



## **WORKING PAPER NO. 181**

### *How Does Liquidity Affect Government Bond Yields?*

**Carlo Favero, Marco Pagano, Ernst-Ludwig von Thadden**

**June 2007**



**University of Naples Federico II**



**University of Salerno**



**Bocconi**

**Bocconi University, Milan**



## **WORKING PAPER NO. 181**

### ***How Does Liquidity Affect Government Bond Yields?***

**Carlo Favero<sup>♦</sup>, Marco Pagano<sup>♥</sup>, Ernst-Ludwig von Thadden<sup>♠</sup>**

**June 2007**

#### **Abstract**

The paper explores the determinants of yield differentials between sovereign bonds in the Euro area. There is a common trend in yield differentials, which is correlated with a measure of aggregate risk. In contrast, liquidity differentials display sizeable heterogeneity and no common factor. We propose a simple model with endogenous liquidity demand, where a bond's liquidity premium depends both on its transaction cost and on investment opportunities. The model predicts that yield differentials should increase in both liquidity and risk, with an interaction term of the opposite sign. Testing these predictions on daily data, we find that the aggregate risk factor is consistently priced, liquidity differentials are priced for a subset of countries, and their interaction with the risk factor is in line with the model's prediction and crucial to detect their effect.

**JEL classification:** E43, G12.

**Acknowledgements:** We gratefully acknowledge the comments of Stefan Gerlach, Eugene Kandel, Philippe Martin, Henri Pages, Eli Remolona, Raman Uppal and seminar participants at Bocconi University, the Banque de France, CEMFI (Madrid), the Bundesbank and the ECB-CFS Research Network on "Capital Markets and Financial Integration in Europe". We are grateful to the Fondation Banque de France for providing most of the funding for this project, and to Inquire for additional financial support. Euro MTS, and in particular its CEO Gianluca Garbi, have kindly supplied data and useful comments. Stephan Maier and Michele Galietta provided able research assistance.

<sup>♦</sup> Università Bocconi, IGIER, CSEF and CEPR.

<sup>♥</sup> Università di Napoli Federico II, CSEF and CEPR

<sup>♠</sup> Universität Mannheim and CEPR



# Table of contents

*1. Introduction*

*2. Related literature*

*3. Data and stylized facts*

*4. Theoretical framework*

*5 Empirical evidence*

5.1 The baseline model

5.2 Robustness

*6 Conclusions*

*References*



# 1 Introduction

The European Monetary Union (EMU) in 1999 brought to life an integrated market for fixed-income securities in the Euro-area. EMU eliminated currency risk within this area, and standardization of bond conventions by Euro-area sovereign issuers made public bonds issued by different Euro-area governments close substitutes. Consequently, yield spreads on Euro-area government bonds converged significantly, narrowing from highs in excess of 300 basis points, for certain maturities, to less than 30 basis points across the maturity spectrum less than a year after the introduction of EMU. Yet, despite such convergence, euro-zone government bonds are still not regarded as perfect substitutes by market participants: non-negligible differences in yield levels across countries have remained, to different extents for different issuers and maturities, and they fluctuate over time. Even the bonds issued by AAA-rated issuers are not regarded as perfect substitutes of each other, so that for example French bonds traded in the cash market are not considered as a perfect hedge for positions in Bund futures.<sup>1</sup>

What is the reason for these persistent differentials? One possible explanation is persistent risk differences. Different sovereign issuers are perceived as having different solvency risks, in spite of the provisions of the Stability Pact. A second possible explanation is liquidity. This is indeed the explanation that is often advanced by practitioners.

However, a look at the time-series behavior of Euro-area yield differentials suggests that neither one of these two factors in isolation is likely to provide the full answer. First, as shown below, the yield differentials relative to the German Bund tend to fluctuate together, much more than measures of liquidity, such as bid-ask spreads, do. This suggests that liquidity alone cannot be the full answer, and that there must be other factors driving the differentials' time-series behavior. Such factors are likely to be related to international investment opportunities or risk perceptions. For instance, even if the default risk of the Italian and French governments relative to the German one were very stable over time, a changing world price for risk could induce the implied yield differentials to correlate over time. But this cannot be the full story either. Sizable yield differentials have been observed for several years even within the group of AAA-rated euro-zone countries, even

---

<sup>1</sup>See Pagano and von Thadden (2004) for an account of the integration of European bond markets and for a survey of the relevant literature.

though they have generally narrowed considerably over time. Still, as late as 2002, 10-year AAA-rated Finnish debt yielded on average 20 basis points more than the 10-year German Bund. This suggests that indeed liquidity differences may play a role, as practitioners claim.<sup>2</sup>

As a possible explanation of this phenomenon we develop a simple asset-pricing model with exogenous transactions costs and endogenous liquidity demand. The model is kept deliberately simple in order to isolate the implications of our key assumption of price-elastic liquidity demand, which can be seen already under risk neutrality and using first moments. A more general model with risk and risk aversion is outlined in the appendix. The most important insight of our theoretical analysis is that liquidity matters for pricing, but that it interacts with aggregate factors in a way that is different from what traditional CAPM-like asset pricing models would predict.

CAPM models of liquidity such as Acharya and Pedersen (2004) assume that the demand for liquidity is inelastic, which implies that increased illiquidity must be compensated by higher returns and that this effect is amplified by the impact of market risk on returns. Our model instead assumes that the demand for liquidity is price-elastic and decreases if market risk increases. As a result, it predicts that, although illiquidity increases returns (as in any sensible pricing model), its effect is dampened by an increase in risk. Intuitively, the reason is that in our setting an increase in aggregate risk worsens the risk-return profile of alternative investments (such as private equity) compared to that of marketable ones: these alternative assets are real investment opportunities, for which no price effect compensates for the increased risk. So at times of high risk, investors become less inclined to liquidate marketable assets to move into alternative investments. As the demand for liquidity falls, the more liquid of the marketable assets end up earning a lower premium than in normal times.

---

<sup>2</sup>For instance, the increase of yield differentials relative to the Bund rate in late 1999 was explained as follows: “after having tested the waters of Europe’s smaller bond markets, institutional investors are deciding they’ve had enough . . . declining liquidity in the smaller debt markets is boosting the premiums these countries are having to pay investors compared with the core euro-zone nations” (Wall Street Journal Europe, November 3, 1999). Market practitioners clearly attribute remaining yield differentials to liquidity premia, which are held to be larger in thinner markets, irrespective of their credit rating: “Peripheral issuers in Europe are in trouble: They’re paying a huge liquidity premium’ says Steven Mayor, chief bond strategist at ING Barings in London. He says that their problem comes down to the fact that some still only represent 1% to 2% of the euro-zone issuance” (ibidem).



The empirical implication is that, while both increases in risk and illiquidity should reduce asset prices and drive up their returns, their interaction should have the opposite sign – that is, increase asset prices and dampen the increase in their required return. An estimation that ignores the effect caused by the interaction of liquidity with aggregate risk is likely to underestimate the direct impact of liquidity (as well as that of risk) on prices. We bring these ideas to the data using two years of daily observations on yields and liquidity variables for Euro-area sovereign bonds at the 5-year and 10-year maturities. The results show that a standard proxy for aggregate risk – the yield difference between U.S. corporate bonds and U.S. government bonds at the corresponding maturity – is the single most important explanatory variable for Euro-area yield differentials. Liquidity differentials – as proxied by the difference between the local and the relevant reference bid-ask spread – play a role only in a subset of countries. Whenever it appears with a statistically significant coefficient, the bid-ask spread impacts positively the corresponding yield relative to that of the benchmark, as any asset pricing model would predict. However, different from standard pricing models, its interaction with the aggregate risk factor is negative and precisely estimated. In other words, (i) illiquidity appears to command a premium, as in most of the literature following Amihud and Mendelson (1986), and (ii) the size of such premium is reduced by covariation between the cost of illiquidity and aggregate risk.

The structure of the remainder of this paper is as follows. Section 2 sets the paper in the context of the relevant literature. Section 3 presents the data and describes the stylized facts that emerge from them. Section 4 lays out the model and its predictions. Section 5 presents and discusses the estimation results. Section 6 concludes. In an appendix, we generalize the basic model used in the main text to include uncertainty and risk aversion. A short second appendix describes our data.

## 2 Related literature

This paper adds to a considerable literature on the relation between returns and liquidity. At a theoretical level, two main views have been advanced to explain why liquidity should be priced by financial markets: illiquidity (i) creates trading costs, and (ii) can itself create additional risk. These views are not mutually exclusive, although they have emerged sequentially

in the literature. This paper builds on the first view and develops it in a new direction, which is similar in spirit to that of the second view.

The “trading cost view” holds that illiquid securities must provide investors with a higher expected return to compensate them for their larger transaction costs, controlling for fundamental risk. The prediction here is a cross-sectional one: risk-adjusted expected return must be higher for less liquid securities. This view, first proposed and tested by Amihud and Mendelson (1986), has been the basis of a vast empirical literature. Many subsequent studies of stock-market data have confirmed a significant cross-sectional association between liquidity (as measured by the tightness of the bid-ask spread or trading volume) and asset returns, controlling for risk: among these are Brennan and Subrahmanyam (1996), Chordia, Roll and Subrahmanyam (2000), and Datar, Naik and Radcliffe (1998).

Other studies have focussed on liquidity effects in fixed-income security markets. Here, too, the initiators were Amihud and Mendelson (1991), who showed that the yield to maturity of treasury notes with six months or less to maturity exceeds the yield to maturity on the more liquid treasury bills. Studies on U.S. public debt by, e.g., Warga (1992), Daves and Ehrhardt (1993), Kamara (1994) and Krishnamurthy (2002) confirmed these findings, although using more recent data Strebulaev (2001) found that the yield spread between bills and matched notes is much smaller than previously found, especially when bills are on-the-run. Goldreich, Hanke and Nath (2002) have refined this line of analysis by investigating the impact of expected liquidity on securities’ prices. They analyze the prices of Treasury securities as their liquidity changes predictably, in the transition from on-the-run to the less liquid off-the-run status, and show that the liquidity premium depends on the expected future liquidity over their remaining lifetime rather than on their current liquidity.

The “liquidity risk view”, developed in particular by Pastor and Stambough (2003), highlights that liquidity is priced not only because it creates trading costs, but also because it is itself a source of risk, since it changes unpredictably over time. Since investors care about returns net of trading costs, the variability of trading costs affects the risk of a security. In an important paper, Acharya and Pedersen (2004) show theoretically in a CAPM framework with overlapping generations of investors that liquidity risk should be priced to the extent that it is correlated across assets and with asset fundamentals, and uncover evidence consistent with this prediction. Similarly, Ellul and Pagano (2006) show that the initial underpricing

of IPO shares should compensate investors also for the expected illiquidity and for the liquidity risk that investors face in after-market trading, and not only for fundamental risk and adverse selection problems. Also Gallmeyer, Hollifield and Seppi (2004) propose a model of liquidity risk where traders have asymmetric knowledge about future liquidity, so that less informed investors try to learn from the amount of current trading volume how much liquidity there may be in the future. They show that current liquidity is a predictor of future liquidity risk, and therefore is priced.

Our paper puts forward what may be labeled the “risk-liquidity interaction view”: we point out that liquidity alters the impact of changes in risk on current prices and yields. So here the emphasis is not on liquidity risk (indeed in this approach future liquidity is perfectly anticipated), but rather on the interaction between liquidity and aggregate risk. In our model, changes in aggregate risk affect the liquidity premium that assets with lower transactions costs command. This parallels the work by Vayanos (2004), who also works with constant exogenous transactions costs. In his model, fund managers are subject to withdrawals when their performance falls below a given threshold, and therefore are more likely to liquidate at times of high volatility. This increases the demand for liquidity, and therefore the liquidity premium in times of high volatility. Hence, not only do increased transactions costs increase an asset’s required yield, but this effect is stronger for higher aggregate risk. Our model has the same prediction regarding the direct effect of transaction costs, but the opposite prediction with respect to the interaction term. As explained in the previous section, in our setting higher aggregate risk makes investors less eager to trade away from the existing portfolio of marketable assets, and therefore reduces liquidity demand and liquidity premia. Hence, while Vayanos (2004) predicts that high volatility amplifies the effect of illiquidity, our models predicts a dampening.<sup>3</sup>

On the empirical front, our analysis adds to a growing recent literature on Euro-area yield differentials. Codogno, Favero and Missale (2003) estimate models of Euro-area differentials with both monthly and daily data. Their estimates based on monthly data show that for most countries only international risk factors, and not domestic ones, have explanatory power (the former being proxied by U.S. bond yield spreads and the latter by national

---

<sup>3</sup>It is useful to emphasize that this effect arises from the interaction of liquidity and the common risk factor of government bonds. Concerning asset-specific risk, our model does not deliver more than the standard result that higher default risk increases the yield.

debt/GDP ratios). In their estimates of daily data (that refer to 2002 only), macroeconomic variables are not included because they move too slowly to allow the estimation of the impact of the domestic risk factor. Again, the international factor is statistically significant for most countries, while liquidity (as measured by trading volume) is significant only for France, Greece, the Netherlands and Spain.

Geyer, Kossmeyer and Pichler (2004) estimate with weekly data a multi-issuer state-space version of the Cox-Ingersoll-Ross (1985) model of bond yield spreads (over Germany) for four EMU countries (Austria, Belgium, Italy, and Spain). They find that idiosyncratic country factors have almost no explanatory power, and yield-spread data reflect mainly a single (“global”) factor, whose variation can, to a limited extent, be explained by EMU corporate bond risk (as measured by the spread of EMU corporate bonds over the Bund yield), but by nothing else – in particular not by measures of liquidity. Their measurement of liquidity variables is, however, at best indirect, as they do not use data on bid-ask spreads, but rather derived measures of liquidity, such as issue size and the yield differential between on-the-run and off-the-run bonds. Despite the considerable differences in the methodology and data used, both, Geyer, Kossmeyer and Pichler (2004) and Codogno, Favero, and Missale (2003) agree on the finding that yield differentials under EMU are driven mainly by a common risk (default) factor, related to the spread of corporate debt over government debt, and suggest that liquidity differences have at best a minor role in the time-series behavior of yield spreads. As we shall see, our results, which rely on a more direct measure of liquidity (daily bid-ask spreads), confirm the former result, but also highlight that the effect of liquidity cannot be properly gauged without taking into account its interaction with changes in the common risk factor.

Beber, Brandt, and Kavajecz (2007) complement the MTS data on market prices and bid-ask spreads with daily data on credit default swaps for European sovereign bonds to disentangle empirically the problems of flight to quality and of flight to liquidity. They show that credit risk and liquidity are positively correlated across European sovereign issuers and find, consistent with previous studies, that liquidity typically is of minor importance in explaining yields compared to credit quality. Yet, liquidity becomes an important determinant of yields in times of high market volatility, measured by a U.S. index (VIX) or by a European one (VSTOXX). This result contrasts with our evidence that the liquidity premium on Euro government debt tends to be lower when aggregate risk is higher. This difference in results may arise

from the fact that Beber, Brandt, and Kavajecz (2006) control for country-specific risk, but do not consider aggregate risk factors.<sup>4</sup> In contrast, we model risk as being driven by a common factor, consistent with the results by Codogno, Favero, and Missale (2003) and Geyer, Kossmeyer, and Pichler (2004).<sup>5</sup>

### 3 Data and stylized facts

The data that we use in the empirical analysis concern benchmark bonds' prices and liquidity indicators for the Euro area, observed at daily frequencies for the period from 1 January 2002 to 23 December 2003. The data are collected from the Euro MTS Group's European Benchmark Market trading platform, and refer to a snapshot taken at 11 a.m. (Central European Time) in all market days for the Telematico cash markets. The database contains: (i) the best five bid and ask prices across all markets, (ii) the aggregate quantity of all the outstanding proposals made at the best bid and best ask prices, and (iii) the daily trading volume of each bond on the EBM. From these data we calculate redemption yields, maturities and a range of liquidity-related variables described in the Appendix. We consider Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Portugal and Spain. We do not include Greece and Ireland in the sample, because in 2002 the convergence process to EMU was still ongoing for Greece, while the Euro MTS data for Ireland become available only at a very late stage of our sample.

Table 1 provides descriptive statistics for the yield differentials relative to Germany (for 10-year bonds) and France (for 5-year bonds) and the bid-ask spreads by country. For 10-year benchmark bonds, average yield differentials range from 4.16 and 6.94 basis points for France and the Netherlands to 14.47 and 15.50 basis points for Italy and Portugal respectively, while the range of variation is smaller for 5-year bonds. In both cases, the standard deviation indicates that yield differentials feature considerable time-series variability. The statistics reported in the lower panel indicate that bid-ask spreads are all very tight and stable over time. For 10-year benchmark bonds, average

---

<sup>4</sup>Other work has investigated the high-frequency impact of credit risk on corporate bond spreads: see Longstaff, Mithal, and Neis (2005) and Ericsson and Renault (2006).

<sup>5</sup>Another difference between the two studies lies in the data set, which refers to a largely non-overlapping different time period, and to a set of securities that is not identical.

bid-ask spreads range from 2.52 and 2.86 ticks for Italy and France to 4.60 and 4.87 for Austria and Finland, respectively. German Bunds are the third most liquid bonds after Italian and French ones in the cash market, with a spread of 3.25 ticks. The situation is similar for 5-year bonds.

Figure 1 illustrates the time variation of 10 year yield differentials between each country in our sample and Germany, taken as the reference country. For clarity, we report separately the data for the Netherlands, France and Austria in the upper panel, and for all the remaining countries in the lower panel of the Figure. Yield differentials have a clear tendency to comove. The presence of comovement is confirmed by Table 2, which reports the correlation between yield differentials over the sample period and presents a principal-components analysis. Correlations are very high both within and between groups, and the principal-components analysis shows that the first principal component explains above 90 percent of the variance of the series. Liquidity indicators behave differently. Figure 2 shows the difference in bid-ask spreads observed for benchmark bonds relative to German ones, for the same groupings of countries as those used in Figure 1. The figure reports five-days moving averages of the daily observations to smooth volatility. Clearly, liquidity indicators have a different time pattern from yield differentials. This is confirmed by the correlations and principal-components analysis shown in Table 2. The correlation between differentials in liquidity indicators is much lower than that between yields differentials. Moreover, the principal-components analysis reveals that for liquidity indicators at least six components are needed to explain the same proportion of the total variance as that explained by the first component in the case of yield differentials.

The principal-components analysis of Table 2 shows clearly that there is a common international factor in yield differentials in Europe. In Figure 3 we display the behavior of a variable that is often proposed in the literature as a proxy for this factor: the spread between the yield on 10-year fixed interest rates on swaps and the yield on 10-year US government bonds. There is ample evidence of a common trend in international bond spreads (see, for example, Dungey et al., 2000). The empirical literature on sovereign bond spreads in emerging markets shows that the yield of US government bonds, the slope of the US yield curve and risk indicators on the US bond markets, are the main determinants of sovereign spreads (see, for example, Eichengreen and Mody, 2000; Barnes and Cline, 1997, and Kamin and Von Kleist, 1999, Arora and Cerisola, 2001). Blanco (2001) and Codogno et al. (2003) use proxies for global credit risk derived from the US yield curve in their models

of euro-zone government security yields. Consistent with these findings and with the results of Geyer, Kossmeier and Pichler (2004), Figure 3 shows that this international risk factor is strongly correlated with the first principal component of yield differentials in the Euro area.

## 4 Theoretical framework

We consider a partial-equilibrium model with three dates  $t = 0, 1, 2$ . There are two traded bonds, denoted by  $A$  and  $B$ , and a riskless asset that yields a net return of  $r$  per period. Bond  $i = A, B$  pays its face value  $V$  with probability  $q_i$  and 0 with probability  $1 - q_i$  at date 2 and nothing at date 1. Without loss of generality we assume that  $q_B \leq q_A$  and interpret bond  $A$  as the benchmark. The repayment probabilities of the two bonds are driven by a common factor  $\alpha$ . When this factor increases, both bonds are less likely to repay:  $dq_i/d\alpha \leq 0$ .<sup>6</sup> The purchase price of bond  $i$  at date 0 is  $p_{0i}$ . The bonds can be re-traded at date 1 at a bid price of  $(1 - t_i)p_{1i}$ , and ask price of  $p_{1i}$ , where  $t_i$  is a proportional transactions cost.

A continuum of investors  $h \in [0, 1]$  are ready to invest at date 0 in order to consume at date 2. Investors are risk neutral and maximize date-2 consumption. Investors may want to liquidate their bonds at date 1 to invest in alternative investment opportunities, which comprises non-marketable private equity and human capital. The expected return of these real investments is negatively affected by aggregate factor  $\alpha$ :

$$\bar{r} - e^h \alpha,$$

where  $\bar{r} > 0$  is the maximal expected return and  $e^h \in [0, e]$ , is a person-specific parameter that captures investor  $h$ 's exposure to the aggregate factor. In our risk-neutral setting, the aggregate factor can be anything that depresses the profitability of real investment opportunities as well as reducing the solvency of marketable debt, such as the likelihood or severity of macroeconomic downturns. Accordingly, the individual factor  $e^h$  can be thought as the sensitivity of a person's private investment opportunities to this common factor, as determined for instance by the sector of her family firm. In a model with risk-averse investors and risky investments, the common factor  $\alpha$  would

---

<sup>6</sup>Note that since we work in a risk-neutral setting, there is no need to specify the variance-covariance structure of bond returns. We describe such an extension for the case of risk-averse investors in the appendix.

instead capture aggregate risk, while the individual factor  $e^h$  would measure the investor's exposure to it. We sketch such an extension of the model in the appendix.

The assumption that the common factor  $\alpha$  affects both the returns to marketable assets (in our case, the occurrence of default) and those to non-marketable investments accords with a substantial body of evidence that finds a strong positive correlation between them, starting with the seminal paper on private equity returns by Moskowitz and Vissing-Jorgensen (2002). In our empirical analysis, we will proxy the factor  $\alpha$  by the difference between the U.S. corporate swap rate and the treasury yield, a measure typically used to describe aggregate corporate risk (although it also captures other macroeconomic factors).

As the investor's exposure to the aggregate factor,  $e^h$ , increases over its support, the expected return of  $h$ 's investment opportunity falls. We assume that  $e^h$  satisfies the law of large numbers, so that uncertainty washes out in the aggregate and individual probabilities are the same as aggregate frequencies. Let us denote by  $G$  the differentiable cumulative distribution function of  $e^h$  and by  $g = G'$  its density function.

At date 1, an investor will liquidate his holdings of bond  $i$  to invest in his outside investment opportunity if and only if the latter's expected return exceeds the expected return of the bond over its residual life, net of transactions costs:

$$\bar{r} - e^h \alpha > \frac{q_i V}{(1 - t_i) p_{1i}}. \quad (1)$$

Hence, investor  $h$  will sell his holdings of bond  $i$  with ex-ante probability

$$\pi_i = \text{prob} \left( e^h < \frac{\bar{r}}{\alpha} - \frac{q_i V}{\alpha(1 - t_i) p_{1i}} \right) \quad (2)$$

$$= G \left( \frac{\bar{r}}{\alpha} - \frac{q_i V}{\alpha(1 - t_i) p_{1i}} \right). \quad (3)$$

Note that here investors liquidate their assets as a function of changing investment opportunities, and therefore as a function of market prices and yields. In other words, in this model demand for liquidity is price-elastic.

At the interim date, investors will buy bond  $i$  only if (1) does not hold and if the bond does not yield less than the safe investment, i.e. if its price is sufficiently low:

$$p_{1i} \leq \frac{q_i V}{1 + r} \equiv v_i. \quad (4)$$



Hence,  $p_{1i} = v_i$  is the maximum possible price for an equilibrium at date 1 to exist. Necessary for the existence of an equilibrium is therefore that aggregate demand for liquidity at this price is positive, i.e. that  $\bar{r}$  is sufficiently large. If  $\bar{r}$  were too small, even the investors with the most profitable investment opportunity would value them less than their bond holdings and no investor would ever want to sell bonds at date 1.

No-arbitrage implies that the expected yields of all assets are equalized in equilibrium. Hence (4) must hold with equality,<sup>7</sup> implying:

$$p_{1i} = v_i, i = A, B, \quad (5)$$

$$\pi_i = G\left(\frac{\bar{r}}{\alpha} - \frac{1+r}{\alpha(1-t_i)}\right). \quad (6)$$

In particular, the discount rate between dates 1 and 2 is  $r$ , the same as between 0 and 1. The expected payoff of bond  $i$  at date 0 therefore is

$$p_{0i} = \pi_i \frac{(1-t_i)p_{1i}}{1+r} + (1-\pi_i) \frac{q_i V}{(1+r)^2} = \frac{(1-\pi_i t_i)q_i V}{(1+r)^2},$$

because investors have rational expectations over their future decision to either sell (which occurs with probability  $\pi_i$ ) or hold on to the bond (which occurs with probability  $1-\pi_i$ ). So the bond's pledged yield to maturity is

$$1 + Y_i = \frac{V}{p_{0i}} = \frac{(1+r)^2}{(1-\pi_i t_i)q_i}$$

The yield ratio between the two bonds is simpler to calculate than the yield differential. It is given by

$$\frac{1 + Y_B}{1 + Y_A} = \frac{1 - \pi_A t_A}{1 - \pi_B t_B} \cdot \frac{q_A}{q_B}.$$

By using the approximation  $\ln(1+x) \approx x$  the yield differential at date 0 can be approximated simply by:

$$\Delta Y = Y_B - Y_A \approx \pi_B t_B - q_B - \pi_A t_A + q_A. \quad (7)$$

This expression for the yield differential is intuitive. First, there is a direct positive impact of transactions costs. If  $\pi_i$  were constant, that is,

---

<sup>7</sup>In a closed economy with no short-selling allowed there could be an equilibrium in which (4) holds with strict inequality. Clearly, this is of no interest in our setting.

the probability of liquidation were not endogenously chosen by investors, then transaction costs would, as usual, drive up yields. Equivalently, greater bond liquidity would be associated with lower required yields. This is clear from (7): if  $\pi_i$  is constant and the cost  $t_B$  of trading bond  $B$  increases, its yield increases relative to that of bond  $A$ . The reason is that the buyer of the asset anticipates trading costs that must be compensated. As these costs only materialize if the holder trades at date 1,  $t_B$  is weighed with the probability of this event, which is  $\pi_B$ .

The second feature of (7) is that the yield differential increases in fundamental risk: the higher the risk of default of, say, bond  $B$ , which is  $1 - q_B$ , the higher its required yield compared to bond  $A$ . Given the absence of risk aversion in our model, this is not a risk premium but simply reflects the reduction in the discounted expected payoff of the bond. The impact of fundamental risk and of liquidity costs on yield spreads in (7) is standard and in line, for example, with the empirical findings of Beber, Brandt, and Karavaciz (2006) for Euro government bonds.

However, the response of bond yields to changes in liquidity and risk become more interesting once one takes into account that the probability of liquidation is endogenous in this model. This probability itself reacts to changes in both transaction costs (the terms at which the “supply of liquidity” is available) and outside investment opportunities (that determine the “demand for liquidity” by market participants). Indeed, as we shall see, it also depends non-trivially on the interaction between the two.

As one would expect, both higher transaction costs (higher  $t_i$ ) and less attractive aggregate investment opportunities (higher  $\alpha$ ) reduce a given bond’s probability of liquidation ( $\pi_i$ ):

$$\frac{\partial \pi_i}{\partial t_i} = -\frac{1+r}{\alpha(1-t_i)^2}g\left(\frac{\bar{r}}{\alpha} - \frac{1+r}{\alpha(1-t_i)}\right) < 0, \quad (8)$$

$$\frac{\partial \pi_i}{\partial \alpha} = -\frac{1}{\alpha^2}\left(\bar{r} - \frac{1+r}{1-t_i}\right)g\left(\frac{\bar{r}}{\alpha} - \frac{1+r}{\alpha(1-t_i)}\right) < 0. \quad (9)$$

Intuitively, if transactions costs increase, i.e. the bond becomes less liquid, then liquidating it becomes less attractive, hence the probability of selling decreases. Similarly, if the aggregate risk factor increases, the investors’ market investment opportunities become less attractive, which again decreases their probability of selling the bond.<sup>8</sup>

---

<sup>8</sup>We do not report the joint effect of an increase in transaction costs and in aggregate

Equipped with these results, we can now explore how risk and liquidity affect bond yields in our simple model. For simplicity, we perform the comparative statics of the yield differential (7) with respect to  $t_B$  only (the effect of  $t_A$  is analogous):

$$\frac{\partial \Delta Y}{\partial t_B} = \pi_B + \frac{\partial \pi_B}{\partial t_B} t_B, \quad (10)$$

$$\frac{\partial \Delta Y}{\partial \alpha} = \frac{\partial \pi_B}{\partial \alpha} t_B - \frac{\partial \pi_A}{\partial \alpha} t_A + \frac{\partial (q_B - q_A)}{\partial \alpha}, \quad (11)$$

$$\frac{\partial^2 \Delta Y}{\partial t_B \partial \alpha} = \frac{\partial \pi_B}{\partial \alpha} + \frac{\partial^2 \pi_B}{\partial t_B \partial \alpha} t_B. \quad (12)$$

The first term in (10) is positive, the second negative. Hence, the sign is a priori ambiguous. This ambiguity reflects two opposing effects – a direct and an indirect one. The direct effect of higher transactions costs is to drive up the price by a factor  $\pi_B$ : with exogenous liquidation probability, this would be the only effect, as observed above in discussing (7). But when liquidation is endogenously determined, this effect is at least partly counteracted by a reduction of the trading probability, as described in (8). As this probability decreases, the relative price of the bond increases, because liquidation costs are incurred less often. Hence, this effect is proportional to  $t_B$ . To the extent that empirically bid-ask spreads in the bond market are very small (see Table 1), we expect the latter effect to be small in absolute terms and the overall effect to be positive.

The impact of risk on yield differentials in (11) has first two terms of small size and opposite sign, so that one would expect its overall sign to depend mainly on the sign of the third term  $\partial(q_A - q_B)/\partial\alpha$ , which measures the impact of aggregate risk on the fundamental risk of the two bonds. Two possibilities exist in principle. First, the effect of aggregate risk is similar for both bonds, in which case the term is approximately zero. Alternatively, the riskier bond is more sensitive to aggregate risk than the safer one, in which case we have  $\partial(q_A - q_B)/\partial\alpha > 0$ . In this case, we have the well-known “flight to quality”: an increase in aggregate risk makes the safer bond more

---

risk,  $\partial^2 \pi_i / \partial t_i \partial \alpha$ . In our model, this cross-derivative cannot be signed unambiguously, but is positive for several examples of distributions  $G$ , including the uniform distribution. If it is positive, an increase in transactions costs (i.e. a decrease in liquidity) reduces the absolute value of the impact of the aggregate factor on trading volume  $\pi_i$ , hence, there is a flight to liquidity. Yet, as we shall see, this term will be of second order in the derivation of the relevant derivatives for yield spreads and can be neglected in a first approximation.

attractive than the riskier one and hence drives up the yield differential. This is the typical effect stressed by practitioners. Our empirical strategy is to let the data tell us the sign of the derivative. In the appendix, we show how the positive sign arises in a simple model of imperfect but constant correlation of bond returns.

Finally, the joint effect of changes in risk and in liquidity is again ambiguous, but has one dominant element. As seen above, while the direct effect of aggregate risk on the probability of liquidation – the first term in (12) – is always negative, the second term is ambiguous. This second term is the product of the flight to liquidity in terms of order flow and the transactions cost  $t_B$ . Since  $t_B$  is small, we expect the negative first term to dominate.

We summarize the previous discussion in the following testable predictions regarding the effects of risk and illiquidity on the yield differential, where, recalling that changes in  $t_B$  are mirror images of those in  $t_A$ , we measure illiquidity by the transaction cost differential  $\Delta t$ :

**Hypothesis 1.** *The yield differential is insensitive to aggregate risk if aggregate risk affects bonds of different fundamental value identically. It is increasing in aggregate risk if the fundamentals of riskier bonds react more to aggregate risk than those of less risky ones:*

$$\frac{\partial \Delta Y}{\partial \alpha} \geq 0. \quad (13)$$

**Hypothesis 2.** *The yield differential between the two bonds depends positively on their transactions cost differential:*

$$\frac{\partial \Delta Y}{\partial \Delta t} > 0. \quad (14)$$

**Hypothesis 3.** *The positive effect of transactions costs on the yield differential is dampened by aggregate risk:*

$$\frac{\partial^2 \Delta Y}{\partial \Delta t \partial \alpha} < 0. \quad (15)$$

While the first two predictions are in line with those of other existing models of asset pricing and liquidity, the third prediction is specific to our model. If, for example, bond trades are driven by consumption or endowment

shocks, as in the OLG model of Acharya and Pedersen (2004) or the theoretical liquidity literature building on Diamond-Dybvig (1983), then the  $\pi_i$  would be constant and, from (7), yield differentials would only depend on liquidity and aggregate risk directly. The cross-effect identified in (15) would be zero in this case. If bond sales were triggered by changes in consumption opportunities or other “supply side” considerations as in the delegated-portfolio management model of Vayanos (2004), rather than from changes in the risk-return profile of alternative investments, the  $\pi_i$  would depend positively on  $\alpha$  rather than negatively, as in our model, and the cross-effect (15) would be positive. Then aggregate risk would amplify rather than dampen liquidity effects.

## 5 Empirical evidence

The empirical strategy used to test the predictions of Section 4 is based on the estimation of a simultaneous equation model for yield differentials in the Euro area at different maturities.

We measure the aggregate risk factor by the spread between  $j$ -year fixed interest rates on U.S. swaps,  $R_{SWUS,t}^j$ , and the yield on  $j$ -year U.S. government bonds,  $R_{US,t}^j$ . We opt for this measure because of its high correlation with all U.S.-based measures of risk and because of its availability at different maturities. In the next section we report estimates obtained using alternative measures of risk and show that our results are robust to the choice of risk measure. We measure the liquidity factor by the bid-ask spread of each bond. We considered a range of alternative liquidity indicators and selected the bid-ask spread as the most significant measure.

In taking the model to the data, we take the following specification strategy. First, we chose as benchmarks German bonds for the ten-year maturity and French bonds for the five-year maturity. Our choice is supported by the econometric evidence provided by Dunne, Moore and Portes (2002) and by the fact that traders regard French OATs as the 5-year Euro-area benchmark in the same way as they regard the 10-year Bunds as the 10-year Euro-area benchmark, because French bonds are considered as particularly liquid for the 5-year maturity bucket.

Second, as yield spreads in the euro area are very persistent and as our predictions are derived within a static framework, we posit the following

dynamic partial adjustment model for yield differentials:

$$Y_{i,t} - Y_{b,t} = \rho_i (Y_{i,t-1} - Y_{b,t-1}) + (1 - \rho_i) (Y_{i,t} - Y_{b,t})^* + u_{i,t},$$

where  $u_{i,t}$  are independently identically distributed shocks and  $(Y_{i,t} - Y_{b,t})^*$  is the theory-consistent long-run equilibrium value for yield differentials. Third, we augment the specification with the differentials in the residual maturity of the benchmark bonds in country  $i$  and the benchmark bonds to filter out of the data the effect introduced by the different maturity of benchmark bonds and the effect of changes in benchmarks occurring at different dates for different countries in the sample period.<sup>9</sup>

## 5.1 The baseline model

To sum up, we estimate as a baseline model the following eight-equation model, where the dependent variables are the yield differentials relative to a benchmark government bond for the other eight countries listed in Section 3:

$$\begin{aligned} (Y_{i,t}^j - Y_{b,t}^j) &= \rho_i (Y_{i,t-1}^j - Y_{b,t-1}^j) + (1 - \rho_i) (Y_{i,t}^j - Y_{b,t}^j)^* + \\ &\quad + \delta_i (M_{i,t}^j - M_{b,t}^j) + u_{i,t} \\ (Y_{i,t} - Y_{b,t})^* &= \beta_{1,i} (c_{i,t} - c_{b,t}) + \beta_{2,i} (R_{SWUS,t}^j - R_{US,t}^j) \\ &\quad - \beta_{3,i} (c_{i,t} - c_{b,t}) (R_{SWUS,t}^j - R_{US,t}^j). \end{aligned}$$

The index  $i$  varies across countries and the index  $j$  varies across maturities (five and ten years). The estimation is performed by Seemingly Unrelated Regression (SUR), and the empirical results are shown in Tables 4.1 and 4.2.

The estimates for the 10-year maturity yield differential are presented in Table 4.1. The coefficient of the lagged dependent variable is always significant and close to unity, which indicates strong persistence of yield differentials. Also the coefficient of the maturity differential variable is uniformly

---

<sup>9</sup>We also tried different methods of dealing with these problems such as omitting from the sample dates in which benchmarks are changed or constructing constant maturity yields. We favour the use of the maturity differentials in that it is a natural way of correcting the differentials and it allows our liquidity indicator to operate during episodes in which liquidity might highly matter, such as at dates when benchmarks are changed.

An alternative to the maturity differential is the duration differential. However, the difference between these two measures is not very relevant in our case given that they both act as dummies to model the same jump in duration and maturity occurring in occasion of benchmark changes.

positive and significant, confirming the importance of this correction. The corresponding results for the 5-year maturity are shown in Table 4.2. Again, the coefficient of the lagged dependent variable is positive and significant, but it is smaller for all eight countries, indicating lower persistence in the time-series behavior of 5-year yield differentials. Also the maturity correction coefficient stays positive and significant for all eight countries.

More importantly, the coefficient of the aggregate risk factor is positive and significantly different from zero for all eight countries in both maturities, in accordance with Hypothesis 1 of Section 4. It ranges between 0.3 and 0.6 for the 10-year bonds and between 0.23 and 0.68 for the 5-year bonds (except in the latter case for Germany, where the coefficient is virtually zero). Interpreting these positive coefficients from the perspective of the theory summarized in Hypothesis 1 leads us to conclude that the fundamental risk of non-benchmark bonds is perceived to be more strongly affected by changes in aggregate risk than that of benchmark bonds. So, higher aggregate risk – as proxied by our U.S. swap yield differential – is correlated with wider Euro-area yield differentials relative to the Bund, resp. the OAT.

Turning to the liquidity variable, we note that at the 5-year maturity the coefficients of the liquidity variables are positive and significant in the case of Austria, Belgium, Spain, Italy, Netherlands and Portugal, in line with Hypothesis 2 of Section 4.

Finally, in line with Hypothesis 3, the interaction between liquidity and the aggregate risk factor has always a negative impact on the yield differentials. This finding is substantially confirmed at the 10-year maturity, although the results are weakened in the case of Spain and Italy. This evidence illustrates the importance of non-linearities in the effect of liquidity indicators on yield differentials. Interestingly, the coefficient of the liquidity differential variable becomes significant only when the interaction between liquidity indicators and the aggregate risk factor is also included in the regression. If the coefficient of the interaction is constrained to zero, then also the level of the liquidity indicator becomes insignificant.

This evidence does not simply reflect the fact that for less liquid bonds prices take more time to absorb the change in risks. In fact, we control for different dynamic effects across countries of the variables included in our model by having potentially different coefficients on the lagged dependent variable. Moreover a simple check, effected by adding further lags of the included variables, delivers non-significant parameters for higher order dynamics.

It could be observed that our SUR estimation is inefficient when valid

cross-equation restrictions can be imposed on our model. This argument is strengthened in the context of our theoretical model where the cross-equation restrictions on the coefficients on the measure of liquidity and on the interaction between the aggregate risk factor and this measure are indeed implied by theory.

In Table 5, we explore this possibility by imposing cross-equation restrictions on our estimated models for 5-year and 10-year differentials. We test for the validity of cross-equation restrictions on each coefficient separately and on the full set of coefficients. Interestingly, the Wald statistics reported in Table 5 illustrate that the panel restrictions can only be validly imposed on the liquidity indicators at the 10-year maturity. When these restrictions are imposed, the effect of the liquidity variables is significant and in line with the prediction of the theory. Yet, this result does not carry over to the 5-year maturity, where the panel restrictions cannot be imposed on the liquidity variables. Furthermore, the panel restrictions on the coefficient of the aggregate risk factor are always rejected, in line with the predictions of the model where the impact of international risk on fundamental risk is heterogenous across different bonds.

Summing up our empirical results are generally supportive for the implications of our theoretical framework: in particular, the aggregate risk factor always has a positive and significant effect on yields, and there is an important interaction between liquidity and the international factor.

## 5.2 Robustness

Swap spreads can be considered a good measure of risk, for a number of reasons. First, being differentials between bonds of the same maturity, they are not affected by the path of expected future risk-free rates and, differently from term spreads, they reflect only risk premia, as they are unaffected by expected monetary policy. Second, they are available at the different maturities relevant to our study, thus enabling us to account for a non-flat term structure of risk premia. Third, U.S. swap spreads provide a non-European measure of risk and therefore are much more likely to be an exogenous variable for the estimation of parameters of interest than any measure based on European yields. Fourth, as a spread between homogenous types of bonds, they are a superior measure of risk to the spread between Treasury bonds



and corporate bonds.<sup>10</sup>

However, it must be recognized that swap spreads are a special measure of risk, in that they include the counterparty risk of swap dealers,<sup>11</sup> and on some occasions they might reflect factors not related to aggregate risk. A close examination of Figure 3 reveals that the positive and strong comovement between the first principal component of yield differentials in the Euro area and our measure of risk has a clear exception in late July 2003. At this date, swap spreads suddenly increased for reasons related to the hedging of mortgage-backed securities and hence little related to international factors. It is therefore important to assess the robustness of our results.

We provide the relevant evidence in Table 6, where we report the results of estimating our model for the 10-year yields differentials on a shorter sample, which excludes the July 2003 episode. The table also reports the evidence obtained by augmenting the baseline regression with two alternative measures of risk. The first is the yield spread between BBB long-term corporate bonds and AAA long-term corporate bonds, the second is an indicator based on the European equity market: the implied volatility from options on the Eurostoxx 50. The results show that our estimates are robust both to the choice of the sample size and to the use of different measures of risk. In particular, the results on the shorter sample fully confirm the evidence from our full sample with some slight modification of the original point estimates. Augmentation of the model with alternative measures of risk shows that, although all alternative measures of risk are significant, their inclusion does not affect the significance of all variables included in the original model. Overall, the significance of risk factors is more robust than that of the liquidity factor and of the interaction term. We perform similar robustness checks for the 5-year differentials, but for brevity we do not report the corresponding results, which confirm those obtained for 10-year spreads.

In the case of the 5-year bonds we also re-estimated the model with the German Bund as a benchmark instead of the French OAT. This modification led to much less precise estimates of all relevant parameters and to a set of results that were much less consistent with those obtained for the 10-

---

<sup>10</sup>Duffee (1998) noted that the spread between Treasury bonds and corporate bonds is a spread between callable bonds and a mixture of callable and non-callable bonds. Given that the response of callable and non-callable bonds to shocks in the level of the term structure is different, the government-corporate spread is sensitive to the level of the term structure.

<sup>11</sup>Although, in practice this effect is minimal (see, for example, Duffie and Huang(1996))

year differentials. We take this as confirmatory evidence of the econometric analysis by Dunne, Moore and Portes (2002) that clearly indicates the OAT as the preferred choice of benchmark for the 5-year maturity.

Since we have chosen a U.S. based measure of aggregate risk, a final robustness check concerns the inclusion of the dollar-euro exchange rate as an independent source of risk: in all specifications, this variable is insignificant.

## 6 Conclusions

This paper explores the determinants of observed yield differentials between long-term sovereign bonds in the Euro area. Inspection of daily data for the early EMU period indicates that there is a strong comovement in yield differentials of benchmark bonds, and that their first principal components explains more than ninety per cent of the variance. This common trend appears highly correlated with measures of aggregate risk. In contrast, liquidity differentials – proxied, for example, by bid-ask spread differentials – feature sizeable heterogeneity and no common factor.

To generate testable predictions about the relation between yield differentials, fundamental risk, and liquidity, we develop a simple model of the interplay between aggregate risk and transactions costs. Our model has two key novel features. First, the demand for liquidity responds to the magnitude of transaction costs and to changes in investors' opportunity set (rather than being determined by exogenous liquidation needs à la Diamond-Dybvig). Specifically, investors value liquidity less when alternative investment opportunities become less attractive. Second, if the payoff of alternative investments covaries with that of marketable assets, being both driven by a common factor, the equilibrium value of liquidity tends to be lower in worse aggregate states, as determined by this common factor.

The model predicts that bond yield differentials should increase in both liquidity and aggregate risk, with a non-linear term that captures the effect on required bond returns due to the interaction between liquidity and the aggregate factor. We test these predictions on daily data for the Euro-area sovereign yield differentials in 2002 and 2003. The econometric results show that the aggregate risk factor is consistently priced, while liquidity differentials are priced only for a subset of nine country/maturity pairs (out of a total of sixteen), and that the interaction of liquidity differentials with the risk factor is always negative when significant, in line with the prediction of

the model.

Our findings are consistent with those of recent papers that find no economically important effect of liquidity variables on bond returns. In fact, when the coefficient of the interaction term between aggregate risk and liquidity is constrained to be zero, liquidity becomes insignificant throughout our regressions.

Our data do not allow us to draw conclusions regarding the effect of cross-sectional differences in macroeconomic conditions or fiscal policies. For example, simple cross-country regressions show that on average over the sample period, (average) yield spreads are positively correlated with (average) government debt/GDP ratios, which in turn are negatively correlated with (average) bid-ask spreads. But, being based on nine data points only, these cross-sectional estimates have little reliability, and in our time-series analysis we do not have sufficient variation in macroeconomic variables (such as debt/GDP ratios) to draw conclusions about possible macroeconomic determinants of the variables that we observe.

The implications of our analysis for policy-makers and for portfolio managers are rather more subtle. From a policy-making standpoint, the empirical estimates highlight the importance of the aggregate risk factor in determining bond yield spreads, and thus underscore the importance of good macroeconomic fundamentals to minimize exposure to the aggregate risk factor. Instead, there seems to be little need for further action on the liquidity side, because bid-ask spreads are already rather uniform and very small across European bond markets, at least for benchmark bonds. Instead, the lesson for portfolio management is that liquidity can affect the risk sensitivity of the assets being held, and that this interaction depends on the covariance of illiquidity costs with aggregate risk. In this sense, the lesson of our model – in spite of its simplicity – may be considerably more general than our specific application to Euro-area bond markets.

## Appendix 1: Extension to Risky Portfolios

In this appendix, we sketch an extension of the model in which the investors' investment opportunities are risky and imperfectly correlated. As in the main text of the paper, the two bonds can either repay ( $\tilde{V}_i = V$ ) or default ( $\tilde{V}_i = 0$ ). The probabilities are

$$(\tilde{V}_A, \tilde{V}_B) = \begin{cases} (V, V) & \text{with probability } \gamma + \alpha, \\ (V, 0) & \text{with probability } p_{V0}, \\ (0, V) & \text{with probability } 1 - \alpha - \gamma - p_{V0}, \\ (0, 0) & \text{with probability } \alpha. \end{cases}$$

It is straightforward to show that the correlation between the two returns is

$$\rho = \frac{(\alpha + \gamma + p_{V0})(p_{V0} + \alpha) - p_{V0}}{\sqrt{(\alpha + \gamma + p_{V0})(1 - \alpha - \gamma - p_{V0})(p_{V0} + \alpha)(1 - p_{V0} - \alpha)}}.$$

Holding the correlation constant, for small values of  $\alpha$  (which is the relevant case), an increase in  $\alpha$  increases the variance of both bonds, hence  $\alpha$  can be viewed as an indicator of aggregate risk. If we hold  $\rho$  constant, this structure has two free parameters, namely the aggregate factor  $\alpha$  and the relative attractiveness of bond  $A$ , which can be measured by  $\gamma$ .

The assumption  $q_A \geq q_B$  (bond  $A$  is the benchmark) implies that  $\alpha + \gamma + 2p_{V0} \geq 1$ . Under this assumption, for  $\rho$  constant, it is straightforward (though tedious) to calculate that  $\frac{\partial}{\partial \alpha}(q_A - q_B) \geq 0$  for small values of  $\alpha$ , as postulated in Section 4.

The alternative investment opportunity of investor  $h$  pays off as follows:

$$\tilde{R}^h = \begin{cases} \bar{r} - \sqrt{e^h} & \text{with probability } \alpha \\ \bar{r} & \text{with probability } s \\ \bar{r} + \sqrt{e^h} & \text{with probability } \alpha \end{cases}$$

where  $s > 0$ ,  $s + 2\alpha = 1$ ,  $0 \leq e^h \leq e$ . For each  $h$  we have  $E\tilde{R}^h = \bar{r}$  and  $var(\tilde{R}^h) = 2\alpha e^h$ . Hence, higher  $\alpha$  means higher risk, and the investor bears more of the risk the greater her  $e^h$ . If she is risk-averse, she will therefore behave as if she maximized the certainty equivalent  $\bar{r} - \alpha e^h$  considered in the main text.

If risk-averse, the investor's objective is to maximize  $Eu(c)$ , where  $u$  is increasing and concave,

$$c = x_A \tilde{V}_A + x_B \tilde{V}_B + x_C(1 + r) + x_R \tilde{R}^h$$

is final consumption, and the  $x_i$  are the investor's end-of-period-1 holdings of the four assets.

Because of transactions costs, the first-order condition of this problem depends on whether the investor buys or sells in equilibrium. As an example, consider the case where the investor sells both bonds in order to invest alternatively. In this case, her first-order conditions are

$$\begin{aligned} E(\tilde{V}_A - (1+r)(1-t_A)p_{1A})u'(c) &= 0, \\ E(\tilde{V}_B - (1+r)(1-t_B)p_{1B})u'(c) &= 0, \\ E(\tilde{R}^h - (1+r))u'(c) &= 0. \end{aligned}$$

Hence,

$$\begin{aligned} (1-t_i)p_{1i}E[\tilde{R}^h u'(c)] &= E[\tilde{V}_i u'(c)], \quad i = A, B \\ \Leftrightarrow \text{cov}\left(\frac{\tilde{V}_i}{p_{1i}} - (1-t_i)\tilde{R}^h, u'(c)\right) &= \left(\frac{\tilde{V}_i}{p_{1i}} - (1-t_i)\bar{r}\right) Eu'(c) \end{aligned}$$

This condition on how the the excess return of each bond over the investor's (transactions costs adjusted) alternative return must covary with marginal utility is a standard CAPM-type condition, with the new feature that the alternative return is investor-specific. This "investor-specific  $\beta$ " gives rise to a supply curve of bonds that is increasing in  $p_{1i}$ ,  $i = A, B$ , and decreasing in aggregate risk  $\alpha$ , as used in the main text.

## Appendix 2: Data Description

The data for 5-year and 10-year-maturity bonds from 1 January 2002 to 23 December 2003 are collected from Euro MTS Group's European Benchmark Market (EBM) trading platform, at 11 a.m. CET during all market days in the Telematico cash markets. The database contains the best bid or ask prices across all markets, the aggregate quantity of all of the outstanding proposals on basis of the best bid and best ask prices, and the daily trading volume of each bond on the EBM. From these data we calculate redemption yields, maturities and a set of time-varying liquidity variables for the benchmark bonds of each country in our sample. The countries are Austria, Belgium, Finland, France, Germany, Italy, the Netherlands, Portugal and Spain. We constructed the following liquidity variables (in all cases as the difference between the relevant country's value and the value observed for the benchmark, which was Germany for the 10-year bucket and France for the 5-year one):

- 5-day-moving-average of the bid-ask spread (in ticks);
- trading volume for the benchmark bond, in million of Euros;
- bid-side market depth, defined as the difference between bid and mid price, divided by the bid quantity;
- ask-side market depth, defined as the difference between mid price and ask price, divided by the ask quantity;
- maximum quantity available at the best 5 prices.

After experimentation, we selected the bid-ask spread as the most significant liquidity indicator, and reported the estimates of our non-linear empirical model only for this liquidity indicator.<sup>12</sup>

---

<sup>12</sup>This is confirmed, e.g., in the study by Beber, Brandt and Kavajecz (2007), who find no significant difference in the impacts of the four liquidity variables they construct from the MTS data base.

## References

- [1] Acharya, Viral V., and Lasse Heje Pedersen (2004), “Asset Pricing with Liquidity Risk,” *Journal of Financial Economics*, 52.
- [2] Amihud, Yakov, and Haim Mendelson (1986), “Asset pricing and the bid-ask spread,” *Journal of Financial Economics* 17, 23-249.
- [3] Amihud, Yakov, and Haim Mendelson (1991), “Liquidity, maturity, and the yields on U.S. Treasury securities,” *Journal of Finance* 46, 1411-1425.
- [4] Arora, Vivek, and Martin Cerisola (2001), “How Does U.S. Monetary Policy Influence Sovereign Spreads in Emerging Markets?” *IMF-Staff-Papers* 48(3): 474-98.
- [5] Barnes, Kevin, and William Cline (1997), “Spreads and Risks in Emerging Markets Lending,” Institute of International Finance Working Paper No.97-1.
- [6] Beber, Alessandro, Michael Brandt, and Ken Kavajecz (2007), “Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market,” *Review of Financial Studies*, forthcoming.
- [7] Blanco Roberto (2001), “The Euro-Area Government Securities Market. Recent Developments and Implications for Market Functioning,” Banco de Espana, Servicio de Estudios Working Paper 0120.
- [8] Brennan, Michael J., and Avanidhar Subrahmanyam (1996), “Market microstructure and asset pricing: On the compensation for illiquidity in stock returns,” *Journal of Financial Economics* 41, 441-464.
- [9] Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam (2000), “Market liquidity and trading activity,” *Journal of Finance* 56, 501-530.
- [10] Codogno, Lorenzo, Carlo Favero, and Alessandro Missale (2003). “Government bond spreads,” *Economic Policy* 18(37), 503-532.
- [11] Datar, Vinay, Naik Narayan, and Robert Radcliff (1998), “Liquidity and stock returns: An alternative test,” *Journal of Financial Intermediation* 1, 203-219.

- [12] Daves, Philip R., and Michael C. Erhardt (1993), "Liquidity, reconstitution, and the value of U.S. Treasury strips," *Journal of Finance* 48(1), 315-329.
- [13] Diamond, Douglas W., and Dybvig, Philip H. (1983), "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy* 91(3), 401-419.
- [14] Duffie, Darrell and Ming Huang (1996) "Swap Rates and Credit Quality," *Journal of Finance* 51, 921-950
- [15] Dungey, Mardi, Vance L. Martin, and Adrian R. Pagan (2000), "A Multivariate Latent Factor Decomposition of International Bond Yield Spreads," *Journal of Applied Econometrics* 15, 697-715.
- [16] Dunne, Peter G., Michael J. Moore and Richard Portes (2002), "Defining Benchmark Status: An Application Euro-Area Bonds," Discussion Paper 2002/10, CEPR DP 3490, NBER WP 9087, June. Revised version as 'Price Discovery in the European Bond Market', December 2005.
- [17] Eichengreen, Barry, and Ashoka Mody, (2000), "What Explains Changing Spreads on Emerging Market Debt?" in *Capital flows and the emerging economies: Theory, evidence, and controversies*, S. Edwards ed., NBER Conference Report series, Chicago and London: University of Chicago Press, 2000, 107-34.
- [18] Ellul, Andrew, and Marco Pagano (2006), "IPO underpricing and after-market liquidity," *Review of Financial Studies* 19, 381-421.
- [19] Ericsson, Jan, and Olivier Renault (2006), "Liquidity and Credit Risk," *Journal of Finance* 61, 2219-2250.
- [20] Gallmeyer, Michael, Burton Hollifield and Duane Seppi (2004), "Liquidity Discovery and Asset Pricing," Carnegie Mellon University Working Paper.
- [21] Goldreich, David, Bernd Hanke, and Purnendu Nath (2005), "The price of future liquidity: Time-varying liquidity in the U.S. Treasury market," *Review of Finance*, 9(1), 1-32.



- [22] Kamara, Avraham (1994), "Liquidity, taxes, and short-term Treasury yields," *Journal of Financial and Quantitative Analysis* 29(3), 403-417.
- [23] Krishnamurthy, Arvind (2002), "The bond/old-bond spread," *Journal of Financial Economics* 66, 463-506.
- [24] Francis A. Longstaff, Sanjay Mithal, and Eric Neis (2005), "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *Journal of Finance* 55, 2213-2253.
- [25] Moskowitz, Tobias J. and Annette Vissing-Jorgensen (2002), "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?," *American Economic Review* 92(4), 745-778.
- [26] Pagano, Marco, and Ernst-Ludwig von Thadden (2004), "The European Bond Markets under EMU," *Oxford Review of Economic Policy* 20(4), 531-554.
- [27] Pastor, Lubos, and Robert F. Stambough (2003), "Liquidity Risk and Expected Stock Returns," *Journal of Political Economy* 111, 642-685.
- [28] Strebulaev, Ilya (2001), "Market imperfections and pricing in the U.S. Treasury securities market," London Business School Working Paper.
- [29] Vayanos, Dimitri (2004), "Flight to Quality, Flight to Liquidity, and the Pricing of Risk," NBER Working Paper No. 10327.
- [30] Warga, Arthur, 1992, "Bond return, liquidity, and missing data," *Journal of Financial and Quantitative Analysis* 27, 605-617.

**Table 1. Descriptive statistics by country**

*Panel A. Euro-area yield differentials relative to Germany, 10 year maturity  
(resp. France, 5 year maturity), in basis points*

	10-year benchmark bonds			5-year benchmark bonds		
Country	Mean	Median	St. dev.	Mean	Median	St. dev.
Austria	10.05	9.46	7.19	3.35	0.74	9.22
France	4.16	5.62	4.36	3.57	2.37	4.70
Netherlands	6.94	6.92	4.48	6.07	5.60	6.87
Belgium	13.45	11.79	6.80	4.78	4.40	8.09
Spain	9.72	8.06	7.44	-2.16	-0.42	10.13
Finland	10.88	9.34	8.30	6.48	5.82	11.18
Italy	14.47	15.70	4.88	7.97	8.34	8.01
Portugal	15.50	14.48	7.73	6.46	12.03	16.76

*Panel B. Bid-ask spreads in ticks*

	10-year benchmark bonds			5-year benchmark bonds		
Country	Mean	Median	St. dev.	Mean	Median	St. dev.
Austria	4.60	4.4	1.10	4.11	4.00	0.64
France	2.86	2.80	0.46	2.52	2.60	0.34
Netherlands	3.55	3.60	0.50	3.75	3.80	0.45
Belgium	3.47	3.40	0.53	2.71	2.60	0.31
Spain	3.47	3.20	0.80	2.94	2.60	0.78
Finland	4.87	4.60	1.09	4.07	3.80	0.81
Italy	2.52	2.40	1.37	2.12	2.00	0.43
Portugal	4.33	4.40	0.69	3.16	3.00	0.51
Germany	3.25	3.00	0.67	3.20	3.20	0.45

**Table 2. Correlation and principal components of Euro-area yield differentials, 10 year bonds, relative to Germany**

*Panel A. Correlation matrix*

Country	AT	FR	NL	BE	ES	FI	IT	PT
Austria	1	-	-	-	-	-	-	-
France	0.65	1	-	-	-	-	-	-
Netherlands	0.51	0.48	1	-	-	-	-	-
Belgium	0.88	0.72	0.61	1	-	-	-	-
Spain	0.88	0.67	0.58	0.94	1	-	-	-
Finland	0.84	0.81	0.73	0.93	0.90	1	-	-
Italy	0.75	0.84	0.52	0.82	0.80	0.89	1	-
Portugal	0.92	0.69	0.61	0.87	0.89	0.88	0.78	1

*Panel B. Principal components*

Component	1	2	3	4	5	6	7	8
Eigenvalue	7.28	0.26	0.16	0.13	0.06	0.05	0.03	0.01
Proportion of variance	0.91	0.03	0.02	0.016	0.008	0.006	0.004	0.001
Cumulative proportion	0.91	0.94	0.96	0.98	0.988	0.99	0.998	1

**Table 3. Correlation and principal components of Euro-area bid-ask spread differentials relative to Germany**

*Panel A. Correlation matrix*

Country	AT	FR	NL	BE	ES	FI	IT	PT
Austria	1.00	-	-	-	-	-	-	-
France	0.22	1.00	-	-	-	-	-	-
Netherlands	0.49	0.51	1.00	-	-	-	-	-
Belgium	0.39	0.46	0.44	1.00	-	-	-	-
Spain	0.58	0.26	0.60	0.35	1.00	-	-	-
Finland	0.48	0.21	0.54	0.26	0.76	1.00	-	-
Italy	0.09	0.37	0.19	0.26	-0.08	-0.11	1.00	-
Portugal	0.22	0.50	0.29	0.56	0.20	0.19	0.24	1.00

*Panel B. Principal components*

Component	1	2	3	4	5	6	7	8
Eigenvalue	3.46	1.76	0.80	0.61	0.48	0.38	0.29	0.19
Proportion of variance	0.43	0.22	0.10	0.08	0.06	0.05	0.04	0.03
Cumulative proportion	0.43	0.65	0.75	0.83	0.89	0.93	0.97	1

**Table 4.1 Estimation of a system of simultaneous equations for Euro-area 10-year yield differentials**

The equations are estimated by SURE, on a sample of daily observations from 1/1/2002 to 23/12/2003. The Panel shows the coefficient estimates for the 10-year maturity, spreads are on German bonds. Standard errors are reported within brackets below the coefficient estimates. An asterisk (\*) and a cross (†) indicate that the corresponding coefficient is significantly different from zero at the 5 and 10 percent level, respectively.

Variable	Constant	Own lag	Maturity	Risk factor	B-A spread	Interaction
Austria	-0.167* (0.026)	0.857* (0.016)	0.280* (0.034)	0.546* (0.060)	0.043* (0.014)	-0.077* (0.026)
Belgium	-0.129* (0.021)	0.936* (0.007)	0.357* (0.040)	0.497* (0.043)	0.052* (0.022)	-0.099* (0.048)
Spain	-0.135* (0.034)	0.867* (0.018)	0.349* (0.061)	0.485* (0.077)	0.007 (0.024)	-0.009 (0.047)
Finland	-0.159* (0.049)	0.956* (0.006)	0.207* (0.045)	0.467* (0.118)	-0.01 (0.024)	-0.025 (0.079)
France	-0.119* (0.038)	0.945* (0.01)	0.184* (0.077)	0.321* (0.072)	0.016 (0.038)	-0.025 (0.079)
Italy	-0.077* (0.021)	0.912* (0.01)	0.288* (0.037)	0.290* (0.047)	0.017 (0.018)	-0.042 (0.043)
Netherlands	-0.076* (0.019)	0.891* (0.012)	0.314* (0.029)	0.305* (0.042)	0.034* (0.016)	-0.052† (0.032)
Portugal	-0.150* (0.044)	0.920* (0.010)	0.384* (0.052)	0.633* (0.099)	0.080* (0.033)	-0.139* (0.07)

**Table 4.2 Estimation of a system of simultaneous equations for Euro-area 5-year yield differentials**

The equations are estimated by SURE, on a sample of daily observations from 1/1/2002 to 23/12/2003. The Panel shows coefficients estimates results for the 5-year maturity, spreads are on French Bonds. Standard errors are reported within brackets below the coefficient estimates. An asterisk (\*) and a cross (†) indicate that the corresponding coefficient is significantly different from zero at the 5 and 10 percent level, respectively.

Variable	Constant	Own lag	Maturity	Risk factor	B-A spread	Interaction
Austria	-0.251* (0.039)	0.833* (0.017)	0.170* (0.011)	0.679* (0.09)	0.079* (0.023)	-0.184* (0.048)
Belgium	-0.082* (0.015)	0.774* (0.016)	0.214* (0.008)	0.297* (0.034)	-0.022 (0.018)	-0.033 (0.042)
Spain	-0.143* (0.013)	0.693* (0.021)	0.210* (0.007)	0.337* (0.03)	0.048* (0.020)	-0.095* (0.041)
Finland	-0.106* (0.017)	0.606* (0.022)	0.205* (0.005)	0.258* (0.041)	-0.018 (0.012)	-0.025 (0.024)
Germany	-0.017 (0.015)	0.742* (0.018)	0.168* (0.007)	0.01 (0.03)	-0.007 (0.012)	0.004 (0.026)
Italy	-0.043* (0.016)	0.584* (0.03)	0.172* (0.006)	0.231* (0.028)	0.107* (0.017)	-0.208* (0.032)
Netherlands	-0.123* (0.016)	0.563* (0.021)	0.191* (0.04)	0.317* (0.036)	0.017* (0.009)	-0.045* (0.020)
Portugal	-0.122* (0.022)	0.853* (0.010)	0.240* (0.006)	0.458* (0.05)	0.052* (0.018)	-0.125* (0.04)

**Table 5. Testing panel restrictions**

The table is based on a fixed-effects panel estimates for the 10-year and 5-year yield differentials. The p-value of the Wald test of the identity restriction of individual coefficients for all eight countries is shown on the right of the relevant coefficient. The p-value of the Wald test of the identity restriction of all the coefficients for all eight countries is shown in the bottom row. Standard errors are reported within brackets below the coefficient estimates. An asterisk (\*) and a cross (†) indicate that the corresponding coefficient is significantly different from zero at the 5 and 10 percent level, respectively.

Variable	10-year yield differentials		5-year yield differentials	
	Coefficient and S.E.	Wald p-value	Coefficient and S.E.	Wald p-value
Own Lag	0.956* (0.006)	0.000	0.853* (0.006)	0.000
Maturity	0.269* (0.041)	0.000	0.232* (0.003)	0.000
Risk factor	0.172* (0.063)	0.000	0.372* (0.032)	0.000
Bid-ask spread	0.047* (0.021)	0.207	0.039* (0.008)	0.000
Interaction	-0.064* (0.033)	0.192	-0.087* (0.018)	0.000
Panel restriction		0.000		0.000

**Table 6. Robustness Analysis**

The table reports robustness analysis for the SURE system on 10-year yield differentials. We consider three alternative Risk Factors. R F 1 is the swap spread, R F 2 is the differential between yields on seasoned US BAA bonds and seasoned US AAA bonds (the source for these data is the FRED database), R F 3 is the implied volatility in options on the EUROstoxx 50. The source for these data is Datastream.

Variable	Sample	Constant	Own lag	Maturity	R F 1	R F 2	R F 3	Bid-ask	Interaction
Austria	02-01-03:06	-0.135*	0.775*	0.285*	0.503*			0.05*	-0.089*
		(0.021)	(0.022)	(0.027)	(0.046)			(0.011)	(0.020)
	02-01-03:12	-0.324*	0.811*	0.237*	0.388*	0.193*		0.035*	-0.062*
		(0.034)	(0.018)	(0.026)	(0.052)	(0.034)		(0.010)	(0.019)
	02-01-03:12	-0.199*	0.813*	0.308*	0.492*		0.178*	0.031*	-0.051*
		(0.021)	(0.018)	(0.027)	(0.047)		(0.032)	(0.010)	(0.020)
Belgium	02-01-03:06	-0.081*	0.906*	0.317*	0.433*			0.081*	-0.14*
		(0.018)	(0.011)	(0.034)	(0.035)			(0.019)	(0.039)
	02-01-03:12	-0.252*	0.925*	0.327*	0.384*	0.148*		0.042*	-0.08*
		(0.039)	(0.009)	(0.043)	(0.053)	(0.041)		(0.019)	(0.039)
	02-01-03:12	-0.177*	0.917*	0.343*	0.488*		0.174*	0.042*	-0.077*
		(0.019)	(0.008)	(0.031)	(0.033)		(0.003)	(0.019)	(0.037)
Spain	02-01-03:06	-0.10*	0.77*	0.324*	0.465*			0.034 <sup>†</sup>	-0.06 <sup>†</sup>
		(0.017)	(0.02)	(0.045)	(0.059)			(0.019)	(0.034)
	02-01-03:12	-0.33*	0.82*	0.239*	0.284*	0.242*		0.02	0.006
		(0.05)	(0.02)	(0.05)	(0.073)	(0.053)		(0.02)	(0.035)
	02-01-03:12	-0.17*	0.83*	0.343*	0.417*		0.203*	-0.002	0.025
		(0.027)	(0.048)	(0.048)	(0.059)		(0.055)	(0.019)	(0.04)
Finland	02-01-03:06	-0.110*	0.953*	0.127 <sup>†</sup>	0.433*			-0.02	-0.017
		(0.069)	(0.009)	(0.07)	(0.15)			(0.03)	(0.06)
	02-01-03:12	-0.377*	0.944*	0.166*	0.312*	0.250*		-0.006	0.02
		(0.07)	(0.01)	(0.043)	(0.101)	(0.07)		(0.02)	(0.03)
	02-01-03:12	-0.190*	0.950*	0.282*	0.527*		0.01	-0.01	0.03
		(0.045)	(0.006)	(0.05)	(0.107)		(0.08)	(0.02)	(0.04)



**Table 6. continued**

Variable	Sample	Constant	Own lag	Maturity	R F 1	R F 2	R F 3	Bid-ask	Interaction
France	02-01-03:06	-0.035	0.945*	0.053	0.184*			0.042	-0.062
		(0.039)	(0.02)	(0.073)	(0.069)			(0.034)	(0.067)
	02-01-03:12	-0.176*	0.944*	0.212*	0.293*	0.057		0.01	-0.02
		(0.069)	(0.01)	(0.08)	(0.09)	(0.07)		(0.04)	(0.07)
	02-01-03:12	-0.169*	0.930*	0.163*	0.307*		0.188*	0.01	-0.02
		(0.033)	(0.012)	(0.06)	(0.056)		(0.056)	(0.03)	(0.06)
Italy	02-01-03:06	-0.027	0.88*	0.263*	0.242*			0.02	-0.042
		(0.021)	(0.02)	(0.040)	(0.043)			(0.016)	(0.039)
	02-01-03:12	-0.108*	0.89*	0.270*	0.215*	0.114*		0.01	-0.03
		(0.038)	(0.02)	(0.040)	(0.043)	(0.03)		(0.01)	(0.03)
	02-01-03:12	-0.06*	0.86*	0.237*	0.292*		0.187*	0.014	-0.036
		(0.015)	(0.01)	(0.02)	(0.03)		(0.023)	(0.01)	(0.028)
Netherl.	02-01-03:06	-0.076*	0.88*	-0.07*	0.329*			0.046*	-0.072*
		(0.02)	(0.012)	(0.026)	(0.037)			(0.016)	(0.032)
	02-01-03:12	-0.183*	0.86*	0.28*	0.180*	0.130*		0.028*	-0.042 <sup>†</sup>
		(0.03)	(0.013)	(0.025)	(0.04)	(0.03)		(0.013)	(0.026)
	02-01-03:12	-0.097*	0.88*	0.33*	0.338*		0.333*	0.028	-0.031*
		(0.02)	(0.012)	(0.031)	(0.033)		(0.043)	(0.034)	(0.014)
Portugal	02-01-03:06	-0.110*	0.890*	0.406*	0.598*			0.098*	-0.173*
		(0.013)	(0.038)	(0.044)	(0.083)			(0.029)	(0.06)
	02-01-03:12	-0.327*	0.90*	0.329*	0.436*	0.224*		0.06*	-0.10 <sup>†</sup>
		(0.057)	(0.012)	(0.046)	(0.095)	(0.06)		(0.027)	(0.057)
	02-01-03:12	-0.215*	0.870*	0.386*	0.589*		0.293*	0.049*	-0.082*
		(0.029)	(0.013)	(0.032)	(0.061)		(0.038)	(0.020)	(0.04)

Figure 1. 10-year yield differentials in the euro area, relative to German bonds

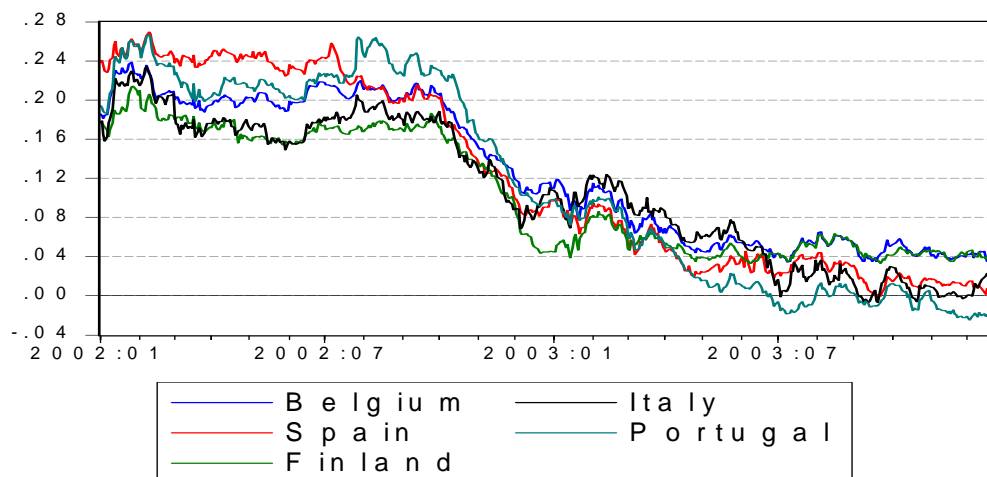
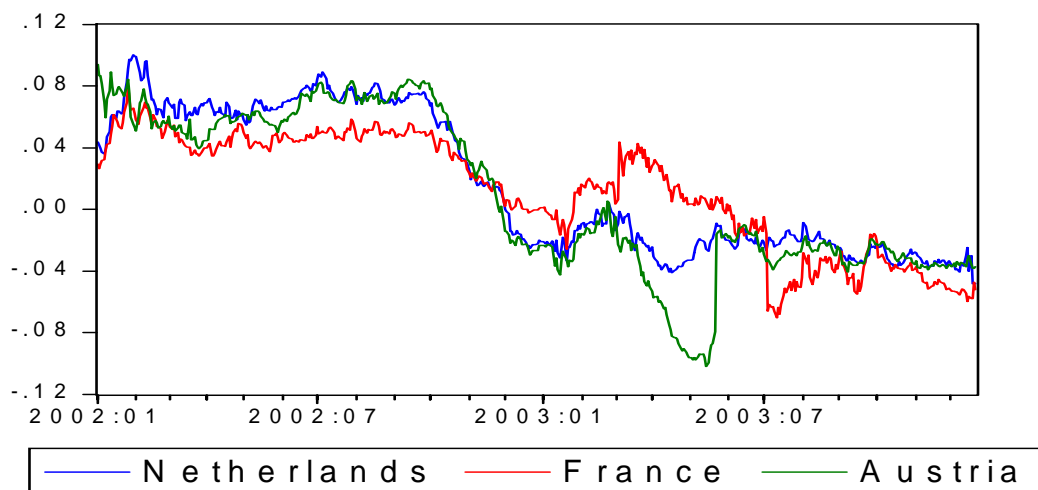
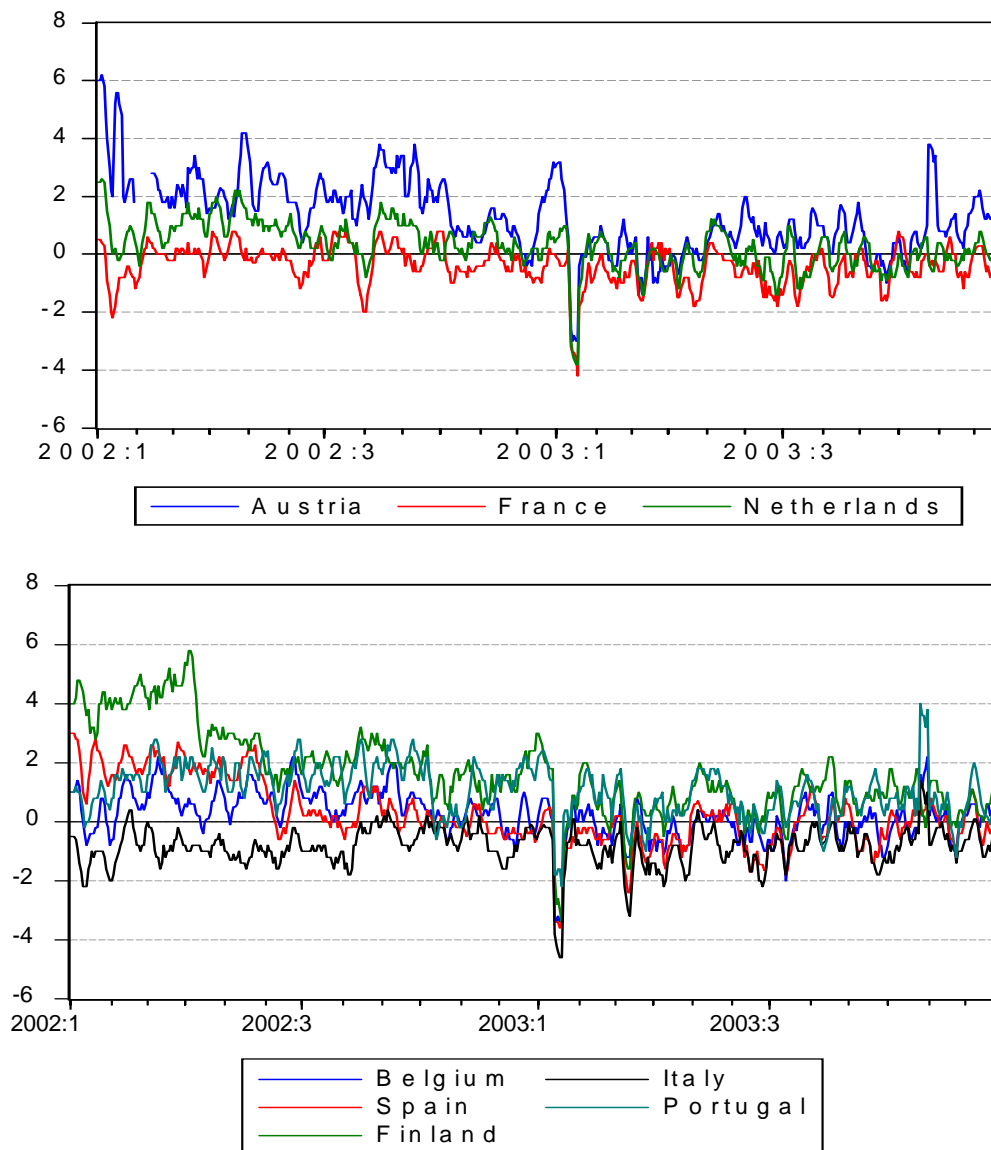


Figure 2. Bid-ask spread differentials in the Euro area, relative to German bonds (10-year benchmark bonds, in cents)



**Figure 3. First principal components of Euro-area yield differentials and the spread between the 10-year fixed interest rate on swaps and US government bond yield**

