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Firms, Shareholders, and Financial Markets

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Abstract:
We study the influence of the financial market on the decisions of firms in the real market. To that end, we present a model in which the shareholders portfolio selection of assets and the decisions of the publicly-traded firms are integrated through the market process. Financial access alters the objective function of the firms, and the market interaction of shareholders substantially influences firms behavior in the real sector. After characterizing the unique equilibrium, we show that the financial sector integrates the preferences of all shareholders into the decisions for production and ownership structure. The participation from investors in the financial market also limits the firms ability to manipulate real prices, i.e., there is a loss of market power in the real sector. Note that, while the loss of market power changes expected profits, it is not detrimental to shareholders since the expected return of equity share depends on the variance (and not the mean) of profits. Indeed, any change in expected profits is absorbed by the financial price. We also show that financial access increases production, thereby altering the distribution of profits. In particular, financial access induces firms to take on more risk. Finally, financial access renders the relationship between risk-aversion and risk-taking ambiguous. For example, it is possible that an increase in risk-aversion leads to more risk-taking, i.e., the variance of real profits increases.

Keywords: Financial sector, Firm behavior, Market power, Monopoly, Perfect competition, Publicly-traded firm, Shareholder behavior.

JEL Classification: D21, D41, D42, D80, G32, L1

1 Introduction

While the bulk of economic activity comes from publicly-owned firms, the standard framework for a firm in industrial organization focuses on privately-owned firms. Indeed, the real and financial sectors are usually studied independently.[1] However, the real and financial functions of the firm are strongly linked, as the prices of financial instruments are closely related to the profits, and, thus, the prices of goods in the real sector. This paper studies the influence of the financial market on the decisions of firms in the real market. To that end, we present a model in which the shareholders' portfolio selection of assets and the decisions of the firms are integrated through the price in the financial market process. Financial access alters the objective function of the firms, and the market interaction of shareholders influences firms' behavior in the real sector. In particular, the financial sector integrates the preferences of all shareholders into the decisions for production and ownership structure. The participation from investors in the financial market also limits the firms' ability to manipulate real prices, i.e., there is a loss of market power in the real sector.

Before proceeding with a presentation and a discussion of the results, we provide an overview of the model. In order to obtain a clear exposition of the link between the real and financial sectors, we consider an economy with one firm and two shareholders. We later show in Appendix B that our results are robust to a model with several firms and many shareholders. In our model, the firm supplies a good in the real market, generating random profit. In addition, equity shares, which are risky assets linked to the random real profit, are sold in the financial market. The decisions of the firm are influenced by the decisions of the shareholders, whose objective is to maximize expected utility of final wealth over assets. The group of shareholders is composed of one entrepreneur and a continuum of investors. The entrepreneur is the founder of the firm and the original claimant of the real profit. The entrepreneur is also the managing shareholder of the firm who undertakes

[1] One important exception concerns corporate finance which focuses on moral hazard problems. See Tirole (2006).

a risky project in the real sector and interacts with the investors in the financial market in order to allocate risk, i.e., the random profit.[2] While the entrepreneur allocates the profits of the firm among the shareholders, the entrepreneur retains control of the firm's decisions. Specifically, subject to real and financial demands, the entrepreneur decides both the level of output and the ownership structure of the firm. Yet the entrepreneur's decisions are influenced by the preferences of the investors through the price of the risky asset. Indeed, the resulting market price of the risky asset combines the optimal behavior of the shareholders and is instrumental in influencing the decisions of the firm.

The role of the financial sector is two-fold. First, the decisions of the firm (both the level of output and the ownership structure) reflect the preferences of all the shareholders.[3] The financial price provides an incentive for the managing shareholder (the entrepreneur) to act on behalf of all shareholders. Specifically, the interaction of the entrepreneur and the investors in the financial market yields an output that takes account of the preferences of all the shareholders. Second, the interaction of the shareholders in the financial market limits the firm's ability to exercise market power in the real sector. With the introduction of a financial market, the objective of the firm is altered because it accounts for financial revenues from issuing equity shares. While the financial price depends on the real price through the expected payoff of the risky asset, the firm has no control over it. In other words, the firm cannot manipulate the real demand implicit in the financial price.[4] The reason is that the real demand implicit in the financial price depends upon the beliefs of the investors about the expected payoffs (i.e., the investors' reservation price of the risky asset) and not the actual choice of the firm. In equilibrium, these beliefs are consistent with the choice of the firm. Note that, while the loss of market power changes expected profits, it is not detrimental to shareholders since the expected return of equity share depends on

[2]In Appendix B, we consider an economy with N_E entrepreneurs/firms and N_I investors.

[3]Here, preferences refer to the coefficients of risk-aversion of the shareholders.

[4]Recall that the value of a share depends on the real profit, and, thus, on the real demand.

the variance (and not the mean) of profits. Indeed, any change in expected profits is absorbed by the financial price.

We also study the effect of financial access on the level of output and the distribution of profits. Access to the financial market induces hybrid behavior for a monopolist, that is, a convex combination of monopoly and perfect competition and, thus, a loss of market power in the real sector. This loss of market power implies that the monopolist behaves more competitively, which yields a higher level of output, closer to that of the competitive equilibrium in the real sector. Moreover, the allocation of risk among shareholders reduces the cost of risk for the entrepreneur. Indeed, the cost of bearing risk by a privately-owned monopolist is higher than the cost of sharing risk among several shareholders in a publicly-owned monopolist.[5] The combination of the loss of market power with the spreading of risk among the shareholders implies an increase in both output and risk-taking. In other words, financial access increases the variance of the real profit of the firm.

Finally, financial access alters the relationship between risk-aversion and risk-taking (or the amount of risk undertaken by the firm). This is best seen in studying the effect of risk-aversion of the entrepreneur on the decisions of the firm. Without a financial sector, an increase in risk-aversion induces the firm to decrease output, which decreases the amount of risk (i.e., the variance of real profit) undertaken. However, with financial access, this effect is altered by the link between risk-aversion, ownership structure, and market power. Specifically, a more risk-averse entrepreneur decreases the level of output, and, thus, decreases the variance of the real profit. However, due to the presence of the financial sector, an increase in the entrepreneur's risk aversion induces a higher participation in the financial sector (in order to share risk) which further limits the exercise of market power, i.e., output (and thus the variance of real profit) increases due to a more competitive behavior in the real sector. While these two effects of risk-aversion (loss of market power and reduction of risk) pull in opposite direction, the market power effect is always stronger. That is, an increase in risk-aversion increases risk-taking.

[5] Since shareholders are risk-averse, the cost of risk is captured by the risk premium.

Our work is related to several strands of the literature. First, we study the interplay between the firm and risk-averse shareholders. Previous work has only studied the behavior of risk-averse firms maximizing the expected utility of profit. See Baron (1970) and Sandmo (1971) for the competitive firm, and Baron (1971) for an imperfectly competitive market. Leland (1972) provides a general treatment of a risk-averse firm facing demand uncertainty under both perfect and imperfect competition. See also Hawawini (1978) for a geometric exposition using the mean-variance framework.[6] In this literature, while firms take account of risk, i.e., decision-making is influenced by the riskiness of profits, the effect of financial access on the firms' control over the amount of risk through the market process is absent. Specifically, although risk-averse shareholders have an aversion for risk, their rewards (expected return) depend positively on the amount of risk the firm takes. In other words, the higher the risk premium of an investor, the higher the premium (in terms of expected returns) given to a shareholder to bear part of the risk of the firm. This conflict between shareholders' disdain for risk and the increase in the payment when risk increases impacts real decisions. In other words, merely assuming risk-aversion of the firm without studying the underlying risk-taking process yields results in which the firm takes on less risk, which necessarily reduces how much investors are rewarded. In this paper, we extend this literature by linking the behavior of the firms to the portfolio selection of risk-averse shareholders. We show that risk-sharing among risk-averse shareholders induces the firms to take on more risk.

Second, our work combines aspects of the industrial organization literature and the financial literature. Real and financial decisions have been integrated in other contexts. The relationship between real and financial sectors have been studied in the context of the debt-equity positions of firms. See Dotan and Ravid (1985), and Prezas (1988) for cases of a single firm, and Brander and Lewis (1986) and Showalter (1995) for studies in an oligopolistic environment. Also, the relationship between real and financial markets has been studied in the context of insider trading. In particular, Jain and Mir-

[6] The behavior of risk-averse firms facing uncertainty has also been extended to the oligopolistic framework. See Asplund (2002) for a general treatment and references therein.

man (2000) explicitly links real and financial sectors through insider trading. See also Leland (1992), Dow and Rahi (2003), and Medrano and Vives (2004) for a welfare analysis of insider trading in the presence of real investment. In our model, the entrepreneur makes no choice of debt-equity positions, and has no informational motivations. Our interest lies in studying the nature of the firm when the behavior of shareholders is explicitly integrated into the model.

Finally, our work should be distinguished from a principal-agent model.[7] Here, we are concerned with the influence of the shareholders interacting in a financial market, and not with the problem of incentives in the presence of moral hazard and asymmetric information. In a principal-agent problem, the principal (the shareholder) hires an agent (a manager) to run his firm. Because the shareholder is unable to perfectly monitor the manager, conflicts of interests arise between the principal and the agent. Thus, the shareholder offers the manager a contract that provides the manager appropriate incentives to run the firm in the principal's interests. In our model, the deviation from maximizing expected profit arises from the explicit treatment of shareholders' portfolio selection through the market process and its relation to the firm's decisions. In other words, we study the market interaction of a group of risk-averse shareholders, i.e., one entrepreneur and a continuum of investors. This interaction yields decisions on the type of risk to undertake, the ownership structure of the firm to allocate that risk, and the characteristics of the risky asset issued by the firm. Although it is the entrepreneur that makes the decisions of the firm, the entrepreneur is provided with a *market* incentive to perform on behalf of all shareholders, including himself, through the price of the risky asset. The implementation of the optimal level of output by an agent is not the purpose of this paper.

The paper is organized as follows. In Section 2, we present the model of one firm and two shareholders.[8] In Section 3, we define and characterize the

[7]See Wilson (1968), Ross (1973), Harris and Raviv (1978), Holmström (1979), Shavell (1979), Grossman and Hart (1983), and Mirrlees (1999) for the principal-agent literature. See also Admati et al. (1994), and DeMarzo and Urošević (2006) for models in which the monitoring of a large shareholder affects the securities' expected payoffs.

[8]See Appendix B for the model of N_E entrepreneurs/firms and N_I investors.

equilibrium. Several aspects of the optimal behavior of the monopolist with shareholders are then presented. Section 4 discusses the role of the financial sector for the behavior of the firm. Section 5 studies the effect of financial access on the optimal level of output. Section 6 analyses the influence of risk-aversion on the amount of risk undertaken by the firm. Finally, Section 7 concludes and provides a discussion of possible extensions.

2 The Model

We present a model of a firm owned by shareholders. The firm supplies a good in the real market, generating random profit. In addition, equity shares, which are risky assets linked to the random real profit, are sold in the financial market. The market price of the risky asset reflects the optimal behavior of the shareholders and integrates the preferences of all shareholders, which is instrumental in influencing the decisions of the firm. We first present the model of the firm, and then describe the behavior of the shareholders.

The analysis in the body of the paper focuses on a model with one firm and two shareholders. The simplification, abstracting from several firms and many shareholders, yields a clear exposition of the structure of the model. In Appendix B, we show that the effects of the financial sector on the real sector, e.g., the inability of the firms to fully exercise market power, remain valid in a general model with several firms (and, thus, several risky assets). and many shareholders. Moreover, risk remains relevant with a large number of firms and shareholders, even in the limit.

2.1 The Firm

The firm is a monopoly in a real market with access to the financial market.[9] In the real market, the firm chooses the level of output $q \geq 0$, facing a random demand function. Specifically, the random price (net of cost) corresponding to supplying q units is $\tilde{p}_R = P_R(q) + \tilde{\varepsilon}$,[10] where $P_R(q)$ is the expected inverse

[9] The adjective *real* refers to the sector of goods and services other than those of financial nature.

[10] Alternatively, marginal cost is constant and $\tilde{p}_R$ is the random profit per unit.

demand and $\tilde{\varepsilon}$ is a shock with zero mean.[11] The presence of the shock is due to both systematic and nonsystematic risk in the economy. The profit of the firm is thus $\pi(q, \tilde{\varepsilon}) = (P_R(q) + \tilde{\varepsilon})q$. The expected profit is strictly concave in the level of output.

Assumption 2.1. $P_R''(q)q + 2P_R'(q) < 0$.

In the financial sector, the firm issues equity shares (a risky asset) at unit price p_F.[12] Each share is a claim on the profit corresponding to one unit of output. Hence, q shares of the risky asset are issued by the firm, and the random payoff of each share is $\tilde{p}_R$.[13] Finally, the firm chooses the fraction $1 - \omega \in [0, 1]$ of the shares sold in the financial market. Hence, the variable ω defines the ownership structure of the firm, which specifies the allocation of the random profit among the shareholders.

While the real inverse demand $\tilde{p}_R = P_R(q) + \tilde{\varepsilon}$ is given, and, thus, exogenous, the financial inverse demand $p_F = P_F((1 - \omega)q)$ is determined endogenously by the behavior of shareholders, which we describe next.

2.2 The Shareholders

The objective of each shareholder is to maximize the expected utility of final wealth. To that end, each shareholder diversifies wealth between the risky asset issued by the firm and a risk-free asset with a rate of return normalized to one. This interaction of the shareholders in the financial market determines both the amount and the share of risk taken on the the firm, and, therefore it determines the real output of the firm.

In our model, there are two shareholders: an entrepreneur and a continuum of investors of mass one.[14] The entrepreneur is the founder of the firm and the original claimant of the profit generated by his entrepreneurial prospects. The entrepreneur is also the managing shareholder of the firm,

[11] The subscript R refers to the *real sector* and the tilde sign differentiates a random variable from its realization.

[12] The subscript F refers to the *financial sector*.

[13] The fact that there are q shares when q units of output are sold is a normalization. Shares are assumed to be infinitely divisible entities.

[14] See Appendix B for the model with several firms and many shareholders.

making the output decision, retaining part of the risky asset, and selling the remaining shares to the investors. The proceeds from the sale of shares is invested at a risk-free rate. Unlike the entrepreneur, the investors do not have entrepreneurial prospects and have no direct control over the decisions of the firm. However, each investor has initial wealth $W_I > 0$, which is used to purchase shares of the risky asset and the risk-free asset.

We now derive the final wealth of the entrepreneur and the investors. The entrepreneur's random final wealth combines the payoffs from both the real and financial sectors:

$$\tilde{W}'_E = \omega \tilde{p}_R q + (1-\omega) p_F q, \tag{1}$$

where ω is the entrepreneur's level of ownership. The expression $\omega \tilde{p}_R q$ is the entrepreneur's portion of the random profit of the firm, while $(1-\omega)p_F q$ is the wealth generated from selling claims to the profit of the remaining $(1-\omega)q$ units of output at price p_F, and investing $(1-\omega)p_F q$ in a risk-free asset. Here the rate of return of the risk free asset is normalized to one. The final wealth of each investor is derived by diversifying the initial wealth W_I between m shares of the risk-free asset and z shares of the risky asset issued by the firm. Given the budget constraint of an investor, $W_I = m + p_F z$, his random final wealth is

$$\tilde{W}'_I = W_I + (\tilde{p}_R - p_F) z. \tag{2}$$

Here, $W_I - p_F z$ is invested in the risk-free asset and $\tilde{p}_R z$ is the random payoff corresponding to z shares of the risky asset. Note that the return on a share of the firm is $\tilde{p}_R - p_F$.

Next, we present the objective functions of the shareholders. Each shareholder maximizes the expected utility of final wealth defined by (1) or (2). The shareholders are assumed to be risk-averse in final wealth, which yields portfolio diversification. For tractability, we assume that the systematic shock is normally distributed and the shareholders' preferences for final wealth exhibit constant absolute risk aversion (CARA).

Assumption 2.2. $\tilde{\varepsilon} \sim N(0, \sigma^2)$.

Assumption 2.3. *The coefficients of absolute risk aversion are* $a_E > 0$ *and* $a_I > 0$ *for the entrepreneur and the investors, respectively.*[15]

Assumptions 2.2 and 2.3 combined with the general structure of the real sector yields a closed-form characterization of the shareholders' maximization problems. It also yields a strictly monotonic relation between expected utility and a certainty equivalent. Hence, maximizing expected utility of final wealth is equivalent to maximizing the certainty equivalent. The certainty equivalent approach is valid and is used throughout the paper.

From (1), given that $\tilde{p}_R = P_R(q) + \tilde{\varepsilon}$, the certainty equivalent of the entrepreneur is[16]

$$CE_E = \omega P_R(q)q + (1 - \omega)p_F q - a_E \sigma^2 \omega^2 q^2/2. \tag{3}$$

Here, $\omega P_R(q)q + (1 - \omega)p_F q$ is the expected payoff to the entrepreneur from the real and financial sectors weighted by the level of ownership. The term $a_E \sigma^2 \omega^2 q^2/2$ is the risk premium of the entrepreneur. The risk premium plays the role of a cost, due to risk aversion, imposed on the entrepreneur for bearing part of the risk. From (2), the certainty equivalent of each investor is

$$CE_I = W_I + (P_R(q) - p_F)z - a_I \sigma^2 z^2/2. \tag{4}$$

Here, $W_I + (P_R(q) - p_F)z$ is the expected mean of final wealth and $a_I \sigma^2 z^2/2$ is the risk premium.

Both the real and financial sectors are integrated because the payoff of the risky asset depends on the level of output, and reflects the uncertainty of the real sector. Also, the level of output depends on the amount of risk assumed by the investors. This link between the decisions of the firm and the behavior of the shareholders is shown in (3). Note that the decisions of the firm are derived directly from the behavior of the entrepreneur, but the

[15] In other words, utility functions for final wealth x are exponential: $u(x; a) = -e^{-ax}, a \in \{a_E, a_I\}$.

[16] The expected utility of the entrepreneur is $\mathbb{E}u(\tilde{W}_E, a_E) = e^{-a_E CE_E}$, where $\mathbb{E}$ is the expectation operator.

influence of the investors on the firm's decisions is indirect. It comes through the demand for the risky asset, i.e., $p_F = P_F((1-\omega)q)$. Indeed, investors' optimal behavior influences the financial price, which, in turn, has an effect on the behavior of the firm via the entrepreneur's maximization problem.

3 The Equilibrium

Having described the model, we now define and characterize the equilibrium. The entrepreneur and the investors move simultaneously. Hence, the entrepreneur takes as given the financial demand of the investors, while the investors take as given the decisions of the firm made by the entrepreneur. Moreover, the entrepreneur can manipulate the price of the real good, while the investors are price-taker in the financial market. The extent to which the entrepreneur can manipulate the financial price is discussed after Definition 3.1.

The equilibrium consists of the firms' decisions made by the entrepreneur $\{q^*, \omega^*\}$, the investors' demand for the risky asset $z^*(p_F)$ and the financial price p_F^*.

Definition 3.1. *The tuple $\{q^*, \omega^*, z^*(p_F), p_F^*\}$ is an equilibrium if*

1. *Given q^* and p_F^*, the investors' quantity demanded for the risky asset is*
$$z^*(p_F^*) = \arg\max_{z \geq 0} W_I + (P_R(q^*) - p_F^*)z - a_I \sigma^2 z^2/2. \tag{5}$$
2. *Given $z^*(p_F)$,*
$$\{q^*, \omega^*\} = \arg\max_{q \geq 0, \omega \in [0,1]} \omega P_R(q)q + (1-\omega)P_F^*((1-\omega)q)q - a_E \sigma^2 \omega^2 q^2/2, \tag{6}$$
where $P_F^*((1-\omega)q)$ is the inverse function of the investors' financial demand $z^*(p_F)$ when $(1-\omega)q$ shares of the risky asset are sold.
3. *Given q^*, ω^*, and $z^*(p_F)$, $p_F^* = P_F^*((1-\omega^*)q^*)$.*

Before proceeding with the characterization of the equilibrium, we comment on an important aspect of the equilibrium. In an investor's maximization problem, a conjecture about the expected payoff of the share of the risky asset is formed, i.e., $P_R(q^c)$ where c stands for *conjecture*. Since the conjecture is consistent with the behavior of the firm in equilibrium, $P_R(q^c) = P_R(q^*)$ as written in (5). The investors form a conjecture because the decisions and payoffs of the entrepreneur (or the firm) are *unobservable* by the investors at the time of decision-making. In other words, the investors purchase stocks of the firm based on his beliefs about payoffs that are yet to be realized. Since the beliefs of the investors are not under the control of the entrepreneur, they constitute an externality, which influences the behavior of the firm.

Proposition 3.2 states that there exists a unique equilibrium. We then characterize and discuss the equilibrium. All proofs are relegated to Appendix A.

Proposition 3.2. *There exists a unique equilibrium.*

We begin with the behavior of the (price-taking) investors, which defines the demand for the stock of the firm. This helps us to understand the extent to which the entrepreneur can manipulate the financial price. Proposition 3.3 provides the equilibrium demand of the investors for the risky asset.

Proposition 3.3. *In equilibrium, the investors' demand for the risky asset is*

$$z^*(p_F) = \frac{P_R(q^*) - p_F}{a_I \sigma^2}, \tag{7}$$

where $P_R(q^*) - p_F$ *is the expected return of a share of the risky asset.*

Financial demand depends positively on the *conjectured* expected payoff $P_R(q^*)$, and negatively on both the risk-aversion of the investors, and the variance of the real shock (i.e., the riskiness of the project). Using (8), Remark 3.4 establishes the inverse demand of the risky asset faced by the entrepreneur (or the firm) in the financial sector.

Remark 3.4. *The investors' inverse demand for the risky asset is*

$$P_F^*((1-\omega)q) = P_R(q^*) - a_I \sigma^2 (1-\omega) q, \tag{8}$$

when $(1-\omega)q$ *shares are sold.*

The price of a share of the risky asset is equal to the expected price of the real good minus the term $a_I\sigma^2(1-\omega)q$, which is closely related to the investors' risk premium in (4). Here, the entrepreneur sells shares of the risky asset at a price below the expected payoff in order to induce the investors to bear some of the risk. For each share sold, the risk-averse entrepreneur incurs a cost for reducing his own risk.

The price of the risky asset is at the core of the market allocation of risk between the entrepreneur and the investors. The financial market brings together the agents' diverse interests for the risky asset and determines the risk faced by each agent. The price function depends not only on the investors' optimal behavior, but also on the entrepreneur's decisions. For instance, the lower the entrepreneur's level of ownership, the lower the price of the risky asset.

Finally, the price function for the risky asset establishes the link between the real and financial sectors. Here, the reservation price of the risky asset is the expected real price conjectured by the investors. Any change in the real market that increases the demand for the real good translates into a higher price of the risky asset through a higher reservation price.

Having characterized the demand for the risky asset, we next turn to the optimal behavior of the entrepreneur. Proposition 3.5 characterizes the equilibrium decisions of the firm made by the entrepreneur corresponding to (6). From (8), the entrepreneur partially controls the financial price through the quantity supplied $(1-\omega)q$.[17] However, the entrepreneur has no influence on the reservation price of the risky asset because it refers to the beliefs of the investors about the expected payoff of the risky asset. However, in equilibrium, the expected payoff of the risky asset does depend on the entrepreneur's choice of output, i.e., $P_R(q^*)$ is function of the equilibrium level of output.

[17] In the general model with a continuum of firms (and, thus, risky assets), financial markets are perfectly competitive and the entrepreneurs have no control over financial prices.

Proposition 3.5. *In equilibrium, output q^* satisfies*

$$\omega^* \left(P_R'(q^*)q^* + P_R(q^*)\right) + (1-\omega^*)P_R(q^*) = \omega^{*2} a_E \sigma^2 q^* + 2(1-\omega^*)^2 a_I \sigma^2 q^* \quad (9)$$

and

$$\omega^* = \frac{2a_I}{2a_I + a_E}. \quad (10)$$

Two comments regarding proposition 3.5 are warranted. First, consider expression (9).[18] The left-hand side is the *generalized* marginal revenue of output. It admits all the possibilities of financial participation, including the limiting case of no financial participation, i.e., $\omega^* = 1$.[19], i.e., a firm that is owned and managed by a single agent. The right-hand side is the marginal cost of sharing risk. The first term of the right-hand side is related to the entrepreneur's risk aversion and reflects the cost of bearing risk corresponding to a fraction ω^* of additional output. The second term depends on the investors' risk aversion and characterizes the entrepreneur's cost of reducing risk corresponding to a fraction $1 - \omega^*$ of additional output. The cost of reducing risk is related to the payment to investors for bearing some of the additional risk.

Next, consider expression (10). Note that the equilibrium ownership structure is independent of the real sector. Indeed, using (6) and (8), the first-order condition with respect to ω is $P_R(q)q - p_F^* q + a_I \sigma^2 (1-\omega) q^2 = a_E \sigma^2 \omega q^2$ evaluated at $q = q^*$ and $\omega = \omega^*$. Since, using (8), the equilibrium financial price is

$$p_F^* = P_R(q^*) - a_I \sigma^2 (1 - \omega^*) q^*, \quad (11)$$

it follows that the first-order condition with respect to ω is rewritten as

$$2a_I \sigma^2 (1 - \omega) q^2 = a_E \sigma^2 \omega q^2, \quad (12)$$

evaluated at $q = q^*$ and $\omega = \omega^*$. From (12), the real price plays no role in determining ω^*. To understand why, observe that the expected return

[18] If $P_R(q) = \theta - \gamma q$, $\theta, \gamma > 0$, then, from (9) and (10), $q^* = \theta / \left((1 + \omega^*)\gamma + \omega^* a_E \sigma^2\right)$.

[19] From (10), $\omega^* = 1$ when the entrepreneur is risk-neutral or, alternatively, $a_I \to \infty$, which is equivalent to the the case in which there is no investor in the market, as he cannot bear any risk.

of a share is independent of the real demand, i.e., from (11), $P_R(q^*) - p_F^* = a_I\sigma^2(1-\omega^*)q^*$.[20] In other words, the expected profit corresponding to a share (i.e., the real mean price) has no effect on the expected return. This is due to the fact that any changes in the expected profit is absorbed by the financial price, and, thus, the difference between expected payoff and financial price is independent of expected profit.

In addition, ω^* is independent of σ^2. Indeed, an increase in σ^2 increases the entrepreneur's payment to the investors, which induces the entrepreneur to retain more ownership. At the same time, a riskier real profit increases the entrepreneur's cost of bearing risk, inducing less ownership on the part of the entrepreneur. In equilibrium, these two effects pull in opposite directions in equal strength, and, thus, cancel each other. Similarly, from (12), ω^* is independent of the entrepreneur's output choice. Therefore, the equilibrium allocation of equity among the entrepreneur and the investors is independent of the real profit of the firm.

In the next three sections, several aspects of the behavior of the monopolist with shareholders are presented. First, the role of the financial sector for the behavior of the firm is discussed. Second, we study the effect of financial access on the optimal level of output. Finally, the influence of risk-aversion on the amount of risk undertaken by the firm is analyzed.[21]

4 The Role of the Financial Sector

In this section, we show that the financial sector influences the behavior of the firm in two ways. First, the interaction of shareholders limits the firm's ability to exercise market power in the real sector. Second, the price of the stock determined in the financial sector integrates the preferences of all the shareholders for both the choice of output and the allocation of the risk among the shareholders (ownership structure). Both influences of the

[20] More precisely, the expected return of a share does not depend directly on the real demand, although it does depend on it indirectly through the policy functions.

[21] The amount of risk is measured by the variance of the real profit, $\mathbb{V}\pi_R(q, \tilde{\varepsilon}) = \sigma^2 q^2$, where $\mathbb{V}$ is the variance operator.

financial market on the entrepreneur goes through the price of the risky asset, which appears in the payoff function of the entrepreneur.

4.1 Market Power

The extent to which the firm is able to manipulate the real price is captured by the left-hand side of (9). If the firm is completely owned by the entrepreneur, the real demand is the sole basis for the maximization profit, i.e., the marginal revenue from the real sector determines the output of the firm. However, given the optimal ownership structure $\omega^* \in [0, 1]$, the firm can only manipulate a fraction ω^* of the real demand. The remaining fraction $1-\omega^*$ of the real demand cannot be manipulated by the entrepreneur since, although the real demand is implicit in the financial price, it refers to the beliefs of the investors and not the actual choice of the entrepreneur about expected payoffs, (i.e., the investors' reservation price of the risky asset). The more ownership relinquished by the entrepreneur, the weaker the degree of market power exercised by the firm. By selling more shares to the investors, the entrepreneur forgoes market power in the real sector in exchange for financial revenue as well as a reduction in risk.

The loss of market power is due solely to the strategic interaction of the shareholders and has nothing to do with the market structure of the financial sector. Specifically, the inability of the entrepreneur to manipulate the real price remains whether the entrepreneur is a monopoly (as in this model) or perfectly competitive in the financial sector (as shown in the general model with several risky assets in Appendix B).[22]

The ability to manipulate the real price can be thought of as the ability to charge a markup above the marginal cost. If the demand belongs to a class of demands that can be ordered by their elasticity of demand, e.g., $P_R(q) = q^{-\frac{1}{\eta}}$, where $\eta > 1$ is the elasticity, then (9) is rewritten as

$$\frac{P_R(q^*) - MC(q^*, \omega^*)}{P_R(q^*)} = \frac{\omega^*}{E_d}, \tag{13}$$

[22]Monopoly in the financial sector means that the entrepreneur has an effect on the financial price through the quantity supplied of shares as shown in (11).

which relates the Learner index to the ownership level of the managing shareholder and the elasticity of demand. Here, $MC(q^*, \omega^*)$ refers to the right-hand side of (9), $\omega^* = 2a_I/(2a_I + a_E)$, and $E_d = \eta$ is the elasticity of demand. For any given level of elasticity, a higher level of financial integration (a lower ω^*) reduces the firm's ability to charge a markup, which illustrates our result. In other words, under financial access, the markup depends not only on the real demand, but also on the intensity of financial participation.

4.2 Shareholders' Preferences

The financial sector integrates the preferences of all the shareholders into the decisions of the firm. Regarding the choice of output, the marginal cost of risk sharing (the right-hand side of (9)) combines the diverse preferences of the shareholders (both the entrepreneur and the investors). In particular, the firm takes account of the investors' preferences through the cost of reducing risk, which reflects the investors' willingness to bear part of the firm's risk. The more ownership acquired by the investors, the more relevant their preferences are to the firm.

As for the ownership structure defined by (12), the preferences of the shareholders are at the core of the allocation of risk among the shareholders. Indeed, a more risk-averse entrepreneur increases the cost of bearing risk, which induces a reduction of his exposure to risk, $\partial\omega^*/\partial a_E < 0$.[23] More risk-averse investors increases the cost of reducing risk, which reduces the entrepreneur's incentive to relinquish ownership of the firm, $\partial\omega^*/\partial a_I > 0$.

5 The Effect of Financial Access

Having described the role of the financial price on the behavior of the firm, we now turn to the effect of financial access on the optimal behavior of the firm. To that end, we introduce two benchmark models of no financial access. The first characterizes the optimal behavior of a monopolist facing no

[23]From (12), the cost of reducing risk depends on the expected return of a share of the risky asset held by an investor.

risk. The second concerns the optimal behavior of a monopolist facing risk. Comparing the two benchmark models allows us to study the effect of risk on the optimal output of the firm. Comparing our model with the benchmark models allows us to study the effect of financial access on the optimal output. Since the distribution of real profit is directly tied to the level of output, i.e., $\pi_R(q, \tilde{\varepsilon}) \sim N(P_R(q)q, \sigma^2 q^2)$, we are also able to study the effect of financial access on the amount of risk undertaken by the firm.

5.1 Benchmark Models of No Financial Access

We first present the two benchmark models, and, then, study the effect of risk on the optimal output.[24] No financial access implies that the entrepreneur retains ownership of the firm, i.e., $\omega = 1$.

No Risk. If there is no risk, i.e., $\sigma^2 = 0$, the entrepreneur maximizes expected profit, so that (6) evaluated at $\omega = 1$ and $\sigma^2 = 0$ is $\max_{q \geq 0} P_R(q)q$. Optimal output, q', satisfies the first-order condition

$$MR(q') = 0. \tag{14}$$

Condition (14) is also valid for a risk-neutral firm facing risk, i.e., $a_E = 0$ and $\sigma^2 > 0$.

Risk. Under risk and risk-aversion, but no financial access, i.e., $a_E, \sigma^2 > 0$ and $\omega = 1$, (6) is rewritten as $\max_{q \geq 0} P_R(q)q - a_E \sigma^2 q^2/2$. The last term of the objective function is the risk premium of the entrepreneur. The risk premium plays the role of a cost, due to risk aversion, imposed on the entrepreneur for bearing all the risk. Optimal output, $\hat{q}$, satisfies the first-order condition

$$MR(\hat{q}) = a_E \sigma^2 \hat{q}. \tag{15}$$

The Effect of Risk under Risk-Aversion. From (14) and (15), the presence of risk induces a risk-averse firm to decrease output, as shown in Figure 1. Specifically, Figure 1a depicts the case of a monopolist facing no risk. The optimal output q' sets the marginal revenue equal to zero, as

[24] See Leland (1972) for an in-depth analysis of a risk-averse firm facing risk.

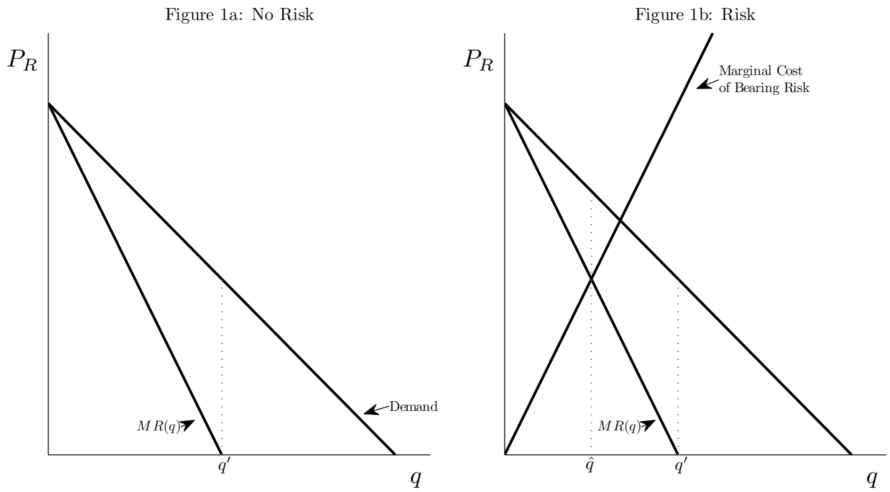

Figure 1: The Effect of Risk on a Risk-Averse Monopolist

in (14). Figure 1b depicts the case of a monopolist facing risk, under the same demand. Because the entrepreneur is risk-averse, the optimal output $\hat{q}$ sets the marginal revenue equal to the marginal cost of bearing risk, the right-hand side of (15), which has the effect of reducing output ($\hat{q} < q'$ in Figure 1b), and, thus, the amount of risk undertaken by the firm (the variance of the real profit) is reduced.

5.2 Financial Access

Having described the benchmark models of no financial access, we next study the effect of financial access on optimal output by comparing a monopolist facing risk with and without financial access. Formally,

Proposition 5.1. *From* (9) *and* (15)*, financial access increases output.*

The effect of financial access is shown in Figure 2. Specifically, Figure 2a (identical to Figure 1b) depicts the optimal behavior of a risk-averse monopolist without financial access supplying $\hat{q}$, where the marginal revenue is equal to the marginal cost of bearing risk. In Figure 2b, a firm with financial access (owned by several shareholders) supplies q^*, where the generalized marginal

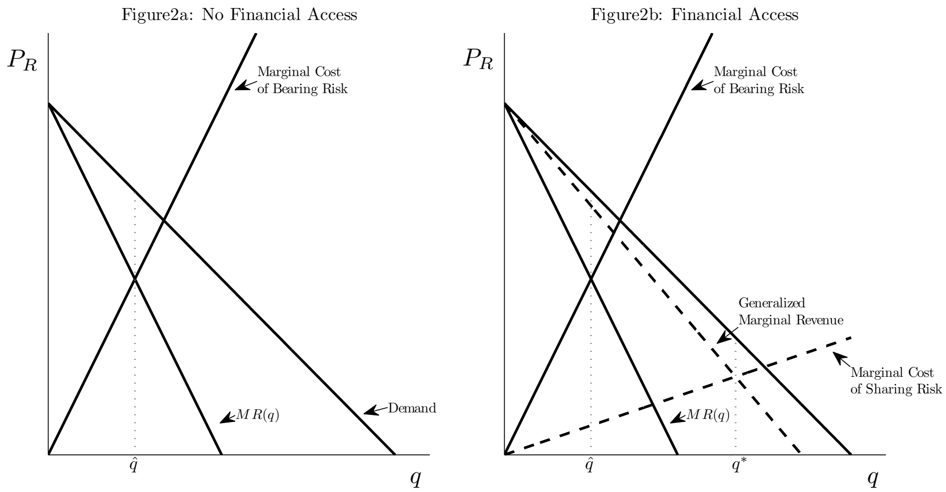

Figure 2: The Effect of Financial Access on a Risk-Averse Monopolist

revenue is equal to the marginal cost of sharing risk, as in (9). Consistent with Proposition 5.1, $\hat{q} < q^*$. It follows immediately that financial access increases both the mean and variance of the real profit.

While risk decreases output for a risk-averse firm, the allocation of risk (via the financial market) reverses the effect, i.e., increases output. The effect of financial access on output is two-fold. First, the monopolist behaves more competitively in the real sector. In particular, the investors' conjecture about the payoffs of the risky asset is an externality that induces more perfectly competitive behavior in the real sector. This implies a higher level of output. Second, the interaction of the shareholders in order to share the risk of the firm reduces the cost of risk, which also implies a higher level of output.[25]

Both effects are restated formally in Remarks 5.2 and 5.3. Proposition 5.1 follows immediately from Remarks 5.2 and 5.3.

Remark 5.2. *Access to a financial market induces hybrid behavior for a monopolist, that is, a convex combination of monopoly and perfect competition in the real sector.*[26]

[25]The cost of risk refers to the cost of bearing risk for a firm without financial access, and refers to the cost of sharing risk for a firm with financial access.

[26]Formally, with no financial access, optimal output of a monopolist satisfies (15), while

The influence of the financial market induces the firm to increase output, because the generalized marginal revenue, the left-hand side of (9), lies above the marginal revenue in Figure 2b.

The reduction in the cost of risk is shown in Figure 2b, where, for any level of output, the marginal cost of bearing risk lies above the marginal cost of sharing risk. This decrease in cost also induces the firm to supply more.

Remark 5.3. *Financial access lowers the cost of risk.*

To understand Remark 5.3, simplify expression (9):[27]

$$\omega^* \left(P_R'(q^*)q^* + P_R(q^*)\right) + (1-\omega^*)P_R(q^*) = \omega^* a_E \sigma^2 q^*. \tag{16}$$

From (10), $\omega^* \in [0,1]$. Hence, the cost of bearing risk (with no financial access) is always greater than the cost of sharing risk with investors. Specifically, for any level of output, the right-hand side of (15) is greater than the right-hand side of (16).[28]

One implication of Proposition 5.1 is worth mentioning. An increase in output due to financial access is beneficial to the investors because expected return

$$\left(P_R(q^*) - p_F^*\right) z^* = a_I \sigma^2 \left(1-\omega^*\right)^2 q^{*2} \tag{17}$$

increases with more financial access. This is due to the fact that the expected return of a share to the investors is **unrelated** to the expected profits of the firm. Rather, the expected return depends on the variance (risk) of real profits. In other words, the investors are rewarded for the amount of risk borne. Consequently, the loss of market power discussed in the previous

optimal output of a perfectly competitive firm satisfies $P_R(q) = a_E\sigma^2 q$. Hence, the left-hand side of (9) combines both monopoly and perfectly competitive behavior.

[27]Plugging (10) into (9) and rearranging yields (16). In other words, given (10), $\omega^{*2} a_E \sigma^2 q^* + (1-\omega^*)^2 a_I \sigma^2 q^* = \omega^* a_E \sigma^2 q^*$.

[28]Remark 5.3 is only true because the value of ω^* makes it possible to rewrite (9) as (16). In other words, the endogenization of the ownership structure always lowers the cost of risk, and, thus, makes the effect of financial access unambiguous. If ω were a parameter instead of a control variable, then Remark 5.3 would not hold. Specifically, if $\omega = \bar{\omega} \in [0,1]$ was fixed, then, for any level of output, the marginal cost of sharing risk under an exogenous ownership structure, i.e., $\bar{\omega}^2 a_E \sigma^2 q + (1-\bar{\omega})^2 a_I \sigma^2 q$ and the marginal cost of bearing risk (the right-hand side of (15)) cannot be ordered.

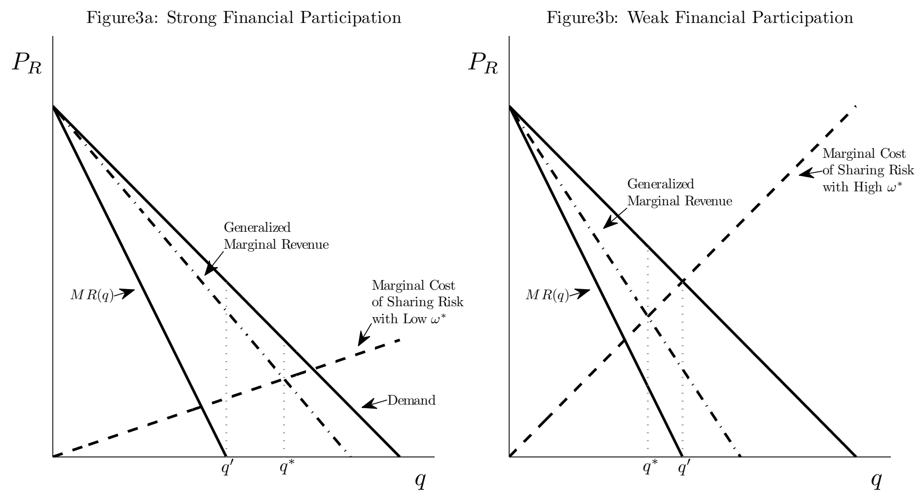

Figure 3: The Strength of Financial Access on a Risk-Averse Monopolist

section does not imply a loss for the investors. Any changes in the expected profit is absorbed by the financial price, i.e., $\partial p_F^* / \partial P_R(q^*) = 1$. Hence, the difference between expected payoff and financial price is independent of expected profit.

Having established that financial access increases the output of a risk-averse monopolist, we next study the strength of this effect. The extent to which behavior is altered by access to the financial market depends on the intensity of financial participation, i.e., the portion of the firm sold to investors. Indeed, a lower ω^* yields more financial participation, which, in turn, intensifies the effect of financial access by inducing the monopolist to behave more like a perfectly competitive firm in the real sector, as well as by lowering the cost of sharing risk.

Proposition 5.4. *From* (10)*, the strength of the effect of financial access on optimal output is*

1. *increasing in a_E, but*
2. *decreasing in a_I.*

The strength of financial access can be significant as illustrated in Figure 3. Indeed, depending on the ownership structure, financial access tempers or even reverses the effect of risk on output.[29] To see this, compare output q' under no risk and output q^* under financial access. On the one hand, strong financial participation (low ω^*) reverses the effect of risk on optimal output, i.e., $q^* > q'$ in Figure 3a. On the other hand, weak financial participation (high ω^*) tempers the effect of risk, i.e., $q^* < q'$ in Figure 3b. It follows that, depending on the strength of financial access, expected profit $P_R(q)q$ might increase or decrease. Specifically, weak financial participation unambiguously increases expected profit as optimal output approaches the maximizer. Strong financial profit might decrease expected profit as optimal output is much closer to the one chosen by a perfectly competitive firm.

6 Risk-Aversion and Risk-Taking

In this section, we show that access to the financial sector alters the effect of risk-aversion on risk-taking (or the amount of risk undertaken by the firm). i.e., the effects of a_E and a_I on $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon}) = \sigma^2 q^{*2}$. Specifically, with no financial sector, an increase in risk-aversion induces the firm to decrease output, which decreases the amount of risk undertaken.[30] However, with financial access, the effect is altered by the link between risk-aversion, ownership structure, and market power. Proposition 6.1 summarizes the effects of a_E and a_I on the variance of real profit under financial access. In particular, the effect of a_E on $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon})$ depends on the difference between the slopes of the real and financial demands.[31]

Proposition 6.1. $\frac{\partial \mathbb{V}\pi_R(q^*,\tilde{\varepsilon})}{\partial a_E} > 0$, *and* $\frac{\mathbb{V}\pi_R(q^*,\tilde{\varepsilon})}{\partial a_I} < 0$.

[29] Recall that risk decreases output ($\hat{q} < q'$ in Figure 1), while, from Proposition 5.1, financial access increases output ($\hat{q} < q^*$ in Figure 2).

[30] From (15), an increase in a_E increases the cost of bearing risk, which induces the firm to reduce output, and, thus, the amount of risk undertaken. In other words, being more-risk averse induces the firm to reduce the variance of the real profit.

[31] From (8), $a_I\sigma^2$ is the slope of the demand for shares of the risky asset.

Proof. Recall (16):

$$\omega^* \left(P_R'(q^*)q^* + P_R(q^*)\right) + (1-\omega^*)P_R(q^*) = \omega^* a_E \sigma^2 q^*, \tag{18}$$

where, from (10), $\omega^* = 2a_I/(2a_I + a_E)$. Differentiating (18) with respect to a_E and a_I yields $\partial q^*/\partial a_E > 0$ and $\partial q^*/\partial a_I < 0$. Since $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon}) = \sigma^2 q^{*2}$, Proposition 6.1 follows. □

Consider first the effect of a_E on $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon})$. Note that an increase in a_E induces the entrepreneur to sell a larger fraction of the firm, i.e., ω^* decreases. This, in turn, has an effect not only on the cost of risk, but also on the firm's ability to exercise market power. Specifically, from the left-hand side of (18), an increase in a_E reduces the monopolist's ability to manipulate the price. This implies a higher level of output and thus a higher level of risk-taking. From the right-hand side, an increase in a_E increases the cost of risk, which induces the firm to decrease the level of output, and, thus, to decrease the amount of risk undertaken.[32] Hence, the two effects of a_E on q^* (and $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon})$) pull in opposite directions. However, the effect of a_E is stronger on the exercise of market power than on the cost of risk. Hence, a more risk-averse entrepreneur increases the amount of risk undertaken.

Next, consider the effect of a_I on $\mathbb{V}\pi_R(q^*, \tilde{\varepsilon})$. Proposition 6.1 states that an increase in a_I leads to a decrease in the amount of risk undertaken by the firm. Note that having more risk-averse investors limits the entrepreneur's participation in the financial sector, i.e., ω^* increases. This, in turn, has an effect not only on the cost of risk, but also on the firm's ability to exercise market power. Indeed, from the left-hand side of (18), an increase in a_I reduces the influence of the perfectly competitive financial market, which implies a decrease in output. From the right-hand side, an increase in a_I increases the cost of risk, which implies a decrease in output as well. Hence, the amount of risk undertaken by the firm decreases.

[32] Note that the effect of a_E on the right-hand side of (18) is two-fold. First, as a_E increases, the right-hand side increases directly. Second, an increase in a_E increases the cost of risk, which gives the entrepreneur an incentive to relinquish a higher fraction of the firm, i.e., ω^* decreases. The overall effect of a_E on the cost of risk via the financial market is nonetheless always positive.

7 Final Remarks

In this paper, financial access is shown to reduce a monopolist's ability to exercise market power in the real sector, regardless of the market structure in the financial sector. Indeed, if a monopolist is privately owned, it fully exercises market power. Once a monopolist goes public, the strategic interaction of the shareholders limits the firm's ability to manipulate the real price. Moreover, the ability to exercise market power in the real sector is reduced as financial participation increases. Consequently, more financial participation reduces welfare loss in the real sector as the monopolist behaves more like a perfectly competitive firm. While not discussed in the paper, this result has implications regarding the extent to which public policy is used to avoid the negative consequences of market power.

In order to study the effect of the financial sector and the market interaction of the shareholders on the behavior of the firm, we have abstracted from two important aspects. First, there is no asymmetric information in our model. In fact, asymmetric information is ubiquitous among shareholders. For instance, consider a situation in which some investors do not know the true distribution of the risky asset's payoff. Learning occurs because the price of the risky asset is used as a signal by uninformed investors to infer the unknown distribution. Second, in our model, the motivation for the interaction of shareholders is solely risk sharing. Extending the model in which firms can raise money in order to buy capital adds another layer of complexity.

A Proof

Using Definition 3.1, we characterize the equilibrium. Given q^* and p_F^*, the first-order condition corresponding to (5) is $P_R(q^*) - p_F^* = a_I\sigma^2 z$, so that the quantity demanded (by a continuum of investors of mass one) for the risky asset is

$$z^*(p_F^*) = \frac{P_R(q^*) - p_F^*}{a_I\sigma^2}, \tag{19}$$

as in (7). The second-order condition is satisfied. Given demand $z^*(p_F) = \frac{P_R(q^*) - p_F}{a_I\sigma^2}$ or inverse demand

$$P_F^*((1-\omega)q) = P_R(q^*) - a_I\sigma^2(1-\omega)q, \tag{20}$$

when $(1-\omega)q$ shares of the risky asset are sold, the first-order conditions corresponding to (6) are

$$q : \omega\left(P_R'(q)q + P_R(q)\right) + (1-\omega)\left(P_F^*((1-\omega)q) + (1-\omega)P_F^{*\prime}((1-\omega)q)q\right) = a_E\sigma^2\omega^2 q, \tag{21}$$

$$\omega : P_R(q)q - P_F^*((1-\omega)q)q - (1-\omega)P_F^{*\prime}((1-\omega)q)q^2 = a_E\sigma^2\omega^2 q. \tag{22}$$

Using (20),[33] (21) simplifies to (9) evaluated at $q = q^*$ and $\omega = \omega^*$ and solving (22) yields (10). The second-order condition is satisfied. Given q^* and ω^*, the financial price is $p_F^* = P_F^*((1-\omega^*)q^*) = P_R(q^*) - a_I\sigma^2(1-\omega^*)q^*$, as in (11).

B Extension

In this appendix, we extend the model to several firms and many shareholders. We first show that our results are robust. In particular, the interaction of shareholders leads to a loss of market power in the real sector and the financial market integrates the shareholders' preferences into the decisions of the firms. We also show that risk and uncertainty are still relevant with a larger number of agents, even in the limit. We first describe the model and

[33] Here, $P_F^{*\prime}((1-\omega)q) = -a_I\sigma^2$.

define the equilibrium. We then characterize and discuss the equilibrium.

B.1 Model and Equilibrium

The Firms. Consider an economy with $N_E \geq 1$ firms. Firm j is a monopolist in the real market and has access to a competitive financial market. In other words, each firm is a local monopoly in the real sector facing global competition in the financial sector.[34]

In the real market, firm j chooses the level of output $q_j \geq 0$. At the time of choosing production, demand is uncertain. Specifically, the random price (net of cost) corresponding to supplying q_j units in market j is $\tilde{p}_{R,j} = P_R(q_j) + \tilde{\varepsilon}_j$, where $P_R(q_j)$ is the expected inverse demand and $\tilde{\varepsilon}_j$ is a demand shock. The profit of firm j is $\pi(q_j, \tilde{\varepsilon}_j) = (P_R(q_j) + \tilde{\varepsilon}_j)q_j$.[35] The demand shock is assumed to have both systematic and nonsystematic components. Formally,

Assumption B.1. *For all $j, k = 1, \ldots, N_E$, $j \neq k$, $\tilde{\varepsilon}_j = \tilde{\lambda} + \tilde{\eta}_j$, where $\tilde{\lambda} \sim N(0, \sigma_\lambda^2)$ and $\tilde{\eta}_j \sim N(0, \sigma_\eta^2)$ such that $\mathbb{E}\tilde{\lambda}\tilde{\eta}_j = 0$ and $\mathbb{E}\tilde{\eta}_j\tilde{\eta}_k = 0$.*

In the financial sector, firm j issues equity shares. Each share is a claim on the profit corresponding to the sale of one unit of output by firm j. Hence, q_j shares of a risky asset are issued by firm j, and the random payoff of each share is $\tilde{p}_{R,j}$. Finally, firm j chooses the fraction $1 - \omega_j \in [0, 1]$ of the shares sold in the financial market. The variable ω_j defines the ownership structure of firm j, which specifies the allocation of the random profit among the shareholders. Let $p_{F,j}$ be the price of a share of the risky asset issued by firm j.

The Shareholders. The group of shareholders is composed of $N_E \geq 1$ entrepreneurs and $N_I \geq 1$ investors. Entrepreneur j is the founder of firm j

[34] The hybrid market structure conveys the idea that any publicly-traded firm has, in general, less ability to manipulate prices of financial instruments than prices of real goods. Indeed, while a firm can be a monopolist in the real sector due to barriers to entry, the financial market is by nature more competitive. For instance, consider two firms, each selling a different product with little substitution or complementarity with the other product. While firms face no competition in the real sector, their respective equity are in fact similar, i.e., each is a claim to profit. Hence, even if they are complementary due to portfolio diversification, there is more competition in the financial market relative to the real market.

[35] Assumption 2.1 holds.

and the original claimant of the real profit generated by his entrepreneurial prospects. Entrepreneur j is also the managing shareholder of firm j, making the output decision, retaining part of the risky asset, and selling the remaining shares to investors. Investors, on the other hand, do not have entrepreneurial prospects and have no direct control over the decisions of the firms. However, they do have initial wealth, which they use to purchase shares of the risky assets and the risk-free asset.

The final wealth of each of the N_I investors is

$$\tilde{W}_I' = W_I + \sum\nolimits_{j=1}^{N_E} \left(\tilde{p}_{R_j} - p_{F_j}\right) z_j, \tag{23}$$

where W_I is initial wealth and z_j is the number of shares for risky asset j with random per-share return $\tilde{p}_{R_j} - p_{F_j}$.[36] Given CARA preferences, the certainty equivalent of an investor is

$$\begin{aligned} CE_I =& W_I + \sum\nolimits_{j=1}^{N_E} \left(P_R(q_j) - p_{F_j}\right) z_j - a_I \left(\sigma_\lambda^2 + \sigma_\eta^2\right) \sum\nolimits_{j=1}^{N_E} z_j^2/2 \\ & - a_I \sigma_\lambda^2 \sum\nolimits_{j=1}^{N_E} \sum\nolimits_{k \neq j} z_j z_k/2. \end{aligned} \tag{24}$$

The final wealth of entrepreneur j is

$$\tilde{W}_j' = \omega_j \tilde{p}_{R_j} q_j + p_{F_j}(1-\omega_j) q_j + \sum\nolimits_{k \neq j} \left(\tilde{p}_{R_k} - p_{F_k}\right) y_{jk}, \tag{25}$$

where ω_j is entrepreneur j's level of ownership for firm j. The term $\omega_j \tilde{p}_{R_j} q_j$ is the portion of real profit from firm j to which entrepreneur j is entitled. The term $p_{F_j}(1-\omega_j)q_j$ is the wealth generated from selling claims to real profit of the remaining $(1-\omega_j)q_j$ units of output at price p_{F_j}, and diversifying among the remaining risky assets (issued by firms $k \neq j$) and the risk-free asset. Specifically, entrepreneur j buys y_{jk} shares of the risky asset issued by firm k at unit price p_{F_k} with random payoff $\tilde{p}_{R_k} y_{jk}$. Finally, the remaining $p_{F_j}(1-\omega_j)q_j - \sum_{k \neq j} p_{F_k} y_{jk}$ is invested in the risk-free asset with a rate of return normalized to one. Given CARA preferences, the certainty equivalent

[36] Since investors are identical, there is no index for a particular investor.

of investor j is

$$
\begin{aligned}
CE_j =& \omega_j P_R(q_j)q_j + p_{F_j}(1-\omega_j)q_j + \sum\nolimits_{k\neq j}\left(P_R(q_k) - p_{F_k}\right)y_{jk} \\
&- a_E\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\omega_j^2 q_j^2/2 - a_E\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\sum\nolimits_{k\neq j} y_{jk}^2/2 \\
&- a_E\sigma_\lambda^2 \sum\nolimits_{k\neq j}\sum\nolimits_{\substack{\kappa\neq k\\ \kappa\neq j}} y_{jk}y_{j\kappa}/2 - a_E\sigma_\lambda^2\omega_j q_j \sum\nolimits_{k\neq j} y_{jk}.
\end{aligned}
\tag{26}
$$

We now define the equilibrium, which is analogous to Definition 3.1. To simplify the analysis, the financial sector is assumed to be perfectly competitive, i.e., both entrepreneurs and investors take the financial prices as given. The equilibrium consists of, for all j, firm j's decisions $\left\{q_j^*, \omega_j^*\right\}$, entrepreneur j's demands for risky assets $k\neq j$ $\left\{y_{jk}^*(p_{F_k}, \mathbf{p}_{F_{-k}})\right\}_{k\neq j}$, investors' demand for risky asset j, $z_j^*(p_{F_j}, \mathbf{p}_{F_{-j}})$, and the financial price $p_{F_j}^*$. Here, $\mathbf{p}_{F_{-j}} = \{p_{F_1}, ..., p_{F_{j-1}}, p_{F_{j+1}}, ..., p_{F_{N_E}}\}$.

Definition B.2. *The tuple* $\left\{q_j^*, \omega_j^*, \left\{y_{jk}^*(p_{F_k}, \mathbf{p}_{F_{-k}})\right\}_{k\neq j}, z_j^*(p_{F_j}, \mathbf{p}_{F_{-j}}), p_{F_j}^*\right\}_{j=1}^{N_E}$ *is an equilibrium if*

1. *Given* $\left\{q_k^*, p_{F_k}^*\right\}_{k=1}^{N_E}$,

$$
\begin{aligned}
\left\{z_j^*(p_{F_j}^*, \mathbf{p}_{F_{-j}}^*)\right\}_{j=1}^{N_E} =& \arg\max_{\{z_j\}_{j=1}^{N_E}} \Big\{W_I + \sum\nolimits_{j=1}^{N_E}\left(P_R(q_j^*) - p_{F_j}^*\right)z_j \\
&-a_I\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\sum\nolimits_{j=1}^{N_E} z_j^2/2 - a_I\sigma_\lambda^2\sum\nolimits_{j=1}^{N_E}\sum\nolimits_{k\neq j} z_j z_k/2.\Big\}
\end{aligned}
\tag{27}
$$

2. *Given* $\{q_k^*\}_{k\neq j}$ *and* $\left\{p_{F_j}^*\right\}_{j=1}^{N_E}$, *for all* j,

$$
\begin{aligned}
\left\{q_j^*, \omega_j^*, \left\{y_{jk}^*\right\}_{k\neq j}\right\} =& \arg\max_{q_j, \omega_j, \left\{y_{jk}\right\}_{k\neq j}} \Big\{\omega_j P_R(q_j)q_j + p_{F_j}^*(1-\omega_j)q_j + \sum\nolimits_{k\neq j}\left(P_R(q_k^*) - p_{F_k}^*\right)y_{jk} \\
&- a_E\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\omega_j^2 q_j^2/2 - a_E\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\sum\nolimits_{k\neq j} y_{jk}^2/2 \\
&-a_E\sigma_\lambda^2\sum\nolimits_{k\neq j}\sum\nolimits_{\substack{\kappa\neq k\\ \kappa\neq j}} y_{jk}y_{j\kappa}/2 - a_E\sigma_\lambda^2\omega_j q_j\sum\nolimits_{k\neq j} y_{jk}\Big\}.
\end{aligned}
\tag{28}
$$

3. *Given* $\left\{q_j^*, \omega_j^*, \left\{y_{jk}^*(p_{F_k}, \mathbf{p}_{F_{-k}})\right\}_{k \neq j}, z_j^*(p_{F_j}, \mathbf{p}_{F_{-j}})\right\}_{j=1}^{N_E}$, $p_{F_j}^*$ *clears the financial market for risky asset* j, $j = 1, 2, ..., J$.

Note that the financial sector is perfectly competitive, i.e., the financial prices are given, and, thus, the entrepreneurs cannot take into account the effect of their decisions on the financial prices via the quantities supplied. The assumption that the financial market is oligopolistic has no effect on our results, especially on the loss of market power in the real sector. Indeed, regardless of the market structure in the financial sector (monopoly, oligopoly, or perfect competition), expected payoffs must be conjectured by the investors, thereby preventing the entrepreneurs from manipulating the financial prices via the expected payoffs. Hence, the loss of control over the expected payoff, and, thus, over the real prices implicit in the financial prices is due solely to the nature of a Nash game.

B.2 Characterization and Discussion

Proposition B.3 states that there exists a unique equilibrium. Propositions B.4 and B.5 characterize the symmetric equilibrium.[37] The proof is relegated to the next section.

Proposition B.3. *There exists a unique equilibrium.*

Proposition B.4. *In equilibrium, output* q^* *satisfies*

$$\begin{aligned}
&\omega^* \left(P_R'(q^*)q^* + P_R(q^*)\right) + (1-\omega^*)P_R(q^*) \\
&= a_E\left(\sigma_\lambda^2 + \sigma_\eta^2\right)\omega^{*2}q^* + \frac{a_I a_E \sigma_\lambda^2\left(\sigma_\lambda^2 + N_E\sigma_\eta^2\right)}{a_E\sigma_\lambda^2 N_I + a_I(N_E-1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E\geq 2]}\right)}(1-\omega^*)^2 q^* \\
&+ \frac{a_I a_E (N_E-1)\sigma_\lambda^4}{a_E\sigma_\lambda^2 N_I + a_I(N_E-1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E\geq 2]}\right)}(1-\omega^*)\,\omega^* q^* \\
&+ a_E\sigma_\lambda^2\omega^*(N_E-1)\frac{\left(a_I\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E\geq 2]}\right)(1-\omega^*) - \frac{a_E\sigma_\lambda^2 N_I \sigma_\lambda^2\omega^*}{\sigma_\lambda^2 + N_E\sigma_\eta^2}\right)q^*}{a_E\sigma_\lambda^2 N_I + a_I(N_E-1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E\geq 2]}\right)} \qquad (29)
\end{aligned}$$

[37] Note that $N_E = N_I = 1$ cannot yield the same exact result obtained previously as we assume a perfectly competitive financial sector in the appendix.

and

$$\omega^* = \frac{\left((N_E-1)\left(\sigma_\lambda^2+\sigma_\eta^2\right)+\left(\sigma_\lambda^2+N_E\sigma_\eta^2\right)\right)a_I\sigma_\lambda^2}{\left(\left((N_E-2)\sigma_\lambda^2-N_E\sigma_\eta^2\right)\sigma_\lambda^2-(N_E-1)\left(\sigma_\lambda^2+\sigma_\eta^2\right)\sigma_\eta^2\right)a_I+\left(\frac{(N_E-1)\sigma_\lambda^4}{\sigma_\lambda^2+N_E\sigma_\eta^2}-\left(\sigma_\lambda^2+\sigma_\eta^2\right)\right)\sigma_\lambda^2 N_I a_E}, \tag{30}$$

Proposition B.5. *In equilibrium, the financial price is*

$$\begin{aligned} p_F^* =& P_R(q^*) \; - \frac{a_I a_E \sigma_\lambda^2\left(\sigma_\lambda^2+N_E\sigma_\eta^2\right)}{a_E\sigma_\lambda^2 N_I + a_I(N_E-1)\left(\sigma_\lambda^2+\sigma_\eta^2 1_{[N_E\geq 2]}\right)}(1-\omega^*)q^* \\ & - \frac{a_I a_E (N_E-1)\sigma_\lambda^4}{a_E\sigma_\lambda^2 N_I + a_I(N_E-1)\left(\sigma_\lambda^2+\sigma_\eta^2 1_{[N_E\geq 2]}\right)}\omega^* q^*. \end{aligned} \tag{31}$$

In view of Proposition B.4, expression (29) is analogous to 9 in the simpler model. Indeed, the left-hand side of (29) shows that there is a loss of market power. Moreover, the first two terms on the right-hand side of (29) shows that the financial markets aggregate preferences of the investors. The additional last two terms on the right-hand side of (29) represent the aggregation of preferences of the other entrepreneurs, only present when there is more than one entrepreneur. Indeed, the financial market integrates preferences across sectors. In other words, entrepreneur j affects the behavior of entrepreneur $k \neq j$ in the real sector.

The final point we wish to make concerns the relevance of risk with a large number of shareholders. It is often claimed that a large number of agents in the financial market removes any concern for risk, i.e., the firms become risk-neutral. The risk-neutrality remark is a restatement of the Arrow-Lind Theorem (Arrow and Lind, 1970): if the number of agents sharing risk is large, behavior toward risk is almost risk-neutral. One must be careful, as shown in (31), when taking limits. While

$$\lim_{N_I\to\infty} p_F^* = P_R\left(\lim_{N_I\to\infty} q^*\right) \tag{32}$$

implies risk-neutrality behavior, the limiting case occurs only if the fraction of entrepreneurs to investors (or the fraction of entrepreneurship to investment activities) goes to zero.[38] Moreover, the limiting case of

$$\lim_{N_E \to \infty} p_F^* = P_R\left(\lim_{N_E \to \infty} q^*\right) - \frac{a_E \sigma_\lambda^4}{\sigma_\lambda^2 + \sigma_\eta^2} \lim_{N_E \to \infty} q^* \tag{33}$$

shows that risk remains relevant even though investors become insignificant.[39] Finally, assuming that neither group of shareholders disappears in the limit, i.e., $\lim_{N_E, N_I \to \infty} N_E/N_I = c > 0$ exists, then (31) does not go to $P_R\left(\lim_{N_E, N_I \to \infty : N_E/N_I \to c} q^*\right)$, implying that risk remains relevant. Moreover, all results hold in this limiting case.

B.3 Proof

The first-order conditions corresponding to (27) are

$$z_j : P_R(q_j^*) - p_{F_j}^* = a_I\left(\sigma_\lambda^2 + \sigma_\eta^2\right) z_j + a_I \sigma_\lambda^2 \sum\nolimits_{k \neq j}^{N_E} z_k, \tag{34}$$

[38]Using (29) and (30), it can be shown that $\lim_{N_I \to \infty} q^*$ exists and $\lim_{N_I \to \infty} \omega^* = 0$.
[39]Using (29) and (30), it can be shown that $\lim_{N_E \to \infty} q^*$ exists and $\lim_{N_E \to \infty} \omega^* = 1$.

$j = 1, \ldots, N_E$, so that[40]

$$z_j^* \left(p_{F_j}^*, \mathbf{p}_{F_{-j}}^*\right) = \frac{\left(\sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2\right)\left(P_R\left(q_j^*\right) - p_{F_j}^*\right) - \sigma_\eta^2 \sum_{k \neq j}\left(P_R\left(q_k^*\right) - p_{F_k}^*\right)}{a_I \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)}, \tag{37}$$

$j = 1, \ldots, N_E$. Therefore, the demand of each investor for risky asset j is

$$z_j^* \left(p_{F_j}, \mathbf{p}_{F_{-j}}\right) = \frac{\left(\sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2\right)\left(P_R\left(q_j^*\right) - p_{F_j}\right) - \sigma_\eta^2 \sum_{k \neq j}\left(P_R\left(q_k^*\right) - p_{F_k}\right)}{a_I \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)}, \tag{38}$$

$j = 1, \ldots, N_E$.

The first-order conditions corresponding to (28) are

$$q_j : \omega_j \left(P_R'(q_j) q_j + P_R(q_j)\right) + p_{F_j}^*(1 - \omega_j) = a_E \left(\sigma_\lambda^2 + \sigma_\eta^2\right) \omega_j^2 q_j + a_E \sigma_\lambda^2 \omega_j \sum_{k \neq j} y_{jk}, \tag{39}$$

$$\omega_j : P_R(q_j) q_j - p_{F_j}^* q_j = a_E \left(\sigma_\lambda^2 + \sigma_\eta^2\right) \omega_j q_j^2 + a_E \sigma_\lambda^2 q_j \sum_{k \neq j} y_{jk}, \tag{40}$$

as well as

$$y_{jk} : P_R\left(q_k^*\right) - p_{F_k}^* = a_E \left(\sigma_\lambda^2 + \sigma_\eta^2\right) y_{jk} + a_E \sigma_\lambda^2 \sum_{\substack{\kappa \neq k \\ \kappa \neq j}} y_{j\kappa} + a_E \sigma_\lambda^2 \omega_j q_j \tag{41}$$

[40] In matrix form,

$$\begin{bmatrix} P_R(q_1^*) - p_{F_1}^* \\ P_R(q_2^*) - p_{F_2}^* \\ \vdots \\ P_R(q_{N_E}^*) - p_{F_{N_E}}^* \end{bmatrix} = a_I \begin{bmatrix} \sigma_\lambda^2 + \sigma_\eta^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\lambda^2 + \sigma_\eta^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_\eta^2 \\ \sigma_\eta^2 & \cdots & \sigma_\eta^2 & \sigma_\lambda^2 + \sigma_\eta^2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{N_E} \end{bmatrix}, \tag{35}$$

so that

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{N_E} \end{bmatrix} = \frac{\begin{bmatrix} \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 \end{bmatrix} \begin{bmatrix} P_R(q_1^*) - p_{F_1}^* \\ P_R(q_2^*) - p_{F_2}^* \\ \vdots \\ P_R(q_{N_E}^*) - p_{F_{N_E}}^* \end{bmatrix}}{a_I \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)}. \tag{36}$$

for $k = 1, \ldots, N_E$, $k \neq j$. Using (41), entrepreneur j's quantity demanded for shares of risky asset $k \neq j$ is[41]

$$y_{jk}^*\left(p_{F_j}^*, \mathbf{p}_{F_{-j}}^*\right) = \frac{\left(\sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2\right)\left(P_R\left(q_k^*\right) - p_{F_k}^*\right) - \sigma_\eta^2 \sum_{\substack{\kappa \neq k \\ \kappa \neq j}} \left(P_R\left(q_\kappa^*\right) - p_{F_\kappa}^*\right)}{a_E \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} - \frac{\sigma_\lambda^2 \omega_j q_j}{\sigma_\lambda^2 + N_E \sigma_\eta^2}. \tag{44}$$

[41]Without loss of generality, consider entrepreneur $j = 1$'s quantity demanded of risky asset $k \neq 1$. In matrix form,

$$\begin{bmatrix} P_R\left(q_2^*\right) - p_{F_2}^* \\ P_R\left(q_3^*\right) - p_{F_3}^* \\ \vdots \\ P_R\left(q_{N_E}^*\right) - p_{F_{N_E}}^* \end{bmatrix} = a_E \begin{bmatrix} \sigma_\lambda^2 + \sigma_\eta^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\lambda^2 + \sigma_\eta^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_\eta^2 \\ \sigma_\eta^2 & \cdots & \sigma_\eta^2 & \sigma_\lambda^2 + \sigma_\eta^2 \end{bmatrix} \begin{bmatrix} y_{12} \\ y_{13} \\ \vdots \\ y_{1N_E} \end{bmatrix} + a_E \sigma_\lambda^2 \begin{bmatrix} \omega_1 q_1 \\ \omega_1 q_1 \\ \vdots \\ \omega_1 q_1 \end{bmatrix} \tag{42}$$

so that

$$\begin{bmatrix} y_{12} \\ y_{13} \\ \vdots \\ y_{1N_E} \end{bmatrix} = \frac{\begin{bmatrix} \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 \end{bmatrix} \begin{bmatrix} P_R\left(q_2^*\right) - p_{F_2}^* \\ P_R\left(q_3^*\right) - p_{F_3}^* \\ \vdots \\ P_R\left(q_{N_E}^*\right) - p_{F_{N_E}}^* \end{bmatrix}}{a_E \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} - \frac{\begin{bmatrix} \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sigma_\eta^2 \\ -\sigma_\eta^2 & \cdots & -\sigma_\eta^2 & \sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2 \end{bmatrix} \begin{bmatrix} \omega_1 q_1 \\ \omega_1 q_1 \\ \vdots \\ \omega_1 q_1 \end{bmatrix}}{\sigma_\lambda^2 + N_E \sigma_\eta^2}. \tag{43}$$

Hence, entrepreneur j's demand for risky asset $k \neq j$ is

$$\begin{aligned} y_{jk}^{*}\left(p_{F_j}, \mathbf{p}_{F_{-j}}\right) =& \frac{\left(\sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2\right)\left(P_R\left(q_k^*\right) - p_{F_k}\right) - \sigma_\eta^2 \sum_{\substack{\kappa \neq k \\ \kappa \neq j}} \left(P_R\left(q_\kappa^*\right) - p_{F_\kappa}\right)}{a_E \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} \\ & - \frac{\sigma_\lambda^2 \omega_j q_j}{\sigma_\lambda^2 + N_E \sigma_\eta^2}. \end{aligned} \tag{45}$$

In order to fully characterize equilibrium output and ownership structure defined by (39) and (40), we now characterize the equilibrium financial prices $\left\{p_{F_j}^*\right\}_{j=1}^{N_E}$. Financial prices are determined by market-clearing conditions, i.e.,

$$\underbrace{N_I z_j^*\left(p_{F_j}^*, \mathbf{p}_{F_{-j}}^*\right)}_{\text{Investors' Demand}} + \sum\nolimits_{j \neq k} \underbrace{y_{kj}^*\left(p_{F_j}^*, \mathbf{p}_{F_{-j}}^*\right)}_{\text{Demand of entrepreneur } k \text{ for risky asset } j} = \underbrace{\left(1 - \omega_j^*\right) q_j^*}_{\text{Supply by entrepreneur } j} \tag{46}$$

for $j = 1, ..., N_E$. Using (38) and (45), and considering a symmetric equilibrium, (46) is rewritten as

$$\begin{aligned} & N_I \frac{\left(\sigma_\lambda^2 + \left(N_E - 1\right)\sigma_\eta^2\right)\left(P_R\left(q^*\right) - p_F^*\right) - \sigma_\eta^2\left(N_E - 1\right)\left(P_R\left(q^*\right) - p_F^*\right)}{a_I \sigma_\lambda^2\left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} \\ & + \frac{\left(N_E - 1\right)\left(\sigma_\lambda^2 + (N_E - 1)\sigma_\eta^2\right)\left(P_R(q^*) - p_F^*\right) - \sigma_\eta^2\left(N_E - 1\right)\min\left\{N_E - 2, 0\right\}\left(P_R\left(q^*\right) - p_F^*\right)}{a_E \sigma_\lambda^2\left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} \\ & - \frac{\left(N_E - 1\right)\sigma_\lambda^2 \omega^* q^*}{\sigma_\lambda^2 + N_E \sigma_\eta^2} = \left(1 - \omega^*\right) q^*. \end{aligned} \tag{47}$$

Solving the above expression for p_F^* yields

$$\begin{aligned} p_F^* =& P_R\left(q^*\right) - \frac{a_I a_E \sigma_\lambda^2\left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)}{a_E \sigma_\lambda^2 N_I + a_I(N_E - 1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)}\left(1 - \omega^*\right) q^* \\ & - \frac{a_I a_E (N_E - 1)\sigma_\lambda^4}{a_E \sigma_\lambda^2 N_I + a_I(N_E - 1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)}\omega^* q^*, \end{aligned} \tag{48}$$

as in (31), and where $1_{[N_E \geq 2]}$ is an indicator function equal to one when $N_E \geq 2$ and zero when $N_E = 1$.

Having characterized the equilibrium financial prices, we now turn to the behavior of the firms. To that end, plugging (31) into (45) yields

$$y^*(p_F^*) = \frac{\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)\left(P_R(q^*) - p_F^*\right)}{a_E \sigma_\lambda^2 \left(\sigma_\lambda^2 + N_E \sigma_\eta^2\right)} - \frac{\sigma_\lambda^2 \omega^* q^*}{\sigma_\lambda^2 + N_E \sigma_\eta^2}, \tag{49}$$

$$= \frac{\left(a_I \left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)(1 - \omega^*) - \frac{a_E \sigma_\lambda^2 N_I \sigma_\lambda^2 \omega^*}{\sigma_\lambda^2 + N_E \sigma_\eta^2}\right) q^*}{a_E \sigma_\lambda^2 N_I + a_I (N_E - 1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)}, \tag{50}$$

in a symmetric equilibrium.

Finally, plugging (31) and (50) into (39) yields (29) for the equilibrium level of output. Plugging (31) and (50) into (40) yields

$$P_R(q^*) q^* - p_F^* q^* = a_E \left(\sigma_\lambda^2 + \sigma_\eta^2\right) \omega^* q^{*2} + a_E \sigma_\lambda^2 q^* (N_E - 1) y^*(p_F^*),$$

$$= a_E \left(\sigma_\lambda^2 + \sigma_\eta^2\right) \omega^* q^{*2} \tag{51}$$

$$+ a_E \sigma_\lambda^2 (N_E - 1) \frac{\left(a_I \left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)(1 - \omega^*) - \frac{a_E \sigma_\lambda^2 N_I \sigma_\lambda^2 \omega^*}{\sigma_\lambda^2 + N_E \sigma_\eta^2}\right) q^*}{a_E \sigma_\lambda^2 N_I + a_I (N_E - 1)\left(\sigma_\lambda^2 + \sigma_\eta^2 1_{[N_E \geq 2]}\right)} q^*, \tag{52}$$

which simplifies to (30) for the equilibrium entrepreneur's level of ownership.

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