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Abstract

We consider the strategic choice between product innovation and logistic optimization in a novel urban framework where consumers are distributed across the city and have different incomes depending on their location in the town. Depending on the relative efficiency of the product innovation process and the logistic innovation process as well as on the degree of spatial symmetry between the firms, both symmetric and asymmetric business strategy equilibria may arise, as well as both unique and multiple business strategy equilibria.

JEL codes: L13; R12; R41

Keywords: Product innovation; logistic optimization; linear town.

1. Introduction

A frequent decision that firms face in their day-by-day business life concerns which type of innovation to engage on. On one hand, firms would like to improve the quality of their products, as a higher-quality good can be sold at a higher price. The efforts made by a firm in order to improve the quality of the product are usually labelled as "product innovation". Product innovation frequently absorbs a relevant share of the R&D activities of the firms (Cohen and Klepper, 1994). Moreover, product innovation is commonly considered within the business community as a fundamental instrument to increase market share and firms' profits. On the other hand, firms would like to reduce their costs too, as this, all else being equal, directly translates into higher profits. Nowadays, one of the most relevant components of firms' expenditures is represented by transportation costs, that constitute on average one of the top five expenditures for firms,[1] and are accounted for nearly 60% of the overall logistic costs of firms (Hesse and Rodrigue, 2004). A firm that works to minimize its own transportation costs is said to engage in "logistic optimization". Needless to say, transportation costs and logistic optimization are

[1] See for instance www.hitachiconsultin.org and http://www.tompkinsinc.com/.

central issues when firms operating in a spatial environment are considered.

Given that firms' resources are limited, the firms have often to choose whether to engage in product innovation or logistic optimization. Such decision has important implications in terms of how the firms compete in a spatial market. The aim of this paper is to investigate the strategic interaction between the decision to engage in product innovation and the decision to reduce transportation costs when two firms compete in a urban-type framework. While product innovation has been widely investigated in its relation with process innovation (i.e., the reduction of the production costs)[2], as far as we know, it has never been studied in its relation with logistic optimization (i.e., the reduction of the transportation costs)[3]. This represents the first main novelty of our contribution. In order to study the strategic interaction between product innovation and logistic optimization, a standard game-theoretic approach is adopted. Two firms, which are located in two different points of the city (the locations are kept as general as possible), first simultaneously and non-cooperatively decide their business strategy (product innovation or logistic optimization) and then simultaneously set the price. We want

[2] See, among the others, the contributions by Lin and Saggi (2002), Rosenkranz (2003) and Weiss (2003).

[3] Quite surprisingly, logistic optimization itself has received scarce attention by economists even if it is a core strategy for every firm. The only papers considering transportation costs reducing practices by competing firm we are aware of are Dos Santos Ferreira and Thisse (1996) and Hendel and Neiva de Figueiredo (1997).

to shed light on the relationship between the locational distribution of the firms across the city and their decision to invest in high quality products or to decrease the transportation costs. Similarly, we shall show how the degree of spatial symmetry between the two firms impacts over the business strategy equilibrium arising in the game.

The second and novel contribution of this article consists in the urban framework we adopt to investigate the choice between product innovation and logistic optimization. We develop an extension of the familiar Hotelling linear-city model that includes a double heterogeneity of residents. The first dimension of heterogeneity is standard in spatial models: consumers/residents live in different points of the city, as someone is located near to the city centre, while some others are located at the peripheries of the town. The second dimension of heterogeneity concerns the income of the residents, which is supposed to vary across the city's zones. This reflects a common feature of modern towns. The cost of the houses and of the rents varies across the different zones of the town. Allegedly, the income of people living in prestigious zones is expected to be higher than the income of those citizens living in the poorest areas of the town. In this sense, the model we propose here has both a horizontal component and a vertical component: it is horizontal as the residents are distributed along the space; it is vertical as the income is distributed along the residents. Moreover, the distribution of the income is related to the distribution of the residents across the urban space.

We obtain the following results. When the efficiency of product innovation is low with respect to the efficiency of the logistic optimization process, multiple asymmetric business strategy equilibria arise: one firm chooses product innovation, while the rival chooses logistic optimization, and this may happen even if the firms are symmetrically localized in the city. When the efficiency of the product innovation is moderate, only one asymmetric business strategy equilibrium arises, where the firm located at the periphery of the city chooses logistic optimization, while the firm located in the central zone chooses product innovation. Instead, when the efficiency of the product innovation is sufficiently high, as expected, both firms choose product innovation. An implication of these results is that the peripheral firm is more prompt than the central firm to choose logistic optimization instead than product innovation. Moreover, we study the impact of the degree of spatial asymmetry between the firms on the business strategy equilibrium arising in the market. The main result is the following: when the degree of spatial asymmetry between the firms is low, both firms are more likely to choose product innovation, while when the degree of spatial asymmetry is high asymmetric business strategy equilibria are more likely to arise.

Further, we analyse the case of a monopolistic firm endowed with two plants localized in the city and facing the dilemma about where to engage on product innovation and where to engage in logistic optimization. We find that when the efficiency of the product innovation is sufficiently low, the profits maximizing configuration is characterized by the central plant engaging in product innovation and the peripheral plant engaging in logistic optimization.

Finally, we discuss the welfare implications of the different business strategy configurations. We show that the business strategy configuration that maximizes welfare is represented by the symmetric configuration where both firms sell the high-quality good, provided that the efficiency of the product innovation is high with respect to the efficiency of the logistic optimization process. Instead, if the efficiency of the product innovation is low with respect to the efficiency of the logistic optimization process, the optimal configuration consists in the central firm engaging in product innovation, while the peripheral firm should save on the transportation costs.

The rest of the article is structured as follows. In Section 2 we introduce the model. In Section 3 we study the second-stage equilibrium when both firms choose the same business strategy, while in Section 4 we study the second-stage equilibrium when the firms choose different business strategies. In Section 5, we characterize the sub-game Nash perfect equilibrium. In Section 6 we modify the model to consider the case of a multi-plant monopolist. Section 7 considers the welfare implications. Section 8 concludes. All the proofs are relegated in the Appendix.

2. The model

In this section we develop an urban-type framework, where residents are differentiated both in terms of their location in the city and in terms of their income. We consider a linear town, where residents are uniformly distributed. Denote by $x \in [0,1]$ the location of each resident. In the city, two firms (firm *A* and firm *B*) are active. Fixed and marginal costs of both firms are constant and normalized to zero. The location of firm *A* in the city is indicated by $s-d$, while the location of firm *B* is indicated by $s+d$, where $s \in (0,1/2]$ and $d \in (0,s]$. Therefore, the firms' locations in the town are weakly asymmetrically distorted to the left. Parameter *s* measures the degree of symmetry between firms: for given *d*, the higher is *s*, the higher is symmetry: firms are symmetrically localized in the town when $s=1/2$. Parameter *d* instead measures (half of) the distance between the firms: for given *s*, the higher is *d*, the more the firms are distant in the city. Therefore, when $s=1/2$ and $d=0$ the two firms are located in the city centre, when $s=d=0$ both firms are located at the left periphery of the city, and so on. In the rest of the article, we shall often refer to firm *A* (resp. *B*) as the "peripheral" (resp. "central") firm. Firms sustain linear transportation costs to ship the good from the plant to the residents. To ship one unit of the product from plant $s-d$ (resp. $s+d$) to resident *x*, firm *A* (resp. *B*) pays a transport cost equal to: $t|s-d-x|$ (resp. $t|s+d-x|$), where t is the strictly positive unit transport cost. Firms may decide to engage in logistic optimization (L), with to purpose to reduce the transportation costs. Examples of logistic optimization practices include: adopting cost-saving carriers, investing in faster transportation technologies, optimizing the shipment plan… We assume that when a firm engages in L, its transportation costs drop to zero.[4] In this sense, parameter t measures also the efficiency of the logistic optimization process: the higher is t, the stronger is the reduction of the transportation costs due to the logistic optimization process.

As standard in shipping models, firms are able to discriminate among residents (Hamilton et al., 1989). Arbitrage among residents is assumed to prohibitively costly, so it is excluded. Let $p_J(x)$ denote the price schedule charged by firm $J = A, B$. A price schedule refers to a positive valued function $p_J(.)$, with $J = A, B$, defined on $[0,1]$ that specifies the price $p_J(x)$ at which firm J is willing to sell one unit to resident x. In the rest of the article the argument of the price schedules shall be omitted in order to save on notation.

[4] The case where the transportation costs diminish still being positive can be analysed within the same framework. This case would correspond to the more realistic situation where there are factors that are exogenous to the firms, like public expenditure in infrastructure or oil prices. However, assuming that in the case of logistic optimization the transportation costs become zero just simplifies the calculations without affecting qualitatively the results, and therefore this assumption has been maintained throughout the article.

Each resident buys no more than one unit of the good. The utility function of resident x when he buys from firm A (resp. B) is given by: $v + I_A \gamma f(x) - p_A(x)$ (resp. $v + I_B \gamma f(x) - p_B(x)$). Parameter v is assumed to be sufficiently high, so that in equilibrium all residents are served. I_J takes value 1 if firm $J = A, B$ engages in product innovation (*P*), while takes value 0 if firm $J = A, B$ does not engage in product innovation. Therefore, product innovation is modelled here as an improvement of the existing product, such that, all else being equal, the willingness to pay of the residents for the good increases when a firm engages in *P*. Parameter γ is a measure of the product innovation efficiency: all else being equal, the higher is γ, the higher is the increase of the willingness to pay of the residents due to product innovation. We assume the following functional form for function $f(.)$: $f(x) = \begin{cases} x & if \quad 0 \le x \le 1/2 \\ 1 - x & if \quad 1/2 \le x \le 1 \end{cases}$. Function $f(.)$ allows to model different marginal utilities of quality. In particular, the residents located at the periphery of the city ($x = 0$ and $x = 1$) receive zero utility from quality, while the marginal utility of quality is highest for the resident located in the town centre ($x = 1/2$). As shown by Tirole (1988), high-income consumers have a higher marginal utility from quality than low-income consumers. Therefore, the difference of the marginal utility from quality across residents described by $f(.)$ reflects the difference of income across the residents in the city: the inverse U-shape functional form of $f(.)$ adopted here describes a situation where high-income residents reside in the city centre while low-income residents live in the peripheral areas.[5] We assume that R&D costs of improving the quality of the product (i.e. engaging in product innovation) and the R&D costs of reducing the transportation costs (i.e. engaging in logistic optimization) are fixed and equal.[6] As fixed and equal costs do not play any role in the strategic choice between the two business strategies (see later), they shall be disregarded throughout the article.

The timing of the game is the following. In stage 1 the two firms choose simultaneously whether to engage in *P* or *L*,[7] while in stage 2

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[5] This type of vertical differentiation within a horizontal differentiation set-up has been proposed for example by Brekke et al. (2010). Note that in that case, the consumers were not heterogeneous with respect to their willingness-to-pay for quality, as in other models of "pure" vertical differentiation (see for example, Motta, 1993). In contrast, our model is more complete, as we allow also for vertical differentiation in the sense that the residents obtain different marginal utility from quality, while maintaining the heterogeneity with respect to the horizontal dimension (the "space"). In Section 8 we shall discuss the implications of different functional forms of $f(x)$, that is, different income distributions across the city.

[6] Even if not realistic, this assumption allows focusing on the strategic choice between the two managerial decisions (logistic optimization *vs* product innovation) without caring about the relative costs of the two strategies. In fact, the higher is the cost of product innovation, the more likely is logistic optimization in equilibrium, and vice-verse. To avoid taking into account such trivial effects, we simply assume that both strategies have the same implementation costs.

[7] We do not allow for the possibility to engage simultaneously in both types of business strategies. This can be rationalized by assuming that the fixed costs are too high to allow for two profitable innovations (logistic and prod-

the two firms choose simultaneously the price schedule. The subgame perfect Nash equilibrium concept is used in solving the game. As usual, we shall proceed by backward induction.

Finally, we introduce the following assumption on the parameters of the model to maintain the analysis analytically tractable. Let us define $\vartheta \equiv \gamma/t$. Parameter ϑ measures the efficiency of product innovation with respect to the efficiency of the logistic optimization process. We assume that product innovation is neither too efficient nor too inefficient with respect to the efficiency of the logistic optimization process. In particular, we assume that: $\vartheta \in [1-2(s-d), 1]$. That is, we consider a moderately efficient product innovation process with respect to the logistic optimization process. This allows the logistic-optimizing firm to have a positive demand in equilibrium when the rival engages in product innovation (upper bound), and guarantees that the market area of at least one firm is continuous (lower bound).[8]

uct), but are sufficiently low to allow for one profitable innovation in equilibrium. Also, it can be easily shown that the decision not to innovate is always dominated by the decision to engage in one of the two strategies, so it can be excluded.

[8] See later in Section 4 and in the Appendix for more details.

3. Symmetric strategies

In this section, we define the equilibrium price and profits in the second stage of the game when the two firms have chosen the same business strategy in the first stage of the game.

Case (P,P). We consider the case where both firms have chosen P in stage 1 of the game. The equilibrium prices are described in the following proposition:

Proposition 1. *In the case* (P,P)*, the equilibrium prices are*:

$$p_A^{PP}* = p_B^{PP}* = \max\{t|s-d-x|;t|s+d-x|\} \quad (1)$$

Proof. See the Appendix. ■

The equilibrium profits are therefore:

$$\Pi_A^{PP}* = \int_0^s (p_A^{PP}* - t|s-d-x|)dx = td(2s-d) \quad (2)$$

$$\Pi_B^{PP}* = \int_s^1 (p_B^{PP}* - t|s+d-x|)dx = td(2-d-2s) \quad (3)$$

Case (L,L). We consider now the case where both firms have chosen logistic optimization in stage 1 of the game. If both firms choose to engage in logistic optimization, the transportation costs of both firms fall to zero. It follows that no firm has a locational advantage over the other firm, whatever is the location of the residents. It follows that standard Bertrand competition drives the equilibrium price at any location to zero. As a consequence, the equilibrium profits of both firms in the case (L,L) are simply $\Pi_A^{LL}* = \Pi_B^{LL}* = 0$.

4. Asymmetric strategies

In this section, we define the equilibrium price and profits in the second stage of the game, when the two firms have chosen different business strategies in the first stage of the game.

Case (L,P). We first consider the case where the peripheral firm (firm *A*) has chosen *L*, while the central firm (firm *B*) has chosen *P* in the first stage of the game. The following proposition defines the equilibrium price schedules:

Proposition 2. *In the case* (L,P)*, the equilibrium prices are:*

$$p_A^{LP}* = \begin{cases} t|s+d-x| - \gamma f(x) & \text{if} \quad t|s+d-x| \geq \gamma f(x) \\ 0 & \text{if} \quad t|s+d-x| \leq \gamma f(x) \end{cases} \tag{4}$$

$$p_B^{LP}* = \begin{cases} t|s+d-x| & \text{if} \quad t|s+d-x| \geq \gamma f(x) \\ \gamma f(x) & \text{if} \quad t|s+d-x| \leq \gamma f(x) \end{cases} \tag{5}$$

Proof. See the Appendix. ■

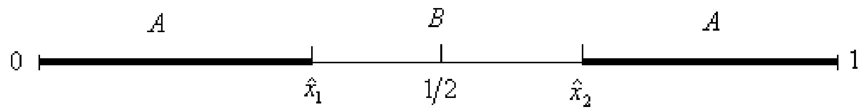
The following proposition determines the urban area served by each firm in equilibrium in case (L, P):

Proposition 3. *Under* (L, P), *firm A serves the residents located at* $x \leq \hat{x}_1$ *and* $x \geq \hat{x}_2$, *while firm B serves the residents located at* $x \in [\hat{x}_1, \hat{x}_2]$, *where* $\hat{x}_1 = \dfrac{t(s+d)}{\gamma + t}$ *and* $\hat{x}_2 = \dfrac{t(s+d)+\gamma}{\gamma + t}$.

Proof. See the Appendix. ■

Figure 1 illustrates the market areas of the two firms in equilibrium: firm *A* serves the residents located at the left of $\hat{x}_1$ and at the right of $\hat{x}_2$, while firm *B* serves the residents located between $\hat{x}_1$ and $\hat{x}_2$.

Figure 1



The demand of firm *A* is therefore: $D_A^{LP} = \hat{x}_1 + 1 - \hat{x}_2 = \dfrac{1-\gamma}{\gamma + t}$, while the demand of firm *B* is: $D_B^{LP} = \hat{x}_2 - \hat{x}_1 = \dfrac{\gamma}{\gamma + t}$. Note that the demand of the logistic-optimizing firm in equilibrium is positive. Also, both $\hat{x}_1$ and $\hat{x}_2$ are interior solutions, as $1 \geq \hat{x}_2 \geq \dfrac{1}{2} \geq \hat{x}_1 \geq 0$. Such equilibrium distribution of the residents between the two firms is due to the fact that when firm *A* engages in logistic optimization, it is equally efficient in serving all residents, whatever is their location in the city. Therefore, firm *A* is able to serve residents both at the left and at the right of firm *B*, but not in the proximity of firm *B*'s location, as here firm *B*'s transportation costs are low too (even if positive). It follows that the equilibrium profits of the two firms are:

$$\Pi_A^{LP} * = \int_0^{\hat{x}_1} p_A^{LP} * dx + \int_{\hat{x}_2}^{1} p_A^{LP} * dx = \frac{t^2[1 + 2d^2 - 2s + 2s^2 - 2d(1-2s)]}{2(\gamma + t)}$$

(6)

$$\Pi_B^{LP}* = \int_{\hat{x}_1}^{\hat{x}_2} (p_B^{LP}* - t|s+d-x|)dx = \frac{\vartheta[\vartheta - t(1-2d-2s)^2]}{4(\gamma+t)} \qquad (7)$$

Case (P, L). We consider now the case where the peripheral firm (firm *A*) has chosen product innovation, while the central firm (firm *B*) has chosen logistic optimization in the first stage of the game. The following proposition defines the equilibrium price schedules:

Proposition 4. *In the case* (P, L) *, the equilibrium prices are:*

$$p_A^{PL}* = \begin{cases} \gamma f(x) & if \quad t|s-d-x| \leq \gamma f(x) \\ t|s-d-x| & if \quad t|s-d-x| \geq \gamma f(x) \end{cases} \qquad (8)$$

$$p_B^{PL}* = \begin{cases} 0 & if \quad t|s-d-x| \leq \gamma f(x) \\ t|s-d-x| - \gamma f(x) & if \quad t|s-d-x| \geq \gamma f(x) \end{cases} \qquad (9)$$

Proof. The proof proceeds as the proof of Proposition 2, therefore it has been omitted. ■

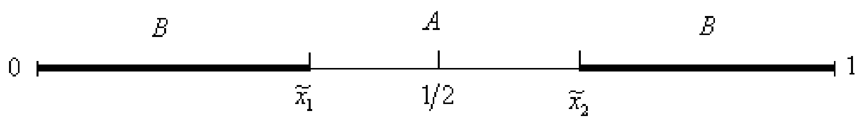
The following proposition determines the urban area served by each firm in equilibrium in case (P, L) :

Proposition 5. *Under* (P, L), *firm B serves the residents located at* $x \le \tilde{x}_1$ *and* $x \ge \tilde{x}_2$, *while firm A serves the residents located at* $x \in [\tilde{x}_1, \tilde{x}_2]$, *where* $\tilde{x}_1 = \frac{t(s-d)}{\gamma + t}$ *and* $\tilde{x}_2 = \frac{t(s-d)+\vartheta}{\gamma + t}$.

Proof. See the Appendix. ■

Figure 2 illustrates the market areas of the two firms in equilibrium: firm *B* serves the residents located at the left of $\tilde{x}_1$ and at the right of $\tilde{x}_2$, while firm *A* serves the residents located between $\tilde{x}_1$ and $\tilde{x}_2$.

Figure 2



The demand of firm *A* is therefore: $D_A^{PL} = \tilde{x}_2 - \tilde{x}_1 = \frac{\gamma}{\gamma + t}$, while the demand of firm *B* is: $D_B^{PL} = \tilde{x}_1 + 1 - \tilde{x}_2 = \frac{1-\gamma}{\gamma + t}$. Note that the demand of the logistic-optimizing firm is positive. Moreover, both $\tilde{x}_1$

and $\tilde{x}_2$ are interior solutions, as $1 \geq \tilde{x}_2 \geq \frac{1}{2} \geq \tilde{x}_1 \geq 0$. Therefore, the equilibrium profits of the two firms are:

$$\Pi_A^{PL}* = \int_{\tilde{x}_1}^{\tilde{x}_2} (p_A^{PL} * -t|s-d-x|)dx = \frac{\gamma[\gamma - t(1+2d-2s)^2]}{4(\gamma+t)} \quad (10)$$

$$\Pi_B^{PL}* = \int_0^{\tilde{x}_1} p_B^{PL} * dx + \int_{\tilde{x}_2}^{1} p_B^{PL} * dx = \frac{t^2[1+2d^2+2d(1-2s)-2s+2s^2]}{2(\gamma+t)}$$
(11)

5. The sub-game perfect equilibrium

We consider now the first stage of the game, where the firms have to choose simultaneously whether to engage in logistic optimization or product innovation. The following pay-off table (Table 1) illustrates the four possible cases.

The following proposition defines the business strategy equilibrium, that is, the business strategy chosen by each firm in the first stage of the game:

Table 1

Π^B \ Π^A	P	L
P	$td(2s-d)$; $td(2-d-2s)$	$\dfrac{\gamma[\gamma-t(1+2d-2s)^2]}{4(\gamma+t)}$; $\dfrac{t^2[1+2d^2+2d(1-2s)-2s+2s^2]}{2(\gamma+t)}$
L	$\dfrac{t^2[1+2d^2-2s+2s^2-2d(1-2s)]}{2(\gamma+t)}$; $\dfrac{\gamma[\gamma-t(1-2d-2s)^2]}{4(\gamma+t)}$	0 ; 0

Proposition 6. *Consider the first stage of the game. Define* $\vartheta_1 \equiv \dfrac{1-2d+4d^2-2s+2s^2}{2d(2s-d)}$ *and* $\vartheta_2 \equiv \dfrac{1-2d+4d^2-2s+2s^2}{2d(2-d-2s)}$*, where* $\vartheta_1 \geq \vartheta_2$*. When* $\vartheta \leq \vartheta_2$*, there are two business strategy equilibria,* (P,L) *and* (L,P)*; when* $\vartheta \in [\vartheta_2, \vartheta_1]$*, there is a unique business strategy equilibrium,* (L,P)*; when* $\vartheta \geq \vartheta_1$*, there is a unique business strategy equilibrium,* (P,P)*.*

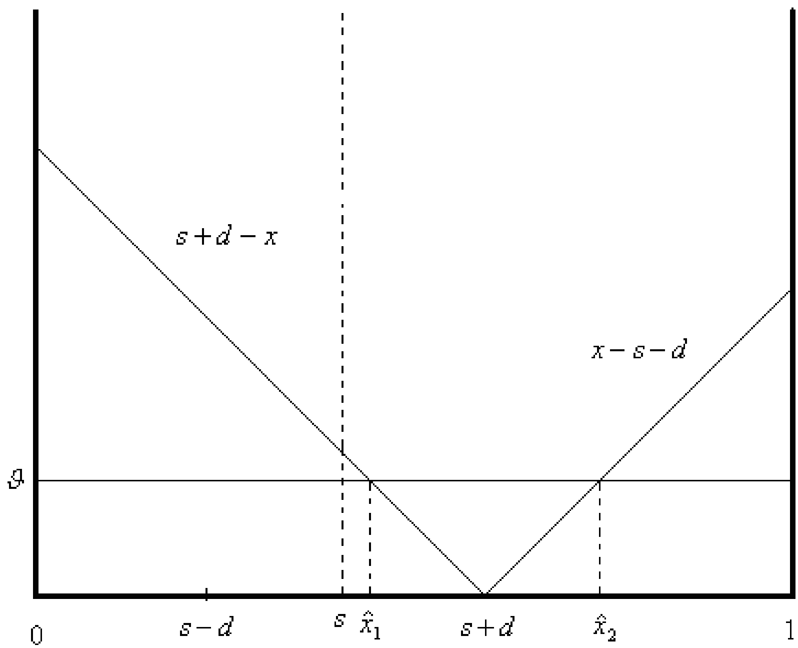
Proof. See the Appendix. ■

Proposition 6 states that when the efficiency of the product innovation is low with respect to the efficiency of the logistic optimization process ($\vartheta \leq \vartheta_2$), we have multiple equilibria. Interestingly, we do not have symmetric business strategy equilibria as one may expect: instead, one firm chooses product innovation, while the rival chooses logistic optimization. Therefore, two *asymmetric* innovation equilibria arise. In particular, note that this happens even if the two firms are symmetrically localized in the city, provided that product innovation is not too efficient. When the efficiency of the product innovation is moderate ($\vartheta_2 \leq \vartheta \leq \vartheta_1$), only one asymmetric business strategy equilibrium arises: the peripheral firm (firm *A*) chooses logistic optimization, while the central firm (firm *B*) chooses product innovation. Finally, when the efficiency of the product innovation is high ($\vartheta \geq \vartheta_1$), only one symmetric innovation equilibrium arises, where both firms choose product innovation.

The intuition is the following. First, suppose that one firm chooses *L*. The dominant strategy for the rival consists in choosing *P*. This is due to the fact that if both firms engage in *L*, spatial differentiation disappears (transportation costs fall to zero, so the firms compete *vis-à-vis* in all points of the city). As a consequence, Bertrand competition determines zero prices everywhere. Suppose now that one firm chooses *P*. Which is the best-reply of the rival? If the rival chooses *P*, the same conditions of a no-innovation equilibrium are obtained, as both firms improve the quality of their product by the same amount. Instead, if the rival chooses *L*, a different situation arises. In order to evaluate the opportunity for an asymmetric business choice by a firm when the other chooses *P*, consider the equilibrium prices as defined in Proposition 2 and Proposition 4. Clearly, the higher is the efficiency of product innovation with respect to the efficiency of logistic optimization (that is, the higher is ϑ), the lower is the price that the firm choosing *L* has to set in order to compensate for the quality difference between the two products. Therefore, all else being equal, the higher is ϑ, the lower is the mark-up that the firm choosing *L* obtains at each location in the city. We call this effect as *price effect*: the *price effect* suggests that one firm should replicate the business strategy of the other firm when it chooses *P*. In particular, the higher is ϑ, the stronger is the *price effect*. However, the *price effect* is not the only effect at work. Consider the market area of the firm choosing *L*. If firm *B* chooses *P* and firm *A* chooses *L*, the market area of firm *A* goes from 0 to $\hat{x}_1$ and from $\hat{x}_2$ to 1.[9] Instead, if firm *A* chooses *P* as the rival, its market area goes from 0 to *s*. Figure 3 illustrates the market area of firm *A* in case (L, P) and in case (P, P).

[9] The case where firm *A* chooses *P* and firm *B* chooses *L* is similar, and therefore it is not discussed here.

Figure 3



In particular, the market area of firm A under asymmetric business strategies is always higher than under symmetric business strategies.[10] This is not a surprise. When firm A chooses L, it can serve

[10] In fact, the market area of firm A in case (L,P) amounts to $\hat{x}_1 + 1 - \hat{x}_2 = 1 - \frac{\gamma}{\gamma + t}$, which is always larger than s, which is the market area of firm A in case (P,P). To prove this, we have to show that

each location of the city with lower transportation costs, and this allows firm *A* to serve also those locations at the right of firm *B*. We call this effect the *demand effect*: the *demand effect* suggests that one firm should not replicate the business strategy of the other firm when the rival chooses *P*. Note that the higher is ϑ, the weaker is the *demand effect*: when product innovation is particularly efficient, firm *A* cannot obtain a large market share even if it does not sustain transportation costs. Therefore, we observe two contrasting effects when a firm has to choose whether to engage in product innovation or in logistic innovation given that the rival is engaging in product innovation. If the product innovation efficiency is high, the *price effect* is strong and the *demand effect* is weak. Therefore, the firm replicates the business strategy of the rival. On the other hand, if the product innovation efficiency is low, the *price effect* is weak and the *demand effect* is strong. Therefore, the firm chooses to engage in logistic optimization when the rival chooses product innovation.

A direct implication of Proposition 6 is that the peripheral firm is more prompt than the central firm to choose logistic optimization when the rival chooses product innovation. In fact, we observe that

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$1-\frac{\gamma}{\gamma+t}>s$, or $1-s>\frac{\gamma}{\gamma+t}$. As the left-hand-side of the last inequality decreases with *s*, we may consider only $s=\frac{1}{2}$. Substituting into the inequality, we have $\frac{1}{2}>\frac{\gamma}{\gamma+t}$, or $1>\frac{\gamma}{t}$, which is true by assumption.

$\vartheta_1 \geq \vartheta_2$, which implies that there is a parameter space where firm *A* chooses *L* when firm *B* chooses *P*, while firm *B* chooses *P* when firm *A* chooses *P*. The higher propensity of the peripheral firm to differentiate its business strategy from the rival follows from the discussion above. Firm *A* is disadvantaged with respect to firm *B* by its peripheral location in the city. Therefore, it benefits more than firm *B* by the reduction of the transportation costs due to the logistic optimization, which allows it to serve also distant residents. In other words, the *demand effect* is stronger for firm *A* than for firm *B*.

We investigate now the effects of spatial symmetry/asymmetry between firms on the equilibrium arising in the market. Therefore, we consider the impact of higher *s* (higher spatial symmetry in the city between the firms) on the critical values ϑ_1 and ϑ_2. We have:

$$\frac{\partial \vartheta_1}{\partial s} = -\frac{1+4d^2-3d+2sd+2s^2}{d(d-2s)^2} \tag{12}$$

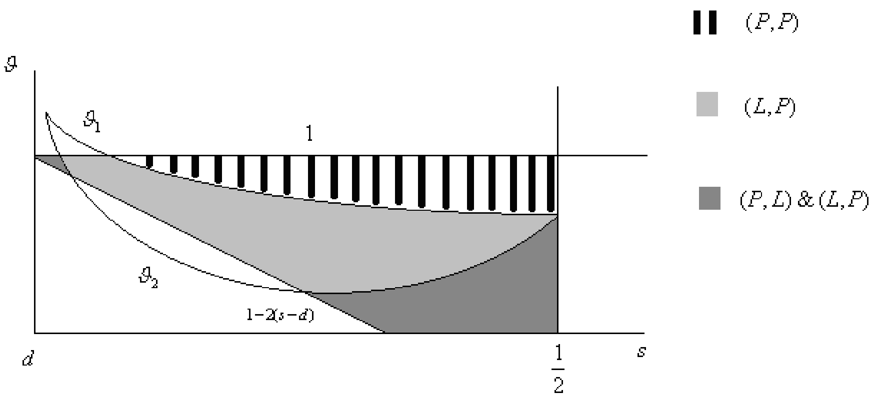
$$\frac{\partial \vartheta_2}{\partial s} = -\frac{1+4d^2-4s+d-2sd+2s^2}{d(2-d-2s)^2} \tag{13}$$

We have that $\partial\vartheta_1/\partial s \leq 0 \;\; \forall s, d$, while $\partial\vartheta_2/\partial s \leq (\geq)\, 0$ if and only if $s \leq (\geq)\, (2-d-\sqrt{2-6d+9d^2})/2$. Moreover, we observe that:

$\vartheta_1(s=1/2)=\vartheta_2(s=1/2)$. Figure 4 illustrates the shape of ϑ_1 and ϑ_2 as a function of s. In Figure 4, the downward-sloping curves represent ϑ_1 and ϑ_2. The horizontal straight line and the downward-sloping straight line limit the admissible values of ϑ (i.e. $\vartheta \in [1-2(s-d),1]$). Figure 4 shows that, depending on the level of spatial symmetry, the conditions on ϑ for the emerging of symmetric or asymmetric business strategy equilibria may vary. When the firms are strongly asymmetric, only asymmetric business strategy equilibria may arise. In particular, if product innovation is efficient enough (that is, $\vartheta \geq \vartheta_2$), there is a unique asymmetric equilibrium, where the peripheral firm chooses logistic optimization, while the central firm chooses the product innovation. Instead, if ϑ is sufficiently low (that is, $\vartheta \leq \vartheta_2$), multiple asymmetric business strategy equilibria are possible. When the firms are sufficiently symmetric in the city, even a symmetric business strategy equilibrium where both firms choose product innovation may arise, provided that product innovation is efficient enough with respect to logistic optimization (that is, $\vartheta \geq \vartheta_1$). However, multiple asymmetric business strategy equilibria or a unique asymmetric business strategy equilibrium are still possible (when $\vartheta \leq \vartheta_1$). Importantly, note that, as ϑ_1 decreases with s and ϑ_2 increases with s when s is sufficiently high, the parameter space supporting the unique asymmetric equilibrium (L,P)

becomes narrower for higher levels of symmetry, and at the limit collapses to zero when firms are perfectly symmetric. On the other hand, the parameter space supporting the symmetric equilibrium (P, P) becomes larger when the firms are more symmetrically localized in the town.

Figure 4



We can summarize the impact of firms' symmetry as follows:

Result 1. *The symmetric business strategy equilibrium where both firms choose product innovation is more likely when the degree of spatial symmetry between the firms is high, while asymmetric business strategy equilibria are more likely to arise when the degree of spatial symmetry between the firms is low. Multiple equilibria may* *happen when firms are strongly symmetric or strongly asymmetric in the city.*

The intuition behind the impact of the degree of symmetry over the critical values of ϑ is the following. Consider ϑ_1: if $\vartheta \geq \vartheta_1$, firm *A* chooses *P* when firm *B* chooses *P*; if $\vartheta \leq \vartheta_1$, firm *A* chooses *L* when firm *B* chooses *P*. As we already pointed out, the choice between product innovation and logistic optimization by firm *A* depends on the relative strength of the *price effect* and the *demand effect*. We also noted that the *demand effect* is weaker the more firm *A* is located near to the city centre. It follows that the higher is *s* the lower is the *demand effect*. Therefore, the higher is *s*, the larger is the parameter space supporting (P, P). Consider now ϑ_2: if $\vartheta \geq \vartheta_2$, firm *B* chooses *P* when firm *A* chooses *P*; if $\vartheta \leq \vartheta_2$, firm *B* chooses *L* when firm *A* chooses *P*. As for firm *A*, the *demand effect* is weaker the more firm *B* is located near to the city centre. It follows that that the higher is *s* the stronger is the *demand effect*. Hence, the higher is *s*, the more is likely that firm *B* differentiates the business strategy from firm *A* when the latter chooses product innovation.[11]

[11] One should note that also a "second-order" effect emerges when *s* increases, and this effect is responsible for the decreasing pattern of ϑ_2 for low levels of *s*. To see this, consider the equilibrium prices of firm *B* (Proposition 2) and the equilibrium market areas (Proposition 3). When *s* increases, the identity of the residents served in equilibrium by firm *B* changes: in fact, the market area at the right of firm *A* shrinks, while the

Figure 5

The analysis developed in this section has clear implications for the locational distribution of firms across the city. For example, Proposition 6 implies that, all else being equal, the peripheral firm benefits more than the central firm from logistic optimization, while the op-

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market area at the left of firm *A* enlarges: that is, residents located at the right of firm *A* are substituted by residents located at the left of firm *A*. Therefore, as the equilibrium prices of firm *B* are higher when the residents are more distant from firm *A*, it turns out that, for low levels of *s*, more profitable residents are substituted by less profitable residents. It follows that the equilibrium profits of firm *B* when it chooses *P* decreases with the degree of spatial symmetry.

posite holds with regard to product innovation. In other words, one should expect to observe high-quality firms located in the centre of the city, while low-quality (but cheaper) firms located in the peripheral zones. This is commonly observed in those cities where the high-income residents live in the city centre. Let us consider for example the distribution across periphery and centre of furniture shops in the city of Milan. In Figure 5 we insert the locations of outlets/showroom of the cheapest furniture (kitchen) firms operating in Italy,[12] and the most expensive furniture firms.[13] The location of low-quality and low-price furniture outlets is indicated with the red arrow in the map, while the location of high-quality and high-price outlets is indicated with the blue arrow. We observe that the firms locating at the peripheral areas of Milan are those firms committed to charging consumers with low prices and offer advantageous conditions for furniture transportation (cheap low-quality firms). On the other hand, the firms having their outlets located at the centre of Milan are those firms which offer high-quality furniture at higher prices (expensive high-quality firms).[14]

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[12] Ikea, with three outlets in the Milan area, Mondo Convenienza, with two outlets, and Record and Classika, with one outlet each.

[13] Boffi, Gaggenau, Strato, Ernesto Meda, Binova and Bulthaup with one outlet each, and Scavolini with two outlets.

[14] It may be noted that some of the firms indicated in the example (e.g. Ikea) offer a transport service in change of a fixed payment within a well-defined geographical area, which configures the model as a mixture of shipping and shopping models, while other firms (e.g. Scavolini) sustain entirely the transportation costs (pure shipping model).

6. Multi-plant monopolist

In this section, the model introduced in Section 2 is slightly modified to consider the case of a monopolistic firm endowed with two plants that has to decide where to develop product innovation and where to engage in logistic optimization.[15] We proceed by considering case by case.

Case (L, L). Suppose that the two plants engage in logistic optimization. It is immediate to see that the optimal price at each location is the price that extracts the whole consumer surplus, which is given by v. As the transportation costs are zero for both plants, from the point of view of the maximization of the joint profits each location can be served indifferently by one of the two plants. It follows that the total profits are given by:

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[15] Note that the monopolist multi-plant firm can engage in both business strategies, while each duopolistic firm can engage in only one strategy (footnote 7). The reason is that even if the fixed costs duplicate when the monopolist engages in two different strategies, its profits are higher too, and we assume they are sufficient to cover the fixed costs in equilibrium. A more subtle question instead is the following: if one plant invests in *P* and improves the quality of the product, why can't it transfer its knowledge to the other plant (the same for the investment in *L*)? We rationalize this assumption by noticing that knowledge is often plant-based and it is costly to transfer it to other plants. Such costs may consists in moving qualified workers and/or machines that allow the product or the logistic innovation. We assume that the costs of sharing knowledge and innovation are sufficiently high to prevent both plants of the multi-plant monopolist to engage in both business strategies.

$$\overline{\Pi}^{LL} = \int_0^1 v dx = v \tag{14}$$

Case (P, P). We consider now the case where both plants are devoted to product innovation. The optimal price at each location is again the price that extracts the whole consumer surplus. In this case the price is higher, as residents are prompt to pay a higher price in order to buy the high-quality product. In particular, the price is now: $v + \gamma f(x)$. Moreover, the two plants sustain positive transportation costs. Therefore, the total profits are maximized when every location is served by the nearer plant. It follows that the expression of the total profits is given by:

$$\overline{\Pi}^{PP}_{s+d \geq 1/2} = \int_0^{s-d} [v + \gamma x - t(s - d - x)]dx +$$
$$+ \int_{s-d}^{s} [v + \gamma x - t(x - s + d)]dx + \int_s^{\frac{1}{2}} [v + \gamma x - t(s + d - x)]dx +$$
$$+ \int_{\frac{1}{2}}^{s+d} [v + \gamma(1 - x) - t(s + d - x)]dx + \int_{s+d}^{1} [v + \gamma(1 - x) - t(x - s - d)]dx$$

if $s + d \geq 1/2$, and by:

$$\overline{\Pi}^{PP}_{s+d\le 1/2} = \int_0^{s-d} [v+\gamma x - t(s-d-x)]dx +$$
$$+\int_{s-d}^{s} [v+\gamma x - t(x-s+d)]dx + \int_s^{s+d} [v+\gamma x - t(s+d-x)]dx +$$
$$+\int_{s+d}^{\frac{1}{2}} [v+\gamma x - t(x-s-d)]dx + \int_{\frac{1}{2}}^{1} [v+\gamma(1-x) - t(x-s-d)]dx$$

if $s+d \le 1/2$. In both cases, the total profits are:

$$\overline{\Pi}^{PP} = \frac{4v+\gamma-2t(1-2d+4d^2-2s+2s^2)}{4} \tag{15}$$

Case (L,P). We consider now the case where the peripheral plant is devoted to logistic optimization, while the central plant is devoted to product innovation. In this case, the optimal price set by plant *A* is *v*, while the optimal price set by plant *B* is $v+\gamma f(x)$. As plant *B* sustains the transportation costs, the mark-up obtained at each location if it is served by the product innovating plant is: $v+\gamma f(x)-t|s+d-x|$. At the opposite, the mark-up obtained at each location if it is served by plant *A* is simply *v*, as plant *A* does not sustain any transportation cost. By comparing the two mark-ups, we get that, from the point of view of the total profits, plant *A* should serve the residents located at the left of $\hat{x}_1$ and at the right of $\hat{x}_2$, while plant *B* should serve the residents located between $\hat{x}_1$ and $\hat{x}_2$, where $\hat{x}_1$ and $\hat{x}_2$ have been defined in Section 4. It follows that the expression of the total profits is given by:

$$\overline{\Pi}^{LP}_{s+d\geq 1/2}=\int_0^{\hat{x}_1} vdx+\int_{\hat{x}_1}^{\frac{1}{2}}[v+\gamma x-t(s+d-x)]dx+$$
$$+\int_{\frac{1}{2}}^{s+d}[v+\gamma(1-x)-t(s+d-x)]dx+$$
$$+\int_{s+d}^{\hat{x}_2}[v+\gamma(1-x)-t(x-s-d)]dx+\int_{\hat{x}_2}^{1} vdx$$

if $s+d\geq 1/2$, and by:

$$\overline{\Pi}^{LP}_{s+d\leq 1/2}=\int_0^{\hat{x}_1} vdx+\int_{\hat{x}_1}^{s+d}[v+\gamma x-t(s+d-x)]dx+$$
$$+\int_{s+d}^{\frac{1}{2}}[v+\gamma x-t(x-s-d)]dx+$$
$$+\int_{\frac{1}{2}}^{\hat{x}_2}[v+\gamma(1-x)-t(x-s-d)]dx+\int_{\hat{x}_2}^{1} vdx$$

if $s+d\leq 1/2$. In both cases, the total profits are:

$$\overline{\Pi}^{LP}=\frac{4vt+\gamma^2+\gamma[4v-t(1-2d-2s)^2]}{4(t+\gamma)} \tag{16}$$

Case (P, L). Finally, we consider the case where the peripheral plant engages in product innovation, while the central plant engages in logistic optimization. In this case, the optimal price set by firm *A* is $v + \gamma f(x)$, while the optimal price set by firm *B* is *v*. Following the same reasoning introduced above, it is immediate to note that, in order to maximize the joint profits, plant *B* should serve the residents located at the left of $\tilde{x}_1$ and at the right of $\tilde{x}_2$, while plant *A* should serve the residents located between $\tilde{x}_1$ and $\tilde{x}_2$, where $\tilde{x}_1$ and $\tilde{x}_2$ have been defined in Section 4. Therefore, the maximum total profits are given by:[16]

$$\overline{\Pi}^{PL} = \int_0^{\tilde{x}_1} v dx + \int_{\hat{x}_1}^{s-d} [v + \gamma x - t(s - d - x)]dx +$$
$$+ \int_{s-d}^{\frac{1}{2}} [v + \gamma x - t(x - s + d)]dx +$$
$$+ \int_{\frac{1}{2}}^{\tilde{x}_2} [v + \gamma(1 - x) - t(x - s + d)]dx + \int_{\tilde{x}_2}^{1} v dx =$$
$$= \frac{4vt + \gamma^2 + \gamma[4v - t(1 + 2d - 2s)^2]}{4(t + \gamma)} \tag{17}$$

We consider now which type of business strategy configuration should be chosen in order to maximize the profits of the multi-plant firm. We state the following proposition:

[16] Note that only the case $s - d \le 1/2$ is possible under (P, L).

***Proposition** 7. If $\vartheta \leq \vartheta_1$, the multi-plant firm chooses (L,P); if $\vartheta \geq \vartheta_1$, the multi-plant firm chooses (P,P).*

Proof. See the Appendix. ■

Therefore, Proposition 7 shows that, depending on parameter ϑ, two profits maximizing configuration may arise: if ϑ is sufficiently high, both plants should engage in product innovation, while if ϑ is sufficiently low, the central plant should engage in product innovation, while the peripheral plant should engage in logistic optimization. By comparing this result with the competitive business strategy equilibrium (Section 5), it is immediate to note that a situation where the central firm engages in logistic optimization while the peripheral firm engages in product innovation is never expected to arise. The intuition is the following. When there is a multi-plant firm, two incentives are at work. On one hand, both plants would like to set a high price. Product innovation, by developing a high-quality product, allows firms to set a higher price and extract more consumer surplus. On the other hand, the plants would like also to save on transportation costs. Logistic optimization, by reducing the cost of shipping the good across the city, decreases the transportation costs. When the increase of quality is sufficiently high relatively to the reduction of the transportation costs ($\vartheta \geq \vartheta_1$) the first effect dominates, and both plants develop a high-quality good in order to extract more consumer surplus, while when the increase of quality is low ($\vartheta \leq \vartheta_1$) the second effect dominates, and only the central plant develops a high-quality good. Clearly, a situation where only the peripheral plant develops a high-quality good is never expected to arise. In fact, it would imply that only the peripheral residents are served with a high quality product. But this is certainly not profit-maximizing, since the residents with the highest willingness to pay for quality are the richest residents, which are located at the city centre. Therefore, in the case where only one plant engages in product innovation, it must be the central plant, as the central residents are the most profitable consumers. Note that in the case of competition between firms, also a situation where only the peripheral firm develops a high-quality product may arise in equilibrium (when $\vartheta \leq \vartheta_2$, see Proposition 6). This is due to the fact that, when the central firm engages in logistic optimization, it is optimal for the peripheral firm to engage in product innovation, *and*, when the peripheral firm engages in product innovation, it is optimal for the central firm to engage in logistic optimization (see the discussion in Section 5). However, as shown in Proposition 7, this situation does not maximize the joint profits and represents a prisoner dilemma, as when $\vartheta \leq \vartheta_2$ (and then also when $\vartheta \leq \vartheta_1$) the joint-profits maximizing configuration is (L, P) .

On the other hand, it is worth to note that the two configurations (L, P) and (P, P) occur under the same circumstances both when the two firms compete and when the two firms act as one (in fact, the threshold of parameter ϑ is ϑ_1 in both cases). In this sense, the result that when product innovation is not extremely efficient the peripheral firm engages in logistic optimization while the central firm engages in product innovation is robust to the case where both plants belong to the same firm.

7. Welfare

In this section we discuss the implications of the different business strategy configurations on welfare. We consider the competitive set-up developed in Section 3, 4 and 5. First, we calculate the consumer surplus under each configuration. Then, we shall evaluate which business strategy configuration maximizes the consumer surplus and welfare.

We consider first the case where both firms engage in logistic optimization, (L, L). As in equilibrium both firms set a price equal to zero at each location, it turns out that the total consumer surplus is given by:

$$CS^{LL} = \int_0^1 v dx = v \tag{18}$$

In the case of asymmetric business strategy configurations, the equilibrium prices are described in Proposition 2 and 4. First, we consider the case where only the central firm engages in product innovation, (L,P). The total consumer surplus is the following:

$$CS^{LP} = \int_0^{\hat{x}_1} (v - p_A^{LP}*)dx + \int_{\hat{x}_1}^{\hat{x}_2} (v + \gamma f(x) - p_B^{LP}*)dx + \\ + \int_{\hat{x}_2}^{1} (v - p_A^{LP}*)dx = v - \frac{t^2[1+2d^2-2s+2s^2-2d(1-2s)]}{2(t+\gamma)} \quad (19)$$

Two things are worth to note. First, consider the market area served by the product-innovating firm, that is, the central urban area between $\hat{x}_1$ and $\hat{x}_2$. The residents located in this area buy a higher quality product. However, they do not enjoy any additional utility from consuming the higher quality product. In fact, the higher surplus due to the fact that the product sold by firm *B* has an higher quality than the product sold by firm *A* is completed offset by the fact that the price set by firm *B*, $\gamma f(x)$, has increased. This is due to the fact that, when the transportation costs of firm *B* are lower than the additional willingness to pay stemming from higher quality, the product-innovating firm can serve the residents by applying a price which is the difference between the willingness to pay for an high-quality product and a low-quality product: that is, $\gamma f(x)$. Second, consider the market areas served by the logistic-optimizing firm, that is, the peripheral urban areas from 0 to $\hat{x}_1$ and from $\hat{x}_2$ to 1. The consumer surplus of a resident located at the left periphery of the city is given by: $v-t(s+d-x)+\gamma x$. It is immediate to see that it continuously increases with x, and that when $x=\hat{x}_1$ it is equal to v. Similarly, the consumer surplus of a resident located at the right periphery of the city is given by: $v-t(x-s-d)+\gamma(1-x)$. The consumer surplus decreases continuously with x, and is equal to v when $x=\hat{x}_2$. At the endpoints of the city, $x=0$ and $x=1$, the consumer surplus is equal to: $v-t(s+d)$ and $v-t(1-s-d)$ respectively. Figure 6 describes the consumer surplus as a function of the location in the city in the case (L,P).

Now, we consider the case where only the peripheral firm engages in product innovation, (P,L). The total consumer surplus is the following:

$$CS^{PL}=\int_0^{\tilde{x}_1}(v-p_B^{PL}*)dx+\int_{\tilde{x}_1}^{\tilde{x}_2}(v+\gamma f(x)-p_A^{PL}*)dx+ \\ +\int_{\tilde{x}_2}^{1}(v-p_B^{PL}*)dx=v-\frac{t^2[1+2d^2-2s+2s^2+2d(1-2s)]}{2(t+\gamma)} \tag{20}$$

Figure 6

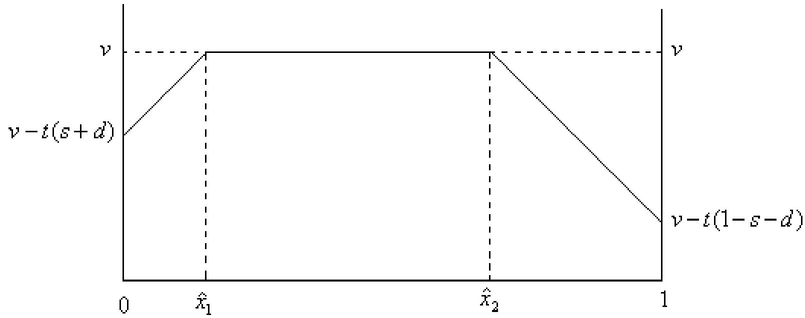
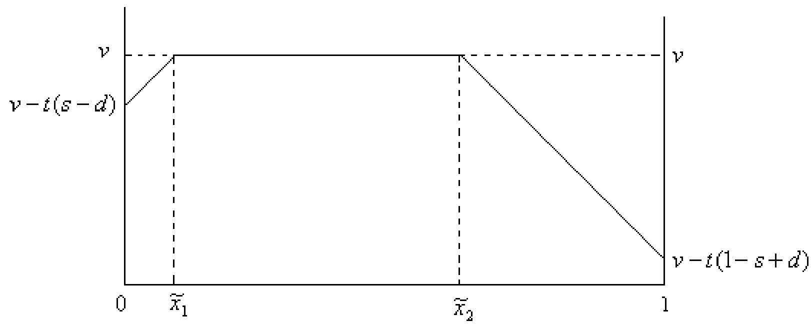


Figure 7



Even in case (P,L), the residents which are served by the product-innovating firm (which now is firm *A*) receives a utility which is equal to *v*, independently on their location in the city. On the other hand, the utility of the residents served by the logistic-optimizing firm (firm *B*) increases with *x* for the residents located between 0 and

$\widetilde{x}_1$, and decreases with x for the residents located between $\widetilde{x}_2$ and 1. Figure 7 represents the consumers' surplus as a function of x in the case (P, L).

We are now in the position to compare the consumer surplus under the different business strategy configurations. First, we can compare CS^{LP} with CS^{PL}. Note that $\widetilde{x}_1 \leq \hat{x}_1$ and $\widetilde{x}_2 \leq \hat{x}_2$. In other words, Figure 7 is simply Figure 6 shifted to the left. It follows that the consumer surplus in case (L, P) is higher than the consumer surplus in case (P, L), or $CS^{LP} \geq CS^{PL}$. The reason is immediate. The surplus of the residents served by the product-innovating firm is the same in both cases. Instead, the surplus of the residents served by the logistic-optimizing firm is different. In fact, in case (P, L), the market area served by the logistic-optimizing firm is more distorted to the right with respect to the market area in case (L, P). As the equilibrium price set by the logistic-optimizing firm increases with the distance between the resident and the product-innovating firm, it follows that the equilibrium prices at the right periphery of the city are higher when the logistic-optimizing firm is firm B. Clearly, the opposite holds at the left periphery of the city. However, as the model is spatially distorted to the left, the reduction of the consumer surplus at the right periphery dominates over the increase of the consumer surplus at the left periphery. It follows that the consumer surplus is higher in case (L, P) than in case (P, L). Consider now the consumer surplus in case (L, L). As the surplus consumer of each resident is equal to *v*, it follows that the total consumer surplus is higher under (L, L) than under (L, P). The reason is the following. Consider the residents located at the peripheries of the city. The equilibrium price is higher under (L, P) than under (L, L). In fact, when both firms engage in logistic optimization, Bertrand competition drives the prices to zero everywhere. Instead, when firm *B* sustains positive transportation costs, firm *A* is able to obtain positive profits over the peripheral residents. Therefore, the equilibrium surplus of the peripheral residents is higher under (L, L) than under (L, P), while the equilibrium surplus of the central residents is *v* under both cases.

We can sum up in the following proposition:

Proposition 8. *The consumer surplus in case* (L, L) *is higher than the consumer surplus in case* (L, P)*, which in turn is higher than the consumer surplus in case* (P, L).

It remains to consider the case where both firms engages in product innovation, (P, P). The equilibrium prices are indicated in Proposition 1. Therefore, the total consumer surplus is given by:

$$CS^{PP} = \int_0^s (v + \gamma f(x) - p_A^{PP}*)dx + \int_s^1 (v + \gamma f(x) - p_B^{PP}*)dx =$$
$$= \frac{4v + \gamma - 2t(1 + 2d - 2s + 2s^2)}{4} \tag{21}$$

In this case, the surplus of the residents buying from firm *A* (i.e. the residents located at $x \leq s$) increases with *x*, as the marginal utility from quality is higher for more central residents, and the equilibrium price is lower due to fiercer competition between the firms for central residents. Symmetrically, the surplus of the residents buying from firm *B* (i.e. the residents located at $x \geq s$) decreases with *x*, as the marginal utility from quality is lower for more peripheral residents, and the equilibrium price is higher due to less fierce competition between the firms for the residents located at the peripheries of the town.[17] Figure 8 illustrates the consumer surplus as a function of *x* when both firms sell a product of higher quality. Figure 8 shows that the consumer surplus is maximum at $x = s$. At the left of *s*, the consumer surplus increases with *x*, while at the right of *s*, the consumer surplus decreases with *s*. More importantly, for the residents

[17] More precisely, for a subset of residents (i.e. the residents located between *s* and 1/2), both the marginal utility from quality and the equilibrium price *increase* with *x*. In terms of consumer surplus, the impact of an higher marginal utility from quality amounts to γ, while the impact of the higher equilibrium price is *t*. Therefore, given the assumption on ϑ, the reduction of the utility due to higher price dominates the increase of utility due to higher marginal utility from quality. This implies that at the right of *s*, the consumer surplus function is piecewise linear (see later Figure 8).

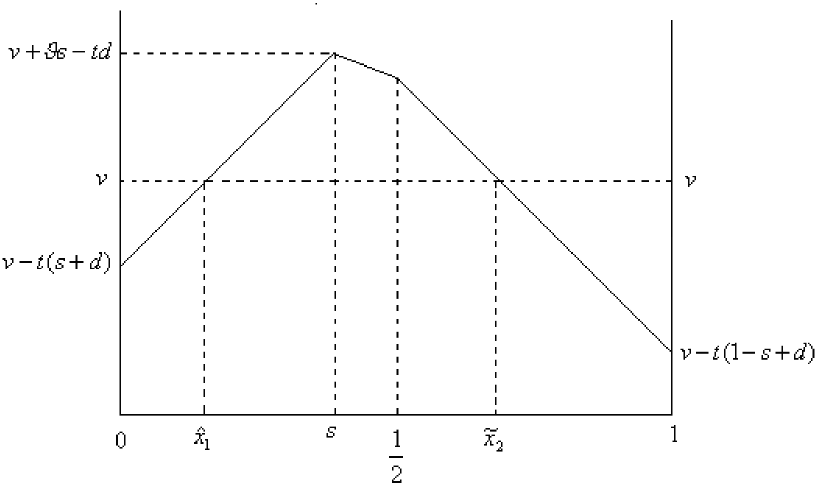
located between $\widetilde{x}_1$ and $\hat{x}_2$, the surplus is higher than *v*, which is the surplus of all residents in case (L, L) and the maximum consumer surplus under the asymmetric configurations (L, P) and (P, L).

The following proposition completes the analysis by comparing the consumer surplus in the case (P, P) with the consumer surplus in the case (L, L):

Proposition 9. *The consumer surplus is higher in case* (L, L) *than in case* (P, P).

Proof. See the Appendix. ■

Figure 8



Proposition 9 shows that total consumer surplus is higher when both firms engages in logistic optimization with respect to the case where both firms engage in product innovation. This is due to the fact that, even if the richer residents located near to the city centre are better off when both firms product a high-quality good, the poorer residents located at the peripheries of the town would prefer a situation where both firms save on the transportation costs (logistic optimization). The latter effect dominates, thus determining that the total consumer surplus is higher in case (L, L) than in case (P, P).[18] Together, Proposition 8 and Proposition 9 show that the best business strategy configuration in terms of total consumer surplus (but not in terms of each resident) is the situation where both firms engage in logistic optimization.

In the last part of this section, we consider welfare, which is defined as the sum of total consumer surplus and the profits of both firms. Using the equilibrium consumer surplus equations ((18), (19), (20) and (21)) and the equilibrium profits equations ((2), (3), (6), (7), (10) and (11)), we have the following welfare equation for each business strategy configuration:

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[18] No simple comparison can be performed between (P, P) and (L, P), and between (P, P) and (P, L).

$$W^{LL} = CS^{LL} + \Pi_A^{LL}* + \Pi_B^{LL}* = v \tag{22}$$

$$\begin{aligned} W^{PP} &= CS^{PP} + \Pi_A^{PP}* + \Pi_B^{PP}* = \\ &= v + \frac{\gamma - 2t(1 - 2d + 4d^2 - 2s + 2s^2)}{4} \end{aligned} \tag{23}$$

$$W^{LP} = CS^{LP} + \Pi_A^{LP}* + \Pi_B^{LP}* = v - \frac{\gamma[t(1 - 2d - 2s)^2 - \gamma]}{4(t + \gamma)} \tag{24}$$

$$W^{PL} = CS^{PL} + \Pi_A^{PL}* + \Pi_B^{PL}* = v - \frac{\gamma[t(1 + 2d - 2s)^2 - \gamma]}{4(t + \gamma)} \tag{25}$$

The following proposition compares the welfare under the different configurations:

Proposition 10. *The welfare in case (L, P) is higher than the welfare in case (P, L), which in turn is higher than the welfare in case (L, L); the welfare in case (L, P) is higher (lower) than the welfare in case (P, P) when $\vartheta \leq (\geq) \vartheta_1$.*

Proof. See the Appendix. ■

Proposition 10 shows that the business strategy configuration that maximizes welfare is represented by the symmetric configuration where both firms sell the high-quality good, provided that the parameter measuring the ratio between the efficiency of the product innovation and the efficiency of the logistic optimization process is sufficiently high. Instead, if the efficiency of the product innovation is particularly low with respect to the efficiency of the logistic optimization process, the optimal business strategy configuration in terms of welfare is asymmetric, with the central firm that should produce the high-quality good, while the peripheral firm should save on the transportation costs. It is immediate to see that the equilibrium competitive equilibrium (Section 5) maximizes welfare, provided that $\vartheta \geq \vartheta_2$.[19] Also, the equilibrium emerging in the case of a multi-plant monopolist (Section 6) always maximizes welfare. However, it should be noted that such welfare-maximizing result of the competitive equilibrium and the monopolistic equilibrium is driven by fact that total profits are maximized. On the contrary, as shown by Proposition 8 and 9, consumer surplus maximization would require a symmetric business strategy equilibrium where both firms engage in logistic optimization.

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[19] If $\vartheta \leq \vartheta_2$, also an asymmetric equilibrium where only the peripheral firm produces the high-quality good may arise: this business strategy configuration is not welfare maximizing, as shown in Proposition 10.

8. Conclusions

This paper investigates the strategic choice between product innovation (developing a higher quality product) and logistic optimization (reducing the transportation costs) within a novel urban-type framework, where residents are distributed along the city and have different income depending on their location in the town. We analyse a two-stage model where the firms first decide which type of business strategy to adopt and then set the prices. We show that when the efficiency of the product innovation process is low with respect to the efficiency of the logistic optimization process, multiple asymmetric business strategy equilibria arise. When the efficiency of the product innovation process is moderate, only an asymmetric business strategy equilibrium arises, where the firm located at the periphery of the city chooses logistic optimization, while the firm located in the central zone chooses product innovation. Instead, when the efficiency of the product innovation process is sufficiently high, both firms in equilibrium choose product innovation. Moreover, we show that the peripheral firm is more prompt than the central firm to choose logistic optimization instead than product innovation. We investigate also the case of a multi-plant monopolist, and we show that when the efficiency of the product innovation process is sufficiently low with respect to the efficiency of the logistic optimization process, the central plant engages in product innovation while the peripheral plant engages in logistic optimization. Finally, we consider the welfare implications, and we show that total welfare is maximized when both firms engages in product innovation (if the efficiency of product innovation is high) or when the central firm engages in product innovation and the peripheral firm engages in logistic optimization (if the efficiency of product innovation is low).

Before concluding, it is worth to note that the functional form of the income distribution across the zones of the city plays a relevant role for the business strategy equilibrium that arises in the first stage of the game. In particular, the fact that high-income residents are located at the centre of the city increases the incentive for the more central firm to engage in product innovation relatively to logistic optimization, as nearer residents are those consumers with the highest willingness to pay for high quality. Therefore, an interesting extension of the present bi-dimensional framework would consist in allowing for more general forms of function $f(x)$. Even if an explicit solution of the model for other functional forms of $f(x)$ goes behind the aim of this article, we can argue the following. Provided that the average income in the city is constant,[20] the flatter is function $f(x)$ (i.e., the more equal is the distribution of income across the city's zones) the larger will be the parameter space supporting the

[20] Clearly, if the average income of the town increases, we shall expect that both firms are more prompt to engage in product innovation, as the higher quality of the products would be evaluated more.

asymmetric business strategies equilibria where one firm engages in product innovation while the other engages in logistic optimization. This is due to the fact that the main incentive of each firm would consist in differentiating from the rival. Indeed, the *price effect* (see Section 5) would be weaker, as the quality difference between the products would evaluated less on average by residents. On the other hand, if function $f(x)$ is U-shape (i.e., high-income consumers reside at the peripheries of the town, while low-income consumers live in the city centre), we expect a situation opposite to the one illustrated in this article: in particular, for intermediate levels of the efficiency of the product innovation process with respect to the efficiency of the logistic optimization process, we should expect that the peripheral firm engages in product innovation, while the central firm engages in logistic optimization.

Clearly, the model we proposed here is open to many improvements and extensions. For example, an interesting extension would be the following. Suppose a dynamic model that runs ad as follows. The firms in the first stage of the model choose the business strategy; in the second stage, they use the first' period profits to cover the fixed costs needed to engage in the business strategy not adopted in the first period. In this case we should expect that the firms change the business strategy in the second period. The interesting question is which business strategy is chosen first: that is, firms are expected to produce "elite" goods at high prices and then try to improve the sell services (by reducing the transportation costs) in order to capture low-income residents or, on the other hand, the firms are expected to start from low prices and then engage to improve the quality of their product to enter in high-income residents segment? The answer to this question needs further research.

Appendix

Proof of Proposition 1. Suppose that *x* is near to firm *A*, that is, $x < s$. Without loss of generality, we assume that if the utility of a consumer is the same when he buys from firm *A* and when he buys from firm *B*, the consumer buys from the nearer firm.[21] This assumption is standard in spatial models, and allows avoiding the technicality of ε-equilibria without affecting the results. Consider firm *B*. First, we show that $p_B^x > t|s+d-x|$ cannot be an equilibrium. When $p_B^x > t|s+d-x|$, the best-reply of firm *A* consists in setting $p_A^x = p_B^x$: the resident *x* buys from firm *A*, which obtains a positive mark-up, as $t|s+d-x| > t|s-d-x|$ since $x < s$. Firm *B* has the incentive to undercut firm *A* by setting a price equal to: $p_B^{x'} = p_B^x - \varepsilon$, where ε is a positive and small number. Since p_B^x is

[21] For more details about this assumption, see for example Hamilton et al. (1989).

higher than $t|s+d-x|$ by hypothesis and ε is a positive and small number by definition, $p_B^{x'}$ is higher than the transportation costs sustained by firm *B* (i.e. the mark-up of firm *B* on consumer *x* is positive). Therefore, $p_B^x > t|s+d-x|$ cannot be an equilibrium, because firm *B* would obtain higher profits by setting $p_B^{x'}$. We now show that $p_B^x = t|s+d-x|$ is an equilibrium. The best-reply of firm *A* is still $p_A^x = p_B^x$. With such a price firm *B* obtains zero profits from resident *x*, which buys from firm *A*, but it has no incentive to change the price, because by increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss (the mark-up would be negative). It follows that $p_A^x = p_B^x = t|s+d-x|$ represents the (unique) price equilibrium. The proof for $x > s$ is symmetric. Finally, when the resident is equally distant from the two firms ($x = s$), the standard Bertrand's result holds: both firms set a price equal to the marginal cost. ■

Proof of Proposition 2. Recall that in the case (L, P) the utility of a resident buying from firm *A* is $v - p_A^x$, while the utility of a resident buying from firm *B* is $v + \gamma f(x) - p_B^x$. Suppose that *x* is such that $t|s+d-x| \geq \gamma f(x)$. Without loss of generality, we assume that in case of equal utility the resident buys from the firm with lower transportation costs, i.e. firm *A*. This assumption – as the successive for the case $t|s+d-x| \leq \gamma f(x)$ – allows avoiding ε-equilibria, but it is without any other consequence. Consider firm *A*. By solving $v - p_A^x = v + \gamma f(x) - p_B^x$, we get that firm *A* serves resident *x* if $p_A^x \leq p_B^x - \gamma f(x)$. First, we show that $p_B^x > t|s+d-x|$ cannot be an equilibrium. When $p_B^x > t|s+d-x|$, the best-reply of firm *A* consists in setting $p_A^x = p_B^x - \gamma f(x)$: the resident *x* buys from firm *A*, which obtains a non-negative mark-up, as $t|s+d-x| - \gamma f(x) \geq 0$ by assumption. Firm *B* has the incentive to undercut firm *A* by setting a price equal to: $p_B^{x'} = p_B^x - \varepsilon$, where ε is a positive and small number. Since p_B^x is higher than $t|s+d-x|$ by hypothesis and ε is a positive and small number by definition, $p_B^{x'}$ is higher than the transportation costs sustained by firm *B* (i.e. the mark-up of firm *B* on resident *x* is positive). Therefore, $p_B^x > t|s+d-x|$ cannot be an equilibrium, because firm *B* would obtain higher profits by setting $p_B^{x'}$. We show that $p_B^x = t|s+d-x|$ is an equilibrium. The best-reply of firm *A* is still: $p_A^x = p_B^x - \gamma f(x)$. With such a price firm *B* obtains zero profits from resident *x*, which buys from firm *A*, but it has no incentive to change the price, because increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss (the mark-up would be negative). It follows that $p_A^x = t|s+d-x| - \gamma f(x)$ and $p_B^x = t|s+d-x|$ represents the (unique) price equilibrium. Suppose now that x is such that $t|s+d-x| \leq \gamma f(x)$. Without loss of generality, we assume that in case of equal utility the resident buys from the firm with the higher quality, i.e. firm *B*. Again, this allows avoiding ε-equilibria, but it is without any other consequence. First, we show that $p_B^x > \gamma f(x)$ cannot be an equilibrium. When $p_B^x > \gamma f(x)$, the best-reply of firm *A* consists in setting $p_A^x = p_B^x - \gamma f(x) - \omega$, with ω being a positive and small number: the resident x buys from firm *A*, which obtains a positive mark-up as $p_B^x > \gamma f(x) > 0$ by assumption. Firm *B* has the incentive to undercut firm *A* by setting a price equal to: $p_B^{x'} = p_A^x - \varepsilon$, where ε is a positive and small number. Since $p_B^x > \gamma f(x) \geq t|s+d-x|$ and ε is a positive and small number by definition, $p_B^{x'}$ is higher than the transportation costs sustained by firm *B* (i.e. the mark-up of firm *B* on residents x is positive). Therefore, $p_B^x > \gamma f(x)$ cannot be an equilibrium, because firm *B* would obtain higher profits by setting $p_B^{x'}$. We show instead that $p_B^x = \gamma f(x)$ is an equilibrium. The best-reply of firm *A* is: $p_A^x = p_B^x - \gamma f(x) = 0$. With such a price firm *A* obtains zero profits from resident x but it has no incentive to change the price, because increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss (the mark-up would be negative). It follows that $p_B^x = \gamma f(x)$ and $p_A^x = 0$ represents the (unique) price equilibrium. ■

Proof of Proposition 3. By solving $t|s+d-x| = \gamma f(x)$, we derive the "limit" residents, i.e. those residents that determine the urban area served by each firm. Four cases are possible:

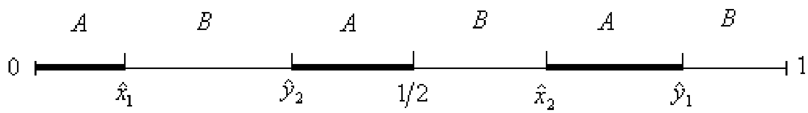
1) $t(s+d-x) = \gamma x$, which yields $\hat{x}_1 = \dfrac{t(s+d)}{\gamma + t}$;

2) $t(s+d-x) = \gamma(1-x)$, which yields $\hat{y}_1 = \dfrac{t(s+d)-\gamma}{t-\gamma}$;

3) $t(x-s-d) = \gamma x$, which yields $\hat{y}_2 = \dfrac{t(s+d)}{t-\gamma}$;

4) $t(x-s-d) = \gamma(1-x)$ which yields $\hat{x}_2 = \dfrac{t(s+d)+\gamma}{\gamma + t}$.

We want to prove that when $\vartheta \in [1-2(s-d),1]$, cases 2) and 3) are not possible. This guarantees that the market area of the product-innovating firm is continuous. Consider case 2). For it to be possible, it must be that $\hat{y}_1 \geq \frac{1}{2}$, or $2(s+d)-1 \geq \vartheta$, which is impossible when $1-2(s-d) \leq \vartheta$, since $1-2(s-d) \geq 2(s+d)-1$. Consider case 3). For it to be possible, it must be that $\hat{y}_2 \leq \frac{1}{2}$, or $1-2(s+d) \geq \vartheta$, which is impossible when $1-2(s-d) \leq \vartheta$, since $1-2(s-d) \geq 1-2(s+d)$. Note that if case 2) and case 3) were possible, the market areas of the two firms would be:



The assumption on ϑ allows avoiding such piecewise configuration of the market areas, while determining a more tractable configuration where the market area of firm *B* is continuous. ■

Proof of Proposition 5. By solving $t|s-d-x| = \gamma f(x)$, we derive the "limit" residents. Four cases are possible:

1) $t(s-d-x) = \gamma x$, which yields $\tilde{x}_1 = \frac{t(s-d)}{\gamma + t}$;

2) $t(s-d-x) = \gamma(1-x)$, which yields $\tilde{y}_1 = \frac{t(s-d)-\gamma}{t-\gamma}$;

3) $t(x-s+d) = \gamma x$, which yields $\tilde{y}_2 = \frac{t(s-d)}{t-\gamma}$;

4) $t(x-s+d) = \gamma(1-x)$ which yields $\tilde{x}_2 = \frac{t(s-d)+\gamma}{\gamma + t}$.

We want to prove that when $\vartheta \in [1-2(s-d),1]$, cases 2) and 3) are not possible. Consider case 2). For it to be possible, it must be that $\tilde{y}_1 \geq \frac{1}{2}$, or $2(s-d)-1 \geq \vartheta$, which is impossible when $1-2(s-d) \leq \vartheta$, since $1-2(s-d) \geq 2(s-d)-1$ or $\frac{1}{2} \geq s-d$. Consider case 3). For it to be possible, it must be that $\tilde{y}_2 \leq \frac{1}{2}$, or $1-2(s-d) \geq \vartheta$, which is impossible. Therefore, the market area of the product-innovating firm is continuous. ■

Proof of Proposition 6. First, we show that if firm *A* chooses *L*, then firm *B* chooses *P*. This amounts to require that $\frac{\gamma[\gamma - t(1-2d-2s)^2]}{4(\gamma+t)} \geq 0$, or $\vartheta \geq (1-2d-2s)^2$, which is always satisfied. Similarly, we show that if firm *B* chooses *L*, then firm *A* chooses *P*. This amounts to require that $\frac{\gamma[\gamma - t(1+2d-2s)^2]}{4(\gamma+t)} \geq 0$, or $\vartheta \geq (1+2d-2s)^2$, which is always satisfied. Suppose now that one firm chooses *P*. We look for the conditions under which choosing the same business strategy, *P*, is the best-reply for the rival. When such conditions are not satisfied, the best-reply is *L*. Suppose first that firm *B* chooses *P*. Let us define the following function:

$\Gamma_1 \equiv \Pi_A^{PP}* - \Pi_A^{LP}*$. Solving $\Gamma_1 \geq 0$ with respect to ϑ, we get that $\Gamma_1 \geq 0$ when $\vartheta \geq \vartheta_1$ and $\Gamma_1 \leq 0$ otherwise, where the root is the following: $\vartheta_1 = \dfrac{1-2d+4d^2-2s+2s^2}{2d(2s-d)}$.

Suppose now that firm *A* chooses *P*. Let us define the following function: $\Gamma_2 \equiv \Pi_B^{PP}* - \Pi_B^{PL}*$. Solving $\Gamma_2 \geq 0$ with respect to ϑ, we get that $\Gamma_2 \geq 0$ when $\vartheta \geq \vartheta_2$ and $\Gamma_2 \leq 0$ otherwise, where the root is the following: $\vartheta_2 = \dfrac{1-2d+4d^2-2s+2s^2}{2d(2-d-2s)}$.

Now we compare the critical values ϑ_1 and ϑ_2. We want to show that $\vartheta_1 \geq \vartheta_2$. As the numerators of the two critical values coincide, it is sufficient to require that the denominator of ϑ_1 is lower than the denominator of ϑ_2. This amounts to require that $2s-d \leq 2-d-2s$, or $s \leq 1/2$, which is always true by assumption. ■

Proof of Proposition 7. First, we show that (P,L) is always dominated by (L,P). Consider the difference between $\overline{\Pi}^{LP}$ and $\overline{\Pi}^{PL}$: $\overline{\Pi}^{LP} - \overline{\Pi}^{PL} = \dfrac{2dt\gamma(1-2s)}{t+\gamma} \geq 0$. Next, we show that (L,L) is always dominated by (L,P). Consider the difference between $\overline{\Pi}^{LP}$

and $\overline{\Pi}^{LL}$: $\overline{\Pi}^{LP}-\overline{\Pi}^{LL}=\dfrac{\gamma[\gamma-t(1-2d-2s)^2]}{4(t+\gamma)}$. We want to show that the numerator is always positive, that is: $\vartheta\geq[1-2(s+d)]^2$. As $\vartheta\geq 1-2(s-d)$, in order to prove that $\overline{\Pi}^{LP}-\overline{\Pi}^{LL}\geq 0$, it is sufficient to show that: $1-2(s-d)\geq[1-2(s+d)]^2$. Note that the left-hand-side of the last inequality increases with *d*, while the right-hand-side decreases with *d*. Therefore, if the inequality holds for the lowest admissible value of *d*, it must hold also for the other values. Substituting $d=0$, we get: $1-2s\geq 0$, which is always verified. Finally, let us define the following function: $\Gamma_3\equiv\overline{\Pi}^{PP}-\overline{\Pi}^{LP}$. Solving $\Gamma_3\geq 0$ with respect to ϑ, we get that $\Gamma_3\geq 0$ when $\vartheta\geq\vartheta_1$ and $\Gamma_3\leq 0$ otherwise. ■

Proof of Proposition 9. We want to prove that $CS^{LL}\geq CS^{PP}$, or $\dfrac{4v+\gamma-2t(1+2d-2s+2s^2)}{4}\geq v$, which amounts to: $\vartheta\geq 2(1+2d-2s+2s^2)$. As $\vartheta\geq 1-2(s-d)$, a sufficient condition for $CS^{LL}\geq CS^{PP}$ is that $1-2(s-d)\geq 2(1+2d-2s+2s^2)$. After simplifications, this condition reduces to: $3+6d-6s+6s^2\geq 0$ which is always verified. ■

Proof of Proposition 10. First, we show that the welfare in case (L,P) is always higher than in case (P,L). This amounts to require that $W^{LP} \geq W^{PL}$, which, after simplifications, reduces to: $(1-2d-2s)^2 \leq (1+2d-2s)^2$, which is always verified. Next, we show that the welfare in case (P,L) is always higher than in case (L,L). This amounts to require that $W^{PL} \geq W^{LL}$, which, after simplifications, reduces to: $\frac{\gamma[\gamma - t(1-2s+2d)^2]}{4(\gamma+t)} \geq 0$. For the last inequality to be verified, it is sufficient that $\vartheta \geq [1-2(s-d)]^2$. As $\vartheta \geq 1-2(s-d)$, it is sufficient to prove that $1-2(s-d) \geq [1-2(s-d)]^2$, which is always true. It remains to compare W^{LP} with W^{PP}. By solving $W^{LP} - W^{PP} = 0$, we obtain the root ϑ_1. In particular, when $\vartheta \leq \vartheta_1$, we have $W^{LP} - W^{PP} \geq 0$, while the opposite holds when $\vartheta \geq \vartheta_1$. ■

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