented (4.85%), less concentrated in the South and the Islands (55,5%) and households in the other areas are more evenly distributed over these areas and over time (there are always more than 10 observations in any area per year). Therefore, it may be reasonable to consider this result not as a model failure but as the outcome of a poor representation in the younger RP data sample of couples with three children.

Apart from this specific case, the younger RP model gives results similar to those obtained from the older RP one. A substantial increase in the scale values for households with children (insignificant since the largely overlapping corresponding confidence interval) would indicate a higher cost of children for younger couples. A straightforward explanation would be the lower average age of children in younger RP families: as shown by Menon and Perali (2009), a young child is more expensive than an older one. Other explanations based on parents' age would also be possible, such as a better employment condition generally in force for older workers. However, these conjectures may only be considered as clues for further research, without any conclusive claim.

To summarize, our results show that the strength and direction of the scale dependence on expenditure varies with the presence and number of children, and with the working condition of the household members, sketching out a more articulated picture than Donaldson and Pendakur (2006). However, for the fraction of families' population considered in this work (couples with or without children), the prevailing pattern remains that of equivalence scales decreasing with expenditure. This implies that, if children are present, scale economies in current consumption are lower for families with poor expenditure capacities.

An additional set of results concerns the effect of households' geographical location. Such results are inconclusive, since the involved confidence intervals are overlapping, but some indication may still be drawn.

Figure 2 shows the estimated 95% confidence intervals of GAESE scale values for childless couples with one employed member, located in the North-West, North-East, and the South and the Islands. The Centre is the reference one (its value is fixed at 1). Values from both the younger and the older RP models are reported, calculated at the sample mean of all continuous variables.

Living in the North seems to be a substantial advantage for younger couples, with scale values significantly lower than 1. For the South and the Islands, the confidence interval is around 1, so that this area seems rather homogeneous with the Centre. The estimated cost increase for couples living in the Centre and South-Islands with respect to those living in the North is more than 20%.

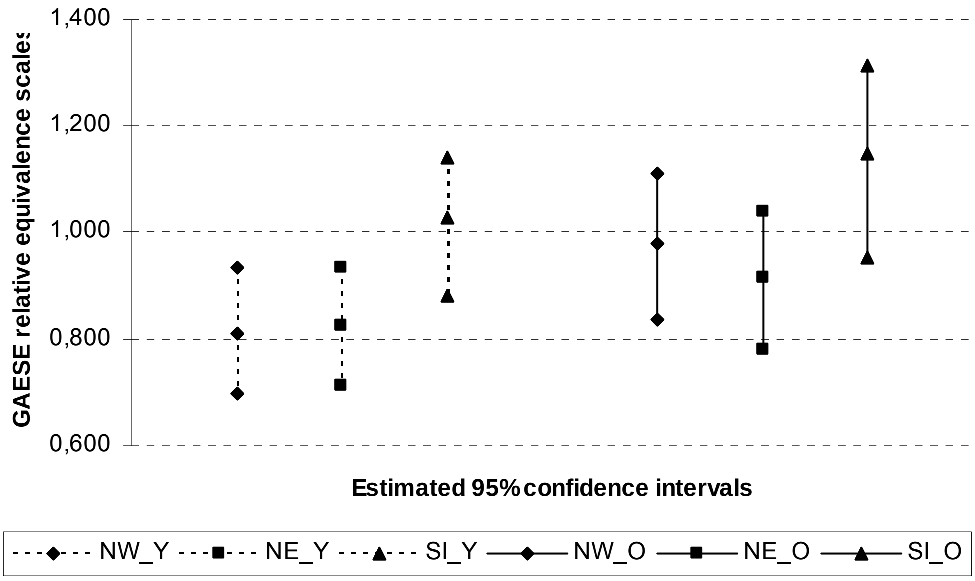

Figure 2. Estimated 95% confidence intervals of GAESE equivalence scales at the mean values of continuous variables, for childless couples with one member employed, in different areas. Values from the younger and the older RP models.

For older couples, the scale referred to the North-West area is around the unit reference value assigned to the Centre. However, the North-East seems to maintain a certain advantage, with a lower scale value, but not significantly lower at the 95% confidence level. On the other hand, the value for the South and Islands is higher than 1, indicating a substantial 15% cost increase. This would imply a clear disadvantage for couples living in such an area, but the scale value is not significantly different from 1 at the 95% confidence level.

Even though separate tests are statistically inconclusive, indications from the two models converge towards a coherent picture. Couples living in the South and Islands suffer a substantial additional cost to get, *ceteris paribus*, the same well-being of those living in the North; couples living in the Centre stay in an intermediate condition, that seems to be closer to the southern one for the younger, and closer to the northern one for the older.

This picture is relevant to the discussion about the fiscal reform that seems close to pass in Italy. The reform, which has a specific federal content, will have a deep and long-term impact on the distribution of resources among all Italian administrations, with presumably strong effects on the presence and the quality of public services. In particular, our results may be of help to evaluate the impact of interregional reallocations.

6. EQUIVALENT EXPENDITURES AND MEASURED INEQUALITY

Figure 3 shows the Gini coefficients calculated for equivalent expenditures of Italian couples whose RP is older than 40, for the main macro-areas of Italy. The values are obtained using both GAESE and ISTAT equivalence scales (ISTAT, 2010), with the latter not accounting for expenditure dependence.

Figure 3. Gini coefficients calculated for equivalent expenditures of Italian couples with RP>40, using ISTAT and GAESE scales. Annual data, per macro-area.

Concentration coefficients under GAESE are always higher than those obtained using the ISTAT scale: differences range from +10% (North-West, 1997) to +55% (Centre, 1997)[7] and are especially marked for the North-East and the Centre. Therefore, accounting for expenditure dependence when this pattern is declining results in higher measured inequality. In the Italian case such understatement seems to be substantial, even though this finding only applies to the portion of the population considered in this work (couples with and without children).

Gini indexes referred to the first three years under investigation are, on average, lower than those referred to the subsequent four years, for all areas of Italy and for both GAESE and ISTAT scales. This is an indication that inequality was higher between 2000 and 2003 than in the previous three years. Such a trend seems to reverse in 2004, however a similar conjecture has to be confirmed over a longer period[8].

7. CONCLUDING REMARKS

We have estimated GAESE equivalent expenditure functions and equivalence scales from Italian consumption data. Our contributions to the existing literature are the following.

First, the consumption behavior of Italian households is significantly consistent with GAESE equivalent-expenditure functions which are expenditure-dependent.

Second, responses from the three estimated models seem qualitatively rather similar as long as elasticities and other standard outputs are considered. When scale calculations come into play, one of them fails to generate a GAESE equivalence scale lying into an economically plausible range. This failure seems to reveal a specific sensitivity of estimated equivalence scales to the model specification and (or) to the estimation strategy.

Third, the strength and the direction of scales' dependence on expenditure varies with the presence and the number of children, and with the working condition of the household members. However, for the fraction of families considered in this work (couples with or without children), the prevailing pattern, affecting all couples with children, is an equivalence scale decreasing with expenditure. This implies that scale economies in current consumption are lower for families with poor expenditure capacities. If the number of children increases, this pattern tends to become stronger.

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[7] Corresponding Gini coefficients under GAESE and under the ISTAT scale are, respectively, .408 and .371 in North-West, year 1997; .297 and .192 in the Centre, same year.

[8] The ISTAT sample is selected according to a stratified design changing every year, but our inference is done considering it as a simple random sample. This makes our finite population inference rather approximate. In addition, in 2004 a relevant size reduction of the sample (more than 10% with respect to the previous year) has occurred.

Fourth, families living in the South and the Islands suffer a substantial additional cost to achieve, *ceteris paribus*, the same well-being of those living in the North; families in the Centre are in an intermediate condition, closer to the South's condition for the younger households, and closer to the North's condition for the older.

Finally, ignoring the expenditure-dependence of equivalence scales results in a significant understatement of measured inequality.

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