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Denoising Techniques Based on the Multiresolution Representation

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So far, considerable research efforts have been invested in the are of using statistical methods for image processing purposes yielding to a significant amount of models that aim to improve as much as possible the still existing and currently used processing techniques, some of them being based on using wavelet representation of images. Among them the simplest and the most attractive one use the Gaussian assumption about the distribution of the wavelet coefficients. This model has been successfully used in image denoising and restoration. The limitation comes from the fact that only the first-order statistics of wavelet coefficients are taking into account and the higher-order ones are ignored. The dependencies between wavelet coefficients can be formulated explicitly, or implicitly. The multiresolution representation is used to develop a class of algorithms for noise removal in case of normal models. The multiresolution algorithms perform the restoration tasks by combining, at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. Our work reports two attempts in using the multiresolution based algorithms for restoration purposes in case of normally distributed noise. Several results obtained using our new restoration algorithm are presented in the final sections of the paper.
Keywords: *multiresolution support, wavelet transform, filtering techniques, statistically significant wavelet coefficients.*

1 Introduction

The effectiveness of restoration techniques mainly depends on the accuracy of the image modeling. A long series of image degradation models have been proposed under various working assumptions. One of the most popular degradation models is the linear continuous image-degradation where it is assumed that the image blur can be modeled as a superposition with an impulse response that may be space variant and its output is subject to an additive noise.

The restoration can be viewed as a process that attempts to reconstruct or recover a degraded image using some a priori knowledge about the degradation mechanism.

Thus restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image. This approach usually involves formulating a criterion of goodness that will yield some optimal estimate of the desired result.

Generally speaking, the multiresolution algorithms perform the restoration tasks by combining, at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. The multiresolution support set is a data structure suitable for developing noise removal algorithms that perform the restoration tasks by combining,

at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image.
So far, considerable research efforts have been invested in the are of using statistical methods for image processing purposes yielding to a significant amount of models that aim to improve as much as possible the still existing and currently used processing techniques, some of them being based on using wavelet representation of images. Among them the simplest and the most attractive one use the Gaussian assumption about the distribution of the wavelet coefficients. This model has been successfully used in image denoising and restoration. The limitation comes from the fact that only the first-order statistics of wavelet coefficients are taking into account and the higher-order ones are ignored. The dependencies between wavelet coefficients can be formulated explicitly, or implicitly. Moreover, most wavelet models can be loosely classified into two categories: those exploiting inter-scale dependencies and those exploiting intra-scale dependencies [7].
Typically, on each resolution level, the magnitudes of wavelet coefficients corresponding to images are strongly correlated [8]. The wavelet coefficients can be thought on a quad-tree-like structure such that when a parental node is of small magnitude, those of its descendents are very likely to be small as well. A working assumption is that the wavelet coefficient distributions are Gaussian mixture for all subbands.
The spatial clustering trend of wavelet coefficients are extensively exploited by different compression algorithms as, for instance, the EQ coder and the morphological coder [6]. For example, in case of the EQ coder, the resulted wavelet coefficients are independent of zero mean and slow varying variance. This technique proved useful in developing denoising applications in signal processing, where local statistics are estimated from the data [9]. Also, methods were spatially varying variances were assumed have been proposed, these models being able to take into account the inter-scale dependencies.

2. Image Denoising Using a Scale-Space Mixture Model

A model explicitly combining the inter-scale and intra-scale dependencies of image wavelet coefficients was proposed by Liu and Moulin [9]. The model uses a simple classification technique and is based on the following empirical observations: wavelet coefficients of large magnitude are typically representative for edges, textures as well as noisy areas, while those of small magnitude rather correspond to smooth regions.
The design of this model is motivated by the zero tree coding technique. Let T be a significance threshold. In each subband except the first fine scale, the wavelet coefficients are partitioned into two classes based on the magnitude of their parents: W_{sig} is the set of coefficients that have significant parents (>T) and W_{insig} is the set of coefficients that have insignificant parents. Hence, the size of the each of the two classes is controlled by the significance threshold T. However, the two classes have quite different statistics. Since the histogram of the coefficients in W_{insig} is highly concentrated around zero, while the histogram of W_{sig} is more spread out.
The described statistical model can be applied to image denoising as follows. Assume that the initial clean image I is disturbed by additive white noise of zero mean and variance σ^2, producing the image $I' = I + \eta$. The goal is to determine a good estimation of I given the image I'.
The estimation problem can be formulated in the wavelet domain: the image coefficients $\widetilde{I}$ have to be estimated using the empirical coefficients $\widetilde{I}' = \widetilde{I} + \eta$. The algorithm can be described as follows.
For each wavelet coefficient, depending on the subband it belongs to, execute:
Step 1. For each of the first three fine subbands (with horizontal, vertical and diagonal

orientations), wavelet coefficients within the subband are modeled as identically distributed Laplacian with mean zero and variance $\sigma^2_{\tilde{I}j}$, where *j* stands for the index of the selected subband. The variance estimate is computed as,

$$\sigma^2_{\tilde{I}j} = \max\left\{0, Var\left\{\tilde{I}', I \in subbandj\right\} - \sigma^2\right\} \quad (1)$$

Consequently the maximum a posteriori (MAP) estimates of $\tilde{I}$ result by applying a soft threshold $\lambda = \frac{\sqrt{2\sigma^2}}{\sigma^2_{\tilde{I}j}}$ to each noisy coefficient.

Step 2. For each of the higher subbands, wavelet coefficients are clustered into one of the classes W_{sig} and W_{insig} according to the magnitude of their estimated parent with respect to T. Coefficients in W_{sig} are modeled as identically distributed Laplacian of zero mean. Their estimated variances are,

$$\sigma^2_{\tilde{I}j} = \max\left\{0, Var\left\{\tilde{I}', I \in W_{sig}\right\} - \sigma^2\right\} \quad (2)$$

The wavelet coefficients belonging to W_{insig} have small magnitudes and they represent smooth image areas. Assuming that $\hat{\sigma}_i^2$ is the true variance of $\tilde{I}_i$, the MAP estimation is,

$$\hat{I}_i = \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \hat{\sigma}^2}\tilde{I}'_i \quad (3)$$

Note that the course band coefficients are not processed because of their very high SNR.

The approach can be described as a top-down denoising process. Initially, the coarse scale coefficients are identified, then the algorithm propagates from parental nodes to their descendent subbands until the highest subband is reached. The coefficient estimates and the parental node significance information are used to process the next subband.

3. The Proposed Denoising Wavelet Based Model

Our attempt aims the development of a denoise intra-scale algorithm based on the multiresolution support set of the images.

The multiresolution representation is used to develop a class of noise removal algorithms in case of normal models. The multiresolution algorithms perform the restoration tasks by combining, at each resolution level, according to a certain rule, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. Our work reports two attempts in using the multiresolution based algorithms for restoration purposes in case of normally distributed noise. Several results obtained using our new restoration algorithm are presented in the final sections of the paper.

The multiresolution support set is a data structure suitable for developing noise removal algorithms. The multiresolution algorithms perform the restoration tasks by combining, according to a certain rule and at each resolution level, the pixels of a binary support image. The values of the support image pixels are either 1 or 0 depending on their significance degree. At each resolution level, the contiguous areas of the support image corresponding to 1-value pixels are taken as possible objects of the image. The multiresolution support is the set of all support images. The multiresolution support can be computed using the statistically significant wavelet coefficients.

Let j be a certain multiresolution level. Then, for each pixel (x, y) of the input image I, the multiresolution support at the level j is,

$M(I; j, x, y) = 1 \Leftrightarrow$ I contains significant information at the level j and the pixel (x,y).

If we denote by ψ be the mother wavelet function, then the generic computing scheme of the multiresolution support set is described as follows.

Step 1. Compute the wavelet transform of the input image using ψ .

Step 2. Compute $M(I; j, x, y)$ using the statistically significant wavelet coefficients for each resolution level j and for each pixel

(x,y).
The computation of the wavelet transform of a one dimensional signal is performed by the algorithm "À Trous". The algorithm can be extended to perform this computation in case of two-dimensional signals as, for instance, image signals. Let f be the continuous signal function and let ϕ be a low pass filter having the dilatation property

$$\frac{1}{2}\phi\left(\frac{x}{2}\right)=\sum_{l} h(l)\phi(x-l) \quad (4)$$

where h is a discrete valued low-pass filter. If we denote by $\{c_0(k)\}$ the sampled signal f computed via φ, then

$$c_0(k)=\langle f(x),\phi(x-k)\rangle . \quad (5)$$

The set $\{c_j(k)\}$ at the resolution level j is given by,

$$c_j(k)=\frac{1}{2^j}\left\langle f(x),\phi\left(\frac{x-k}{2^j}\right)\right\rangle , \quad (6)$$

therefore, we get

$$\frac{1}{2}\varphi\left(\frac{x-k}{2^j}\right)=\sum_{l} h(l)\varphi\left(\frac{x-k}{2^{j-1}}-l\right) \quad (7)$$

and

$$c_j(k)=\frac{1}{2^{j-1}}\left\langle f(x),\sum_{l} h(l)\varphi\left(\frac{x-k}{2^{j-1}}-l\right)\right\rangle=\sum_{l} h(l)\frac{1}{2^{j-1}}\left\langle f(x),\varphi\left(\frac{x-k}{2^{j-1}}-l\right)\right\rangle$$
$$=\sum_{l} h(l)c_{j-1}\left(k+2^{j-1}l\right).$$

The wavelet coefficients ω_j are computed using the terms c_{j-1} and c_j as

$$\omega_j(k)=c_{j-1}(k)-c_j(k). \quad (8)$$

The wavelet coefficients can be also expressed as [12],

$$\omega_j(k)=\frac{1}{2^j}\left\langle f(x),\psi\left(\frac{x-k}{2^j}\right)\right\rangle \quad (9)$$

where $\frac{1}{2}\psi\left(\frac{x}{2}\right)=\phi(x)-\frac{1}{2}\phi\left(\frac{x}{2}\right)$.

Using the resolution levels $1,2,...,p$, where p is a given parameter, the "À Trous" algorithm computes the wavelet coefficients according to the following scheme [12].

Input: The sampled signal $\{c_0(k)\}$

For j=0,1,…,p **do**

Step 1. j=j+1; compute $c_{j-1}(k)$,

$$c_j(k)=\sum_{l} h(l)c_{j-1}\left(k+2^{j-1}l\right).$$

Step 2. Compute $\omega_j(k)=c_{j-1}(k)-c_j(k)$

End-for

Output: The set $\{\omega_j(k),c_p\}_{j=1,...,p}$.

Note that the computation of $c_j(k)$ carried out in Step 1 imposes that either the periodicity condition $c_j(k+N)=c_j(k)$ or the continuity property $c_j(k+N)=c_j(N)$ holds.

Since the representation of the original sampled signal is

$$c_0(k)=c_p(k)+\sum_{j=1}^{p}\omega_j(k) \quad (10)$$

in case of images, the values of c_0 are computed for each pixel (x,y) as follows,

$$c_0(x,y)=c_p(x,y)+\sum_{j=1}^{p}\omega_j(x,y). \quad (11)$$

If the input image I encodes a noise component η, then the wavelet coefficients also encode some information about η. A label procedure is applied to each $\omega_j(x,y)$ in order to remove the noise component from the wavelet coefficients computed for I. In case for each pixel (x,y) of I, the distribution of the coefficients is available, the significance level corresponding to each component $\omega_j(x,y)$ can be established using a statistical test. We say that I is local constant at the resolution level j in case the amount of noise in I at this resolution level can be neglected. Let H_0 be the hypothesis H_0: I is local constant at the resolution level j. In case there is significant amount of noise in I at the resolution level j, we get that the alternative hypothesis $H_0 : \omega_j(x,y) \sim N\left(0,\sigma_j^2\right)$. In order to define the critical region W of the statistical test we proceed as follows. Let $0<\varepsilon<1$ be the a priori selected significance level and let z_ε be such that when H_0 is true,

$$1-\varepsilon = \mathcal{P}rob\left(\left|\omega_j(x,y)\right| < z_\varepsilon\right) = \frac{1}{\sqrt{2\pi}\sigma_j}\int_{-z_\varepsilon}^{z_\varepsilon} \exp\left\{-\frac{t^2}{2\sigma_j^2}\right\}dt \ . \ (12)$$

In other words, the probability of rejecting H_0 (hence accept H_0) when H_0 is true is ε and consequently, the critical region is $W=\left[-z_\varepsilon, z_\varepsilon\right]$. Accordingly, the significance level of the wavelet coefficients is given by the rule: $\omega_j(x,y)$ is a significant coefficient if and only if $\omega_j(x,y) \notin W$.

Usually, z_ε is taken as $k\sigma_j$, where k is a selected constant $k \approx 3$, because

$$P\left(\left|\omega_j(x,y)\right| > k\sigma_j\right) = P\left(\omega_j(x,y) > k\sigma_j\right) + P\left(\omega_j(x,y) < -k\sigma_j\right) =$$

$$= 2P\left(\omega_j(x,y) > k\sigma_j\right) = 2\left(1-P\left(\omega_j(x,y) \le k\sigma_j\right)\right)$$

$$z_{k\sigma_j} < \varepsilon \Rightarrow P\left(\left|\omega_j(x,y)\right| > k\sigma_j\right) \ge 2(1-\varepsilon)$$

Using the significance level, we set to 1 the statistically significant coefficient and respectively we set to 0 the non-significant ones. The restored image $\widetilde{I}$ is computed as

$$\widetilde{I}(x,y) = c_p(x,y) + \sum_{j=1}^{p} g\left(\sigma_j, \omega_j(x,y)\right)\omega_j(x,y) \ (13)$$

where g is defined by $g\left(\sigma_j,\omega_j(x,y)\right) = \begin{cases} 1, & \left|\omega_j(x,y)\right| \ge k\sigma_j \\ 0, & \left|\omega_j(x,y)\right| < k\sigma_j \end{cases}$.

4. The Filtering Technique of the Images Distorted by General Normal Distributed Noise

Let g be the original "clean" image, $\eta \sim N\left(m,\sigma^2\right)$ and the analyzed image $f = g+\eta$. The sampled variants of f, g and η obtained using the two-dimensional filter φ are given by,

$$c_0(x,y) = \left\langle f(l,c), \varphi(x-l, y-c)\right\rangle,$$

$$I_0(x,y) = \left\langle g(l,c), \varphi(x-l, y-c)\right\rangle,$$

$$E_0(x,y) = \left\langle \eta(l,c), \varphi(x-l, y-c)\right\rangle,$$

$$c_0 = I_0 + E_0 .$$

Consequently, the wavelet coefficients of c_0 computed by the algorithm "À Trous" are

$$\omega_j^{c_0}(x,y) = \frac{1}{2^j}\left\langle f(l,c), \psi\left(\frac{l-x}{2^j}, \frac{c-y}{2^j}\right)\right\rangle = \frac{1}{2^j}\left\langle g(l,c)+\eta(l,c), \psi\left(\frac{l-x}{2^j}, \frac{c-y}{2^j}\right)\right\rangle =$$

$$= \frac{1}{2^j}\left\langle g(l,c), \psi\left(\frac{l-x}{2^j}, \frac{c-y}{2^j}\right)\right\rangle + \frac{1}{2^j}\left\langle \eta(l,c), \psi\left(\frac{l-x}{2^j}, \frac{c-y}{2^j}\right)\right\rangle = -= \omega_j^{I_0}(x,y) + \omega_j^{E_0}(x,y),$$

where $\frac{1}{2}\psi\left(\frac{x}{2}\right) = \phi(x) - \frac{1}{2}\phi\left(\frac{x}{2}\right)$. For any pixel (x,y), we get

$$c_p(x,y) = \frac{1}{2^p}\left\langle f(l,c), \varphi\left(\frac{l-x}{2^p}, \frac{c-y}{2^p}\right)\right\rangle = I_p(x,y) + E_p(x,y).$$

The representation of the image c_0 is given by $c_0(x,y) = c_p(x,y) + \sum_{j=1}^{p}\omega_j^{c_0}(x,y) =$

$$=I_p(x,y)+E_p(x,y)+\sum_{j=1}^{p}\omega_j^{I0}(x,y)+\sum_{j=1}^{p}\omega_j^{E0}(x,y).\ (14)$$

Note that only $E_p(x,y)$ and $\sum_{j=1}^{p}\omega_j^{E0}(x,y)$ include noise component in (14). The mean of the noise can be decreased using the following algorithm.

Step1. Determine the images $E^{(i)}$, $1\le i\le n$, by superimposing noise sampled from $N(m,\sigma^2)$ on the "white wall" image.

Step2. For all *j*, $1\le j\le p$, compute c_j, $E_j^{(i)}$, $1\le i\le n$ and the coefficients $\omega_j^{c_0},\omega_j^{E^{(i)}}$ using the "À Trous" algorithm, according to,

$$c_j(x,y)=\sum_l\sum_c h(l,c)c_{j-1}\left(x+2^{j-1}l,y+2^{j-1}c\right)$$

$$E_j^{(i)}(x,y)=\sum_l\sum_c h(l,c)E_{j-1}^{(i)}\left(x+2^{j-1}l,y+2^{j-1}c\right)$$

$\omega_j^{c0}(x,y)=c_{j-1}(x,y)-c_j(x,y)$ and

$$\omega_j^{E^{(i)}}(x,y)=E_{j-1}^{(i)}(x,y)-E_j^{(i)}(x,y)$$

Step 3. Compute the image $\tilde{I}$ by

$$\tilde{I}(x,y)=\frac{1}{n}\sum_{i=1}^{n}\left[c_p(x,y)-E_p^{(i)}(x,y)+\sum_{j=1}^{p}\left(\omega_j^{c0}(x,y)-\omega_j^{E^{(i)}}(x,y)\right)\right].$$

Step 4. Compute a variant of the original image I_0 using the multiresolution filtering based on the statistically significant wavelet coefficients.

Note that $\tilde{I}$ computed at Step 3 is $\tilde{I}=I_0+E'$, where $E'\sim N\left(m',\sigma'^2\right)$, $m'\approx 0$ and $E\left(\sigma'^2\right)\approx\sigma^2$.

4. Concluding Remarks and Suggestions for Further Work

A series of experiments were performed, different 256 gray level images being preprocessed aiming the contrast enhancement, increasing enlighting and noise removing by filtering them. Our experiments use the averaging and respectively binomial filtering techniques. The parameters involved in the mentioned algorithm were tuned taking into account the following factors: the distortion degree of the inputs, the particular smoothing filter, the volume of the resulting accepted data.

The implementation of the proposed algorithm used the masks

$$h_1=\begin{pmatrix}\frac{1}{256}&\frac{1}{64}&\frac{3}{128}&\frac{1}{64}&\frac{1}{256}\\ \frac{1}{64}&\frac{1}{16}&\frac{3}{32}&\frac{1}{16}&\frac{1}{64}\\ \frac{3}{128}&\frac{3}{32}&\frac{9}{64}&\frac{3}{32}&\frac{3}{128}\\ \frac{1}{64}&\frac{1}{16}&\frac{3}{32}&\frac{1}{16}&\frac{1}{64}\\ \frac{1}{256}&\frac{1}{64}&\frac{3}{128}&\frac{1}{64}&\frac{1}{256}\end{pmatrix}\text{ and } h_2=\begin{pmatrix}\frac{1}{20}&\frac{1}{10}&\frac{1}{20}\\ \frac{1}{10}&\frac{2}{5}&\frac{1}{10}\\ \frac{1}{20}&\frac{1}{10}&\frac{1}{20}\end{pmatrix}.$$

Some of the results produced by the proposed algorithm are depicted in Table 1. Our tests were performed on distorted images processed by the masks h_1 and h_2 and the Gaussian distributions N(40,100) and respectively N(50,200), assumed to model the additive noise.

In our opinion, the performance of efficiency of the proposed algorithm can be significantly improved by narrowing the class of processed nodes, namely by taking into account and process only the coefficients belonging to the set W_{sig}. Also, improvements are expected to come from different refinements of the proposed algorithm.

Table 1.

Restoration algorithm	Type of noise	Mean error/pixel
Using the mask h_1	N(40,100)	12.6
Using the mask h_2		10.53
Using the mask h_1	N(50,200)	16.16
Using the mask h_2		12.74

Extensions of this methodology in case of more general assumptions concerning the statistical properties, as for instance modeling in case of generalized Gaussian mixtures are aimed. These developments are still in progress and the results are going to be reported elsewhere.

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