Equity market interdependence: the relationship between European and US stock markets

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In this article, the degree of interdependence between European and US stock markets is measured by the conditional correlation between stock returns: the correlation coefficient is estimated using a model describing the variations over time in a number of variables (returns and volatility, for example), and its estimate takes account of all available information at a given time. We estimate conditional variance in the same way. Moreover, two statistical tools, recently introduced in applied finance, are combined. The first, developed by Engle in 2001 – an original specification of the conditional correlations in multivariate models – enables us to describe time-varying correlations between two or more assets. The second tool, copula functions, allows us to apply distributions that are more consistent with the stylised facts observed on financial markets than those commonly used.

The approach used in this study is original in that it combines both the above tools. Using a multivariate model implies rejecting the two assumptions traditionally adopted in empirical studies in finance: correlations between assets are presumed to be constant; asymmetry or the presence of rare events are not taken into account in asset price distributions. Consequently, our empirical findings corroborate the assumption that correlations vary over time and validate the choice of an asymmetric joint distribution integrating the presence of rare events. We also observe the presence of periods of strong and weak correlations and similar periods for volatility. Furthermore, our results highlight a close link between the correlations and volatilities observed on the different equity markets: in phases of high volatility, the correlation tends to rise above its medium-term average; inversely, in phases of low volatility, markets seem to display greater independence. Lastly, the correlation coefficient of close to 1 confirms that French and German stock market indices have been converging in recent years. This may reflect the growing integration of these two markets and of the economies of these two countries within Economic and Monetary Union.

Equity markets are assumed to be interdependent but the instruments used for measuring this relationship over time are often unsophisticated. Indeed, not only can several models be applied to monitor and determine variations in volatility over time, in particular when the latter is calculated on the basis of available information (conditional volatility), but furthermore relatively few approaches exist for measuring time-varying interdependence between markets.

In keeping with empirical literature available on volatility (see Bollerslev, Engle and Nelson, 1994, or Gouriéroux, 1992), we observe that the degree of interdependence of markets may be higher during periods of "crisis" or euphoria than in "normal" times. This appears to be due to the fact that financial markets overreact, in general, to very bad or very good news (but not necessarily in a symmetrical way).[1] Consequently, when analysing several equity markets (multivariate analysis), we can reasonably assume that strong and weak links alternate.

Moreover, the globalisation and market integration process, which has been underway for the past two decades, suggests that interdependence between stock markets reflects structural changes in the global financial system in the long term and developments in the financial environment in the short term. This process may have heightened the risks of contagion between financial markets and, more specifically, between equity markets.

We use the conditional correlation as a measure of the interdependence or of the degree of linkage between two or more variables. In other words, the correlation coefficient is estimated using a model that describes notably stock return dynamics and time-varying volatility.

This study sets out to ascertain whether the assumption that correlations vary over time is true and whether they have similar properties to those of conditional volatilities. In addition to equations describing time-varying correlations, we also analyse stock return equations (calculated as relative changes in prices) and volatility (here, conditional variances) on US and European stock markets. We can thus compare time-varying volatilities estimated on these markets, on the one hand, and volatilities and correlations, on the other.

To make these comparisons, we study time-varying conditional correlations between the two main euro area stock markets (Paris and Frankfurt) and the US stock market by combining properties from the Dynamic Conditional Correlation Multivariate Model (see Appendix 1), developed by Engle, with those obtained using copula theory (see Appendix 2), which make it possible to carry out an appropriate decomposition of the joint distribution of a number of variables.

As we will see further on, the approach developed by Engle (2001, 2002) elaborates on his earlier work that set out to simultaneously explain stock return dynamics and time-varying volatility, for example. Engle's models, more commonly known as ARCH models (*i.e.* AutoRegressive Conditional Heteroscedasticity), are extremely useful in applied finance studies. Amongst other things, on the basis of these models we can reject the assumption that conditional variance does not vary over time. ARCH models simultaneously describe the dynamics of stock returns and time-varying volatility. To a certain extent, the assumption that correlations between endogenous variables vary over time extends ARCH models to the multivariate case and enables us to add equations describing relationships of interdependence between these variables.

Until recently, due to the complexity of the analytical expression of the joint distributions relating to a multivariate analysis, we had only used a limited range of joint distributions in empirical studies. The copula function approach allows us to avoid this problem and access a broader range of joint distributions.

This article is organised as follows: Section One gives a brief overview of the means used to define the conditional correlation; Section Two sets out the presentation and treatment of data and the presentation and interpretation of the results; it also includes an example of the application of the time-varying correlation coefficient assumption using a capital asset pricing model (CAPM). Finally, the main conclusions are presented at the end of the article.

[1] *Empirical results (see De Bondt and Thaler, 1985, for example) have revealed periods of equity market overreaction resulting,* inter alia, *from successive bouts of optimism or pessimism on the part of market participants. These results, which have given rise to much debate, can be found mainly in literature on market efficiency tests.*

1| Brief methodological overview

1|1 General framework

A number of models can be used to calculate the volatility on a given market. They may be either of a structural (explication by fundamentals), or a statistical nature. Without doubt, in the past few decades, the greatest developments have been seen in statistical models. ARCH models come within this latter category. They have been applied to financial markets to take account of certain stylised facts (fat tails, asymmetric effects, etc.). Moreover, thanks to recent progress in econometrics their estimates have become more robust. ARCH models are a key focal point of this study.

When there is only one endogenous variable (one equity market for example), aside from the stock return equation, the ARCH model (or GARCH model, *i.e.* generalised ARCH) allows us to define an explanatory equation of the conditional variance based on three factors: the lagged value of this variance, which introduces a phenomenon of momentum (or persistence) in the equation; recent shocks, represented here by the difference between the estimated and the observed values of the variable studied; and a constant factor (in fact, the constant of the equation). Therefore, if the conditional variance is assumed to be constant (*i.e.* if the equation is reduced to a single constant), the coefficients of the first two factors (the persistence effect and the recent shock effect) are zero.

When we attempt to simultaneously analyse several variables or markets (multivariate analysis), one of the trickiest problems stems from the fact that the number of unknown parameters increases in line with that of the variables or the markets. Furthermore, this analysis imposes additional constraints, in particular in terms of the signs or values of the parameters. This general difficulty relating to multivariate models also concerns ARCH or GARCH models. If we take three markets assumed to be dependent for example, in addition to the parameters of the stock return equations, it is also necessary to introduce three correlation coefficients, three conditional variances as well as parameters specific to the joint distributions of these variables. If we attempt to describe the variances and correlations using equations, it would be extremely difficult to simultaneously estimate all the equations, unless we only use very simple explanatory equations.

In the case of multivariate ARCH models, several studies have focused on the particular specifications that make it possible to both reduce the number of parameters and limit the size of the constraints, while maintaining a relatively rich dynamic structure of the model. One approach would be to assume that there are one or more explanatory factors common to the different markets (see Diebold and Nerlove, 1989). The main difficulty of this approach, which is more centred on finding a structural explanation, is to identify the factors when they can be observed and to estimate them when they cannot. Consequently, the complexity of estimation methods of this kind of model does not generally yield results that are as robust as evidence would suggest. Another approach would consist in using purely statistical models such as ARCH univariate models (Baba, Engle, Kraft and Kroner, 1987, for example).

The Constant Conditional Correlations ARCH (CCC-ARCH) model developed by Bollerslev in 1987 – one of the approaches representative of this category – takes conditional variances to be time-varying but maintains constant correlations. This model considerably reduces the number of parameters to be estimated but the assumption of constant correlations does not reflect the reality. Therefore, researchers have aimed to "retain" the main properties of Bollerslev's model (simplicity of implementation, flexible framework, etc.) by adding a more realistic assumption about the behaviour of the correlations.

Engle (2001, 2002), Engle and Sheppard (2001) and Tse and Tsui (2002) proposed an original dynamic specification of the conditional correlations in multivariate GARCH or ARCH models, the DCC-GARCH model. In relation to Bollerslev's approach, the DCC-GARCH model introduces equations that describe time-varying correlation coefficients similar in their conception to those of the conditional variances discussed above (see Appendix 1).

Indeed, in the same way as with the conditional variances, these coefficients can be explained by three main factors: their own lagged values, with a view to taking account of persistence phenomena; a factor representing the effect of recent shocks; and a constant. If the assumption that correlation coefficients vary over time is rejected, their equations are thus reduced to constant parameters (this is then tantamount to a CCC-ARCH model).

This approach is more realistic than that put forward by Bollerslev, which does not stand up to empirical evidence, in particular if it derives from equity market analysis. Moreover, the implementation of DCC-GARCH models is relatively simple thanks to recent advances in econometrics. It is also adaptable in respect to a certain number of tests including that to ascertain whether correlation coefficients are constant.

1|2 The contribution of copula functions

Copula functions have recently been used in applied finance to obtain a broader and more realistic range of joint distributions of stock returns on a number of markets. Indeed, prior to this, in order to take account of certain stylised facts (the presence of rare events and asymmetric effects) in multivariate models, it was necessary to know the analytical expression of distributions or how easy or difficult they were to implement. This, for example, was the case for the Student distribution, which is symmetric but does not take account of the presence of rare events. This greatly limited the scope of the simultaneous modelling of markets. A number of difficulties, arising in particular from the choice of joint distribution, the sharp increase in the number of unknown parameters and, at times, the decrease in the amount of data available, have penalised multivariate empirical models. Copula functions can be used, in part, to deal with these problems.

Under easily-verifiable conditions, copula functions make it possible to make a single decomposition of a given joint distribution of a number of variables into two components. The first is a function, or structure, of dependence, which is characterised by a set of parameters, termed dependence measures or parameters. These parameters include the correlation coefficient, which is one of the measures of interdependence. The second component is a term corresponding to the product of the marginal distributions of the variables studied; if we take the case of the two variables for example, this term will correspond to the marginal distribution of the first multiplied by that of the second, see Appendix 2 or Patton (2001) or Rockinger and Jondeau (2001).

Thanks to this decomposition, if we know the dependence structure and the marginal distributions, we can obtain that of the joint distribution, which is defined as the product of its two components. Consequently, it is no longer necessary to know the exact analytical expression of this distribution. We can for example choose asymmetric marginal distributions and/or fat tails (presence of rare events) combined with a dependence structure that makes it possible to establish links between extreme events (upward or downward price spikes). Furthermore, copula functions facilitate multivariate model estimations; they make the implementation of these models more flexible.

1|3 Brief description of the estimated model

The model used for the applications contains equations describing returns, variances and conditional correlations (see Appendix 3). In particular, the variance equation makes it possible to differentiate between the positive and negative shock effects (asymmetric effects). This distinction was introduced in order to take into account the stylised fact according to which financial markets react more violently to bad news.[2]

As we mentioned above, copula functions make it possible to decompose the joint distribution, which facilitates the implementation of the model. In the context of this study, after preliminary tests (see Avouyi-Dovi and Neto, 2003), the most appropriate distribution on each market must be asymmetric and allow the presence of rare events. The Pearson-IV distribution has the above properties and has recently been tested with success in other studies. We have used this distribution here.

Empirical results (see Avouyi-Dovi and Neto, 2003, and Longin and Solnik, 1998, or Mashal and Zeevi, 2002) have shown that the dependence structure of the joint distribution of returns should allow a marked dependence for both fatter and thinner tails, *i.e.* that rare events (upward or downward price spikes) must be linked. The choice of this structure must therefore be restricted to the family of

[2] *The asymmetric reaction to the signs of shocks could be explained by market participants' long positions on equity markets that would make them more sensitive to negative shocks.*

functions liable to correspond to the previous property. This is the case for the dependence structure of the Student distribution. Moreover, the dependence parameters of the latter are correlations (see Appendices 1 and 3), which is exactly what we set out to define here using the DCC-GARCH model.

At this stage, we should specify that:

- the correlation coefficients analysed here are calculated between pairs of markets. They do not concern the relationship between the volatilities observed on the different markets. For simplicity's sake, these coefficients may be interpreted as measures of the relationship between stock returns;
- we can verify the existence of the relationship between indices for instance by introducing the return on the US market in the equation of its French counterpart. In order to test the variation over time of the intensity of the relationship, we would then have to assume that the coefficient of the US index, as an explanatory factor of the French index, varies over time. We did not choose this option here because it is not an easy exercise given that series of variances and correlations were not observable *(ex ante)*. In this study, we assumed that the lagged values of each return explains its current dynamics.[3]

2| The results and their interpretation

2|1 A brief descriptive analysis of the data

The intensity of the relationship between French, German and US stock markets (*i.e.* the CAC40, the DAX, and the Dow Jones) is studied here using daily data for the period from 31 December 1993 to 30 July 2002 (*i.e.* 2,238 points for each series). For reasons of homogeneity, we used the narrow indices of these stock markets. The series are derived from Datastream databases; closing (c) and opening (o) index values are available for the three markets. The data associated with particular closing days such as public holidays specific to each country have been replaced by moving averages centred on the missing points.

In order to take account of exceptional closures (the three days following 11 September 2001 for example), dummy variables were introduced into the models. Stock returns, calculated as the first difference of the log of the daily indices multiplied by 100 (*i.e.* $100^*(1\mathrm{n}P_t - 1\mathrm{n}P_{t-1})$ where 1n denotes the log), are analysed at the same time as their volatilities and correlations.[4]

Table 1
Coefficients of correlation between returns

	CAC^o_t/ DAX^o_t	CAC^c_t/ DAX^c_t	CAC^o_t/ CAC^c_t
DJ^c_{t-1}	**0.58381/0.56418**	0.27497/0.31071	—
DJ^o_t	—	0.30219/0.33656	—
DJ^c_t	—	0.38640/0.39480	—
DAX^o_t/ DAX^c_t	—	0.38640/0.39480	**0.73513/0.73526**

In the analyses of the relationship between US and European markets, we generally compare returns on European markets with that on their US counterpart lagged by one period in order to take account of the time difference between Europe and the United States. By analysing the (non-conditional) correlation coefficients between the returns on the CAC, the DAX and the Dow Jones, estimated at t or at t – 1, with the opening or closing indices (see Table 1), we observe that:

- the closing value of the US market at t – 1 appears to most strongly influence European markets at opening at t (Table 1). The correlation coefficients between the returns on European and US markets

[3] *Our findings would have probably been more relevant had intraday data been available, but this was not the case.*

[4] *The returns are centred (of zero average) to avoid the problem of identifying constants in a trivariate model. These returns have the statistic properties (stationarity) that make it possible to preclude false relationships. Moreover, as we mentioned above, we reject the assumption that the joint distribution is normal.*

come to 0.58 for the CAC and the Dow Jones, and 0.56 for DAX and Dow Jones. These coefficients show that there is a relatively close relationship between the European indices at opening at t and US indices at closing at t – 1;

— the correlation coefficients calculated between the returns observed at t at closing in Europe and at opening and closing in the United States (0.39 at closing for the pairs CAC/Dow Jones and DAX/Dow Jones; 0.30 and 0.34 at opening for the same pairs) as well as those estimated at closing between European returns at t and that of Wall Street at t – 1 (0.28 and 0.31) are relatively weak and may indicate that the intensity of the relationship between these markets is low.

In general, when opening indices are not available a comparison is made between the returns at closing in Europe at t and those of the United States at t–1. Clearly, this greatly underestimates the relationship between the European and US markets. For the pair CAC and Dow Jones for example, the correlation coefficient falls from 58% to 27%.

Based on the results of the descriptive statistics, returns on European markets at opening at t will therefore be compared with the return on the US market at closing at t–1 in the trivariate model used in this article.

2|2 A study of conditional correlations

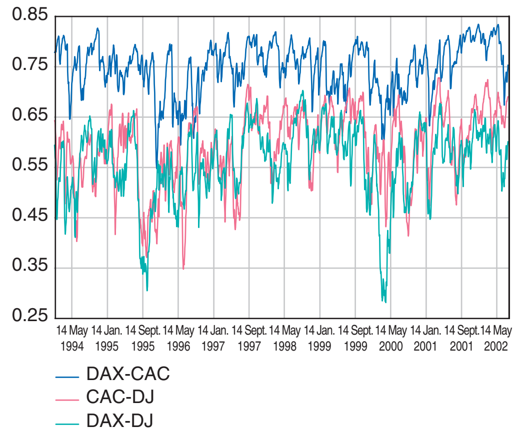
From analysing the variation over time of the correlations between the different pairs of stock returns (CAC-Dow Jones, CAC-DAX, DAX-Dow Jones, see Chart 1) we conclude that:

— irrespective of the return pair studied, "packets" of strong and weak correlations appear. This only reflects the persistence phenomenon mentioned above;

— the correlation coefficients calculated for the pairs CAC-Dow Jones and DAX-Dow Jones are, unsurprisingly, very close (both qualitatively and quantitatively speaking). For example, we observe "peaks" in the correlations around the recent crisis periods (the Asian and Russian crises and the bursting of the tech bubble) and troughs at the first signs of the cyclical turnaround in the United States in 2000. The same is true for 1996 when the first warning signs appeared pointing to an overvaluation of the US stock market;

— aside from some rare exceptions, correlation coefficients of the returns on the CAC and on the DAX are computed at between 70% and 80% over the entire study period with a slightly more marked upward trend between the third quarter of 1999 and the first quarter of 2002. Despite the fact that we observe a slight drop towards the very end of the period, no doubt caused by cyclical differences between the two countries, the high levels of the correlation coefficients probably reflect the growing integration of these two markets and, beyond this, of the French and German economies within Economic and Monetary Union.

Chart 1
Conditional correlations
(as a %; daily data)



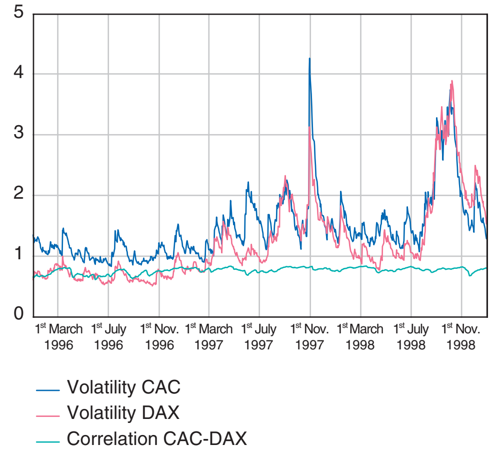
Source: Datastream and Banque de France calculations

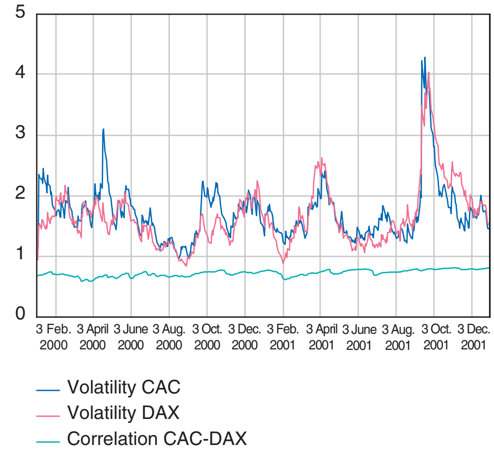
If we compare the patterns of correlations with those of conditional variances (for example the variances of the CAC and of the Dow Jones and the correlation coefficient between the two returns), and if we take the two sub-periods (1996-1998 and 2000-2001), for greater clarity, with the conclusions remaining true for the whole period, we note that (see Chart 2):

— the correlation coefficients increase as soon as one of the markets becomes relatively volatile; when both markets display high levels of volatility, the previous trend (increasing correlations) becomes more pronounced (see Asian and Russian crises or 11 September 2001). The magnitude of the variations of the correlation coefficients depends on the strength of these volatilities;

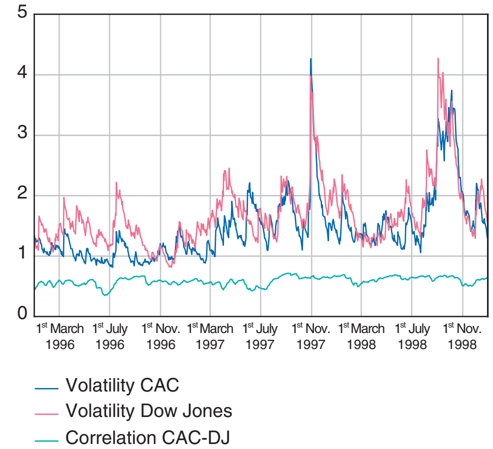
Chart 2
Conditional volatilities and correlations
(as a %; daily data)

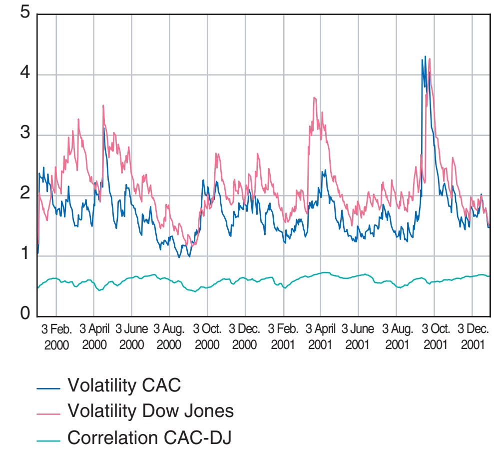
CAC-DAX



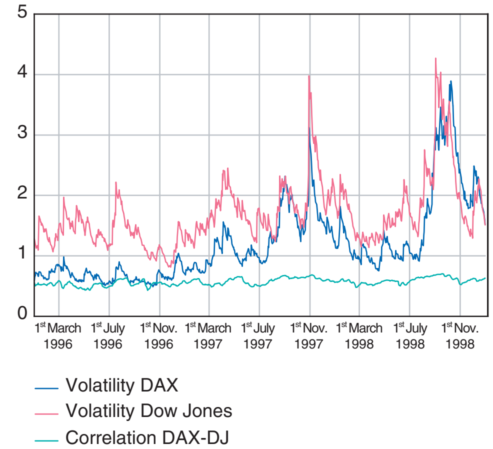


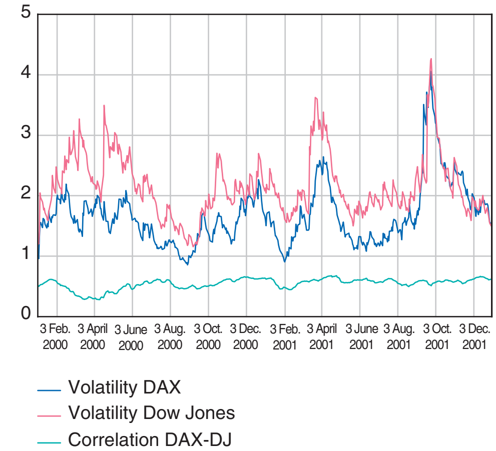
CAC-Dow Jones





DAX-Dow Jones





Source: Datastream and Banque de France calculations.

— conversely, during periods of steady rises or falls in volatility or of low volatility, the correlation coefficients tend to decline or stagnate.

From this graphical study, it is difficult to substantiate the assumption that correlations do not vary over time. Furthermore, in view of the fact that agents operating on these markets may interpret information differently, the variation in correlations and in volatilities is not abnormal. We will now analyse the results of the estimates in order to assess their quality, in particular in statistical terms.

2|3 Some comments on the results of the estimates

The estimate was carried out in two stages, using copula functions (see Appendix 3): in the first stage, we estimated the marginal distribution parameters as well as those of the equations describing the dynamics of stock returns and volatilities (EGARCH process) and, in the second, we estimated the coefficients of the dependence structure and the parameters of the equations of the correlations. The detailed findings are presented in Appendix 3. In general, the parameters estimated in both stages are all significantly different from zero.

For those estimated in the first stage, we note that:

— the parameter of the stock return equations (reduced here to a single endogenous variable coefficient lagged by one period, φ_1, *i.e.* the autoregression coefficient, see Appendix 3) is, generally, low in absolute terms. However, while it remains significantly different from zero for the European markets, it is almost zero for the US market. This means that the weight of past returns is less significant in the calculation of returns for the Dow Jones than for the European indices. This difference in the method of calculating returns on European and US indices may stem from the differences in the behaviour of market participants in particular in terms of the speed of reaction to information that can influence the price discovery process. This observation should however be treated with caution as it only focuses on narrow indices studied over a specific period;

— the conditional variance parameters of the three markets are relatively close. In particular, we note a very strong persistence of volatility (the coefficient β is close to 1 in the three cases and varies from 0.971 for the CAC to 0.986 for the DAX). This suggests the presence of a traditional persistence phenomenon, in particular in the case of equity markets. Moreover, the use of an EGARCH specification (see Appendix 3) appears to be relevant. Indeed, the impacts of the positive and negative shocks on volatility seem asymmetric: the coefficient of the sensitivity of volatility to negative shocks ($\gamma - \alpha_1$, see Appendix 3) amounts to -0.363 for the Dow Jones, and -0.244 and -0.230 for the CAC and the DAX respectively; the coefficient of the sensitivity of volatility to positive shocks ($\gamma + \alpha_1$, see Appendix 3) is around 0.10 for the European markets and only 0.025 for the US market. As the confidence intervals of the coefficients do not overlap, we can consider them as statistically different. As expected, equity markets therefore react more strongly to negative shocks.

For example, a significant rise in unemployment in the United States, perceived as a negative signal, would lead to a relatively large increase in volatility whereas a significant fall in unemployment (positive shock) would result in a decline in volatility of a lesser magnitude. Furthermore, the asymmetry seems much more marked in the United States. We also observe a similarity in the behaviour of French and German markets whose coefficients of the sensitivity of volatility to shocks are very similar.

We must reject the assumption that the empirical results relating to the joint distributions of the returns on the three markets are symmetrical. In order to make this assumption, the parameter representing the degree of asymmetry (or symmetry "checking" parameter, δ, see Appendix 3) would have to be statistically zero, which is not the case. This confirms our assumption as to the choice of an asymmetric distribution in the specification of the model and shows that modelling asymmetry using a EGARCH process is not sufficient when analysing the returns on these indices. Likewise, we accept the presence of fat tails (rare events) in the distributions of returns on European and US markets.

As regards the coefficients estimated in the second stage, we can make the following comments:

— the averages of conditional correlation coefficients are not significantly different from the values obtained in Table 1. For the CAC and the Dow Jones, the averages were 0.59 compared with 0.58, and 0.75 compared with 0.74 for the CAC and DAX; lastly, they were 0.562 compared with 0.564 in the case of the DAX and the Dow Jones. In the long run, the impacts of these positive and negative shocks might therefore have been offset or adjusted;

— the presence of persistence phenomena in the time-varying correlation matrix between the returns is confirmed. Indeed, the closer the parameter measuring the degree of persistence (here θ_2, see Appendix 3) is to 1, the longer the impact of shocks persists in time-varying correlations (*i.e.* when a correlation coefficient reaches a given level due to a shock, it remains there for a certain time). Here, this coefficient is 0.935. This corroborates the empirical findings showing a marked persistence of volatility, which is an indicator of the same nature as covariance (or correlations). It is not surprising that persistence phenomena, considered to be stylised facts in the analysis of the dynamics of stock markets, are also borne out by the correlations;

— we note the high significance of the parameters of recent shocks (θ_1, see Appendix 3) on the correlations. As we have just seen, the shocks alone do not explain the variation of the correlation coefficients over time.

The result we obtained was therefore very much in line with expectations: the intensity of the relationship between stock markets is not constant over time. This finding can be set against that relating to conditional volatilities.

2|4 Application of the DCC-GARCH in the framework of the capital asset pricing model (CAPM)

Adopting the assumption that correlations vary over time, we show that the "beta" (*i.e.* the measure of the volatility of a risky asset relative to the overall market), evaluated by the CAPM, also varies over time (see Box). To illustrate this, we take an investor who holds a risky asset (German stock market index), a risk-free asset (seven-day money market rate), and a benchmark asset (World ex. EMU-Datastream Market Price Index[5]). The data are derived from Datastream. The conditional correlations and variances are obtained from the estimate of a DCC-GARCH model.[6]

By taking only the curves reflecting the variation over time of the "beta" and the correlation coefficient between the return on a risky and a benchmark asset (Chart 3), we note that there is a marked similarity between these two curves. In particular, the troughs and the peaks coincide. The periods in which the return on the risky asset amplifies (or dampens) significantly the shocks affecting the market are associated with marked increases (or decreases) in the correlations. This variation of the "market beta" is passed on to the variation of the systematic risk (see Box)[7] which increases or decreases according to that of the "beta".

This example shows that, in the analysis of financial stability through the study of risks, correlations should be considered to be time-varying. We observed that systematic risk is far from invariable as traditional analysis, in which variances and correlations are assumed to be constant, may suggest. The assumption that correlations are time-varying offers a more dynamic and realistic interpretation of the "beta" and the risk. Indeed, in this context, we can observe a series of phases of amplification ($\beta > 1$) or damping ($\beta < 1$), by the asset, of the shocks arising from the market.

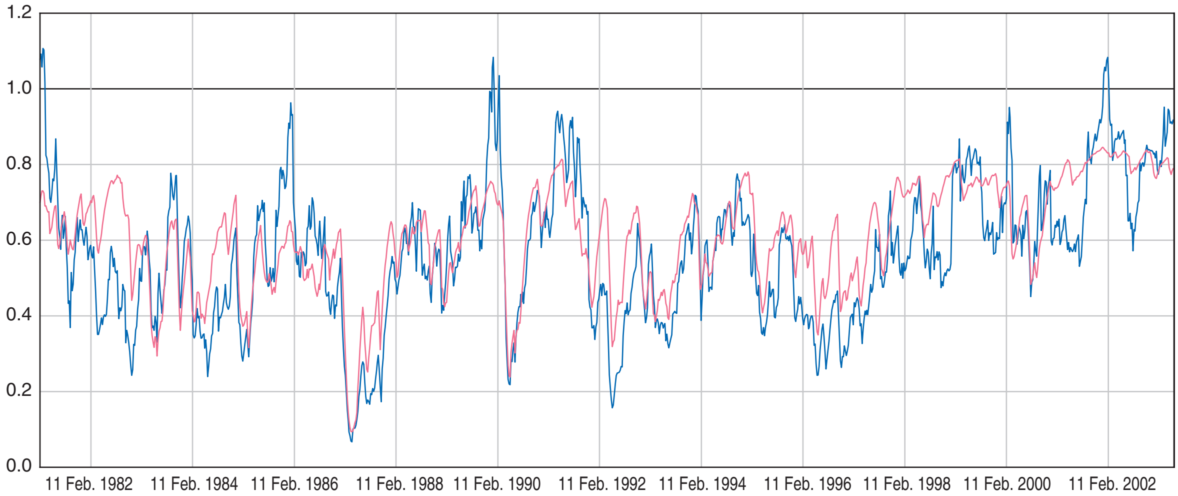
5 *The index used is calculated by Datastream for the world excluding European Monetary Union. The sum of the weights of the United States, the United Kingdom and Japan makes up around 80% of this index.*

6 *The results of the estimates may be obtained from the authors.*

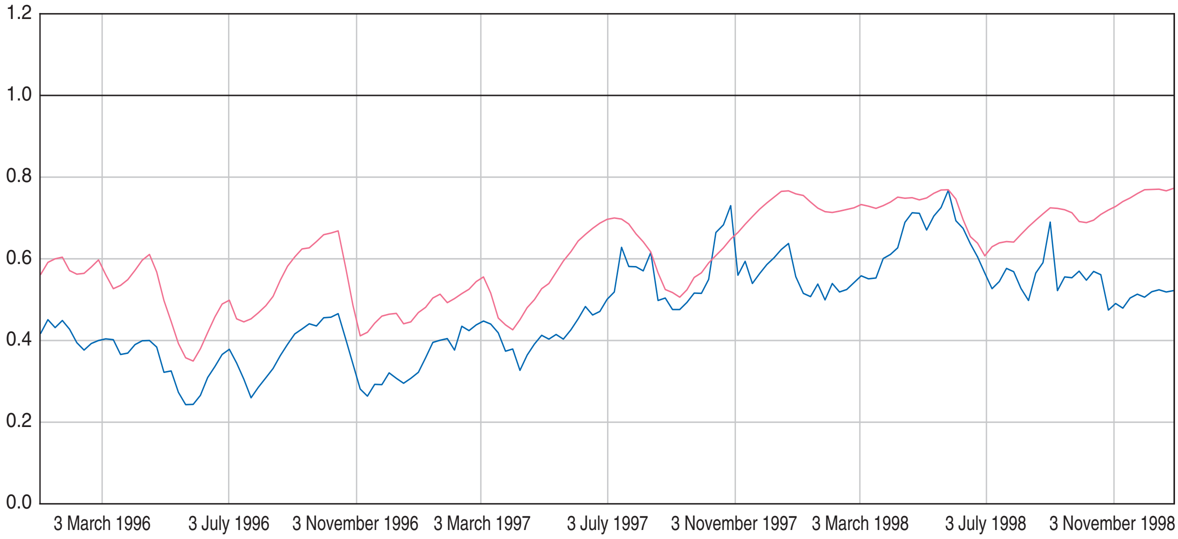
7 *Systematic risk is the component of total risk attributable to the "system", that is to say to the economic environment* i.e. *that which cannot be diversified.*

Charts 3 – Beta and correlation

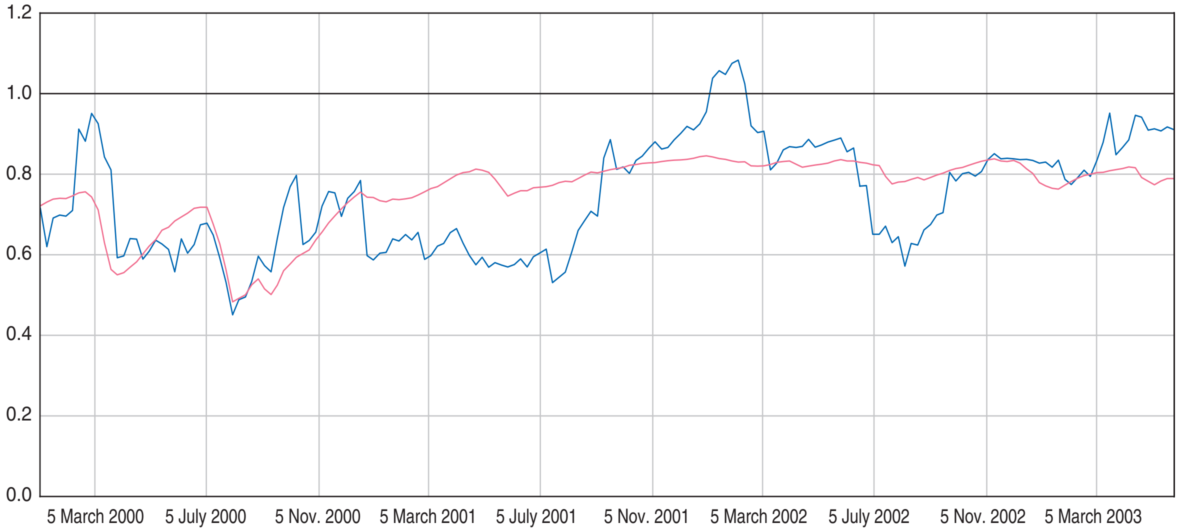
From 11 February 1981 to 28 May 2003



From 1st January 1996 to 31 December 1998



From 1st January 2000 to 28 May 2003



— Beta — Rho

Source: Datastream and Banque de France calculations.

Application of the DCC-GARCH in the framework of the CAPM

The CAPM was developed by Sharpe (1964) and Lintner (1965). It builds on Markowitz's portfolio selection theory of 1952. The CAPM is based on the following assumptions: investors are risk averse and use the mean-variance criterion to select an efficient portfolio; they all opt for the same probability distribution of returns (informational efficiency of the market); the market is perfect (there are no transaction costs, assets are infinitely divisible, short selling is allowed); the market is competitive (agents are price takers); there is a finite number of linearly independent assets.

If $r_{j,t}$ *denotes the return on a risky asset* j, r_f *the return on a risk-free asset,* r_m *the return on the market portfolio and if* $E[r_{j,t} - r_f]$, $V[r_{m,t} - r_f]$ *and* $COV[(r_{j,t} - r_f),(r_{m,t} - r_f)]$ *are respectively the operators of expectation, variance and covariance, the fundamental result of the CAPM is:*

$$E[r_{j,t} - r_f] = \frac{COV[(r_{j,t} - r_f),(r_{m,t} - r_f)]}{V[r_{m,t} - r_f]} E[r_{m,t} - r_f]$$

When the expectation, variance and covariance of the spreads vary over time, the CAPM is expressed as follows:

$$E_t[r_{j,t} - r_f] = \frac{COV_t[(r_{j,t} - r_f),(r_{m,t} - r_f)]}{V_t[r_{m,t} - r_f]} E_t[r_{m,t} - r_f]$$

where the operators E_t, V_t *and* COV_t *are respectively the expectation, variance and covariance conditional on the information set available at time* t. *Equation [1'] is expressed as follows:*

$$E_t[r_{j,t} - r_f] = \beta_t E_t[r_{m,t} - r_f]$$

β_t *measures the relative volatility of the asset* j *to the market. When it is higher (lower) than 1, asset* j *amplifies (dampens) the shocks that affect the market. When it is equal to 1, the fluctuations of the risky asset replicate those of the market. Under the assumption that correlations vary over time,* β_t *can be expressed as follows:*

$$\beta_t = \frac{COV_t[(r_{j,t} - r_f),(r_{m,t} - r_f)]}{\sqrt{V_t[r_{m,t} - r_f]V_t[r_{j,t} - r_f]}} \sqrt{\frac{V_t[r_{j,t} - r_f]}{V_t[r_{m,t} - r_f]}} = \rho_t \sqrt{\frac{V_t[r_{j,t} - r_f]}{V_t[r_{m,t} - r_f]}}$$

ρ_t, *the correlation between asset* j *and the market, may be generated by a DCC-GARCH model (see Appendix 1):*

$$\rho_t = (1 - \theta_1 - \theta_2)\rho^\circ + \theta_1 \psi_{t-1} + \theta_2 \rho_{t-1}$$

The conditional systematic risk is expressed as $\sigma_{m,t} = \sqrt{\beta_t^2 V_t[r_{m,t} - r_f]}$.

By combining the conditional correlations defined by Engle (2001) with copula functions, we were able to study, in a flexible way, the dynamics of the dependence between European and US equity markets. The specification adopted allows us to easily model, despite the difficulties of multivariate statistical analysis, the conditional correlation between the returns on the three markets, taken in pairs. Moreover, we were able to test and reject the assumption that correlations are constant over time. Furthermore, thanks to the recent application of copula functions in empirical analysis in finance, a broader range of joint distributions was tested by using, inter alia*, a copula that allows dependence between extreme events (bubbles and crises). Several empirical studies, notably on developed stock markets, corroborate the relevance of these findings (Longin and Solnik, 1998).*

The observation that correlations vary over time calls into question many models in which they are assumed to be constant. Such is the case, for example, in Markowitz's portfolio selection model, the capital asset pricing model (CAPM) and multivariate Value at Risk (VaR) models. In the case of the CAPM, for instance, if the correlation and the "beta" are assumed to be constant, ceteris paribus*, the risky asset may constantly amplify (or dampen) the shocks affecting the market as a whole. If, however, the correlation coefficient is assumed to vary over time, the "beta" could fluctuate and display phases corresponding to high values (*i.e. *amplification of shocks) or phases associated with lower values (*i.e. *dampening of shocks).*

Moreover, if we integrate the dynamic interdependence of stock markets in the previous models we should be able to take better account of spillover effects, which are a significant component of overall risk.

Two key findings of this study could influence modelling in applied finance:

— *first, the assumption that correlations are constant is totally ruled out. While the interdependence of markets is naturally taken into account in the models of international portfolio diversification, we should introduce the notion that this interdependence varies over time; a factor that in itself could justify the need for more or less large and frequent portfolio shifts. Yet, the variation of correlations over time has been largely ignored in empirical studies due to the complexities it introduces. Besides, by carrying out a combined analysis of the volatility and the conditional correlation we observed a clear relationship between these two variables: in periods of high volatility, the correlation tends to rise above its "normal" level, symmetrically, in periods of low volatility, markets seem to be more independent;*

— *second, there is a marked persistence in time-varying correlations. This can be explained by the existence of cycles (a succession of packets of phases of rises or falls) in the formation dynamics of the interdependence indicator of equity markets. Another explanation for this persistence phenomenon is the heterogeneous behaviour of agents operating on the markets studied. Nevertheless, to validate this assumption more in-depth analysis is called for.*

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Appendix 1

Specification of dynamic conditional correlations

As an illustration, let us take two dependent financial markets. On market i, $i=1,2$, $r_{j,t}$, $\varepsilon_{i,t}$, m_i and I_t, denote the return, the random variable, the conditional expectation and the information set available at time t respectively. For simplicity's sake, let us assume that the returns follow a normal joint distribution, of dimension 2 (bivariate) and with a time-dependent conditional variance-covariance matrix H_t:

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} | I_t \sim N(0,H_t).$$

For each i, $i=1,2$, $r_{i,t}$ is generated by an AR [1] process. Therefore, for each t, $t=1,\dots,T$ (T being the total number of observations), the model is expressed as follows:

$$\begin{aligned} r_{1,t} &= m_1 + \varphi_1 r_{1,t-1} + \varepsilon_{1,t} \\ r_{2,t} &= m_2 + \varphi_2 r_{2,t-1} + \varepsilon_{2,t} \end{aligned}$$

$$[1] \quad \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} | I_t \sim N(0,H_t) \text{ avec } \mathrm{H_t} = \begin{pmatrix} h_{1,t} & h_{12,t} \\ h_{12,t} & h_{2,t} \end{pmatrix}$$

We shall now specify the equations describing the elements of the variance-covariance matrix H_t (*i.e.* equations describing the dynamics of the $h_{i,t}$, $i=1,2$, and of $\rho_{12,t}$, respectively, the conditional correlation and variances). H_t may be broken down into a product of matrices: $H_t = D_t R_t D_t$ where:

- D_t is a diagonal matrix whose non-zero elements are the square roots of the conditional variances (or volatilities) $h_{i,t}$, $i=1,2$;

- the definition of D_t enables us to consider R_t as a matrix of correlations whose elements of the main diagonal equal 1 (if $R_t = R$, *i.e.* $\rho_{12,t} = \rho$, we obtain Bollerslev's constant conditional correlation model, 1987). More specifically, H_t is expressed as:

$$[2] \quad H_t = \begin{pmatrix} \sqrt{h_{1,t}} & 0 \\ 0 & \sqrt{h_{2,t}} \end{pmatrix} \begin{pmatrix} 1 & \rho_{12,t} \\ \rho_{12,t} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{h_{1,t}} & 0 \\ 0 & \sqrt{h_{2,t}} \end{pmatrix}.$$

— $h_{i,t}$, $i=1,2$, are assumed to be described by *GARCH (p, q)* processes. If $p = q = 1$, we obtain: $h_{i,t} = \alpha_0 + \alpha_{i1}\varepsilon^2_{i,t-1} + \beta_{i1} h_{i,t-1}$ with $\alpha_0 > 0$, α_{i1} and $\beta_{i1} \geq 0$ such that α_{i1} et $\beta_{i1} < 1$.

— The conditional correlations are described by an autoregressive process, the Dynamic conditional correlations (DCC), originally developed by Engle and Sheppard (2001) and subsequently built on by Tse and Tsui (2002):

$$[3] \quad \mathrm{R_t} = (1 - \sum_{i=1}^{P} \theta_{1,i} - \sum_{j=1}^{Q} \theta_{2,j})\overline{\mathrm{R}} + \sum_{i=1}^{P} \theta_{1,i} \Psi_{t-i} + \sum_{j=1}^{Q} \theta_{2,j} \mathrm{R}_{t-j}$$

where :

- $\overline{\mathrm{R}}$, the matrix of the non-conditional correlations calculated over the period, is expressed as:

$$\overline{R} = \begin{pmatrix} 1 & \mathring{\rho}_{1,2} \\ \mathring{\rho}_{1,2} & 1 \end{pmatrix};$$

- Ψ_t is a matrix whose elements are empirical correlations calculated at t on a window of given length m (1, 2, 5, ..., days):

$$\Psi_t = Q_t^{-1} M_t M_t' Q_t^{-1} \;; \mathrm{M_t} = \left(\frac{\xi_{1,t}}{\sqrt{h_{1,t}}}, \frac{\xi_{2,t}}{\sqrt{h_{2,t}}} \right)';$$

$$\xi_{i,t} = \left(\varepsilon_{i,t-m}, \dots, \varepsilon_{i,t} \right)'; \; \mathrm{Q_t} = \mathrm{diag}\left(\sum_{i=1}^{m} \frac{\varepsilon^2_{1,t-i}}{h_{1,t}}, \sum_{i=1}^{m} \frac{\varepsilon^2_{2,t-i}}{h_{2,t}} \right)^{0,5}$$

where *diag* denotes the operator defining a diagonal matrix.

Engle and Sheppard showed that

if $\theta_{1,i}$ and $\theta_{2,j} \geq 0 \qquad (\forall i,\ 1 \leq i \leq P \text{ and } j, 1 \leq j \leq Q)$

and $\sum_{i=1}^{P} \theta_{1,i} + \sum_{j=1}^{Q} \theta_{2,j} < 1$,

the matrix R_t is positive at all points in time. As in the case of models, the sum of the parameters measures the degree of persistence of the correlation.

Comments

The previous equations of the conditional variances and correlations define a DCC-GARCH (1,1) model.

In the bivariate case, the equation describing the time-varying correlation matrix can be reduced to an equation explaining the variation of the correlation coefficient between the two markets.

We can assume that the joint distribution of returns is non-normal and thus take into account the stylised facts (presence of fat tails and/or asymmetry in the distribution of returns) of the financial markets.

In their studies, Engle and Sheppard on the one hand, and Tse and Tsui on the other assumed that the joint distribution was normal. This assumption is not realistic in the case of financial asset prices. Consequently, in this study, we opted for a Pearson IV distribution which enables us to verify both the asymmetry and the presence of extreme values. Tests carried out *ex post* show that the adjustment to this distribution is of excellent quality.

Appendix 2

Copula functions

Copula functions have recently been used in applied finance to obtain greater flexibility in multivariate modelling (wider choice of joint distributions, larger diversity of dependence functions, increased choice of distribution functions, greater ease of implementation, etc., see Nelsen, 1998, or Avouyi-Dovi and Neto, 2004). They make it possible to take better account of the events observed on financial markets. We shall define these functions for the two markets; the generalisation to n markets is immediate. Let two random variables X_1 and X_2 of distribution functions F_1 et F_2, be defined by the vector of parameters θ_i, $i=1,2$. Let H be the joint distribution of X_1 and X_2 of the vector of parameters θ_H.

The parametric copula of family Q, denoted C_Q and of dependence parameter matrix θ_c, is a link function between H and the marginal functions F_1 and F_2 with a value in the interval [0,1], defined by:

[1] $$H(X_1,X_2;\theta_H)=C_Q(F_1(X_1;\theta_1),F_2(X_2;\theta_2);\theta_c)$$

According to Sklar's theorem if F_1 and F_2 are continuous, then the above decomposition is unique. From equation [1], we derive an equivalent expression that enables us to define the copula from the joint distribution (assuming that $u_1=F_1(X_1;\theta_1)$ and $u_2=F_2(X_2;\theta_2)$:

[2] $$C_Q(u_1,u_2;\theta_c)=H(F_1^{-1}(u_1;\theta_1),F_2^{-1}(u_2;\theta_2);\theta_H)$$
$$(u_1,u_2)\in[0,1]^2$$

By differentiating H [1] with respect to each of the variables, we obtain a relationship between the joint density, h, (the derivative of H) and the densities C_Q (the derivative of C_Q) and f_i ($i=1,2$, the derivatives of functions F_i). The joint density function is therefore equal to the product of the density functions f_i, $i=1,2$, and of a dependence function C_Q, thus:

[3] $$c_Q(u_1,u_2;\theta_c)\times f_1(X_1;\theta_1)\times f_2(X_2;\theta_2)=h(X_1,X_2;\theta_H).$$

By definition,

$$c_Q=(u_1,u_2;\theta_c)=\frac{\partial^2 C_Q(u_1,u_2;\theta_c)}{\partial u_1\partial u_2},$$

$$f_i(X_i;\theta_i)=\frac{\partial F_i(X_i;\theta_i)}{\partial X_i}$$

and $$h(X_1,X_2;\theta_H)=\frac{\partial^2 H(X_1,X_2;\theta_H)}{\partial X_1\partial X_2}.$$

This decomposition of the joint distribution is particularly appropriate: it enables us to carry out an estimate in two stages (known as the *Inference Function for Margins* approach, Joe, 1997) which means that we can solve, at least in part, the problem of the number of unknown parameters.

Moreover, it allows us to use a more general joint distribution because we are no longer limited by difficulties relating to the analytical expression of this distribution. We can therefore select any distribution functions (provided that they are continuous) combined with a very general dependence structure.

In this empirical study, the marginal distributions are Pearson IV distributions, while the dependence function is a Student copula that allows dependence in tails (dependence between rare events of the same nature). Furthermore, the dependence parameter matrix is the matrix of the correlations in this case.

Appendix 3

The model: specifications and estimates

As we noted in Appendix 2, copula functions make it possible to separate margins and the dependence structure corresponding to the joint distribution. More specifically, the two stages (Inference Function of Margins) consist in firstly estimating the parameters of the marginal functions and then those of the dependence structure, taking account of the parameters estimated in the first stage.

Specification of the marginal functions

The returns and conditional variances of the financial assets are modelled in such a way as to take account of the stylised facts observed on the markets (presence of asymmetry and fat distribution tails, etc.). Asymmetry is checked for in two ways: differentiating between effects of the shocks on variance by their signs (using a Exponential GARCH, EGARCH process); using an asymmetric distribution. Rare events are taken into account using a fat tail distribution. The Pearson IV distribution (generalised Student or gamma distributions, for example) can be used to check for the presence of fat tails. This distribution was used here due to the results in recent literature. Expected returns are determined autoregressively and the variance of errors is modelled by a traditional :

$$r_{i,t} = m_i + \varphi_i r_{i,t-1} + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} = (h_{i,t})^{1/2}\eta_{i,t}$$

$$\ln h_{i,t} = \alpha_0 + \beta \ln(h_{i,t-1}) + \gamma\eta_{i,t-1} + \alpha_1\left[\left|\eta_{i,t-1}\right| - \sqrt{\frac{2}{\pi}}\right]$$

$$\eta_{i,t} \sim P_{IV}(.;a,q,\delta,\sigma)$$

$$\beta < 1, \forall i = 1,2,3$$

Where: φ_i $(i = 1,2,3)$ are the autoregressive coefficients; β is the parameter that measures the persistence effect of the variance; the influence of the positive (negative) shocks (represented by $\eta_{i,t-1}$), on the variance is measured by $\gamma + \alpha_1$ (resp. $\gamma - \alpha_1$), which represents the coefficient of the $\eta_{i,t-1}$. $|X|$ denotes the absolute value of X and $P_{IV}(.;a,q,\delta,\sigma)$ denotes the Pearson IV distribution. Its reduced centred density is expressed as:

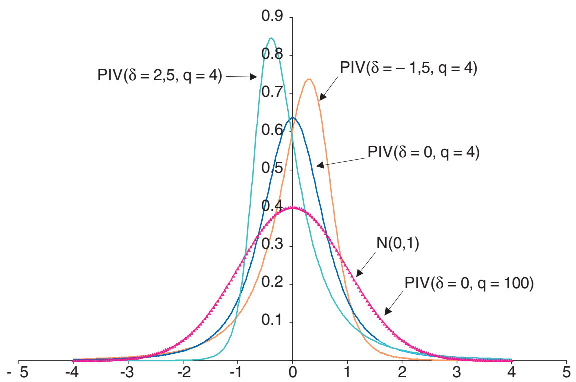
$$f_{IV}(\eta_{i,t};a,q,\delta,\sigma) = \kappa^{-1}\frac{\sigma}{\sqrt{h_{i,t}}}\left[1 + \frac{\sigma}{a}\eta_{i,t} + \frac{\mu}{a}\right]^{2-\frac{2+q}{2}} \exp\left[\delta \tan^{-1}\left(\frac{\sigma}{a}\eta_{i,t} + \frac{\mu}{a}\right)\right]$$

where κ is a normalisation constant

$$\kappa = a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^q(\omega)\exp(\delta\omega)d\omega,\ \mu = \frac{\delta a}{q} \text{ and } \sigma^2 = \frac{a^2(q^2+\delta^2)}{q^2(q-1)}.$$

Comments

- Pearson IV has a parameter, δ, that checks for asymmetry and a parameter q that provides indications of the thickness of distribution tails;
- if $\delta = 0$, the distribution is symmetric; if, furthermore, $q \to \infty$, then $f_{IV}(.;a,q,0,\sigma) \to N(0,1)$;
- parameter a, which appears in the normalisation constant, may have to be fixed at 1 (due to difficulties in identifying the parameters of the model). In fact, we can either fix the constant m_i at zero or fix the parameter a at 1 $(\log(a) = 0)$. We opted for the former solution;
- examples of Pearson IV density functions defined with respect to the values of the parameters q and δ (see Chart) are given here in order to visualise the properties of this distribution:



Specification of the dependence structure

Recent empirical studies (see Longin and Solnik 1998, for example) have shown that some financial markets, equity markets for example, were characterised by a dependence between extreme events (or distribution tails). We thus take into account the fact that bubbles (positive tails) or, inversely, crashes (negative tails) observed on the different markets are interrelated. A copula function that allows dependence between rare events such as the Student function (but not the Gaussian function) is therefore appropriate. Furthermore, the dependence parameter matrix of the Student copula (as well as that of the Gaussian function) is interpreted as a correlation matrix. Consequently, it seems appropriate to choose the latter for analysing the interdependence of markets as it meets the criteria for both the dependence in tails and the interpretation of the dependence parameter matrix. In the case of two markets, the Student function takes the following standard form (see Avouyi-Dovi and Neto (2003) for a more general expression of this function):

$$c_{St} = st(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2); R_t, \nu)$$

where: $st(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2); R_t, \nu)$ denotes the density function of the Student copula, R_t and ν represent the dependence parameter matrix and the degree of freedom (when $\nu \rightarrow \infty$, then the Student copula converges towards a Gaussian copula); t_ν^{-1} the inverse of the univariate Student distribution function and lastly u_1 and u_2 the distribution functions (empirical and theoretical) and the marginal functions (here Pearson IV distributions).

The time-varying correlation matrix is described by an Engle and Sheppard DCC completed by Tse and Tsui (see Appendix 1). In the applications presented here, we have used a DCC model with an autoregressive process of order 1 ($Q=1$) and an empirical correlation matrix lag ($P=1$). R_t is thus expressed as: $R_t = (1-\theta_1-\theta_2)\overline{R} + \theta_1\Psi_{t-1} + \theta_2 R_{t-1}$.

The empirical correlations are calculated on a window with a length of five working days ($m=5$). The estimate of the model is made in two stages: estimate of the parameters of Part 1, followed by the estimate of the parameters of the dependence function taking into account the results of the first stage. We can check the constancy of conditional correlations using the probability ratio test. The null hypothesis H_0 is defined by H_0: $H_0: \ \theta_1 = \theta_2 = 0 \Rightarrow R_t = \overline{R}$. Under H_0, test statistic (W, see table below) follows a Chi2 distribution with 2 degrees of freedom.

Estimate of the EGARCH (1,1) model with a Pearson distribution (a)

	DJ^c_{t-1}	CAC^o_t	DAX
φ_i	0.02790 (0.01996)	– 0.05017 (0.02330)	0.05854 (0.02009)
α_0	0.09068 (0.05833)	0.03599 (0.09894)	0.01829 (0.04358)
α_1	0.19382 (0.06217)	0.17430 (0.06808)	0.17036 (0.04540)
β	0.97989 (0.00557)	0.97141 (0.01076)	0.98598 (0.00413)
γ	– 0.16900 (0.05575)	– 0.07412 (0.01752)	– 0.06263 (0.01871)
a	0.82659 (0.50371)	2.37343 (1.31127)	2.78262 (1.46125)
q	7.53926 (1.59160)	9.36610 (2.37405)	11.09497 (3.25626)
δ	– 1.18037 (0.54115)	– 2.53695 (0.91643)	– 2.44632 (1.14878)

(a) *(o) opening and (c) closing. The figures in brackets represent the standard deviations.*

Estimate of the Student dependence structure (a)

m = 5	DJ^c_{t-1}, CAC^o_t, DAX^o_t
$\rho^\circ_{\text{CAC-DJ}}$	0.64606 (0.02185)
$\rho^\circ_{\text{DAX-CAC}}$	0.77978 (0.01428)
$\rho^\circ_{\text{DJ-DAX}}$	0.60997 (0.02276)
θ_1	0.02638 (0.00539)
θ_2	0.93475 (0.01519)
ν	19.56372 (3.35417)
Moy($\rho_{\text{CAC-DJ.t}}$)	0.59332
Moy($\rho_{\text{DAX-CAC.t}}$)	0.74918
Moy($\rho_{\text{DJ-DAX.t}}$)	0.56164
W	79.28
P-*Value*	0.000

(a) *Estimates are obtained using the IFM method* (Inference Function for Margins) *described in Joe (1997).*

The values of the elements of the correlation matrix associated with the constant correlation assumption H_0, (elements $\rho^\circ_{i,j}$, $i \neq j$, i, j = CAC, DAX, Dow Jones of matrix $\overline{R}$, see Appendix 3), are relatively strong (all greater than 0.50) and significantly different from zero. More specifically, they are all greater than or equal to the averages of the time-varying correlations. If we had chosen constant correlation matrices, and did not reject H_0, we would have obtained *de facto* higher coefficients and overestimated, on average, the degree of correlation between markets. However, the contribution of this matrix (corresponding to H_0) to time-varying correlations is modest due to the weakness of its coefficient in the equation describing $R_t (1 - \theta_1 - \theta_2 = 0.042)$.

The comments regarding the similarity between the averages of non-conditional correlation coefficients and those of the conditional correlation coefficients confirmed the need to test the constant correlation assumption H_0.[8] The results of this test presented in the Appendix (see the statistic W and P *-value*) enable us to reject H_0 at the 1% threshold.

[8] *As constant correlation models and time-varying correlation models are overlapping, we can use a probability ratio test to check H_0 (see Avouyi-Dovi and Neto, 2003).*