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DISCUSSION PAPER SERIES

10 – 06

AN ALGORITHMIC MEASUREMENT OF TECHNICAL PROGRESS

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DECEMBER 2010

ABSTRACT

In this paper we propose a measure of technological progress which is based on the information embedded in standard input–output tables. Well known duality properties enables one to establish a connection between the quantities necessary as inputs and the associated output and some auxiliary prices (like the *wage-profit curves*). We claim that properly tailored *wage-profit frontiers* may provide a basis for the measurement of technological progress. But the computation of these *wage-profit frontiers* is not trivial.

A brute force algorithm for the computation of the *wage-profit frontiers* has high combinatorial complexity that would make its precise computation intractable. But thanks to an efficient algorithm that we have been able to devise we can now compute it. We consider this to be an important and original contribution. Here we present and apply this algorithm. Due to this improvement we can now use these *wage-profit frontiers* as benchmarks against which to measure technological progress: two new indices have been defined.

These new tools have have been applied to the OECD input–output data 1970–2005 and the reslts are presented here.

Keywords: Technological Change, Convergence, Input–output analysis, Technological Frontier, Computational Techniques

JEL classifications: C61, C63, C67, O47

1 Introduction

In this paper we propose a measure of technological progress which is based on the standard input–output tables. We consider the specific individual country combinations of inputs as representing a specific method of production which is implemented to produce an individual output.

Due to well known duality properties we establish a connection with the quantities necessary as inputs and the associated output and the *wage-profit curve.* Extending the work by Bruno (1969) we define and interpret the outer envelope of all the possible *wage-profits curves* as the *wage-profit frontier.* We use this *wage-profit frontier* as a benchmark against which to measure technological progress. Hence, in the context of this paper *wage-profit frontier* and the *technological-frontier* are dual with each-others.

Hence, we will assume that technological progress is represented as a change in the different methods – as it appears from the information embedded in the input–output tables. By comparison between country specific input–output tables we will be able to construct a set of discrete methods of productions [1]. In heterogeneous production whether a new technique is superior with respect to previous one would depend also on the prices of the other inputs and hence also on the production methods available for their production. Hence the duality between quantities and prices is inevitably of essential importance.

In Section 2 we will review the relation existing between *wage-profit frontier* the *factor-price frontier*, the *optimal-transformation-frontier* and claim their duality. In Section 3 we will define the *technological frontier* as the outer envelope computed from all the possible *wage-profit curves.* The mathematical notion of an envelope is conceptually straightforward, but the *brute force* algorithm associated with the computation of such an envelope is for every single point computationally infeasible. Thanks to an efficient algorithm which exploits a result by Bharadwaj (1970) that we have been able to devise, we can compute several versions of the *wage-profit frontier.* This algorithm is described in Section 4.

The new properly tailored *wage-profit frontiers* are then used to compute two new indices of technological progress. These indices are defined in Section 5. The OECD data set is described in Section 6 and in the Appendix A. In Section 7 the results of the computations are presented. Section 8 summarizes the main findings, while Section 9 concludes the paper. Appendix B reports additional results.

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[1] As observed, among others, by Bruno (1969, p. 51) ”any neo-classical technology could be simulated by a ’very dense’ spectrum of discrete techniques”. Hence, as we will see below 3 we can establish a relationship which could be a fairly unproblematic one (or one satisfactory w.r.t. the current state of the art)

2 Technological Progress and the Quantity–Price Dualities

Here we base our analysis on input-output models. We will follow standard assumptions and assume that there are different production methods (activities) available for the production of a single output. These methods are extracted from the set of available input-output tables. We will assume here that b_{ii} of commodity i can be produced with t_i different alternative methods.

$$\phi(z_i, :, i) : a_{i1}^{z_i}, a_{i2}^{z_i}, \dots, a_{in}^{z_i}, \ell_i^{z_i} \mapsto b_{ii}^{z_i} \tag{2.1}$$

where: $i = 1, \dots, n; j = 1, \dots, n; \quad z_i = 1, ..., s_i$; and $a_{ij}^{z_i} \in \mathbb{Q}$. s_i is the number of available methods for the production of good i and n is the number of goods.

The set of methods for the production of good i can be represented in matrix notation as:

$$\Phi(1 : s_i, 1 : (n+2), i) = \begin{bmatrix} a_{i1}^1 & a_{i2}^1 & \dots & a_{in}^1 & \ell_i^1 & b_{ii}^1 \\ a_{i1}^2 & a_{i2}^2 & \dots & a_{in}^2 & \ell_i^2 & b_{ii}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1}^{s_i} & a_{i2}^{s_i} & \dots & a_{in}^{s_i} & \ell_i^{s_i} & b_{ii}^{s_i} \end{bmatrix} \tag{2.2}$$

Obviously the cardinality t_i of the above set of methods can be very large and subsets of the above methods can have in principle a great variety of topological properties. For example, subsets of the above methods can be such as to fulfill standard neoclassical properties or such as to contradict them. After a moment of reflection one can see that the above set of discrete methods can approximate a flexible coefficients production function as well as a fix coefficients production function. It all depends on the numerosity and on the structure of these alternative methods. As correctly pointed out by Bruno (1969, p. 51) *"any neo-classical technology could be simulated by a 'very dense' spectrum of discrete techniques"*. In essence whether the overall production system approximates a neoclassical production function depends on the actual structure of Φ.

The set of all the available methods is given by the following set of activities $\mathbf{\Phi} = \{\Phi(:, :, 1) \cup \Phi(:, :, 2) \dots \cup \Phi(:, :, t_i) \dots, \Phi(:, :, n)\}$.[2]. Hence an

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[2] Alternatively one can see $\mathbf{\Phi}$ as a multiple dimensional array whose maximum number of rows is given by $max\{s_1, s_2, \dots, s_i, \dots, s_n\}$, the number of columns is $n+2$ (the n inputs, labour and output) and the number of metrices which is equal to the number of goods. Each matrix $\Phi(:, :, i)$ contains information about all the possible discrete methods. The users of Matlab and/or Mathematica will be familiar of the notation and of the structure presented here

n-commodities output vector can be generated by using one combination of the methods which belongs to set $\mathbf{\Phi}$. There are a total $\boldsymbol{s} = \prod_{i=1}^{n} s_i$ of these combinations. Given one of these combinations, $\bar{\mathbf{z}} = [\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n]'$, we have one production possibility which can be represented by the following

$$\mathbf{A}^{\bar{\mathbf{z}}} = \begin{bmatrix} a_{11}^{\bar{z}_1} & a_{12}^{\bar{z}_1} & \ldots & a_{1n}^{\bar{z}_1} \\ a_{21}^{\bar{z}_2} & a_{22}^{\bar{z}_2} & \ldots & a_{2n}^{\bar{z}_2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}^{\bar{z}_n} & a_{n2}^{\bar{z}_2} & \ldots & a_{nn}^{\bar{z}_n} \end{bmatrix} ; \mathbf{L}^{\bar{\mathbf{z}}} = \begin{bmatrix} \ell_1^{\bar{z}_1} \\ \ell_2^{\bar{z}_2} \\ \vdots \\ \ell_n^{\bar{z}_n} \end{bmatrix} ; \mathbf{B}^{\bar{\mathbf{z}}} = \begin{bmatrix} b_{11}^{\bar{z}_1} & 0 & \ldots & 0 \\ 0 & b_{22}^{\bar{z}_2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & b_{nn}^{\bar{z}_n} \end{bmatrix} ;$$

$$\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}, \mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}} \mapsto \mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}} \tag{2.3}$$

Where $\mathbf{X}$ is a semipositive diagonal matrix which represents the intensity of the utilization of the methods used (the activity levels). The system is defined as being productive for all those cases in which $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{X}$. is such that $\mathbf{e}'(\mathbf{XB} - \mathbf{XA}) \geq 0$, where $\mathbf{e}$ is the unit or summation vector.

For a productive system and for a given endowment of the factors of production, we can construct the economy n–dimensional *production possibility frontier*.[3]

Moreover, there exist exchange rates, i.e. prices, that would assure the system to be productive also for the periods to come.

$$(\mathbf{I} + \mathbf{R})\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}\mathbf{p} + \mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}}w = \mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}}\mathbf{p} \tag{2.4}$$

where $\mathbf{I}$ is the nxn identity matrix and $\mathbf{R}$ is a diagonal nxn matrix where each diagonal element represents a profit rate, r_{ii}.

For a given matrix $\mathbf{R}$ and uniform wage rate w there exists a price vector $\mathbf{p}$ which would allow the system to be productive also the subsequent period.

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}}w \tag{2.5}$$

An important result is that the re-proportion matrix $\mathbf{X}$ is in-influent for the determination of the price vector $\mathbf{p}$

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}w \tag{2.6}$$

Equations 2.5 2.6 encapsulate the very important result known as the *non-substitution-theorem* which is the fact that the relative prices for a given

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[3] Moreover, once a proper subset of $\mathbf{B}$ is defined as being consumption goods one can construct the n-dimensional *optimal transformation frontier* which is the *"... dynamic analog of an economy's production possibility frontier, namely the locus of maximal combinations of the* per capita *consumption and the rates of growth of the various capital goods (Bruno, 1969, p.39)"*. The dynamic characterization of growth and/or of change is given by the sequence $\{\mathbf{X}_t, \mathbf{X}_{t+1}, \cdots, \mathbf{X}_T\}\}$ from where individual rates of growth, g_i, can be derived

system $(\mathbf{X}\mathbf{A}^{\bar{\mathbf{z}}}, \mathbf{X}\mathbf{L}^{\bar{\mathbf{z}}}, \mathbf{X}\mathbf{B}^{\bar{\mathbf{z}}})$ are independent of the intensities of the different activities, $\mathbf{X}$ [4]

Once the choice of a *numéraire* $\eta'\mathbf{p} = 1$, is made we have that the wage rate is given by:

$$w^{\bar{\mathbf{z}}} = [\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{2.7}$$

Substituting 2.7 into 2.6 we obtain the price vector

$$\mathbf{p}^{\bar{\mathbf{z}}} = [\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}[\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - (\mathbf{I} + \mathbf{R})\mathbf{A}^{\bar{\mathbf{z}}}]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{2.8}$$

The price vector $\mathbf{p}^{\bar{\mathbf{z}}}$ is a function of the particular set of methods $\bar{\mathbf{z}}$ and of the profit rates r_{11}, r_{22}, $\ldots$, r_{nn}. Obviously these prices are not market prices, but are auxiliary prices, that is prices that would allow the *accounting* equilibrium between buyers and sellers of the factors of productions such that the same production activity could take place during next production cycle [5].

3 The Technological Frontier

Bruno (1969) has demonstrated an important dual relation between the auxiliary prices and the methods of productions (and quantities, i.e., the *production possibility frontier*). We can now attempt a measurement of technological progress by comparing the prices associated with the employment of old methods with respect to the prices associated to the employment of new ones. Equation 2.7 is meaningful only when the n profit rates, the matrix $\mathbf{R}$, are given explicit numerical values. A simplifying and meaningful special case is the one in which the rates of profit are uniform so that $r = r_{11} = r_{22} = \ldots = r_{nn}$. [6].

Equation 2.7 is then simplified into:

$$w^{\bar{\mathbf{z}}} = [\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - \mathbf{A}^{\bar{\mathbf{z}}}(1 + r)]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{3.1}$$

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[4] On the origins of the *non-substitution-theorem* see Arrow (1951), Koopmans (1951), Samuelson(1951). A more recent treatment is given in MasColell et al. (1995), pp.159-60. See also Zambelli (2004).

[5] They can be interpreted in many different ways. They can be seen as: Adam Smith's *natural prices*; Ricardo-Marx"s *production prices*; somewhat analogous to Seton's eigen-prices; lomg term competitive equilibrium prices; Walrasian market clearing prices; shadow prices and so on and so forth. In order not to attach to them any particular interpretation we have chosen to refer to them as *auxiliary prices*

[6] Clearly there is a cloud of possible values that the individual profit rates could take and that would guarantee a set of values for which the reproduction of the system could take place. The choice of the *uniform rate of proifit* finds its principlal justification from the fact that it allows to work in a two dimensional space. In practice the graphical representation of of the *wage-profit rates frontier* collapses from a $n + 1$ hyperspace to a 2 dimensional space

where $r = \{r \in \mathbb{Q} : 0 \leq r \leq R\}$ and R, the maximum rate of profit

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$$w^{\bar{\mathbf{z}}} = [\eta'[\mathbf{B}^{\bar{\mathbf{z}}} - \mathbf{A}^{\bar{\mathbf{z}}}(1+r)]^{-1}\mathbf{L}^{\bar{\mathbf{z}}}]^{-1} \tag{3.2}$$

where $r = \{r \in \mathbb{Q} : 0 \leq r \leq \mathcal{R}^{\bar{\mathbf{z}}}\}$. $\mathbb{Q}$ is the set of rational numbers and $\mathcal{R}^{\bar{\mathbf{z}}}$ is the maximum rate of profit of system $\bar{\mathbf{z}}$.

Although 3.2 is a well known relation its empirical importance may have been somewhat underestimated. The above is the *wage-profit curve.* To each combination of methods $\bar{\mathbf{z}}$, there corresponds a *wage-profit curve.* The outer envelope of all possible combinations of methods is the *wage-profit frontier.*

The *wage profit curves* and *frontiers* are scale independent. This is a result of the of the *non-substitution* theorem. Hence two different productive systems, let us say the one associated with a small country and the one associated with a big country, can be compared using the same framework.

Comparison between two *wage profit frontiers* is independent of the cardinality of they productive systems. Two systems which have different cardinality, let us say n and m, can still be compared as long as they have the same *numéraire.* The only requirement is that the *numéraire* is a transformation based on the subset of commodities which are common to both systems.

The *wage profits curve*, 2.7 or 3.2, is dual with respect to the *production possibilities frontier* 2.3. Clearly for a given set of profit rates if $w^{\bar{\bar{\mathbf{z}}}} > w^{\bar{\mathbf{z}}}$ this means that the $w^{\bar{\bar{\mathbf{z}}}}$ has, for the associated auxiliary prices, a higher purchasing power with respect to $w^{\bar{\mathbf{z}}}$. This is possible only if the *production possibilities frontier* associated with the methods $\bar{\bar{\mathbf{z}}}$ is superior with respect to the *production possibilities frontier* associated with the method $\bar{\mathbf{z}}$.

All the possible linear combinations of two methods $\bar{\bar{\mathbf{z}}}$ and $\bar{\mathbf{z}}$ will result in a set of *wage profit curves* or *frontiers* which will be dominated either by $w^{\bar{\bar{\mathbf{z}}}}$ or by $w^{\bar{\mathbf{z}}}$. This exstends to any subset of the methods in $\mathbf{\Phi}$

Given any subset $\boldsymbol{E} = \left\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_\mathrm{m}\right\}$ of $\mathbf{\Phi}$ the

$$w^{\mathrm{TF}}_{\boldsymbol{E}} = \max\left\{w^{\mathbf{z}_1}, w^{\mathbf{z}_2}, ..., w^{\mathbf{z}_\mathrm{m}}, \right\} \tag{3.3}$$

An example of the *technological frontier* is illustrated in Figure 3.1.

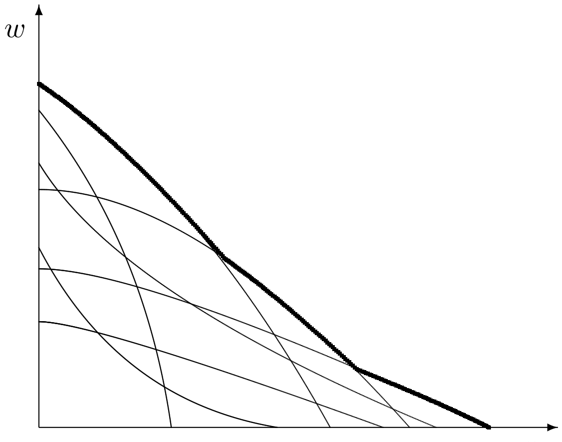

Fig. 3.1: *The technological frontier*

Clearly, not all the *wage-profit curves* associated to $\boldsymbol{E}$ contribute to the formation of the *technological frontier*, $w_{\boldsymbol{E}}^{\mathrm{TF}}$. The subset of methods of $\boldsymbol{E}$ which enters the frontier, $\boldsymbol{E}^{\mathrm{TF}}$, represents the most productive system of methods, which is a combination of different sets. This is, in our view, the composite bentchmark commodity to be used in order to measure increases of productivity due to technological progress [7].

Whether the *technological frontier*, w^{TF}, is consistent with the neo-classical framework or not will depend on the particular structure of the set of methods. Hence this approch is general.

4 The Velupillai-Fredholm-Zambelli *technological frontier* - Algorithm

The computation of the *wage profit frontier* is not a simple matter. There exist a brute force algorithm which allows a precise computation of the w_{Φ}^{TF}. But the implementation of this algorithm (see below) becomes computationally intractable as the cardinality of the set of methods increases. However

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[7] Velupillai and Zambelli (1993) is a first search in this direction. Velupillai (1993, 5-6) states: *Production structures carry with them natural prices corresponding to particular analytical assumptions about the economics of the production system. What is needed is a device to extract these prices from the observed data of a functioning economy. Thus the natural questions for the production approach relate to indices of productivity and the optimality of price systems supporting production structures from particular economic viewpoints. The measures encapsulate the price-mediated interaction between resource allocation and income distribution for efficient production. The conceptional tools include the* optimal transformation frontier *. . . and the* factor price frontier. *The framework yields efficiency indices of different production systems and, by implication, real output comparisons between different economies. Constructing indices for these conceptual categories, as remarked earlier, is the main task of this paper.* That is the 1993 paper. We think that this task, whose foundations were set in 1993 is fully completed in 'this' paper

we have been able to a tractable algorithm that allows a drastic reduction of the computational effort. For example with the cardinality of the data set that we use in this paper, see below, the computation of $w_{\mathbf{\Phi}}^{\mathrm{TF}}$ with a desktop computer and using the brute force algorithm would take several decades, while the use of our algorithm allows its computation in just a few hours. Given that we think that this is an important contribution of the present paper we have taken the liberty of calling it the VFZ-*algorithm* which leads to the computation of the VFZ-*technological frontier* which allows the computation of two indices of technological progress: the VFZ-*index* and the VFZ-*rankings.*

4.1 A brute-force algorithm

1. input data, i.e. individual input–output tables and organize it into the multiple dimension array, $\mathbf{\Phi}$ (see equation 2.2)
2. enumerate all possible combinations of methods $\boldsymbol{E}_{\mathbf{\Phi}} = \left\{\mathbf{z}_{\mathrm{j}}\right\}$ with $j = 1, ..., \boldsymbol{s}$ with $\boldsymbol{s} = \prod_{i=1}^{n} s_i$
3. compute sequentially, for $j = 1$ to $\boldsymbol{s}$, the *wage profit frontier*, $w^{\mathbf{z}_j}$ 3.2 and retain the value for wages w which dominates the previously computed *wage profit frontiers.* That is, compute (see 3.3)

$$w_{\boldsymbol{E}_{\{j\}}}^{\mathrm{TF}} = \max\left\{w_{\boldsymbol{E}_{\{j-1\}}}^{\mathrm{TF}}, w^{\mathbf{z}_{\mathrm{j}}}\right\}$$

However, when using the above algorithm the computational complexity of the problem implies that it is practicalyl impossible to compute the technological frontier for even small datasets, since for each rate of profit all possible combinations of techniques must be evaluated. Using the *Big-O notation* the time-complexity is (at least) $O(\boldsymbol{s})$. This implies that no matter how powerful a computer that will be developed within, say the next century, it will always be possible to include additional available data, such that the algorithm will not halt within any reasonable time frame[8].

4.2 The VFZ-*algorithm* and the VFZ-*technological frontier*

The computational complexity can however be drastically reduced (in the order of $\boldsymbol{s}$ to $\boldsymbol{s}^{1/n}$) by exploiting an important result concerning switch points. Bharadwaj (1970) has shown that:

i) *"At a switch point the adjacent production system differ in the method of production for* only one *of the commodities common to them (Bharadwaj, 1970, p.423, emphasis added)"*;

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[8] With N input-output tables, n sectors, the total number of systems, *wage-profit frontiers* are N^n. The OECD input-output tables used below is formed of $N = 64$ tables with $n = 23$ sectors, which means $64^{23} \approx 3.5 \cdot 10^{41}$ unique systems. Running a whole year, the computer must evaluate $1.1 \cdot 10^{34}$ systems per second, each including several matrix operations. Not anything near such a computer exists today or will within any reasonable time frame.

ii) *"The choice of the value unit* [the numéraire] *does not affect the maximum number of switching possibilities* [and their correspondence to the profit rate] *(Bharadwaj, 1970, p.424)"*

Using any point on any frontier the following procedure, so to say, climbs the individual wage-profit frontiers using the switch points as stepping stones. This is how we reduce drastically the computational time.

1. import data and convert it into matrices of technical coefficients
2. choose an initial point on any frontier
3. from this point, while $r > 0$, lower the profit rate one increment[9] and compute the wage rate without changing the techniques, save this as $\overline{w}$
    (a) one by one, change the techniques (piecemeal), i.e., $n \cdot (N-1)$ times, and for each system
        i. if the profit rate is smaller than the maximum profit rate, compute the wage rates
        ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use $\overline{w}$
4. Now reverse the procedure, while $w > 0$, increase the profit rate one increment and compute the wage rate without changing the techniques, save this as $\overline{w}$
    (a) one by one, change the techniques and for each system:
        i. if the profit rate is smaller than the maximum profit rate, compute the wage rates.
        ii. if this wage rate is greater than $\overline{w}$, then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use $\overline{w}$
5. go to point # 3 as long as loop # 3 and 4 do not produce identical results, else terminate and collect the results

Both algorithms can be implemented with no serious demand for the available memory, but unlike the brute-force algorithm the VFZ-*algorithm* cannot be run parallel.

An easy way to verify the outcome from the Piecemeal algorithm is to apply the two algorithms on a tractable subset and check that they yield identical results. This has been done with positive results.[10]

The full set of results based on the eight OECD countries for eight time periods can be computed within a few hours, with the VFZ-*algorithm* using a standard desktop computer.

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[9] In the actual computation the step-size is fixed at $\frac{1}{1000}$. Between $\frac{1}{500}$ and $\frac{1}{1000}$ the number of switch point increased, which implies that the algorithm missed some switch points. No changes in the results are found when narrowing the step-size to $\frac{1}{2000}$.

[10] There exist one potential problem; it is theoretical possible, by some fluke, that the envelope is not connected by intersections with the initially chosen frontier. However, the probability of this occurring tends to zero as the number of techniques tends to infinity.

5 Two measures of technological progress based on the VFZ-*technological frontier*

The VFZ-*technological frontier* can be used to measure the technological progress and the relative economic performances of the different economic systems, countries. We have constructed two different indices of performance: the VFZ-*index* and the VFZ-*ranking.*

The VFZ-*index* measures the level of development as the ratio between the system specific *wage-profit curve* and the VFZ-*technological frontier.* The VFZ-*index* is dependent on the choice of the *numéraire*, but has the advantage of assessing the degree of economic backwardness or forwardness in terms of the globally efficient production frontier captured by the VFZ-*technological frontier.* In essence it is an assessment of the actual development of the particular national system with respect to the benchmark represented by the VFZ-*technological frontier.*

The VFZ-*ranking* computes the relative performances based on the contribution of the economic systems to the formation of the efficient global VFZ-*technological frontier.* As Bharadwaj (1969) has shown the switch points of the *wage-profit frontier* are independent of the *numéraire* and hence the contributions of the economic systems do not change with it. A ranking between the different systems can be made by exploiting this fact. Obviously an economic system that contribute substantially and more than others to the formation of the VFZ-*technological frontier* can be considered as being forward in technological development with respect to those not contributing at all. This does not mean that we have to expect that the economic system necessarily performs better than others. Whether this technological forwardness is actually exploited so as to assure, for example, full employment level or high level of per-capita output or income is another matter which is not discussed in this paper [11].

It has to be stressed that the VFZ-*index* is an 'absolute' measurement of actual potential economic performance, while the VFZ-*ranking* is a 'relative' measure of the access to more advanced, and potentially more productive, industry level production methods. The computations of these two indices require the computation of the VFZ-*technological frontier.* Hence, for the reason explained above they have never been computed before.

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[11] This becomes obvious when one considers the fact that to any *wage-profit curve* there is associated an infinity of re-proportioning or level of activities matrix $\mathbf{X}$. Clearly, a great variety of employment levels and per-capita incomes can be consistent with different activity levels, $\mathbf{X}$, it all depends on the actual structure and demand level of the regions involved

5.1 The VFZ-*index*

Given a set of systems $\boldsymbol{E}$ (derived from combinations af the available methods, $\boldsymbol{\Phi}$), the VFZ-*index* provides a measure of the average efficiency relative to the VFZ-*technological frontier* , $w_{\boldsymbol{E}}^{\mathrm{VFZ}}$, associated to the subset.

For the jth country at time t the VFZ-*index* is computed as:

$$VFZ_{j,t}^{index} = \frac{1}{m} \sum_{i=1}^{m} \left[w_{j,t}(r_i)/w_{\boldsymbol{E}_t}^{\mathrm{VFZ}}(r_i) \right] \tag{5.1}$$

$$j = 1, 2, ..., N, \quad t = 1, 2, ..., T$$

where: $r_i = \{0 \leq r_i \leq r_m = \mathcal{R}_{\boldsymbol{E}_t}^{\mathrm{VFZ}}, i = 1, \ldots, m\}$; $\mathcal{R}_{\boldsymbol{E}_t}^{\mathrm{VFZ}}$ is the maximun rate of profit of VFZ-*technological frontier*; m is the number of points of the rate of profits domain of the VFZ-*technological frontier* $w_{\boldsymbol{E}_t}^{\mathrm{VFZ}}$;

The closer the index is to unity the more efficient is the technology used in the single country relative to the theoretical maximum computed from the entire set of production activities.

The advantages of the VFZ-*index* over conventional ones is:

1. The method is non-parametric and non-stochastic.
2. Technology, value, and aggregation are fully integrated through the auxiliary prices, hence to some extend circumvents standard index number and value problems.
3. The indices are time-invariant, i.e., they are fully determined within single accounting period.[12]
4. The stability of the switch points greatly limits the sensitivity of changes in the *numéraire.*
5. The interdependence among industries is endogenously captured by changes in the prices of production.
6. The indices will not change as a consequence of simple changes in the scale of production in the single industries, but only if real technological innovations are observed in one or more industries.[13]
7. In the study of convergence, the benchmark/reference point is determined from the system as a whole and not simple a 'leading country'.

5.2 The VFZ-*ranking*

The VFZ-*technological frontier* is a piecemeal function formed with v intervals, where for each interval a fixed combination of methods, $\bar{\mathbf{z}}$, holds. This

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[12] However, updating the entire dataset with new data, say the 2010 OECD tables, will almost certainly change the intertemporal technological frontier, but the within-period ranking will remain unaffected.

[13] By real technological innovations we mean changes in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.

is independent of any *numéraire*. We have exploited this fact to construct a *numéraire-free* index of performance. Our approach is to consider the level of forwardness and of backwardness of economic regions to be related also with respect to the contributions that the methods of the region give to the formation of the global VFZ-*technological frontier*. Here it is not, as in the case of the VFZ-*index*, the distrance with respect to the VFZ-*technological frontier* that matters, but the actual number of methods belonging to a region that contribute to the computation of the VFZ-*technological frontier*.

In order to take account also of methods that are not the most 'efficient' ones, but that are almost as efficient as the most efficient, we have generated a scheme in which methods can be ordered as being first, second, third, ... and last (N^{th}). A method would be ranked second when the method ranked first is removed from the set of methods and it is the one that would contribute to the new, and lower, VFZ-*technological frontier* [14] ; it would be ranked third when the methods ranked first and second are removed and would contribute to the new VFZ-*technological frontier* and so on.

Once these rankings have been generated they are aggregated using the Borda Counts weights (see for example Saari, 1985). That is the first would weighted with value 1, the second with value 1/2, the third with value 1/3 ... the N^{th} with value $1/N$. These values are used to determine the ranking of the different regions by summing all the values associated to the methods of the region. Clearly if the methods employed in a region are all superior with respect to the others the highest value would be equal to the number of commodities. Hence it is appropriate to normalize this value with respect to the number of commodities, i.e industries or sectors. In this way the highest possible performance value, as in the case of VFZ-*index* would be 1, but in this case a high perfomance of one region would imply a much lower performance of the other regions [15]. This is not so in the case of VFZ-*index*.

This index has been called VFZ-*ranking*. Being totally independent form the choice of the *numéraire*, we think that this is a very strong measure of economic performance.

6 Data and the Choice of *Numéraire*

We study three versions of the VFZ-*technological frontier*; the contemporary $w^{\text{VFZ}}_{\text{CTF}}(r_i, \boldsymbol{E}_t)$, the rolling $w^{\text{VFZ}}_{\text{RTF}}(r_i, \boldsymbol{E}_1 \cup \boldsymbol{E}_2 \cup ... \cup \boldsymbol{E}_t)$, the intertemporal

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[14] Here we would like to stress that the calculations of a new VFZ-*technological frontier* each time that a method is 'removed' would not be tractable if we had not found an efficient way to compute it.

[15] In the very unlikely case in which a region dominates in all the industries the value would be 1, in the case in which it would alwasys perform seconfd best, the value would be 1/2, in the case in which all the industries would berform third best the value would be 1/3 and so on.

$w^{\mathrm{VFZ}}_{\mathrm{ITF}}(r_i, \boldsymbol{\Phi})$, where $\boldsymbol{E}_t$ denotes the set of techniques used at time t and $\boldsymbol{E}_{\boldsymbol{\Phi}}$ the total set of systems made from all the combinations of techniques available. An obvious analytical property of these three versions of the technological frontier is that[16]:

$$w^{\mathrm{VFZ}}_{\mathrm{CTF}}(r_i, \boldsymbol{E}_t) \leq w^{\mathrm{VFZ}}_{\mathrm{RTF}}(r_i, \boldsymbol{E}_1 \cup \boldsymbol{E}_2 \cup ... \cup \boldsymbol{E}_t) \leq w^{\mathrm{VFZ}}_{\mathrm{ITF}}(r_i, \boldsymbol{E}_{\boldsymbol{\Phi}}) \tag{6.1}$$

For the actual computation of the technological frontiers we have chosen the OECD 1970–2005 input–output tables from the US, Germany, the UK, France, Canada, Denmark, Japan, and Australia. All based on the the ISIC 2 or ISIC 3 classifications with respectively 35 and 48 industries.[17] The tables contain both the domestic interindustrial flow and industry-specific imports of capital goods.

Some problems of comparability exist between the two methods of classification, but steps have been taken to minimize these problems. The initial 48 and 35 industries have been aggregated into 23 industries following standards of national accounting. The main reasons for doing so is that there are differences in the specific input-output tables due to different national statistical bureaus' practices. We think and hope that our aggregation furthers comparability over time and across time [18].

Unfortunately, tables are not available for all countries for all time periods. To further increase comparability we have chosen to substitute the missing tables with the most commensurable table, typically the table from the previous accounting period in the same country. For details, see Table A.1 in Appendix A.

As labour inputs we use data from the OECD on the industry-level 'compensation of employees' and use this to distribute the total employment in hours to the single industries. When available we use detailed industry-level employment data from The Groningen Growth and Development Centre.[19]

There is a fundamental problem related to the units of accounting, since the tables are denominated in current values of the national currency. Macro-industry deflators have been computed as the differences between macro-industry GDP denomination in respectively current and base period prices, and used to deflate the value denominated tables. This is probably the best available proxy for the physical flow among industries found in the OECD input–output tables. Appendix A contains additional information on the data used.

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[16] Since $\boldsymbol{E}_t \subseteq \{\boldsymbol{E}_1 \cup \boldsymbol{E}_2 ... \cup \boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}_{\boldsymbol{\Phi}}\} \ \forall \ t = 1, 2, ..., T$

[17] See www.OECD.org.

[18] We have investigated whether different aggregations would change the qualitative results presented here. For aggregations that are consistent with the ISIC 2 or ISIC 3 in the sense of aggragating 'neighbouring' industries we found that the qualitative results, i.e. the relative positions, do not change significantly

[19] See www.GGDC.net.

As a *numéraire* we chose the vector of domestic net product of the USA from the base year 2000 normalised with the total hours worked.

7 Efficiency, Technological Change, and Convergence

This analysis is both from a theoretical and empirical point of view 'average', as opposed to 'marginal', since it deals with average costs, returns, revenues, etc. Given that marginal magnitudes can hardly be observed, but average magnitudes can, the use of input-output tables is in our view very appropriate.

A general problem associated with the measurement of technological progress is due to the well known observation that different production activities use different sets of factors of production. For example one can consider the production of energy; nuclear energy, wind mills, hydro-power, solar-energy, oil, coal, gas, etc. Without a robust measure of efficiency, to assess which production process is most efficient is almost always impossible.

There might be reasons different from technological superiority that may lead to the adoption of a specific production structure. But the VFZ-*technological frontier*, being the outer frontier of all possible combinations of the methods of production of all the economic systems involved is a robust benchmark against which we measure the technological efficiency of the different individual regional or national systems.

7.1 The empirical technological frontiers

Figure 7.1 shows the complete collection of contemporary and rolling VFZ-*technological frontiers*. Analogous to the study of the *wage-profit curves* for the individual countries, an outward shift of the VFZ-*technological frontiers* imply, in the context of the approach presented here, unambiguous technological progress. Only in the case in which two *wage-profit curves* or *frontiers* intersect, it cannot unambiguously be determined whether or not a higher level of productivity for the whole system has been reached.[20]

The contemporary VFZ-*technological frontiers*, $w^{\mathrm{VFZ}}_{\mathrm{CTF}}(r_i, \boldsymbol{E}_t)$, show a clockwise and steady shift outwards, while the rolling VFZ-*technological frontiers*, $w^{\mathrm{VFZ}}_{\mathrm{RTF}}(r_i, \boldsymbol{E}_1, \boldsymbol{E}_2, ..., \boldsymbol{E}_t)$, show a more parallel shift. This difference provides a first-hand insight into the nature of the global technological progress. Here

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[20] It is not always the case that the adoption of a new method indicates that the method is superior, simply because its evaluation in terms of cost-benefits would depend on the adoption of the methods of the other industries. Furthermore, both the market prices and the auxiliary prices do depend on the methods used for the whole system (and other very important contingent factors).

we can interpret the clockwise shifts of the contemporary technological frontiers as a global labour-saving technological progress. Since the value of the circulating capital does not necessarily change monotonically with the profit rate, this interpretation is not fully unambiguous.

In the context of this paper we assume, following standard practices, that the index numbers representing the inputs and the output do represent the same class of commodities. As time evolves new combinations of inputs do allow the production of the outputs. The complexity of interactions is very high and hence it is not at all clear that a particular combination would be efficient when inserted in a different context where other methods are used for the production of other commodities. The production structure of a national system is the result of a complex set of events and hence a particular combination of inputs used in the past may not be realized at a later point in time. The rolling technological frontier $w^{\mathrm{VFZ}}_{\mathrm{RTF}}$ does capture technological progress. This is particularly so when the relation between inputs and output is considered as a method of production.

The problem of intersection(s) between frontiers does not exist for the rolling technological frontier, since these by construction will never intersect. Consequently, together with the other frontiers, this property makes the rolling technological frontier a strong analytical tool.

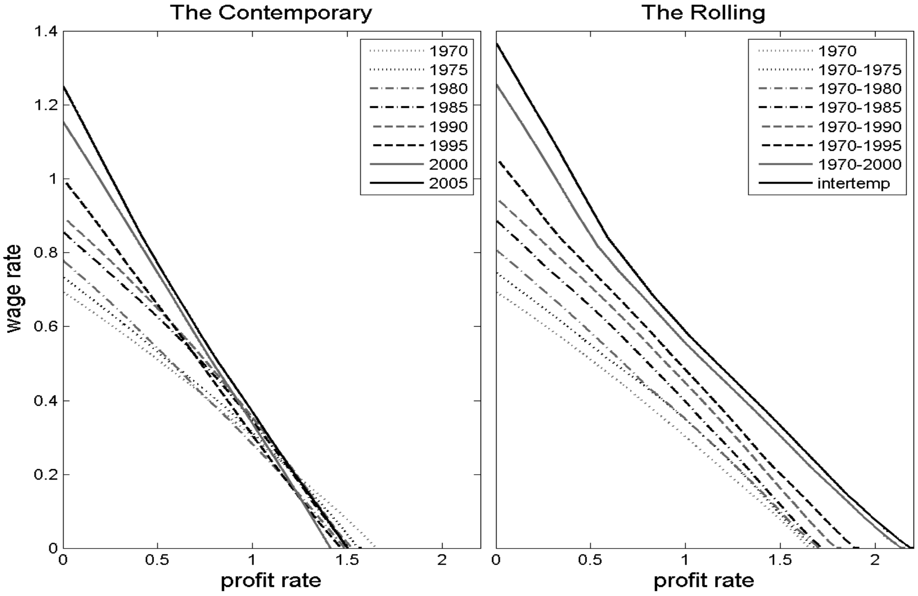

Fig. 7.1: *The contemporary, w^{VFZ}_{CTF}, and rolling w^{VFZ}_{RTF}, technological frontiers*

An observed difference between the contemporary and rolling frontiers implies that there exist some combinations of the old and new production techniques, which are more productive than all combinations of the techniques currently used.[21].

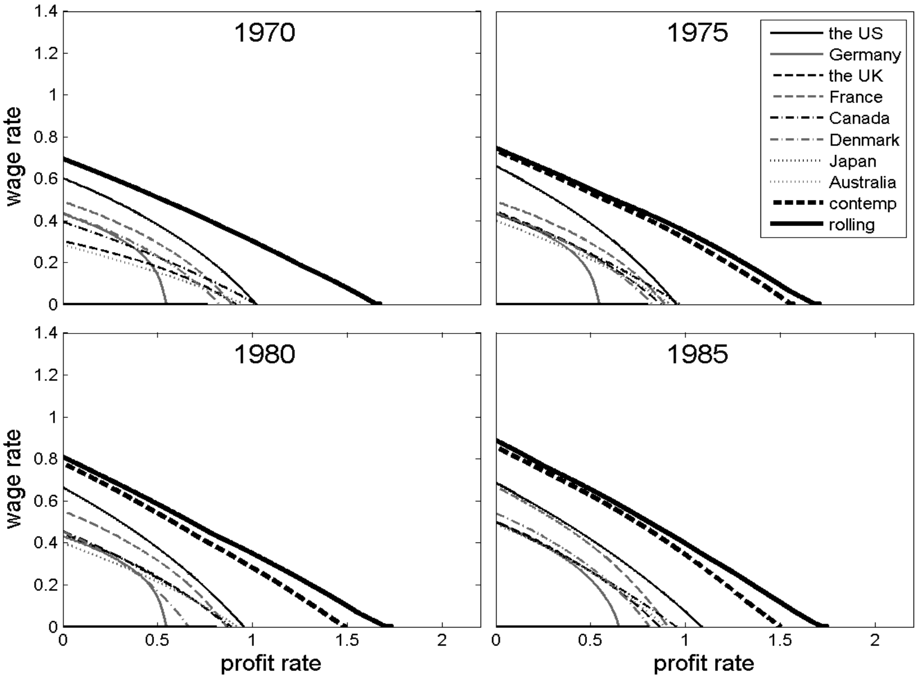

Fig. 7.2: *Contemporary, rolling and country specific wage-profits technological frontiers: 1970–1985*

Figure 7.2 and 7.3 show the wage-profit curves for the individual countries together with the contemporary and rolling technological frontiers. Figure 7.2 for the period 1970–1985 and Figure 7.3 for 1990–2005. As expected the US is the leading country from the 1970s, but the US wage-profit frontiers do not shift as much as the other countries' frontiers in the 1970s, i.e., evidence of a slowdown in the US and catching up by the other countries.[22] See also Figure B.2 and B.3 in the statistical companion, where the frontiers are presented country-by-country.

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[21] However, it could be argued that some old techniques of production should be discarded from the set of techniques forming the rolling (and inter-temporal) technological frontier. These could be techniques that are both (under some circumstances) superior to contemporary techniques, but practically obsolete. And hence *de facto* no longer exist in the *book of available blueprints.* But this type of analysis, although very relevant, goes beyond the scope of the present study

[22] See Degasperi and Fredholm (2010) for a discussion of the US productivity slowdown.

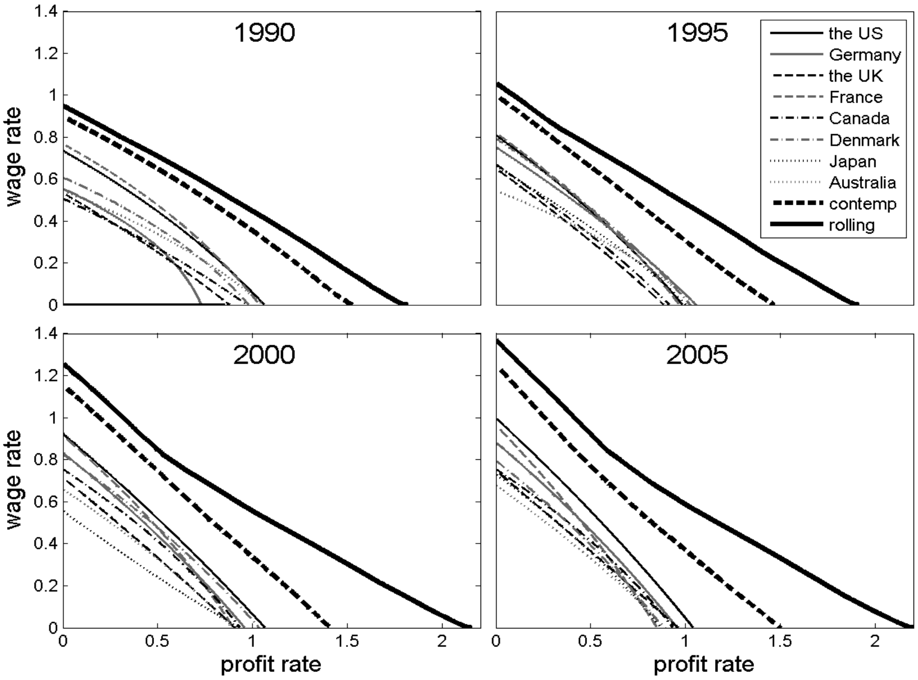

Fig. 7.3: *Contemporary, rolling and country specific wage-profits technological frontiers: 1990–2005*

7.2 The VFZ-*index* applied to the OECD input-output data set

The values of the VFZ-*index* computed for the eight countries and the economy as a whole are collected in Tables[23] 7.1, 7.2, 7.3, 7.4 . Table 7.1 reports the values of the VFZ-*index* when the benchmark is the contemporary wage-profit frontier, $w_{\mathrm{CTF}}^{\mathrm{VFZ}}(r_i, \boldsymbol{E}_t)$. Clearly, as shown also in the figures of the previous section, the contemporary wage-profit frontier evolves through time. Particularly interesting is the stable values exhibited by the United States that, for the whole sample period, maintain a value between 0.66 and 0.69. Meanwhile Germany, France and Denmark, which started at a much lower level, in the twenty years going from 1970 to 1990-95 have reached the United States levels. Germany has gone from 0.37 to 0.64; France from 0.52 to 0.70; Denmark from 0.43 to 0.67. Other countries, according to the results presented here, have moved slowly upward, but far from exhibiting a clear catching up. Particularly interesting is also the observation that for the period going from 1995 to 2005 it is only the United States (and at a lower level Canada) that keeps the same distance with respect to the contemporary technological frontier: all the other countries fall. This does also indicate the technological development could have been driven, in the recent

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[23] The values inside brackets of these table are relative to missing input-output tables, they are computed anyway using the method described in Appendix A

past and to a large extent, by the United States.

This observation is confirmed when we measure the country performances having as benchmark the inter-temporal technological frontier, $w^{\mathrm{VFZ}}_{\mathrm{ITF}}$ $(r_i, \boldsymbol{E_\Phi})$, see Table 7.3. The VFZ-*index* for the United States shows that throughout the 80s the United States have shown no sign of technological development, a minor improvement is observed from 1985 to 1995, but the real expansion is captured by the sharp increase exhibited from 1995 to 2005: jumping from 0.39 (1995) to 0.49 (2005).

The last row of Table 7.3 reports the measure of distance to the inter-temporal VFZ-*technological frontier* of the contemporary technological frontier. From this index it is clear that the technological progress has taken place in two jumps: the first from 1980 to 1985 (going from 0.47 to 0.55) and the second from 1995 to 2000 (going from 0.56 to 0.64). While the technological progress observed from 1980 to 1985 was paralleled by all the countries with improvements of similar magnitudes, the jump observed from 1995 to 2000 is to be attributed principally to the United States (and Canada).

When we analyze the data reported in Table 7.3 we observe that the contemporary technological frontier has, for the period going from 1970 to 2005, increased from 0.45 to 0.66. This corresponds to a compounded growth rate of 1.1 percent per year, which is far less than the often reported 1.5–2 percent. Over a period of 35 years the difference between a growth rate of 1.1 and 2.0 percent corresponds to an increase of a factor 1.5 and 2, respectively.

For the single countries the difference between the level of 1970 and 2005 corresponds to a compounded growth rate of: the US 1.0, Germany 1.2, the UK 2.0, France 1.2, Canada 1.5, Denmark 1.4 and Australia 2.1 percent per year, these growth rates are surprisingly small. Especially, the 1.0 percent annual growth for US. [24]

If these results — as here implicitly claimed — provide an alternative measure with respect to the usual indices of technological progress, then they are indeed interesting.

An interesting point is that the US - as well as the other countries - seem not to be facing a technology constraint. The distance between the actual economic structures and the VFZ-*technological frontier* is noticeable. Naturally, this depends on the availability of the foreign production techniques. Some techniques might be country-specific, i.e, cannot be transferred; a great deal will probably not be transferable, but internal to multinational

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[24] The OECD data bank does not report all the input-output tables. Appendix A contains additional information on the data used. With respect to Japan, we have found a great gap between the input-output tables available before 1995 and those available starting from 1995. We consider the values computed up to 1990 to be unreliable, and hence its value is here not reported. It must also be noted that between 1990 and 1995 the input–output tables change from the ISIC 2 to the present ISIC 3 standard of accounting. Whether or not this greatly influence our results is *pro tempore* unknown.

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.68	0.69	0.68	0.66	0.67	0.66	0.69	0.68
Germany	(0.37)	(0.36)	0.36	0.39	0.43	0.64	0.59	0.57
the UK	0.32	(0.44)	0.43	0.43	0.41	0.48	0.47	0.46
France	(0.52)	0.51	0.55	0.60	0.70	0.67	0.63	0.58
Canada	0.42	0.44	0.43	0.44	0.42	0.51	0.54	0.50
Denmark	(0.43)	0.43	0.38	0.46	0.54	0.67	0.62	0.49
Japan	...	...	...	...	...	0.54	0.35	0.45
Australia	0.29	0.40	0.39	0.42	0.50	0.48	0.44	0.42

Table 7.1: *The VFZ-index values: Contemporary,* $w^{VFZ}_{CTF}(r_i, \boldsymbol{E}_t)$

	1970	1975	1980	1985	1990	1995	2000	2005
the US	1	1	1	1	2	3	1	1
Germany	(5)	(7)	7.00	7.00	5.00	4.00	4.00	3.00
the UK	6.00	(4)	4.00	5.00	7.00	8.00	6.00	6.00
France	(2)	2.00	2.00	2.00	1.00	1.00	2.00	2.00
Canada	4.00	3.00	3.00	4.00	6.00	6.00	5.00	4.00
Denmark	(3)	5.00	6.00	3.00	3.00	2.00	3.00	5.00
Japan	...	...	...	...	...	5.00	8.00	7.00
Australia	7.00	6.00	5.00	6.00	4.00	7.00	7.00	8.00

Table 7.2: *The VFZ-index positions: Contemporary,* $w^{VFZ}_{CTF}(r_i, \boldsymbol{E}_t)$

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.32	0.33	0.32	0.36	0.38	0.39	0.46	0.49
Germany	(0.16)	(0.16)	0.16	0.20	0.24	0.38	0.39	0.41
the UK	0.15	(0.20)	0.20	0.23	0.23	0.28	0.31	0.34
France	(0.24)	0.24	0.26	0.33	0.40	0.40	0.42	0.41
Canada	0.20	0.21	0.21	0.23	0.24	0.30	0.36	0.36
Denmark	(0.20)	0.20	0.18	0.24	0.31	0.40	0.41	0.35
Japan	...	...	...	...	...	0.32	0.23	0.32
Australia	0.13	0.19	0.19	0.23	0.29	0.29	0.29	0.30
Contemporary	0.45	0.48	0.47	0.55	0.57	0.56	0.64	0.66

Table 7.3: *The VFZ-index values: Benchmark, Inter-temporal VFZ*-technological frontier, $w^{VFZ}_{ITF}(r_i, \boldsymbol{\Phi})$

	1970	1975	1980	1985	1990	1995	2000	2005
the US	1.00	1.00	1.00	1.00	2.00	3.00	1.00	1.00
Germany	(5)	(7)	7.00	7.00	6.00	4.00	4.00	3.00
the UK	6	(4)	4.00	5.00	7.00	8.00	6.00	6.00
France	(2)	2	2	2	1	2	2	2
Canada [25]	4	3	3	4	5	6	5	4
Denmark	(3)	5	6	3	3	1	3	5
Japan	...					5	8	7
Australia	7	6	5	6	4	7	7	8

Table 7.4: *The VFZ-index positions: Benchmark, Inter-temporal VFZ*-technological frontier, $w_{ITF}^{VFZ}(r_i, \mathbf{\Phi})$

corporations, which at least limits the transferability; and some (if not most) production techniques require a great deal of human capital which in one way or another also must be transferred. In any case, we observe that the US - as well as the other countries - from the 1980s has been approaching the inter-temporal technological frontier, but also that there is, still potentially, a long way to go [26].

The same goes for Germany and Denmark. While the UK and Australia remain behind.

7.3 The VFZ-*ranking* applied to the OECD input-output data set

The VFZ-*ranking* is a *numéraire* free measure of performance. As explained above, subsection 5.2 being a measurement of the performance of the individual industries in terms of first, second and n-th best contributions to the formation of the VFZ-technological frontier, it is not a measure of the actual state of the implementation of the technological progress, which is captured by the VFZ-*index* discussed in the previous section, but does measure the contribution of the economic region with respect to the formation of the most efficient global outer frontier represented by the VFZ-*technological frontier*, $w_{\mathrm{CTF}}^{\mathrm{VFZ}}(r_i, \boldsymbol{E}_t)$. It is also a concise measure of the relative position of the regions. A sure *'winning'* region is the one which contributes-dominates all the others because the methods of production for each sector dominates

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[26] Particularly striking is the very low position of the actual wage-profit frontier of Japan. For the year 2000 it was the worst in terms of the wage-profit frontier and for the year 2005 only Australia had an actual wage-profit frontier that was lower than that of Japan. In terms of technological progress we have that the VFZ-*index* measures the actual technological progress, while the VFZ-*rankings* the relative position with respect the potential contribution to the worldwide potential frontier of production. As we will see below, for the year 2005, Japan performs in 2005 rather poorly, but second in the relative positions

the methods of all the other regions. This case of total dominance would mean that the value of the VFZ-*ranking* would be 1.

This being the case the highest possible VFZ-*ranking* for the second region, where all the methods of this region are second best, would be 1/2. This being the case the highest possible value for the VFZ-*ranking* for the country performing third-best would be 1/3 and so on. Clearly what just described is a case of perfect relative dominant ordering. But the situation is rarely clear cut and the VFZ-*ranking* values are weighted averages. A value between 1 and 1/2 means that on the average the methods are between being first best and second best, a value between 1/2 and 1/3 between second and third best and so on.

The VFZ-*index* is useful in order to assess the actual development of an economic region, but, as explained above, the VFZ-*ranking* does indicate the degree of technological forwardness or backwardness in terms of the importance of the individual industries.

From the values reported in Table 7.5 and the relative positions,Table 7.6, with respect to the VFZ-*rankings* we can conclude that Canada has experienced a remarkable fall in the level of technological progress, going from 0.6 in 1970 and falling to 0.36 in 1995. Although Canada remains above average the loss in position is noticeable. The US has experienced from the 70s to the whole 90s a fall in the technological performance. But has kept an important relative position as second best. It is only in the years 2000 and 2005 that the United States does express unambiguous leadership. The VFZ-*ranking* relative to 2000 is back to the levels shown in 1970. Contrary to what one would expect, Germany does exhibit a modest performance in both the VFZ-*ranking* values (Table 7.5) and relative position (Table 7.6).

Particularly striking is also the poor performance of the UK and Australia. Clearly, given that the VFZ-*ranking* is a measurement of relative performance we have that for as given year high values for one region imply low levels for the others and hence there must be some which exhibit relative backwardness. This result, also compared with the values of the VFZ-*index* does indicate a problematic performance. Another interesting result is the fact that France, which was performing well in terms of the VFZ-*index* in the case of the VFZ-*ranking* is not performing as well. This can be interpreted as a well balanced domestic interdependent production structure. In particular France has reached a leadership role in 1990, role overtaken by Denmark in 1995. An inspection to the VFZ-ranking of 1990 and 1995 shows that all countries were performing rather poorly - or alternatively - that there was not a clear indication of leadership. When we compare the VZF-*ranking* yearly highest values we see that the leader of 1990 (France, 0.48) and of 1995 (Denmark, 0.45) are those having the lowest VFZ-*ranking* [27].

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[27] 1970 Canada 0.6, 1975 Canada 0.58, 1980 Canada 0.52, 1985 Canada 0.52, 1990 France

	1970	1975	1980	1985	1990	1995	2000	2005
the US	**0.50**	**0.45**	**0.44**	**0.43**	**0.41**	**0.42**	**0.53**	**0.51**
Germany	(0.38)	(0.37)	**0.39**	**0.35**	0.30	0.32	0.31	0.31
the UK	0.18	(0.24)	0.23	0.25	0.23	0.20	0.29	0.28
France	(0.35)	**0.34**	**0.35**	**0.41**	**0.48**	0.31	0.29	0.32
Canada	**0.60**	**0.58**	**0.52**	**0.52**	**0.47**	**0.36**	**0.40**	**0.37**
Denmark	(0.32)	0.31	**0.35**	**0.38**	**0.39**	**0.45**	**0.37**	0.30
Japan	...	...	...	...	...	**0.35**	0.28	**0.39**
Australia	0.24	0.31	(0.31)	0.24	0.31	0.30	0.24	0.24

Table 7.5: *The VFZ-ranking values*

	1970	1975	1980	1985	1990	1995	2000	2005
the US	2	2	2	2	3	2	1	1
Germany	(3)	(3)	3	5	6	5	4	5
the UK	7	(7)	7	6	7	8	6	7
France	(4)	4	4	3	1	6	5	4
Canada	1	1	1	1	2	3	2	3
Denmark	(5)	6	5	4	4	1	3	6
Japan	...	...	...	...	...	4	7	2
Australia	6	5	6	7	5	7	8	8

Table 7.6: *The VFZ-ranking values. Relative positions*

7.4 Single industry contributions to the formation of the VFZ-*technological frontier*

Figure 7.4 shows the (unweighted) average industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

Even though the US is considered the leading country, it is only in few cases the country that is contributing most to the technological frontiers (including the inter-temporal which is the rightmost of the rolling frontiers). This indicates that the US in a few industries strongly dominates, i.e., all or most segments of the envelope include a particular US technique, and that these industries must play a vital role for the economy as a whole. By inspecting Figures B.4–B.26 for the single industries in the statistical companion, it is found that the US dominates in 'Construction'; 'Machinery and equipment, nec'; and 'Business activities (finance, real estate, and R&D)'. Together with Germany the US also dominates in 'Manufacturing, nec'. Germany dominates in 'Electrical machinery and apparatus'; 'Transport equipment'; and 'Manufacturing nec; recycling (include Furniture)'. Canada in 'Other non-metallic mineral products'; 'Metals'; 'Fabricated metal products,

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0.47, 1995 Denmark 0.45, 2000 United States 0.53, 2005 United States 0.51

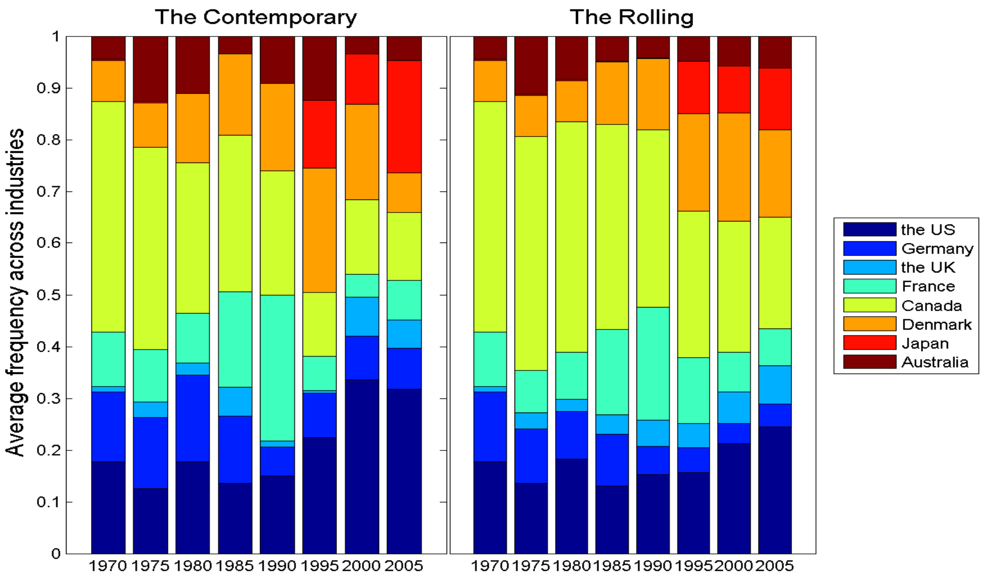

Fig. 7.4: *The composition of the contemporary and rolling technological frontiers*

except machinery and equipment'. Denmark, however insignificant on the world marked, dominates in 'Mining and quarrying' and 'Food products, beverages, and tobacco'.

An important general point is that no country at a single point in time dominates the entire technological frontier (contemporary or rolling). Hence, all countries could, at any point in time, potentially gain from further global integration, either through increased trade or transfer of production techniques (including human capital) or that there are additional potential gains from specialization due to potential comparative advantages.

The statistical companion contains detailed empirical evidence on the country/industry specific contributions to the different technological frontiers. From these results it is possible to go deeper into an analysis of the displacement of the production techniques over time. In particular, it would be interesting to study the difference between the displacement of techniques in the contemporary and rolling technological frontiers, since this can tell us to what extent new more productive techniques have been introduced as oppose to new combinations of old techniques of production.

8 A summing up: technological progress and catching up

The value-added of this paper is the discovery of the VFZ-*algorithm*, see section 4, which allows the computing of actual technological frontiers from huge collections of production techniques. This is done without going through stochastic *ad hoc* and hence unsatisfactory short cuts. This is an important breakthrough that allows the computation of powerful measures of systemic technological progress.

An important property of the wage-profit frontiers and consequently also of the the VFZ-*technological frontier* is that they are scale independent, see section 2, page 6. This means that the performance of small economic regions as well as of big ones can all be measured with the same method.

Based on this powerful tool, we have tried to develop synthetic indices of economic progress: the VFZ-*index* and the VFZ-*ranking.* These indices are useful to measure the intra-temporal economic progress and the inter-temporal technological development.

Here we have applied these new tools to the OECD input-output data so as to measure the performance of 8 leading countries for the period going from 1970 to 2005. We can conclude the following:

1. The global technological progress has been, for the period considered, around 1.1% per year.
2. This evolution can be divided into two periods (see Table 7.3, p.20):
    (a) Catching-up. Period 1970-1995, characterized by a low global overall technological progress, 0.88:%
        i. During this period the United States have maintained the high technological level reached at the beginning of the period, 1970, but with a very low growth rate, 0.79%;
        ii. Practically all the countries have converged towards the United States level;
            A. A set of countries have, by the year 1995, reached the United States levels: Germany (6% growth rate) ; France (2.6% growth rate); Denmark (2.8% growth rate);
            B. Another set of countries have exhibited catching-up, but has never reached the United States levels: Canada (2% growth rate) has in 1995 reached a VFZ-*index value* of 0.3 while the US had 0.39; United Kingdom (2.5% growth rate) has in 1995 reached a VFZ-*index value* of 0.28 while the US had 0.39; Japan has in 1995 reached a VFZ-*index value* of 0.32 while the US had 0.39[28]; Australia (3.3% growth rate) has in 1995 reached a VFZ-*index value* of 0.29 while the US had 0.39 .
    (b) Restart. Period 1995-2005, characterized with higher technological progress, 2% ;

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[28] As mentioned above, the data for Japan antecedent the period 1995-2005 seems to be somewhat flawed and hence it is here not considered in terms of its rate of growth

   i. During these years, after having kept for a long period the position gained 25 years before, the United States have exhibited a sustained growth where the VFZ-*index* has gone from the 1995 level of 0.39 to the 2005 level of 0.49, implying a rate of growth of 2.3%;
   ii. A clear indication of leadership his shown also by the United States high value of the VFZ-ranking; the values for the years 2000 and 2005 are above the already high value of 1970 (see Table 7.5, p.23);
   iii. Apart from Canada, which has continued in catching-up, all the other countries have remained in 2005 at the same level as they had in 1995 and in one case, Denmark, it has exhibited a definite and sharp fall.

3. An inspection of VFZ-*index* values of Table 7.1 p.20 where the benchmark is the contemporary VFZ-*technological frontier* and of the values of Table 7.3, p.20, where the benchmark is the inter-temporal VFZ-*technological frontier* indicate that although there is a clear leadership in technological progress, it is also the case that the *wage-profit curves* of the individual countries are substantially below the global *wage-profit frontiers*. Hence it seems that there is scope for improvements.

9 Concluding Remarks

In this paper we have presented a computationally efficient algorithm - the VFZ-*algorithm* - that allows the computation of the efficient *wage-profit frontier*, here called the VFZ-*technological frontier*. As shown the VFZ-*technological frontier* is a theoretical construction that, due to duality properties, can be used to measure technological progress in terms of the changes of these auxiliary prices. Once the computation of VFZ-*technological frontier*, was made tractable it has been straightforward to define two important measures of technological performance, the VFZ-*index* and the VFZ-*ranking*.

We have applied the above tools to the OECD input-output data bank. This has led to important empirical results which we have summarized in the previous section. The results appear interesting and somewhat illuminating and a review comparing results from other studies could be useful.

Another important direction to be investigated further is the comparison of the productivity of the different sectors that can be conducted by extracting information with respect to the auxiliary prices associated to it. An interesting characteristic of the method used here is that the prices are all measured in terms of the per-capita Net National Product of the United States, year 2000, but their values vary as a function of the specific individual local structures. Hence, the virtual purchasing power (or virtual exchange values) of the commodities produced in one system might be very different form the values produced in another system. This information can be used further to measure the sectoral technological progress. Further research on this direction would be necessary and, we think, very rewarding. A similar attempt has been made, 25 years ago, by Wassily Leontief (1985) himself. Thanks to the algorithms available and constructed for this paper we are confident that interesting results would emerge.

Related with the above there is also the issue connected with the difference that there is between actual market prices and our virtual prices. As we have pointed out in Section 2 the assumption of a uniform rate of profit, although very standard, is convenient assumption that allows us to work with a simple two dimensional space - instead of an n-dimensional one.

Furthermore, the VFZ-*technological frontier* can be seen as a tool for the study of comparative and absolute advantages. This too would be the scope for further research.

A last word. The VFZ-*algorithm* does exploit an important result found in Bharadwaj (1970). That paper was written as a contribution to what is often considered an important, but maybe useless, theoretical field: capital theory. To be able to use the result presented there for an empirical application is a further element in support of pure theory. Whether a theoretical result might or might not have a practical application cannot be determined *a priori*. We hope that this paper is a further example of a useful, but unexpected and hence not programmable, link between pure theory and applications.

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A Data Description

Table A.1 shows which OECD input–output tables that are available from the period 1970–2005. Tables are not necessarily available from the exact five year intervals, e.g., the US tables here labelled 1970 and 1975 are actually the 1972 and 1977 tables, respectively.[29]

	1970	1975	1980	1985	1990	1995	2000	2005
the US	×	×	×	×	×	×	×	×
Germany			×	×	×	×	×	×
the UK	×		×	×	×	×	×	×
France		×	×	×	×	×	×	×
Canada	×	×	×	×	×	×	×	
Denmark		×	×	×	×	×	×	×
Japan	×	×	×	×	×	×	×	×
Australia	×	×		×	×		×	×

Table A.1: *Available input–output tables*

The list below shows how the original tables have been aggregated down to the 23×23 used in this study. The numbers in the brackets refer to their respective ISIC 2 and ISIC 3 classification, *viz.* {[ISIC 3],[ISIC 2]}.

1. Agriculture, hunting, forestry, and fishing {[1],[1]}
2. Mining and quarrying {[2–3],[2]}
3. Food products, beverages, and tobacco {[4],[3]}
4. Textiles, textile products, leather, and footwear {[5],[4]}
5. Wood and products of wood and cork {[6],[5]}
6. Pulp, paper, paper products, printing, and publishing {[7],[6]}
7. Coke, refined petroleum products, and nuclear fuel {[8],[9]}
8. Chemicals {[9–10],[7–8]}
9. Rubber and plastics products {[11],[10]}
10. Other non-metallic mineral products {[12],[11]}
11. Metals {[13–14],[12–13]}
12. Fabricated metal products, except machinery and equipment {[15],[14]}
13. Machinery and equipment, nec {[16],[15]}
14. Electrical machinery and apparatus {[17–20],[16–18]}
15. Transport equipment {[21–25],[19–22]}
16. Manufacturing nec; recycling (include furniture) {[25],[23–24]}
17. Production and distribution of electricity, gas, and water {[26–29],[25]}
18. Construction {[30],[26]}
19. Wholesale and retail trade {[31],[27]}
20. Service activities (transport, hotels and restaurants) {[32–36],[28–29]}
21. Post and telecommunications {[37],[30]}
22. Business activities (finance, real estate, and R&D) {[38–43],[31–32]}
23. Public administration, education and health {[44–48],[33–35]}

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[29] The full list of available tables are: the US {1972, 1977, 1982, 1985, 1990, 1995, 2000, 2005}, Germany {1978, 1986, 1990, 1995, 2000, 2005}, the UK {1968, 1979, 1984, 1990, 1995, 2000, 2003}, France {1977, 1980, 1985, 1990, 1995, 2000, 2005}, Canada {1971, 1976, 1981, 1986, 1990, 1995, 2000}, Denmark {1977, 1980, 1985, 1990, 1995, 2000, 2004}, Japan {1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005}, and Australia {1968, 1974, 1986, 1989, 1999 ,2005}.

Table A.2–A.9 show the macro-industry deflators used to convert the tables denominated in current prices (possible domestic currency) into tables denominated in fixed US 2000 prices. The transition to the EURO has been taken into account in the tables below. The deflators are computed as the ratio between GDP in constant prices and GDP in current prices and when necessary also divided by the dollar-domestic currency exchange rate (www.sourceoecd.org). The missing values marked with a '–' correspond with the unavailable OECD tables.

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	1.23	0.84	0.71	0.74	0.65	0.67	1.00	0.91
II Industry including energy	2.83	1.89	1.14	1.09	1.02	0.96	1.00	0.93
III Construction	4.97	3.28	2.03	1.79	1.43	1.26	1.00	0.71
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.13	1.53	1.12	1.05	0.97	0.87	1.00	0.98
V Financial intermediation; real estate, business activities	4.79	3.45	2.34	1.88	1.47	1.23	1.00	0.88
VI Other service activities	5.15	3.57	2.37	1.98	1.53	1.24	1.00	0.82

Table A.2: *Macro-industry deflators for the US 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	–	0.53	0.60	0.64	1.00	1.03	1.39
II Industry including energy	–	–	0.81	0.60	0.57	1.02	1.03	1.01
III Construction	–	–	1.15	0.84	0.71	1.02	1.03	0.98
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	–	0.78	0.63	0.60	1.01	1.03	1.03
V Financial intermediation; real estate, business activities	–	–	1.01	0.69	0.61	1.02	1.03	0.94
VI Other service activities	–	–	0.98	0.75	0.67	1.08	1.03	0.97

Table A.3: *Macro-industry deflators for Germany 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	5.97	–	2.26	2.43	1.40	1.06	1.57	1.37
II Industry including energy	11.3	–	3.65	2.21	1.86	1.64	1.57	1.57
III Construction	18.4	–	4.44	3.14	2.15	2.00	1.57	1.35
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	13.2	–	4.41	2.87	1.91	1.72	1.57	1.51
V Financial intermediation; real estate, business activities	14.8	–	4.50	2.87	2.01	1.71	1.57	1.34
VI Other service activities	21.5	–	5.81	3.84	2.33	1.94	1.57	1.38

Table A.4: *Macro-industry deflators for the UK 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	0.25	0.23	0.17	0.15	0.98	1.06	1.09
II Industry including energy	–	0.34	0.26	0.17	0.16	1.04	1.06	1.12
III Construction	–	0.56	0.38	0.26	0.21	1.19	1.06	0.86
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	0.39	0.30	0.20	0.17	1.05	1.06	0.95
V Financial intermediation; real estate, business activities	–	0.53	0.42	0.29	0.22	1.22	1.06	0.94
VI Other service activities	–	0.58	0.41	0.26	0.21	1.18	1.06	0.91

Table A.5: *Macro-industry deflators for France 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.53	1.63	1.00	0.96	0.95	0.78	0.81	–
II Industry including energy	3.95	2.37	1.38	1.18	1.04	0.94	0.81	–
III Construction	3.33	1.89	1.31	1.18	0.92	0.87	0.81	–
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.63	1.76	1.20	0.96	0.85	0.82	0.81	–
V Financial intermediation; real estate, business activities	4.26	2.50	1.58	1.17	0.94	0.87	0.81	–
VI Other service activities	4.40	2.58	1.71	1.29	1.02	0.90	0.81	–

Table A.6: *Macro-industry deflators for Canada 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	.104	.094	.074	.078	.101	.119	.143
II Industry including energy	–	.305	.246	.178	.149	.140	.119	.113
III Construction	–	.360	.290	.222	.173	.147	.119	.104
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	.244	.210	.134	.120	.117	.119	.109
V Financial intermediation; real estate, business activities	–	.329	.253	.179	.150	.128	.119	.109
VI Other service activities	–	.360	.285	.197	.151	.136	.119	.103

Table A.7: *Macro-industry deflators for Denmark 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	.0135	.0083	.0067	.0063	.0060	.0056	.0065	.0071
II Industry including energy	.0111	.0078	.0063	.0058	.0058	.0059	.0065	.0074
III Construction	.0313	.0152	.0104	.0087	.0073	.0066	.0065	.0066
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	.0123	.0082	.0067	.0062	.0062	.0061	.0065	.0068
V Financial intermediation; real estate, business activities	.0177	.0119	.0091	.0078	.0069	.0064	.0065	.0068
VI Other service activities	.0288	.0137	.0104	.0086	.0075	.0066	.0065	.0067

Table A.8: *Macro-industry deflators for Japan 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.32	2.09	–	0.98	0.73	–	0.88	0.75
II Industry including energy	3.78	2.76	–	0.94	0.88	–	0.80	0.55
III Construction	4.45	2.72	–	1.05	0.88	–	0.77	0.65
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	5.35	3.42	–	1.02	0.84	–	0.77	0.69
V Financial intermediation; real estate, business activities	7.30	4.98	–	1.34	0.99	–	0.80	0.65
VI Other service activities	5.04	2.96	–	1.20	1.01	–	0.79	0.61

Table A.9: *Macro-industry deflators for Australia 1970–2005*

B Additional Results

B.1 The Wage-profit and Intertemporal Technological Frontiers

Figure B.1 shows the wage-profit frontiers forming the intertemporal technological frontier, and Figure B.2 and B.3 show the wage-profit frontiers for the individual countries together with the intertemporal technological frontier.

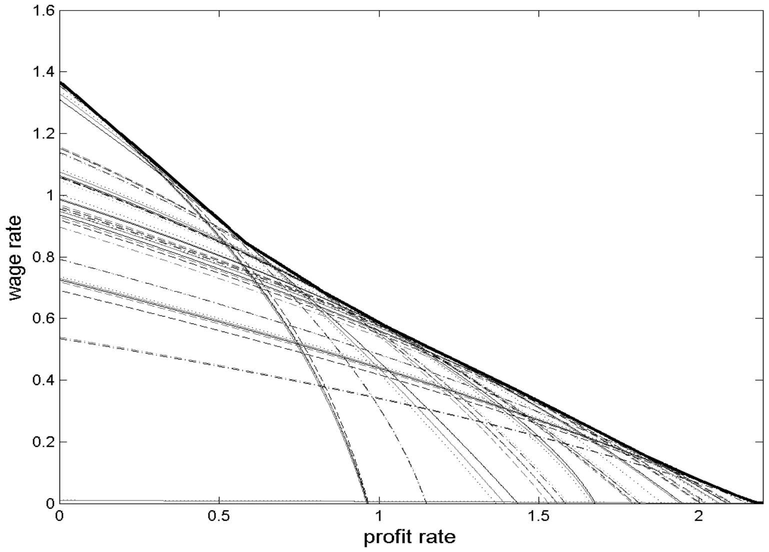

Fig. B.1: *The intertemporal technological frontier*

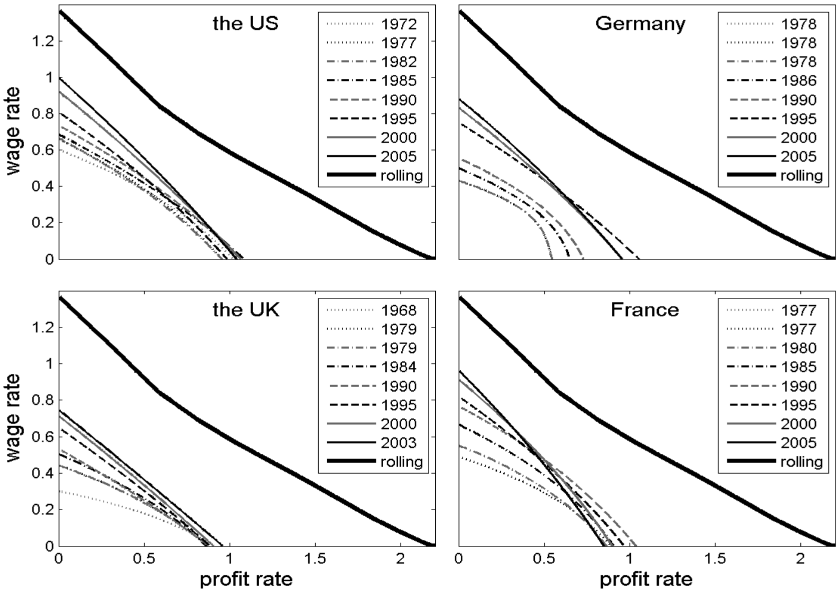

Fig. B.2: *Wage-profit frontiers and the intertemporal technological frontier: the US, Germany, the UK, and France*

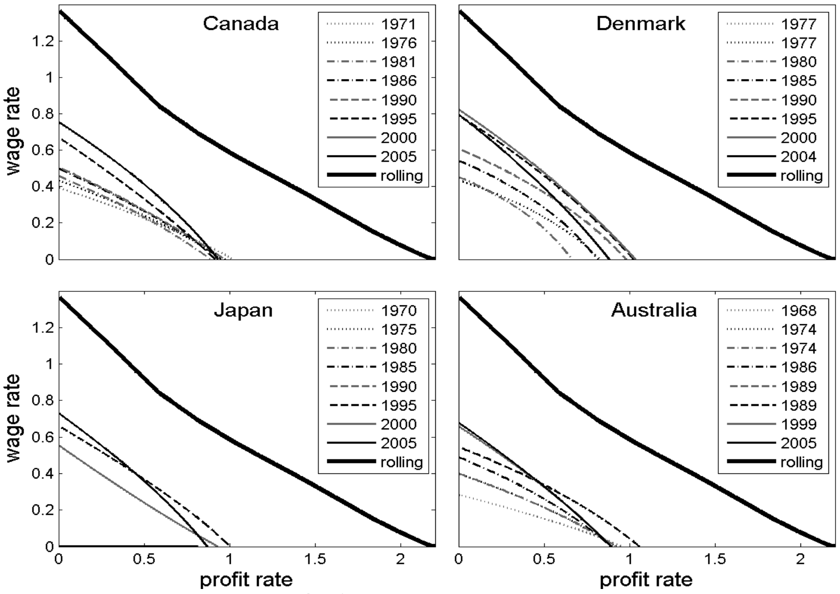

Fig. B.3: *Wage-profit frontiers and the intertemporal technological frontier: Canada, Denmark, Japan, and Australia*

B.2 Industrylevel Frequency of the Single Countries Contribution to the Contemporary Technological Frontiers

The following 23 figures show the industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

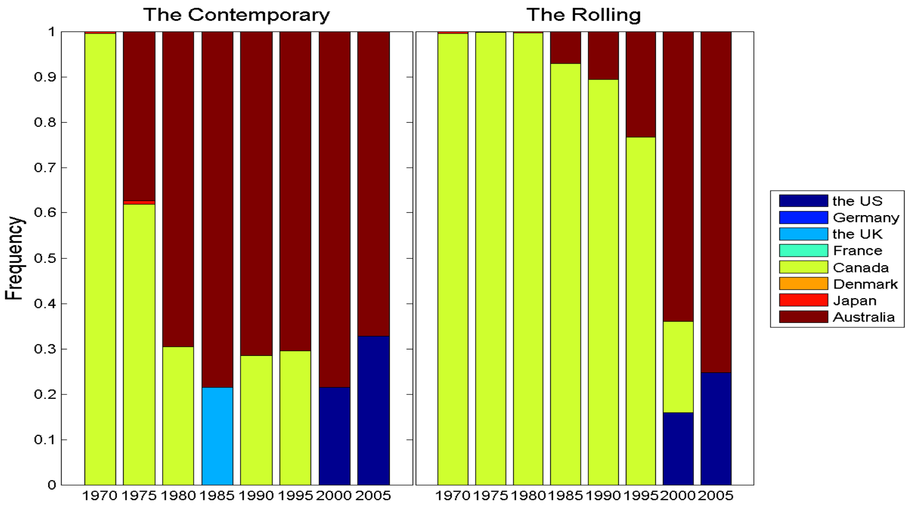

Fig. B.4: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Agriculture, hunting, forestry, and fishing*

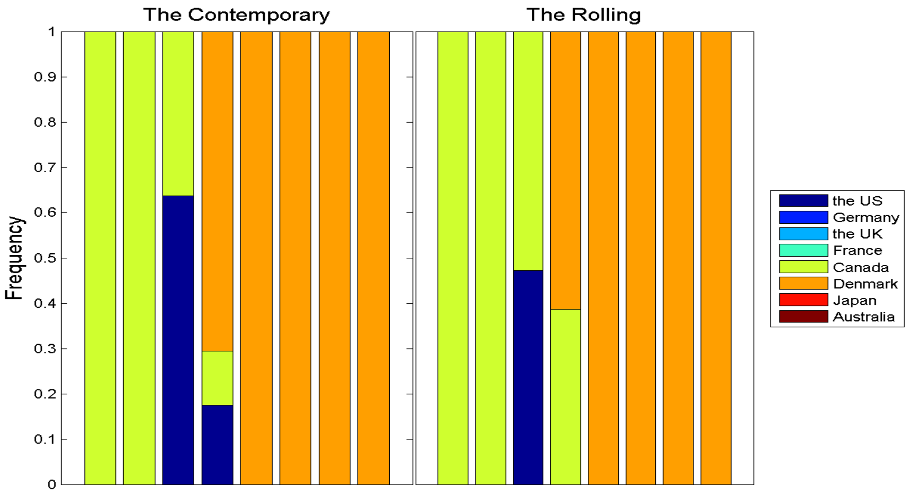

Fig. B.5: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Mining and quarrying*

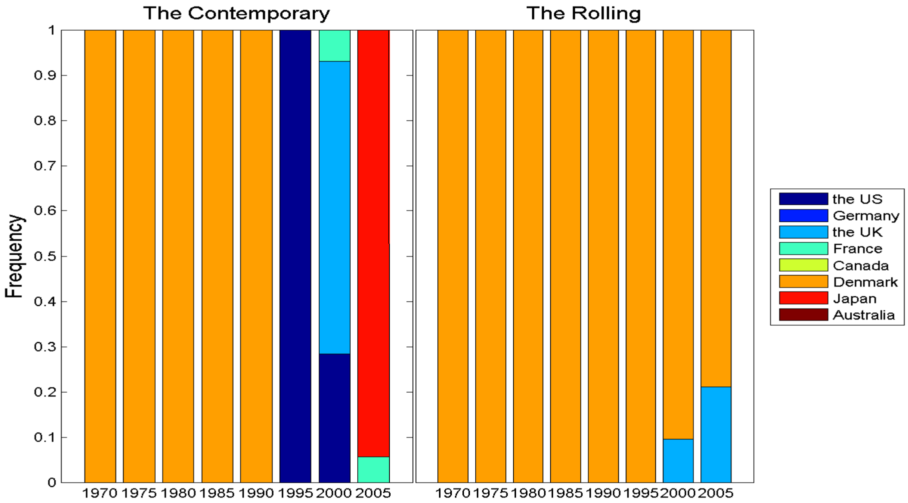

Fig. B.6: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Food products, beverages, and tobacco*

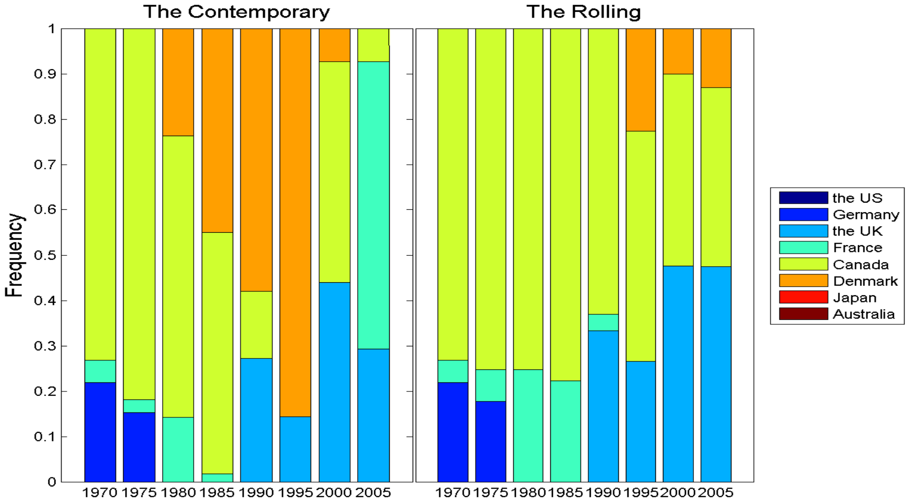

Fig. B.7: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Textiles, textile products, leather, and footwear*

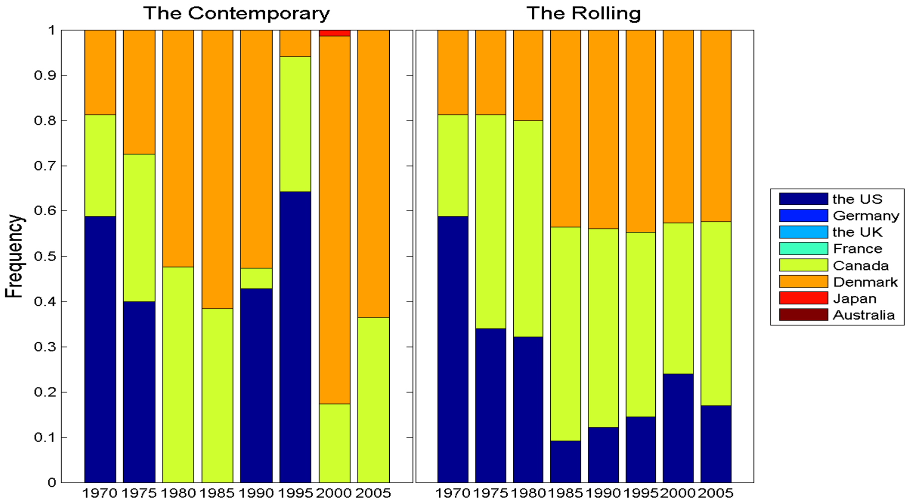

Fig. B.8: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wood and products of wood and cork*

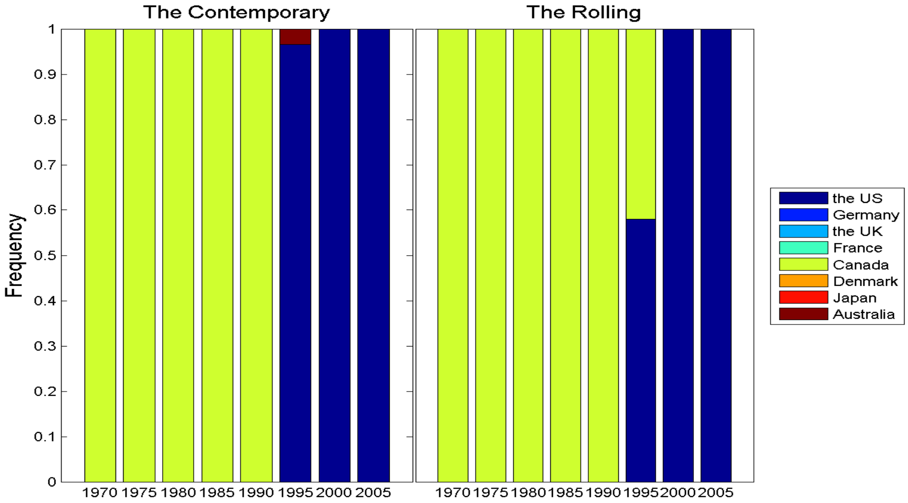

Fig. B.9: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Pulp, paper, paper products, printing, and publishing*

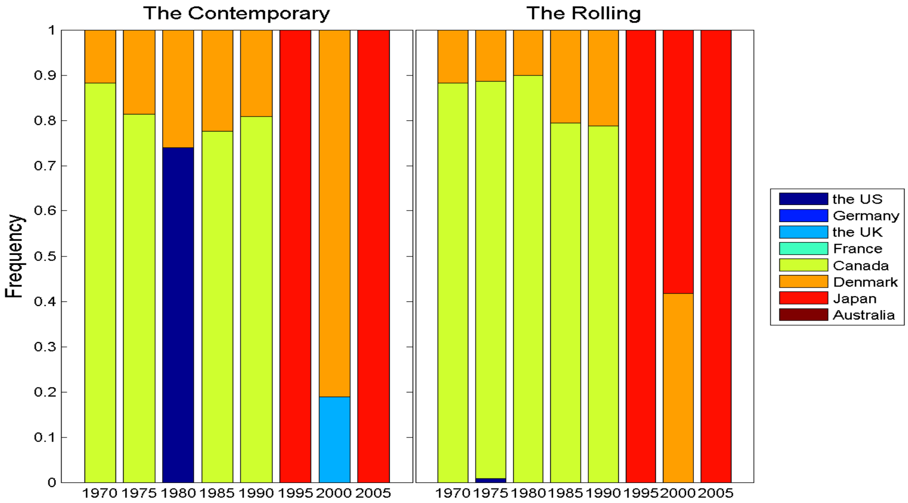

Fig. B.10: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Coke, refined petroleum products, and nuclear fuel*

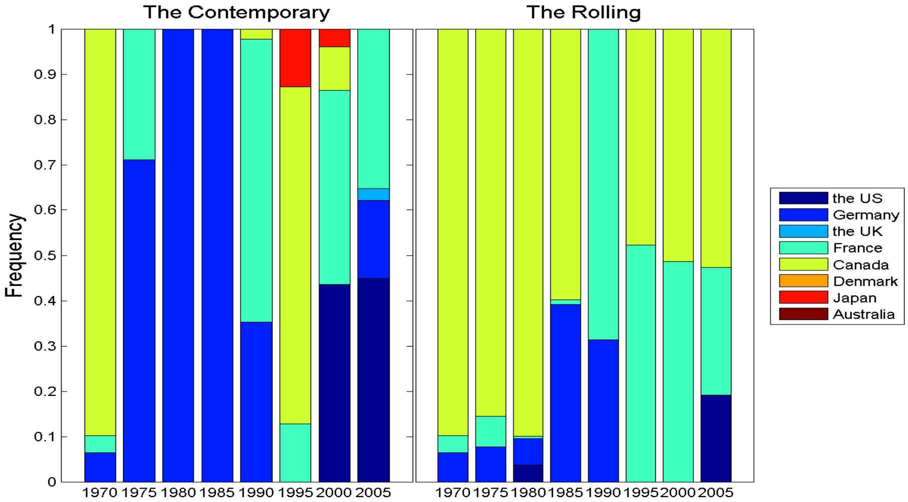

Fig. B.11: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Chemicals*

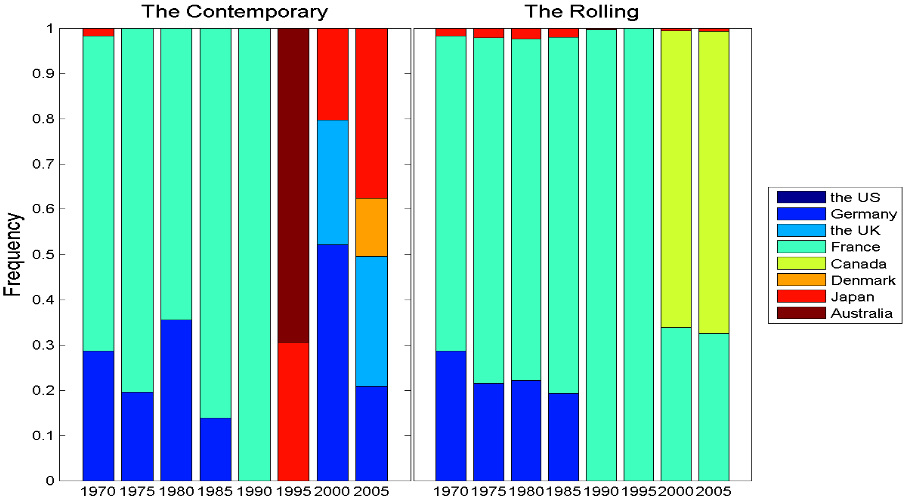

Fig. B.12: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Rubber and plastics products*

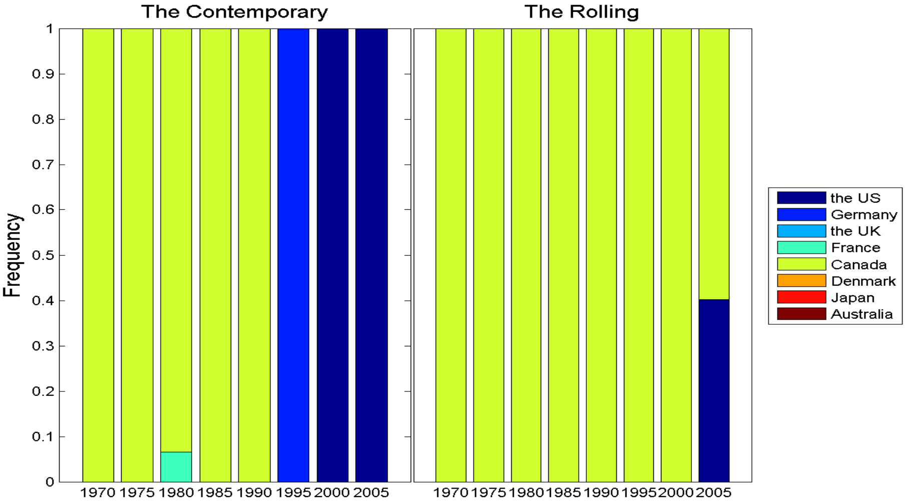

Fig. B.13: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Other non-metallic mineral products*

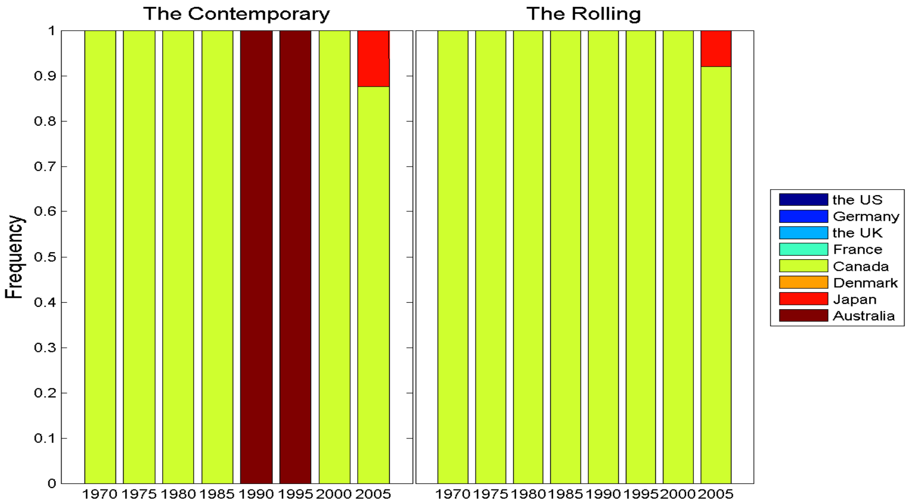

Fig. B.14: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Metals*

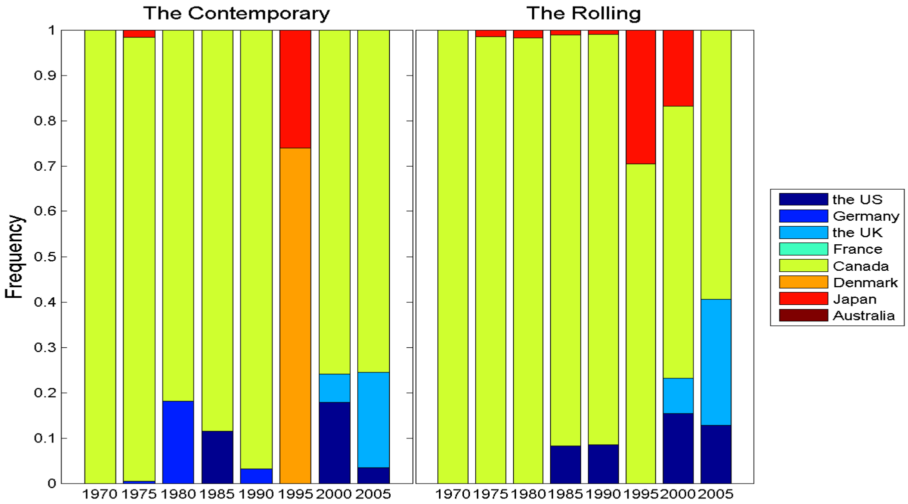

Fig. B.15: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Fabricated metal products, except machinery and equipment*

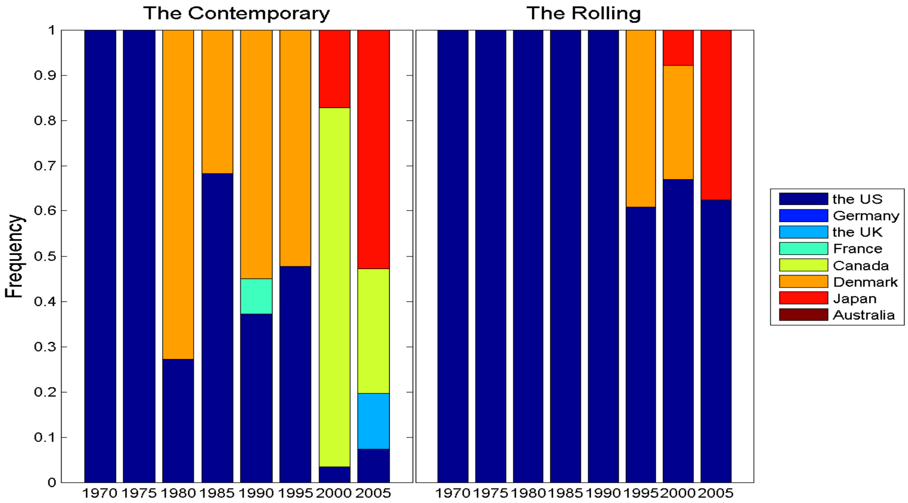

Fig. B.16: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Machinery and equipment, nec*

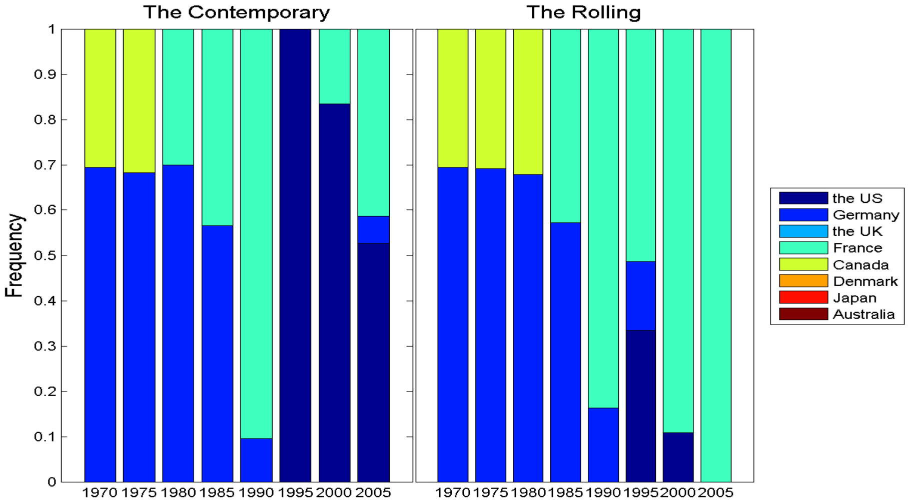

Fig. B.17: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Electrical machinery and apparatus*

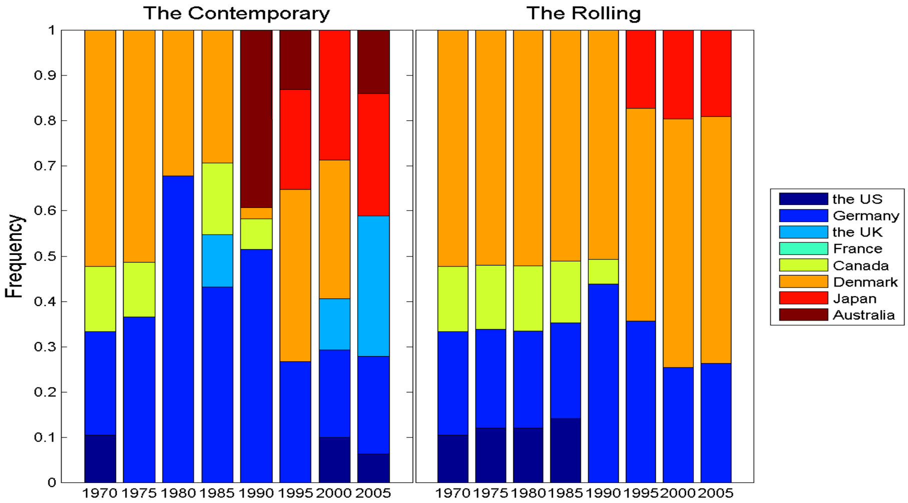

Fig. B.18: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Transport equipment*

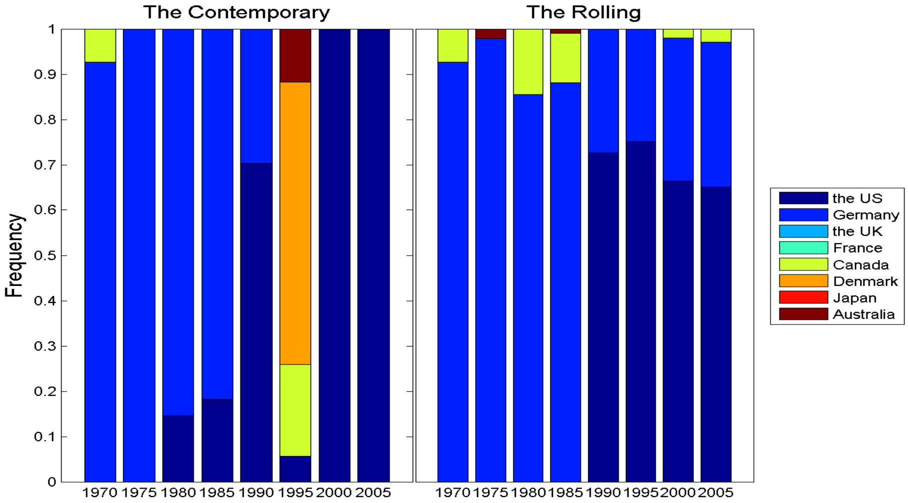

Fig. B.19: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Manufacturing nec; recycling (include Furniture)*

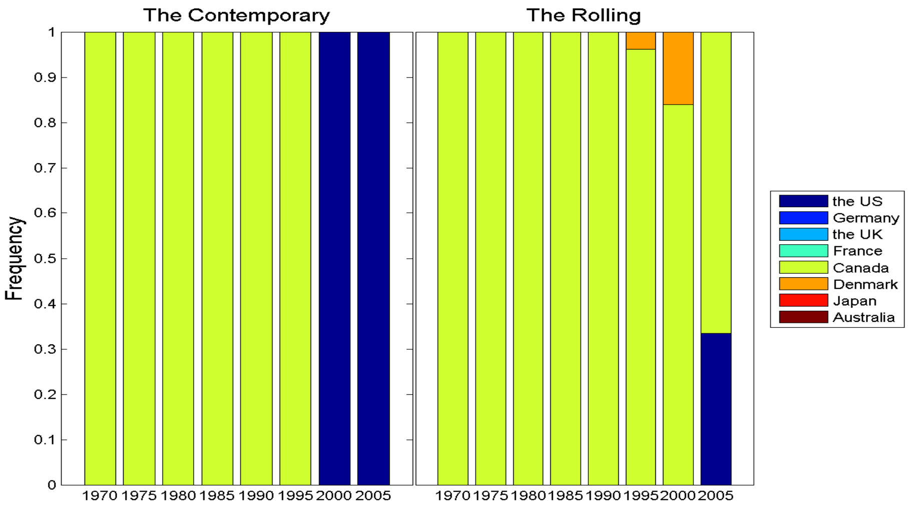

Fig. B.20: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Production and distribution of electricity, gas, and water*

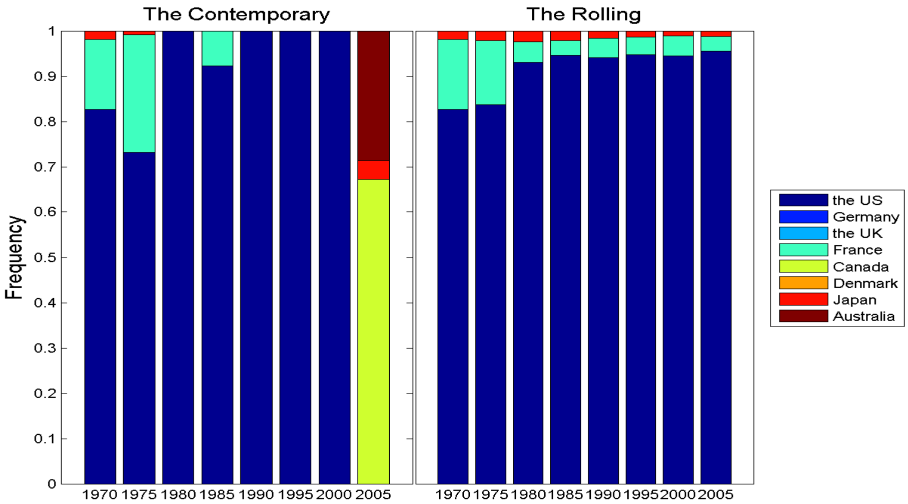

Fig. B.21: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Construction*

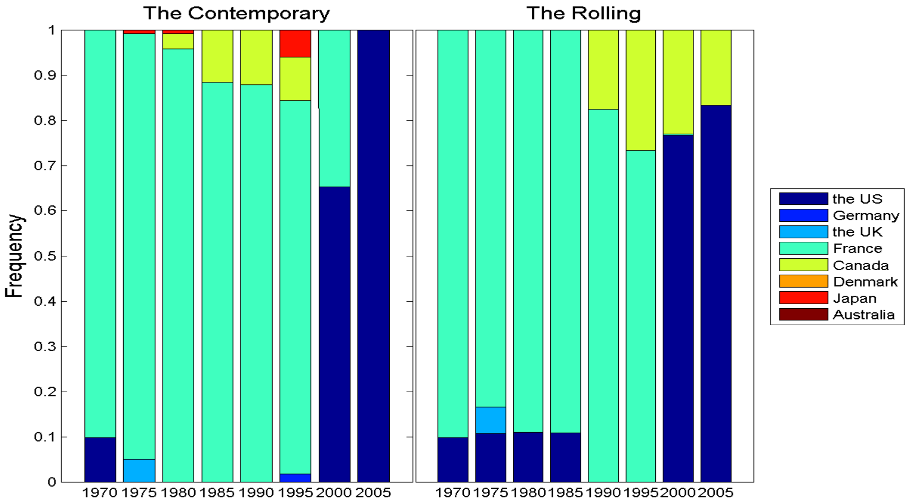

Fig. B.22: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wholesale and retail trade*

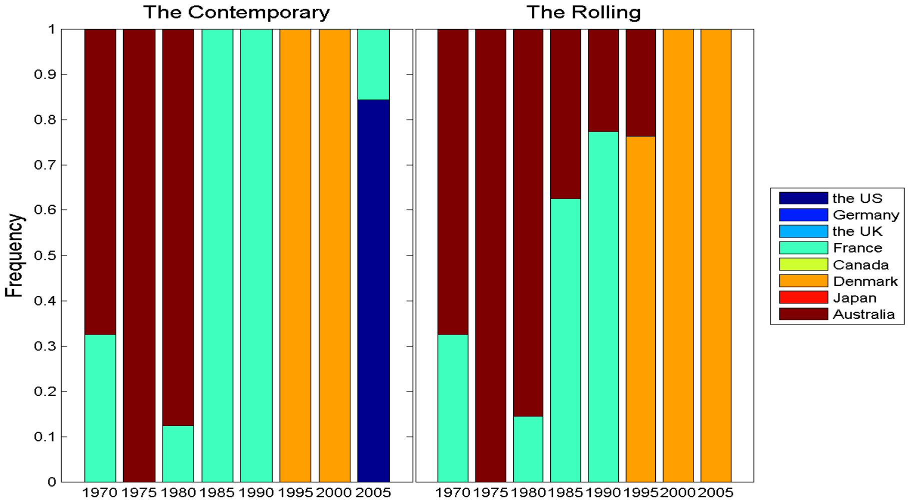

Fig. B.23: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Service activities (transport, hotels and restaurants)*

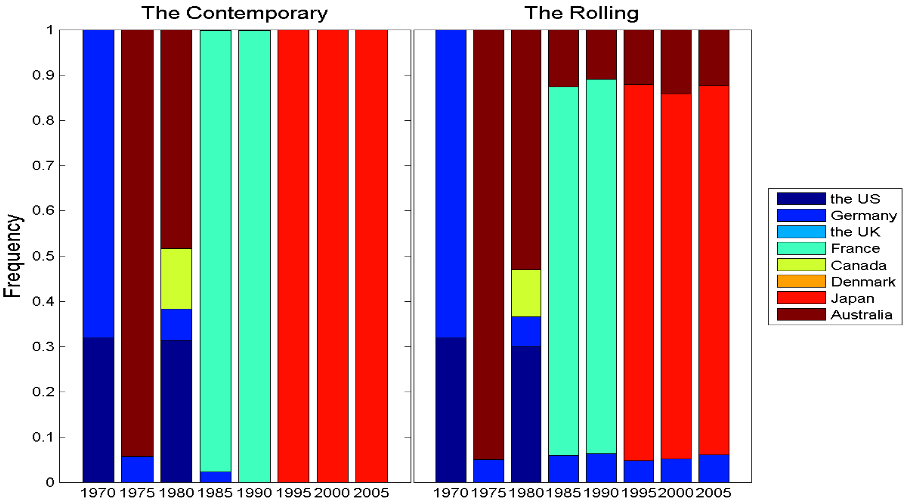

Fig. B.24: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Post and telecommunications*

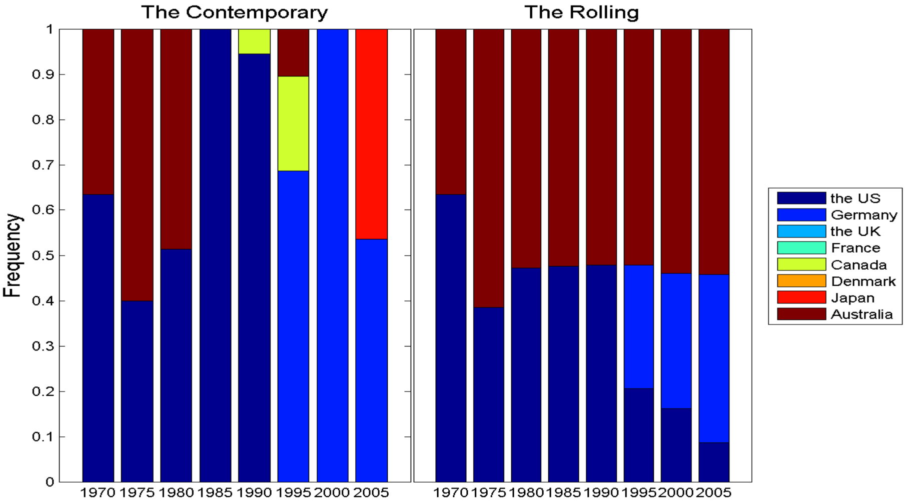

Fig. B.25: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Business activities (finance, real estate, and R&D)*

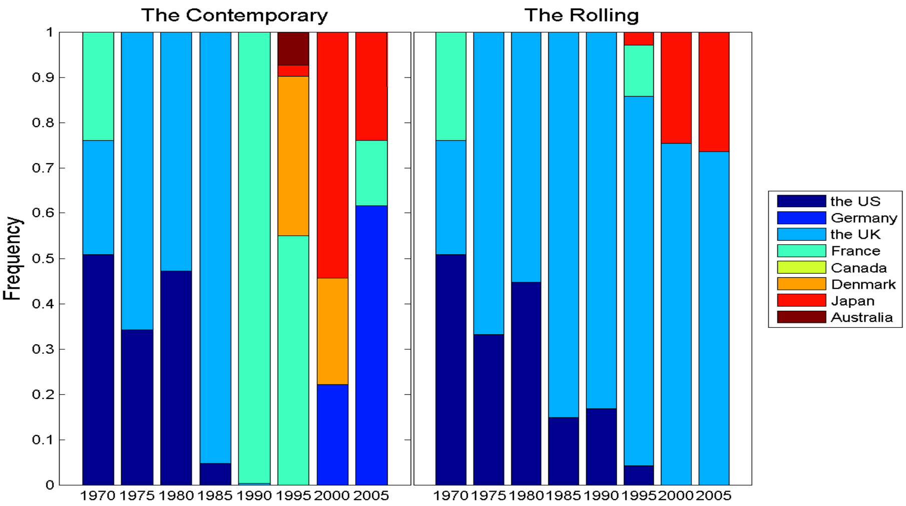

Fig. B.26: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Public administration, education and health*

B.3 Country Specific Contributions to the Contemporary and Rolling Technological Frontiers

The following 8 figures show the country specific contributions to the contemporary and rolling technological frontiers. The maximum value is equal to the number of industries, 23, and would imply that the given country's wage-profit frontier coincided with the technological frontier.

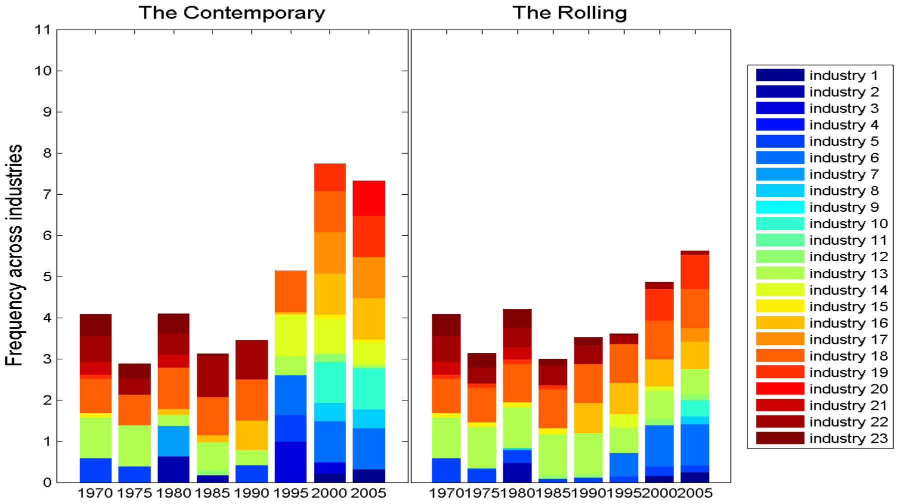

Fig. B.27: *Country specific contributions to the contemporary technological frontiers, the US*

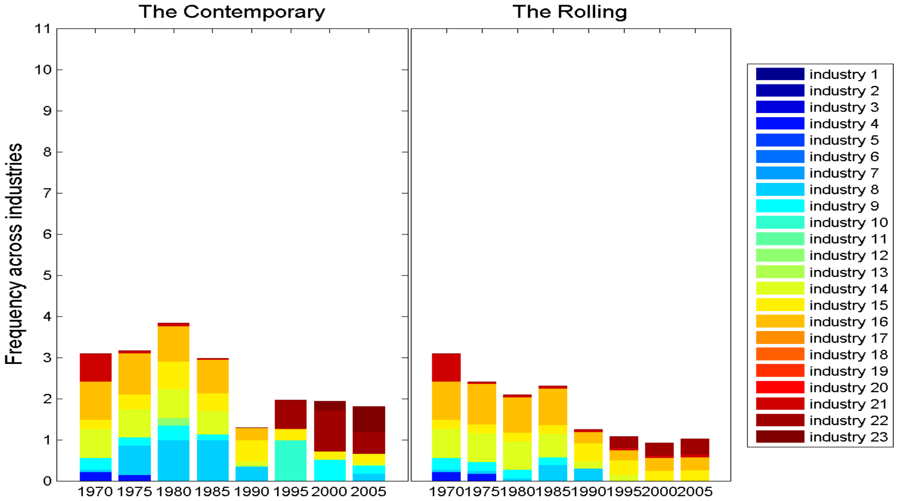

Fig. B.28: *Country specific contributions to the contemporary technological frontiers, Germany*

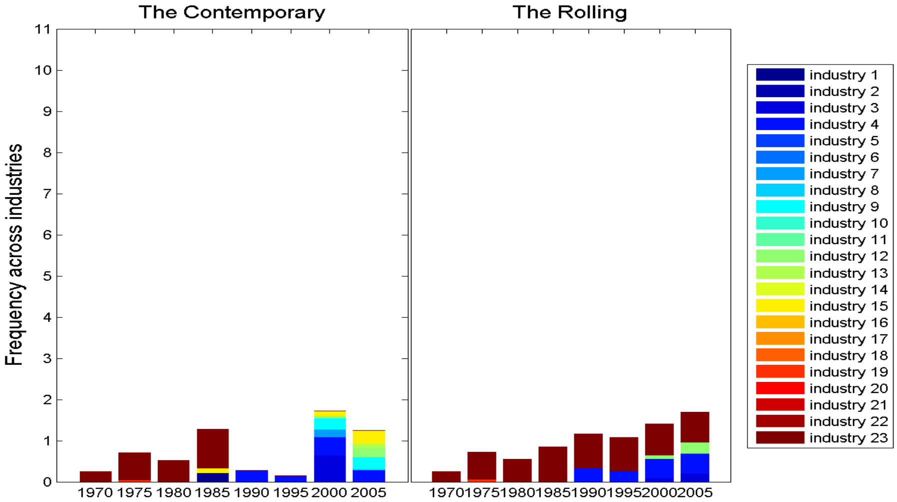

Fig. B.29: *Country specific contributions to the contemporary technological frontiers, the UK*

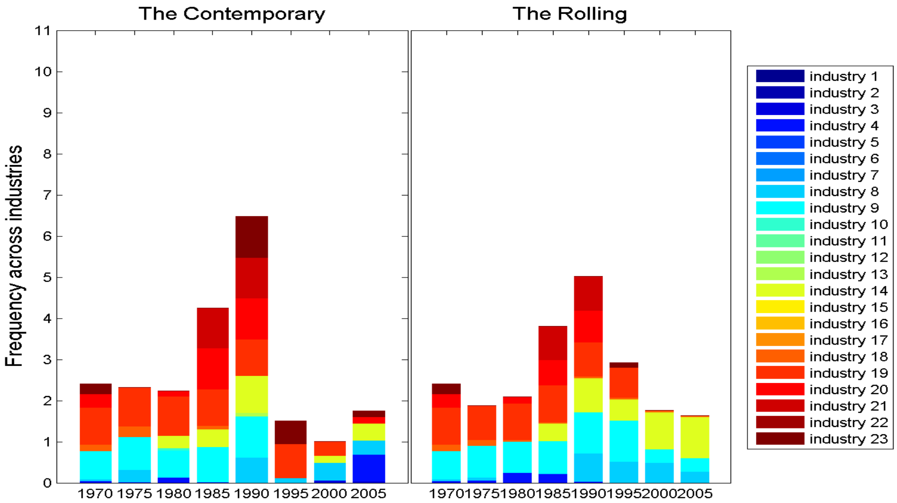

Fig. B.30: *Country specific contributions to the contemporary technological frontiers, France*

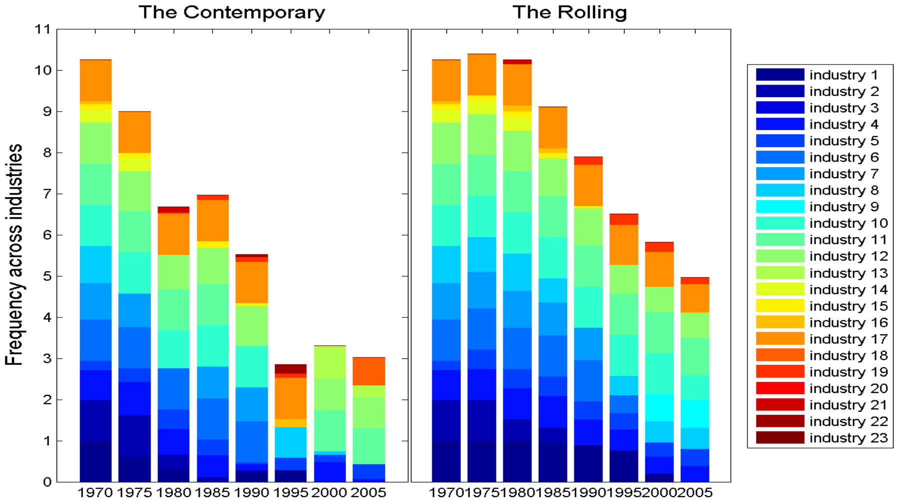

Fig. B.31: *Country specific contributions to the contemporary technological frontiers, Canada*

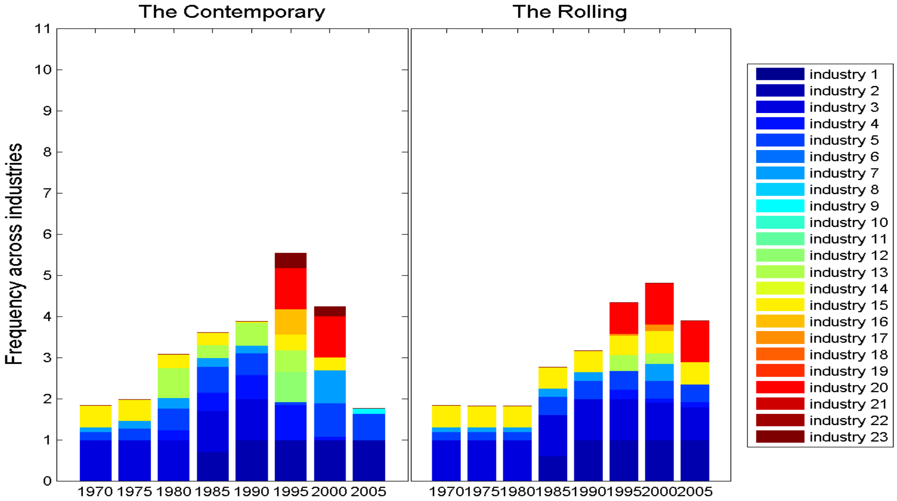

Fig. B.32: *Country specific contributions to the contemporary technological frontiers, Denmark*

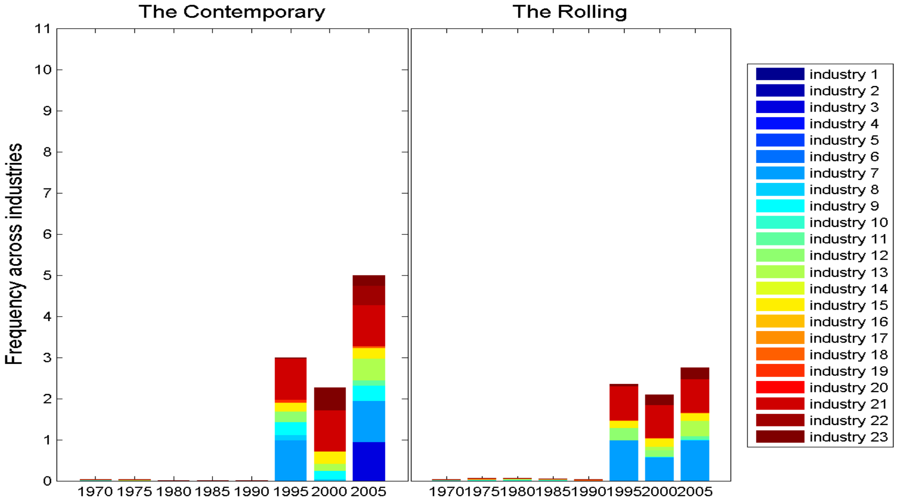

Fig. B.33: *Country specific contributions to the contemporary technological frontiers, Japan*

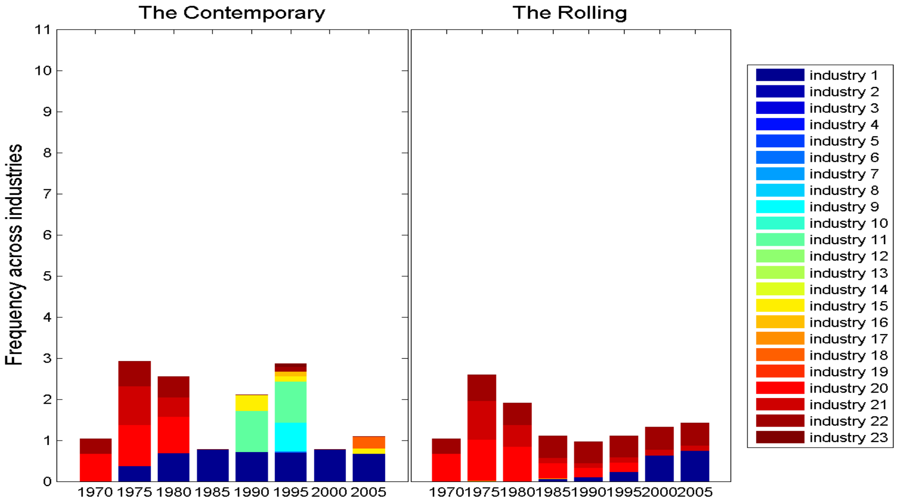

Fig. B.34: *Country specific contributions to the contemporary technological frontiers, Australia*