Journal of International Economics 14 (1983) 367–380. North-Holland Publishing Company

FACTOR REWARDS AND THE INTERNATIONAL MIGRATION OF UNSKILLED LABOR: A MODEL WITH CAPITAL MOBILITY

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Received August 1980, revised version received March 1982

Conventional economic wisdom holds that the migration of unskilled labor from less developed countries to neighboring developed countries should be expected to narrow the wage gap between those countries, and thereby reduce the incentive for further migration. If capital is mobile internationally this reasoning may be inappropriate. Instead, emigration of unskilled labor out of the less developed country provides an incentive for capital to leave the country, too. As a consequence, wage rates move in the same direction in each country, and the gap between wage rates across countries even may increase.

1. Introduction

The Guest Worker programs initiated by Western European nations and the more recent surge of illegal immigration into the United States from Latin America are but two examples of movements of predominately unskilled workers from less developed countries to developed countries. Conventional economic wisdom regarding the effects of such labor movements suggests that the wages paid to unskilled workers in the receiving country should fall and that the wages paid to their counterparts in the country of emigration should rise. This seemingly incontrovertible statement appears to explain why proposed liberalizations of immigration restrictions in developed countries often meet with strenuous objections from labor groups while government officials in less developed countries tend to view emigration as a vent for surplus unskilled labor. If capital is internationally mobile, however, this casual application of economic theory may lead to erroneous predictions about the behavior of wage rates in the face of labor movements between countries. Also, the focus on an alleged conflict of interest among members of the unskilled worker group may only serve to draw attention away from the probable increase in the returns experienced by capital owners world-wide.

*We would like to thank the referees for their perceptive suggestions, and also Professor Bhagwati for his patience and encouragement.

The aim of this paper is to show, in the context of a static, general equilibrium model, that when unskilled workers move from a less developed country to a developed country: (1) wages paid to this type of labor are likely to fall in both countries while the returns to all capital owners rise, and (2) the developed country accumulates capital at the expense of the less developed country. Moreover, production technologies may differ sufficiently across countries that the absolute disparity between the wage rates paid to unskilled labor in the two countries actually may increase. These results indicate that the current dilemma regarding illegal immigration into the United States may be persistent. Even leaving aside the compounding factor of divergent population growth rates, incentives for entry brought about by international wage rate differences may not tend to disappear when emigration to the United States occurs.

The remainder of the discussion is organized into three sections. Section 2 presents the model used while the results stated above are developed in greater detail in section 3. Implications and conclusions are drawn out in section 4.

2. The model

In this model, two countries, A and B, are assumed to make up a closed economic system. Country A is assumed to represent an economically developed nation while B represents one that is economically less developed. These countries produce two goods, X_1 and X_2, using three factors, unskilled labor, L, skilled labor, S, and capital, K. Output prices are determined through trade between the two countries. In addition, capital is assumed to be fixed in supply to both countries taken together but perfectly mobile internationally, while each country's supply of labor is completely inelastic. At first, these assumptions regarding international factor mobility may seem ill-suited to a study of labor migration issues. Labor, however, is subject to comparatively more restrictions on its international mobility as compared with capital and these assumptions do capture the essence of this distinction. Labor migration is handled in this formulation by parametrically shifting unskilled labor across national boundaries, where such shifts reflect assumed changes in immigration policy. The factor mobility assumptions adopted here have the important implication, which is roughly consistent with the empirical findings of Harberger (1980), that returns to capital owners must be internationally identical, while if technologies differ across countries, wages paid to labor will be higher in one country than another.

Turning next to the structure of production, country A is assumed to produce both goods, the outputs of which are denoted as X_{1A} and X_{2A}, and has available quantities of all three factors, L_A, S_A, and K_A. More specifically, the two types of labor are specific factors in that L_A is used only in the production of X_{1A} and S_A is used only in the production of X_{2A}. Capital is intersectorally mobile and may therefore be used in either X_{1A} or X_{2A} production. In contrast to country A, country B is assumed to be completely specialized in the production of X_1. This output, denoted X_{1B}, is obtained using unskilled labor, L_B, and capital, K_B, available in country B. Since B is assumed to possess no skilled labor, production of X_2 in this country is impossible. Consequently, the pattern of trade involves B exporting X_{1B} and importing X_{2A}.

This simplistic characterization of production and trade patterns captures several important aspects of economic relations between developed and neighboring less developed countries, although it admittedly ignores others which may be critical in the context of different policy questions. Production of a set of high technology goods in the developed country alone, where skilled labor is much more readily available, does not conflict too sharply with actual experience. The greater diversification of the developed country's economy, implicit in this formulation, would appear to be a warranted simplification as well.[1] The assumption of full employment in the developing country may seem less appropriate. However, if wage rigidity and unemployment were assumed to exist instead, the set of questions addressed would deal with the net impact on unemployment as a consequence of the initial migration to the developed country, and the answers should rest on the same factors identified here.[2] While less developed countries do in reality have skilled portions of their work-forces, this proportion generally is much smaller than in developed countries. More importantly, the immigration from neighboring less developed countries which has occurred in the European and U.S. cases cited in the introduction is dominated by the movement of unskilled labor.[3]

The equations describing the production side of the model are quite similar to those found in Jones (1965, 1971). Underlying assumptions are: (1) production functions for each good produced exhibit constant returns to scale, and (2) factor and commodity markets are perfectly competitive. Together, these two assumptions imply that all factors of production are fully employed and that enterpreneurs earn zero profits. For country A, the full employment conditions are:

$$C_{L1}^{A} X_{1A} = L_A, \tag{1}$$

[1]By allowing for a single sector in the less developed country, any consideration of internal migration from one sector to another in that country is ruled out. For contributions which address that issue, see Harris and Todaro (1970) and Bhagwati and Srinivasan (1974).

[2]As shown in the next section of the paper, a key causal factor in the model is that the greater availability of unskilled labor in the developed country attracts capital out of the developing country. Whether that capital movement leads to a fall in wages or a fall in employment opportunities in the developing country depends upon the initial assumption of flexible or rigid wages, but clearly either outcome presents social problems in the developing country.

[3]For an excellent collection of papers dealing with the emigration of skilled labor from less developed countries to developed countries, see Bhagwati (1976).

$$C^{A}_{S2}X_{2A}=S_{A}, \tag{2}$$

$$C^{A}_{K1}X_{1A}+C^{A}_{K2}X_{2A}=K_{A}, \tag{3}$$

and the zero profit conditions are:

$$C^{A}_{L1}W_{A}+C^{A}_{K1}r=P_{1}=1, \tag{4}$$

$$C^{A}_{S2}q+C^{A}_{K2}r=P_{2}=P, \tag{5}$$

where C^{A}_{ij} denotes the input–output coefficient describing the average quantity of factor i ($i=L_A, S_A, K_A$) used to produce one unit of commodity j ($j=1,2$) in country A, W_A denotes the wage paid to unskilled labor in A, q denotes the wage paid to skilled labor in A, r denotes the common return to the owners of capital in both countries, and $P=P_2/P_1$ denotes the common commodity price ratio prevailing in both countries.

In country B, the full employment and zero profit equations are analagous to those for A. Specifically:

$$C^{B}_{L1}X_{1B}=L_{B}, \tag{6}$$

$$C^{B}_{K1}X_{1B}=K_{B}, \tag{7}$$

$$C^{B}_{L1}W_{B}+C^{B}_{K1}r=1, \tag{8}$$

where all variable definitions parallel those given in the model for country A. Note, however, that C^{B}_{L1} and C^{B}_{K1} would differ from their counterparts for country A due to international differences in technology which result in differences in the wage–rental ratio. Alternatively, such differences would arise if distortions in the output market meant that different relative prices were faced in the two countries.[4]

Finally, the production side of the model is completed by: (1) constraining the sum of K_A and K_B to be fixed at K,

$$K_{A}+K_{B}=K, \tag{9}$$

and (2) constraining the sum of L_A and L_B to be fixed at L,

$$L_{A}+L_{B}=L. \tag{10}$$

[4]If technologies used in the production of X_1 are identical in each country, then if the same product prices and returns to capital are faced in each country, the wage rates in each country must be identical. An initial wage gap and the economic incentive for migration internationally does not exist in that situation.

The demand side of the model insures that commodity markets are always in equilibrium and that payments balance for both countries. Commodity market equilibrium equations may be expressed as:

$$D_1 = D_1(P, Y) = X_1 = X_{1A} + X_{1B}, \tag{11}$$

$$D_2 = D_2(P, Y) = X_{2A}, \tag{12}$$

$$Y = Y_A + Y_B, \tag{13}$$

where D_j denotes the total demand for commodity j in countries A and B taken together and Y_i denotes real income in country i ($i = \mathrm{A, B}$). These two equations embody the assumptions that tastes in the two countries are identical and homothetic, and that total demand for each commodity must equal total supply. Eqs. (11), (12), and (13), however, do not guarantee balance of payments equilibrium for either country. Consequently, eqs. (14) and (15) are added in order to insure this outcome:

$$(X_{1A} - D_{1A}) + P(X_{2A} - D_{2A}) + r(\kappa_A - K_A) = 0, \tag{14}$$

$$(X_{1B} - D_{1B}) - PD_{2B} + r(\kappa_B - K_B) = 0, \tag{15}$$

where D_{ji} denotes the demand for commodity j in country i and κ_i denotes the quantity of capital owned by residents of country i. Eq. (14), then, requires that, for country A, the value of net exports plus net foreign earnings must equal zero. Note that this formulation does not take into account remittances sent by workers back to the developing country, a distinction which is unimportant as long as identical homothetic tastes are assumed. Taken together, the two balance of payments restrictions imply that one of the commodity market equilibrium equations is redundant, and eq. (12) is dropped from further consideration.

In summary, the model to be applied in the following section consists of eqs. (1) through (15), excepting (12). Attention there is directed to the case where the international difference in technology applied in X_1 production results in $W_A > W_B$. The effects on W_A, W_B and r resulting from a simultaneous increase in L_A and decrease L_B satisfying eq. (10) are of primary interest. That parametric shift in unskilled labor supplies, which is economically consistent with the assumed direction of the initial wage gap, could arise if A liberalized immigration policy toward B or reduced the vigor with which existing immigration statutes are enforced.

3. Effects of unskilled labor migration in factor rewards

This section focuses on changes in: (1) the nominal rewards paid to unskilled labor in countries A and B, (2) the nominal rewards paid to owners of capital in both countries, and (3) the quantity of capital employed in both countries in the face of an international movement of unskilled labor. Effects on the remaining endogenous variables are not of major interest in this study, but are given in the appendix.

The existence of capital mobility in this model has some rather unexpected implications for the behavior of wage rates paid to unskilled labor in both countries. In particular, rewriting the zero profit equations for $X_{1\mathrm{A}}$ and $X_{1\mathrm{B}}$ in relative rates of change and then solving for W_{A}^* and W_{B}^* yields:

$$W_{\mathrm{A}}^* = -(\theta_{K1}^{\mathrm{A}}/\theta_{L1}^{\mathrm{A}})r^*, \tag{16}$$

$$W_{\mathrm{B}}^* = -(\theta_{K1}^{\mathrm{B}}/\theta_{L1}^{\mathrm{B}})r^*, \tag{17}$$

where $Z^* = \mathrm{d}Z/Z$, $\theta_{K1}^{\mathrm{A}} = C_{K1}^{\mathrm{A}}r$, and the remaining θ_{ij}^k are similarly defined. Eqs. (16) and (17) show that in response to a movement of unskilled workers from the less developed country B to the developed country A: (1) W_{A} and W_{B} move in the same, rather than in the opposite direction, (2) both W_{A} and W_{B} move in the opposite direction from the change in r, and (3) since, as will be demonstrated below, r may move in either direction, both W_{A} and W_{B} could actually rise. These results contrast with the impressionistic conjectures mentioned in the introduction regarding the probable behavior of these two variables. Apparently, these conjectures are based upon an assumption of capital immobility between the two countries. That is, if capital were immobile, then the wage rate paid to L_{B} would rise because fewer workers would be employed there using a capital stock of a fixed size.

The direction of change in the rental rate of return to capital is unambiguously positive (negative) if the signs of the two expressions shown in eqs. (18) and (19) are both positive (negative):

$$(K_{1\mathrm{A}}/L_{\mathrm{A}}) - (K_{\mathrm{B}}/L_{\mathrm{B}}) \gtreqless 0, \tag{18}$$

$$(X_{1\mathrm{A}}/L_{\mathrm{A}}) - (X_{1\mathrm{B}}/L_{\mathrm{B}}) \gtreqless 0. \tag{19}$$

Therefore, if the capital–labor ratio in the production of $X_{1\mathrm{A}}$ exceeds that for $X_{1\mathrm{B}}$ *and* if average productivity of unskilled labor is higher in country A than in B, $r^*/L_{\mathrm{A}}^* > 0$. To illustrate conditions under which both eqs. (18) and (19) would be positive or negative, consider the special case where $X_{1\mathrm{A}}$ and $X_{1\mathrm{B}}$ are produced according to different Cobb–Douglas type production functions. Suppose

$$X_{1\mathrm{A}} = MK_{1\mathrm{A}}^{\alpha}L_{\mathrm{A}}^{1-\alpha}, \tag{20}$$

$$X_{1\mathrm{B}} = NK_{\mathrm{B}}^{\beta}L_{\mathrm{B}}^{1-\beta}, \tag{21}$$

where $M \neq N$ and/or $\alpha \neq \beta$. The marginal products of unskilled labor in the two countries are

$$MPL_A = (1-\alpha)X_{1A}/L_A = (1-\alpha)M(K_{1A}/L_A)^{\alpha}, \tag{22}$$

$$MPL_B = (1-\beta)X_{1B}/L_B = (1-\beta)N(K_B/L_B)^{\beta}. \tag{23}$$

The previously stated assumptions of: (1) perfect competition, (2) $W_A > W_B$, and (3) identical commodity prices in both countries, guarantee that $MPL_A/MPL_B > 1$. The condition $MPL_A/MPL_B > 1$, however, does not imply that the differences shown in eqs. (18) and (19) will be either positive or negative. That proposition easily can be demonstrated by multiplying expressions for MPL_A/MPL_B and $(X_{1A}/L_A)/(X_{1B}/L_B)$ by the ratio of the marginal products of capital in X_{1A} and X_{1B} production. Since the rental rate of return to capital is identical in both countries, $MPK_{1A}/MPK_B = 1$ and the multiplications described produce:

$$W_A/W_B = (MPL_A/MPL_B)(MPK_{1A}/MPK_B) > 1 \tag{24}$$

$$= (1-\alpha)\beta(K_{1A}/L_A)/(1-\beta)\alpha(K_B/L_B) > 1, \tag{25}$$

$$(X_{1A}/L_A)/(X_{1B}/L_B) = M(K_{1A}/L_A)^{\alpha}/N(K_B/L_B)^{B} \tag{26}$$

$$= \beta(K_{1A}/L_A)/\alpha(K_B/L_B). \tag{27}$$

In relating these results to (18) and (19), one obvious situation to consider is where $\alpha = \beta$ (the case where the technological differences favoring the developed country A are factor neutral). More specifically, if $\alpha = \beta$, eqs. (25) and (27) show that unambiguously, $(K_{1A}/L_A) - (K_B/L_B) > 0$ and $(X_{1A}/L_A) - (X_{1B}/L_B) > 0$, and, as noted above, $r^*/L_A^* > 0$. Additionally, that same sign pattern for eqs. (18) and (19) must hold if the output elasticity of capital is larger in country A than in B, i.e. $\alpha > \beta$. However, if $\beta > \alpha$, then possibly eqs. (18) and (19) both will be negative or will have opposite signs.

To establish an economic rationale underlying the behavior of r, assume that sufficient conditions exist guaranteeing a positive sign on both eqs. (18) and (19). Under those conditions, when unskilled labor is transferred from country B to country A, world output of X_1 rises by more than any possible increase in X_2 production, thus driving up $P = P_2/P_1$ to clear world markets. The distributional effects that will be simultaneously observed in this situation can be deduced from a more general result developed by Batra and Casas (1976) in their comprehensive analysis of production relationships in a three-factor–two-good setting. Their theorem 9 states that a rise in the relative price of a commodity will lower the real reward of the factor used relatively intensively by the other good. In the present situation, then, when the relative price of X_2 rises, the return to unskilled labor falls. Hence, from zero-profit eq. (16), the return to capital rises.

Interestingly, in the broader case where eqs. (18) and (19) are not unambiguously positive and the sign of r^*/L_A^* may be positive, negative, or indeterminate, capital *must* flow out of country B and into country A whenever there is an international migration of unskilled labor in that same direction. To understand this result, let r_A and r_B denote respectively capital's rental rate in countries A and B, where in equilibrium $r_A - r_B = 0$. If K_B is held constant, then a small increase in L_A, and corresponding reduction in L_B, would increase W_B relative to W_A.[5] From eqs. (16) and (17), then $r_A - r_B$ would rise. Also, if L_B is held constant, then a small increase in K_A, and corresponding reduction in K_B, would lower $r_A - r_B$. Thus, the maintenance of the equilibrium condition $r_A - r_B = 0$ implies that a small increase in L_A must be accompanied by a corresponding increase in K_A, a result indicating that when country A liberalizes immigration policy or relaxes border enforcement, B loses both labor and capital.

Consider again the case where a movement of unskilled workers from country B to country A unambiguously results in a decline in both W_A and W_B. If W_A and W_B both fall, the possibility exists that the gap between W_A and W_B may actually increase in the face of a transfer of unskilled workers from B to A. A necessary, although not sufficient, condition for this situation to arise is that the percentage decline in W_B be greater than the percentage decline in W_A. From eqs. (16) and (17) that case can be seen to occur when

$$(\theta_{K1}^A \theta_{L1}^B - \theta_{K1}^B \theta_{L1}^A)/\theta_{L1}^A \theta_{L1}^B < 0. \tag{28}$$

This expression depends upon factor intensity measures expressed in value terms. Such a comparison, based on the ratio of capital's share versus labor's share of output in the production of X_1 across both countries, contrasts with the definitions in physical terms which appeared in eq. (19). The two measures may give different results when $W_A > W_B$. A similar distinction between physical intensities and value intensities has played a key role in the voluminous literature on factor market distortions.[6] The present focus is slightly different, since attention here is paid to differences across countries in the two intensity definitions, and not differences across industries within a country.

How likely is this condition to be met? Recall the particular example mentioned above, which rested on Cobb–Douglas production functions and factor-neutral technological superiority in country A. Under these conditions

[5]This result can be derived from theorem 1 of Batra and Casas (1976).
[6]See Magee (1973) for a summary of the development of this literature.

where the value shares of capital and labor are identical in each country, then eqs. (16) and (17) show that wages change by the same percentage in each country and the wage gap remains constant in percentage terms. Additionally, while the extent of country A's technological superiority helps determine the size of the initial wage gap, that factor does not affect the percentage wage changes which result from labor migration. This outcome of a constant proportional wage gap suggests little reason for optimism with respect to the potential equilibrating tendency of greater immigration into advanced countries to cause the current wage gap to disappear. In the more general case, if capital's share of X_1 output in country B exceeds that of country A, an even more pessimistic conclusion might follow, namely that the wage gap could increase.[7] That is, if $\beta > \alpha$, then r^*/L_A^* still could be positive and, in that case, the percentage decline in W_B would be larger than the percentage decline in W_A.

4. Summary

This paper has projected some of the likely impacts on income distribution of the inflow of unskilled labor into developed countries from neighboring less developed countries. Because internationally mobile capital is incorporated into the static, general equilibrium model used, the analysis develops several counter-intuitive implications of greater labor migration. The key behavioral factor is that the greater availability of unskilled labor in the developed country will attract capital out of the less developed country. Even though the direction of this capital flow can be stated unambiguously, the effect on wage rates and returns to capital cannot be predicted without additional information. When both the capital–labor ratio and output labor ratio in the developed country are greater than the less developed country, wage rates in both countries will fall. Furthermore, the gap between the wage rates in the two countries even may increase.

Appendix

The purpose of this appendix is to provide algebraic detail regarding some of the results presented in the text. To solve for the direction of change in the endogenous variables with respect to an increase in unskilled workers from country B entering country A, the equations of the model are rewritten in relative rates of change. For the production side in country A:

$$X_{1A}^* + \gamma_L^A (r^* - W_A^*) = L_A^* \tag{A.1}$$

[7]These comments are based on an illustrative example only, which nevertheless demonstrates that even in a fairly structured situation, few unambiguous results hold. When other restrictions are relaxed, such as the assumed elasticity of factor substitution of one in a Cobb–Douglas world, even more ambiguities are introduced.

$$X_{2A}^* + \gamma_S^A(r^* - q^*) = S_A^*, \tag{A.2}$$

$$\lambda_{K1}^A X_{1A}^* + \lambda_{K2}^A X_{2A}^* + \gamma_{KL}^A(W_A^* - r^*) + \gamma_{KS}^A(q^* - r^*) = K_A^*, \tag{A.3}$$

$$\theta_{L1}^A W_A^* + \theta_{K1}^A r^* = 0, \tag{A.4}$$

$$\theta_{S2}^A q^* + \theta_{K2}^A r^* = P^*, \tag{A.5}$$

where $Z^* = \mathrm{d}Z/Z$, $\lambda_{Kj}^A = C_{Kj}^A X_{jA}/K_A$ denotes the fraction of K_A employed in the production of commodity j, $\theta_{K2}^A = C_{K2}^A r/P$ denotes capital's share of the output of commodity 2, and the remainder of the λ_{ij} and θ_{ij} are similarly defined. Also, $\gamma_L^A = \theta_{K1}^A \sigma_1^A$, $\gamma_S^A = \theta_{K2}^A \sigma_2^A$, $\gamma_{KL}^A = \lambda_{K1}^A \theta_{L1}^A \sigma_1^A$, $\gamma_{KS}^A = \lambda_{K2}^A \theta_{S2}^A \sigma_2^A$, and $\gamma_{KK}^A = \gamma_{KL}^A + \gamma_{KS}^A$, where σ_j^A denotes the elasticity of substitution in the production of commodity j in country A.

Expressions for country B's production relations may be presented in an analogous fashion. In relative rates of change the full employment equations are:

$$X_{1B}^* + \gamma_L^B(r^* - W_B^*) = L_B^*, \tag{A.6}$$

$$X_{1B}^* + \gamma_K^B(W_B^* - r^*) = K_B^*, \tag{A.7}$$

where $\gamma_L^B = \theta_{K1}^B \sigma_1^B$, $\gamma_K^B = \theta_{L1}^B \sigma_1^B$, and $\lambda_{L1}^B = \lambda_{K1}^B = 1$, while the zero profit equation is:

$$\theta_{L1}^B W_B^* + \theta_{K1}^B r^* = 0. \tag{A.8}$$

The full employment equations for country B are tied to those for country A via the factor supply relations:

$$K^* = k_A K_A^* + k_B K_B^* = 0, \tag{A.9}$$

$$L^* = l_A L_A^* + l_B L_B^* = 0, \tag{A.10}$$

where $l_j = L_j/L$ and $k_j = K_j/K$ for $j = $ A, B.

Finally, the demand side of the model may be compressed to one reduced form equation. First, observe that when no net saving takes place in either country, the total value of expenditures by a country's residents must equal the total value of national income:

$$Y_A = D_{1A} + PD_{2A} = X_{1A} + PX_{2A} + r(\kappa_A - K_A), \tag{A.11}$$

$$Y_B = D_{1B} + PD_{2B} = X_{1B} + r(\kappa_B - K_B). \tag{A.12}$$

Therefore, both balance of payments and commodity market equilibrium are guaranteed by substituting eqs. (13), (A.11), and (A.12) into eq. (11), and then expressing the result in relative rates of change as:

$$\Pi_{1A}X_{1A}^* + \Pi_{1B}X_{1B}^* - \Pi_{2A}X_{2A}^* - \alpha_p P^* = 0, \tag{A.13}$$

where $\Pi_{1A} = X_{1A}(Y - \eta_Y X_1)/X_1 Y$, $\Pi_{1B} = X_{1B}(Y - \eta_Y X_1)/X_1 Y$, $\Pi_{2A} = \eta_Y P X_{2A}/Y$, α_p denotes the income compensated elasticity of demand for X_1 calculated with respect to a change in $P = P_2/P_1$, and η_Y denotes the income elasticity of demand for X_1 which is equal to unity under the assumption of homothetic tastes. Since α_p is assumed to be positive and since the difference $(Y - \eta_Y X_1)$ is necessarily non-negative, the coefficients Π_{1A}, Π_{1B}, and Π_{2A} are non-negative as well.

A more complete derivation of eq. (A.13) is as follows. Write eq. (11) in relative rates of change as:

$$X_1^* = (X_{1A}/X_1)X_{1A}^* + (X_{1B}/X_1)X_{1B}^* = \alpha_p' P^* + \eta_y Y^*, \tag{A.14}$$

where α_p' denotes the ordinary price elasticity of demand for X_1. Next, write the income equations in (A.11) and (A.12) in relative rates of change as:

$$Y_A^* = (X_{1A}/Y_A)X_{1A}^* + (P/X_{2A})(X_{2A}^* + P^*)$$
$$+ [r(\kappa_A - K_A)/Y_A]r^* - (rK_A/Y_A)K_A^*, \tag{A.15}$$

$$Y_B^* = (X_{1B}/Y_B)X_{1B}^* + [r(\kappa_B - K_B)/Y_B]r^* - (rK_B/Y_B)K_B^*. \tag{A.16}$$

Substituting (A.15) and (A.16) into (A.17):

$$Y^* = (Y_A/Y)Y_A^* + (Y_B/Y)Y_B^*. \tag{A.17}$$

And then substituting this result into (A.14) produces the expression reported in eq. (A.13).

The algebraic results from the model not presented in the text are:

$$|D| = -\alpha_p\theta_{L1}^A\theta_{S2}^A(\gamma_K^B + \gamma_L^B) - \alpha_p\theta_{L1}^A\theta_{L1}^B k_{AB}\lambda_{K2}^A\gamma_S^A$$
$$-\alpha_p\theta_{L1}^B\theta_{S2}^A k_{AB}\gamma_L^A\lambda_{K1}^A - \alpha_p\theta_{L1}^B k_{AB}(\theta_{L1}^A\gamma_{KS}^A + \theta_{S2}^A\gamma_{KL}^A)$$
$$-\theta_{L1}^A k_{AB}\gamma_L^A\gamma_S^A(\lambda_{K1}^A\Pi_{2A} + \lambda_{K2}^A\Pi_{1A}) - k_{AB}\theta_{L1}^B\gamma_L^A\Pi_{1A}\gamma_{KS}^A$$
$$-\theta_{L1}^A k_{AB}\gamma_L^B\Pi_{1B}(\gamma_{KS}^A + \lambda_{K2}^A\gamma_S^A) - \theta_{L1}^A k_{AB}\Pi_{2A}\gamma_S^A\gamma_{KL}^A$$
$$-\theta_{L1}^A\gamma_S^A\Pi_{2A}(\gamma_K^B + \gamma_L^B) \leqq 0, \tag{A.18}$$

$$X^*_{1A}/L^*_A = \{-\alpha_p\theta^B_{L1}\theta^A_{S2}\gamma^A_L l_{AB} - \alpha_p\theta^A_{S2}\theta^A_{L1}(\gamma^B_K + \gamma^B_L)$$

$$-\alpha_p\theta^B_{L1}\theta^A_{L1}\gamma^A_S\lambda^A_{K2}k_{AB} - \alpha_p\theta^B_{L1}\theta^A_{L1}k_{AB}\gamma^A_{KS}$$

$$-\alpha_p\theta^B_{L1}\theta^A_{S2}k_{AB}\gamma^A_{KL} - \theta^A_{L1}\gamma^A_S\Pi_{2A}(\gamma^B_L + \gamma^B_K)$$

$$-\theta^A_{L1}\gamma^A_S\lambda^A_{K2}k_{AB}\Pi_{1B}\gamma^B_L - \theta^A_{L1}\gamma^A_{KS}k_{AB}\Pi_{1B}\gamma^B_L - \theta^B_{L1}\gamma^A_L l_{AB}\gamma^A_{KS}\Pi_{1B}k_{AB}$$

$$-\theta^B_{L1}l_{AB}\gamma^A_L\gamma^A_S(\lambda^A_{K2}\Pi_{1B}l_{AB} + \Pi_{2A}) - \theta^B_{L1}\Pi_{2A}\gamma^A_S\gamma^A_{KL}l_{AB}\}/|D| \gtreqless 0, \tag{A.19}$$

$$X^*_{1B}/L^*_A = \{\alpha_p\theta^B_{L1}\theta^A_{L1}k_{AB}l_{AB}\lambda^A_{K2}\gamma^A_S + \alpha_p\theta^B_{L1}\theta^A_{S2}k_{AB}l_{AB}\lambda^A_{K1}\gamma^A_L$$

$$+\alpha_p\theta^A_{S2}\theta^A_{L1}(\lambda^A_{K1}k_{AB}\gamma^B_L + l_{AB}\gamma^B_K) + \alpha_p\theta^B_{L1}\theta^A_{L1}\theta^A_{K2}k_{AB}l_{AB}\gamma^A_{KS}$$

$$+\alpha_p\theta^B_{L1}\theta^A_{K1}\theta^A_{S2}k_{AB}l_{AB}\gamma^A_{KL} + \alpha_p\theta^B_{L1}\theta^A_{K1}\theta^A_{L1}k_{AB}l_{AB}\gamma^A_{KK}$$

$$+\theta^B_{L1}\theta^A_{K1}k_{AB}l_{AB}\Pi_{2A}\gamma^A_{KL}\gamma^A_S + \theta^B_{L1}\theta^A_{K1}k_{AB}l_{AB}\gamma^A_L$$

$$\times[\gamma^A_S(\lambda^A_{K1}\Pi_{2A} + \Pi_{1A}\lambda^A_{K2}) + \Pi_{1A}\gamma^A_{KS}]$$

$$+\theta^B_{K1}\theta^A_{L1}\Pi_{2A}\gamma^A_S[\lambda^A_{K1}k_{AB}\gamma^B_L + l_{AB}\gamma^B_K] + \theta^B_{KL}\theta^A_{L1}\gamma^B_L k_{AB}\Pi_{1A}(\lambda^A_{K2} + \gamma^A_{KS})$$

$$+\theta^B_{L1}\theta^A_{L1}\gamma^B_K[k_{AB}(\gamma^A_{KS} + \lambda^A_{K2}\gamma^A_S) + \Pi_{2A}\gamma^A_S] + \theta^B_{L1}\theta^A_{L1}k_{AB}\Pi_{2A}l_{AB}\gamma^A_S\gamma^A_{KL}$$

$$+\theta^B_{L1}\theta^A_{L1}k_{AB}\lambda^A_{K1}\Pi_{2A}\gamma^A_S(\gamma^B_L + l_{AB}\gamma^A_L) + \theta^B_{L1}\theta^A_{L1}k_{AB}\Pi_{1A}\gamma^B_L(\gamma^A_{KS} + \lambda^A_{K2}\gamma^A_S)$$

$$+\theta^B_{L1}\theta^A_{L1}k_{AB}\Pi_{1A}l_{AB}\gamma^A_L(\gamma^A_{KS} + \lambda^A_{K2}\gamma^A_S)\}/|D| \leqq 0, \tag{A.20}$$

$$X^*_{2A}/L^*_A = \{-\alpha_p\theta^B_{L1}\theta^A_{L1}\gamma^A_S(l_{AB} - \lambda^A_{K1}k_{AB})$$

$$+\theta^B_{L1}\gamma^A_S[\gamma^A_L l_{AB}(\lambda^A_{K1}\Pi_{1B}k_{AB} - \Pi_{1A})$$

$$+\gamma^A_{KL}k_{AB}(\Pi_{1B}l_{AB} - \Pi_{1A})]$$

$$+\theta^A_{L1}\gamma^A_S[\gamma^B_L(\lambda^A_{K1}\Pi_{1B}k_{AB} - \Pi_{1A})$$

$$+\gamma^B_K(\Pi_{1B}l_{AB} - \Pi_{1A})]\}/|D| \gtreqless 0, \tag{A.21}$$

$$q^*/L_A^* = \{\theta_{L1}^B \gamma_L^A l_{AB}[\Pi_{1B}\lambda_{K1}^A k_{AB} - \Pi_{1A}]$$

$$+ \theta_{L1}^B \theta_{L1}^A k_{AB}\gamma_{KK}^A[\Pi_{1B} l_{AB} - \Pi_{1A}] + \theta_{L1}^A \gamma_K^B[\Pi_{1B} l_{AB} - \Pi_{1A}]$$

$$+ \theta_{L1}^A \gamma_L^B[\Pi_{1B}\lambda_{K1}^A k_{AB} - \Pi_{1A}] + \theta_{L1}^B \theta_{K1}^A \gamma_{KL}^A k_{AB}[\Pi_{1B} l_{AB} - \Pi_{1A}]$$

$$+ \theta_{L1}^B \theta_{L1}^A \gamma_S^A[\Pi_{2A}(l_{AB} - \lambda_{K1}^A k_{AB})$$

$$+ (k_{AB}\lambda_{K2}^A(\Pi_{1B} l_{AB} - \Pi_{1A}))]\}/|D| \gtreqless 0, \tag{A.22}$$

$$r^*/L_A^* = \{\theta_{L1}^A \theta_{L1}^B(\alpha_p \theta_{S2}^A + \Pi_{2A}\gamma_S^A)(l_{AB} - \lambda_{K1}^A k_{AB})$$

$$+ \theta_{L1}^A \theta_{L1}^B k_{AB}(\gamma_{KS}^A + \lambda_{K2}^A \gamma_S^A)(\Pi_{1B} l_{AB} - \Pi_{1A})\}/|D| \gtreqless 0, \tag{A.23}$$

$$K_A^*/L_A^* = \{-\alpha_p \theta_{S2}^A \theta_{L1}^A \lambda_{K1}^A(\gamma_L^B + \gamma_K^B) - \alpha_p \theta_{L1}^B \theta_{L1}^A l_{AB}\lambda_{K2}^A \gamma_S$$

$$- \alpha_p \theta_{L1}^B \theta_{S2}^A l_{AB}\lambda_{K1}^A \gamma_L^A - \alpha_p \theta_{L1}^B \theta_{S2}^A l_{AB}\gamma_{KL}^A$$

$$- \alpha_p \theta_{L1}^A \theta_{L1}^B l_{AB}\gamma_{KS}^A - \theta_{L1}^A \gamma_S^A(\lambda_{K1}^A \Pi_{2A}$$

$$+ \Pi_{1A}\lambda_{K2}^A)(\gamma_K^B + \gamma_L^B) - \theta_{L1}^B l_{AB}\Pi_{2A}\gamma_{KL}^A \gamma_S^A$$

$$- \theta_{L1}^B \Pi_{1A}\gamma_{KS}^A \gamma_L^A l_{AB} - \theta_{L1}^B \gamma_S^A \gamma_L^A l_{AB}(\lambda_{K1}^A \Pi_{2A} + \Pi_{1A}\lambda_{K2}^A)$$

$$+ \theta_{L1}^A \gamma_{KS}^A l_{AB}\Pi_{1B}\gamma_K^B + \theta_{L1}^A \gamma_K^B l_{AB}\Pi_{1B}\lambda_{K2}^A \gamma_S^A$$

$$- \theta_{L1}^A \gamma_K^B \Pi_{1A}\gamma_{KS}^A - \theta_{L1}^A \Pi_{1A}\gamma_{KS}^A \gamma_L^B\}/|D| \geqq 0. \tag{A.24}$$

The fact that $K_A^*/L_A^* \geqq 0$ may be proved as follows. Consider the sum of the positively signed terms in the numerator:

$$\theta_{L1}^A \gamma_K^B \gamma_S^A \lambda_{K2}^A \Pi_{1B} l_{AB} + \theta_{L1}^A \gamma_K^B \gamma_{KS}^A \Pi_{1B} l_{AB} = \theta_{L1}^A \gamma_K^B l_{AB}\Pi_{1B}\lambda_{K2}^A \sigma_2^A. \tag{A.25}$$

Next combine this result with two negatively signed terms to form

$$S = \theta_{L1}^A \gamma_K^B l_{AB}\Pi_{1B}\lambda_{K2}^A \sigma_2^A - \theta_{L1}^A \gamma_S^A(\lambda_{K1}^A \Pi_{2A}$$

$$+ \Pi_{1A}\lambda_{K2}^A)(\gamma_L^B + \gamma_K^B) - \theta_{L1}^A \Pi_{1A}\gamma_{KS}^A(\gamma_L^B + \gamma_K^B). \tag{A.26}$$

Substituting for the γ_{ij}^{k} produces:

$$S=\theta_{L1}^{\mathrm{A}}\theta_{L1}^{\mathrm{B}}l_{\mathrm{AB}}\Pi_{1\mathrm{B}}\lambda_{K2}^{\mathrm{A}}\sigma_1^{\mathrm{B}}\sigma_2^{\mathrm{A}}-\theta_{L1}^{\mathrm{A}}\theta_{K2}^{\mathrm{A}}(\lambda_{K1}^{\mathrm{A}}\Pi_{2\mathrm{A}}+\Pi_{1\mathrm{A}}\lambda_{K2}^{\mathrm{A}})\sigma_1^{\mathrm{B}}\sigma_2^{\mathrm{A}}$$
$$-\theta_{L1}^{\mathrm{A}}\Pi_{1\mathrm{A}}\lambda_{K2}^{\mathrm{A}}\theta_{S2}^{\mathrm{A}}\sigma_1^{\mathrm{B}}\sigma_2^{\mathrm{A}}, \qquad (\mathrm{A}.27)$$

which can be rewritten as:

$$S=\sigma_1^{\mathrm{B}}\sigma_1^{\mathrm{A}}\theta_{L1}^{\mathrm{A}}[\theta_{L1}^{\mathrm{B}}l_{\mathrm{AB}}\Pi_{1\mathrm{B}}\lambda_{K2}^{\mathrm{A}}-\theta_{K2}^{\mathrm{A}}\lambda_{K1}^{\mathrm{A}}\Pi_{2\mathrm{A}}-\Pi_{1\mathrm{A}}\lambda_{K2}^{\mathrm{A}}] \qquad (\mathrm{A}.28)$$

or

$$S=\frac{\sigma_1^{\mathrm{B}}\sigma_2^{\mathrm{A}}\theta_{L1}^{\mathrm{A}}K_{2\mathrm{A}}}{K_{\mathrm{A}}YX_1}[W_{\mathrm{B}}L_{\mathrm{A}}(Y-X_1)-rK_{1\mathrm{A}}X_1-X_{1\mathrm{A}}(Y-X_1)]\leqq 0, \quad (\mathrm{A}.29)$$

since by assumption $W_{\mathrm{B}}<W_{\mathrm{A}}$ and $W_{\mathrm{A}}L_{\mathrm{A}}\leqq X_{1\mathrm{A}}$:

$$P^*/L_{\mathrm{A}}^*=\{\theta_{L1}^{\mathrm{B}}\theta_{L1}^{\mathrm{A}}\gamma_S^{\mathrm{A}}[k_{\mathrm{AB}}\lambda_{K2}^{\mathrm{A}}(\Pi_{1\mathrm{B}}l_{\mathrm{AB}}-\Pi_{1\mathrm{A}})$$
$$+\Pi_{2\mathrm{A}}(l_{\mathrm{AB}}-\lambda_{K1}^{\mathrm{A}}k_{\mathrm{AB}})]$$
$$+\theta_{L1}^{\mathrm{B}}\theta_{L1}^{\mathrm{A}}\gamma_{KS}^{\mathrm{A}}[k_{\mathrm{AB}}(\Pi_{1\mathrm{B}}l_{\mathrm{AB}}-\Pi_{1\mathrm{A}})]$$
$$+\theta_{S2}^{\mathrm{A}}\theta_{L1}^{\mathrm{A}}[\gamma_K^{\mathrm{B}}(\Pi_{1\mathrm{B}}l_{\mathrm{AB}}-\Pi_{1\mathrm{A}})+\gamma_L^{\mathrm{B}}(\lambda_{K1}^{\mathrm{A}}k_{\mathrm{AB}}\Pi_{1\mathrm{B}}-\Pi_{1\mathrm{A}})]$$
$$+\theta_{L1}^{\mathrm{B}}\theta_{S2}^{\mathrm{A}}l_{\mathrm{AB}}\gamma_L^{\mathrm{A}}(\lambda_{K1}^{\mathrm{A}}k_{\mathrm{AB}}\Pi_{1\mathrm{B}}-\Pi_{1\mathrm{A}})$$
$$+\theta_{L1}^{\mathrm{B}}\theta_{S2}^{\mathrm{A}}\gamma_{KL}^{\mathrm{A}}k_{\mathrm{AB}}(\Pi_{1\mathrm{B}}l_{\mathrm{AB}}-\Pi_{1\mathrm{A}})\}/|D|\gtreqless 0. \qquad (\mathrm{A}.30)$$

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