Interactions between business cycles, stock market cycles and interest rates: the stylised facts

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In this paper, we study the co-movements between stock market indices and real economic activity over the business cycle in France, Germany, Italy, the United Kingdom and the United States. Working on the premise that there is neither a single definition of the business cycle, nor a single method for studying it, we use two complementary approaches in our analysis.

First, we identify the turning points in real economic indicators and stock market indices and determine the extent to which these series co-move, i.e. *are regularly and significantly in the same phase of the cycle. Second, we decompose the series studied into a cyclical part and a structural part in order to calculate the correlations between the cyclical components of real economic indicators and excess returns, on the one hand, and the correlations between the structural components of these indicators, on the other. We then analyse the co-movements between three-month interest rates and the cyclical and structural components of real economic and stock market indices.*

Two main conclusions can be drawn from these different analyses: (i) there does not appear to be a strong dependence between stock prices and the level of real activity in the short term, except in the United States; (ii) in the longer term, real activity and stock prices seem to share the same determinants. However, it seems difficult to clearly identify an impact of asset price movements on the conduct of monetary policy, represented here by three-month money market interest rates. In general, we do not detect a significant relationship between the cyclical components of excess returns and those of money market rates; nor do we find a significant link between the structural components of these variables.

The spectacular rise in asset prices up to 2000 in most developed countries has attracted much attention and has re-opened the debate over whether these prices should be taken into account in monetary policy strategies. Some observers see asset price developments, in particular those of stock prices, as being uncorrelated with those of economic fundamentals, *i.e.* a speculative bubble. This interpretation carries with it a range of serious consequences arising from the bursting of this bubble: scarcity of financing opportunities, a general decline in investment, a fall in output, and finally a protracted contraction in real activity. Other observers believe that stock prices are likely to impact on goods and services prices and thus affect economic activity and inflation.

These theories are currently at the centre of the debate on whether asset prices should be taken into account in the conduct of monetary policy, *i.e.* as a target, or as an instrument [1]. However, the empirical link between asset prices and economic activity on the one hand, and the relationship between economic activity and interest rates or between stock prices and interest rates, on the other, are not established facts. This study therefore sets out to identify a number of stylised facts that characterise this link, using a statistical analysis of these data (economic activity indicators, stock prices and interest rates).

More specifically, we study the co-movements between stock market indices, real activity and interest rates over the business cycle. Assuming that there is not a single definition of the business cycle, we adopt an agnostic approach in our methodology.

The traditional approach characterises the cycle as a series of phases of expansion and contraction. Formally, expansion phases are defined as the periods of time separating a trough from a peak; conversely, contraction phases correspond to periods separating a peak from a trough. In this respect, it is vital to define and accurately identify peaks and troughs.

Although this view of the cycle fell out of fashion after the 1970s, it has recently come back into focus thanks to a number of studies, in particular by Harding and Pagan (2002a,b) [2] who proposed a simple method for analysing the concordance between macroeconomic variables. By definition, the concordance index represents the average number (standardised) of periods in which two variables (*e.g.* GDP and a stock market index) coincide at the same phase of the cycle.

The traditional approach defines the business cycle directly by analysing changes in the level of a variable, *e.g.* GDP. The modern approach, as we mentioned above, enables us to split a variable into two components, one cyclical or short-term, and the other permanent or structural, using appropriate statistical techniques (filtering). As its name suggests, the cyclical component can be associated with the business cycle. Note that it is not possible to detect a trend in the latter. Consequently, we can calculate the correlations between the cyclical components of two variables in order to study their co-movement (*i.e.* the similarity of their profile). However, we show that the structural component of a trending variable is also driven by a trend. Therefore, so as not to obtain spurious relationships, we study the growth rate of the structural components. We can also calculate the correlations between the growth rate of the structural components of two trending variables in order to study their co-movement.

As the notions of concordance and correlation do not have an identical scope, it is useful to use both of these tools when attempting to characterise the business cycle stylised facts.

The first part of this study is devoted to the empirical analysis of the concordance indicator; the second part firstly describes changes in the variables studied (real activity, stock prices and interest rates) by separating the cyclical (or short-term) components from the structural (or long-term) components, and then compares the variables using the dynamic correlations of their corresponding components (*i.e.* cyclical, cyclical and structural, and structural).

In both parts, we compare the results obtained on the business and stock market cycles to the monetary policies applied over the period studied: first, we analyse the behaviour of short-term interest rates over the phases of expansion and contraction of real activity and stock prices; second, we calculate the correlations between the cyclical components of real activity, stock prices and interest rates on the one hand, and the correlations between the structural components of these variables, on the other.

1 *Much theoretical literature has recently been published on this subject. See Bernanke and Gertler (2001), Bullard and Schalling (2002), Filardo (2000) and the references cited in these papers.*

2 *For a recent application on euro data, see Artis* et al. *(2003).*

1| Concordance between business cycles and stock market cycles: an empirical analysis

As a concordance indicator, we use a descriptive statistic recently developed by Harding and Pagan (2002a,b) and utilised at the IMF by Cashin *et al.* (1999) and McDermott and Scott (2000). Cashin *et al.* applied this method to the analysis of the concordance of goods prices while McDermott and Scott used it to study the concordance of business cycles in major OECD countries.

The underlying method is based on studies by the National Bureau of Economic Research (NBER) and consists in dating the turning points in cycles. On the basis of these points, we can associate a contraction period with the lapse of time that separates a high point (peak) from a low point (trough). We can then define the contraction and expansion phases for one or more variables and thus define the concordance statistic that indicates the average number (standardised) of periods in which two variables (*e.g.* GDP and a stock market index) coincide at the same phase of the cycle [3]. There is a perfect concordance between the series (perfect juxtaposition of expansions and contractions) if the index is equal to 1 and perfect disconcordance (a marked lead or lag or out of phase) if the index is equal to 0.

1|1 Presentation of data

We set out to study the relationship between business cycles and stock market cycles in France, Germany, Italy, the United Kingdom and the United States [4]. Stock prices are obtained from composite indices calculated by Morgan Stanley (MSCI), deflated by the consumer price index. These variables are available at a quarterly and a monthly frequency. We use three variables to define the business cycle: at a quarterly frequency, market GDP and household consumption (these variables are taken from the OECD database over the study period from Q2 1978 to Q3 2002); and at a monthly frequency, retail sales (in volume terms, over the period January 1978-December 2002). As this series is only available as of 1990 for Italy, we therefore do not take this country into account in our analysis of monthly data. Moreover, the monthly sales index displays a highly erratic pattern that could conceal some turning points. In order to avoid this, we prefilter these data [5] in order to strip out the most erratic parts of these series and focus the analysis on an adjusted version of these variables.

Box 1

Empirical data

The data used in this study are explained below.

- *Financial data: Morgan Stanley Capital International (MSCI) indices obtained from Datastream. In order to calculate excess returns, we use the nominal yield on government bonds (annualised) for France, the United Kingdom, and the United States, the interbank rate for Germany and the money market interest rate for Italy. For all of these countries, we use the three-month money market interest rates as indicators of monetary policy. These data are obtained from the IMF database.*

- *Real economic data: real market GDP and real private consumption are expressed in volume terms at 1995 prices. Real sales are obtained from the real retail sales index (1995 base). These data are obtained from the OECD database. We also use the consumer price index from the same database to deflate the stock market indices.*

[3] *See Appendix 2(A) for further details.*
[4] *For a presentation of the data, see Box 1.*
[5] *See Watson (1994).*

1|2 Results

The turning points in real GDP, real private consumption and MSCI at a quarterly frequency indices are shown in Charts 1, 2, and 3, respectively (see Appendix 1). Those of the retail sales index and MSCI indices at a monthly frequency are given in Charts 4 and 5, respectively.

At a quarterly frequency, results derived from the charts relating to real activity variables (Charts 1 and 2) are compatible overall and consistent with the analysis of McDermott and Scott (2000) and with that of Artis *et al* (2003). Naturally, we do not detect a perfect identity between the cycles described by real GDP and real private consumption. In France, for example, a short contraction can be observed in 1995 when we study real private consumption data, whereas the French economy was in a phase of expansion according to GDP data. When studying the turning points observed in stock markets, we note in particular that they are more frequent than in the real economy, irrespective of the country considered in our sample. The long phase of expansion in the 1990s is clearly visible in all countries. Some pronounced leads or lags are observed between the phases of the business and stock market cycles, in particular in Europe, especially at the start of the 2000s.

We note that the retail sales index is a more or less reliable indicator of private consumption and is more volatile than the latter. Nevertheless, these are the two indicators that must be compared. We therefore compare the turning points derived from the analysis of these two variables. Overall, in sales indices we observe the same marked contractions as those in consumption, as well as more occasional contractions, consistent with the high volatility of sales indices. We can carry out the same analysis on stock market indices at two frequencies: all pronounced contractions at a quarterly frequency can also be observed at a monthly frequency; here too, more contractions are detected at a monthly frequency.

These initial findings obtained from analysing the charts naturally call for a more in-depth study of the co-movements of real economy and stock market variables.

Table 1 (see Appendix 1) lists the intra-country index of concordance between MSCI indices and the three real activity indicators used.

The United States appears to be characterised by a significant concordance between the level of real activity and stock prices. Indeed, this is the case for the three real activity indicators used, which is not surprising in view of the role of stock markets in the investment and financing behaviour of US economic agents. The same is not true of the other countries in the sample. In particular, we do not observe this concordance of cycles in EU countries.

Business and stock market cycles do not occur at the same frequencies and furthermore may be uncorrelated, with the exception of the United States. Indeed, an analysis of Charts 1 (or 2) and 3 shows that the duration of a stock market expansion is generally shorter than that of GDP or consumption. This difference naturally contributes to reducing the degree of concordance between real activity and stock markets.

Nevertheless, the lack of significant concordance in most countries under review does not necessarily mean that business and stock market cycles are different or uncorrelated phenomena. The result obtained simply highlights the fact that the periods of expansion and contraction of GDP and stock prices for example do not coincide.

We observe that the start of US stock market contractions (*i.e.* the dates of peaks) precede contractions in real activity measured by real GDP [6]. The lead oscillates between one and four quarters. We also note that not all stock market contractions result in contractions in real activity. In particular, when they are very short like in 1987, they do not seem to spill over into activity. A similar phenomenon can be detected in European countries such as France and Italy. Like in the United States, but to a lesser degree, the start of GDP contractions are preceded by stock market contractions. Likewise, most stock market contractions in these two countries did not result in contractions in real activity.

This rule does not apply to Germany and the United Kingdom. Stock market contractions may precede or follow contractions in real activity by more than a year.

Therefore, contrary to conventional wisdom, it does not always appear relevant to use negative turning points in stock markets as leading indicators of the start of a contraction phase of GDP or consumption.

[6] *To date, statistics for testing the significativity of these leads do not exist.*

Turning now to the relationship between monetary policy and business and stock market cycles, we observe a relative decoupling between certain contraction periods of real activity or stock markets and money market rate developments, used here as indicators of monetary policy (Chart 6). No clear rule emerges from a comparison between stock markets and money markets: for the business cycle, a decline in rates more or less coincides with a contraction but, here too, it is difficult to establish a general rule. This chart suggests that the reaction of money market rates to turnarounds in real activity or stock markets was not systematic or coincident in the countries studied. This corresponds in theory to the mandate of monetary authorities as well as to the way in which monetary policy rules are modelled in recent macroeconomic studies [7].

Concordance indices have enabled us to measure the degree of "juxtaposition" between two chronological series, without having to consider whether there is a trend in the variables (non-stationarity). It should nevertheless be noted that only one aspect of the notion of cycles is taken into account here.

It could therefore be useful to broaden the study by retaining the concepts of phase and duration, but without limiting ourselves to such restrictive indicators as concordance indices. To do this, we decompose, in Part two, the different series studied in order to isolate the long-term (or structural) components and the short-term (or cyclical) components; the latter correspond to the business cycle concept put forward by the NBER.

2| Correlations of cyclical and structural components

On the basis of NBER studies, we identify business cycles with all movements whose recurrence period is between 6 and 32 quarters. This corresponds to business cycle frequencies. Furthering this approach, recent macroeconomic literature defines the movements of a variable (a_t) in terms of the recurrence period of its components. That corresponding to the business cycle is determined as the residual obtained after stripping out long movements, imputable to structural economic factors (τ_t) [8]. By construction, the residual variables ($a_t - \tau_t$) obtained by robust statistical techniques (filtering) are detrended (stationary). We can thus calculate the correlations between the corresponding components of the series in the hope of isolating a set of statistical regularities or stylised facts that characterise the business cycle.

The analysis of these components is based on the assumption that it is possible to isolate them from each other. To this end, we use a very robust technique recently put forward by Christiano and Fitzgerald (2003) (CF filter) [9]. For each country and each variable (a_t), we thus define the short-term (or cyclical, a_t^{ct}) components and the long-term (or structural, a_t^{lt}) components and calculate the correlations between the corresponding components.

2|1 Application of the method

The different real activity indicators are logarithms of real market GDP and real private consumption; for the financial sphere, we consider the excess returns on stocks relative to the risk-free interest rate [10]. Here, the analysis is limited to quarterly frequencies.

We propose two applications. For each country, we calculate the correlation between the cyclical (short-term) components of the variables studied and the correlation between the structural (long-term) components. In the latter case, we do not deal with real activity indicators and measures of returns in the same way. Indeed, these activity indicators are characterised by trends and therefore do not have the required statistical properties (they are not stationary) for calculating the correlations [11]. We show that their long-term components are non-stationary too. Consequently, we focus on the growth rate of the

7 *See, in particular, studies in the collective work produced by Taylor (1999).*
8 *This is the approach generally adopted following the seminal contribution of Kydland and Prescott (1982).*
9 *See appendix 2 (B) for further details.*
10 *Excess returns are defined as the difference between the nominal returns on stocks and on three-month government securities.*
11 *The notion of correlation is only defined for stationary variables. Where non-stationarity is present, the analysis of correlations yields spurious relationships.*

structural components that are, in general, stationary (in particular, they are not characterised by a trend). Conversely, the excess returns on stocks relative to the risk-free interest rate and their components are stationary. We can therefore study these variables in level form. For further details, see Box 2.

2|2 Results

From Tables 2 and 3 (see Appendix 1), we cannot conclude that there is a strong link between the cyclical components of GDP or consumption and those of excess returns in the different countries reviewed.

However, in the United States, France and Germany, the correlation between y^{ct}_{t+k} and x^{ct}_{t} is significantly positive for $k=2$ or 3 quarters. This means that a positive variation of the cyclical component of GDP at $t+2$ or at $t+3$ is associated with a positive variation of the cyclical component of excess returns at t. In other words, a positive variation of the cyclical component of GDP follows an increase in the cyclical component of excess returns with a lag of two or three quarters [12]. Even though the share of equities in household wealth differs on both sides of the Atlantic [13] the reactions of the three economies display a certain convergence. A similar link is observed for the cyclical component of consumption, although the lag in the correlation appears to be closer to three quarters.

However, the correlations between the growth rate of the structural component of GDP and the structural component of excess returns are significantly positive for all countries, at a fairly short horizon (Tables 4 and 5). The structural determinants of excess returns appear to covary positively with those of real activity. This result is broadly borne out when consumption is used as a real activity indicator, at least for short horizons [14].

If we compare the cyclical and structural components of the real activity indicator, stock prices and interest rates, we see that in most countries studied (Table 6), with the notable exception of France, the correlation between the cyclical component of GDP and that of the nominal interest rate is positive for negative k and negative for positive k. These results seem to point to a stabilising monetary policy: temporary rises in the

Box 2

Determining the components

In order to determine the cyclical components, we adopt the traditional definition of the cycle presented above. For all the variables studied, the business cycle is identified with all movements whose recurrence period is between 6 and 32 quarters. In order to isolate the structural components, we apply the CF filter so as to strip out the cyclical movements with a recurrence period of less than 32 quarters. We then calculate the difference between the initial series and the filtered series in order to obtain the structural component.

Let y_t *denote the log of real GDP at* t *and* x_t *the excess return at* t. *For each country* i *(*i = France, Germany, Italy, United Kingdom and United States*), we calculate the following correlations:*

– *the correlation between the cyclical component of GDP and excess returns,* $y_{t+k}^{ct}(i)$ *and* $x_t^{ct}(i)$*, for* $k = -3,...,3$*;*

– *the correlation between the growth rate of the structural component of GDP,* $\Delta y^{lt}_{t+k}(i)$ *and the structural component of excess returns,* $x_t^{lt}(i)$ *for* $k=-3,...,3$*;*

where Δ *is the first difference operator (*$\Delta a_t = a_t - a_{t-1}$*). We establish* k *as ranging from -3 to 3 as is the usual practice in studies of US data. For the purposes of symmetry, we adopt the same horizon for the other countries. As mentioned above, the exponent* ct *denotes the short-term component and the exponent* lt *denotes the long-term component.*

We estimate these correlations using a robust econometric method: the Generalised Method of Moments (GMM) completed with the HAC procedure developed by Andrews and Monahan (1992).

We use the same methods for real private consumption, replacing y_t *by* c_t*, the logarithm of consumption.*

[12] *This result must however be considered with caution as the sign of the correlation coefficient sometimes changes with* k *in some countries (see the line corresponding to the United States).*

[13] *See Odonnat and Rieu (2003).*

[14] *We can compare these conclusions with those of Daniel and Marshall (1998). These authors show that it is not possible to reject an augmented version of the Consumption-based Capital Asset Pricing Model (C-CAPM) when consumption and excess returns have been stripped of their short-term cyclical movements.*

level of real activity are followed by temporary increases in the money market interest rate, which precede a decline in the cyclical component of GDP. The difference in the French case may be due, *inter alia*, to the implementation of the "strong franc" policy at the start of the 1980s, which introduced a break.

We do not, however, detect a significant relationship between the cyclical component of excess returns and that of money market interest rates (Table 7), except in the United Kingdom: overall, short-term fluctuations in excess returns appear in some respects to be independent of those in money market interest rates. If we use these rates to represent monetary policy, this analysis does not rule out the possibility that monetary authorities may have reacted to some stock market events, but it indicates that, in general, stock price fluctuations do not play a determining role in the conduct of their policy.

Table 8 suggests that there is a negative relationship between the long-term component of the money market interest rate and that of real GDP in the United States, France, Germany (where we observe a lag). This relationship means that a lasting rise in the money market interest rate results in a fall in the growth rate of the long-term component of GDP. We could enhance the interpretation of this result by comparing the long-term components of real activity with those of real interest rates, calculated *ex-ante*, in keeping with economic theory. However, this exercise is not easy because no simple and reliable measure of this interest rate is available.

Lastly, we do not detect a significant link between the long-term component of the money market interest rate and that of the excess returns (Table 9), except in the United Kingdom and to a lesser extent in the United States. The long-term component of interest rates therefore does not appear to react to the structural component of excess returns, expect in the United Kingdom and the United States, no doubt owing to the weight of equities in household wealth that characterises these countries.

In order to understand the link between business cycles and stock market cycles and use it to improve the conduct of monetary policy, it is first necessary to identify the stylised facts underlying this relationship.

In practice, we set out to study the links between business and stock market cycles by using two complementary approaches that enable us to measure the co-movements between these phenomena.

Firstly, in the tradition of the NBER, we defined the business cycle as a succession of phases of expansion and contraction in order to compare the cycles based on two variables by calculating their concordance index. Above all, this exercise allowed us to identify significant concordance between the business and stock market cycles in the United States.

*Secondly, using the predominant methodology in applied macroeconomics, we analysed this link by decomposing the variables studied into short- and long-term components and by calculating the correlations between corresponding components (*i.e. *cyclical, cyclical and structural, and structural).*

We draw two conclusions from the various analyses carried out: (i) *there does not seem to be a strong dependence link between stock prices and the level of real activity at business cycle frequencies, except in the United States;* (ii) *in the longer term, it appears that real activity and stock prices share the same determinants. At any rate, we cannot clearly identify an impact of asset prices on three-month interest rates, used to represent monetary policy in the countries studied. In general, we do not detect a significant relationship between the cyclical components of excess returns and money market rates, nor do we observe a significant link between the structural components of these same variables*

These conclusions appear to be robust. However, it may be useful to further investigate the dichotomy between the short- and long-term using an approach based on a behavioural analysis of agents (or a microeconomic analysis of markets). In particular, we will attempt to identify the transmission mechanisms that enable us to detect links between business and stock market cycles.

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Appendix 1

Table 1
Concordance between business cycles and stock market cycles

	United States	France	Germany	United Kingdom	Italy
GDP	0.68687 (*)	0.61616	0.62626	0.58586	0.54545 (*)
Consumption	0.64646 (*)	0.60606	0.66667 (*)	0.59596	0.53535
Sales	0.73874 (*)	0.54655	0.56456	0.62462 (*)	–

Notes: A 5% significant coefficient is indicated by an asterisk. The degree of significativity of concordance indices is calculated using the method developed by Harding and Pagan (2002b). See Appendix 2(B) for further details.

Table 2
Short-term correlation between real GDP and excess returns on stocks (above the risk-free interest rate)

k	-3	-2	-1	0	1	2	3
United States	-0.0097	-0.1872	-0.2940	-0.2835	-0.1528 (*)	0.0493	0.2461 (*)
France	-0.0020	0.1015	0.2178	0.2884	0.2729 (*)	0.1789 (*)	0.0377
Germany	-0.1131	-0.1129	-0.0438	0.0656	0.1666 (*)	0.2357 (*)	0.2625 (*)
United Kingdom	0.1215	0.1276	0.0875	0.0070	-0.0675	-0.1023	-0.0938
Italy	0.1279	0.1631	0.1647	0.1381	0.0997	0.0769	0.0731

Notes: Correlation between y^{ct}_{t+k} *(i) and* x^{ct}_{t} *(i), where i is the country in the first column. A 5% significant coefficient is indicated by an asterisk.*

Table 3
Short-term correlation between real private consumption and excess returns on stocks (above the risk-free interest rate)

k	-3	-2	-1	0	1	2	3
United States	-0.1076	-0.1958	-0.2181	-0.1530	-0.0165	0.1352	0.2368 (*)
France	-0.2315	-0.0839	0.0949	0.2280	0.2929 (*)	0.2659 (*)	0.1707
Germany	-0.1902	-0.2442	-0.2528	-0.2024	-0.0995	0.0502	0.2125 (*)
United Kingdom	0.0208	-0.0262	-0.0816	-0.0975	-0.0609	0.0012	0.0248
Italy	-0.0323	0.0018	0.0369	0.0793	0.1251	0.1830 (*)	0.2362 (*)

Notes: Correlation between c^{ct}_{t+k} *(i) and* x^{ct}_{t} *(i). A 5% significant coefficient is indicated by an asterisk.*

Table 4
Long-term correlation between real GDP and excess returns on stocks (above the risk-free interest rate)

k	-3	-2	-1	0	1	2	3
United States	0.6243 (*)	0.6528 (*)	0.6665 (*)	0.6653 (*)	0.6415 (*)	0.6073 (*)	0.5641 (*)
France	0.1872 (*)	0.3062 (*)	0.4179 (*)	0.5197 (*)	0.5997 (*)	0.6650 (*)	0.7143 (*)
Germany	0.0622	0.1381	0.2128	0.2845	0.3265 (*)	0.3663 (*)	0.4029 (*)
United Kingdom	0.6161 (*)	0.6242 (*)	0.6175 (*)	0.5965 (*)	0.5586 (*)	0.5093 (*)	0.4501 (*)
Italy	0.4909 (*)	0.5735 (*)	0.6424 (*)	0.6959 (*)	0.7254	0.7423	0.7462

Notes: Correlation between $\Delta y^{lt}_{t+k}(i)$ *and* $x_t^{lt}(i)$. *A 5% significant coefficient is indicated by an asterisk.*

Table 5
Long-term correlation between real private consumption and excess returns on stocks (above the risk-free interest rate)

k	-3	-2	-1	0	1	2	3
United States	0.3898	0.4041	0.4091 (*)	0.4054 (*)	0.4060	0.3989 (*)	0.3850 (*)
France	0.0629	0.1698 (*)	0.2714 (*)	0.3653 (*)	0.4580 (*)	0.5369 (*)	0.6006 (*)
Germany	0.0974	0.1675	0.2362	0.3019	0.3425 (*)	0.3804 (*)	0.4149 (*)
United Kingdom	0.3423	0.3855	0.4175	0.4380	0.4556 (*)	0.4602 (*)	0.4522 (*)
Italy	0.3377 (*)	0.4391 (*)	0.5305 (*)	0.6098 (*)	0.6598 (*)	0.6991 (*)	0.7266 (*)

Notes: Correlation between $\Delta c^{lt}_{t+k}(i)$ *and* $x_t^{lt}(i)$. *A 5% significant coefficient is indicated by an asterisk.*

Table 6
Short-term correlation between real GDP and money market interest rates

k	-3	-2	-1	0	1	2	3
United States	0.5341 (*)	0.6218 (*)	0.6334 (*)	0.5430 (*)	0.3629 (*)	0.1096	-0.1750 (*)
France	0.1775	0.1996	0.1827	0.1188	0.0219	-0.0801	-0.1720
Germany	0.7303 (*)	0.7233 (*)	0.6299 (*)	0.4475 (*)	0.2020 (*)	-0.0585	-0.2846 (*)
United Kingdom	0.5535 (*)	0.5172 (*)	0.3870 (*)	0.1663	-0.0904	-0.3187 (*)	-0.4740 (*)
Italy	0.5129 (*)	0.5983 (*)	0.5702 (*)	0.4524 (*)	0.2644	0.0973	-0.0137

Note: A 5% significant coefficient is indicated by an asterisk.

Table 7
Short-term correlation between excess returns on stocks (above the risk-free interest rate) and money market interest rates

k	-3	-2	-1	0	1	2	3
United States	-0.0115	-0.1372	-0.2137 (*)	-0.2298	-0.1842	-0.1009	-0.0007
France	-0.1078	-0.1159	-0.0643	-0.0195	-0.0058	-0.0222	-0.0417
Germany	0.0796	0.0778	0.0580	0.0235	-0.0111	-0.0231	-0.0071
United Kingdom	-0.1632	-0.0729	0.1482	0.3792 (*)	0.4989 (*)	0.4289 (*)	0.2083 (*)
Italy	-0.0950	-0.0931	-0.0750	-0.0301	0.0367	0.1051	0.1381 (*)

Note: A 5% significant coefficient is indicated by an asterisk.

Table 8
Long-term correlation between real GDP and money market interest rates

k	-3	-2	-1	0	1	2	3
United States	-0.2332	-0.2493	-0.2600 (*)	-0.2646 (*)	-0.2761 (*)	-0.2776 (*)	-0.2685 (*)
France	-0.2404	-0.2906 (*)	-0.3363 (*)	-0.3764 (*)	-0.4187	-0.4549	-0.4835
Germany	0.1101	0.0233	-0.0612	-0.1417	-0.2272	-0.3044 (*)	-0.3715 (*)
United Kingdom	-0.3266	-0.3582	-0.3824	-0.3986	-0.4026	-0.3929	-0.3691
Italy	0.1183	0.0932 (*)	0.0732	0.0587	0.0309	0.0086	-0.0077

Note: A 5% significant coefficient is indicated by an asterisk.

Table 9
Long-term correlation between excess returns on stocks (above the risk-free interest rate) and money market interest rates

k	-3	-2	-1	0	1	2	3
United States	0.0312	0.0615	0.0895	0.1155 (*)	0.0606	0.0112	-0.0316
France	-0.1670	-0.1386	-0.0995	-0.0497	-0.0618	-0.0630	-0.0528
Germany	-0.2636	-0.2238	-0.1724	-0.1097	-0.1036	-0.0860	-0.0571
United Kingdom	0.2013 (*)	0.2068 (*)	0.2163 (*)	0.2305 (*)	0.1796	0.1347	0.0971
Italy	0.0489	0.1047	0.1693	0.2421	0.2326	0.2276	0.2270

Note: A 5% significant coefficient is indicated by an asterisk.

Chart 1
Turning points in real GDP, period 1978 (T1)-2002 (T3)

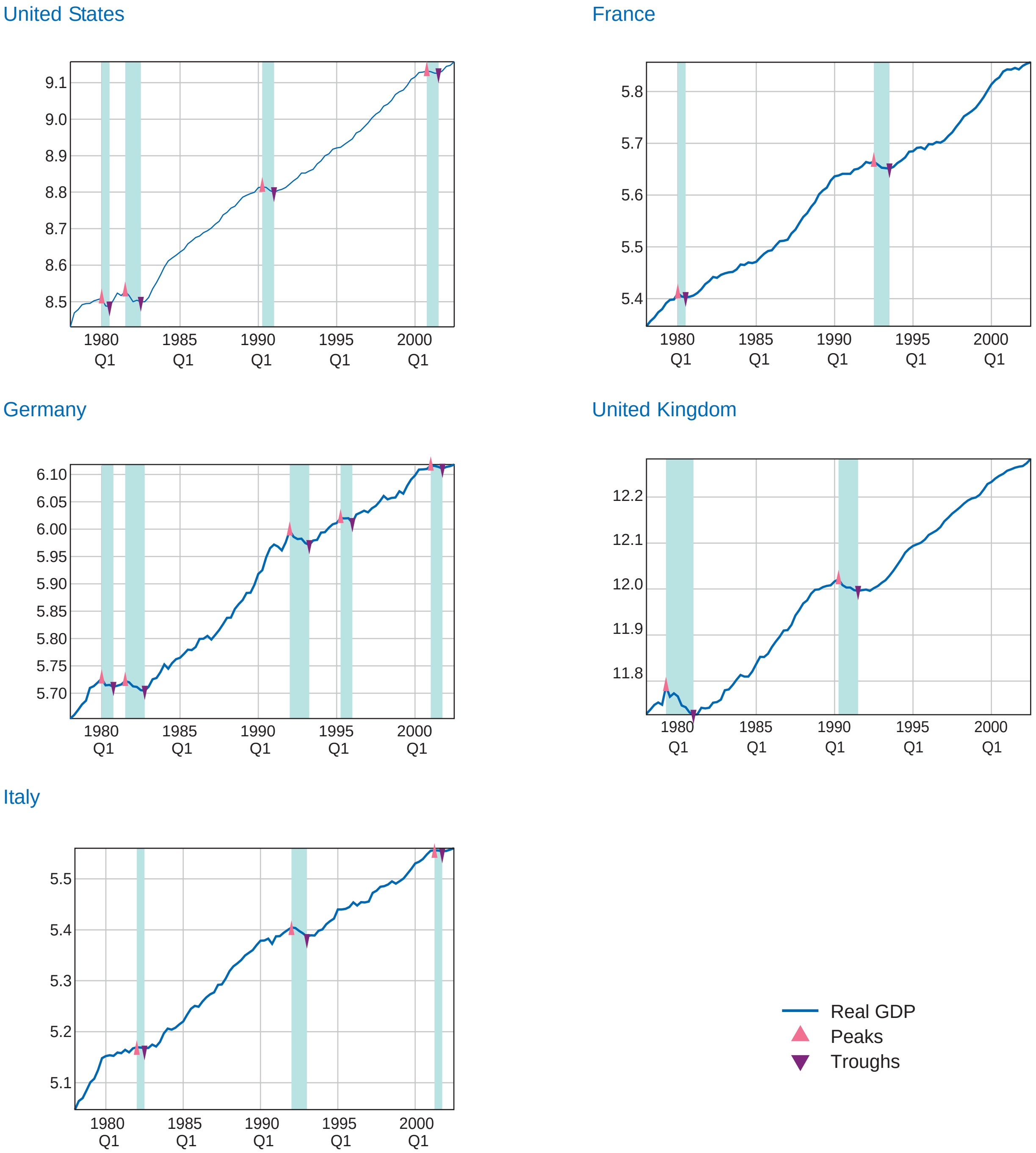


Chart 2
Turning points in real private consumption, period 1978 (T1)-2002 (T3)

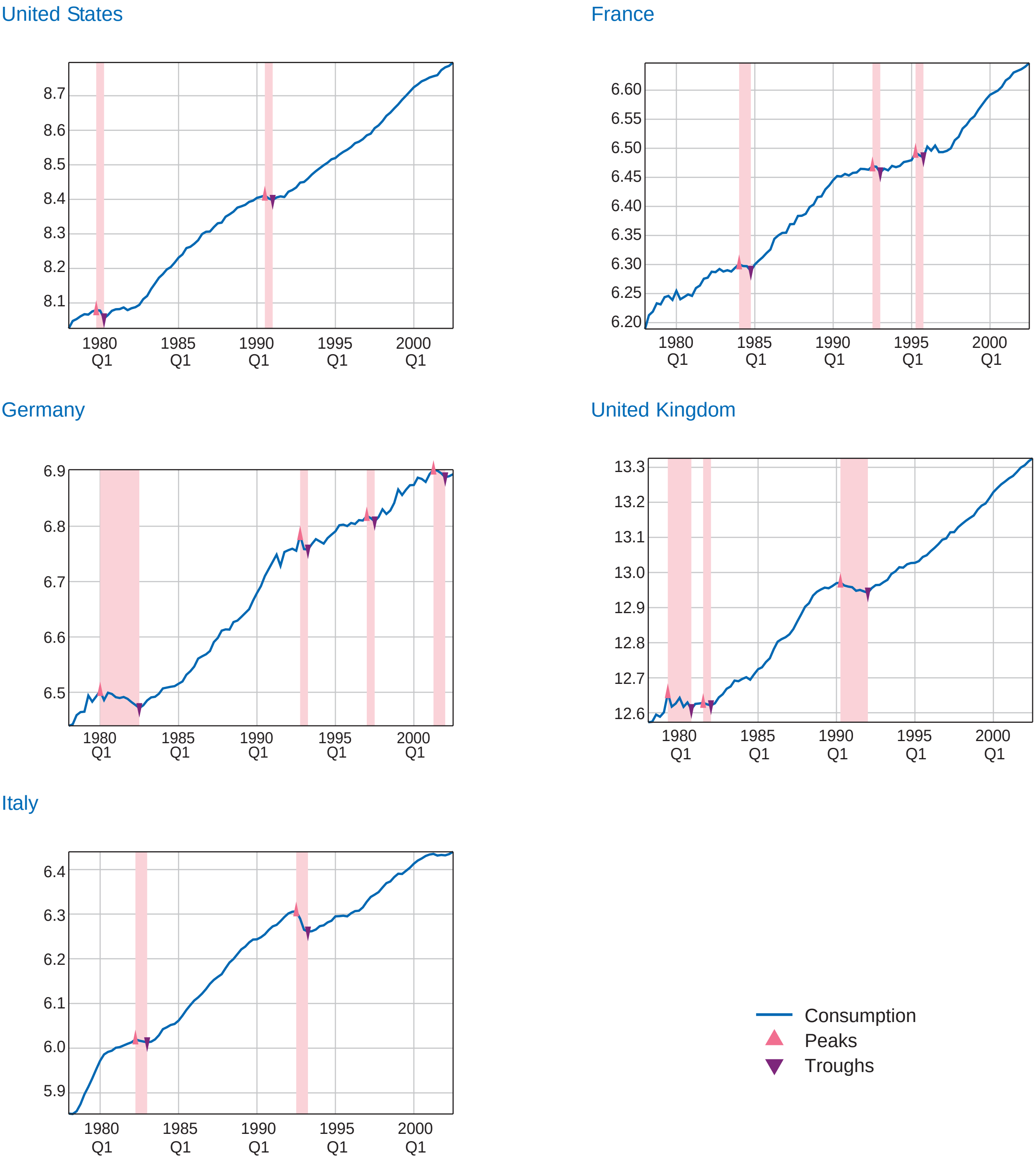


Chart 3
Turning points in MSCI indices, period 1978 (T1)-2002 (T3)

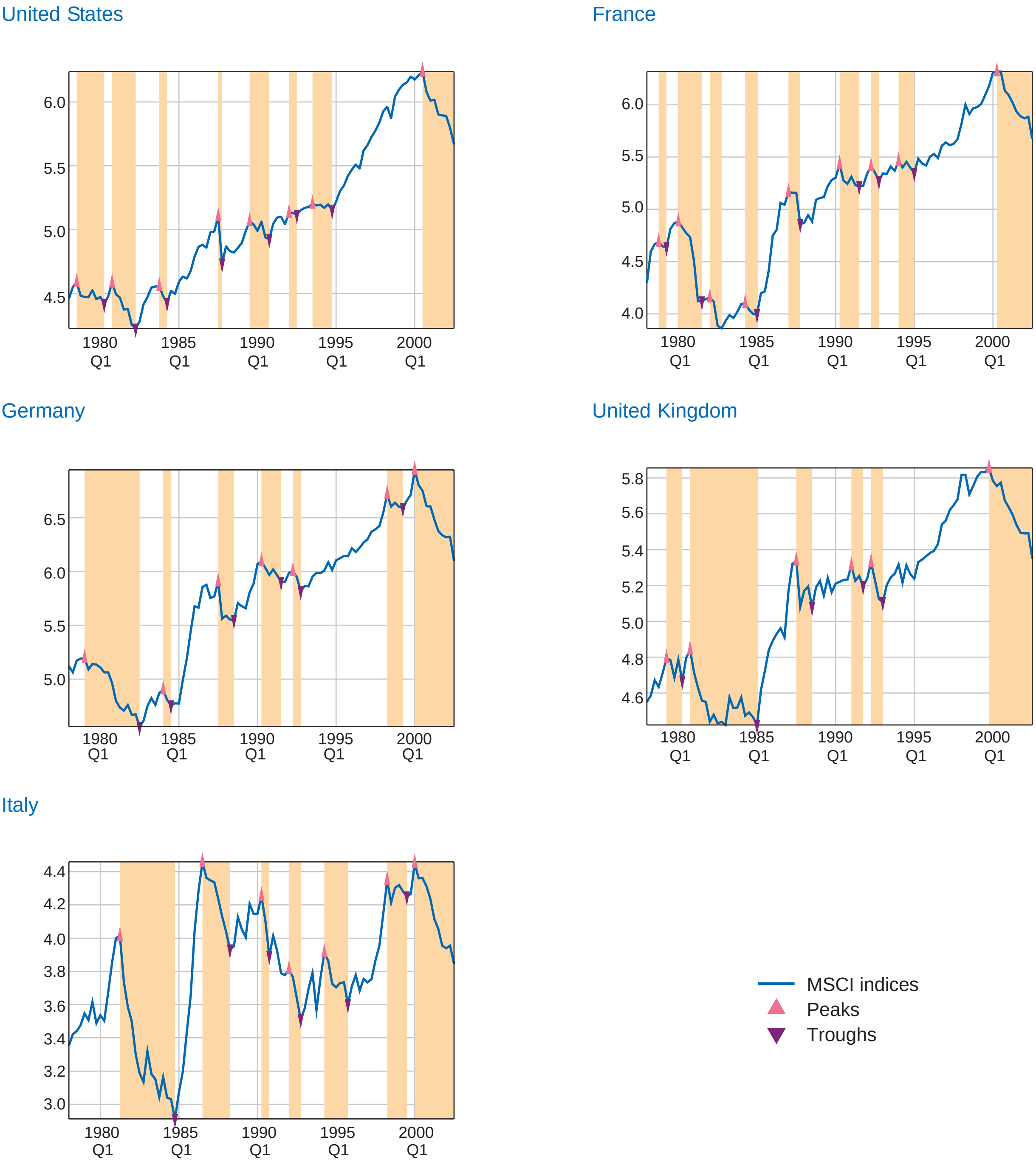


Chart 4
Turning points in the retail sales index (filtered), period 1978 (January)-2002 (September)

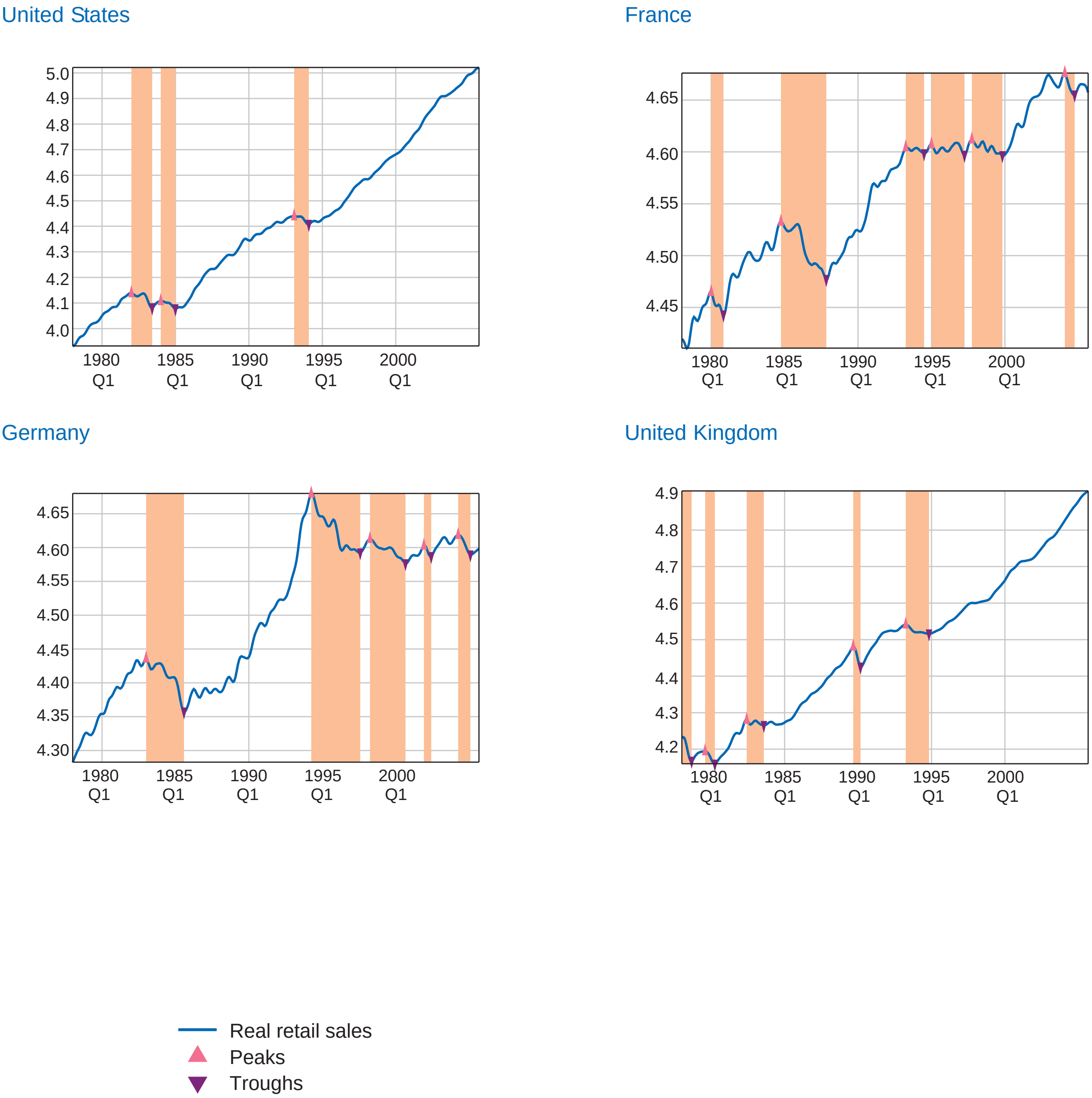
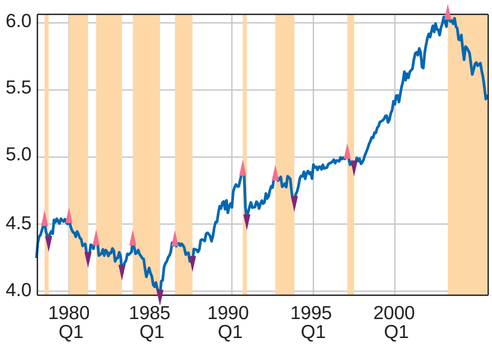
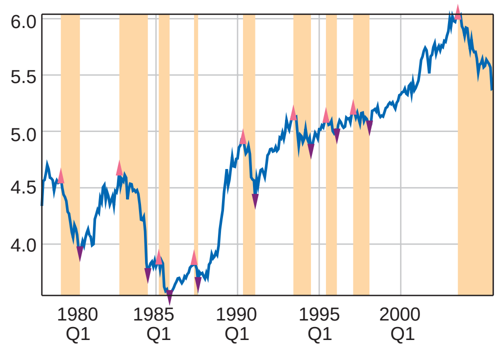


Chart 5
Turning points in MSCI indices, period 1978 (January)-2002 (September)

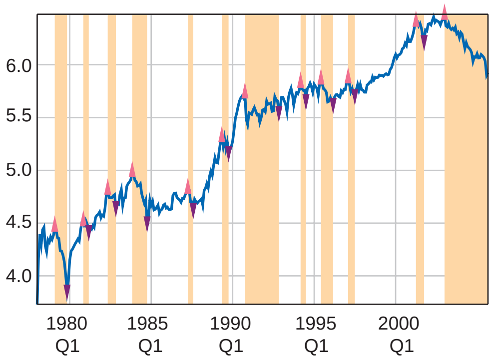
United States



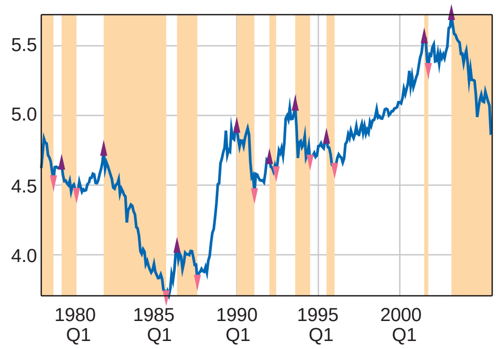
France



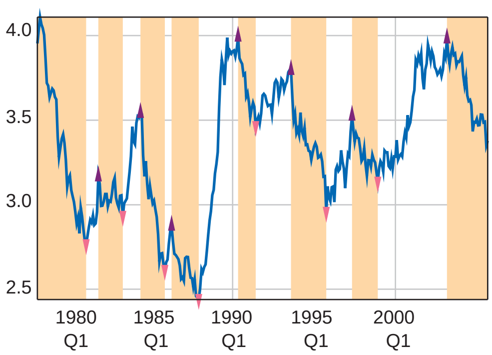
Germany



United Kingdom



Italy



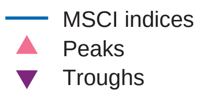
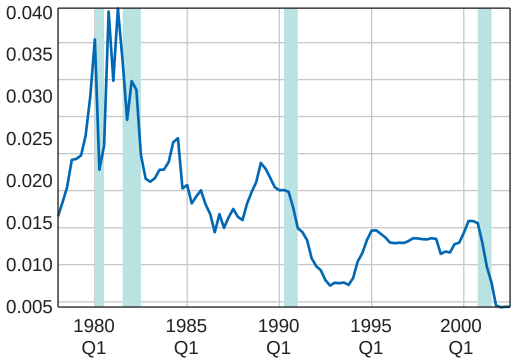
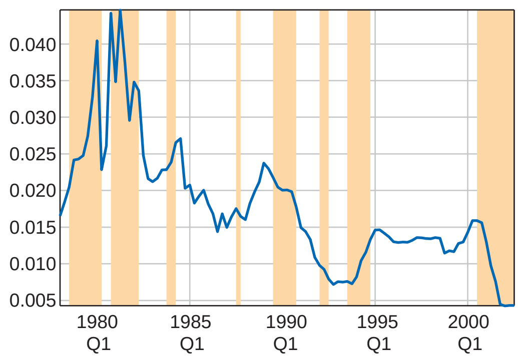


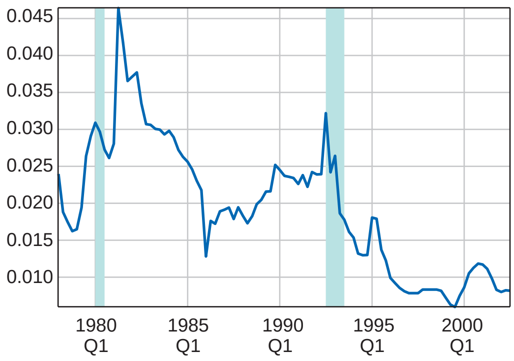
Chart 6
Money market interest rates and turning points in real GDP (Charts on left-hand side) and MSCI indices (Charts on right-hand side), period 1978 (T1)-2002 (T3)

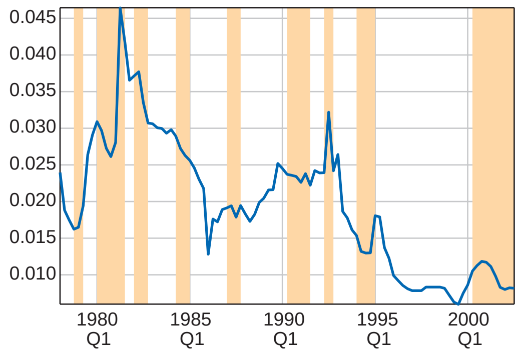
United States



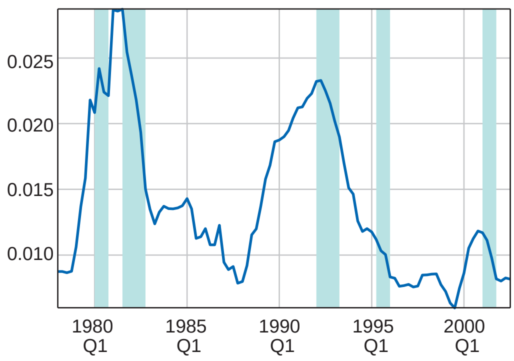


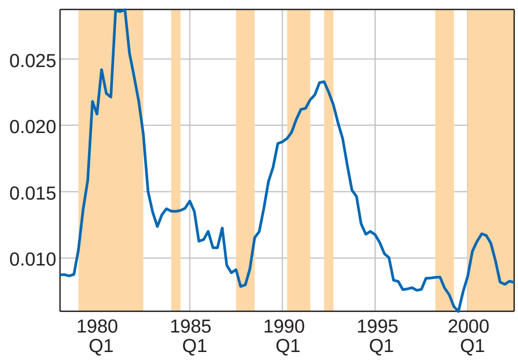
France





Germany

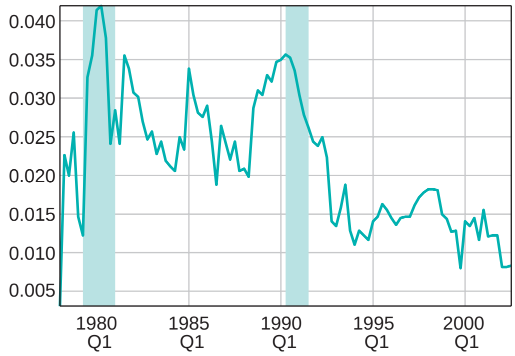


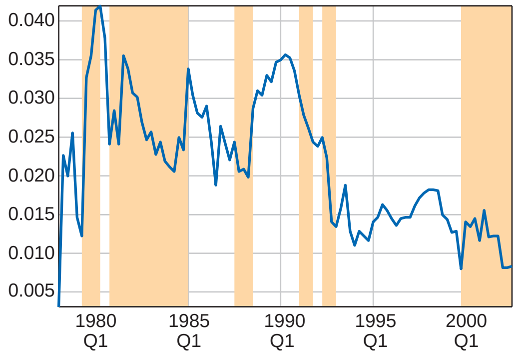


Money market rates
Turning points in real GDP
Turning points in MSCI indices

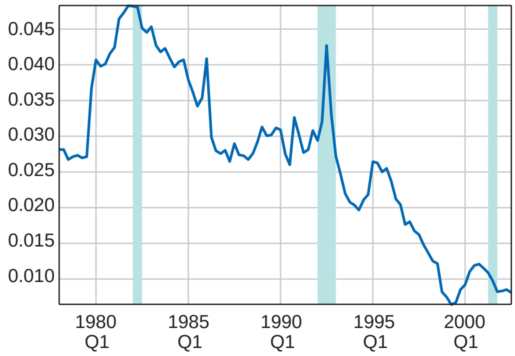
NB: In decimal points, not as a %. The figures are quarterly rates.

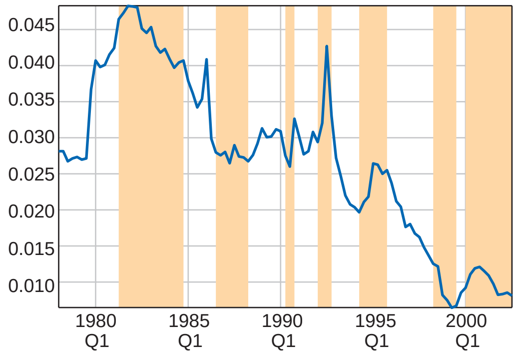
United Kingdom





Italy





Money market rates
Turning points in real GDP
Turning points in MSCI indices

NB: In decimal points, not as a %. The figures are quarterly rates.

Appendix 2

A. Turning points and concordance

Bry and Boschan (1971) determined an algorithm that made it possible to replicate the contraction start dates identified by a committee of experts from the NBER. We used a variation of this algorithm, developed by Harding and Pagan (2002a,b), whose steps are as follows.

- A peak/trough is reached at t if the value of the series at date t is superior/inferior to the previous k values and to the following k values, where k is a natural integer that varies according to the type of series studied and its sampling frequency[15].

- A procedure is implemented to ensure that peaks and troughs alternate, by selecting the highest/lowest consecutive peaks/troughs[16].

- Cycles whose duration is shorter than the minimum time m are stripped out, as are cycles whose complete recurrence period (number of periods separating a peak from a peak or a trough from a trough) is lower than the prespecified number of periods M.

- Complementary rules are applied:

  - the first peak/trough cannot be lower/higher than the first point in the series, and the last peak/trough cannot be lower/higher than the last point in the series;

  - the first/last peak/trough cannot be positioned at less than e periods from the first/last point in the series studies.

The monthly sales index is prefiltred using a Spencer curve, in accordance with the usual procedure described in the literature. Note that, like Pagan and Sossounov (2003), we do not prefilter the monthly financial series. Moreover, in the latter case, imposing a minimum phase m may be restrictive. Pagan and Sossounov (2003) therefore propose relaxing the constraint on the minimum phase where a fall or a rise in excess of 20% is present in a period. We adopt this procedure here.

A contraction/expansion phase is thus defined as the time separating a peak/trough from a peak/trough, when the sequence of peaks and troughs meets all the identification rules listed above. We can hence define the variable $s_{y,t}$ as equal to 1 if y is in expansion at t and 0 if not. We apply the same logic to the variable x, for which we define $s_{x,t}$.

The concordance index between x and y, c_{xy}, is defined as the average number of periods where x and y are simultaneously in the same phase, and is expressed as follows:

$$c_{xy} = \frac{1}{T}\sum_{t=1}^{T}[s_{x,t}\, s_{y,t} + (1 - s_{x,t})\,(1 - s_{y,t})].$$

It is equal to 1 if x and y are always in the same phase and to 0 if x and y are always in opposite phases.

In general, the distribution properties of c_{xy} are unknown. In order to calculate the degrees of significativity of these indices, we use the method suggested by Harding and Pagan (2002b) given below.

Let μ_{s_i} and σ_{s_i}, $i = (x, y)$, denote the empirical average and the empirical standard deviation of $s_{i,t}$, respectively. If ρ_s denotes the empirical correlation between $s_{x,t}$ and $s_{y,t}$, we demonstrate that the concordance index can be expressed as follows:

$$c_{xy} = 1 + 2\,\rho_s \sigma_{s_x}\sigma_{s_y} + 2\mu_{s_x}\mu_{s_y} - \mu_{s_x} - \mu_{s_y}. \quad \text{(A.1)}$$

According the equation (A.1), c_{xy} and ρ_s are linked in such a way that either of these two statistics can be studied to the same effect. In order to calculate ρ_s, Harding and Pagan estimate the linear relationship:

$$\left(\frac{s_{y,t}}{\sigma_{s_y}}\right) = \eta + \rho_s\left(\frac{s_{x,t}}{\sigma_{s_x}}\right) + u_t, \qquad \text{(A.2)}$$

where η is a constant and u_t a residual.

The estimation procedure of equation (A.2) must be robust vis-à-vis the residual serial correlation, as u_t inherits the serial correlation properties of $s_{y,t}$ under the null hypothesis $\rho_s = 0$. The ordinary least squares method augmented by the HAC procedure is therefore used here for estimating equation (A.2).

[15] *In this method used for identifying turning points, it is not necessary to assume that the series studied is stationary.*
[16] *This criterion is not always adopted in the literature (see Canova, 1999).*

B. The Band Pass Filter

The ideal band pass filter used to isolate cyclical movements, whose recurrence periods are between the interval $[b_i, b_s]$, is defined by the following equation:

$$y_t^{ct} = B(L)y_t, \quad B(L) = \sum_{k=-\infty}^{k=+\infty} B_k L^k, \quad L^k y_t = y_{t-k},$$

where B_k are expressed as:

$$B_k = \frac{\sin(2k\pi / b_i) - \sin(2k\pi / b_s)}{\pi k}.$$

In order to interpret the role played by the filter, we introduce the concept of *spectral density*. The spectral density of the stationary stochastic process y_t, denoted $S_y(\omega)$, is interpreted as the decomposition of the variance of y_t in the frequency domain. As y_t can be decomposed into a sum of orthogonal cyclical movements that each appear at a different frequency, we can interpret $S_y(\omega)$ as the variance of y_t, explained by the cyclical movements operating at frequency ω.

A classic result of spectral analysis shows us that, under certain conditions, the equation $y_t^{ct} = B(L)\, y_t$ implies that the spectral density of the process y_t^{ct}, $S_{y^{ct}}(\omega)$, is deduced from that of y_t, $S_y(\omega)$, using the formula:

$$S_{y^{ct}}(\omega) = \left\| B(e^{-i\omega}) \right\|^2 S_y(\omega),$$

where $\left\| B(e^{-i\omega}) \right\|^2$ is the squared module of $B(e^{-i\omega})$. Given the definition of B_k, a direct calculation shows that:

$$B(e^{-i\omega}) = \begin{cases} 1 \text{ pour } \omega \in]2\pi / b_s, 2\pi / b_i[\cup]-2\pi / b_i, -2\pi / b_s[\\ 0 \text{ sinon} \end{cases}.$$

From this formula it can be observed that the spectral density of y_t is not zero on the frequency band $]2\pi/b_s, 2\pi/b_s[\cup]-2\pi/b_i, -2\pi/b_s[\subset]-\pi, \pi[$, and zero everywhere else. In other words, all the variance of y_t^{ct} is explained by cyclical movements whose recurrence periods are between b_i and b_s.

The definition of the filter $B(L)$ imposes a major limitation, as it requires a dataset of infinite length. In practice, we work with a finite sample and must therefore make an appropriate approximation of $B(L)$. Starting from a finite number of observations $\{y_1,\dots,y_T\}$ of the stochastic process y_t, Christiano and Fitzgerald (2003) define the optimal linear approximation $\hat{y}_t^{ct}$ of y_t^{ct} as the solution to the problem:

$$\min \mathrm{E}\left[\left(y_t^{ct} - \hat{y}_t^{ct} \right)^2 \middle| \{ y_1, \dots, y_T \} \right]. \qquad \text{(B.1)}$$

The method therefore consists in minimising the mathematical expectation of the square error between the ideally filtered series and the approximately filtered series, where the expectation is conditioned on all the available data.