International Macroeconomic Policy

When Wealth Affects People's Impatience

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<u>Very Preliminary Version</u>

Abstract

Under a modified neo-classical framework, this paper reexamined the effect of international macroeconomic policies by rejecting the routine assumption of a constant rate of time preference. In the model presented here, we suppose the holdings of real financial wealth will affect people's impatience which has far-reaching implications towards various core issues in international macroeconomics. The introducing of wealth into instantaneous discounting function yields intriguing dynamics of consumption, real balances, and foreign bond holdings. One interesting feature of our model is that stationary rate of time preference no longer necessarily equals real interest rate. We also find that central bank's foreign exchange intervention is not super-neutral even if households capitalize all transfers from the government, which contradicts Obstfeld(1981) in that the distribution of the economy's claims on the rest of the world between the public and the central bank is relevant to the economic performance. The monetary policy affects the real factors, but how the economy behaves in the long run and in the short run differs a lot from Uzawa(1968) and Obstfeld(1981).

Key words: international macroeconomic policy, impatience, foreign exchange, monetary policy

JEL classification: D90, F3, F4,

Part I Introduction

The neoclassical theory of optimal growth assumes that people have stationary time preferences in that they discount the future with a constant exponential rate. However, recent studies (Ainslie [1992]) suggest that people are highly impatient about consuming between today and tomorrow but are much more patient about choices advanced further in the future. Motivated by such findings, Laibson have done a series of works examining intertemporal choices of hyperbolic consumers (Laibson [1994], [1996], [1997], [1998]).

The main problem associated with hyperbolic individuals is the fundamental asymmetry between the present and future selves, which is called the time-inconsistent problem. Individuals are assumed to be composed of conflicting selves --- current self and future selves. Each self is tied to choices of all other selves. At an equilibrium, each self choose optimal strategies given the strategies of all other selves.

Barro [1999] incorporates hyperbolic discounting into the standard Ramsey model, with an re-examination of individual choices under different commitment assumptions. He proves that in the case of no commitment and log utility, the equilibrium features a constant effective rate of time preference and is observationally equivalent to the standard Ramsey model. First he guesses the solution with an undetermined parameter, and then solves the parameter under an intra-personal Nash equilibrium. Note that the first-best choice, characterized by the conventional Hamiltonian system, is never a stable one, since future selves will intrinsically not obey the plans made by current self; instead, all future selves have a tendency to deviate from what previous self has planned because they have better choices under their own beliefs. In the sense of such facts, the only stationary choice (or enforceable consumption plan) is given by an intra-personal Nash equilibrium as in Barro [1999]. The method he uses will be summarized in Section 2, which we will use throughout this paper.

Another problem related to hyperbolic representatives is the uniqueness of the

Nash equilibrium, to which Barro [1999] refers as a footnote after he works out the time-consistent solution. Laibson [1996] has proved the uniqueness of the solution in a discrete-time model, given that the utility function is concave, not just for log utility.

In Barro's analysis, however, the long-run discount rate was assumed to a strictly positive constant, and thus could not explain why different countries have various preference structures. For example, empirical studies imply that people in wealthy countries tend to have a higher discount rate than those in poor countries, and wealthy people are more impatient than poor people. Motivated by such evidence, we assume that the long-run discount rate is endogenously determined by capital. Raising the level of real assets increases the rate of time preference and future consumption. This does not contradict the accepted intuition that savings are a decreasing function of financial wealth as described by the Mundell-Tobin effect. Epstein and Hynes [1983] first offered the intuition for using wealth effects to transform time preference into an endogenous function, but it received only a footnote. They argue that monetary growth raises the opportunity cost of holding real balances, which shifts a positively sloped rate of time preference function down along a negatively sloped marginal product of capital locus. This reduces the real interest rate and increases steady state capital according to the Mundell-Tobin effect.

Part II The Model

1. Households' Choice

The firm and the household can be aggregated as a representative private agent in an economy, whose marginal impatience increases[1] as wealth level expands modeled by an endogenous preference structure. Define the financial wealth of households by

$$a_t \equiv \frac{y}{r} + m_t + F_t ,$$

where y is the real output, which is taken to be exogenous and constant; r

[1] For the purpose of a saddle-point stable equilibrium.

represents the constant world bond rate; m_t denotes real balances; F_t is family's net foreign claims. We suppose households capitalize all future real output. And thus the output flow is equivalent to an interest-baring financial asset with face value y. In each period, the households also receive real net transfers of amount τ_t from the government. Thus the resource constraint is given by

$$\dot{m}_t + \dot{F}_t = y + rF_t + \tau_t - c_t - \pi_t m_t .$$

Using our definition of wealth, we can rewrite the resource constraint as

$$\dot{a}_t = ra_t + \tau_t - c_t - (\pi_t + r)m_t .$$

Individual's problem is given by,

$$\max \quad \int_0^\infty [U(c_t, m_t)] \cdot e^{-\int_0^t \rho(a_s)ds} dt$$

$$s.t. \qquad \dot{a}_t = ra_t + \tau_t - c_t - (\pi_t + r)m_t , \quad \rho'(a) > 0 .$$

$$k(0) = k_0, b(0) = b_0 .$$

In addition, we suppose $U(c,m) = u(c) + v(m)$, $(u')^2 + u''u \le 0$, $(v')^2 + v''v \le 0$ and $3(\rho')^2 \le \rho\rho''$.[2] As in Uzawa(1968) and Obstfeld(1981), we simplify the calculations by introducing the psychological time Δ, which is characterized by $d\Delta = \rho(a_t)dt$[3]. The household's problem now may be expressed as choosing flow of consumption and real balances that satisfy

$$\max \quad \int_0^\infty \frac{[U(c,m)]}{\rho(a)} \cdot e^{-\Delta} dt$$

$$s.t. \qquad \frac{da}{d\Delta} = \frac{ra + \tau - c - (\pi + r)m}{\rho(a)} , \quad \rho'(a) > 0 .$$

[2] These restrictions on utility function and discount function will confirm the strict concavity of our objective function.

[3] Suppose $\rho(\cdot)$ is bounded blow by some positive number, say $\underline{\rho} > 0$. Then Δ will go from 0 to ∞ when t evolves from 0 to ∞.

To ensure the problem has an optimal solution, we adopt the standard assumption of a concave utility function as in standard Ramsey-Cass-Koopmans model.

The necessary conditions for optimality are given by the maximum principle. These require that consumption and real balances be chosen so that for each value of the discount factor Δ, the Hamiltonian

$$H = \frac{U(c,m)}{\rho(a)} + \lambda \frac{[ra + \tau - c - (\pi + r)m]}{\rho(a)} \tag{1}$$

is maximized. In the above equation, $\lambda = \lambda_\Delta$ is the co-state variable, which is interpreted as shadow price, in utility terms, of real assets. The short-run equilibrium is characterized by first order conditions (at the interior maximum) as follows:

$$U_c(c,m) = \lambda \tag{2}$$

$$U_m(c,m) = \lambda(r + \pi) \tag{3}$$

The above two equations imply the usual necessary condition of static utility maximization as in Sidrauski(1967),

$$x(c_t, m_t) \equiv U_m(c_t, m_t) / U_c(c_t, m_t) = \pi_t + r\,, \tag{4}$$

in which the right side represents the opportunity cost of holding real balances instead of consumption. In addition, the shadow price evolves according to the law,

$$\frac{d\lambda}{d\Delta} = \frac{\lambda(\rho - r) + \rho' H}{\rho}. \tag{5}$$

Transform the psychological time into real time, and we will have

$$\dot{\lambda}_t = \lambda_t(\rho(a_t) - r) + \rho'(a_t) H_t \tag{6}$$

Transversality condition is given by

$$\lim_{\Delta \to 0} \lambda a e^{-\Delta} = 0. \tag{7}$$

We assume that the central bank varies τ_t in such a way as to hold the rate of monetary growth $\dot{M}_t / M_t$ constant at some level, say μ.

$$\tau_t = \mu m_t + rR - g. \tag{8}$$

Using the necessary conditions, we may solve for the co-state variable λ_t

$$\lambda_t = \lambda(c_t, m_t) \tag{9}$$

Differentiating both sides of the above equation with respect to time yields

$$\dot{\lambda}_t = \lambda_c \dot{c}_t + \lambda_m \dot{m}_t = \lambda_t(\rho(a_t) - r) + \rho'(a_t)H_t, \tag{10}$$

which may be solved for the time path of consumption, by applying the dynamics of co-state variable along with that of real balances,

$$\dot{c}_t = (\lambda_c)^{-1}\{\lambda_t[\rho(a_t) - r] + \rho'(a_t)H_t - \lambda_m m_t[\mu + r - x(c_t, m_t)]\} \equiv \psi(c_t, m_t, F_t) \tag{11}$$

2. Perfect-Foresight Equilibrium Dynamics

As in Brock(1974) and Obstfeld(1981), we first define by a (differentiable) price-level path $\{\hat{P}_t\}$ and the associated path of nominal transfer payments from the government $\{P_t\tau_t\} = \{\mu e^{\mu t} M_0 + P_t rR - P_t g\}$, where M_0 denotes the nominal money stock at $t = 0$.

The differential equation governing the evolution of real balances in perfect-foresight equilibrium is given by

$$\dot{m}_t = [\mu + r - x(c_t, m_t)]m_t\,. \tag{12}$$

Substitute the above equation in to the flow constraint, and we will have

$$\dot{F}_t = y + r(F_t + R) - c_t - g \tag{13}$$

The steady state $(\overline{c}, \overline{m}, \overline{F})$ is characterized by $\dot{c} = \dot{m} = \dot{F} = 0$, which means

$$\psi(\overline{c}, \overline{m}, \overline{F}) = \mu + r - x(\overline{c}, \overline{m}) = y + r(\overline{F} + R) - \overline{c} - g = 0 \tag{14}$$

Adding the assumption that $\mu + r > 0$ will guarantee the existence of the steady state. Linearizing the three-dimension dynamic system around the steady state $(\overline{c}, \overline{m}, \overline{F})$ yields

$$\dot{c} = \overline{\psi}_c (c - \overline{c}) + \overline{\psi}_m (m - \overline{m}) + \overline{\psi}_F (F - \overline{F}), \tag{15}$$

$$\dot{m} = -\overline{x}_c \overline{m}(c - \overline{c}) - \overline{x}_m \overline{m}(m - \overline{m}), \tag{16}$$

$$\dot{F} = -(c - \overline{c}) + r(F - \overline{F}), \tag{17}$$

in which $\overline{\psi}_c = \rho(\overline{a}) - r + \frac{\overline{\lambda}_m \overline{x}_c \overline{m}}{\overline{\lambda}_c}$, $\overline{\psi}_m = \frac{\overline{\lambda}_m}{\overline{\lambda}_c}[\rho(\overline{a}) - r] + \frac{\lambda}{\overline{\lambda}_c}\rho'(\overline{a}) + \frac{\rho''(\overline{a})\overline{H}}{\overline{\lambda}_c} + \frac{\overline{\lambda}_m \overline{x}_m \overline{m}}{\overline{\lambda}_c}$,

$\overline{\psi}_F = \frac{\overline{\lambda}\rho'(\overline{a}) + \rho''(\overline{a})\overline{H} + \rho'(\overline{a})\overline{H}_F}{\overline{\lambda}_c}$.

Rewrite the dynamic system as

$$\begin{pmatrix} \dot{c} \\ \dot{m} \\ \dot{F} \end{pmatrix} = \mathrm{T} \begin{pmatrix} c - \overline{c} \\ m - \overline{m} \\ F - \overline{F} \end{pmatrix}, \tag{18}$$

where $T = \begin{pmatrix} \overline{\psi}_c & \overline{\psi}_m & \overline{\psi}_F \\ -\overline{x}_c \overline{m} & -\overline{x}_m \overline{m} & 0 \\ -1 & 0 & r \end{pmatrix}$. To prove the stability of the steady state, we calculate the trace and determinant of T repectively,

$trace(T) = \rho(\overline{a}) - \overline{x}_m \overline{m} > 0$,

$$Det(T) = \overline{\psi}_c (-\overline{x}_m \overline{m})r + \overline{\psi}_m (-\overline{x}_c \overline{m})r + \overline{\psi}_F (-\overline{x}_m \overline{m})$$

$$= \frac{\overline{\lambda}_c (\overline{\rho} - r)(-\overline{x}_m \overline{m} r) + \overline{\lambda}_c (\overline{\rho} - r) + \overline{\lambda}\overline{\rho}' r \overline{m}(\overline{x}_c r - 2\overline{x}_m) + \overline{\rho}'' \overline{H}(\overline{x}_c r - \overline{x}_m)}{\overline{\lambda}_c} < 0,$$

which implies that among the three characteristic roots of T, one must be negative and the other two must be positive. Thus there must a saddle-point stable path towards the steady state.

Our next step is to prove the uniqueness of the stable path. Let θ_1 denote the negative root with a corresponding eigenvector $\omega = [\omega_1, \omega_2, \omega_3]'$. Since we have $T\omega = \omega\theta_1$, and then $-\omega_1 = (\theta_1 - r)\omega_3$, which implies ω_1 and ω_3 must be of the same sign. Further, we have $-\overline{x}_c \overline{m}\omega_1 - \overline{x}_m \overline{m}\omega_2 = 0$, and then it must be true that ω_1 and ω_2 also have the same sign. Any convergent solution of the linearized system must take the form

$$\begin{aligned} c_t - \bar{c} &= \omega_1 k \exp(\theta_1 t), \\ m_t - \bar{m} &= \omega_2 k \exp(\theta_1 t), \\ F_t - \bar{F} &= \omega_3 k \exp(\theta_1 t), \end{aligned} \tag{19}$$

where k is determined by the value of F at time $t = 0$. Differentiating the above system yields

$$\begin{aligned} \dot{c}_t &= \theta_1 (c_t - \bar{c}) = \theta_1 (\omega_1 / \omega_3)(F_t - \bar{F}), \\ \dot{m}_t &= \theta_2 (m_t - \bar{m}) = \theta_2 (\omega_2 / \omega_3)(F_t - \bar{F}), \\ \dot{F}_t &= \theta_1 (F_t - \bar{F}), \end{aligned} \tag{20}$$

in which intersection of any two hyperplanes is a saddlepath. In our subsequent studies, we will focus on the consumption-real balances plane.[4]

Part III Macro Policy Analysis

1. Foreign Exchange Intervention

In Obstefeld(1981), he concludes that the central bank's purchasing of foreign claims from the public has no real effects on the economy, and money creation accomplished through a purchase of foreign exchange has the same impact as a "helicopter" money-supply change of equal magnitude. These two policies are intrinsically the same even if the households intend to restore their holding of external claims to the original level, because households capitalize all transfers from the government. But this result is based on the assumption that the dynamics of consumption is related to national holdings of foreign assets, $F+R$, while in our case, the change of public holdings of foreign assets, F, will also affect consumption. Then our assumption will yield much different results from those in Obstfeld(1981).

Assume the central bank purchase a certain mount of foreign claims from the public by domestic money, while keeping money growth constant at initial level. We first find the long run relationships between consumption, real balances, public

[4] The saddle-point stable system is no different from that of Obstfeld(1981). But keep in mind that here the dynamics of consumption is affected by F, not $F+R$ as in Obstfeld(1981), and further the stationary utility level is no longer determined by world interest rate solely, so our model will have much different policy results from what Obstfeld implied.

holdings of foreign assets, and central bank reserve. Differentiating the dynamic system at its long run equilibrium yields

$$\bar{\psi}_c d\bar{c} + \bar{\psi}_m d\bar{m} + \bar{\psi}_F d\bar{F} = 0, \tag{21}$$

$$\bar{x}_c d\bar{c} + \bar{x}_m d\bar{m} = 0, \tag{22}$$

$$r d\bar{F} + r dR = d\bar{c}. \tag{23}$$

Solve for $d\bar{c}$, $d\bar{m}$, $d\bar{F}$ in terms of dR, and we will have

$$d\bar{c} = \frac{\bar{\psi}_F}{A} dR, \tag{24}$$

where $A = \bar{\psi}_c - \bar{\psi}_m \dfrac{\bar{x}_c}{\bar{x}_m} + \dfrac{\bar{\psi}_F}{r} = \bar{\rho} - r - \dfrac{\bar{\lambda}_m(\bar{\rho} - r) + \bar{\lambda}\bar{\rho}' + \bar{\rho}''\bar{H}}{\bar{\lambda}_c} \dfrac{\bar{x}_c}{\bar{x}_m} + \dfrac{2\bar{\lambda}\bar{\rho}' + \bar{\rho}''\bar{H}}{\bar{\lambda}_c r} < 0$,

$$d\bar{m} = -\frac{\bar{x}_c}{\bar{x}_m} d\bar{c}, \tag{25}$$

$$d\bar{F} = \frac{d\bar{c}}{r} - dR = (\frac{\bar{\psi}_F}{Ar} - 1) dR. \tag{26}$$

As we know that $dR = \Delta R > 0$, we have $d\bar{c} > 0$ and $d\bar{m} > 0$. By easy calculation, we notice that $\dfrac{\bar{\psi}_F}{Ar} - 1 = \dfrac{\bar{\psi}_F}{r\bar{\psi}_c - r\bar{\psi}_m \dfrac{\bar{x}_c}{\bar{x}_m} + \bar{\psi}_F} - 1 < \dfrac{\bar{\psi}_F}{\bar{\psi}_F} - 1 = 0$, where the inequality comes from the fact that $\bar{\psi}_F < 0$ and $\bar{\psi}_c - \bar{\psi}_m \dfrac{\bar{x}_c}{\bar{x}_m} < 0$. Thus, we have proven that long run level of public holdings of foreign assets and central bank reserve will move in opposite direction, $d\bar{F} < 0$.

As a result, the central bank's foreign exchange intervention will have real effects towards the economy. In our case, real consumption and households' holdings of real balances will increase, while public holdings of foreign assets will decrease in the long run. But the national gross holdings of foreign assets in terms of $F + R$ will increase, because we observe that $d(\bar{F} + R) = d\bar{F} + dR = \dfrac{d\bar{c}}{r} > 0$. In sum, the intervention is no longer superneutral under our wealth-related preference structure.

From above, we may conclude that the instantaneous utility, $U(\bar{c}, \bar{m})$, will increase in the long run. But as for the welfare, $W \equiv \int_0^\infty [U(\bar{c},\bar{m})] \cdot e^{-\int_0^t \rho(\bar{a})ds} dt = \frac{\bar{U}}{\bar{\rho}}$, there are still more to say since the discount factor is no longer an exogenous constant number. We cannot get further information until we have examined the long run locus of households' financial wealth. Given there is no output shock, we have $d\bar{a} = d\bar{m} + d\bar{F}$, whose sign is ambiguous at first glance because we have $d\bar{m} > 0$ and $d\bar{F} < 0$. Luckily, here we have a proposition, which states that financial wealth will decrease as a whole[5], and thus in the long run, the social welfare will increase as a result of the intervention.

Proposition (Welfare implications of central bank's intervention)

Assume the central bank purchases foreign assets from the public by domestic money. Then the households' total capitalized financial wealth will decrease as a whole in the long run. In addition, the social welfare will increase perpetually.

Next, we examine the economy's short-run behavior. As we have proven the uniqueness of the saddle path, we know that the economy's optimal adjustment path is noncyclical. Thus we can easily describe the short-run behavior from the long-run locus. At the time the purchase occurs, public holdings of foreign assets decrease to $\bar{F} - \Delta R$ suddenly, and then F will rise sluggishly to its long-run level $\bar{F}'$, which has the property that $\bar{F} - \Delta R < \bar{F}' < \bar{F}$ because we have $d\bar{F} < 0$ and $d(\bar{F} + R) > 0$. Since on the optimal adjustment path, consumption, real balances and public holdings of foreign assets will move in the same direction, the consumption and real balances must jump to a level lower then their former long-run level, $(\bar{c},\bar{m})$, at the initial stage, and then rise gradually to their new steady state, $(\bar{c}',\bar{m}')$, along the saddle path. Thus compared to the fixed rate of money growth, the exchange rate exhibits overshooting

[5] See appendix for detailed proof.

initially, causing the price level to rise relative to nominal money supply, and then becomes undershooting from some point in the future; in the long run the inflation rate and money growth rate equals. The current account will run a surplus until the economy reaches its new steady state. In sum, the central bank's foreign exchange intervention will cause exchange rate to fluctuate during the process, even in the long run, and cause the current account to run a surplus in the short run.

2. Monetary Policy

Suppose there is an increase in the money growth rate --- from μ to $\mu' > \mu$. Our case differ from that of Obstfeld(1981)'s in that here in the consumption-real balances plane, the slope of $\dot{c} = 0$ is ambiguous around the steady state, while in Obstfeld(1981), the $\dot{c} = 0$ curve is strictly downward sloping. And thus, the long-run perturbation of this policy needs further examination. Using the same methodology as in the last section, we differentiate the right side of the dynamic system at its stationary state,

$$\overline{\psi}_c d\overline{c} + \overline{\psi}_m d\overline{m} + \overline{\psi}_F d\overline{F} = 0, \tag{27}$$

$$\overline{x}_c d\overline{c} + \overline{x}_m d\overline{m} = d\mu, \tag{28}$$

$$r d\overline{F} = d\overline{c}. \tag{29}$$

By routine calculation, we find that

$$d\overline{c} = C^{-1} d\mu, \tag{30}$$

where $C = \overline{x}_c - \overline{x}_m \overline{\psi}_F / (r\overline{\psi}_m) - \overline{x}_m \overline{\psi}_c / (r\overline{\psi}_m)$, whose sign is ambiguous[6] since we do not know for sure the sign of $\overline{\psi}_c$.

Basically, there are two cases concerning the slope of $\dot{c} = 0$ curve around the steady state.

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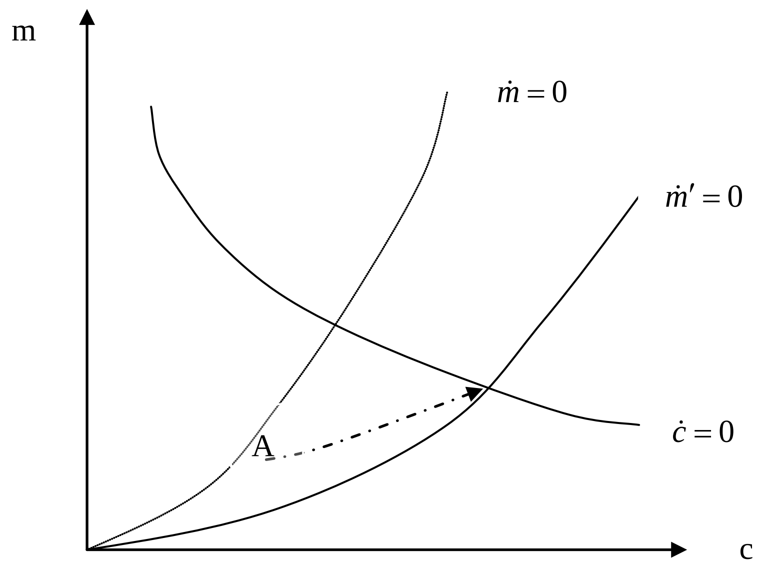
[6] The sum of first two terms of C, $\overline{x}_c - \overline{x}_m \overline{\psi}_F / (r\overline{\psi}_m)$, is strictly positive, while the third term has the same sign as $\overline{\psi}_c$, which is undetermined.

When the policy occurs, the $\dot{m}=0$ will move to the right. Thus we can categorize the new steady state into two cases. If consumption declines in the long run, then the holdings of real balances will also decrease. But if consumption finally rises, whether the steady-state level of real balances will move upward or downward depends on the shape of $\dot{c}=0$ curve between old and new steady states. Figures will make our illustration more apparent.

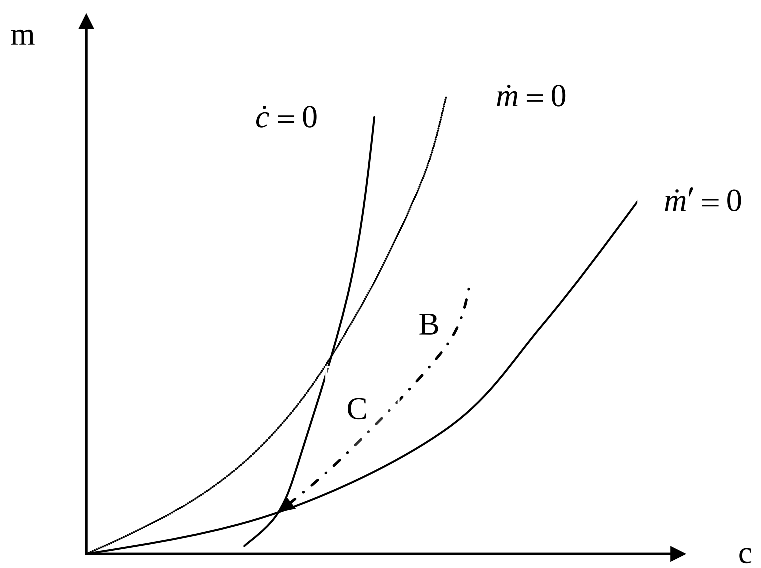
Figure 1.



(Fig. 1) The new steady state has the property that $\overline{c}'>\overline{c}$, and $\overline{m}'<\overline{m}$. We also observe that public holdings of foreign assets will increase to a higher stationary level. Because the variable F cannot jump, the current account must be running a surplus at the initial stage. Thus consumption decreases at first, and then adjusts to the new steady state along the saddle path. As a result, public holdings of real balances also jump initially to a level even lower than the new steady state.

If the foreign exchange rate fully adjusts to the monetary expansion, the real balances will not change. Thus the exchange rate exhibits overshooting during the whole process.

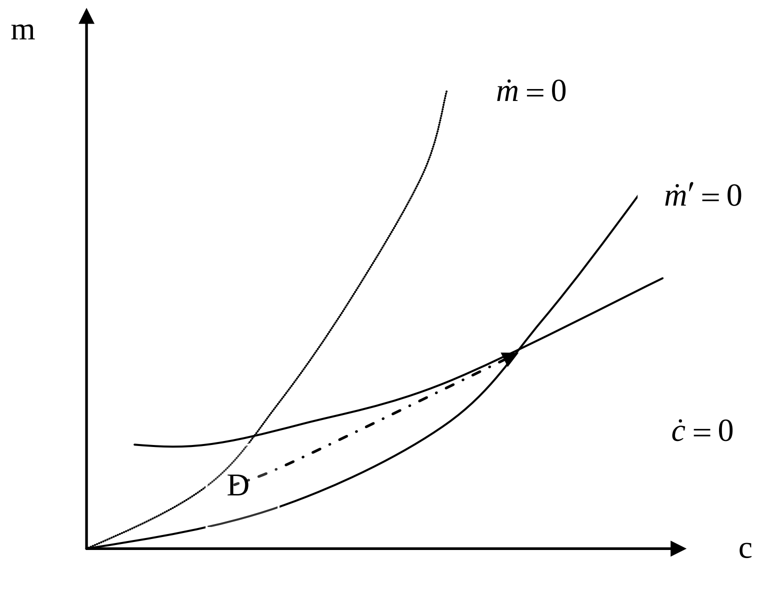
Figure 2.



(Fig. 2) The new steady state has the property that $\bar{c}' < \bar{c}$, and $\bar{m}' < \bar{m}$. Public holdings of foreign assets will decrease to a lower stationary level. Because the variable F cannot jump, the current account must be running a deficit when the monetary policy occurs. Thus consumption jumps to a level higher than $\overline{c}$ at first, and then adjusts to the new steady state along the saddle path. We cannot know for sure the move of the public holdings of real balances at the initial stage, but at the new steady state we have $\overline{m}' < \overline{m}$. The economy adjusts to a point like B or C at time $t = 0$ as the figure shows.

The monetary policy causes consumption to fluctuate during the process, and have ambiguous effect on real balances in the short run. As the real balances decrease at the new steady state, the exchange rate will exhibit overshooting in the long run. It is possible that the exchange rate undershoots in the short run firstly, if the economy moves to a locus like B when the policy takes place. But if the economy is at point C, the exchange rate will exhibit overshooting during the whole process.

Figure 3.



(Fig. 3) The new steady state has the property that $\overline{c}' > \overline{c}$, and $\overline{m}' > \overline{m}$. Public holdings of foreign assets will increase to a higher stationary level. The current account must be running a surplus when the monetary policy occurs. Thus consumption jumps to a level lower than $\overline{c}$ at first, and then adjusts to the new steady state along the saddle path. The real balances will also jump downward at the initial stage, but at the new steady state we have $\overline{m}' > \overline{m}$. The economy adjusts to a point like D at time $t = 0$.

The monetary policy causes both consumption and real balances to fluctuate during the process. As the real balances increase at the new steady state, the exchange rate will exhibit undershooting in the long run. At time $t = 0$, the exchange rate exhibits overshooting.

3. An Increase in Government Consumption

Suppose there is an increase in the government consumption --- from g to $g' > g$. From the dynamic system, we easily see this is equivalent to a permanent rise in central bank reserve --- from R to $R + \Delta g / r$, so the economy will response in a similar manner.

4. Output shock

Suppose there is a permanent increase in the output level. In a traditional Keynes view, consumption, savings and real balances will increase as the disposable income expands. Foreign bond holdings will also increase, which means a current account surplus at the initial stage. To examine whether our model is congruent with traditional results, we differentiate the dynamic system at its long run equilibrium to find how variables respond to the output shock

$$\overline{\psi}_c d\overline{c} + \overline{\psi}_m d\overline{m} + \overline{\psi}_F d\overline{F} + \overline{\psi}_y dy = 0\,, \tag{31}$$

$$\overline{x}_c d\overline{c} + \overline{x}_m d\overline{m} = 0\,, \tag{32}$$

$$dy + rd\overline{F} = d\overline{c}\,. \tag{33}$$

We have the following proposition contradicting to what Keynes told us.

Proposition (Output Shock)[7]

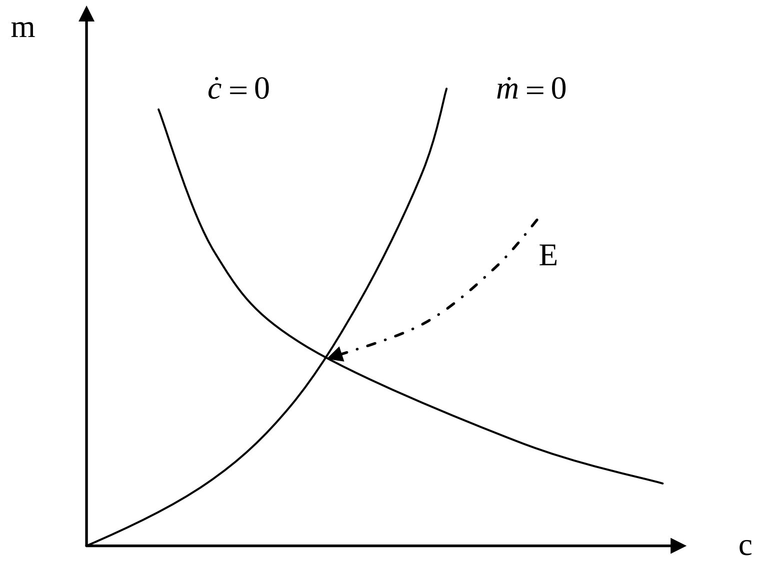
Suppose a permanent positive output shock occurs. This shock will not affect the long run level of consumption and real balances, but has negative effect on the long-run foreign bond holdings.

The intuition behind our findings is that households capitalize all future output flows, which is equivalent to hold an equal amount of another financial asset, say foreign bonds or government bonds, which, in addition, must also earn a fixed interest of r. In our model, the capitalized output flow and foreign bonds are perfect substitutes in that y/r and F appear in the model in terms of $y/r+F$ everywhere. In the short run, foreign bond holdings will adjust slowly towards its new level, but in the long run, the sum of y/r and F will restore to the original level.

Because the foreign bond holdings decrease in the long run, the current account must run a deficit during the adjustment. And thus at the time the output shock occurs, consumption will rise more than the change of output, which is quite different from what traditional Keynes economist told us. The real balances will also jump to a higher level, and then move along the saddle path with consumption towards the original consumption-real balances steady state locus. The point E in figure.4 depicts the first adjustment of the economy. There would be an appreciation of the exchange rate in that price level falls in the short run. To restore the real balances to the original value, the inflation rate will exceed the money growth rate along the adjustment path, which implies high extent exchange rate depreciation.

Figure 4

7 See Appendix for detailed proof.



Part IV Conclusion

Appendix

The second order condition.

Note that the first order condition is sufficient for optimality if $M(c,k) \equiv u(c)/\rho(k)$ and $N(m,k) \equiv v(m)/\rho(k)$ are both concave.

Lemma 1. Suppose the utility function $u(c)$ is strictly increasing and concave. If the instantaneous utility function and discount function further satisfy[8],

$(u')^2 + u''u \le 0$ and $3(\rho')^2 \le \rho\rho''$, then $M(c,k)$ is concave in (c,k).

[8] Note that this condition also implies $\rho'' \ge 0$, and thus the sign of $\rho'(k)$ does not matter for optimality.

Proof: Consider the Hessian matrix of $M(c,k)$

--- $H(M)=\begin{pmatrix} u''/\rho & -u'\rho'/\rho^2 \\ -u'\rho'/\rho^2 & u\cdot[2(\rho')^2-\rho\rho'']/\rho^3 \end{pmatrix}$, whose determinant is given by,

$Det[H(M)]=\{u''u[2(\rho')^2-\rho\rho'']-(u')^2(\rho')^2\}/\rho^4$. From the assumptions in the lemma, we have $-u''u\geq(u')^2\geq 0$ and $\rho\rho''-2(\rho')^2\geq(\rho')^2\geq 0$, which proves that $Det[H(M)]>0$. Along with the condition that $u''<0$, we easily verify the Hessian matrix of $M(c,k)$ is negative definite, which further proves $M(c,k)$ is concave in (c,k).

By the same procedure, we can also show that $N(m,k)\equiv v(m)/\rho(k)$ is concave.

Lemma 2. Suppose the utility function $v(m)$ is strictly increasing and concave. If the instantaneous utility function and discount function further satisfy,

$(v')^2+v''v\leq 0$ and $3(\rho')^2\leq\rho\rho''$, then $N(m,k)$ is concave in (m,k).

From Lemma 1 and Lemma 2, we easily conclude that $[u(c)+v(m)]/\rho(k)$ is concave in (c,m,k) under our assumptions.

Proof of Proposition (Welfare implications of central bank's intervention):

From the definition of households' financial wealth, we have,

$$d\overline{a}=d\overline{m}+d\overline{F}=-\frac{\overline{x}_c}{\overline{x}_m}d\overline{c}+\frac{d\overline{c}}{r}-dR=(-\frac{\overline{x}_c}{\overline{x}_m}+\frac{1}{r}-\frac{\overline{\psi}_c-\overline{\psi}_m\overline{x}_c/\overline{x}_m+\overline{\psi}_F/r}{\overline{\psi}_F})d\overline{c}$$

$$=(-\frac{\overline{x}_c}{\overline{x}_m}-\frac{\overline{\psi}_c-\overline{\psi}_m\overline{x}_c/\overline{x}_m}{\overline{\psi}_F})d\overline{c}=\frac{-\overline{x}_c\overline{\psi}_F-\overline{x}_m\overline{\psi}_c+\overline{\psi}_m\overline{x}_c}{\overline{x}_m\overline{\psi}_F}d\overline{c}\,.$$

Note that the denominator, $\overline{x}_m\overline{\psi}_F$, is strictly positive. Thus we only have to determine the sign of the numerator, which is denoted by B for simplicity,

$$B\equiv-\overline{x}_c\overline{\psi}_F-\overline{x}_m\overline{\psi}_c+\overline{\psi}_m\overline{x}_c=\frac{-2\overline{\lambda}\overline{\rho}'-\overline{\rho}''\overline{H}}{\overline{\lambda}_c}\overline{x}_c-(\overline{\rho}-r)\overline{x}_m+\frac{(\overline{\rho}-r)\overline{\lambda}_m+\overline{\lambda}\overline{\rho}'+\overline{\rho}''\overline{H}}{\overline{\lambda}_c}\overline{x}_c$$

$$= \frac{-\bar{\lambda}\bar{\rho}'}{\bar{\lambda}_c}\bar{x}_c + (\bar{\rho} - r)(\frac{\bar{\lambda}_m}{\bar{\lambda}_c}\bar{x}_c - \bar{x}_m).$$

From the first order conditions, we know that

$x_m = \frac{v''(m)}{u'(c)} = \frac{(\pi + r)\lambda_m}{\lambda}$, and $x_c = \frac{-v'(m)u''(c)}{[u'(c)]^2} = \frac{-(\pi + r)\lambda_c}{\lambda}$. Thus $\frac{\bar{\lambda}_m}{\bar{\lambda}_c}\bar{x}_c = -\bar{x}_m$.

From above, we simplify our calculation as

$$B = \frac{-\bar{\lambda}\bar{\rho}'}{\bar{\lambda}_c}\bar{x}_c - 2\bar{x}_m(\bar{\rho} - r).$$

From $\dot{c} = 0$, we have $\bar{\rho} - r = -\bar{\rho}'\bar{H}/\bar{\lambda} = -\bar{\rho}'(\bar{u} + \bar{v})/(\bar{\rho}\bar{\lambda})$, and thus

$$B = \bar{\rho}'[\frac{\bar{v}'}{\bar{u}'} + 2(\bar{u} + \bar{v})\bar{v}''/(\bar{\rho}\bar{\lambda}\bar{u}')] = \bar{\rho}'[\frac{\bar{\rho}\bar{\lambda}\bar{v}' + 2(\bar{u} + \bar{v})\bar{v}''}{\bar{\rho}\bar{\lambda}\bar{u}'}]$$

$$= \bar{\rho}'[\frac{\frac{\bar{\rho}}{\bar{\pi} + r}(\bar{v}')^2 + \bar{v} \cdot \bar{v}'' + (2\bar{u} + \bar{v})\bar{v}''}{\bar{\rho}\bar{\lambda}\bar{u}'}].$$

Also from $\bar{\rho} - r = -\bar{\rho}'(\bar{u} + \bar{v})/(\bar{\rho}\bar{\lambda}) < 0$, we have $0 < \bar{\rho} < r$, and thus $\frac{\bar{\rho}}{\bar{\pi} + r} < 1$.

Combine the fact that $(v')^2 + v''v \leq 0$, we have

$$B = \bar{\rho}'[\frac{\frac{\bar{\rho}}{\bar{\pi} + r}(\bar{v}')^2 + \bar{v} \cdot \bar{v}'' + (2\bar{u} + \bar{v})\bar{v}''}{\bar{\rho}\bar{\lambda}\bar{u}'}] < \bar{\rho}'[\frac{(\bar{v}')^2 + \bar{v} \cdot \bar{v}'' + (2\bar{u} + \bar{v})\bar{v}''}{\bar{\rho}\bar{\lambda}\bar{u}'}] < 0.$$

We conclude that $d\bar{a}/d\bar{c} < 0$. Using the fact that long-run consumption level rises, we easily see that $d\bar{a} < 0$ --- the households' total capitalized financial wealth decreases in the long run. Since the discount factor is positively related to the financial wealth, we have $d\bar{\rho} < 0$. Note also that $W = \frac{\bar{U}}{\bar{\rho}}$, we conclude that the social welfare will increase.

Proof of Proposition (Output Shock)

Solve for the equations (31), (32) and (33). We will have

$$d\bar{F} = \frac{1}{r}(d\bar{c} - dy), \tag{34}$$

$$d\overline{m} = -\frac{\overline{x}_c}{\overline{x}_m} d\overline{c} \,, \tag{35}$$

$$(\overline{\psi}_c - \overline{\psi}_m \frac{\overline{x}_c}{\overline{x}_m} + \frac{\overline{\psi}_F}{r}) d\overline{c} = (\frac{\overline{\psi}_F}{r} - \overline{\psi}_y) dy \,. \tag{36}$$

But we also have $\overline{\psi}_y = \frac{2\overline{\lambda}\overline{\rho}' + \overline{\rho}''\overline{H}}{r\overline{\lambda}_c} = \frac{\overline{\psi}_F}{r}$, and thus the coefficient of dy is zero in (36). This implies $d\overline{c} = d\overline{m} = 0$, and $d\overline{F} = -dy / r$.