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A Unified Explanation**

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# Financial Development and Economic Volatility: A Unified Explanation\*

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## **Abstract**

Empirical studies showed that firm-level volatility has been increasing but the aggregate volatility has been decreasing in the US for the post-war period. This paper proposes a unified explanation for these diverging trends. Our explanation is based on a story of financial development – measured by the reduction of borrowing constraints because of greater access to external financing and options for risk sharing. By constructing a dynamic stochastic general-equilibrium model of heterogenous firms facing borrowing constraints and investment irreversibility, it is shown that financial liberalization increases the lumpiness of firm-level investment but decreases the variance of aggregate output. Hence, the model predicts that financial development leads to a larger firm-level volatility but a lower aggregate volatility. In addition, our model is also consistent with the observed decline in volatility of private held firms which do not have (or have only limited) access to external funds.

*Keywords:* Great Moderation, Firm-Level Volatility, Irreversible Investment, Borrowing Constraints, Heterogenous Firms, Business Cycle.

*JEL codes:* D21, D58, E22, E32.

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# 1 Introduction

Empirical studies show that firm-level volatility (for publicly traded firms) has been increasing but the aggregate volatility has been decreasing in the U.S. for the post-war period. In particular, using financial data, Campbell, Lettau, Malkiel, and Xu (2001) and Comin (2000) document an increase in volatility of firm-level stock returns. Using accounting data, Chaney, Gabaix, and Philippon (2002), Comin and Mulani (2005, 2006) and Comin and Philippon (2005) show an increase the idiosyncratic variability of capital investment, employment, sales, and earnings across firms. On the other hand, macroeconomic time series exhibit a significant downward trend in the variability of GDP and other major aggregate variables in the post-war period, particularly since the mid 1980's. For example, McConnell and Perez-Quiros (2000) show that the volatility of GDP has declined significantly since the mid 1980's. Blanchard and Simon (2002) document a downward trend in the volatility of GDP beginning in the 50's with an interruption in the 70's. Stock and Watson (2002) confirm that the decline in aggregate volatility is pervasive among almost all macro variables for the post-war period, especially starting in the mid 1980s.

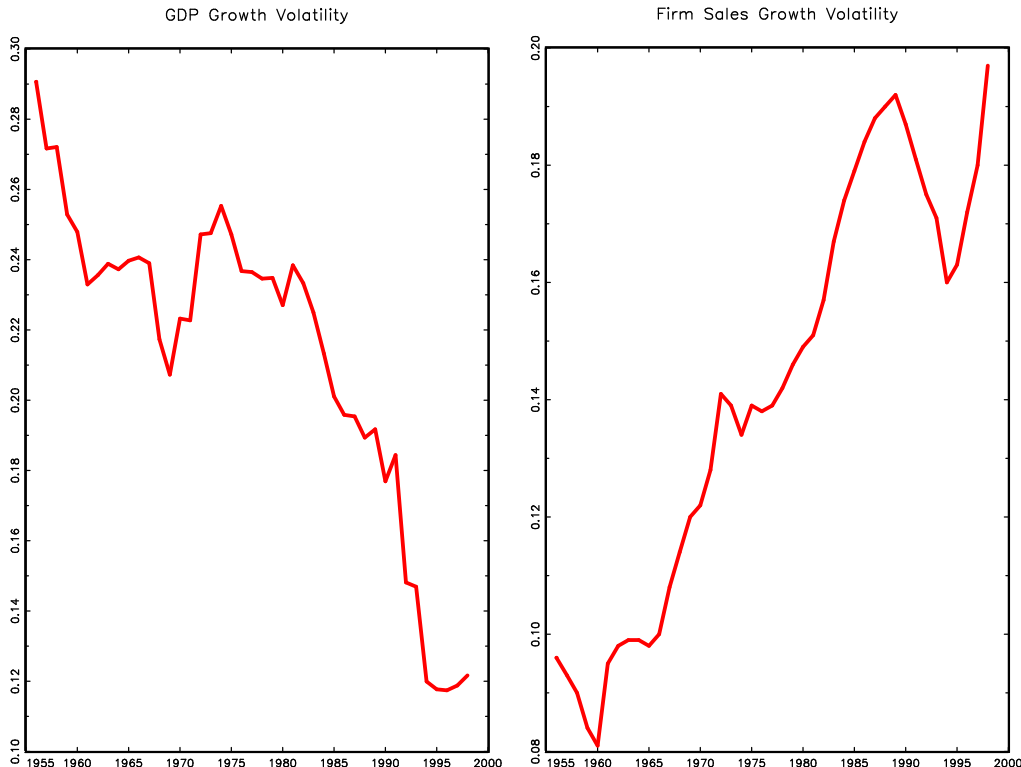


Figure 1. Diverging Trends of Volatility in the U.S..

Figure 1 illustrates the trend dynamics of the U.S. data. The left window shows the standard deviation of annual real GDP growth rate based on a 20-year moving average rolling window (scaled up by 10). The right window shows the standard deviation of firms' sales growth rate (source: Comin and Philippon, 2005, Fig. 1). The diverging trends in micro and macro economic volatility in the post-war period appears puzzling because macro movements are often thought as simple aggregations of micro movements. Namely, a declining trend in macroeconomic volatility should reflect a similar trend at the micro level, instead of the opposite.

In this paper we propose an explanation for this diverging-trend puzzle based on a story of financial development. The financial system has evolved significantly in many ways during the post-war period. Some of this evolution has been market-driven, and some owes to government deregulation policy.<sup>1</sup> In one way or another, financial development reduces borrowing constraints and promotes risk sharing across firms by giving them greater access to external financing and investment options. With less borrowing constraint, irreversible fixed investment at the firm level may become more responsive to idiosyncratic shocks to investment opportunities. Financial development at the same time also promotes better risk sharing and credit-resource allocation so that firms receiving unfavorable productivity shocks can postpone fixed investment and divert resources to savings with better returns through financial intermediation, which in turn allows more productive firms to raise external funds to invest in fixed capital. Thus, with financial development the most productive firms can undertake more fixed investment by raising external funds while the less productive firms can avoid losses by investing (directly or indirectly) in the financial assets of the productive firms,<sup>2</sup> making firm-level capital investment lumpier and more volatile.

On the other hand, better access to external financing implies that aggregate productivity shocks, which affect firms' profits, will have less direct impact on capital investment when firms are less dependent on internal cash flows for financing. Therefore, financial development can increase the variance of firm-level investment but decrease the variance of aggregate investment simultaneously.

The above intuition is illustrated in this paper using a general-equilibrium model of heterogeneous firms facing borrowing constraints and investment irreversibility.<sup>3</sup> We show that, for publicly traded firms who are able to obtain external funds by issuing debt, a reduction in financial market frictions increases the lumpiness of firm-level investment (consequently, dividend and stock prices also become more variable across firms) but decrease the variance of aggregate investment, employment, and output. Hence, the model predicts that financial development leads to a larger firm-level

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<sup>1</sup>See Frame and White (2004) and Dynan, Elmendorf and Sichel (2006a) for literature overview.

<sup>2</sup>Notice that stocks, equities and bonds are more reversible than fixed investment. Hence, they are more liquid and can be more attractive when the returns to fixed investment is low.

<sup>3</sup>Fazzari, Hubbard and Petersen (1988) provide empirical evidence on firms' borrowing constraints and emphasize imperfections in markets for equity and debt.

volatility but a lower aggregate volatility.

We embed the Kiyotaki-Moore (1997) type of financial frictions into an otherwise standard RBC model with heterogenous firms.<sup>4</sup> In particular, the amount of external debt is limited by collateral values. However, an important departure of this paper from Kiyotaki and Moore (1997) is that we do not assume agents to have different time discounting factors in order to engage in lending and borrowing among each other, and we do not require the borrowing constraints to be always binding. This allows us to conduct quantitative business-cycle analyses under aggregate shocks without having to worry about the optimality of an always-binding borrowing constraint assumed in Kiyotaki and Moore (1997) and by most works in this literature.<sup>5</sup>

A methodological contribution of this paper is that we derive analytically tractable decision rules at the firm level despite irreversible investment and borrowing constraints for heterogenous firms. Our method allows us to study the model's aggregate dynamics without having to resort to numerical approximation methods as in Krusell and Smith (1998), which not only is computationally costly but also leaves the intuition of the model in a blackbox. Approximations in our paper are needed only at the aggregate level, for which we can use the standard log-linearization technique based on the method of Blanchard and Kahn (1980). The advantage of being able to solve the decision rules analytically at the firm level is that the economic mechanisms affecting agents' decisions become transparent.<sup>6</sup>

Many theoretical explanations have been proposed in the literature to separately explain the upward trend of volatility at the firm level and the downward trend at the aggregate level. But few of them can explain these two stylized facts simultaneously. In fact, most models are silent on the diverging-trend puzzle, some even imply counterfactual predictions.

For the rise of firm-level volatility, Thesmar and Thoenig (2004) posit that firms can choose the degree of risk inherent to their operation strategies; thus, financial market development, by improving risk sharing between owners of listed firms, increases the willingness of these firms to take risky bets. This in turn increases firm level uncertainty in sales, employment and profits. Using French data, they find empirical support of the view that firm-level volatility increases with financial market development. Comin and Philippon (2005), Irvine and Pontiff (2005) and Philippon (2003) argue that the increased firm-level volatility may be due to higher competition. Comin and Mulani (2005) argue that the increased firm-level volatility is driven primarily by increased R&D innovations. Kaas (2009) develops a real business cycle model with idiosyncratic productivity shocks and collateral-based borrowing constraints to show that an increase in credit

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<sup>4</sup>Alternative approaches to financial frictions include Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (2008).

<sup>5</sup>See, e.g., Pintus and Wen (2008) and the references therein.

<sup>6</sup>Similar methods have been used by Wen (2008) and Wang and Wen (2009) to solve heterogeneous-firm models with inventory investment and borrowing constraints.

market development relaxes borrowing constraints and thereby also increases the spread between internal rates of return across firms. As a result, firm growth rates become more volatile.

For the decline in aggregate volatility, several prominent explanations have been proposed in the literature, including the role played by financial development (Campbell and Hercowitz, 2006; Dynan, Elmendorf and Sichel, 2006a,b; Jermann and Quadrini, 2009), milder economic shocks (Stock and Watson, 2002), improved inventory management (Kahn, McConnel, and Perez-Quiros, 2001), and better monetary policy (Clarida, Gali and Gertler, 2000).

With few exceptions (such as Philippon, 2003; and Comin and Mulani, 2005), none of the aforementioned studies offer a unified framework to simultaneously explain the increasing trend in firm-level volatility and the decreasing trend in aggregate volatility.<sup>7</sup> Philippon's (2003) explanation is based on imperfect competition with sticky prices. Increased competition between firms reduces price stickiness and thus magnifies the effects of idiosyncratic productivity shocks on firm-level activity. This can explain the rise in firm volatility. On the other hand, less price stickiness reduces the impact of aggregate monetary shocks on economy wide activities. This can explain the fall in aggregate volatility. Comin and Mulani (2005) present an endogenous growth model to explain the diverging trends in firm-level and aggregate volatility. In their model, growth is driven by the development of both idiosyncratic R&D innovations and general innovations that can be freely adopted by many firms. Firm-level volatility is affected primarily by R&D innovations while the variance of aggregate productivity growth is determined mainly by the arrival rate of general innovations. In their model, the changes of market shares cause endogenous shift in the allocation of resources from the development of general innovations to the development of R&D innovations, resulting in an increase in firm-level volatility and a decline in aggregate volatility.

Our paper differs from the above two theoretical papers in that we emphasize financial development as an important factor of economic stability and growth, which has received an increasing amount of attention recently from economists (for more recent works along this line, see Wang, 2006; Greenwood, Sanchez, and Wang, 2007; and Jermann and Quadrini, 2009).<sup>8</sup> In addition, we also show that our model has the potential to simultaneously explain a third empirical fact recently discovered in the empirical literature: for privately held firms, volatility and dispersion have declined in the post-war period, in contrast to publicly traded firms (Davis, Haltiwanger, Jarmin and Miranda, 2006). Although to explain this third stylized fact is not the focus of our current

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<sup>7</sup>Also see Wang (2006). But Wang's analysis is based on a partial equilibrium approach.

<sup>8</sup>Jermann and Quadrini (2009) notice firms' financial flows at the aggregate level have become more volatile, in contrast to aggregate output. They propose an explanation for the diverging aggregate trends based on a model of financial innovations. In this paper, we focus on the upward trend in firm-level volatilities. Hence, our model features firm distribution and idiosyncratic shocks, which are absent from Jermann and Quadrini (2009). Wang (2006) presents a partial equilibrium model of the financial sector and argues that advances in financial technology are the main driving force behind the diverging trends. Our paper shares the same view with Wang (2006) but with different modeling strategies.

project, it is nonetheless comforting to know that our model is not inconsistent with this fact. The idea behind our explanation for the third fact is as follows. Financial development in our model improves labor productivity because it enhances aggregate capital accumulation by enabling the most productive firms to undertake capital investment. Consequently, the efficiency of aggregate investment rises. As a result, the aggregate capital-labor ratio and the real wage increase with financial development. The rising real wage may have at least two consequences and both can significantly decrease the volatility of privately held firms. First, higher wage costs generate greater competitive pressure on the survival of privately held firms who face the same pool of labor force with publicly traded firms but do not have the same degree of access to external financing as publicly traded firms. Thus, it becomes much harder for privately-held firms to survive, resulting in a higher degree of homogeneity across the survived privately-held firms. Second, the cost share of wages increases with financial development relative to that of idiosyncratic fixed-labor costs. Given that all firms face more or less identical fixed costs, the larger the firm size, the less important are such fixed costs. Because privately-held firms are much smaller in size than publicly-traded firms, the rising wage share in total costs reduces the sensitivity of firms' labor demand to idiosyncratic labor costs more so for privately-held firms than for publicly-traded firms due to difference in firm size.<sup>9</sup> Consequently, volatility and dispersion across firms decrease more significantly for privately-held firms than for publicly-traded firms as the wage share increases.<sup>10</sup>

Our investment model and solution method are also closely related to the theoretical literature on irreversible investment under uncertainty. In particular, Able and Eberly (1994, 1997) provide competitive equilibrium models with irreversible investment under uncertainty and characterize closed-form solutions for optimal investment decision rules. Miao (2005, 2008) provides general equilibrium models of capital structure with heterogeneous firms facing irreversible investment and borrowing constraints. This literature uses continuous time framework in order to derive analytically closed-form solutions. To our best knowledge, we are the first to derive closed-form solutions in a discrete-time framework with irreversible investment and borrowing constraints.

## 2 The Benchmark Model

### 2.1 The Firm's Problem

There is a continuum of competitive firms indexed by  $i \in [0, 1]$ . Each firm's objective is to maximize its discounted dividend,

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<sup>9</sup>According to Davis, Haltiwanger, Jarmin and Miranda (2006), the average number of works is about 15 for privately held firms and 4000 for publicly traded firms. Hence, the average idiosyncratic fixed costs of labor tend to be larger for small firms than for large firms.

<sup>10</sup>Although these two forces also tend to decrease the volatility of publicly-traded firms, for them the upward trend in volatility due to greater access to external financing is so much stronger than the downward trend. Hence, firm-level volatility still rises for publicly-traded firms.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t d_t(i), \quad (1)$$

where  $d_t(i)$  is the dividend of firm  $i$  in period  $t$  and  $\Lambda_t$  is the representative household's marginal utility which firms take as given. The production technology of each firm  $i$  is given by the CES function,

$$y_t(i) = \{\omega k_t^\sigma(i) + (1 - \omega) [A_t n_t(i)]^\sigma\}^{\frac{1}{\sigma}}, \quad \omega, \sigma \in (0, 1); \quad (2)$$

where  $A_t$  represents aggregate labor-augmenting technology,  $n(i)$  and  $k(i)$  are firm-level employment and capital, respectively. Each firm accumulates capital according to the law of motion,

$$k_{t+1}(i) = (1 - \delta)k_t(i) + \varepsilon_t(i)i_t(i), \quad (3)$$

where  $i(i) \geq 0$  denotes irreversible investment and  $\varepsilon(i)$  is an idiosyncratic shock to the marginal efficiency of investment, which has the cumulative distribution function  $F(\varepsilon)$ . In each period  $t$ , a firm needs to pay wage bill  $w_t n_t(i)$ , decides whether to invest in fixed capital and distribute the dividend  $d(i)$  to households.

Firms' investment is financed by internal cash flows and external funds. Firms raise external funds through borrowing by issuing one-period debt (bond),  $b_{t+1}(i)$ , which pays the competitive market interest rate  $r_t \geq 1$ . We focus on debt financing in this paper because it accounts for 75% to 100% of the total amount of external funds used by corporate firms.<sup>11</sup> This means that a firm can also invest in bonds issued by other firms (i.e., we allow  $b_{t+1}(i)$  to be negative).<sup>12</sup> As a result, at the aggregate we have the bond market clearing condition,

$$\int_0^1 b_{t+1}(i) di = 0. \quad (4)$$

A firm's dividend in period  $t$  is hence given by

$$d_t(i) = y_t(i) + \frac{b_{t+1}(i)}{r_t} - i_t(i) - w_t n_t(i) - b_t(i). \quad (5)$$

We assume that firms cannot pay negative dividend,

$$d_t \geq 0; \quad (6)$$

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<sup>11</sup>Source: Federal Reserve U.S. flow of Funds Data.

<sup>12</sup>Firms do not have to lend to each other directly. Intra-firm lending can be achieved through financial intermediation.



which is the same as saying that fixed investment is financed entirely by internal cash flows,  $y(i) - wn(i)$ , and external funds net of loan payment,  $\frac{b_{t+1}(i)}{r_t} - b(i)$ .

Due to imperfect financial market, firms are borrowing constrained. We impose a borrowing limit as in Kiyotaki and Moore (1997),

$$b_{t+1}(i) \leq \theta k_t(i), \quad (7)$$

which specifies that the new debt issued cannot exceed a proportion of the collateral value of a firms' existing capital stock. Thus, the parameter  $\theta \geq 0$  measures the degree of financial development. Namely, the larger the value of  $\theta$ , the more developed is the financial market. When  $\theta = 0$ , the model is identical to one that prohibits external financing.<sup>13</sup>

Given the real wage,  $w_t$ , the firm's optimal labor demand is determined by the equation,

$$(1 - \omega) \left( \frac{y_t(i)}{n_t(i)} \right)^{1-\sigma} A_t^\sigma = w_t \quad (8)$$

or  $(1 - \omega) \left\{ \omega \left[ \frac{k_t(i)}{n_t(i)} \right]^\sigma + (1 - \omega) A_t^\sigma \right\}^{\frac{1-\sigma}{\sigma}} A_t^\sigma = w_t$ . It follows that both the output-capital ratio  $\frac{y_t(i)}{k_t(i)}$  and the labor-capital ratio  $\frac{n_t(i)}{k_t(i)}$  are independent of the index  $i$  (i.e., identical across firms). Hence, we can define the firm's net revenue as a linear function of its capital stock,

$$y_t(i) - w_t n_t(i) \equiv R(w_t, A_t) k_t(i), \quad (9)$$

where  $R_t$  is a function of the real wage and the technology level. Such a linear relationship between cash flow and the capital stock implies that the aggregate cash flow will depend only on the aggregate capital stock. It means that there is no need to keep track of the distribution of  $k_t(i)$  for aggregate dynamics, thus simplifying our analysis greatly. With the definition in (9), the firm's investment problem can be written as

$$\max_{\{i_t, b_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left( R_t k_t(i) + \frac{b_{t+1}(i)}{r_t} - b_t(i) - i_t(i) \right) \quad (10)$$

subject to

$$k_{t+1}(i) = (1 - \delta) k_t(i) + \varepsilon_t(i) i_t(i), \quad (11)$$

$$i_t(i) \geq 0, \quad (12)$$

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<sup>13</sup>If firms cannot issue bond, then the bond market does not exist. Hence,  $b_{t+1}(i) = 0$  for all  $i$  in equilibrium.

$$i_t(i) \leq R_t k_t(i) + \frac{b_{t+1}(i)}{r_t} - b_t(i), \quad (13)$$

$$b_{t+1}(i) \leq \theta k_t(i). \quad (14)$$

Denote  $\{\lambda(i), \pi(i), \mu(i), \phi_t(i)\}$  as the Lagrangian multipliers of constraints (11)-(14), respectively, the firm's first order conditions for  $\{i_t(i), k_{t+1}(i), b_{t+1}(i)\}$  are given respectively by

$$1 + \mu_t(i) = \varepsilon_t(i) \lambda_t(i) + \pi_t(i), \quad (15)$$

$$\lambda_t(i) = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \{ [1 + \mu_{t+1}(i)] R_{t+1} + (1 - \delta) \lambda_{t+1}(i) + \theta \phi_{t+1}(i) \}, \quad (16)$$

$$\frac{[1 + \mu_t(i)]}{r_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \{ [1 + \mu_{t+1}(i)] \} + \phi_t(i), \quad (17)$$

and the complementarity slackness conditions,  $\pi_t(i) i_t(i) = 0$ ,  $[R_t k_t(i) - i_t(i) - x_t(i) + \frac{b_{t+1}(i)}{r_t} - b_t(i)] \mu_t(i) = 0$  and  $\phi_t(i) [\theta k_t(i) - b_{t+1}(i)] = 0$ . Notice that by equation (16)  $\lambda(i)$  is independent of  $i$  if the idiosyncratic shocks are *iid*, which we assume to be the case in this paper.

## 2.2 Decision Rules for Investment

The decision rules are characterized by an optimal cutoff strategy featuring an endogenous cutoff value for the idiosyncratic shocks,  $\varepsilon_t^*$ , which is time varying but constant across firms (i.e., independent of idiosyncratic uncertainty). This property is crucial for closed-form solutions of firm-level decision rules. Consider two possibilities:

Case A:  $\varepsilon_t(i) \geq \varepsilon_t^*$ . In this case firm  $i$  receives a favorable shock and investment is considered efficient. Suppose this induces the firm to invest, then we have  $i_t(i) > 0$  and  $\pi_t(i) = 0$ . Equations (15) and (16) then become

$$\frac{1 + \mu_t(i)}{\varepsilon_t(i)} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \{ [1 + \mu_{t+1}(i)] R_{t+1} + (1 - \delta) \lambda_{t+1}(i) + \theta \phi_{t+1}(i) \}. \quad (18)$$

Since the multiplier  $\mu_t(i) \geq 0$ , the above equation implies

$$\varepsilon_t(i) \geq \left[ \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \{ [1 + \mu_{t+1}(i)] R_{t+1} + (1 - \delta) \lambda_{t+1}(i) + \theta \phi_{t+1}(i) \} \right]^{-1} \equiv \varepsilon_t^*, \quad (19)$$

where the right-hand side defines the cutoff  $\varepsilon_t^*$ , which is clearly independent of  $i$  because the idiosyncratic shocks are *iid*. Therefore, all firms adopt the same cutoff value so the aggregate dynamics are independent of the distribution of firms. Since  $\pi(i) = 0$ , equation (15) becomes

$1 + \mu_t(i) = \frac{\varepsilon_t(i)}{\varepsilon_t^*}$ . Hence,  $\mu_t(i) > 0$  if and only if  $\varepsilon_t(i) > \varepsilon_t^*$ . It follows that under "case A" firm  $i$  opts to invest at full capacity,

$$i_t(i) = R_t k_t(i) + \frac{b_{t+1}(i)}{r_t} - b_t(i), \quad (20)$$

and retains zero dividend. Also, since  $\mu(i) \geq 0$ , equation (17) implies

$$\phi_t(i) \geq \frac{1}{r_t} - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \{ [1 + \mu_{t+1}(i)] \} \equiv \phi_t^*, \quad (21)$$

where the right-hand side defines the cutoff  $\phi_t^*$ , which is independent of  $i$ . Note  $\phi_t^* \geq 0$  because it is the value of the Lagrangian multiplier when  $\mu(i) = 0$ . Hence, equation (17) can be written as

$$\phi_t(i) = \frac{\varepsilon_t(i) - \varepsilon_t^*}{\varepsilon_t^*} \frac{1}{r_t} + \phi_t^*. \quad (22)$$

Because  $\phi_t^* \geq 0$ , we have  $\phi_t(i) > 0$  when  $\varepsilon_t(i) > \varepsilon_t^*$ ; which means that under "case A" firms are willing to borrow up to the borrowing limit,  $b_{t+1}(i) = \theta k_t(i)$ , to finance investment. Therefore, the optimal investment equation (20) can be rewritten as

$$i_t(i) = \left[ R_t + \frac{\theta}{r_t} \right] k_t(i) - b_t(i). \quad (23)$$

To ensure  $i_t(i) > 0$  in the steady state, we restrict parameter values such that the condition,  $(R + \frac{\theta}{r})(1 - \delta) > \theta$ , is always satisfied.<sup>14</sup>

Case B:  $\varepsilon_t(i) < \varepsilon_t^*$ . In this case firm  $i$  receives a unfavorable shock. Assume that the firm decides to under-invest,  $i_t(i) < R_t k_t(i) + \frac{b_{t+1}}{R_t} - b_t$ , then the multiplier  $\mu_t(i) = 0$ . Equation (15) implies  $\pi_t(i) = \frac{1}{\varepsilon(i)} - \frac{1}{\varepsilon^*} > 0$ . Hence, confirming our assumption, the firm opts not to invest,  $i_t(i) = 0$ . Since  $\int_0^1 b_{t+1}(i) di = 0$ , and  $b_{t+1}(i) = \theta k_t(i) > 0$  when  $\varepsilon_t(i) > \varepsilon_t^*(i)$ , there must exist firms indexed by  $j$  such that  $b_{t+1}(j) < 0$  if  $\varepsilon_t(j) < \varepsilon_t^*$ . It then follows that  $\phi_t(i) = \phi_t^* = 0$  under "case B". That is, firms receiving unfavorable shocks will not invest in fixed capital but opt to invest in financial assets in the bond market by lending a portion of their cash flows to other (productive) firms.

<sup>14</sup>The steady state of the model economy is defined as the situation without aggregate uncertainty. Hence, in the steady state we have  $i_t(i) \geq (R + \frac{\theta}{r}) k_t(i) - \theta k_{t-1}(i)$ . Since  $k_t(i) \geq (1 - \delta) k_{t-1}(i)$ , thus  $i_t(i) \geq 0$  if  $[(R + \frac{\theta}{r})(1 - \delta) - \theta] k_{t-1}(i) \geq 0$ . Notice that this condition is certainly satisfied when  $\theta = 0$ .

A firm's optimal investment strategy is thus given by the decision rule,

$$i_t(i) = \begin{cases} \left[ R_t + \frac{\theta}{r_t} \right] k_t(i) - b_t(i). & \text{if } \varepsilon(i) \geq \varepsilon^* \\ 0 & \text{if } \varepsilon(i) < \varepsilon^* \end{cases}. \quad (24)$$

Since  $\mu(i) = \phi(i) = 0$  if  $\varepsilon(i) < \varepsilon^*$ , equation (17) implies

$$\frac{1}{r_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q(\varepsilon_{t+1}^*), \quad (25)$$

where  $Q(\varepsilon^*) \equiv E [1 + \mu(i)] = F(\varepsilon^*) + \int_{\varepsilon(i) \geq \varepsilon^*} \frac{\varepsilon(i)}{\varepsilon^*} dF(\varepsilon)$  is the option value of one unit of cash flow.

Given one dollar in hand, if not invested, its' value is still one dollar. This case happens with probability  $F$ . On the other hand, if the firm opts to invest, the cash return is  $\frac{\varepsilon(i)}{\varepsilon^*} > 1$  provided  $\varepsilon(i) > \varepsilon^*$ . Hence, the option value  $Q > 1$ .

Equation (25) determines the equilibrium interest rate of bond,  $r_t$ . For firms who decide to lend (investing in bonds), one dollar saving yields  $r_t$  dollars tomorrow, which has the option value of  $Q_{t+1}$ . Notice that  $r < \frac{1}{\beta}$  in the steady state because  $Q(\varepsilon^*) > 1$ . This may help explain the low risk-free rate puzzle of the asset pricing literature. By the definition of the cutoff (19),  $\varepsilon_t^* \equiv \frac{1}{\lambda_t(i)}$ , which is the inverse of the marginal value of capital, the Euler equation (16) implies that the optimal value of the cutoff is determined by the dynamic equation,

$$\frac{1}{\varepsilon_t^*} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ R_{t+1} Q(\varepsilon_{t+1}^*) + \frac{(1-\delta)}{\varepsilon_{t+1}^*} + \frac{\theta}{r_{t+1}} (Q(\varepsilon_{t+1}^*) - 1) \right\}, \quad (26)$$

where  $Q - 1 = E\phi(i) = \int_{\varepsilon(i) \geq \varepsilon^*} \frac{\varepsilon_{t+1}(i) - \varepsilon_{t+1}^*}{\varepsilon_{t+1}^*} dF(\varepsilon)$ . Notice that  $\frac{1}{\varepsilon_t^*}$  is the equity value of one unit installed capital for all firms. Namely, one unit of cash flow can buy  $\varepsilon^*$  units of claims of a firm's capital stock. The right hand side of the above equation has three components. First, one unit of newly installed capital tomorrow can generate  $R$  units of cash flow with an option value of  $RQ$ . Second, this unit of capital has a residual equity value of  $\frac{1-\delta}{\varepsilon^*}$  after depreciation. Third, one additional unit of capital allows the firm to raise external funds by  $\frac{\theta}{r}$  units, which amounts to additional cash value of  $\frac{\theta}{r} (Q - 1)$ , where  $Q - 1$  is the net option value of one unit of loan. Net option value applies here because borrowing must involve repayment. Hence, the right-hand side of equation (26) measures the expected marginal value of new capital (investment), which by arbitrage

must equal the equity price of currently installed capital on the left-hand side.

Notice that if  $Q = 1$ , then equation (26) is reduced to a standard neoclassical equation for investment. This would be the case if there were no idiosyncratic shocks in our model. That is, the option value  $Q > 1$  is the consequence of idiosyncratic shocks and irreversible investment, which induce firms to postpone investment when receiving bad shocks.<sup>15</sup>

Equation (26) and equation (24) are the key equations to understand the model.<sup>16</sup> Equation (24) shows that optimal investment at the firm level is proportional to the firm's existing stock of capital  $k_t(i)$  if we ignore  $b_t(i)$ , with the proportionality depending on  $\left[R_t + \frac{\theta}{r_t}\right]$ . This has the following implications.

1) The volatility and lumpiness of firm-level investment increases with  $\theta$  because  $\left(R_t + \frac{\theta}{r_t}\right)$  measures the responsiveness of a firm's investment rate (or investment-to-capital ratio) to its idiosyncratic shocks. Clearly, the larger the value of  $\theta$ , the higher the investment rate or the investment-to-capital ratio when the firm opts to invest. This suggests that as  $\theta$  increases, investment is more variable at the firm level. Secondly, as will be shown shortly, the cutoff value increases with  $\theta$  (i.e.,  $\frac{d\varepsilon^*}{d\theta} > 0$ ). Hence, the probability that a firm will undertake capital investment will decrease when  $\theta$  increases (because  $i_t(i) > 0$  if and only if  $\varepsilon(i) > \varepsilon^*$ ). This suggests that a firm's investment becomes relatively less frequent with a larger  $\theta$ . Putting together, financial development makes firm-level investment lumpier and more volatile.

2) Since aggregate technology shocks affect firms' investment through cash flow (i.e., through the function  $R_t$ ), aggregate investment  $\int_0^1 i(i) di$  will respond less to aggregate technology shocks when  $\theta$  is larger, because cash flow becomes less important for investment financing when external funds are available. Therefore, the variability of aggregate employment (as well as output) is reduced by financial development.

3) In addition, since financial development promotes investment efficiency by allowing greater degrees of risk sharing and credit-resource allocation across firms, the aggregate capital stock to labor ratio increases with  $\theta$ . Thus, if the production function is CES and the aggregate shocks

<sup>15</sup>Notice that in our model the constraint  $i_t \geq 0$  does not bind with respect to aggregate shocks if the support of idiosyncratic shocks is  $[0, \infty]$ . In this case it is impossible for all firm to have zero investment in the same period no matter how low the aggregate productivity shock is, because there is always a positive fraction of firms with large enough  $\varepsilon(i)$  to undertake investment. Hence, in our model irreversible investment matters only with respect to idiosyncratic shocks. In this regard, the option value  $Q > 1$  implicitly reflects irreversible investment.

<sup>16</sup>In our model the marginal value of installed capital (equity price) is given by  $\lambda_t(i) = \frac{1}{\varepsilon_t^*(\theta)}$  and the marginal cost of investment is given by  $\frac{1}{\varepsilon_t(i)}$ . Hence, Tobin's  $q$  is given by  $q_t(i) = \frac{\varepsilon_t(i)}{\varepsilon_t^*(\theta)}$ , indicating that it is optimal to invest if  $\varepsilon(i) \geq \varepsilon^*$  and not optimal to invest if  $\varepsilon(i) < \varepsilon^*$ . In our general equilibrium model, Tobin's  $q$  is affected by both idiosyncratic shocks and aggregate shocks and is procyclical under both types of shocks (because  $\varepsilon_t^*$  decreases when aggregate productivity increases). However, the degree of procyclicality of the aggregate  $q$  under aggregate shocks depends negatively on the value of  $\theta$ . The more developed is the financial market, the less variations there are in the average (or aggregate) value of  $q$  because  $\varepsilon_t^*$  is less variable.

are from labor-augmenting technologies, then the impact of technology shocks on aggregate output is smaller when the value of  $\theta$  is larger (due to a larger capital-labor ratio), further reducing the importance of aggregate shocks on economic activities.

### 3 General Equilibrium

#### 3.1 Households

To close the model, we add a standard representative household into the model. The representative household chooses consumption  $C_t$  and labor supply  $N_t$  to solve

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{N_t^{1+\gamma}}{1+\gamma} \right\} \quad (27)$$

subject to the budget constraint,

$$C_t \leq w_t N_t + D_t, \quad (28)$$

where  $D_t = \int_0^1 d_t(i) di$  is the aggregate dividend income from firms. Let  $\Lambda_t$  be the Lagrangian multiplier of the budget constraint. Notice that the household has no incentive to buy bonds issued by firms because the equilibrium rate of return to bond is lower than the inverse of the time preference,  $r < \frac{1}{\beta}$ . The first order conditions with respect to consumption and labor are given respectively by

$$\Lambda_t = \frac{1}{C_t}, \quad (29)$$

$$\Lambda_t w_t = N_t^\gamma. \quad (30)$$

#### 3.2 Aggregation

Define  $N_t = \int_0^1 n_t(i) di$ ,  $K_t = \int_0^1 k_t(i) di$ ,  $I_t = \int_0^1 i_t(i) di$  and  $Y_t = \int_0^1 y_t(i) di$ . By the law of large numbers and the fact that  $\varepsilon_t(i)$  is independent of the predetermined variables  $\{b_t(i), k_t(i)\}$  and the aggregate shocks, the aggregate investment is then given by  $I_t = \left(R_t + \frac{\theta}{r_t}\right) K_t [1 - F(\varepsilon_t^*)]$ .

The aggregate capital stock evolves according to  $K_{t+1} = (1 - \delta)K_t + P(\varepsilon_t^*)I_t$ , where  $P(\varepsilon^*) \equiv \left[\int_{\varepsilon \geq \varepsilon^*} \varepsilon dF(\varepsilon)\right] [1 - F(\varepsilon^*)]^{-1}$  measures the average efficiency of investment. Similarly, equation

(8) implies  $(1 - \omega) \left[\frac{Y_t}{N_t}\right]^{1-\sigma} A_t^\sigma = w_t$ . Since the capital-labor ratio is identical across firms,

we must have  $\frac{k(i)}{n(i)} = \frac{K}{N}$ . It follows that the aggregate production function is given by  $Y_t =$

$\{\omega K_t^\sigma + (1 - \omega) [A_t N_t]^\sigma\}^{\frac{1}{\sigma}}$ . By the property of constant returns to scale, the defined function  $R(w, A)$  in (9) is then given by the capital's share coefficient,  $R_t = \omega \left(\frac{Y_t}{K_t}\right)^{1-\sigma}$ , which equals the marginal product of aggregate capital. Because  $\int_0^1 b_t(i) di = 0$ , the aggregate dividend is given by  $D_t = Y_t - I_t - w_t N_t$ . The household resource constraint then becomes  $C_t + I_t = Y_t$ .

In summary, the aggregate variables in this model include  $\{I_t, C_t, Y_t, N_t, R_t, \varepsilon_t^*, r_t, w_t, K_{t+1}\}$  and the system of equations to solve these variables are given by

$$\frac{1}{\varepsilon_t^* C_t} = \beta E_t \frac{1}{C_{t+1}} \left\{ R_{t+1} Q(\varepsilon_{t+1}^*) + \frac{\theta}{r_{t+1}} [Q(\varepsilon_{t+1}^*) - 1] + \frac{(1 - \delta)}{\varepsilon_{t+1}^*} \right\}, \quad (31)$$

$$\frac{1}{r_t} \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} Q(\varepsilon_{t+1}^*), \quad (32)$$

$$C_t + I_t = Y_t, \quad (33)$$

$$Y_t = \{\omega K_t^\sigma + (1 - \omega) [A_t N_t]^\sigma\}^{\frac{1}{\sigma}}, \quad (34)$$

$$I_t = \left( R_t + \frac{\theta}{r_t} \right) K_t [1 - F(\varepsilon_t^*)], \quad (35)$$

$$K_{t+1} = (1 - \delta) K_t + P(\varepsilon_t^*) I_t, \quad (36)$$

$$\frac{w_t}{C_t} = N_t^{1+\gamma}, \quad (37)$$

$$w_t = (1 - \omega) \left( \frac{Y_t}{N_t} \right)^{1-\sigma} A_t^\sigma, \quad (38)$$

$$R_t = \omega \left( \frac{Y_t}{K_t} \right)^{1-\sigma}, \quad (39)$$

where  $Q(\varepsilon_t^*) \equiv \int \max\left(\frac{\varepsilon}{\varepsilon_t^*}, 1\right) dF(\varepsilon)$  and  $P(\varepsilon^*) \equiv \left[ \int_{\varepsilon \geq \varepsilon^*} \varepsilon dF(\varepsilon) \right] [1 - F(\varepsilon^*)]^{-1}$ . Notice that if  $\theta = 0$ , this model reduces to a baseline model without external financing.

### 3.3 Steady State

The steady state of the model is defined as the situation without aggregate uncertainty (i.e.,  $A_t = 1$ ).

In the steady state, equations (32), (35) and (36) become

$$\frac{1}{r} = \beta Q, \quad (40)$$

$$I = \left( R + \frac{\theta}{r} \right) K[1 - F], \quad (41)$$

$$\delta K = PI. \quad (42)$$

Substituting out  $r$  and  $\frac{I}{K}$  in (41) gives  $\frac{\delta}{P} = (R + \beta\theta Q)[1 - F]$ . Hence, the marginal product of capital (or capital-output ratio) is determined by

$$R = \frac{\delta}{P(1 - F)} - \beta\theta Q. \quad (43)$$

Since  $P(1 - F) = \int_{\varepsilon \geq \varepsilon^*} \varepsilon dF(\varepsilon)$ , equation (31) implies

$$\frac{1}{\varepsilon^* Q(\varepsilon^*)} [1 - \beta(1 - \delta)] = \beta \left[ \frac{\delta}{\left[ \int_{\varepsilon \geq \varepsilon^*} \varepsilon dF(\varepsilon) \right]} - \beta\theta \right], \quad (44)$$

which determines the optimal cutoff  $\varepsilon^*$  in the steady state as a function of the structural parameters.

Notice that the following inequalities hold,

$$\frac{dQ}{d\varepsilon^*} < 0, \quad \frac{d[\varepsilon^* Q]}{d\varepsilon^*} > 0, \quad \frac{d[P(1 - F)]}{d\varepsilon^*} < 0. \quad (45)$$

Hence, by equation (44), an increase in  $\theta$  must imply an increase in  $\varepsilon^*$ ,

$$\frac{d\varepsilon^*}{d\theta} > 0. \quad (46)$$

That is, financial development means that the probability of undertaking capital investment ( $\Pr(\varepsilon > \varepsilon^*)$ ) is reduced, making firm-level investment lumpier. This is so because less efficient firms find bonds more attractive than fixed capital to buy. On the other hand, since  $\frac{dQ}{d\varepsilon^*} < 0$ , Equation (40) implies that  $r$  also increases, suggesting that the rate of return to bond increases. Consequently, more firms are willing to invest in bond and this allows the most productive firms to invest in fixed capital by raising debts.

Because the effect of  $\theta$  on the real wage depends on its effect on the aggregate capital stock, which in turn depends both on the efficiency of individual firms' investment and on the number



of investing firms, we need to express the steady-state output-capital ratio as a function of  $\theta$  more analytically so as to conduct comparative statics. For this purpose, we assume Pareto distribution for the idiosyncratic shock,  $F(\varepsilon) = 1 - \varepsilon^{-\eta}$ , with  $\eta > 1$  and the support  $(1, \infty)$ . With the Pareto distribution, we have  $Q = 1 + \frac{1}{\eta-1}\varepsilon^{*-\eta}$  and  $P = \frac{\eta}{\eta-1}\varepsilon^*$ . The equation (43) implies the output-capital ratio,

$$\frac{Y}{K} = \left( \frac{\frac{1}{\beta} - \frac{\delta}{\eta} - (1 - \delta)}{\omega\varepsilon^*(\theta)} \right)^{\frac{1}{1-\sigma}}, \quad (47)$$

which is strictly positive (because  $\beta < 1$  and  $\eta > 1$ ) and decreasing in  $\theta$ . Since  $\delta K = PI$ , the aggregate investment-output ratio is given by

$$\frac{I}{Y} = \left( \frac{\omega\varepsilon^*(\theta)}{\frac{1}{\beta} - \frac{\delta}{\eta} - (1 - \delta)} \right)^{\frac{1}{1-\sigma}} \frac{\delta\eta\varepsilon^*(\theta)}{\eta - 1}, \quad (48)$$

which is increasing in  $\theta$ . These suggest that financial development enhances investment returns and capital accumulation. Hence, both the capital-output ratio and the capital-labor ratio rise with  $\theta$ . As the capital-labor ratio rises, the marginal product of capital declines and the real wage (marginal product of labor) increases.

To sum up, with financial development, two forces are at work to reduce aggregate volatility under aggregate productivity shocks. First, aggregate shocks affect investment mainly through their effect on firms' operating profits (revenues). With the development of the financial market, firms increasingly finance their investment through external borrowing. Hence, their operating profits become less important, leading to a decline in aggregate investment volatility. Second, financial development improves investment efficiency. Hence, capital becomes relatively cheaper than labor. As a result, firms employ more and more capital goods and thus labor augmenting technology plays a smaller and smaller role in production, leading to a decline in aggregate volatility under technology shocks. The first force alone is sufficient for reducing aggregate volatility regardless the technology is labor augmenting or not. But the second force reinforces the first.

Notice that when  $\eta$  approaches infinity, the mean of the Pareto distribution approaches one and the variance approaches zero. In this case, all firms become identical and  $\lim_{\eta \rightarrow \infty} \varepsilon^* = \lim_{\eta \rightarrow \infty} Q = \lim_{\eta \rightarrow \infty} P = 1$ . The benchmark model then degenerates to the standard RBC model.

### 3.4 Calibration and Impulse Responses

Let the time period be a quarter, the time discount rate  $\beta = 0.99$ , the rate of capital depreciation  $\delta = 0.025$ , and the inverse labor supply elasticity  $\gamma = 0$  (indivisible labor). We choose  $\omega = 0.25$

and  $\sigma = 0.175$  so that the implied steady-state capital's income share is about 0.42 when  $\theta = 0$  (our benchmark value).<sup>17</sup> The law of motion for aggregate technology follows an AR(1) process,

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t, \quad (49)$$

where  $\rho = 0.9$ . The Pareto-distribution parameter is set to  $\eta = 2.5$ .<sup>18</sup> The impulse responses of the model to a one-standard-deviation aggregate technology shock are graphed in Figure 2, where the solid lines represent the case with  $\theta = 0$  and the dashed lines the case with  $\theta = 0.5$ . With these calibrated parameter values, the reduction in GDP volatility is about 40%.

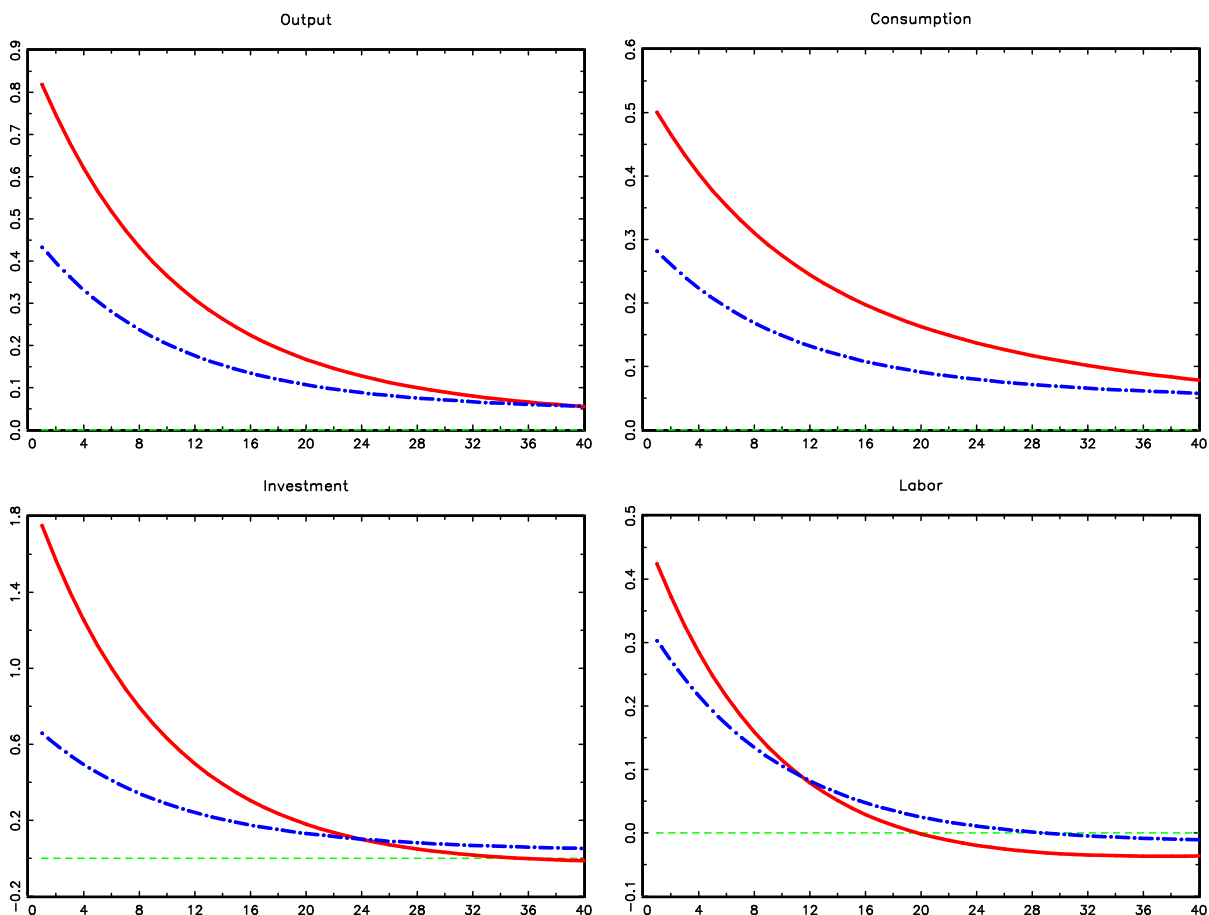


Figure 2. Impulse Responses to Technology Shock (solid line:  $\theta = 0$ ; dashed line:  $\theta = 0.5$ ).

<sup>17</sup> Assuming Cobb-Douglas production function ( $\sigma = 0$ ) gives qualitatively similar results. When the production technology is Cobb-Douglas, labor augmenting technology shocks are identical to TFP shocks. However, the volatility effects of financial development are stronger under labor augmenting technology shocks than under TFP shocks because a larger  $\theta$  implies a larger capital stock to output ratio and a smaller impact of  $A_t$  on production.

<sup>18</sup> The variance of the Pareto distribution is a decreasing function of  $\eta$ . The empirical literature based on distributions of firm size typically finds  $\eta$  close to 1 (see, e.g., Axtell, 2001). The smaller the value of  $\eta$ , the larger is the reduction in aggregate volatility and the increase in firm-level volatility when  $\theta$  increases. Reducing  $\eta$  makes it easier for our model to generate the diverging trends. Hence,  $\eta = 2.5$  is a very conservative number. We choose  $\eta = 2.5$  so that the firm-level volatility is roughly ten times the aggregate volatility, as in the data.

Figure 2 suggests that, with financial development, aggregate output, consumption, investment and employment all become significantly less volatile under technology shocks. For example, the standard deviation of output is reduced by about 40%, consumption by 37%, investment by 57%, and employment by 24%. This suggests that, even if monetary policy had not changed (Clarida, Gali and Gertler, 2000), the inventory management technology had not improved (Kahn, McConnell, and Perez-Quiros, 2001), and the variance of aggregate technology shocks had not reduced (Arias, Hansen and Ohanian, 2007; and Stock and Watson, 2002), the volatility of the U.S. economy would still decrease significantly, simply because of financial developments alone.<sup>19</sup>

## 4 Explaining the Diverging Trends

Note that in our model the measure of firm-level volatility is identical to the measure of dispersion across firms. Following Comin and Philippon (2005), we measure the firm-level volatility in our model by the standard deviation of the median (or average) firm's sales growth. The constant-returns-to-scale production function implies that firm's sales (output) and labor are proportional to capital stock. Hence, we can use capital growth rate,  $g_t(i) = \frac{k_{t+1}(i)}{k_t(i)} - 1$ , as our measure. Since the median-firm's debt level is zero in the model economy, we set  $b_t(i) = 0$  in computing  $g_t(i)$ .<sup>20</sup> Noting  $(R + \frac{\theta}{r}) = \frac{\delta}{P[1-F]}$  in the steady state (see equations 41 and 42), the firm-level growth rate is given by

$$g_t(i) = \begin{cases} -\delta & \text{if } \varepsilon_t(i) \leq \varepsilon^* \\ -\delta + \left[ \delta \left( \frac{\eta-1}{\eta} \right) \varepsilon^{*\eta-1} \right] \varepsilon_t(i) & \text{if } \varepsilon_t(i) > \varepsilon^* \end{cases}. \quad (50)$$

Notice that the mean growth rate is zero,  $\bar{g}(i) = E g_t(i) = 0$ , because it is the same as the aggregate capital growth rate in the steady state,  $\frac{K_{t+1}}{K_t} - 1 = 0$ .<sup>21</sup> Hence, the variance of the average-firm's

<sup>19</sup>The implied aggregate debt to output ratio is about 24% when  $\theta = 0.05$  and about 40% when  $\theta = 0.5$ . In the U.S. economy, non-financial firms' total debt to GDP ratio has doubled from about 23% to 48% in the past half century. Our model predicts that if the debt to output ratio doubles, aggregate output volatility will decrease by about 35%, everything else equal.

<sup>20</sup>Except the firm in the middle, a firm's debt level  $b_t(i)$  is indeterminate in the model. The only thing we know is that  $\int b(i) di = 0$ . Ignoring  $b_t(i)$  tends to under estimate a firm's investment volatility. When  $\varepsilon_t(i) > \varepsilon_t^*$ , the variance of investment is the variance of  $b_t(i)$  plus the variance of the first term in the decision rule (24). Since the variance of firm's investment and borrowing activity increases with  $\theta$ , the variance of  $b_t(i)$  also increases with  $\theta$ .

<sup>21</sup>One can confirm this by computing the true average growth rate,

$$\bar{g}(i) = -\delta + \delta \left( \frac{\eta-1}{\eta} \right) \varepsilon^{*\eta-1} \int_{\varepsilon^*}^{\infty} \varepsilon f(\varepsilon) d\varepsilon = 0.$$

capital growth is given by  $Eg_t^2(i) = \delta^2 \frac{(\eta-1)^2}{\eta(\eta-2)} \varepsilon^{*\eta} - \delta^2$ , and the the standard deviation is

$$\sigma_Y = \delta \sqrt{\frac{(\eta-1)^2}{\eta(\eta-2)} \varepsilon^{*\eta} - 1}. \quad (51)$$

Because  $\frac{d\varepsilon^*}{d\theta} > 0$ , our model implies that firm-level volatility increases with financial development.

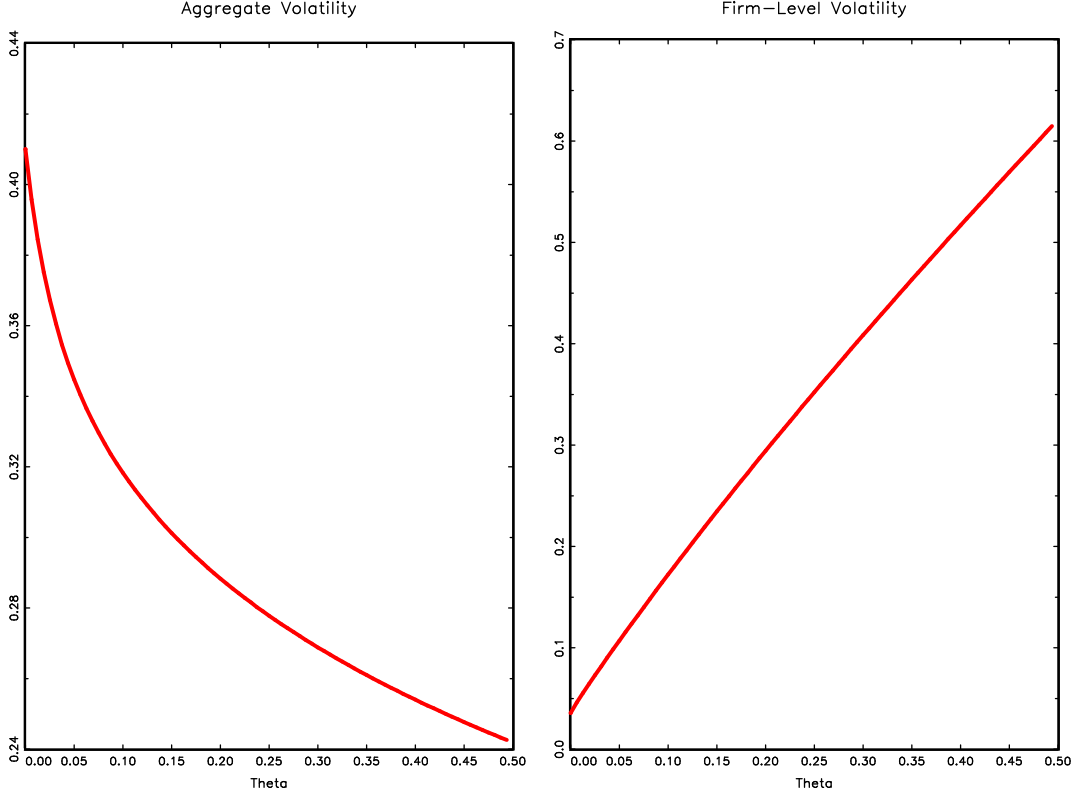


Figure 3. Diverging Trends in Aggregate and Firm-Level Volatility.

In the U.S. data, firm-level volatility is about ten times the aggregate volatility on average over time. Thus we calibrate the variance of idiosyncratic shocks ( $\eta = 2.5$ ) and that of aggregate shocks ( $\sigma_A = 0.02$ ) in our model to match this volatility ratio. Given that the variance of idiosyncratic shocks dominates that of aggregate shocks, the influence of idiosyncratic shocks on firm-level volatility dominates that of aggregate shocks in our model.<sup>22</sup> Thus, ignoring aggregate uncertainty does not have a significant effect on our measure of firm-level volatility. The predicted trends of firm-level volatility and aggregate volatility are plotted in Figure 3, where the left window shows aggregate volatility (scaled up by a factor of ten so as to be comparable to firm-level volatility),

<sup>22</sup>Without idiosyncratic shocks, the time trend of firm-level volatility mimics that of aggregate volatility in our model and the dispersion is always zero.

the right window shows the firm-level volatility, with the horizontal axis indicating the degree of financial development (i.e., the value of  $\theta$ ). It clearly shows that the two trends of volatility are diverging as the financial market develops, consistent with the empirical facts documented by the empirical literature cited in the beginning of this paper.

## 5 Discussion

More recently, Davis, Haltiwanger, Jarmin and Miranda (2006) showed that the increasing trend in firm-level volatility applies only to publicly traded firms who have access to external financing. Using recently developed Longitudinal Business Database (LBD), they found that the opposite is true for privately held firms who do not have excess to outside funds. Namely, for privately held small firms, there has been a large secular decline in the cross firm dispersion of firm growth rates and in the average magnitude of firm-level volatility. This is in sharp contrast to the behavior of publicly traded firms. In this section we offer some preliminary explanations for this phenomenon and argue that our model is not inconsistent with this empirical fact.

Our argument is based on the prediction that financial development raises the real wage. The rising share of wage costs may reduce the sensitivity of firms' labor demand to idiosyncratic labor costs more so for privately held firms than for publicly traded firms due to difference in firm size.<sup>23</sup> Consequently, volatility and dispersion across privately held firms decrease with rising wage costs. This is illustrated in a simple extension of our benchmark model below.

Suppose there is a continuum of privately held firms with measure  $M$  indexed by  $j \in [1, M + 1]$ . According to Davis, Haltiwanger, Jarmin and Miranda (2006), the number of privately-held firms is several hundreds times that of publicly-traded firms in the data; hence, we assume  $M \gg 1$ . To keep the model as stylized as possible, we assume that privately held firms do not accumulate capital. To simplify notation, privately held firms are called sector 2 with output and employment denoted by  $y_{2t}$  and  $n_{2t}$ , respectively, while publicly traded firms are called sector 1 with output and employment denoted by  $y_{1t}$  and  $n_{1t}$ . The production technology of sector 2 is given by

$$y_{2t}(j) = aA_t n_{2t}^\tau(j), \quad 0 < \tau < 1. \quad (52)$$

where  $A$  is the economy-wide aggregate labor-augmenting technology shock common to both sectors. The unit labor cost for sector 2's firm is  $w_t + e_t(j)$ , where  $w$  is the aggregate real wage and  $e(j)$  is an idiosyncratic cost shock to firm  $j$  in sector 2. The firm in sector 2 maximizes profit,  $A_t a n_{2t}^\tau(j) - [w_t + e_t(j)]n_{2t}(j)$ , in each period. Suppose the distribution of  $e(j)$  is binary with

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<sup>23</sup>According to Davis, Haltiwanger, Jarmin and Miranda (2006), the average number of works is about 15 for privately held firms and 4000 for publicly traded firms. Hence, the idiosyncratic fixed costs of labor tend to be more important for small firms than for large firms. For this reason, we introduce an idiosyncratic labor cost only to privately held firms in our extended general-equilibrium model.

$e(j) = \bar{e}$  (with probability  $\pi$ ) and  $e(j) = 0$  (with probability  $1 - \pi$ ). The optimal labor demand in sector 2 is then given by

$$n_{2t}(j) = \begin{cases} \left( \frac{w_t + \bar{e}}{\tau a A_t} \right)^{\frac{1}{\tau-1}}, & \text{with prob. } \pi \\ \left( \frac{w_t}{\tau a A_t} \right)^{\frac{1}{\tau-1}}, & \text{with prob. } 1 - \pi \end{cases}, \quad (53)$$

and the expected output is given by

$$y_{2t}(j) = M \left[ \pi A_t a \left( \frac{w_t + \bar{e}}{\tau a A_t} \right)^{\frac{\tau}{\tau-1}} + (1 - \pi) A_t a \left( \frac{w_t}{\tau a A_t} \right)^{\frac{\tau}{\tau-1}} \right]. \quad (54)$$

The first-order conditions for the publicly traded firms (sector 1) remain the same as before, and so do the household's first-order conditions. The only equations that are changed are the aggregate resource constraints for labor and goods. The labor market clearing condition requires

$$n_1 + M n_2 = N. \quad (55)$$

Since the household owns all firms in sectors 1 and 2, the aggregate budget constraint becomes

$$C_t + I_t + M \pi \bar{e} \left( \frac{w_t + \bar{e}}{\tau a A_t} \right)^{\frac{1}{\tau-1}} = y_{1t} + y_{2t} = Y_t, \quad (56)$$

where the left-hand side is total expenditure (consumption, investment and labor costs), and the right hand side is the total income.

Several steps are needed to determine the steady-state values of the 2-sector model. As in the benchmark model, we first solve the steady-state cutoff value  $\varepsilon^*$  for sector 1, which is the same as in the benchmark model and not affected by the introduction of sector 2. The next step is to determine the real wage in the steady state. Again, the real wage remains the same because of competitive labor market and labor mobility. Given the real wage ( $w$ ), labor demand and output in sector 2 are then determined by equations (53) and (54).

The firm-level volatility for sector 1 remains the same as in the benchmark model. For privately held firms, we compute the volatility of labor growth (which is the same as output growth) in the

steady state as follows. Taking log of equation (53), the employment growth rate is given by

$$g_n = \begin{cases} 0 & \text{with prob. } \pi^2 + (1 - \pi)^2 \\ \frac{1}{\tau-1} [\ln(w) - \ln(w + \bar{e})] & \text{with prob. } \pi(1 - \pi) \\ \frac{1}{\tau-1} [\ln(w + \bar{e}) - \ln(w)] & \text{with prob. } \pi(1 - \pi) \end{cases} \quad (57)$$

Hence the standard deviation of  $g_n$  equals

$$\sigma_n = \frac{\sqrt{2\pi(1-\pi)}}{1-\tau} [\ln(w + \bar{e}) - \ln(w)] \quad (58)$$

It is easy to see  $\frac{\partial \sigma_n}{\partial w} = \frac{\sqrt{2\pi(1-\pi)}}{1-\tau} \left[ \frac{1}{w+\bar{e}} - \frac{1}{w} \right] < 0$ . Hence, the firm-level volatility and dispersion decrease with financial development in sector 2.

Table 1. Parameter Values

Parameter	$\beta$	$\delta$	$\gamma$	$\eta$	$\omega$	$\sigma$	$\tau$	$\pi$	$M$	$\bar{e}$	$a$	$\rho$	$\sigma_A$
Value	0.99	0.025	0	2.5	0.25	0.175	0.9	0.5	100	0.2	1.35	0.9	0.02

We calibrate the structural parameters of the model as in Table 1. These parameter values imply that the ratio of fixed labor cost to the real wage is about 8% when  $\theta = 0$  and about 1% when  $\theta = 0.5$ . These values also imply that the volatility of privately-held firms is several times larger than that of publicly-traded firms when  $\theta$  is small. The predicted trends of growth volatility for sector 1, sector 2, and the aggregate output are graphed in the bottom windows in Figure 4, where the left window shows aggregate volatility (scaled up by a factor of ten) and the right window shows firm-level volatility (green line represents privately-held firms and red line publicly-traded firms). The model clearly captures qualitatively the converging trends in volatility for privately-held firms and publicly-traded firms reported by Davis, Haltiwanger, Jarmin and Miranda (2006, Fig.1 – Fig.10).<sup>24</sup> At the same time, the 2-sector model continues to predict a downward trend in

<sup>24</sup>The top right window is a replication of their Fig. 5.

aggregate volatility under technology shocks.

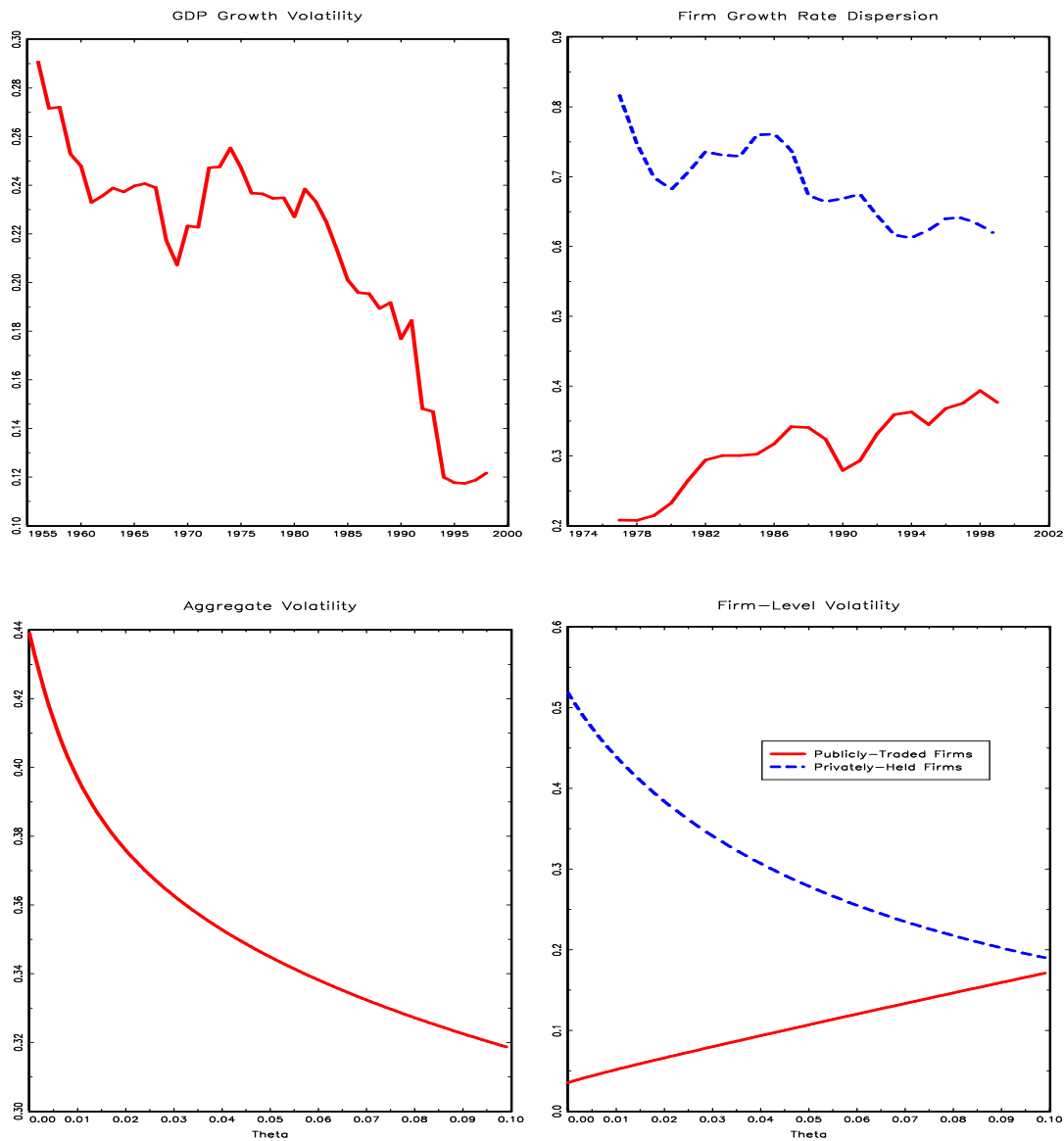


Figure 4. Top Windows: US Data. Bottom Windows: 2-Sector Model.

## 6 Conclusion

Empirical studies found that volatilities have been increasing at the firm level (for publicly-traded firms) but decreasing at the aggregate level. We offer a unified explanation for this diverging trend puzzle. Our explanation is based on a story of financial development that relaxes borrowing constraints and promotes risk sharing across firms. Our dynamic stochastic general equilibrium model predicts that financial liberalization increases firm-level volatility by allowing more produc-



tive firms to expand capital stock through borrowing external funds and less productive firms to reduce losses through savings and investing in bonds, making firm-level investment lumpier, more heterogeneous, and more sensitive to idiosyncratic productivity shocks. At the same time, financial development reduces aggregate volatility by making firms' operations less dependent on internal cash flows; hence, aggregate technology shocks have less impact on firms' investment, production, and employment, leading to a less volatile economy.

There exists another closely related empirical regularity about firm-level volatility: for privately-held firms with little access to external financing, their volatility have been decreasing in the postwar period (Davis, Haltiwanger, Jarmin and Miranda, 2006). Although explaining this third regularity is an challenging and interesting topic for future research, we nonetheless show that our model is not inconsistent with this empirical fact. Our preliminary explanation for this empirical fact is based on a natural extension of our benchmark model in which financial development improves investment efficiency and raises the real wage. The rising share of wage costs may reduce the sensitivity of firms' labor demand to idiosyncratic labor costs more so for privately held firms than for publicly traded firms due to difference in firm size. Consequently, volatility and dispersion across privately held firms decrease more significantly with rising wage costs. Our analysis in this regard is preliminary because we have assumed that privately-held firms (i.e., small firms) are labor intensive and do not accumulate capital. This simplifying assumption simplifies our analysis dramatically but does not allow us to treat big firms and small firms symmetrically. We leave the symmetric analysis to future research.

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