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A Simple Impossibility Result in Behavioral Contract Theory

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Abstract
The paper analyses, within a moral hazard scenario, a contract between an agent with anticipatory emotions and a principal who responds strategically to those emotions. The agent receives a private signal on the profitability of the task he was hired for. If the signal is informative about the return from effort, the agent would benefit from knowing accurate news. However, if the agent derives utility from the anticipation of his final payoff, the suppression of a bad signal may induce a positive interim emotional effect. We show that it may be impossible to achieve the first-best, even though the risk-neutral parties are symmetrically informed at the contracting stage and complete contracts can be written.

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Keywords: Hidden action, anticipatory utility.

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1 Introduction

Forward-looking agents care about expected utility flows, and enjoy anticipatory utility if they are optimistic about the future. Due to imperfect memory, they may then choose their beliefs so as to enjoy the greatest comfort or happiness (cognitive dissonance).[1] Building on the motivated-beliefs and optimal awareness framework, this paper studies a contract between an (endogenously) optimistic agent and a realistic principal. Upon receiving a private signal about the profitability of the task he is hired for, the agent has to choose the level of effort to exert that affects the probability of success of the project. Although the risk-neutral parties are symmetrically informed at the contracting stage and complete contracts can be written, we show that it may be impossible to achieve the first-best, unless the weight on emotions is sufficiently low.

Thus, as several other works in the contract-theoretic literature, this paper also derives an impossibility result (Myerson and Satterthwaite, 1983; Akerlof, 1970, among others). The novelty of the paper is that this result is derived within a behavioral model that in recent years has been applied in a variety of contexts, among which asset-pricing (Caplin and Lehay, 2001; Kuznitz et al., 2008), health economics (Caplin and Lehay, 2004; Koszegi, 2006), policy design (Caplin and Eliaz, 2003), managerial compensation (Immordino et al., 2010) and theories of moral behavior (Bénabou and Tirole, 2010).

2 Model

Consider a setting in which a risk neutral principal hires a risk neutral agent for a project that has three possible outcomes, $\tilde{v} \in \{0, v_L, v_H\}$, with $0 < v_L < v_H$. In carrying out the task he is hired for, the agent chooses a level of effort a affecting the probability of success, with $a \in [0; 1]$. The effort has disutility $c(a)$, with $c(0) = 0$, $c'(a) \geqslant 0$ and $c''(a) > 0$. In order to ensure interior solutions, we also assume that $c'(0) = 0$ and $c'(1) \geq v_H$. After signing the contract but before choosing the

[1] See for instance Akerlof and Dickens (1982), Loewenstein (1987), Caplin and Leahy (2001), Bénabou and Tirole (2002, 2003, 2010), Koszegi (2006), among others.

effort level, the agent receives a private signal $\sigma \in \{L; H\}$ correlated with the project's return $\tilde{v}$. The probability of a good signal is q, with $q \in [0; 1]$. In our setting, good (bad) news means that the outcome is v_H (v_L) or v_0 with probability a and $1 - a$, respectively, i.e.,

$$\Pr(\tilde{v} = v_L | \sigma = H) = \Pr(\tilde{v} = v_H | \sigma = L) = 0.$$

Then, ex-ante v_0, v_L and v_H can each arise with probabilities $1 - a, (1 - q)\, a$ and qa, respectively.

Given that the signal is informative about the return from effort, the agent would benefit from knowing accurate news when choosing a. However, if the agent derives utility from the anticipation of his final payoff, the suppression of a bad signal may induce a positive emotional effect. This is modeled assuming that total utility is a convex combination of actual (as of time 3) and anticipated physical outcomes (as of time 2), with weights $1 - s$ and s, respectively, with $s \in [0, 1]$.

At the time of the effort decision, the recollection of a good signal is always accurate, whilst a bad signal can be forgotten due to voluntary repression. Denote by $\widehat{\sigma} \in \{\widehat{L}; \widehat{H}\}$ the recollection of the news σ and by $\lambda \in [0; 1]$ the probability that bad news will be remembered accurately ($\lambda \equiv \Pr(\widehat{L}|L)$). Finally, denote by Agent 1 the agent's self as of time 1 and by Agent 2 the agent's self as of time 2.

The agent's action is not directly observable by the principal, who can offer a contract $C \equiv \{w_0, w_L, w_H\}$ with rewards contingent on the observable and verifiable project revenues. Finally, we maintain the standard assumption of individuals as rational Bayesian information processors.

The time-line is the following:

t=0: The principal offers a contract to the agent.

t=1: If Agent 1 refuses, the game ends. If he accepts, he observes a private signal σ and, when bad ($\sigma = L$), chooses the probability λ that news will be remembered accurately.

t=2: Agent 2 observes $\widehat{\sigma}$, updates his beliefs on the outcome v, selects the effort level a and enjoys the anticipatory utility.

t=3: The project payoff is realized and the payment is executed.

First-best benchmark. When both the action and the signal are observable, the efficient memory strategy and effort levels are obtained by maximizing the ex-ante total surplus

$$\left(a_{L\hat{L}}^{FB}, a_{L\hat{H}}^{FB}, a_{H\hat{H}}^{FB}, \lambda^{FB}\right) \in \arg\max E_0\left[\tilde{v} - c\left(a\right)|a, \lambda\right] \equiv \tag{1}$$

$$\equiv q\left(a_{H\hat{H}}v_H - c\left(a_{H\hat{H}}\right)\right) + (1-q)\left((1-\lambda)\left(a_{L\hat{H}}v_L - c\left(a_{L\hat{H}}\right)\right) + \lambda\left(a_{L\hat{L}}v_L - c\left(a_{L\hat{L}}\right)\right)\right),$$

where $a_{\sigma\hat{\sigma}}$ is the effort exerted by the agent when observing σ and recollecting $\hat{\sigma}$. Solving problem (1) gives three sets of first-best outcomes

$$\lambda^{FB} \in [0,1], \; a_{L\hat{L}}^{FB} = a_{L\hat{H}}^{FB} = a_L^{FB}, \; a_{H\hat{H}}^{FB} = a_H^{FB}; \tag{2}$$

$$\lambda^{FB} = 0, \; a_{L\hat{L}}^{FB} \in [0,1], \; a_{L\hat{H}}^{FB} = a_L^{FB}, \; a_{H\hat{H}}^{FB} = a_H^{FB}; \tag{3}$$

$$\lambda^{FB} = 1, \; a_{L\hat{L}}^{FB} = a_L^{FB}, \; a_{L\hat{H}}^{FB} \in [0,1], \; a_{H\hat{H}}^{FB} = a_H^{FB}; \tag{4}$$

where a_i^{FB} is such that $c'\left(a_i^{FB}\right) = v_i$, for $i = L, \; H$.

In the following, the effort choice is a hidden action and Agent 1 observes a private signal about the project profitability which he may choose to forget at the time of the effort decision. When effort is chosen by Agent 2 without observing σ, (2) and (3) can never be implemented. Indeed, when the signal is not observable to all parties, effort can only be contingent on signal recollection (and not on the actual signal). Therefore, the first-best could be implemented only if the contract gives Agent 1 the incentive to perfectly recollect the signal as in (4).

Next, we characterize the optimal incentive scheme for each effort level and memory strategy that the principal may want Agent 2 and Agent 1 to select. To this aim we analyze the optimal effort choice a, given Agent 2's beliefs about σ and describe the Perfect Bayesian Equilibrium of the memory game. Finally, we use the agent's optimal effort choice rule and the memory game equilibrium to show our impossibility result.

3 Effort choice and memory strategy

We start by describing the incentive problem faced by the principal to induce Agent 2 to choose the desired level of effort. Denoting by E_2 the expectation at $t = 2$, the intertemporal utility perceived by Agent 2, given the memory $\widehat{\sigma}$ and the contract C, is

$$E_2 [U_3] = -c(a) + E_2 [u(C, a)|\widehat{\sigma}], \tag{5}$$

where $E_2 [u(C, a)|\widehat{\sigma}]$ is the sum of the agent's material payoff, $(1 - s)E_2 [u(C, a)|\widehat{\sigma}]$, and his anticipatory utility (experienced by savoring the future material payoff), $sE_2 [u(C, a)|\widehat{\sigma}]$. This is equal to

$$E_2[u(C, a)|\widehat{L}] = aw_L + (1 - a) w_0,$$

when $\widehat{\sigma} = \widehat{L}$, and to

$$E_2[u(C, a)|\widehat{H}] = a (rw_H + (1 - r)w_L) + (1 - a) w_0,$$

when $\widehat{\sigma} = \widehat{H}$, where r is the posterior probability attached to state H.

Agent 2 chooses the level of effort that maximizes his intertemporal expected utility. Thus, to induce efforts $a_{\widehat{L}}$ and $a_{\widehat{H}}$ from an agent who recalls $\widehat{L}$ and $\widehat{H}$ respectively, the contract has to be such that

$$w_L - w_0 = c' \left(a_{\widehat{L}}\right), \tag{6}$$

and

$$rw_H + (1 - r)w_L - w_0 = c' \left(a_{\widehat{H}}\right). \tag{7}$$

We now consider the incentive problem faced by the principal to induce Agent 1 to correctly recall the signal. To this aim, we describe the Perfect Bayesian Equilibrium of the memory game for given contract C. In the PBE: i) for any realized σ, Agent 1 chooses his message $\widehat{\sigma}$ to maximize his expected utility, correctly anticipating what inferences he will draw from $\widehat{\sigma}$, and what action he will choose; ii) Agent 2 forms his beliefs using Bayes rule to infer the meaning of Agent 1's message, knowing his strategy.

Agent 2 is aware that there are incentives in manipulating memory when the true state is L. Hence, faced with a memory $\widehat{H}$, he assesses its credibility. If he thinks that the bad signal is recalled with probability λ, using Bayes' rule he computes the likelihood of an accurate signal recollection as

$$r(\lambda) \equiv \Pr(H|\widehat{H}, \lambda) = \frac{q}{q + (1-q)(1-\lambda)} \in [q, 1]\,. \tag{8}$$

If Agent 1 observes $\sigma = L$, he chooses the probability of remembering the signal so as to maximize his expected utility

$$\max_{\lambda \in [0,1]} E_1 \left[U(C, a(r, C), \lambda)\right] \equiv E_1[-c\left(a(r, C)\right) + sE_2\left[u(C, a(r, C))\right]] + (1-s)\, E_1\left[u(C, a(r, C))\right], \tag{9}$$

where E_1 denotes expectations at $t = 1$ and $a(r, C)$ is the optimal strategy of Agent 2 for given r and $C \in R^3_+$.

Bayesian rationality implies that Agent 2 knows that Agent 1 is choosing the recalling strategy according to (9), and uses this optimal λ in his inference problem. A PBE of the memory game is a pair $(\lambda^\star; r^\star) \in [0; 1] \times [q; 1]$ that solves (8) and (9).[2]

In order to induce perfect recall ($\lambda = 1$) the contract must be such that

$$E_1\left[U(C, a(r, C), 1)|L\right] \geq E_1\left[U(C, a(r, C), \lambda)|L\right]. \tag{10}$$

At equilibrium $r = r(\lambda)$ and, from (7), the optimal effort when recalling $\widehat{H}$ depends on λ through $r(\lambda)$. Thus, at equilibrium $a(r, C) = a(\lambda, C) \equiv \left\{a_{\widehat{L}}, a_{\widehat{H}}(\lambda)\right\}$ and condition (10) simplifies to

$$\underbrace{c\left(a_{\widehat{H}}(\lambda)\right) - c\left(a_{\widehat{L}}\right)}_{\text{extra cost of effort}} \geq \underbrace{s a_{\widehat{H}}(\lambda) r(\lambda)\left(w_H - w_L\right)}_{\text{emotional gain from forgetting}} + \underbrace{\left(a_{\widehat{H}}(\lambda) - a_{\widehat{L}}\right)\left(w_L - w_0\right)}_{\text{indirect gain from higher effort}}. \tag{11}$$

The agent has an incentive to remember when, for any $\lambda < 1$, the extra-cost he incurs to exert effort $a_{\widehat{H}}(\lambda)$ rather than $a_{\widehat{L}}$ exceeds the emotional gain from forgetting due to the uncertainty about the payment in case of success, plus the gain due to obtaining w_L rather than w_0 with an increased probability $\left(a_{\widehat{H}}(\lambda) - a_{\widehat{L}}\right)$. Notice that the incentive to forget is positively correlated with s.

[2] To simplify notation, we will omit the star superscript.

4 A simple impossibility result

The principal's problem boils down to the choice of effort levels $a_{\widehat{H}}, a_{\widehat{L}}$, recall probability λ, and payments w_H, w_L and w_0 that maximize his expected profit subject to Agent 2's incentive constraints (6) and (7), and Agent 1's recalling constraint (11). Moreover, when the principal makes his offer, the agent does not know σ. To induce him to accept it, the contract has to satisfy the following ex-ante participation constraint

$$\begin{aligned} E_0\left[U\left(C,a,\lambda\right)\right] &= qE_1\left[U(C,a,\lambda)|\sigma=H\right]+(1-q)E_1\left[U(C,a,\lambda)|\sigma=L\right]= \\ &= \left[w_0+qa_{\widehat{H}}\left(w_H-w_0\right)+(1-q)\left(\left(1-\lambda\right)a_{\widehat{H}}+\lambda a_{\widehat{L}}\right)\left(w_L-w_0\right)\right]+ \\ &\quad -\left[(1-\lambda\left(1-q\right))c(a_{\widehat{H}})+\lambda\left(1-q\right)c\left(a_{\widehat{L}}\right)\right]\geq 0, \end{aligned} \tag{12}$$

where E_0 denotes the expectation at $t=0$, $qa_H = \Pr_0\left(v_H|a,\,\lambda\right)$, $(1-q)\left((1-\lambda)\,a_H+\lambda a_L\right) = \Pr_0\left(v_L|a,\,\lambda\right)$, and $\lambda\left(1-q\right)=\Pr_0(\widehat{L}|\lambda)$.

It should be emphasized that if the agent recalls the signal, the first-best is still attainable even when effort is unverifiable, as standard in principal-agent models with risk-neutrality and unlimited liability. The same is true if effort is verifiable but the agent may forget a bad signal. For instance, from (11) it follows that a flat contract, which satisfies the participation constraint at the first-best levels of effort ($w_H = w_L = w_0 = qc(a_H^{FB}) + (1-q)\,c\left(a_L^{FB}\right)$), removes all incentives to forget bad news. Then,

Proposition 1 *With hidden action only or with a forgetful agent only the parties would always write a contract that implements the first-best.*

In contrast, although parties are symmetrically informed at the contracting stage and can write complete contracts specifying state-contingent rewards, effort levels contingent on recollected signals and the memory strategy, there is no contract implementing the first-best. The next proposition states that the simultaneous presence of hidden action and forgetful agent leads to a simple and novel impossibility result.

Proposition 2 *If the weight on anticipatory utility s is sufficiently high, there is no contract that implements the first-best.*

Proof: Substituting $a_{\widehat{H}} = a_H^{FB}$, $a_{\widehat{L}} = a_L^{FB}$ and $\lambda = 1$ in incentive constraints (6) and (7) we work out the premiums for success $\Delta w_H^{FB} = (w_H - w_0)$ and $\Delta w_L^{FB} = (w_L - w_0)$ that implement the first-best levels of effort given perfect recall. Substituting Δw_H^{FB} and Δw_L^{FB} in (11), the recalling constraint becomes

$$c\left(a_{\widehat{H}}(\lambda)\right) - (ca_L^{FB}) \geq s\left[a_{\widehat{H}}(\lambda)c'\left(a_{\widehat{H}}(\lambda)\right) - a_L^{FB}c'(a_L^{FB})\right] + (1-s)(a_{\widehat{H}}(\lambda) - a_L^{FB})c'(a_L^{FB}), \quad (13)$$

where $a_{\widehat{H}}(\lambda)$ is the level of effort chosen by a forgetful agent recalling $\widehat{H}$ and is such that

$$c'(a_{\widehat{H}}(\lambda)) = r(\lambda)\,\Delta w_H^{FB} + (1 - r(\lambda))\,\Delta w_L^{FB}.$$

Then, if $s = 1$, (13) becomes

$$a_L^{FB}c'\left(a_L^{FB}\right) - c\left(a_L^{FB}\right) \geq a_{\widehat{H}}(\lambda)\,c'\left(a_{\widehat{H}}(\lambda)\right) - c\left(a_{\widehat{H}}(\lambda)\right),$$

and this is never true since $a_{\widehat{H}}(\lambda) > a_L^{FB}$ and the function $h(a) = a\,c'(a) - c(a)$ is increasing in a ($h'(a) = a\,c''(a) > 0$ by assumption). By continuity, Agent 1 also prefers to forget bad news for s close to 1. $\square$

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