Revealing Preferences for Fairness in Ultimatum Bargaining*

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Abstract: The ultimatum game has been the primary tool for studying bargaining behavior in recent years. However, not enough information is gathered in the ultimatum game to get a clear picture of responders' utility functions. We analyze a convex ultimatum game in which responders' can "shrink" an offer as well as to accept or reject it. This allows us to observe enough about responders' preferences to estimate utility functions. We then successfully use data collected from convex ultimatum games to predict behavior in standard games. Our analysis reveals that rejections can be "rationalized" with neo-classical preferences over own- and other-payoff that are convex, nonmonotonic, and regular. These findings present a precise benchmark for models of fairness and bargaining.

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1. Introduction

Perhaps the most important tool for studying bargaining over the last 15 years has been the ultimatum game. This game has been explored in hundreds of laboratory experiment, in dozens of countries, and for stakes that range from the trivial to the profound. In each case the results are similar: Fairness in the minds of responders has a significant effect on allocations.[1]

The ultimatum game is a two-stage interaction. In the first stage a proposer offers a division of a pie. In the second stage the responder can accept the proposal or reject it. A rejection means that both players get nothing. Sub-game perfection indicates that all offers should be accepted, and thus the smallest possible offer should be made. Nonetheless, responders tend to reject "unfair" offers, and thus proposers tend not to make them. Recent studies show that behavior remains quite far from the prediction even when using multiple trials and large stakes.[2]

The challenge to economists has been to understand this outcome within the context of an economic model. What kind of preferences or beliefs could result in this type of behavior? While several scholars have proposed models of fairness that could be consistent with the choices of subjects, getting a clear picture of the preferences of bargainers has been elusive. One reason is that an ultimatum game experiment collects too little information from responders to learn about their utility functions. With the dichotomous choice of accept or reject, it is difficult, even over multiple games, to learn enough about the objective functions of bargainers to cleanly test hypotheses about their behavior.

This paper will recover bargainers' preferences by employing a convex ultimatum game.[3] The convex ultimatum game contains the regular ultimatum game

[1]For important early contributions see Werner Guth et al. (1982), Jack Ochs and Alvin E. Roth (1989), Alvin Roth et al. (1991), and Robert Forsythe et al. (1994). For reviews of the literature on bargaining and ulitmatum games, see Alvin E. Roth (1995) and Colin Camerer (2003). For important recent contributions, see Robert Slonim and Alvin E. Roth (1998) on large stakes, Catherine C. Eckel and Philip Grossman (2001) on gender differences, and William Harbaugh et al. (2000) on bargaining of children. Uri Gneezy, Ernan Haruvy, and Roth (2003) look at ultimatum bargaining under a deadline, where proposers' offers can be revised before they are rejected.

[2]See Slonim and Roth (1998) and Roth, Prasnikar, Okunofujiwara and Zamir (1991). These studies show that while repetition and large stakes both move players in the direction of sub-game perfection, few outcomes actually reach the prediction.

[3]The convex ultimatum game was explored by Andreoni, Castillo and Petrie (2003) in a single shot framework. There have been other convex games introduced in the literature. Matthew Rabin (1997) discussed the game theoretically (called the "squishy game"). Ramzi Suleiman

as a special case, has the same subgame perfect equilibrium, but measures much more about responders' preferences than the regular ultimatum game. As before, the proposer offers a split of the pie. Then the responder chooses how much to "shrink" the pie. Not shrinking at all is to accept the offer, shrinking to zero is to reject it, but in the convex game shrinking to some intermediate level is also possible. The advantage of the convex game is that one can learn more about the shape of each bargainer's utility function and estimate whether well-behaved preferences could have generated the data. With multiple observations from each subject, one can get a picture of what these preferences look like.

In this paper, we examine subjects who play a series of 20 regular or convex bargaining games, each with a randomly changing partner. Because the game is repeated and subjects respond to real offers, this gives us the number of observations we need to estimate utility functions based on actual behavior for each of our subjects in the convex game. We find that subjects' choices can be captured by preferences that are convex, regular, but not monotonic. When we ask whether these estimated utility functions can predict the behavior in regular ultimatum games, we get a strikingly strong fit. We also compare the predictions of our model to predictions of two other models of fairness, Fehr and Schmidt (1999) and Charness and Rabin (2002).

The approach highlights the great deal of heterogeneity across subjects. Only about 12 percent of respondents act like money-maximizers, and only 25 percent have "linear" preferences that would lead them to accept or reject (but not shrink) all offers. The rest have strictly convex preferences with indifference curves that, to varying degrees, bend back at the extreme allocations.

Our findings provide a precise benchmark for models of fairness, and can be used to draw distinctions among several alternative models. In addition, the wide heterogeneity of preferences suggests models of incomplete information, along with fairness, may be necessary to understand ultimatum game behavior.[4] It also suggests a possible role for new bargaining institutions that take advantage of natural tastes for fairness to promote economic efficiency.

The paper is organized as follows. The next section outlines the theoretical implications of the convex ultimatum game. Section 3 describes our experiment. Sections 4, 5 and 6 present the results for both proposers and responders, while section 7 is a conclusion.

(1996) presents a game simliar to ours, but instead of letting the responder choose how much to shrink the pie, that is fixed by the experimenter. Marlies Ahlet et al. (2001) also propose a convex game, where the amout of pie shrinkage is bound from above by the show-up fee and offer amount. So, only offers of 50-50 can be shrunk to zero. Bosman and van Winden (2002) use a game similar to ours, but call it a Power-to-Take game, and examine emotion.

[4] See, e.g., Kennan and Wilson (1993) for a review.

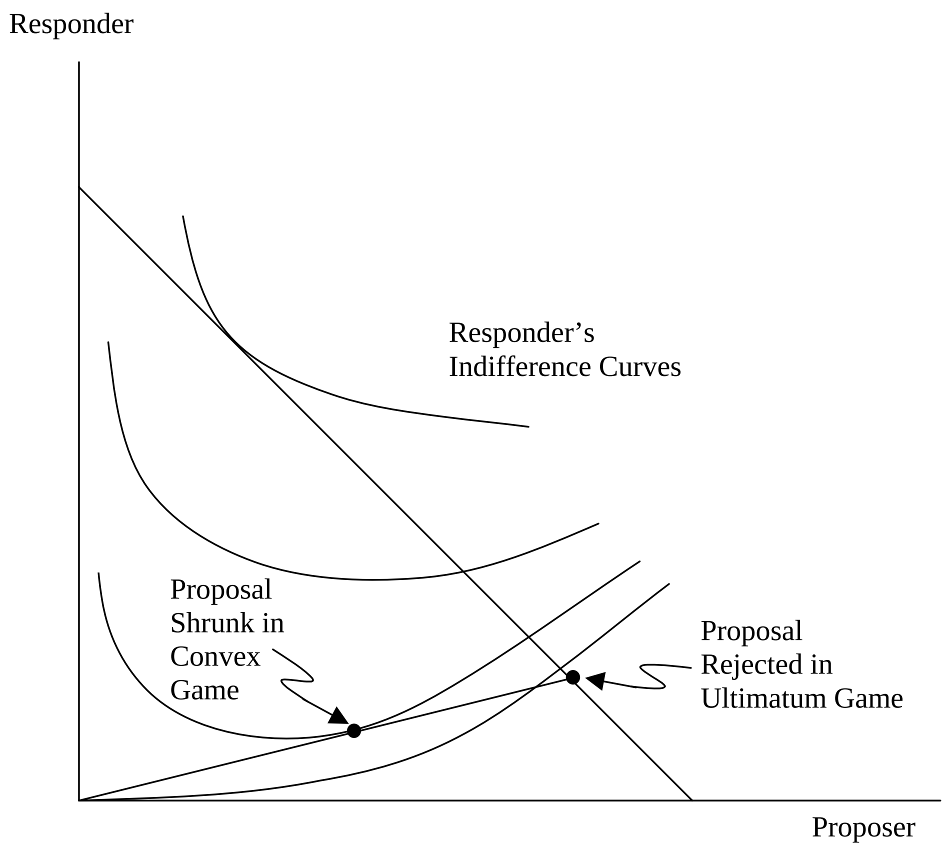


Figure 1: Responder Choices in Convex and Standard Ultimatum Games

2. Theory and Hypotheses

Figure 1 illustrates the convex and regular ultimatum games. Formally, let M be the total number of dollars to be bargained over. The proposer chooses a division $a \in [0, 1]$, and the responder chooses a number of dollars to divide m such that the payoffs for the proposer and responder are, respectively,

$$\begin{aligned} \pi_p &= (1-a) \times m \\ \pi_r &= a \times m. \end{aligned}$$

By restricting m to be either 0 or M we have a regular ultimatum game. That is, the proposal is fully accepted or fully rejected. In the convex ultimatum game, by contrast, we allow m to be any number between 0 and M. Thus, the regular ultimatum game is nested within the convex game. Moreover, the standard subgame perfect equilibrium is the same in both—since no offers should be shrunk, minimal offers should be made.

Figure 1 shows how a responder who dislikes inequality may react differently to the same offer in the two games. It shows that a proposal that would be rejected in the ultimatum game will be shrunk in the convex game. Similarly, some proposals that would be accepted in an ultimatum game will also be shrunk in a convex game.[5]

There are several clear and systematic differences in the two games. These can be seen in Figure 2. First, we can graph the best reply functions of a responder in the two games. We see first that in the ultimatum game there will be a critical offer at which a responder switches from reject to accept. With the convex game the movement between the two extremes is continuous, with some offers being shrunk. Hence, the best-reply function in the convex game will tend to cross that of the regular game from below, as shown in the figure.

Figure 2 also illustrates how the convex game shifts bargaining power to responders. A money-maximizing proposer will tend to make higher offers in the convex game than in the standard game. Notice that, while this means that the responders will always have higher utility in the convex game, they will not necessarily end up with more money. A responder may end up accepting offers in the ultimatum game that she would prefer to shrink were she playing the convex game. Hence, the prediction about relative earnings is ambiguous, even though the prediction on relative utility is not.[6]

[5] The graph also makes the case for the presence of disadvantageous counterproposals (Ochs and Roth, 1989). Notice that when the responder's preferences are nonlinear, but not necessarily monotonic, the responder would prefer an allocation that gives him a smaller amount of money, provided the proposer's payoff is also reduced.

[6] See Matthew Rabin (1997) for more formal consideration of this game. It is also interesting to note that the indifference curves plotted in Figures 1 and 2 would be consistent with preferences proffered by Rabin (1993). This is discussed in more detail in Andreoni, Castillo and Petrie (2003).

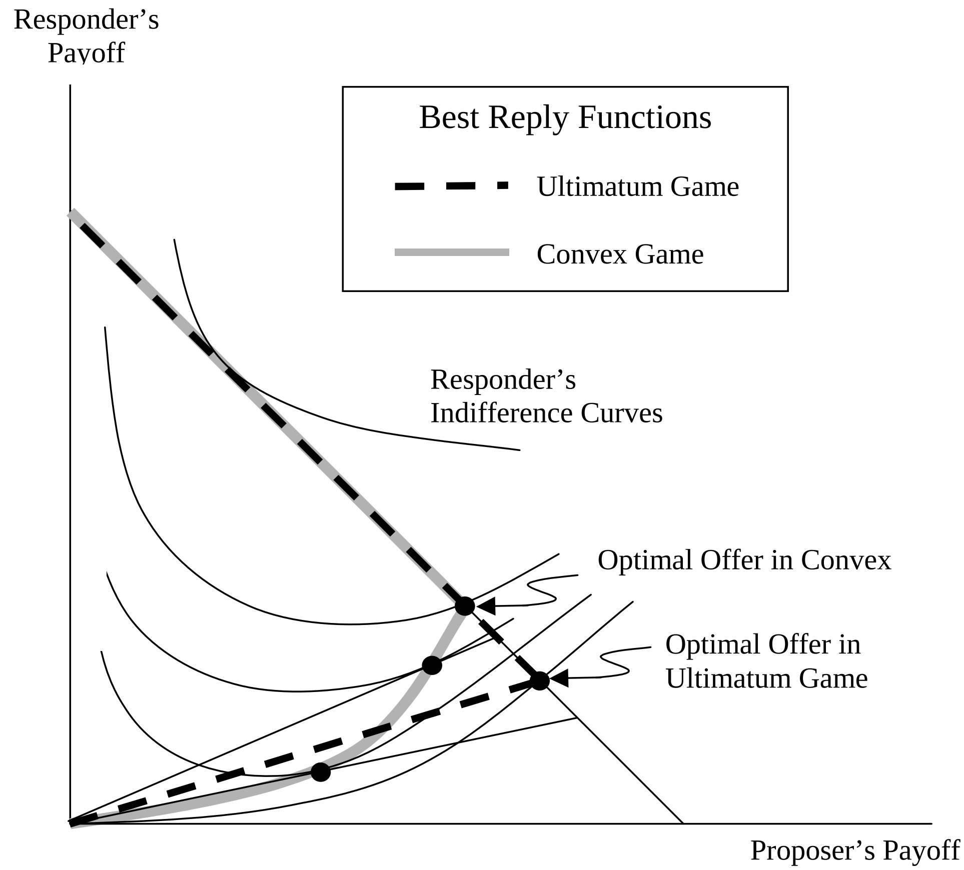


Figure 2: Best Reply Function

Note as well that there is nothing that precludes bargainers who care about inequality of all kinds from shrinking extra-generous offers. A responder who, for instance, is offered 90 percent of the pie may find it immodest to accept this, and may shrink or reject the offer. As we will see, we actually observe behavior consistent with this for some subjects. Such behavior, while uncommon, is consistent with findings elsewhere.[7]

The most important difference in the two games, however, is the added data that is gathered in the convex game. By observing offers that are shrunk we can

[7] Andreoni and Miller (2002) found about 25 percent of subjects disliked even favorable inequality. A similar fraction was found in Andreoni, Castillo and Petrie (2003). Rabin (1993), Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) cite evidence of similar effects in many other experiments.

gain valuable information about the shape of responders' indifference curves that will be useful in uncovering preferences in the standard ultimatum game.

3. Experimental Design

Our data was collected at the University of Wisconsin. Subjects were volunteers from undergraduate economics and business classes. A total of 96 subjects participated in the experiment, 48 in the standard ultimatum game and 48 in the convex ultimatum game. Each session of the experiment lasted about one hour, and subjects earned on average \$17.96 (*s.d.* \$4.50), plus a \$5.00 show-up fee.

Each session of the experiment required 24 subjects. Subjects were assigned one role, proposer or responder, which they kept throughout the experiment. They played 20 rounds of the bargaining game. Each round was played with a randomly chosen anonymous partner. Subjects never knew which person they were playing with, and were guaranteed never to play the same partner more than twice. They were paid in cash at the end of the study.

In each game, the proposer and responder bargain over how to divide 10 quarters (the US 25-cent piece). The proposer offered a division of each quarter, allocating from 0 to 25 cents to the responder and the rest to himself. The division is the same for each quarter. The responder, upon seeing the division offered by the proposer, decides how many of the quarters to divide. In the standard ultimatum game, responders could choose either 0 or 10 quarters. In the convex ultimatum game, the choice was any number of quarters from 0 to 10. With this one exception, the two games were identical. All of the parameters of the experiment were known to all subjects.

Subjects were assigned subject numbers, and names were never recorded, and all interactions took place on a computer network. Subjects read the instructions fully and were taken through several examples of how payoffs were calculated. Their earnings were placed in a closed envelope and presented to them at the end of the study. Full instructions for the games are available from the authors.[8]

Notice that each iteration of the game is a bargain over \$2.50, that is, 10 quarter-dollars. Thus, over the 20 iterations, subjects bargained over the division of \$50.

We ran two sessions of each the regular and convex ultimatum games. Hence, there are 24 proposers and 24 responders in our data from each of the games.

[8] Go to http://www.ssc.wisc.edu/~andreoni/ (Andreoni), http://www.prism.gatech.edu/~mc338/ (Castillo) or http://www.gsu.edu/~ecorap/ (Petrie).

4. Instrument Check

We first check how our repeated standard ultimatum game compares to previous research. The average response function, or the probability of accepting any given offer, is similar to the results from Roth, Prasnikar, Okunofujiwara and Zamir (1991) and Slonim and Roth (1998), who both use a 10-round standard ultimatum game. Acceptance rates in Roth et al. start out at 30 percent for offers of 30 percent and jump to 70 percent for offers of 40 percent. In Slonim and Roth, acceptance rates are 55.5 percent for offers in the range of 30–34.5 percent and 76.3 percent for offers in the range of 40–44.5 percent. In our game, offers around 30 percent are accepted 24 percent of the time, and offers of 40 percent are accepted 78 percent of the time in our study.

The distribution of offers in our standard game is also similar to Roth et al. (1991). The modal offer in Roth et al. is 50 percent, whereas in our game the modal offer is 44 percent (11 of 25 cents).

5. Results for Responders

Figure 3 illustrates the average response to each offer for both the convex and standard ultimatum games. This figure is consistent with the theory presented above. At low offers the ultimatum game leads subjects to reject more than they would like, and at high offers it leads them to accept more than they would like. Indeed, this suggests that the best-reply function (Figure 2) for the convex game is cutting that of the standard game from below.

Table 1 gives more detail. The Table shows that, as in our second prediction, high offers are made more frequently in the convex game. The ability to partially reject offers shifts bargaining power to responders, thus generating higher offers. Notice, however, that we also observe more density at lower offers in the convex game, especially at the 2 to 7 range. This was not anticipated by the theory section above, but is consistent with our findings in the single-shot version of this game (2003). We hypothesized then that lower offers were less risky in the convex game because so many of the responses were shrunk rather than fully rejected, which made these offers more attractive to proposers. We explore this hypothesis again in the section on proposers.

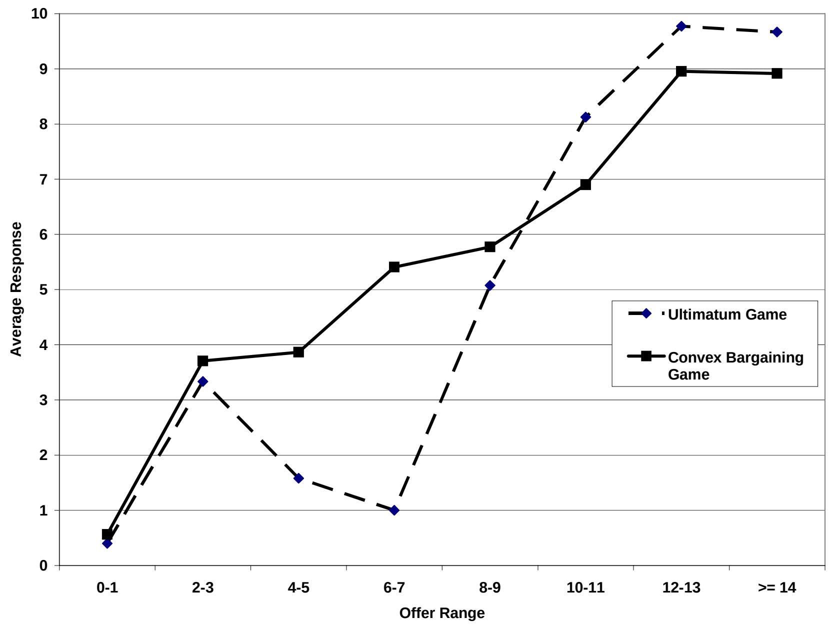


Figure 3: Average Response

A more careful look at Table 1 also reveals some other unexpected observations. First is the curious up-tick in the average response to offers of 2-3 in the ultimatum game. However, since we see only six observations of offers in this range, the effect could simply be due to small numbers. Second, for low offers it seems likely that the conditional probability of choosing 10, that is fully accept, should not be greater in the convex game than in the standard game. However, Table 1 shows that for offers of 0 to 5, for instance, the proportion replying with a 10 is 0.12 in the ultimatum game, but 0.24 in the convex game. It is unclear now whether this result poses a significant deviation from the hypothesis that the same preferences are expressed by responders in both games, or whether it is likely due to random variation that is to be expected when there are only 24 responders in each condition. We will probe this issue more deeply.

Table 1

Frequencies of Offers and Responses in Ultimatum and Convex Bargaining Games

Amount Offered	Ultimatum Game: Freq. of Offer	Frequency of Response: 0	10	Average Response	Convex Bargaining Game: Freq. of Offer	Frequency of Response: 0	$1-9$	10	Average Response
$0-1$	25	24	1	0.4	16	15	1	0	0.6
$2-3$	6	4	2	3.3	17	9	2	6	3.7
$4-5$	19	16	3	1.6	37	15	11	11	3.9
$6-7$	10	9	1	1.0	32	9	8	15	5.4
$8-9$	67	33	34	5.1	35	9	10	16	5.8
$10-11$	192	36	156	8.1	147	13	59	75	6.9
$12-13$	131	3	128	9.8	160	3	29	128	9.0
$14-25$	30	1	29	9.7	36	0	9	27	8.9
Total	480	126	354	7.4	480	73	129	278	7.0

5.1. Are preferences convex and regular?

Next we turn to the individual level data to see if there is some consistency across choices of responders. We see three general hypotheses for the types of preferences that could be underlying choices. First, subjects could have monotonic preferences, that is, preferences that are strictly increasing in own and other's payoffs. These preferences include selfish preferences but also some inequality-averse preferences, as long as indifference curves do not bend back. Monotonicity leads players to accept all offers. Second is non-monotonic utility, but linear indifference curves. For example $U(\pi_s, \pi_o) = \pi_s - \alpha\pi_o$ or $U(\pi_s, \pi_o) = (\pi_s - \pi_o)/\pi_s$ would both have linear indifference curves, the first being parallel lines and the second lines that fan out.[9] These subjects reject offers up to a point and then accept all offers above that point. These players will look the same regardless of whether they play a ultimatum or convex bargaining game. Third is preferences that are strictly convex but not monotonic, as shown in Figures 1 and 2, so subjects may reduce the pie to intermediate levels.

Restricting preferences of the form $u(\pi_s, \pi_o)$ to be strictly convex but not monotonic, however, will not be enough to provide a falsifiable hypothesis about

[9] See Levine (1998), Fehr and Schmidt (1999), and Charness and Rabin (2002) for examples of the first type of fairness, and Bolton and Ockenfels (2000) for models with the second type of assumption.

choice. That is, almost any set of responses could be generated by convex but nonmonotonic preferences. Hence, we add one more bit of structure to preferences that we call regularity.[10] This assumption is a cousin to normal goods or monotonicity in other settings. To define regular, we first assume that responders have a most preferred offer.[11] Then we say preferences are regular if a person does not shrink an offer more the closer it is to the most-preferred offer. This restriction prevents people from, for instance, fully accepting both low and high offers, but rejecting intermediate offers. While we cannot rule out such preferences as "irrational," we will view them as sufficiently implausible as to cast doubt on our approach. Note that both monotonic and linear preferences meet the definition of convex and regular, as do the strictly convex preferences illustrated in Figures 1 and 2 above.

To determine whether preferences are convex and regular, we first take a nonparametric approach. Adopt the maintained assumption that preferences meet the restrictions of convex and regular, but may vary because of decision errors or learning by subjects. If a subject's choices do not precisely fit the definition of regular preferences, then we ask how much would we have to adjust choices to satisfy regularity. For each subject we then calculate the minimum absolute distance from regular preferences. Note that linear preferences, such as those of Fehr and Schmidt (1999) and Charness and Rabin (2002), would have a minimum distance no lower than the distance from regular preferences.

Since we are measuring distance in the choice space, we scaled the differences to be measured in dollar movements. For instance, moving a choice from dividing 7 quarters to dividing 3 would be an absolute distance of 4 quarters, or \$1.00. We add these absolute deviations across all budgets. We call this the absolute distance measure. We also report a reweighting of this measure based on deviations in payoffs. That is, if moving a choice from 7 to 3 occurred when offered \$0.10 of each quarter, this is counted as costing \$0.40, whereas if it happened when offered \$0.20 of each quarter it would cost \$0.80. This measure gives the best sense of the "cost" of the deviation for a subject who truly has the regular preferences we calculate. We can use these distance measures to get a sense of how well a model of regular preferences fits the data.[12] We then further ask whether these regular

[10] We introduced this concept in our earlier paper (2003).

[11] More generally, we can assume a covex set of most preferred offers.

[12] Note that the actual money-equivalent utility cost is likely to be between these two values. Weighting by own-payoff deviations misses the lost utility from altruism or retribution felt by subjects.

preferences are monotonic, linear or strictly convex. The results of this exercise are reported in Table 2.

Table 2

Non-Parametric Categorization of Subjects' Preferences
By Minimum Absolute Distance to Regular Preferences

Ultimatum Game				Convex Bargaining Game			
		Distance Metric				Distance Metric	
Subj	Type	Absolute	Payoffs	Subj.	Type	Absolute	Payoffs
22	Monotonic	$0.00	$0.00	5	Monotonic	$0.00	$0.00
9	Linear	$0.00	$0.00	12	Monotonic	$0.00	$0.00
17	Linear	$0.00	$0.00	15	Monotonic	$0.00	$0.00
20	Near Monotonic	$2.50	$1.50	1	Linear	$0.00	$0.00
2	Near Linear	$2.50	$0.10	10	Linear	$0.00	$0.00
3	Near Linear	$2.50	$0.50	22	Linear	$0.00	$0.00
6	Near Linear	$2.50	$0.80	7	Linear	$0.00	$0.00
7	Near Linear	$2.50	$1.00	24	Strictly Convex	$0.00	$0.00
10	Near Linear	$2.50	$0.80	6	Near Linear	$1.25	$0.35
11	Near Linear	$2.50	$0.80	13	Near Linear*	$3.00	$1.40
12	Near Linear	$2.50	$1.20	21	Near Linear	$3.25	$1.28
13	Near Linear	$2.50	$0.90	2	Near Strictly Convex	$0.75	$0.26
15	Near Linear	$2.50	$0.90	17	Near Strictly Convex	$1.50	$0.59
18	Near Linear	$2.50	$1.20	16	Near Strictly Convex	$2.00	$0.95
23	Near Linear	$2.50	$0.90	19	Near Strictly Convex	$2.25	$0.85
24	Near Linear	$2.50	$1.10	23	Near Strictly Convex	$2.75	$1.19
5	Near Linear	$5.00	$2.20	9	Near Strictly Convex	$3.25	$1.28
8	Near Linear	$5.00	$2.00	3	Near Strictly Convex	$3.75	$1.60
21	Near Linear	$5.00	$2.00	14	Near Strictly Convex	$5.00	$2.49
24	Near Linear	$5.00	$1.10	20	Near Strictly Convex	$5.00	$2.15
19	Near Linear	$7.50	$3.00	11	Near Strictly Convex	$8.25	$2.06
1	Not Regular	$10.00	$4.40	4	Not Regular	$12.25	$5.54
16	Not Regular	$10.00	$3.90	8	Not Regular	$12.50	$4.20
14	Not Regular	$12.50	$4.20	18	Not Regular	$13.00	$4.91

*This subject is equi-distant to linear and strictly convex preferences.

Look first at the results for the ultimatum game, on the left side of the table. Here, only 3 of the 24 subjects exactly met the definition of monotonic or linear for all 20 decisions (that is, distance is zero). Of the remaining subjects, one was closest to monotonic and the rest were closest to linear. Of course, in the standard ultimatum game it is impossible to tell whether preferences are strictly convex or simply linear, since in both models there should be a critical offer at which responders switch from reject to accept. While there is no natural criterion for deciding when the absolute deviation from regular preferences is high enough to reject regularity, the final three subjects listed appeared to stand out. For this reason alone we are willing to call these subjects not regular.

Now turn to the convex game, the right panel of Table 2. By contrast there are eight subjects who exactly fit a utility function over all 20 rounds, three monotonic, four linear and one strictly convex. Contrary to expectations, there were more monotonic and linear subjects in the convex game than in the standard game, even though this gave more opportunity for convex preferences to be expressed. We may, however, be overstating the number of monotonic subjects. The three listed here never received offers below 3 (and for one, 5) so we cannot accurately predict their responses to selfish offers, a point we will return to later. Of the remaining subjects, only three had regular preferences that were closest to linear, while 10 subjects were best described as strictly convex. Overall 46 percent of subjects have preferences that are best measured as strictly convex and regular. As with the standard game, three subjects in the convex game also stood out as not regular.[13]

Figure 4 illustrates four examples of our subject classifications for the convex game. Similar illustrations on all of the subjects are available from the authors' web-sites.[14] The dots are actual choices, while the dotted lines indicate the nearest best reply function under regular preferences. Figure 4*a* shows subject 6 whom we classified as near linear. Here, with the exception of one offer, which was shrunk to 5, choices were exactly linear. Figures 4*b* and 4*c* show two different examples of strictly convex preferences. Subject 2 showed a remarkably smooth pattern of shrinking choices, even many that offered more than 50 percent of the pie.

[13] To get an idea of the order of magnitude of these deviations from regular for the six subjects classified as not regular, we can compare their distance to regular to their earnings in the study. For the ultimatum game the earnings (not including the \$5.00 show-up fee) were \$14.01 (subject 1), \$14.80 (subject 14), and \$6.40 (subject 14). Likewise, for the convex game earnings were \$12.02 (subject 4), \$9.01 (subject 8), and \$10.29 (subject 18).

[14] See footnote 5 above.

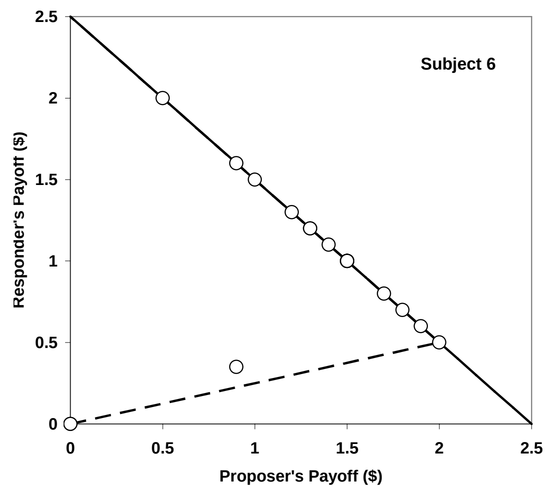
Subject 9 is even more extreme in this behavior. The regular best-reply function that fits this data most closely actually reaches full acceptance at offers of about 50 percent, then shrinks offers above this. Such two-sided inequality aversion, while uncommon, is not without precedent. Our single-shot treatment of this game also found evidence that about 20 percent of subjects had such aversion to even favorable inequality. In this experiment, however, that number is much smaller. Only 8 percent of our subjects showed an aversion to favorable inequality, although this difference could be explained by the small number of favorable offers made.[15] Figure 4*d* shows a subject whose choices, which appear nearly random, were classified as not regular.

We combine the estimates of regular preference of all of our subjects to form a non-parametric estimation of the preference of our subjects as a whole, as represented in best reply functions. This is presented in Figure 5. As can be seen, the average best reply functions match the predictions of the theoretical model, with the curve from the convex game cutting that of the regular game from below. Furthermore, the money maximizing offer in the regular game is 44 percent while in the convex game is 48 percent. Again, this verifies the prediction that the convex game shifts bargaining power to responders.

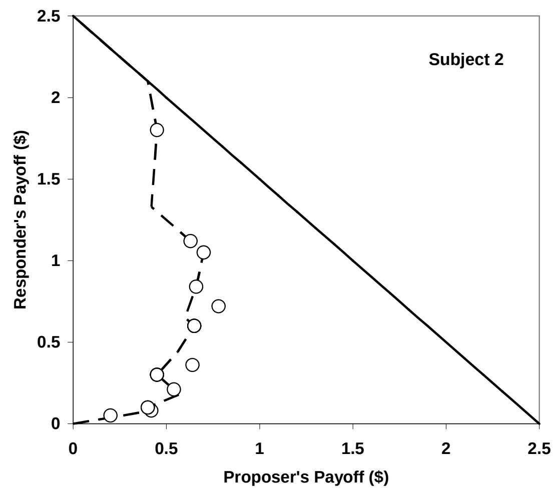
One can note that the best reply function for the convex game does not meet the diagonal line with slope minus one. This is due entirely to 4 subjects who, like subject 9 in figure 4, reduce generous offers.

We conclude from this section that regularity is a reasonable organizing criterion to place on the data. In doing so we verify that preferences that are regular can classify almost 90 percent of our subjects. By considering the convex game, we are able to further verify that, of the subjects with regular preferences, half have either monotonic or linear preferences. The remaining half have strictly convex preferences that are not monotonic.

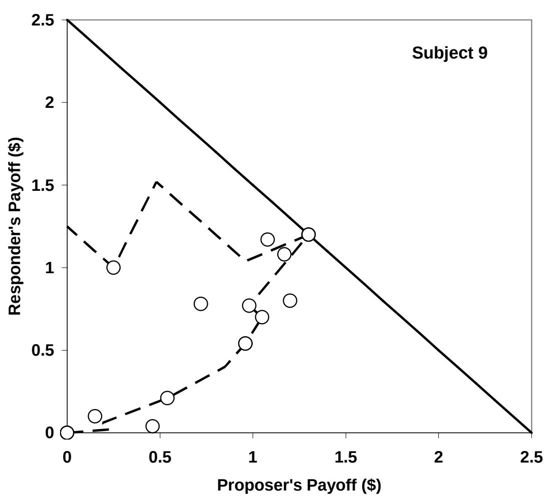
[15] Recall that in the single-shot version of the game, subjects recorded their reactions to all possible offers, not just the offers received. Hence a full spectrum of favorable offers were considered by all subjects. That was not the case here.



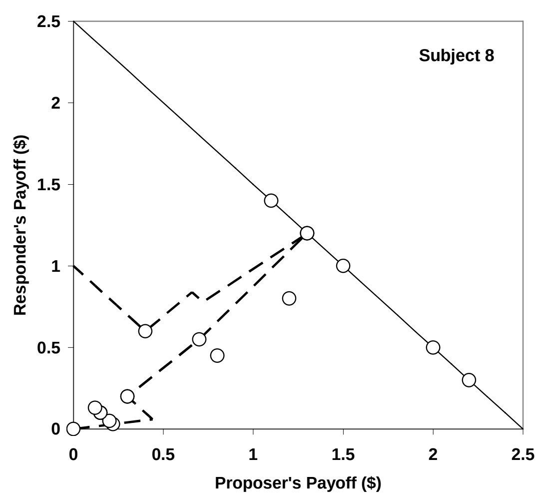
a. Near Linear



b. Near Strictly Convex



c. Near Strictly Convex



d. Not Regular

Figure 4. Examples of Preference Classifications in the Convex Bargaining Game

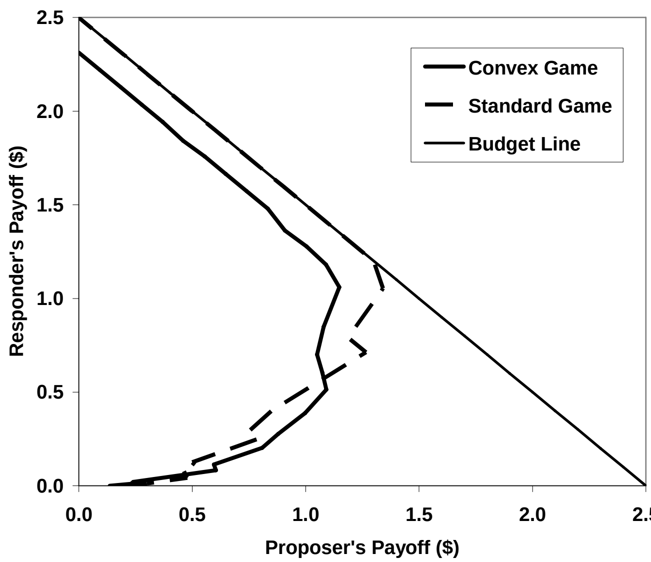


Figure 5: Non-Parametric Response Function

Do the preferences measured in the convex game predict the choices in the ultimatum game? While nonparametric results are encouraging, they do not allow us to compare behavior across treatments. It is impossible to know, for instance, whether a responder in the convex game who shrunk a proposal to 5 would accept or reject the same proposal in the standard ultimatum game. To answer this, we will need to estimate utility functions. Thus, we next turn to parametric analysis of the responders in the convex game.

5.2. Parametric Analysis: Estimating Utility Functions

In this section we estimate utility functions for each of our subjects in the convex ultimatum game. For some of the subjects who fit a classification exactly, such a calculation requires no econometrics. For instance, subjects who are monotonic can be described with any monotonic utility function. For subjects whose preferences are exactly the linear specification, it is sufficient to characterize their "switching point." For the other subjects we minimize the absolute distance between the observed choices and the predicted choices, where the predicted choices are derived from parametric versions of fairness models.

We estimate three models, the Fehr-Schmidt (1999) model, the Charness-Rabin (2002) model, and a flexible utility function which is convex and non-monotonic. The Fehr-Schmidt and Charness-Rabin models use the following utility function:

$$U_s(\pi_s, \pi_o) = (\rho \times 1[\pi_s > \pi_o] + \sigma \times 1[\pi_s < \pi_o]) \times \pi_o + (1 - \rho \times 1[\pi_s > \pi_o] - \sigma \times 1[\pi_s < \pi_o]) \times \pi_s \tag{1}$$

Here π_s (for self) denotes the responder's payoff, π_o (for other) denotes the proposer's payoff, and 1[.] denotes an indicator function. In the estimations, ρ and σ are unconstrained for the Charness-Rabin model. Fehr-Schmidt is identical to Charness-Rabin if conditions $\sigma < 0 \leq \rho < 1$ are imposed. To maintain the Fehr-Schmidt conventions, $\alpha = -\sigma$ and $\beta = \rho$ of the constrained version of the Charness-Rabin model.

The flexible utility function we propose allows backward-bending indifference curves. We estimated the following quadratic utility function for each subject:

$$U_s(\pi_s, \pi_o) = -(\pi_s - \theta_s)^2 - \beta_1(\pi_o - \theta_o)^2 - \beta_2(\pi_s - \theta_s)(\pi_o - \theta_o). \tag{2}$$

This utility function will produce indifference curve that are ovals. We, of course, do not take seriously the interpretation that (θ_s, θ_o) is the "bliss point" of a subject and that, far away from where choices are made, more for both players yields less utility. We are only trying to approximate the frontier that holds locally in the space of the observed subjects' choices by fitting a simple and easily understood function.

We estimate 15 utility functions for each of the models outlined in (1) and (2), including all the Near Strictly Convex, Not Regular, and Near Linear subjects, except subject 6.[16] Appendix 1 explains how the estimates were conducted. Appendix 2 lists the parameter estimates for each subject, as well as the critical switching points estimated for those with Linear or Near Linear preferences. The overall quality of the fit of the estimation is shown in Figure 6 where we plot the average of the predicted and actual choices for the 24 responders in the convex game for all three models. Note that the data itself shows a great deal of variance between the offers of 3 to 5. Nonetheless, the estimated response seems to track

[16] We do not estimate subject 6 because his behavior follows patterns that reflect linear preferences. Also, we do not estimate the only other Strictly Convex subject (Subject 24) because this subject accepts all offers but an offer of one cent, to which he/she responds with dividing 9 of 10 quarters.

the actual choices quite well. All three models track the data similarly, except for offers greater than 13 where the Fehr-Schmidt model predicts complete acceptance. Notice that the prediction fits most poorly at offers from 0–2. This may be due to our decision to assume subjects classified as Monotonic would accept these offers, despite having no observations for them in this range. Assuming instead that they would all reject such offers would lower the predicted response by 1.25, thus improving the fit.

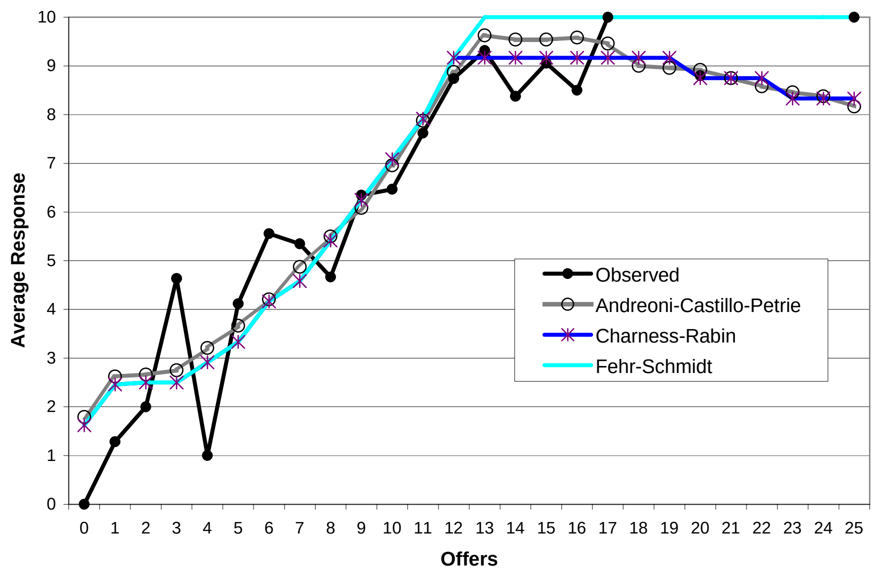


Figure 6: Parametric Prediction and Observed Average Response in Convex Game

As a measure of fit, we calculate the cost to make each of the 15 subjects' observed choices consistent with the estimated parameters for each model. This is shown in Table 3. Table 3 is similar to Table 2, which shows the non-parametrically derived cost to make each subject regular in choices and in payoffs. Table 3 also reveals that the utility function with the lowest cost and the best fit is the quadratic utility function. This is true for every case except subject 18, who was classified as not-regular in the nonparametric analysis.

Table 3
Models of Fairness at the Individual Level
Minimum Absolute Distance

	Fehr-Schmidt		Charness-Rabin		Quadratic Utility	
Subj	Choices	Payoffs	Choices	Payoffs	Choices	Payoffs
2	$16.00	$7.00	$16.00	$7.00	$2.30	$0.90
3	$5.00	$2.10	$5.00	$2.10	$3.80	$1.60
4	$17.30	$7.70	$17.30	$7.70	$12.30	$5.70
8	$18.30	$6.60	$18.30	$6.60	$15.30	$5.30
9	$10.30	$4.50	$10.30	$4.50	$5.50	$2.70
11	$10.50	$2.60	$10.50	$2.60	$8.30	$2.10
13	$3.00	$1.40	$3.00	$1.40	$3.00	$1.40
14	$9.50	$4.90	$7.00	$3.60	$5.00	$2.50
16	$9.30	$4.10	$9.30	$4.10	$2.00	$1.00
17	$6.80	$2.80	$6.80	$2.80	$1.80	$0.70
18	$15.30	$5.10	$15.30	$5.10	$17.30	$6.10
19	$3.80	$1.30	$3.80	$1.30	$2.80	$1.00
20	$7.50	$2.90	$7.50	$2.90	$5.00	$2.20
21	$3.30	$1.30	$3.30	$1.30	$3.30	$1.30
23	$12.30	$4.60	$12.30	$4.60	$4.00	$1.70
Average	$9.90	$3.90	$9.70	$3.80	$6.10	$2.40

Next we apply the estimated utilities of all 24 responders in the convex game for the three models and ask whether this family of utility functions can explain the choices of individuals in the standard ultimatum game. Thus, for each of our estimated utility functions, we determine the probability that the subject would accept or reject any offer presented in the standard ultimatum game, then average this expected response across all 24 responders to get a predicted expected response in the standard game. Figure 7 plots this prediction against the actual choices in the standard ultimatum game. With the notable exception of three points that have few observations, the fit is nearly perfect for all three models, which statistical tests confirm.[17] The Chi-Square test for the differences in distributions shows that the two are statistically indistinguishable ($\chi^2[23] = 22.8$,

[17] This suggests that the observation in Table 1 regarding low offers (that is, the higher frequency of replies of 10 in the convex game than the standard game) is indeed likely due to random variation in small samples. Again, our prediction for the 0–3 range would improve if we assume the three Monotonic subjects would have rejected these offers.

$p = 0.47$ for the quadratic utility model; $\chi^2[23] = 26.4$, $p = 0.29$ for Charness-Rabin; $\chi^2[23] = 20.7$, $p = 0.61$ for Fehr-Schmidt). Thus, while the quadratic model predicts the individual behavior best, its performance in predicting the aggregate data is not superior to the more parsimonious models of Fehr-Schmidt and Charness-Rabin.

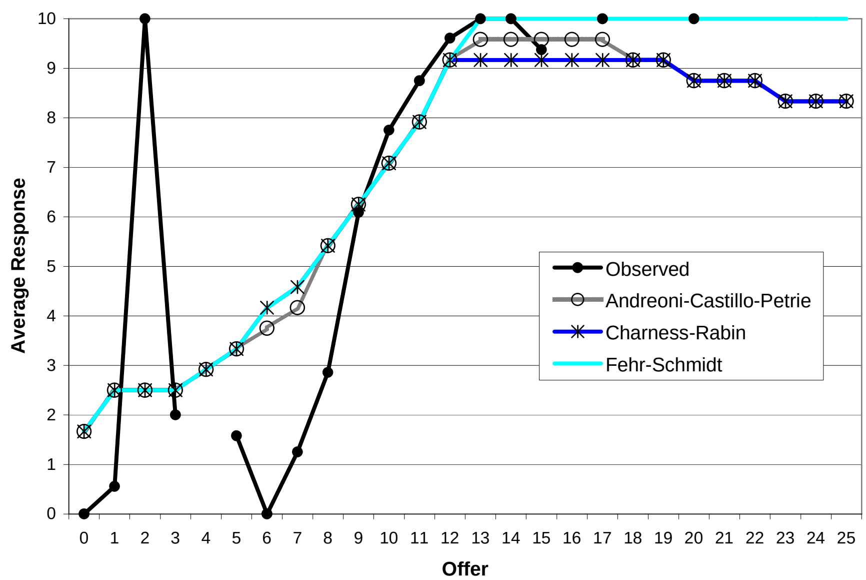


Figure 7: Prediction from Convex Game to Standard Game

6. Proposals and Predicted Earnings

Figure 8 illustrates both the distribution of offers and the predicted earnings, conditional upon those offers, for both convex and standard ultimatum games. The bars show the frequency of offers, and the lines show the predicted earnings conditional on the offer. To predict earnings we used the non-parametric estimate of preferences from section 5.2. We also calculated predicted earnings using the parametric model and got quite similar results.

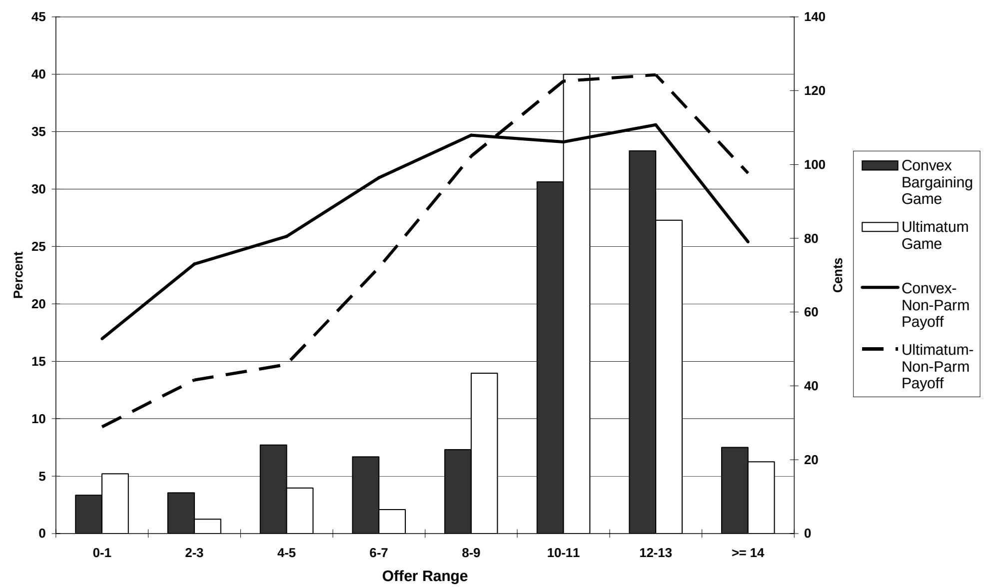


Figure 8: Frequency of Offers (bars, left axis), and Predicted Earnings (lines, right axis)

First, we see that for the convex game the payoffs reach a maximum over a broad range of offers from 8 to 13, and in the standard game the maximum is reached for offers in the 10 to 13 range. By far the most popular offers for both games is either 10-11 or 12-13. Either of these choices is consistent with money maximization.[18]

Another difference in offers can be seen by exploring lower offers. As noted earlier, there are clearly more offers made at the 2-7 range for the convex game than in the standard game. Eighteen percent of all offers are made in this range for the convex game, but only seven percent are for the standard game. What could explain this difference?

One hypothesis is simply that the expected earnings for these offers is much higher for the convex game–in some cases almost twice as high. Hence, subjects

[18]Note that while behavior by responders suggest that money maximization is not a good assumption, models of inequality aversion predict that proposers will behave as if they had monotone preferences over own and other payoffs. Expected payoffs are therefore still relevant.

would not have been so foolish to experiment with these low offers in the convex game. A second hypotheses can be found in our earlier (2003) paper. If the expected return from a low offer were the same in the two games, the offer would still be less attractive in the standard game because it comes with more risk. Similarly in this data, we cannot rule out that greater risk aversion is also making subjects more wary of low offers in the standard ultimatum game.[19]

7. Conclusions

What do responders' preferences look like? This question, which is key to understanding both bargaining and fairness, is nearly impossible to answer using data from an ultimatum game. Here we employed the convex ultimatum game, where responders can "shrink" the proposal as well as accept or reject it. This game collects the information on responders' preferences that allows us to measure preferences and even estimate utility functions for responders. If these estimated preferences can predict the actual choices in the standard ultimatum game, then we have a compelling picture of preferences of responders.

We found that about 90 percent of subjects exhibited well-behaved preferences that were convex and regular over the space of own- and other-payoff. Of these, about 15 percent had monotonic preferences that accepted all offers, and 35 percent had linear preferences that rejected offers up to a point and accepted all above that. The other 50 percent of subjects had strictly convex but not monotonic preferences. These are preferences for which indifference curves bend back at the extremes. We confirm this with both nonparametric estimates of best reply functions and parametric estimation of utility functions.

When applying the estimated preferences from the convex game to predict the play in the standard ultimatum game we get a strikingly close fit—the preferences measured in the convex game overlap nearly perfectly with the actual choices made by subjects in the standard game. We take this out-of-sample prediction as strong support for our approach and for the set of estimated preferences.

What does this research tell us about ultimatum bargaining? Most importantly, it shows that rejections can be "rationalized" with preferences that are convex, nonmonotonic, and that have the added quality of regularity. Regularity,

[19] One difference between the parametric prediction of earnings and the non-parametric prediction is that the two predictions are much closer in the parametric version. This would add greater support to the risk aversion explanation for this difference. However, as a general proposition, nonparametric estimates should be favored, and so we presented them here instead.

a concept akin to normality in demand theory, is remarkably successful in organizing responders' behavior. Our results also point to the importance of individual heterogeneity. While most subjects' choices can be captured by regular preferences, there was great variety in individual utility functions. Accounting for this heterogeneity is key to a successful account of bargaining behavior.

What do our results indicate for models of fairness? This data provides specific information on preferences and sets a clear goal for models of fairness. It establishes that the simplest models of fairness, those that imply linear indifference curves, will capture only a small fraction of the individual data. If we allow these linear models to account for the vast individual heterogeneity, however, the advantage of the more general quadratic utility disappears in predicting the aggregate data.

What do our analyses indicate for future research? An important aspect of our data not yet explored is the rich detail about the heterogeneity of preferences. These findings should perhaps put renewed emphasis on theories and experiments on bargaining under imperfect information. In addition, they suggest that there is potential in new research on institutions that perform well in the presence of heterogeneous populations of fair-minded bargainers.

Appendix 1: Calculating Expected Responses for the Convex Bargaining Game

Estimation: Following the approach proposed in section 5.2, we estimated utility parameters using a minimum distance method. Denote by $u(\pi_s(x,y),\pi_o(x,y))$ the utility function representing a responder's preferences, where $\pi_s(x,y)$ is the responder's payoff when a proposer passes x and a responder chooses y, and by $\pi_o(x,y)$ the corresponding proposer's payoff. Let $y(\theta,x) = \arg\max_y u(\pi_s(x,y),\pi_o(x,y))$ be the responder's optimal action given a utility function $u(\cdot)$. Parameters were obtained by solving the following problem:

$$\widehat{\theta} = \arg\min_{\theta} E_N[|y(\theta,x) - y|]$$

Assuming that responders's utility can be represented by the following quadratic utility function:

$$u(x,y) = -(\pi_s(x,y) - \theta_s)^2 - \beta_1(\pi_o(x,y) - \theta_o)^2 - \beta_2(\pi_s(x,y) - \theta_s)(\pi_o(x,y) - \theta_o)$$

the optimal action is $y(\theta,x) = \max\{0, \min\{[2(x\theta_s + \beta_1(25-x)\theta_o) + \beta_2(x\theta_o + (25-x)\theta_s]/[2x^2 + 2\beta_1(25-x)^2 + 2\beta_2 x(25-x)], 10\}\}$.

For the Fehr-Schmidt and Charness-Rabin estimations, we use the following utility function:

$$\begin{aligned} U(\pi_s(x,y),\pi_o(x,y)) &= (\rho \times 1[\pi_s(x,y) > \pi_o(x,y)] + \sigma \times 1[\pi_s(x,y) < \pi_o(x,y)]) \times \pi_o(x,y) \\ +(1 - \rho \times 1[\pi_s(x,y) &> \pi_o(x,y)] - \sigma \times 1[\pi_s(x,y) < \pi_o(x,y)]) \times \pi_s(x,y) \end{aligned}$$

$1[.]$ denotes an indicator function. In the estimations, ρ and σ are unconstrained for the Charness-Rabin model. Fehr-Schmidt is identical to Charness-Rabin if conditions $\sigma < 0 \leq \rho < 1$ are imposed. To maintain the Fehr-Schmidt conventions, $\alpha = -\sigma$ and $\beta = \rho$ of the constrained version of the Charness-Rabin model.

The optimal action for the Charness-Rabin model is:

$$\begin{aligned} &y = 10 \text{ if } \tfrac{|\sigma|}{1+2|\sigma|} \leq \tfrac{x}{M} \leq \tfrac{\rho}{2\rho-1} \text{ and } \rho > 1 \\ &y = 10 \text{ if } \tfrac{|\sigma|}{1+2|\sigma|} \leq \tfrac{x}{M} \text{ and } \rho \leq 1 \\ &y = 0 \text{ otherwise} \end{aligned}$$

.

Note that Fehr-Schmidt and Charness-Rabin produce step function response functions. This makes it difficult to use normal minimization procedures based on derivatives. To solve this problem, we utilized a function that can be arbitrarely close to a step function. In particular, we used $\Phi(\frac{x}{0.1})$, the cumulative distribution function of a normal random variable, as such an approximation.

The same non-differentiability problem emerges in minimizing absolute distances. Since $|x-y| = (x-y)(2 \times 1[x-y>0]-1)$, where $1[x-y>0]$ is the indicator function, we used $\Phi(\frac{x-y}{0.1})$ instead. I.e., we minimized $(x-y)(2 \times \Phi(\frac{x-y}{0.1})-1)$. This approach is based on the results in Horowitz (1998).

The estimates of Fehr-Schmidt and Charness-Rabin, are not unique. Given the discreteness of the data, a range of parameters predict the same behavior.

Prediction: In order to predict responders' behavior across games, we calculate the individual response to each possible offer when choices are restricted to $y = 0$ and 10. The average across all 24 individuals gives an *expected* response for the ultimatum game. We use the expected response to generate predictions across games in the following way. For every offer made in the ultimatum game, we draw a random number between 0 and 10. If the number falls below the expected response for that particular offer, we predict acceptance. If it falls above, we predict a rejection. This randomly generated data is compared to the actual responses in the ultimatum game. A χ^2 test is then conducted to test the null hypothesis that both populations have the same underlying distribution. The process was repeated 10,000 as shown in Section 5.2.

Appendix 2: Parameter Estimates of Utility Functions for Convex Game

Tables A1 lists the essential preference parameter for those with monotonic, linear and near linear preferences. The critical value listed is that offer, in percentage terms, at which the responder will choose to fully accept offers and below which they will fully reject them. The critical value is derived from the non-parametric estimates. Tables A2 and A3 present the parametric estimates for the 14 subjects whose utility functions were estimated as described in Appendix 1. Table A2 lists the estimates from the quadratic utility model, and Table A3 lists the estimates from the Fehr-Schmidt and Charness-Rabin models.

Table A1

Essential Preference Parameter

Monotonic, Linear and Near Linear Subjects

Subj	Accept all offers of at least:
5	0%
12	0%
15	0%
1	4%
10	32%
22	24%
7	20%
24*	4%
6	20%
21	12%

* Subject 24 is technically classified as Strictly Convex, but only shrank one offer, hence a parametric function could not be estimated. When offered 1 cent, his lowest offer, he chose 9 of 10 quarters to divide. We then approximate his utility with a linear indiffernce curve overlapping the budget at offers of 1.

Table A2

Parametric Estimates of Utility

Near Strictly Convex and Not Regular Subjects

	Quadratic Utility			
Subject	θ_s	θ_o	β_1	β_2
2	188.4	33.7	3.46	-0.26
3	219.6	-62.1	0.55	-0.98
4	227.8	-62.8	0.58	-0.71
8	352.2	2.6	1.11	-1.28
9	405.8	-270.7	0.62	0.40
11	267.6	271.3	0.74	-1.52
13	200.0	-50.0	0.86	-1.73
14	6.5	-6.7	0.84	-1.83
16	503.1	267.8	2.95	-3.15
17	227.1	-28.6	0.76	-1.11
18	306.8	20.5	1.49	-1.27
19	315.7	223.8	0.96	-1.62
20	213.5	-139.8	0.11	-0.63
21	200.0	-50.0	0.04	-0.28
23	279.4	191.6	5.33	-4.62

Table A3

Parametric Estimates of Utility

Near Strictly Convex and Not Regular Subjects

	Fehr-Schmidt	Charness-Rabin	
Subject	α	ρ	σ
2	11.99	1.12	-11.99
3	2.00	0.50	-2.00
4	1.88	0.60	-1.88
8	2.63	16.20	-2.63
9	1.20	1.33	-1.20
11	0.55	0.50	-0.55
13	6.23	0.50	-6.23
14	15.20	1.57e+11	-15.20
16	3.88	0.50	-3.88
17	3.24	0.50	-3.24
18	15.20	0.50	-15.20
19	1.03	0.50	-1.03
20	0.68	0.00	-0.68
21	0.21	0.50	-0.21
23	0.34	0.00	-.034

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