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Policy Games with Liquidity Constrained Consumers

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Abstract

We investigate the optimal responses of policy authorities through a model where the fiscal and the monetary policymakers are independent and play strategically. We allow for the presence of two types of consumers: ‘Ricardians’, who trade in the assets market and ‘liquidity constrained’ consumers, who spend all their disposable labor income for consumption. We find that not only the different game structures but mainly the presence of ‘liquidity constrained’ consumers is crucial in determining the optimal responses of policies. In particular, for high enough fractions of liquidity constrained consumers the way policies react to cope with a mark-up shock change significantly and the role of fiscal policy becomes more relevant.

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1 Introduction

In recent years, many authors concentrated on the issue of heterogeneity of consumers. In particular, they considered the presence of a fraction of consumers which are liquidity constrained, so that the Ricardian equivalence does not hold anymore. This feature brought different results on the dynamics of the economic system with respect to the standard framework. For example, Galí, Lòpez-Salido and Vallés (2007) demonstrate that the presence of liquidity constrained consumers can explain the fact that consumption rises in response to an increase in government spending. Bilbiie (2008) and Di Bartolomeo and Rossi (2007) show that limited asset market participation can lead to an inverted aggregate demand logic (the IS curve has a positive slope). Galí, Lòpez-Salido and Vallés (2004) and Bilbiie (2008) found that in such a context, the so called ‘Taylor principle’ could be also inverted, thus suggesting the relevance of taking into account liquidity constrained consumers in the conduct of monetary policy. Moreover, the estimates of Campbell and Mankiw (1989) pointed to the presence of a significant fraction of this type of consumers, which may increase of importance after financial crises.

This study is focused on the strategic interactions between monetary and fiscal policy in a standard dynamic stochastic general equilibrium model without capital, where distortions are due to monopolistic competition and price stickiness *à la* Rotemberg (1982) and there is limited asset market participation. After considering the Ramsey problem, we build up a strategic game where there are two policy authorities which are independent and do not cooperate, following Adam and Billi (2008). On the contrary, most of the literature which studies the behavior of fiscal and monetary policy assumes that they are both driven by a unique authority. This is clearly not the case nowadays and in particular in the EU context, where the creation of the currency area led to a structure with a unique monetary authority and several independent fiscal authorities. In such a context it is then relevant to investigate the strategic interactions between the Central Bank and the fiscal authorities, as done by Gnocchi (2008), Adam and Billi (2008), Beetsma and Jensen (2007) and many others.

We consider the closed economy model of Adam and Billi (2008) but we allow for hetero-

geneity of households, as done by Galí, Lòpez-Salido and Vallés (2004). In their model two different kinds of households coexist: ‘assets holders’ or ‘Ricardians’ who are forward looking, smooth consumption and are able to trade in all markets for state-contingent securities; and ‘liquidity constrained’ consumers who spend all their current disposable labor income for consumption at time t . This is an important topic because of its empirical relevance and because it has been shown that it alters the standard results on policy design.

The literature on liquidity constrained consumers arose from the need to make fiscal policy have significant effects on the economy. In fact, fiscal policy does not affect real variables since only Ricardian consumers are taken into account so that the Ricardian equivalence holds. For example Galí, Lòpez-Salido and Vallés (2007) and Colciago (2007) model public spending as an exogenous variable and consider fiscal policy rules. Other papers concentrate on monetary policy stance as Galí, Lòpez-Salido and Vallés (2004), Billie (2008), Di Bartolomeo and Rossi (2007). An optimizing framework of policy design with fiscal and monetary policy strategic interaction and households heterogeneity would be new in the literature, up to our knowledge. We will show that this framework can lead to relevant and different results in comparison with the standard models.

We draw on Adam and Billi (2008) but, after calculating the steady state, we analyze the dynamics of our model in comparison with their’s, observing the optimal response of policy variables in face of two kind of shocks (to technology and mark-up). We note that the presence of ‘liquidity constrained’ consumers can alter the reactions of the policy authorities and in particular what is remarkable is that the role of fiscal policy is more relevant, given that it can affect liquidity constrained consumption. This happens mainly in response to a negative mark-up shock. In fact, in face of such a shock, our model calls for a reduction of the interest rate and a rise of public spending if there is a unique committed policy authority (instead of a restrictive monetary policy and a less expansive fiscal policy). When the policy authorities are independent, not committed and play simultaneously, together with an expansive monetary policy, public spending should be reduced significantly when there is limited asset market participation. Moreover, when the monetary authority only cares about inflation and the fiscal authority moves first,

the monetary authority should be restrictive while an expansive fiscal policy should be used more with respect to the model of Adam and Billi (2008) to face the mark-up shock. These are the main results derived from this analysis. Optimal policy reactions change when there is limited asset market participation and the more are liquidity constrained consumers the more these responses are different. The relevance of this study lies in the importance assigned to the presence of limited asset market participation. In fact we think that liquidity constrained consumers may have assumed an increasingly relevant role in the economy, due to the recent financial crisis, because of the worsening of the conditions of access to financial markets. This work provides policy indications for different policy structures: policymakers should first realize which is the policy structure which best describes their economy and secondly carry out the optimal policy needed to cope with a particular shock.

In the next section we introduce the model, section 3 presents the policy problem and the different game structures, in section 4 we show the results from the welfare analysis and section 5 concludes.

2 The model

2.1 Households

The model economy consists of a continuum of infinitely-lived households. Households are divided into a fraction $1 - \lambda$ of ‘Ricardians’ who smooth consumption and have access to assets markets; the remaining fraction λ are the so called ‘liquidity constrained’ consumers who have no assets and spend all their current disposable labor income for consumption each period. Both types of households have the same preferences structure. The utility functions for Ricardians and rule for thumb consumers are then respectively:

$$u(C_t^o, N_t^o, G_t) = \frac{C_t^{o1-\sigma}}{1-\sigma} - \omega_n \frac{N_t^{o1+\varphi}}{1+\varphi} + \omega_g \frac{G_t^{1-\sigma}}{1-\sigma} \quad (1)$$

$$u(C_t^r, N_t^r, G_t) = \frac{C_t^{r1-\sigma}}{1-\sigma} - \omega_n \frac{N_t^{r1+\varphi}}{1+\varphi} + \omega_g \frac{G_t^{1-\sigma}}{1-\sigma} , \quad (2)$$

where C_t^o, N_t^o are Ricardian consumer's consumption and hours worked, C_t^r, N_t^r are liquidity constrained consumer's consumption and hours worked and G_t is public expenditure. Utility is separable in C, N, G and $U_c > 0, U_{cc} < 0, U_n < 0, U_{nn} \leq 0, U_g > 0, U_{gg} < 0$.

Ricardians' budget constraint is:

$$P_t C_t^o + \frac{B_t}{1-\lambda} = R_{t-1} \frac{B_{t-1}}{1-\lambda} + P_t w_t N_t^o - P_t T_t^o + \frac{D_t}{1-\lambda}, \quad (3)$$

where P_t is the nominal price index, R_t is the gross nominal interest rate, B_t are nominal bonds purchased in t and maturing in $t+1$, w_t is the real wage paid in a competitive labor market, T_t^o are lump sum taxes and D_t are profits of monopolistic firms.

The Ricardians' problem consists of choosing $\{C_t^o, N_t^o, B_t\}_{t=0}^{\infty}$ to maximize $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^o, N_t^o, G_t)$ subject to (3), taking as given $\{P_t, w_t, R_t, G_t, T_t, D_t\}^1$. The first order conditions are then:

$$\frac{\omega_n N_t^{o\varphi}}{C_t^{o-\sigma}} = w_t \quad (4)$$

$$\frac{C_t^{o-\sigma}}{R_t} = \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}}. \quad (5)$$

Liquidity constrained consumers each period solve a static problem: they maximize their period utility (2) subject to the constraint that all their labor income is consumed:

$$P_t C_t^r = P_t w_t N_t^r - P_t T_t^r. \quad (6)$$

From the first order conditions we get:

$$\frac{\omega_n N_t^{r\varphi}}{C_t^{r-\sigma}} = w_t. \quad (7)$$

Note that there is a common labor market, so that the real wage is the same for both consumers. This leads to the following condition:

$$\frac{\omega_n N_t^{o\varphi}}{C_t^{o-\sigma}} = \frac{\omega_n N_t^{r\varphi}}{C_t^{r-\sigma}}, \quad (8)$$

¹The no-Ponzi scheme constraint $\lim_{j \rightarrow \infty} E_t \prod_{i=0}^{t+j-1} \frac{1}{R_t} B_{t+j} \geq 0$ and the transversality condition $\lim_{j \rightarrow \infty} E_t \beta^{t+j} C_{t+j}^{o-\sigma} B_{t+j} / P_{t+j} = 0$ hold.

which equals the ratio between the marginal utilities of Ricardian and liquidity constrained consumers respectively.

The aggregate consumption and hours worked are defined as follows:

$$C_t = \lambda C_t^r + (1 - \lambda)C_t^o \quad (9)$$

$$N_t = \lambda N_t^r + (1 - \lambda)N_t^o . \quad (10)$$

2.2 Firms

There is a continuum of intermediate goods, indexed by $i \in [0, 1]$ and a sector of final good which uses the following technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} . \quad (11)$$

The sector of final good operates in perfect competition. Then profit maximization implies $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t$, where ϵ_t represents the elasticity of substitution across varieties and is assumed to be an AR(1) process $\log(\epsilon_t/\epsilon) = \rho_\epsilon \log(\epsilon_{t-1}/\epsilon) + s_t^\epsilon$, with $0 < \rho_\epsilon < 1$ and s_t^ϵ normally distributed innovation with zero mean and standard deviation σ_ϵ . ϵ_t is time-varying, thus induces fluctuations in the monopolistic mark-up charged by firms. P_t is defined as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1 - \epsilon_t} di \right]^{\frac{1}{1 - \epsilon_t}} . \quad (12)$$

The intermediate good sector is characterized by firms producing each a differentiated good with a technology represented by a Cobb-Douglas production function with a unique factor of production (aggregate labor) and constant returns to scale:

$$Y_t(i) = Z_t N_t(i) , \quad (13)$$

where $\log(Z_t/Z) = z_t$ is an aggregate productivity shock with AR(1) process:

$$z_t = \rho_z z_{t-1} + s_t^z . \quad (14)$$

$0 < \rho_z < 1$ and s_t^z is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_z .

In this context each firm i has monopolistic power in the production of its own good and therefore it sets the price. Prices are sticky *à la* Rotemberg (1982) so that firms face quadratic resource costs for adjusting nominal prices according to:

$$\frac{\theta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2, \quad (15)$$

where θ is the degree of price rigidities.

The problem of the firm is then to choose $\{P_t(i), N_t(i)\}_{t=0}^{\infty}$ to maximize the sum of expected discounted profits:

$$\begin{aligned} \max_{N_t(i), P_t(i)} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\gamma_t}{\gamma_0} \left\{ \frac{P_t(i)}{P_t} Y_t(i) - w_t N_t(i) - \frac{\theta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 \right\} \\ \text{s.t. } Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t = Z_t N_t(i), \end{aligned} \quad (16)$$

where $Y_t = C_t + G_t$ and $\gamma_t = C_t^{o-\sigma}$.

In equilibrium all firms will charge the same price, so that we can assume symmetry. After defining mc_t as the real marginal cost, the first order condition are:

$$w_t = mc_t Z_t \quad (17)$$

$$0 = [1 - (1 - mc_t)\epsilon_t] Y_t - \theta(\pi_t - 1)\pi_t + \theta\beta E_t \left(\frac{C_{t+1}^o}{C_t^{o-\sigma}} \right) (\pi_{t+1} - 1)\pi_{t+1}. \quad (18)$$

Combining (17) with (4) and (7) yields to such an expression for the real marginal cost:

$$mc_t = \frac{1}{Z_t} (\lambda\omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda)\omega_n N_t^{o\varphi} C_t^{o\sigma}). \quad (19)$$

Then we combine it with (18) and get:

$$\begin{aligned} C_t^{o-\sigma} (\pi_t - 1)\pi_t = \left[1 - \left(1 - \frac{\lambda\omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda)\omega_n N_t^{o\varphi} C_t^{o\sigma}}{Z_t} \right) \epsilon_t \right] \frac{Z_t N_t C_t^{o-\sigma}}{\theta} \\ + \beta E_t u_{ct+1}^o (\pi_{t+1} - 1)\pi_{t+1}. \end{aligned} \quad (20)$$

2.3 Government

The government is composed by a monetary authority which sets the nominal interest rate R_t and a fiscal authority which determines the level of public expenditure G_t . We assume a balanced budget requirement, supposing public consumption equals lump sum taxes.

$$P_t G_t = P_t T_t . \quad (21)$$

Defining aggregate lump sum taxes as $T_t = \lambda T_t^r + (1 - \lambda)T_t^o$, if the same amount of lump sum taxes is withdrawn from each individual ($T_t^r = T_t^o$), we obtain $G_t = T_t = T_t^r = T_t^o$.

2.4 Equilibrium

To close the model we consider also the goods market clearing condition

$$Z_t[\lambda N_t^r + (1 - \lambda)N_t^o] = \lambda C_t^r + (1 - \lambda)C_t^o + G_t + \frac{\theta}{2}(\pi_t - 1)^2 . \quad (22)$$

A rational expectations equilibrium for the private sector consists of a plan $\{C_t^r, C_t^o, N_t^r, N_t^o, P_t\}$ satisfying (5), (6), (8), (20) and (22), given the policies $\{G_t, T_t, R_t \geq 1\}$, the exogenous processes ϵ_t, Z_t and the initial conditions $\{R_{-1}B_{-1}, P_{-1}\}$.

2.5 Calibration

As a baseline calibration for the following exercise, we follow Adam and Billi (2008). We set $\beta = 0.9913$; the degree of price stickiness θ is 17.5; the intertemporal elasticity of substitution and labor supply elasticity are set to 1. We keep Adam and Billi (2008)'s values for the utility weights ($\omega_n = 26.042$ and $\omega_g = 0.227$) and set the steady state values for Z_t and ϵ_t to 1 and 6 respectively, in line with the literature. The fraction of liquidity constrained consumers is calibrated to be 0.5, following the estimates of Campbell and Mankiw (1989) and Galí, Lòpez-Salido and Vallés (2004). Finally, we set the coefficients of persistence of the shocks to 0.9 and the standard deviations to 0.01.

3 The policy problem

Following Adam and Billi (2008), we suppose that policymakers cannot commit but decide about policies period by period. This behavior is called sequential decision making and is suboptimal because it fails to fully internalize the welfare cost of generating inflation. In fact policymakers are tempted to move output closer to its first-best level but they neglect that the private sector is forward looking in its price decisions and rationally anticipates current inflation.

3.1 The social planner

We start analyzing the first-best allocation, where we omit monopolistic distortions and nominal rigidities. The social planner problem is then to solve:

$$\begin{aligned} \max_{C_t^r, N_t^r, C_t^o, N_t^o, G_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} \\ \text{s.t. } Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] = \lambda C_t^r + (1 - \lambda) C_t^o + G_t \end{aligned} \quad (23)$$

From the first order conditions we obtain:

$$C_t^{r-\sigma} = C_t^{o-\sigma} = \omega_g G_t^{-\sigma} = -\frac{\omega_n N_t^{r\varphi}}{Z_t} = -\frac{\omega_n N_t^{o\varphi}}{Z_t} \quad (24)$$

which in steady state leads to:

$$C^{r-\sigma} = C^{o-\sigma} = \omega_g G^{-\sigma} = \omega_n N^{r\varphi} = \omega_n N^{o\varphi} \quad (25)$$

given that the steady state value of Z_t is 1 (from now on, letters without time subscript represent steady state values). In a way analogous to Adam and Billi (2008), this shows that at a first-best level it is optimal to equate the marginal utility of private and public consumption to the marginal disutility of labor effort. Moreover, the marginal utility of private consumption and the marginal disutility of labor effort are equal for both types of consumers.

3.2 The Ramsey problem

We look now at the Ramsey problem, where the policy authorities fully cooperate and can commit, which means that policymakers determine state-contingent future policies at time zero. The Ramsey allocation also takes into account the presence of sticky prices and monopolistic distortions. The problem is then to solve:

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t, G_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} \\ & \text{s.t. (5), (6), (8), (20), (21), (22) for all } t . \end{aligned} \quad (26)$$

3.2.1 Ramsey steady state

From the first order conditions we derive that the value of π_t in steady state is 1, which imply price stability. Combining this result with the Euler equation we find that $R = 1/\beta$. Imposing the steady state also delivers the following result from (20):

$$w = \left[\lambda \frac{\omega_n N^{r\varphi}}{C^{r-\sigma}} + (1 - \lambda) \frac{\omega_n N^{o\varphi}}{C^{o-\sigma}} \right] = \frac{\epsilon - 1}{\epsilon} \quad (27)$$

In steady state the real wage is constant with respect to the fraction of rule of thumb consumers. Given the technological constraints, the real wage in steady state also corresponds to the real marginal costs, as stated by (17), resembling the equilibrium result under flexible prices, where steady state real marginal costs equal the inverse of the desired mark-up.

Given the complexity of the model, the steady state values of the other variables are obtained through numerical methods, after calibrating parameters. An interesting result is that these steady state values depend on the fraction of liquidity constrained consumers λ . Table 1 resumes steady state values. Our model nests Adam and Billi (2008) model for $\lambda = 0$. This allow us to compare our results to theirs.

We note that C^o is an increasing function of λ and the reason is that when λ increases, the fraction of Ricardians decreases so that per capita profits $D/(1 - \lambda)$ rise, boosting per capita Ricardian consumption. Liquidity constrained consumption slightly increases as λ becomes greater than 0.3 due to a small reduction of G . In fact, given that

$G = T = T^o = T^r$, from (6) we obtain $C^r = wN^r - G$. It is easy to understand that the more than proportional decrease in G with respect to N^r causes C^r to rise, since the steady state value of the real wage is constant. From the policy authority point of view, it is optimal to reduce public spending to maximize welfare when λ increases, because it rises C^r . At the same time both liquidity constrained and Ricardians hours worked decrease, but N^o reduction is more consistent. This leads to a rise of aggregate labor, given that the fraction of liquidity constrained consumers increases, which finally pushes also output up.

3.2.2 Ramsey dynamics

We analyze the model dynamics in the case of Ramsey optimum through impulse response functions. Figure 1 shows the effects of a positive technology shock. When a positive technology shock occurs ($\sigma_t^z = 0.01$), the nominal rate of interest falls and public expenditure rises on impact. Policymakers accommodate the shock to boost the economy by reducing nominal interest rates and raising public expenditure. Thus inflation remains around its steady state (the shock would imply a fall in inflation but the expansive monetary policy works reversely, through a raise of aggregate demand). In this case the authorities commit so that they are completely credible; this is why they can maintain price stability in each period. Labor is not affected by this type of shock, while the real wage and output jump remarkably at time zero. A rise in productivity moves the demand of labor up but the increase of employment is offset by the expansive policy, which pushes Ricardian consumption up; this in turn shifts the labor supply, so that real wages go up while employment remains unchanged. Consumption jumps by the same amount for both consumers. In fact, the real interest rate falls on impact causing Ricardian households to consume more. At the same time liquidity constrained consumption is affected by the fact that real wages react more than public consumption, boosting private consumption. These characteristics are pretty much the same in Adam and Billi (2008) IRFs.

The effects generated by a negative mark-up shock ($\sigma_t^\xi = 0.01$) are more relevant. As shown in figure 2, on impact inflation and the nominal rate of interest drop to increase in the following periods but overshooting the equilibrium values and public spending

jumps. Aggregate consumption and real wages fall slightly while aggregate labor (and consequently output) are almost unchanged on impact. All these variables have an hump-shaped path of adjustment. Liquidity constrained consumption follows a path similar to aggregate consumption while Ricardian consumption has a reversed hump-shaped IRF. N_t^r jumps and then follows a reversed hump-shaped path undershooting its steady state value while N_t^o fall slightly on impact. With a negative mark-up shock, there is a relative pressure on firms' profits which are expected to diminish. This has a negative effect on Ricardian consumption through (3). Monetary policymakers react optimally slightly reducing the interest rate to sustain welfare. Public expenditure rises and, together with real wage initial drop, make C_t^r fall; in the remaining periods the dynamics of C_t^r is determined by that of wages.

There are great differences in comparison with Adam and Billi (2008), with respect to policy optimal reactions. This happens for a big enough value of λ . For $0 \leq \lambda < 0.42$ the nominal interest rate jumps, while when $\lambda \geq 0.42$ the optimal response of policy is to reduce the interest rate. For higher values of λ , real wages are increasingly negatively affected on impact, which dampens the response of consumption. For this reason, the higher is the fraction of liquidity constrained consumers the more the interest rate will be reduced on impact, causing Ricardian consumption to decrease less or even go up for expansive enough monetary policy and Ricardian hours worked to reduce. Moreover, the fact that inflation reduces (more than in Adam and Billi (2008) model) after such a shock leaves room for the expansive monetary policy. At the same time, the impact on public spending is significantly higher than in Adam and Billi (2008) and the effect of the shock is reabsorbed after more periods. Thus, an increasing boost to public expenditure and a greater reduction in interest rates is needed to sustain welfare. In turn, the higher G_t makes liquidity constrained consumers consume less and work more but this effect is counterbalanced by the fact that G_t increases welfare directly and a lower R_t induces Ricardians to consume more.

Result 1. With a committed policymaker, the presence of liquidity constrained con-

sumers induces a different optimal response of policy in face of a negative mark-up shock, calling for a fall of nominal interest rates and a significant rise in public spending.

3.3 The policy game

In this section we impose the game structure to the policy decision-making. Monetary and fiscal policy are independent and cannot commit so that their decisions on the nominal interest rate and public expenditure are taken period by period (discretionally).

3.3.1 Nash game

We start with the case where the policy authorities decide simultaneously at the time of implementation. Moreover, each policymaker takes as given the current policy choice of the other, all future policies and future private-sector choices.

The problem of the fiscal authority is:

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, G_t} E_t \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} \\ & \text{s.t. (5), (6), (8), (20), (21), (22) for all } t \\ & \{ C_{t+j}^r, C_{t+j}^o, N_{t+j}^r, N_{t+j}^o, \pi_{t+j}, R_{t+j-1} \geq 1, G_{t+j} \} \text{ given for } j \geq 1 . \end{aligned} \quad (28)$$

The set of first order conditions define the behavior of the fiscal policymaker and thus, implicitly its reaction function.

Analogously, the monetary authority solves the following problem:

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t} E_t \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} \\ & \text{s.t. (5), (6), (8), (20), (21), (22) for all } t \\ & \{ C_{t+j}^r, C_{t+j}^o, N_{t+j}^r, N_{t+j}^o, \pi_{t+j}, R_{t+j} \geq 1, G_{t+j-1} \} \text{ given for } j \geq 1 . \end{aligned} \quad (29)$$

Calculating the Nash equilibrium with sequential monetary and fiscal policy involves to solve the first order conditions of both policy authorities and the constraints imposed to obtain the policy functions $C^r\{Z_t, \epsilon_t\}$, $C^o\{Z_t, \epsilon_t\}$, $N^r\{Z_t, \epsilon_t\}$, $N^o\{Z_t, \epsilon_t\}$, $\pi\{Z_t, \epsilon_t\}$, $R\{Z_t, \epsilon_t\}$, $G\{Z_t, \epsilon_t\}$.

3.3.2 Nash steady state

We find the steady state of the Nash game through numerical methods. In table 2 these values are grouped.

As pointed out by Adam and Billi (2008), when the policy authorities play simultaneously and under discretion there is an important inflation bias with respect to the Ramsey steady state, which increases as λ increases. The intuition is that inflation is an implicit way to tax Ricardians, whose per capita consumption increases due to a rise of per capita profits $D/(1 - \lambda)$. This inflation bias affects significantly per capita SS profits; in fact this variable tends to increase for λ increasing but for $\lambda > 0.5$ this tendency breaks off and, in particular, per capita SS profits become even negative ($\lambda = 0.7$) because of a too high SS inflation which makes the cost of adjusting prices overcome $Y - wN$. Thus, the rise of Ricardian consumption is dampened in favor of liquidity constrained consumption. Moreover, we also find a government spending bias, which is more important the higher is the fraction of liquidity constrained consumers, but for $\lambda = 0.7$ it reabsorbs.

3.3.3 Nash dynamics

Analyzing the dynamics of the simultaneous game, some differences emerge with respect to the Ramsey problem. With regard to the technology shock (figure 3), we note that the fact that policies do not cooperate produces a rise in inflation. As a matter of fact inflation reaction comes from the fact that the policy game implies sequential policymaking. The monetary policy is not forward looking, it decides period by period and thus generates an inflation bias: the authority is tempted to stimulate demand by lowering interest rates and the additional demand is satisfied by an increase of real wages, which in turn drives inflation up. The nominal interest rate reduction is reflected on C_t^o which jumps. Liquidity constrained labor jumps at impact while Ricardian and aggregate labor fall, in line with the findings of Galí and Rabanal (2004).

The IRFs relative to the mark-up shock are represented in figure 4 and are significantly different from those of the Ramsey problem: they lose the hump-shaped path. A negative mark-up shock affects primarily prices (negatively) and marginal costs (positively). In

fact inflation falls and real wages, which in our model correspond to real marginal costs, rise. The fiscal policy responds by reducing G_t on impact while monetary policymaker decrease the interest rate. An interesting feature of these IRFs is that as long as the private sector is forward looking, it is optimal for a policymaker that is committed (as in Ramsey) to induce a period of inflation following a deflationary shock. In other words it is optimal in these cases to have the inflation rate overshoot its SS: if the private sector understands that the policymaker will act in this way, the future inflation will be incorporated into the private sector's current deflationary expectations. The response of private sector deflationary expectation to a deflationary shock will therefore be less pronounced than it otherwise would have been, which in turn yields less actual deflation in the period of the shock. In contrast, a monetary policymaker which optimizes under discretion (as in the Nash game) cannot take advantage of this effect, since the private sector understands that it will go back on the commitment of rising inflation once the initial reaction to the shock has passed. This inability to carry out earlier commitments lies at the heart of the sub-optimality of discretionary optimization and are clearly detected from inflation IRF. With respect to Adam and Billi (2008) model the effects on policy variables are qualitatively equal, but quantitatively very different. The presence of liquidity constrained consumers ($\lambda = 0.5$) involve a greater reaction of policy decision variables and consequently a greater impact on private sector variables (with the exception of the real wage). Liquidity constrained consumption rises due to the huge reduction of public spending and the expansive monetary policy pushes C_t^o up. This puts pressure on the reduction of hours worked and consequently of output. In particular, as λ increases the stronger reactions of policymakers cause consumption of both types of consumers to rise more and consequently labor supplies to decrease. When $\lambda = 0$ the impact effect on hours and output is positive: the greater increase of real wages and the lower rise of consumption cause labor supply to augment.

Moreover, if we increase the value of λ , we find that public spending jumps on impact instead of reducing. This happens when $\lambda \geq 0.62$. Our intuition about this is that when the fraction of liquidity constrained consumers becomes relevant in the economy, optimal policy implies a sort of social cost to pay in order to maximize welfare. In fact the rise

of G induces a lower response of C^r . At the same time, as λ increases monetary policy becomes more expansive, thus boosting Ricardian consumption. Overall, when the policy authorities face a mark-up shock the rise of the fraction of liquidity constrained consumers causes a redistribution of consumption from liquidity constrained to Ricardians.

Result 2. The presence of liquidity constrained consumers alters quantitatively the reaction of policies when they are independent and play sequentially and simultaneously. After a negative mark-up shock, interest rates have to reduce more while a larger decrease of public consumption is needed. This also leads to a drop of hours worked and output.

In the case where the policy authorities have the same utility function, a game structure such as a Stackelberg game does not matter on equilibrium and dynamics because constraints are the same and are always satisfied. The Stackelberg structure becomes relevant when the utility function of the monetary or the fiscal authority changes. This is what Adam and Billi (2008) suppose in their study. We follow their approach, generalizing their analysis allowing for the presence of liquidity constrained consumers.

3.3.4 Nash game with conservative monetary policy

In this section we suppose that the monetary authority is more inflation averse than society. The utility function of monetary policy becomes then:

$$(1 - \alpha) \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} - \alpha \frac{(\pi_t - 1)^2}{2} \quad (30)$$

where $\alpha \in [0, 1]$ is a measure of monetary conservatism. $\alpha > 0$ means that the monetary authority dislikes inflation more than society and when $\alpha = 1$ the policymaker is fully conservative, so that it only cares about inflation. We may think that this assumption is closer to the ECB's mandate of maintaining price stability. The preferences of the fiscal policymaker remain unchanged.

In this context the timing of the policy moves has an influence on the equilibrium outcome. It is not clear in practice if the authorities act simultaneously or if one of two

moves first, given also that the time of implementation and effectiveness of policies are different.

First, we consider the case of simultaneous decisions. The fiscal problem remains (28), while the monetary authority now solves the following:

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t} E_t \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \alpha) [\lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t)] - \alpha \frac{(\pi_t - 1)^2}{2} \right\} \\ & \text{s.t. (5), (6), (8), (20), (21), (22) for all } t \\ & \{C_{t+j}^r, C_{t+j}^o, N_{t+j}^r, N_{t+j}^o, \pi_{t+j}, R_{t+j} \geq 1, G_{t+j-1}\} \text{ given for } j \geq 1 . \end{aligned} \tag{31}$$

Finding the Nash equilibrium consists of solving the system of the constraints and the first order conditions of fiscal policy and conservative monetary authority.

As before we calculate the steady state values. In this case we may have different values depending on the calibration of λ and α . For brevity, we do not report all the possible combinations, but only the SS values with $\alpha = 0.5$ and λ varying. Actually the presence of a conservative monetary policy ($\alpha = 0.5$) is not so important in determining the SS values: comparing them to those of the Nash equilibrium without conservative monetary policy we can not identify differences before the forth decimal digit or more. Moreover, other values of $\alpha \in (0, 1)$ does not change significantly the steady state. Setting α to 0.5 (so that the monetary authority assigns an equal weight to consumers utility and to inflation stability), the presence of a fraction of liquidity constrained consumers has a certain effect on the SS values (see table 3). Given the (un)importance of α , these values vary in a way very similar to the Nash game without conservative monetary policy. With regard to the dynamics we report IRFs for $\alpha = 0.5$ in figures 5 and 6. We note that variables react to both shocks in a way almost coincident with that of the Nash game of the previous section. As such, in response to a technology shock G_t jumps and R_t drops, determining a higher rise of inflation with respect to Adam and Billi (2008). A mark-up shock causes a greater reaction of policy variables and inflation and lower effect on the real wage, while aggregate labor and output drop instead of rising on impact.

3.3.5 Stackelberg game with conservative monetary policy

Now we consider the situation where the game between the two authorities has a Stackelberg structure. In particular we concentrate on the case in which the fiscal policymaker is the leader while the monetary policymaker is the follower. This means that fiscal policy is determined before monetary policy and thus has to take into account the conservative monetary policy reaction function, which consists of the first order conditions of (31). The fiscal authority's policy problem at time t is thus given by

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t, G_t} E_t \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \} \\ & \text{s.t. (5), (6), (8), (20), (21), (22) FOCs of (31) for all } t \\ & \{ C_{t+j}^r, C_{t+j}^o, N_{t+j}^r, N_{t+j}^o, \pi_{t+j}, R_{t+j} \geq 1, G_{t+j} \} \text{ given for } j \geq 1 . \end{aligned} \tag{32}$$

We find the equilibrium solving for constraints and first order conditions of (32).

3.3.6 Fiscal leader steady state

Calculating the steady state, we observe that the fiscal leadership changes only marginally the SS values with respect to the Nash case when $\alpha = 0.5$. For this reason we do not comment on them. More interesting is the case where $\alpha = 1$, which means that the monetary authority only cares about inflation. In this case the fiscal leadership leads to the Ramsey steady state. This is the same result obtained in Adam and Billi (2008). The fiscal authority takes into account that the monetary policymaker is determined to achieve price stability at all costs, so that if there is a fiscal expansion it will rise interest rate to contain inflationary pressures. The fiscal policymaker benefits of the first move and therefore can internalize this effect, leading to the Ramsey steady state. This also imply that the welfare losses are minimized, as we will show in the next section.

3.3.7 Fiscal leader dynamics

For dynamics we can first observe figures 7 and 8. With $\alpha = 0.5$, there are no significant differences with the Nash game and this is also true for Adam and Billi (2008) model. Therefore, we can assert that nor the presence of a conservative monetary policy nor the type of game played have great influence on the dynamics; still, what is more relevant is the presence of liquidity constrained consumers but in a way very similar to Nash. Instead, when $\alpha = 1$ figure 9 shows that a positive technology shock leads to the Ramsey result. With respect to the non-completely conservative monetary policy, the monetary policymaker reacts with a slightly stronger reduction in interest rates while fiscal policy behaves in the same way. It is interesting to note that the presence of liquidity constrained consumers when $\alpha = 1$ induces a different response of policy in face of a mark-up shock with respect to Adam and Billi (2008) model: in figure 10, G_t rises on impact more than in Adam and Billi (2008) model. In the Nash games public spending went down on impact with both Adam and Billi (2008) and our model, even if with the presence of liquidity constrained consumers the effect was much greater. Moreover, the monetary authority slightly raises the interest rate, while Adam and Billi (2008) model points to a significant reduction. The intuition is that in Adam and Billi (2008) model policymakers look at aggregate variables so that the reduction in interest rates and the small rise of public spending are enough to sustain consumption and welfare, while in our model policymakers look at disaggregate variables: the rise of the interest rate concerns only Ricardians, whose consumption is thus slightly reduced; the fiscal expansion is then greater in order to maximize utility. This is possible because liquidity constrained consumption is supported by the huge increase of real wages (as found by Galí, Lòpez-Salido and Vallés (2007)) so that C_t^r response does not change much and G_t can be used to boost welfare without affecting liquidity constrained consumption. Aggregate hours worked change the direction of impact with respect to the previous games and this is because Ricardian labor rises. It is worth noting that when the monetary policy is fully conservative, price stability is achieved in each period (inflation is not affected by the shock, which is completely reflected in a rise of wages).

By varying the value of λ when $\alpha = 1$ we observe that monetary policy changes reaction when $\lambda \geq 0.49$ making Ricardians consumption fall. With the value of λ increasing monetary policy becomes more and more restrictive, aggregate consumption and labor jump less, while public spending is increasingly used to cope with a negative mark-up shock.

Result 3. When the monetary authority only cares about inflation ($\alpha = 1$), the presence of liquidity constrained consumers in a fiscal leader structure subject to a negative mark-up shock leads to different optimal responses of policies with respect to the standard model of Adam and Billi (2008). In particular G_t rises more on impact and the interest rate is raised instead of reduced for high enough values of λ . This means that fiscal policy is used more.

4 Welfare analysis

We now show a measure for the utility losses associated to a particular game structure following Schmitt-Grohé and Uribe (2006) and Adam and Billi (2008). We calculate the percent loss of each game structure with respect to the Ramsey deterministic steady state. Denote $V^{DSS} = \lambda u(C^r, N^r, G) + (1 - \lambda)u(C^o, N^o, G)$ the period utility for the Ramsey deterministic steady state and V^A the stochastic steady state of the value function of an alternative policy regime. The permanent reduction in private consumption, $\mu^A \leq 0$ (supposing to withdraw the same amount from each type of consumer), that would imply the Ramsey deterministic steady state to be welfare equivalent to the alternative policy regime can be found solving for μ^A the following expression:

$$V^A = \frac{1}{1 - \beta} \left[\lambda (\log(C^r(1 + \mu^A)) - \omega_n \frac{N^{r1+\varphi}}{1 + \varphi} + \omega_g \log G) \right. \\ \left. + (1 - \lambda) (\log(C^o(1 + \mu^A)) - \omega_n \frac{N^{o1+\varphi}}{1 + \varphi} + \omega_g \log G) \right]. \quad (33)$$

Therefore, we obtain:

$$\mu^A = \exp\{(1 - \beta)(V^A - V^{DSS})\} - 1 . \quad (34)$$

Table 6 shows the welfare losses in percentage terms resulting from Adam and Billi (2008) model and the model with liquidity constrained consumers for each policy structure. What we note is that in both cases the simple Nash equilibrium leads to a welfare loss which is bigger than the cases where the monetary policy is conservative about inflation. Secondly, while with Adam and Billi (2008) model there is no difference in losses between the Nash equilibrium with conservative monetary policy and the fiscal leadership with conservative monetary policy when $\alpha = 0.5$, the welfare losses with our model are greater with the fiscal leadership. Finally, the fiscal leader structure with $\alpha = 1$ minimizes the deviation from Ramsey allocations, as we could imagine due to the fact that the Ramsey steady state is reached.

5 Conclusions

In this study we investigated the effects of the presence of a fraction of consumers who cannot smooth consumption and have no access to state-contingent markets nor receive dividends, on policy optimal dynamic responses. We compare our results to the the dynamic responses of the model of Adam and Billi (2008), which does not allow for the presence of ‘liquidity constrained’ consumers. We concentrated on different structures for policy decision making. After considering the Ramsey problem, we skip to game structures, assuming that there are two independent authorities which can play simultaneously or not. The fiscal policy decides on the amount of public expenditure while the monetary authority decides on the level of the nominal interest rate. For each problem we calculated the steady state and then analyzed the dynamics when the model was shocked by a technological and a mark-up shock.

The main result is that the presence of liquidity constrained consumers alters at least quantitatively the optimal response of policies after a negative mark-up shock. When there are liquidity constrained consumers, the optimal responses of the fiscal policy call for a bigger contraction of public spending with sequential policymaking while a higher

fiscal expansion is needed under commitment or with fully conservative monetary authority. With regard to the optimal responses of the monetary policy, the presence of liquidity constrained consumers leads to a reduction of the nominal interest rate, which is more pronounced when policies play under discretion, and to a small rise when the monetary authority is fully conservative and the fiscal policy has the advantage of the first move (while without liquidity constrained consumers in this case the monetary policy is expansive). It is important to note that monetary policy changes response for $\lambda \geq 0.42$ under commitment: while Adam and Billi (2008) model points to a rise of the nominal interest rate, in our model there is a monetary expansion. Moreover, for values of λ higher than 0.49, the fiscal leader structure with fully conservative monetary policy also leads to a different result with respect to the benchmark model, that is to a monetary restriction. When the policy authorities are focused on maximizing social welfare, a reduction in public spending is often needed in order to support liquidity constrained consumption (which would be dampened if not); at the same time, the monetary authority is mainly concerned on sustaining Ricardian consumption. A policy authority which maximizes individuals' utility function has to take into account both the movements in consumption of both consumers and public expenditure, which in turn has a negative effect on liquidity constrained consumption, given that it is set equal to the amount of taxes levied. Precisely, the fiscal policy cannot raise G_t without taking into account that this has a negative effect on liquidity constrained consumption. These are the main channels through which the presence of a limited asset market participation influences the economy and thus the optimal reactions of the two policy authorities: the higher is λ the more public spending reacts.

Moreover, optimal policy responses change direction and intensity with respect to the type of game played. When the policy authorities act independently, simultaneously and under discretion, their optimal reactions are greater on impact because they cannot internalize neither private sector expectations nor the other policymaker's action. With the fiscal leader structure with fully conservative monetary policy, the sole purpose of the monetary authority is to stabilize inflation, without taking into account individuals' utility; this is reflected on a weak response of the interest rate, which makes private sector expectations

on inflation to be zero, thus achieving price stability in each period. Besides, in this case fiscal policy can internalize the reaction of monetary policy, minimizing welfare losses as pointed out in section 4.

We think this paper could give interesting insights on how economic policy should be run, depending on how the structure of policy authorities is and taking into account the presence of limited asset market participation, which may have increased recently due to the financial crisis. Given that the presence of liquidity constrained consumers makes the role of fiscal policy become more relevant, policymakers should probably rely more on fiscal policy in periods of recession, in particular if the downturn originated from a financial crisis, which is often associated with a worsening of the conditions of access to financial markets.

Further developments of this study include the possibility of considering different fiscal structures, given that the balanced budget requirement is a very simplifying assumption and it is not always such a proper description of a country's fiscal structure. First, we would like to allow for different amounts of lump sum taxes to be levied on the different types of consumers; secondly, we could consider the case of distortionary taxation; and finally, it would be interesting to have the fiscal authority choosing the amount of public debt. Besides, we may also think about having a game structure where there are several fiscal authorities playing with a unique monetary authority, which is a case very close to the EU context.

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Technical appendix

A The Ramsey Problem

The Langrangian of the Ramsey problem (26) is

$$\begin{aligned}
\max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t, G_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ & \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda)u(C_t^o, N_t^o, G_t) \\
& + \gamma_t^1 [C_t^{o-\sigma}(\pi_t - 1)\pi_t - \frac{(1 - \epsilon_t)}{\theta} C_t^{o-\sigma} Z_t[\lambda N_t^r + (1 - \lambda)N_t^o] \\
& - \frac{\epsilon_t}{\theta} [\lambda N_t^r + (1 - \lambda)N_t^o] C_t^{o-\sigma} [\lambda \omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda)\omega_n N_t^{o\varphi} C_t^{o\sigma}] \\
& - \beta E_t C_{t+1}^{o-\sigma} (\pi_{t+1} - 1)\pi_{t+1}] \\
& + \gamma_t^2 \left[\frac{C_t^{o-\sigma}}{R_t} - \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}} \right] \\
& + \gamma_t^3 \left[Z_t[\lambda N_t^r + (1 - \lambda)N_t^o] - \lambda C_t^r - (1 - \lambda)C_t^o - G_t - \frac{\theta}{2}(\pi_t - 1)^2 \right] \\
& + \gamma_t^4 [N_t^{o\varphi} C_t^{o\sigma} - N_t^{r\varphi} C_t^{r\sigma}] \\
& + \gamma_t^5 [C_t^r - w_t N_t^r + G_t] \}
\end{aligned}$$

The first order condition w.r.t. $(C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t, G_t)$ respectively are

$$\lambda C_t^{r-\sigma} - \gamma_t^1 \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma} - 1 C_t^{o-\sigma} - \gamma_t^3 \lambda + \gamma_t^4 N_t^{r\varphi} \sigma C_t^{r\sigma-1} + \gamma_t^5 (1 - \omega_n N_t^{o\varphi+1} \sigma C_t^{o\sigma-1}) = 0 \quad (35)$$

$$\begin{aligned}
& - \lambda \omega_n N_t^{r\varphi} + \gamma_t^1 \frac{\lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda)N_t^{o\varphi} C_t^{o\sigma} + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] \\
& + \gamma_t^3 \lambda Z_t + \gamma_t^4 \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^5 \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi + 1) = 0
\end{aligned} \quad (36)$$

$$\begin{aligned}
(1 - \lambda)C_t^{o-\sigma} - (\gamma_t^1 - \gamma_{t-1}^1) \sigma C_t^{o-\sigma-1} (\pi_t - 1)\pi_t - \gamma_t^1 \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] \\
- \gamma_t^2 \sigma \frac{C_t^{o-\sigma-1}}{R_t} + \gamma_{t-1}^2 \sigma \frac{C_t^{o-\sigma-1}}{\pi_t} - \gamma_t^3 (1 - \lambda) - \gamma_t^4 N_t^{o\varphi} \sigma C_t^{o\sigma-1} = 0
\end{aligned} \quad (37)$$

$$\begin{aligned}
& - (1 - \lambda)\omega_n N_t^{o\varphi} + \gamma_t^1 \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^3 (1 - \lambda) Z_t - \gamma_t^4 \varphi C_t^{o\sigma} N_t^{o\varphi-1} = 0
\end{aligned} \tag{38}$$

$$(\gamma_t^1 - \gamma_{t-1}^1) C_t^{o-\sigma} (2\pi_t - 1) + \gamma_{t-1}^2 C_t^{o-\sigma} \pi_t^2 - \gamma_t^3 \theta (\pi_t - 1) = 0 \tag{39}$$

$$-\gamma_t^2 \frac{C_t^{o-\sigma}}{R_t^2} = 0 \tag{40}$$

$$\omega_g G_t^{-\sigma} - \gamma_t^3 + \gamma_t^5 = 0 \tag{41}$$

A.1 Ramsey steady state

We impose the steady state and get

$$\gamma^2 = 0 \tag{42}$$

$$\gamma^3 = \omega_g G^{-\sigma} + \gamma^5 \tag{43}$$

from (40) and (41). Then combining (42) with (39) we obtain

$$\pi = 1 \tag{44}$$

Combining these results with (5) and (20) leads to

$$R = \frac{1}{\beta} \tag{45}$$

and

$$w = \left[\lambda \frac{\omega_n N^{r\varphi}}{C^{r-\sigma}} + (1 - \lambda) \frac{\omega_n N^{o\varphi}}{C^{o-\sigma}} \right] = \frac{\epsilon - 1}{\epsilon} \tag{46}$$

The steady state values of the other variables are obtained through numerical methods.

B Nash policy game

B.1 Fiscal policy problem

The Lagrangian of the fiscal policy problem (28) is:

$$\begin{aligned}
& \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, G_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \\
& + \gamma_t^1 [C_t^{o-\sigma} (\pi_t - 1) \pi_t - \frac{(1 - \epsilon_t)}{\theta} C_t^{o-\sigma} Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] \\
& - \frac{\epsilon_t}{\theta} [\lambda N_t^r + (1 - \lambda) N_t^o] C_t^{o-\sigma} [\lambda \omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) \omega_n N_t^{o\varphi} C_t^{o\sigma}] \\
& - \beta E_t C_{t+1}^{o-\sigma} (\pi_{t+1} - 1) \pi_{t+1}] \\
& + \gamma_t^2 \left[\frac{C_t^{o-\sigma}}{R_t} - \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}} \right] \\
& + \gamma_t^3 \left[Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] - \lambda C_t^r - (1 - \lambda) C_t^o - G_t - \frac{\theta}{2} (\pi_t - 1)^2 \right] \\
& + \gamma_t^4 [N_t^{o\varphi} C_t^{o\sigma} - N_t^{r\varphi} C_t^{r\sigma}] \\
& + \gamma_t^5 [C_t^r - w_t N_t^r + G_t] \}
\end{aligned}$$

The first order condition w.r.t. $(C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, G_t)$ respectively are

$$\lambda C_t^{r-\sigma} - \gamma_t^1 \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma} - 1 C_t^{o-\sigma} - \gamma_t^3 \lambda + \gamma_t^4 N_t^{r\varphi} \sigma C_t^{r\sigma-1} + \gamma_t^5 (1 - \omega_n N_t^{r\varphi+1} \sigma C_t^{r\sigma-1}) = 0 \quad (47)$$

$$\begin{aligned}
& - \lambda \omega_n N_t^{r\varphi} + \gamma_t^1 \lambda \frac{\lambda}{\theta} C_t^{o-\sigma} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] \\
& + \gamma_t^3 \lambda Z_t + \gamma_t^4 \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^5 \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi + 1) = 0
\end{aligned} \quad (48)$$

$$\begin{aligned}
& (1 - \lambda) C_t^{o-\sigma} - \gamma_t^1 \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^1 \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] \\
& - \gamma_t^2 \sigma \frac{C_t^{o-\sigma-1}}{R_t} - \gamma_t^3 (1 - \lambda) - \gamma_t^4 N_t^{o\varphi} \sigma C_t^{o\sigma-1} = 0
\end{aligned} \quad (49)$$

$$\begin{aligned}
& - (1 - \lambda) \omega_n N_t^{o\varphi} + \gamma_t^1 \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^3 (1 - \lambda) Z_t - \gamma_t^4 \varphi C_t^{o\sigma} N_t^{o\varphi-1} = 0
\end{aligned} \quad (50)$$

$$\gamma_t^1 C_t^{o-\sigma} (2\pi_t - 1) - \gamma_t^3 \theta (\pi_t - 1) = 0 \quad (51)$$

$$\omega_g G_t^{-\sigma} - \gamma_t^3 + \gamma_t^5 = 0 \quad (52)$$

B.2 Monetary policy problem

The Lagrangian of the monetary policy problem (29) is:

$$\begin{aligned} & \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \\ & + \gamma_t^1 [C_t^{o-\sigma} (\pi_t - 1) \pi_t - \frac{(1 - \epsilon_t)}{\theta} C_t^{o-\sigma} Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] \\ & - \frac{\epsilon_t}{\theta} [\lambda N_t^r + (1 - \lambda) N_t^o] C_t^{o-\sigma} [\lambda \omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) \omega_n N_t^{o\varphi} C_t^{o\sigma}] \\ & - \beta E_t C_{t+1}^{o-\sigma} (\pi_{t+1} - 1) \pi_{t+1}] \\ & + \gamma_t^2 \left[\frac{C_t^{o-\sigma}}{R_t} - \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}} \right] \\ & + \gamma_t^3 \left[Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] - \lambda C_t^r - (1 - \lambda) C_t^o - G_t - \frac{\theta}{2} (\pi_t - 1)^2 \right] \\ & + \gamma_t^4 [N_t^{o\varphi} C_t^{o\sigma} - N_t^{r\varphi} C_t^{r\sigma}] \\ & + \gamma_t^5 [C_t^r - w_t N_t^r + L_t^r] \} \end{aligned}$$

The first order condition w.r.t. $(C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t)$ respectively are

$$\lambda C_t^{r-\sigma} - \gamma_t^1 \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma} - 1 C_t^{o-\sigma} - \gamma_t^3 \lambda + \gamma_t^4 N_t^{r\varphi} \sigma C_t^{r\sigma-1} + \gamma_t^5 (1 - \omega_n N_t^{r\varphi+1} \sigma C_t^{r\sigma-1}) = 0 \quad (53)$$

$$\begin{aligned} & - \lambda \omega_n N_t^{r\varphi} + \gamma_t^1 \frac{\lambda}{\theta} C_t^{o-\sigma} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] \\ & + \gamma_t^3 \lambda Z_t + \gamma_t^4 \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^5 \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi + 1) = 0 \end{aligned} \quad (54)$$

$$\begin{aligned} & (1 - \lambda) C_t^{o-\sigma} - \gamma_t^1 \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^1 \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] \\ & - \gamma_t^2 \sigma \frac{C_t^{o-\sigma-1}}{R_t} - \gamma_t^3 (1 - \lambda) - \gamma_t^4 N_t^{o\varphi} \sigma C_t^{o\sigma-1} = 0 \end{aligned} \quad (55)$$

$$\begin{aligned}
& - (1 - \lambda)\omega_n N_t^{o\varphi} + \gamma_t^1 \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^3 (1 - \lambda) Z_t - \gamma_t^4 \varphi C_t^{o\sigma} N_t^{o\varphi-1} = 0
\end{aligned} \tag{56}$$

$$\gamma_t^1 C_t^{o-\sigma} (2\pi_t - 1) - \gamma_t^3 \theta (\pi_t - 1) = 0 \tag{57}$$

$$-\gamma_t^2 \frac{C_t^{o-\sigma}}{R_t^2} = 0 \tag{58}$$

In steady state (42) and (43) still hold, but there is no more price stability ($\pi > 1$). In this case all other steady state values are obtained through numerical methods.

C Nash policy game with conservative monetary policy

The fiscal policy problem (28) is unchanged.

Instead, the monetary policy problem becomes:

$$\begin{aligned}
& \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t} E_t \sum_{t=0}^{\infty} \beta^t \{ (1 - \alpha) [\lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t)] - \alpha \frac{(\pi_t - 1)^2}{2} \\
& + \gamma_t^{12} [C_t^{o-\sigma} (\pi_t - 1) \pi_t - \frac{(1 - \epsilon_t)}{\theta} C_t^{o-\sigma} Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] \\
& - \frac{\epsilon_t}{\theta} [\lambda N_t^r + (1 - \lambda) N_t^o] C_t^{o-\sigma} [\lambda \omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) \omega_n N_t^{o\varphi} C_t^{o\sigma}] \\
& - \beta E_t C_{t+1}^{o-\sigma} (\pi_{t+1} - 1) \pi_{t+1}] \\
& + \gamma_t^{13} \left[\frac{C_t^{o-\sigma}}{R_t} - \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}} \right] \\
& + \gamma_t^{14} \left[Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] - \lambda C_t^r - (1 - \lambda) C_t^o - G_t - \frac{\theta}{2} (\pi_t - 1)^2 \right] \\
& + \gamma_t^{15} [N_t^{o\varphi} C_t^{o\sigma} - N_t^{r\varphi} C_t^{r\sigma}] \\
& + \gamma_t^{16} [C_t^r - w_t N_t^r + G_t] \}
\end{aligned}$$

The first order condition w.r.t. $(C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t)$ respectively are

$$(1 - \alpha) \lambda C_t^{r-\sigma} - \gamma_t^{12} \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma} - 1 C_t^{o-\sigma} - \gamma_t^{14} \lambda + \gamma_t^{15} N_t^{r\varphi} \sigma C_t^{r\sigma-1} + \gamma_t^{16} (1 - \omega_n N_t^{r\varphi+1} \sigma C_t^{r\sigma-1}) = 0 \tag{59}$$

$$\begin{aligned}
& - (1 - \alpha)\lambda\omega_n N_t^{r\varphi} + \gamma_t^{12} \frac{\lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] \\
& + \gamma_t^{14} \lambda Z_t + \gamma_t^{15} \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^{16} \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi + 1) = 0
\end{aligned} \tag{60}$$

$$\begin{aligned}
& (1 - \alpha)(1 - \lambda) C_t^{o-\sigma} - \gamma_t^{12} \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^{12} \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] \\
& - \gamma_t^{13} \sigma \frac{C_t^{o-\sigma-1}}{R_t} - \gamma_t^{14} (1 - \lambda) - \gamma_t^{15} N_t^{o\varphi} \sigma C_t^{o\sigma-1} = 0
\end{aligned} \tag{61}$$

$$\begin{aligned}
& - (1 - \alpha)(1 - \lambda) \omega_n N_t^{o\varphi} + \gamma_t^{12} \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^{14} (1 - \lambda) Z_t - \gamma_t^{15} \varphi C_t^{o\sigma} N_t^{o\varphi-1} = 0
\end{aligned} \tag{62}$$

$$\gamma_t^{12} C_t^{o-\sigma} (2\pi_t - 1) - \gamma_t^{14} \theta (\pi_t - 1) - \alpha (\pi - 1) = 0 \tag{63}$$

$$-\gamma_t^{13} \frac{C_t^{o-\sigma}}{R_t^2} = 0 \tag{64}$$

Solving for the steady state we find analogously:

$$\gamma^{13} = 0 \tag{65}$$

$$\gamma^3 = \omega_g G^{-\sigma} + \gamma^5 \tag{66}$$

D Fiscal leadership with conservative monetary policy

The Lagrangian of the fiscal policy problem (32) is:

$$\begin{aligned}
& \max_{C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, G_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \lambda u(C_t^r, N_t^r, G_t) + (1 - \lambda) u(C_t^o, N_t^o, G_t) \\
& + \gamma_t^1 [C_t^{o-\sigma} (\pi_t - 1) \pi_t - \frac{(1 - \epsilon_t)}{\theta} C_t^{o-\sigma} Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] \\
& - \frac{\epsilon_t}{\theta} [\lambda N_t^r + (1 - \lambda) N_t^o] C_t^{o-\sigma} [\lambda \omega_n N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) \omega_n N_t^{o\varphi} C_t^{o\sigma}] \\
& - \beta E_t C_{t+1}^{o-\sigma} (\pi_{t+1} - 1) \pi_{t+1}] \\
& + \gamma_t^2 \left[\frac{C_t^{o-\sigma}}{R_t} - \beta E_t \frac{C_{t+1}^{o-\sigma}}{\pi_{t+1}} \right] \\
& + \gamma_t^3 \left[Z_t [\lambda N_t^r + (1 - \lambda) N_t^o] - \lambda C_t^r - (1 - \lambda) C_t^o - G_t - \frac{\theta}{2} (\pi_t - 1)^2 \right] \\
& + \gamma_t^4 [N_t^{o\varphi} C_t^{o\sigma} - N_t^{r\varphi} C_t^{r\sigma}] \\
& + \gamma_t^5 [C_t^r - w_t N_t^r + G_t] \\
& + \gamma_t^6 [(1 - \alpha) \lambda C_t^{r-\sigma} - \gamma_t^{12} \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma} - 1 C_t^{o-\sigma} - \gamma_t^{14} \lambda + \gamma_t^{15} N_t^{r\varphi} \sigma C_t^{r\sigma-1} \\
& + \gamma_t^{16} (1 - \omega_n N_t^{r\varphi+1} \sigma C_t^{r\sigma-1})] \\
& + \gamma_t^7 [-(1 - \alpha) \lambda \omega_n N_t^{r\varphi} + \gamma_t^{12} \frac{\lambda}{\theta} C_t^{o-\sigma} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] + \gamma_t^{14} \lambda Z_t + \gamma_t^{15} \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^{16} \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi + 1)] \\
& + \gamma_t^8 [(1 - \alpha) (1 - \lambda) C_t^{o-\sigma} - \gamma_t^{12} \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^{12} \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t (\epsilon_t - 1) \\
& - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] - \gamma_t^{13} \sigma \frac{C_t^{o-\sigma-1}}{R_t} - \gamma_t^{14} (1 - \lambda) - \gamma_t^{15} N_t^{o\varphi} \sigma C_t^{o\sigma-1}] \\
& + \gamma_t^9 [-(1 - \alpha) (1 - \lambda) \omega_n N_t^{o\varphi} + \gamma_t^{12} \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t (\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} \\
& + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^{14} (1 - \lambda) Z_t - \gamma_t^{15} \varphi C_t^{o\sigma} N_t^{o\varphi-1}] \\
& \gamma_t^{10} [\gamma_t^{12} C_t^{o-\sigma} (2\pi_t - 1) - \gamma_t^{14} \theta (\pi_t - 1) - \alpha (\pi_t - 1)] \\
& \gamma_t^{11} [-\gamma_t^{13} \frac{C_t^{o-\sigma}}{R_t^2}] \}
\end{aligned}$$

The first order conditions w.r.t. $(C_t^r, N_t^r, C_t^o, N_t^o, \pi_t, R_t, G_t, \gamma_t^{12}, \gamma_t^{13}, \gamma_t^{14}, \gamma_t^{15}, \gamma_t^{16})$ are then respectively

$$\begin{aligned}
& \lambda C_t^{r-\sigma} - \gamma_t^1 \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma-1} C_t^{o-\sigma} - \gamma_t^3 \lambda + \gamma_t^4 N_t^{r\varphi} \sigma C_t^{r\sigma-1} + \gamma_t^5 [1 - N_t^{r\varphi+1} \omega_n \sigma C_t^{r\sigma-1}] \\
& + \gamma_t^6 [-\sigma(1-\alpha) \lambda C_t^{r-\sigma-1} - \gamma_t^{12} \frac{\epsilon_t}{\theta} \lambda N_t C_t^{o-\sigma} \omega_n \sigma N_t^{r\varphi} (\sigma-1) C_t^{r\sigma-2} + \gamma_t^{15} \sigma N_t^{r\varphi} * (\sigma-1) C_t^{r\sigma-2} \\
& - \gamma_t^{16} \omega_n \sigma N_t^{r\varphi+1} (\sigma-1) C_t^{r\sigma-2}] + \gamma_t^7 [-\gamma_t^{12} \frac{\lambda}{\theta} C_t^{o-\sigma} \epsilon_t \omega_n (\varphi N_t N_t^{r\varphi-1} \sigma C_t^{r\sigma-1} + \lambda N_t^{r\varphi} \sigma C_t^{r\sigma-1}) \\
& + \gamma_t^{15} \varphi N_t^{r\varphi-1} \sigma C_t^{r\sigma-1} - \gamma_t^{16} \omega_n N_t^{r\varphi} (\varphi+1) \sigma C_t^{r\sigma-1}] + \gamma_t^8 [\gamma_t^{12} \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} \epsilon_t \lambda \omega_n N_t^{r\varphi} \sigma C_t^{r\sigma-1}] \\
& + \gamma_t^9 [-\gamma_t^{12} \frac{1-\lambda}{\theta} C_t^{o-\sigma} \epsilon_t \omega_n \lambda N_t^{r\varphi} \sigma C_t^{r\sigma-1}] = 0
\end{aligned} \tag{67}$$

$$\begin{aligned}
& - \lambda \omega_n N_t^{r\varphi} + \gamma_t^1 [\frac{\lambda}{\theta} C_t^{o-\sigma} Z_t (\epsilon_t - 1) - \epsilon \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1-\lambda) N_t^{o\varphi} C_t^{o\sigma} + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})] \\
& + \gamma_t^3 \lambda Z_t + \gamma_t^4 \varphi N_t^{r\varphi-1} C_t^{r\sigma} - \gamma_t^5 \omega_n C_t^{r\sigma} N_t^{r\varphi} (\varphi+1) \\
& + \gamma_t^6 [-\gamma_t^{12} \frac{\epsilon_t}{\theta} \lambda C_t^{o-\sigma} \omega_n \sigma C_t^{r\sigma-1} (\lambda N_t^{r\varphi} + N_t \varphi N_t^{r\varphi-1}) + \gamma_t^{15} \sigma C_t^{r\sigma-1} \varphi N_t^{r\varphi-1} \\
& - \gamma_t^{16} \omega_n \sigma C_t^{r\sigma-1} (\varphi+1) N_t^{r\varphi}] \\
& + \gamma_t^7 [-(1-\alpha) \lambda \omega_n \varphi N_t^{r\varphi-1} - \gamma_t^{12} \frac{\lambda}{\theta} C_t^{o-\sigma} \epsilon_t \omega_n (\varphi C_t^{r\sigma} (\lambda N_t^{r\varphi-1} + N_t (\varphi-1) N_t^{r\varphi-2}) + \lambda C_t^{r\sigma} \varphi N_t^{r\varphi-1}) \\
& + \gamma_t^{15} \varphi C_t^{r\sigma} (\varphi-1) N_t^{r\varphi-2} - \gamma_t^{16} \omega_n (\varphi+1) C_t^{r\sigma} \varphi N_t^{r\varphi-1}] \\
& + \gamma_t^8 [\gamma_t^{12} \frac{\sigma}{\theta} C_t^{o-\sigma-1} \epsilon_t \omega_n \lambda C_t^{r\sigma} (\lambda N_t^{r\varphi} + \varphi N_t N_t^{r\varphi-1})] \\
& + \gamma_t^9 [-\gamma_t^{12} \frac{1-\lambda}{\theta} C_t^{o-\sigma} \epsilon \omega_n \lambda C_t^{r\sigma} \varphi N_t^{r\varphi-1}] = 0
\end{aligned} \tag{68}$$

$$\begin{aligned}
& (1 - \lambda)C_t^{o-\sigma} - \gamma_t^1 \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^1 \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}] \\
& - \gamma_t^2 \sigma \frac{C_t^{o-\sigma-1}}{R_t} - \gamma_t^3 (1 - \lambda) - \gamma_t^4 N_t^{o\varphi} \sigma C_t^{o\sigma-1} + \gamma_t^6 [\gamma_t^{12} 2 \frac{\epsilon_t}{\theta} \lambda \omega_n \sigma^2 N_t N_t^{r\varphi} C_t^{r\sigma-1} C_t^{o-\sigma-1}] \\
& + \gamma_t^7 [\gamma_t^{12} \frac{\lambda}{\theta} (-\sigma C_t^{o-\sigma-1} (Z_t(\epsilon_t - 1) - \epsilon_t \omega_n (\varphi N_t N_t^{r\varphi-1} C_t^{r\sigma} + \lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma})) \\
& - C_t^{o-\sigma} \epsilon_t \omega_n (1 - \lambda) N_t^{o\varphi} \sigma C_t^{o\sigma-1})] + \gamma_t^8 [-\sigma (1 - \alpha) (1 - \lambda) C_t^{o-\sigma-1} \\
& + \gamma_t^{12} (\pi_t - 1) \pi_t \sigma (\sigma + 1) C_t^{o-\sigma-2} + \gamma_t^{12} \frac{N_t}{\theta} \sigma (\sigma + 1) (Z_t(\epsilon_t - 1) - \epsilon_t \lambda \omega_n N_t^{r\varphi} C_t^{r\sigma}) C_t^{o-\sigma-2} \\
& + \gamma_t^{13} \sigma (\sigma + 1) \frac{C_t^{o-\sigma-2}}{R_t} - \gamma_t^{15} \sigma N_t^{o\varphi} (\sigma - 1) C_t^{o\sigma-2}] + \gamma_t^9 [\gamma_t^{12} \frac{1 - \lambda}{\theta} (-\sigma C_t^{o-\sigma-1} (Z_t(\epsilon_t - 1) \\
& - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma})) - C_t^{o-\sigma} \epsilon_t \omega_n ((1 - \lambda) N_t^{o\varphi} \sigma C_t^{o\sigma-1} \\
& + \varphi N_t N_t^{o\varphi-1} \sigma C_t^{o\sigma-1})) - \gamma_t^{15} \varphi N_t^{o\varphi-1} \sigma C_t^{o\sigma-1}] - \gamma_t^{10} \gamma_t^{12} (2\pi_t - 1) \sigma C_t^{o-\sigma-1} \\
& + \gamma_t^{11} \gamma_t^{13} \frac{\sigma C_t^{o-\sigma-1}}{R_t^2} = 0
\end{aligned} \tag{69}$$

$$\begin{aligned}
& - (1 - \lambda) \omega_n N_t^{o\varphi} + \gamma_t^1 \frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n [\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\
& + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma}]] + \gamma_t^3 (1 - \lambda) Z_t - \gamma_t^4 \varphi C_t^{o\sigma} N_t^{o\varphi-1} + \gamma_t^6 [-\gamma_t^{12} \frac{\epsilon_t}{\theta} \lambda C_t^{o-\sigma} \omega_n \sigma N_t^{r\varphi} C_t^{r\sigma-1} (1 - \lambda)] \\
& + \gamma_t^7 [-\gamma_t^{12} \lambda \theta C_t^{o-\sigma} \epsilon_t \omega_n (\varphi N_t^{r\varphi-1} C_t^{r\sigma} (1 - \lambda) + (1 - \lambda) C_t^{o\sigma} \varphi N_t^{o\varphi-1})] \\
& + \gamma_t^8 [-\gamma_t^{12} \frac{\sigma}{\theta} C_t^{o-\sigma-1} (1 - \lambda) (Z_t(\epsilon_t - 1) - \epsilon_t \lambda \omega_n N_t^{r\varphi} C_t^{r\sigma}) - \gamma_t^{15} \sigma C_t^{o\sigma-1} \varphi N_t^{o\varphi-1}] \\
& + \gamma_t^9 [-\varphi (1 - \alpha) (1 - \lambda) \omega_n N_t^{o\varphi-1} - \gamma_t^{12} \frac{1 - \lambda}{\theta} C_t^{o-\sigma} \epsilon_t \omega_n ((1 - \lambda) C_t^{o\sigma} \varphi N_t^{o\varphi-1} \\
& + \varphi C_t^{o\sigma} (N_t^{o\varphi-1} (1 - \lambda) + N_t (\varphi - 1) N_t^{o\varphi-2})) - \gamma_t^{15} \varphi C_t^{o\sigma} (\varphi - 1) N_t^{o\varphi-2}] = 0
\end{aligned} \tag{70}$$

$$\gamma_t^1 C_t^{o-\sigma} (2\pi_t - 1) - \gamma_t^3 \theta (\pi_t - 1) + \gamma_t^8 (-\gamma_t^{12} \sigma C_t^{o-\sigma-1} (2\pi_t - 1)) + \gamma_t^{10} (2\gamma_t^{12} C_t^{o-\sigma} - \gamma_t^{14} \theta - \alpha) = 0 \tag{71}$$

$$-\gamma_t^2 \frac{C_t^{o-\sigma}}{R_t^2} + \gamma_t^8 \gamma_t^{13} \frac{\sigma C_t^{o-\sigma-1}}{R_t^2} + \gamma_t^{11} \gamma_t^{13} \frac{2C_t^{o-\sigma}}{R_t^3} = 0 \tag{72}$$

$$\omega_g G_t^{-\sigma} - \gamma_t^3 + \gamma_t^5 = 0 \quad (73)$$

$$\begin{aligned} & -\gamma_t^6 \lambda \frac{\epsilon_t}{\theta} N_t \sigma \omega_n N_t^{r\varphi} C_t^{r\sigma-1} C_t^{o-\sigma} + \gamma_t^7 \frac{\lambda}{\theta} C_t^{o-\sigma} (Z_t(\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} \\ & + \varphi N_t N_t^{r\varphi-1} C_t^{r\sigma})) - \gamma_t^8 \sigma C_t^{o-\sigma-1} (\pi_t - 1) \pi_t - \gamma_t^8 \frac{N_t}{\theta} \sigma C_t^{o-\sigma-1} (Z_t(\epsilon_t - 1) - \epsilon_t \omega_n \lambda N_t^{r\varphi} C_t^{r\sigma}) \\ & + \gamma_t^9 \left[\frac{1 - \lambda}{\theta} C_t^{o-\sigma} [Z_t(\epsilon_t - 1) - \epsilon_t \omega_n (\lambda N_t^{r\varphi} C_t^{r\sigma} + (1 - \lambda) N_t^{o\varphi} C_t^{o\sigma} + N_t \varphi N_t^{o\varphi-1} C_t^{o\sigma})] \right. \\ & \left. + \gamma_t^{10} C_t^{o-\sigma} (2\pi_t - 1) \right] = 0 \end{aligned} \quad (74)$$

$$-\gamma_t^8 \frac{\sigma C_t^{o-\sigma-1}}{R_t} - \gamma_t^{11} \frac{C_t^{o-\sigma}}{R_t^2} = 0 \quad (75)$$

$$\lambda(\gamma_t^7 Z_t - \gamma_t^6) + (1 - \lambda)(\gamma_t^9 Z_t - \gamma_t^8) - \gamma_t^{10} \theta (\pi_t - 1) = 0 \quad (76)$$

$$N_t^{r\varphi} C_t^{r\sigma} \left(\gamma_t^6 \frac{\sigma}{C_t^r} + \gamma_t^7 \frac{\varphi}{N_t^r} \right) - N_t^{o\varphi} C_t^{o\sigma} \left(\gamma_t^8 \frac{\sigma}{C_t^o} + \gamma_t^9 \frac{\varphi}{N_t^o} \right) = 0 \quad (77)$$

$$\gamma_t^6 - \omega_n C_t^{r\sigma} N_t^{r\varphi} \left(\gamma_t^6 \frac{\sigma N_t^r}{C_t^r} + \gamma_t^7 (\varphi + 1) \right) = 0 \quad (78)$$

Tables and graphs

	AB	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$
C^r		0.1445	0.1446	0.1446	0.1446	0.1447	0.1448	0.1449	0.1450
C^o		0.1600	0.1619	0.1642	0.1673	0.1715	0.1776	0.1871	0.2039
C	0.1600	0.1600	0.1602	0.1603	0.1605	0.1608	0.1612	0.1618	0.1627
N^r		0.2214	0.2214	0.2213	0.2213	0.2212	0.2211	0.2209	0.2207
N^o		0.2000	0.1977	0.1949	0.1913	0.1866	0.1802	0.1711	0.1570
N	0.2000	0.2000	0.2001	0.2002	0.2003	0.2004	0.2007	0.2010	0.2016
G	0.0400	0.0400	0.0400	0.0399	0.0398	0.0397	0.0395	0.0393	0.0389
π	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
R	1.0087	1.0087	1.0087	1.0087	1.0087	1.0087	1.0087	1.0087	1.0087
rr	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
Y	0.2001	0.2001	0.2001	0.2002	0.2003	0.2005	0.2007	0.2010	0.2016
w	0.8335	0.8335	0.8335	0.8335	0.8335	0.8335	0.8335	0.8336	0.8336
D^o	0.0333	0.0333	0.0371	0.0417	0.0477	0.0557	0.0669	0.0838	0.1120
V	-354.5	-354.5	-354.6	-354.8	-355.0	-355.2	-355.6	-356.1	-357.0
Z	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003
ϵ	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016

Table 1: Ramsey problem stochastic steady state values

	AB	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$
C^r		0.1448	0.1449	0.1449	0.1450	0.1451	0.1453	0.1455	0.1461
C^o		0.1593	0.1608	0.1625	0.1645	0.1665	0.1675	0.1633	0.1451
C	0.1593	0.1593	0.1592	0.1590	0.1586	0.1579	0.1564	0.1527	0.1458
N^r		0.2215	0.2215	0.2215	0.2215	0.2215	0.2215	0.2215	0.2212
N^o		0.2014	0.1996	0.1975	0.1952	0.1931	0.1921	0.1973	0.2227
N	0.2014	0.2014	0.2017	0.2023	0.2031	0.2044	0.2068	0.2118	0.2216
G	0.0402	0.0402	0.0402	0.0402	0.0402	0.0402	0.0403	0.0403	0.0400
π	1.0146	1.0146	1.0165	1.0189	1.0222	1.0268	1.0341	1.0464	1.0641
R	1.0234	1.0234	1.0253	1.0278	1.0311	1.0358	1.0431	1.0555	1.0733
rr	0.0089	0.0089	0.0089	0.0089	0.0089	0.0090	0.0090	0.0091	0.0093
Y	0.2014	0.2014	0.2018	0.2023	0.2032	0.2045	0.2068	0.2119	0.2217
w	0.8354	0.8354	0.8356	0.8359	0.8364	0.8369	0.8379	0.8393	0.8413
D^o	0.0313	0.0313	0.0343	0.0376	0.0414	0.0451	0.0468	0.0380	-0.0023
V	-355.7	-355.7	-356.1	-356.7	-357.7	-359.1	-361.7	-367.4	-378.9
Z	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003
ϵ	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016

Table 2: Nash game stochastic steady state values

	AB	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$
C^r		0.1448	0.1448	0.1449	0.1450	0.1449	0.1453	0.1456	0.1461
C^o		0.1593	0.1608	0.1625	0.1645	0.1664	0.1677	0.1636	0.1452
C	0.1593	0.1593	0.1592	0.1590	0.1586	0.1578	0.1565	0.1528	0.1458
N^r		0.2215	0.2215	0.2215	0.2215	0.2217	0.2214	0.2214	0.2212
N^o		0.2013	0.1995	0.1975	0.1952	0.1931	0.1918	0.1970	0.2225
N	0.2013	0.2013	0.2017	0.2023	0.2031	0.2046	0.2066	0.2117	0.2216
G	0.0403	0.0403	0.0403	0.0403	0.0403	0.0406	0.0401	0.0403	0.0400
π	1.0144	1.0144	1.0163	1.0187	1.0220	1.0266	1.0338	1.0461	1.0640
R	1.0233	1.0233	1.0252	1.0276	1.0309	1.0356	1.0428	1.0553	1.0732
rr	0.0089	0.0089	0.0089	0.0089	0.0089	0.0090	0.0090	0.0091	0.0093
Y	0.2014	0.2014	0.2018	0.2023	0.2032	0.2046	0.2067	0.2117	0.2216
w	0.8354	0.8354	0.8356	0.8359	0.8363	0.8369	0.8378	0.8393	0.8413
D^o	0.0314	0.0314	0.0343	0.0377	0.0415	0.0454	0.0471	0.0386	-0.0020
V	-355.7	-355.7	-356.1	-356.7	-357.6	-359.0	-361.6	-367.2	-378.8
Z	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003
ϵ	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016

Table 3: Nash game with conservative monetary policy stochastic steady state values ($\alpha = 0.5$)

	AB	$\lambda = 0$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$
C^r		0.1448	0.1449	0.1449	0.1450	0.1451	0.1453	0.1455	0.1461
C^o		0.1593	0.1608	0.1626	0.1645	0.1666	0.1676	0.1636	0.1452
C	0.1593	0.1593	0.1592	0.1590	0.1587	0.1580	0.1565	0.1528	0.1458
N^r		0.2215	0.2215	0.2215	0.2215	0.2215	0.2215	0.2215	0.2212
N^o		0.2013	0.1995	0.1975	0.1952	0.1930	0.1919	0.1970	0.2225
N	0.2013	0.2013	0.2017	0.2023	0.2031	0.2044	0.2067	0.2117	0.2216
G	0.0402	0.0402	0.0402	0.0402	0.0402	0.0402	0.0403	0.0403	0.0400
π	1.0144	1.0144	1.0163	1.0187	1.0220	1.0266	1.0338	1.0461	1.0640
R	1.0233	1.0233	1.0252	1.0276	1.0309	1.0356	1.0428	1.0553	1.0732
rr	0.0089	0.0089	0.0089	0.0089	0.0089	0.0090	0.0090	0.0091	0.0093
Y	0.2014	0.2014	0.2018	0.2023	0.2031	0.2044	0.2067	0.2117	0.2216
w	0.8354	0.8354	0.8356	0.8359	0.8363	0.8369	0.8378	0.8393	0.8413
D^o	0.0314	0.0314	0.0343	0.0377	0.0415	0.0453	0.0471	0.0386	-0.0020
V	-355.7	-355.7	-356.1	-356.7	-357.6	-359.0	-361.6	-367.2	-378.8
Z	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003	1.0003
ϵ	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016	6.0016

Table 4: Fiscal leadership with conservative monetary policy stochastic steady state values ($\alpha = 0.5$)

	AB	$\alpha = 1, \lambda = 0.5$
C^r		0.1448
C^o		0.1776
C	0.1600	0.1612
N^r		0.2211
N^o		0.1802
N	0.2000	0.2007
G	0.0400	0.0395
π	1.0000	1.0000
R	1.0087	1.0087
rr	0.0087	0.0087
Y	0.2001	0.2007
w	0.8335	0.8335
D^o	0.0334	0.0669
V	-354.5	-355.6
Z	1.0003	1.0003
ϵ	6.0016	6.0016

Table 5: Fiscal leadership with conservative monetary policy stochastic steady state values ($\alpha = 1$)

	AB	$\lambda = 0.5$
Ramsey	-0.0009%	-0.0011%
Nash	-1.0314%	-5.1803%
Nash with $\alpha = 0.5$	-1.0129%	-5.0906%
Fiscal leadership with $\alpha = 0.5$	-1.0129%	-5.0953%
Fiscal leadership with $\alpha = 1$	-0.0011%	-0.0021%

Table 6: Welfare losses from Ramsey allocations in consumption equivalents

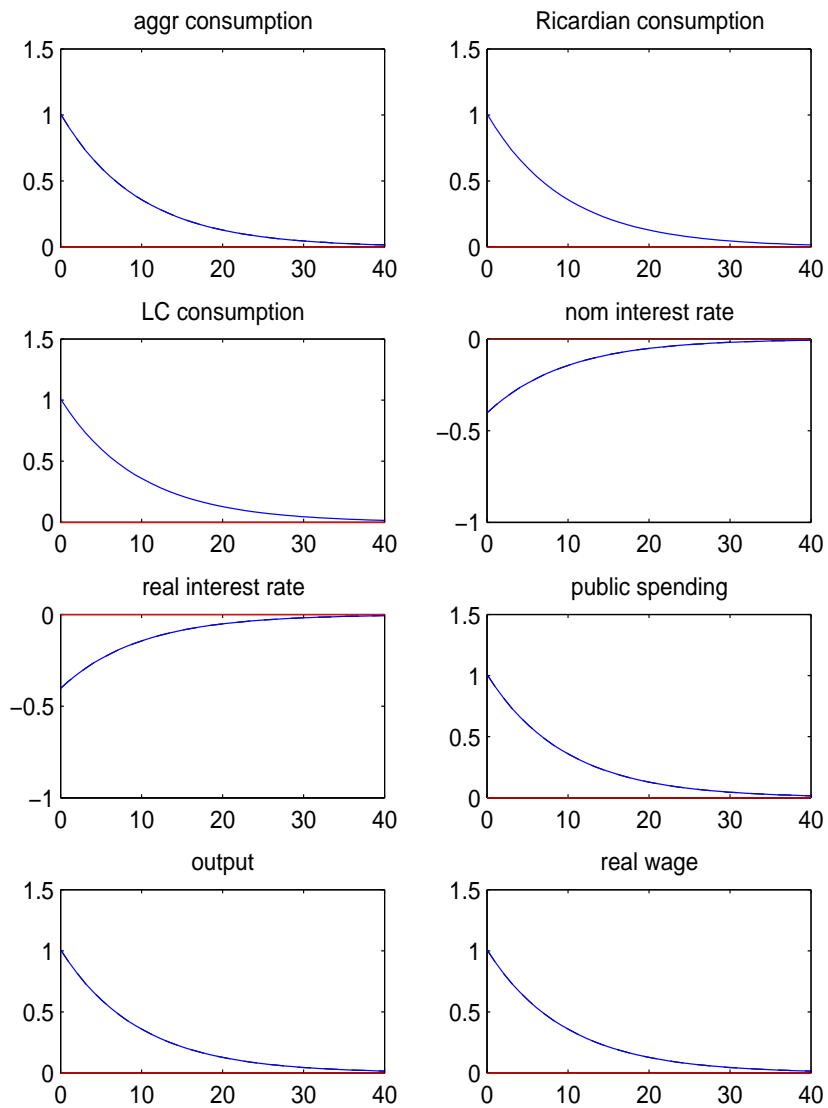


Figure 1: Ramsey IRFs to a positive technology shock

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

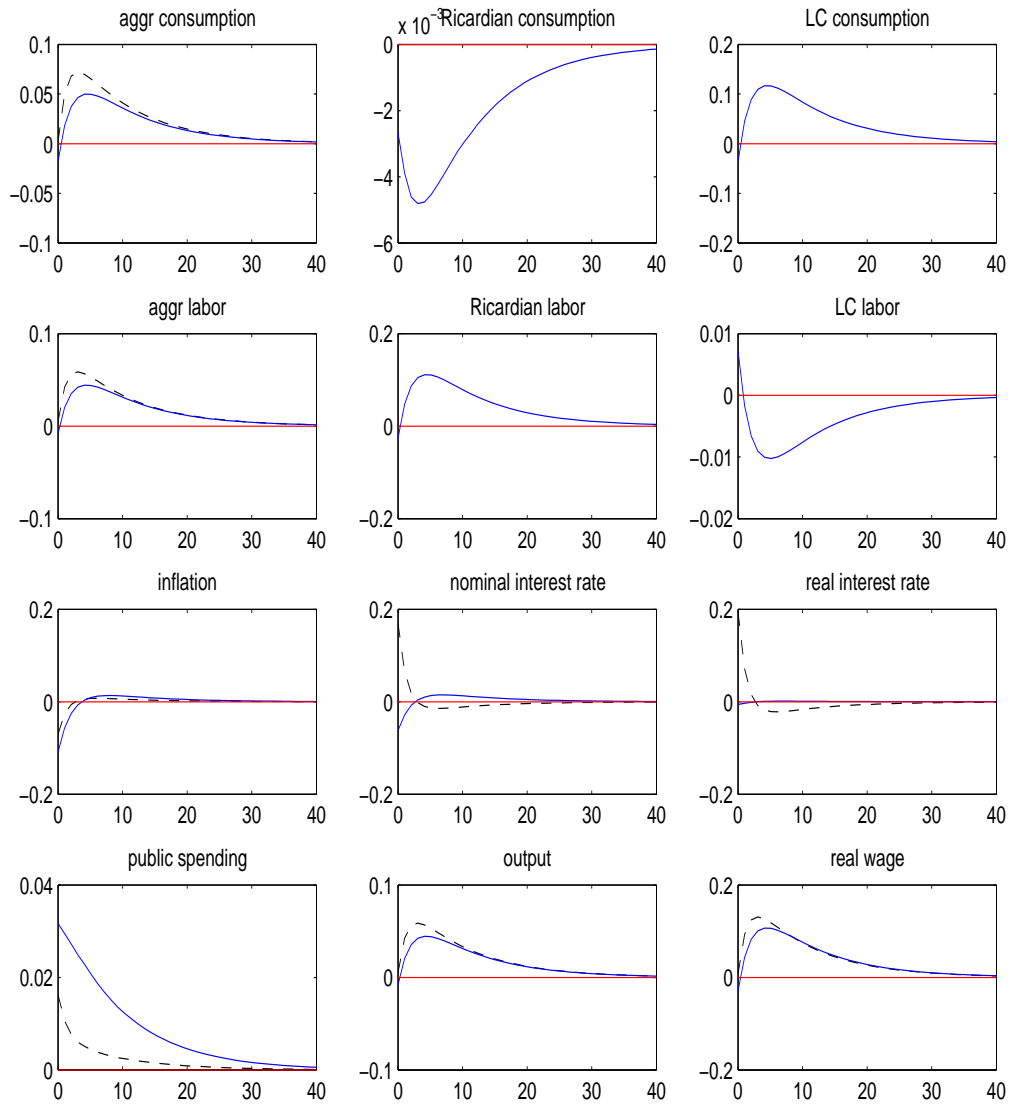


Figure 2: Ramsey IRFs to a negative mark-up shock

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

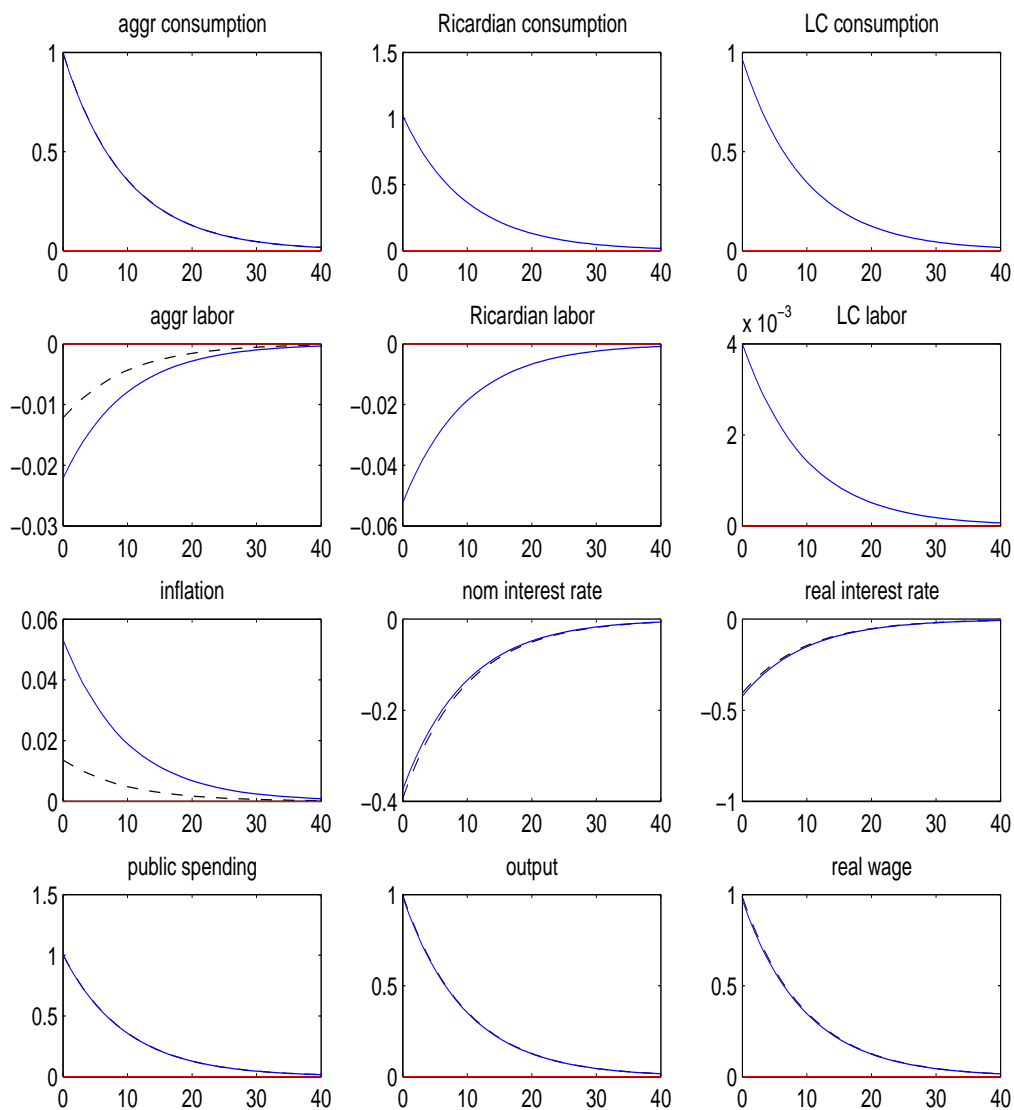


Figure 3: Nash IRFs to a positive technology shock

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

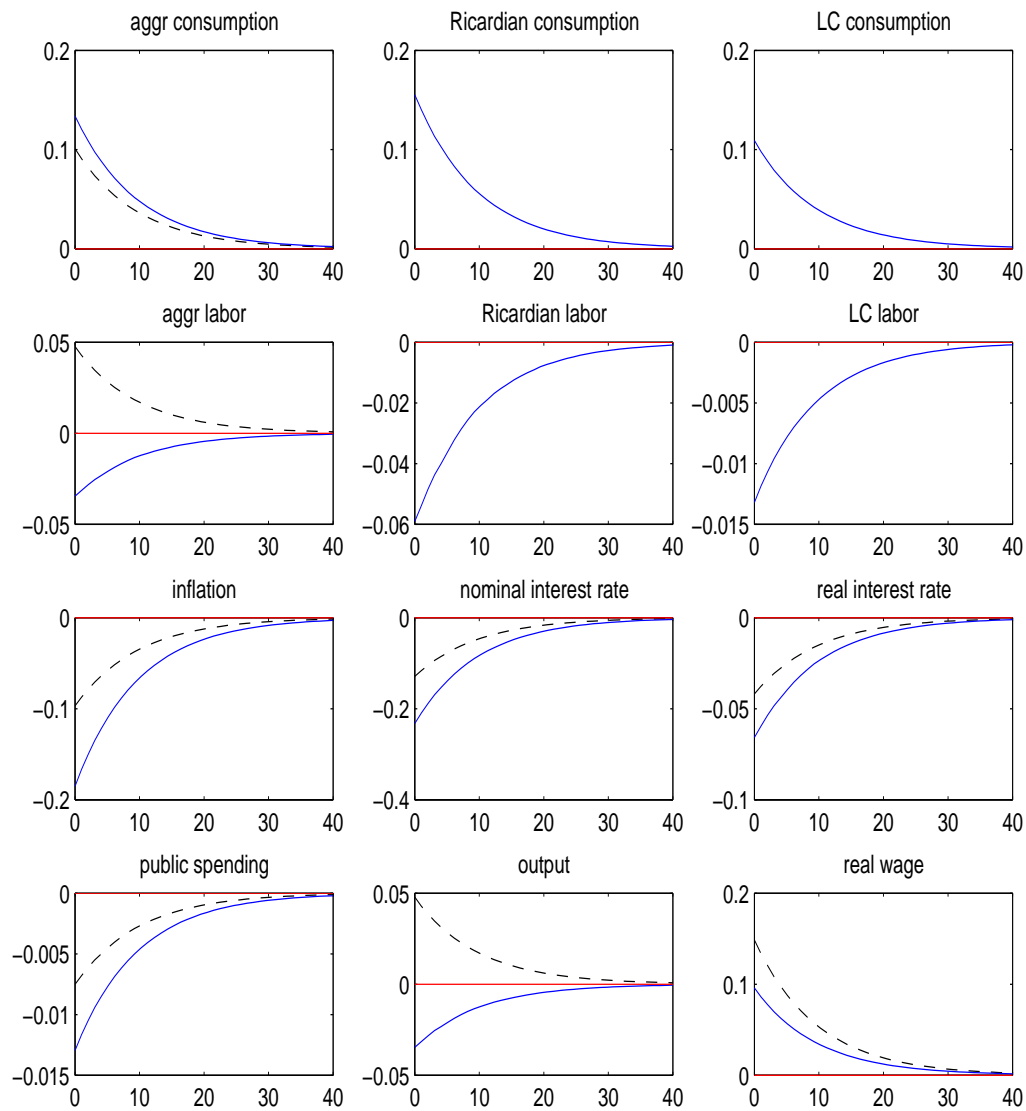


Figure 4: Nash IRFs to a negative mark-up shock

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

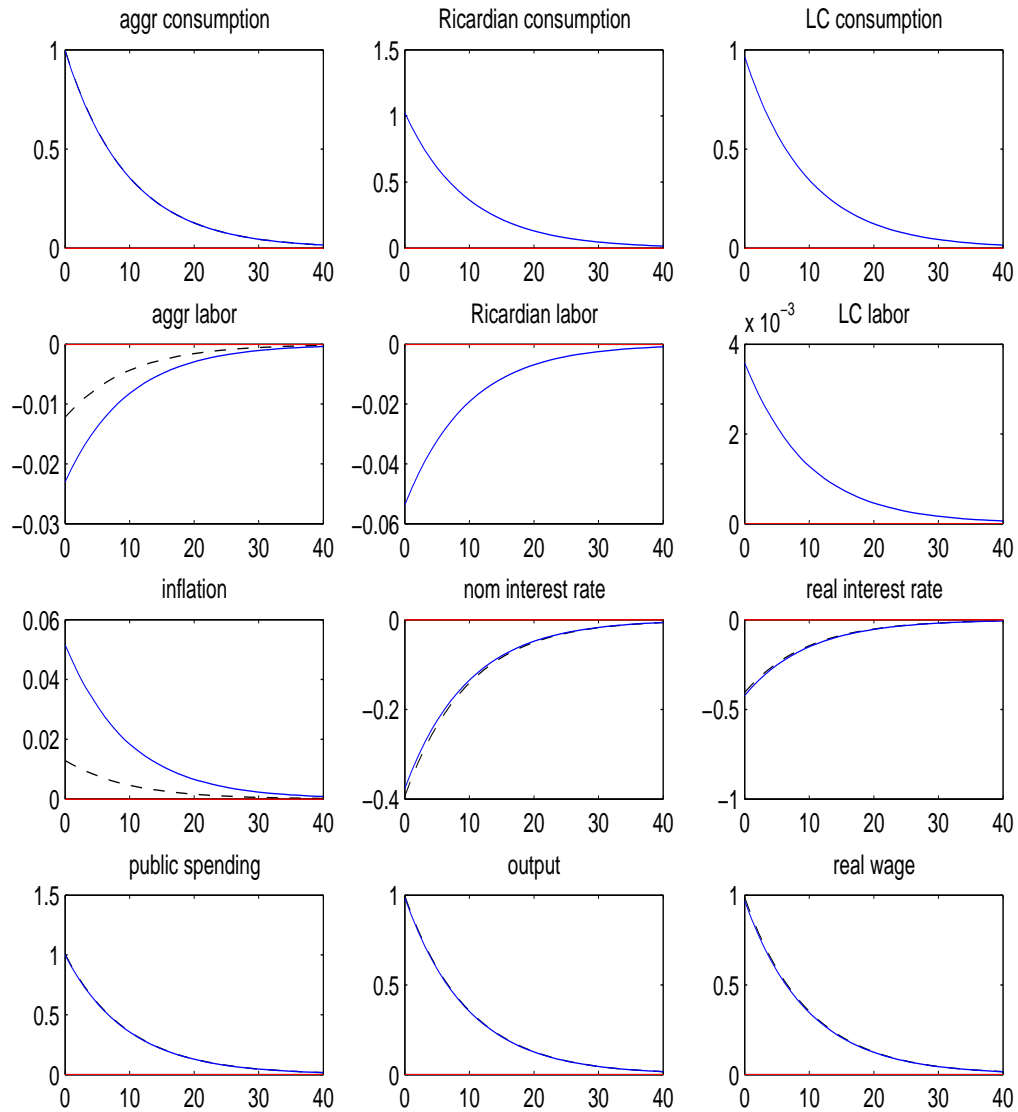


Figure 5: Nash IRFs to a positive technology shock with conservative monetary policy and $\alpha = 0.5$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

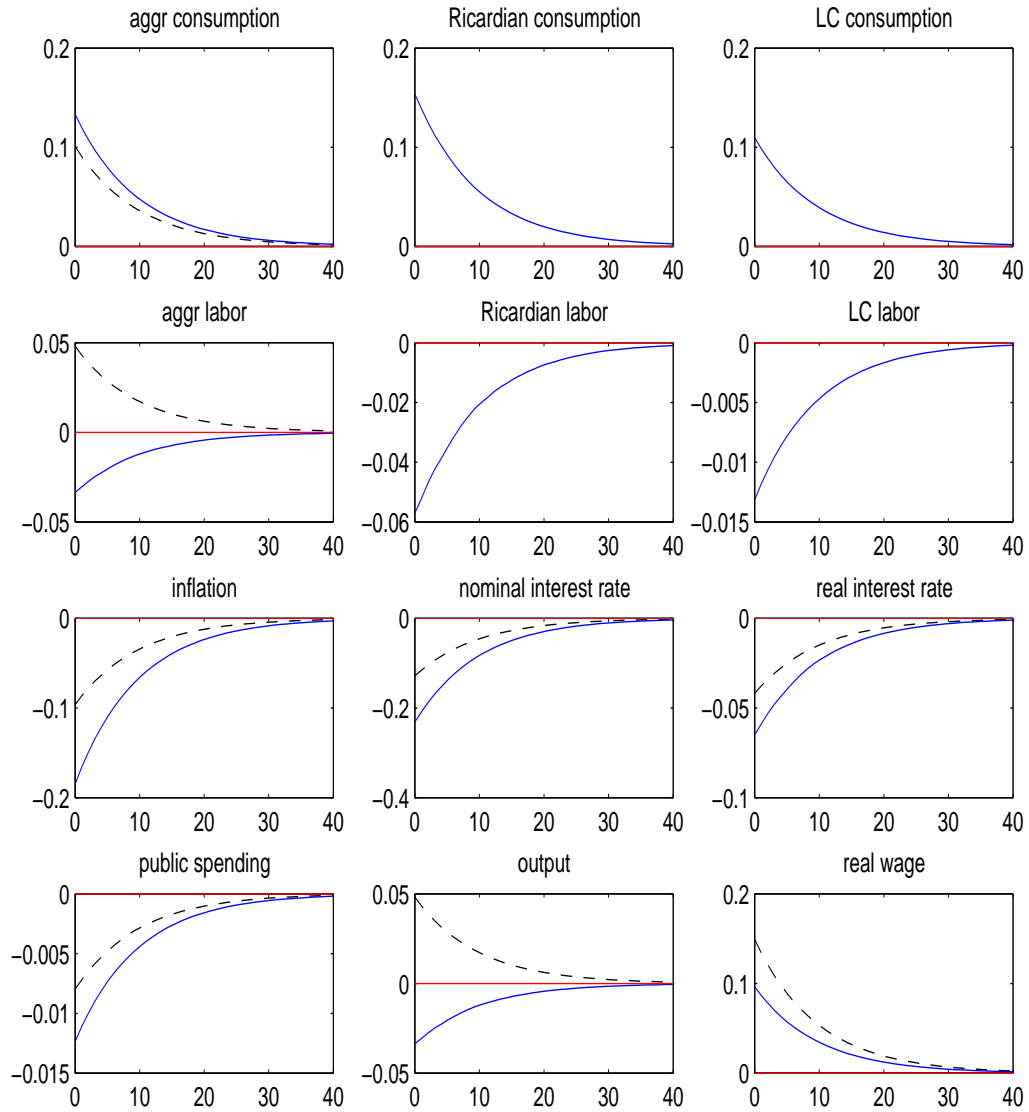


Figure 6: Nash IRFs to a negative mark-up shock with conservative monetary policy and $\alpha = 0.5$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

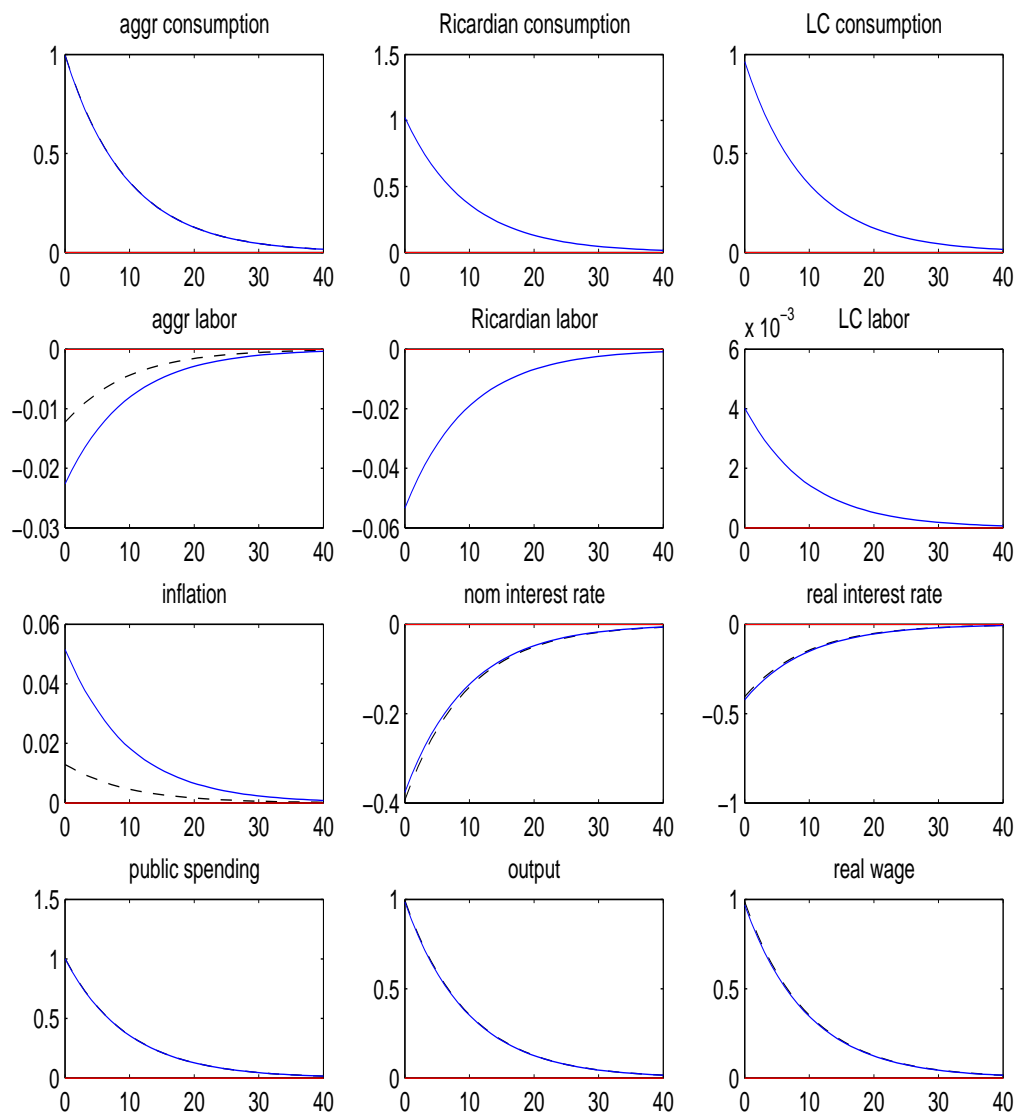


Figure 7: IRFs to a positive technology shock with fiscal leadership, conservative monetary policy and $\alpha = 0.5$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

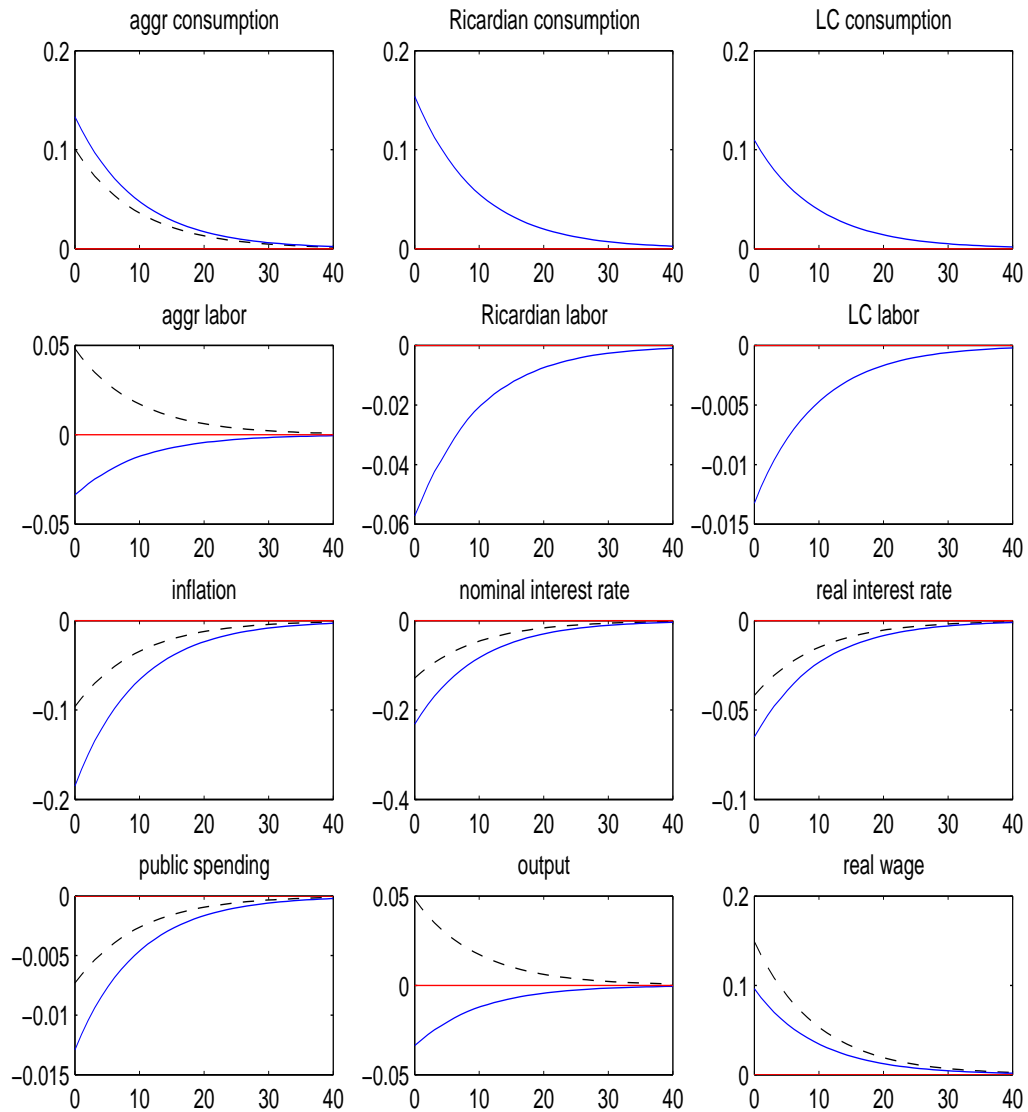


Figure 8: IRFs to a negative mark-up shock with fiscal leadership, conservative monetary policy and $\alpha = 0.5$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

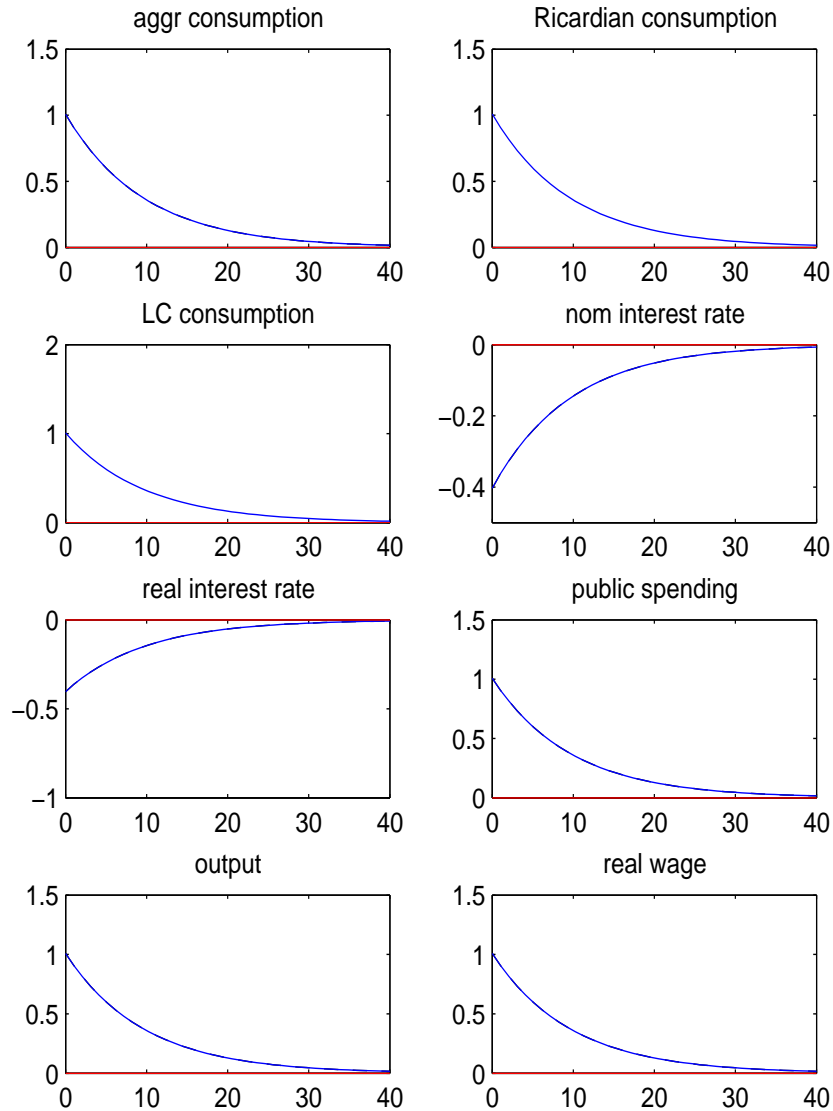


Figure 9: IRFs to a positive technology shock with fiscal leadership, conservative monetary policy and $\alpha = 1$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.

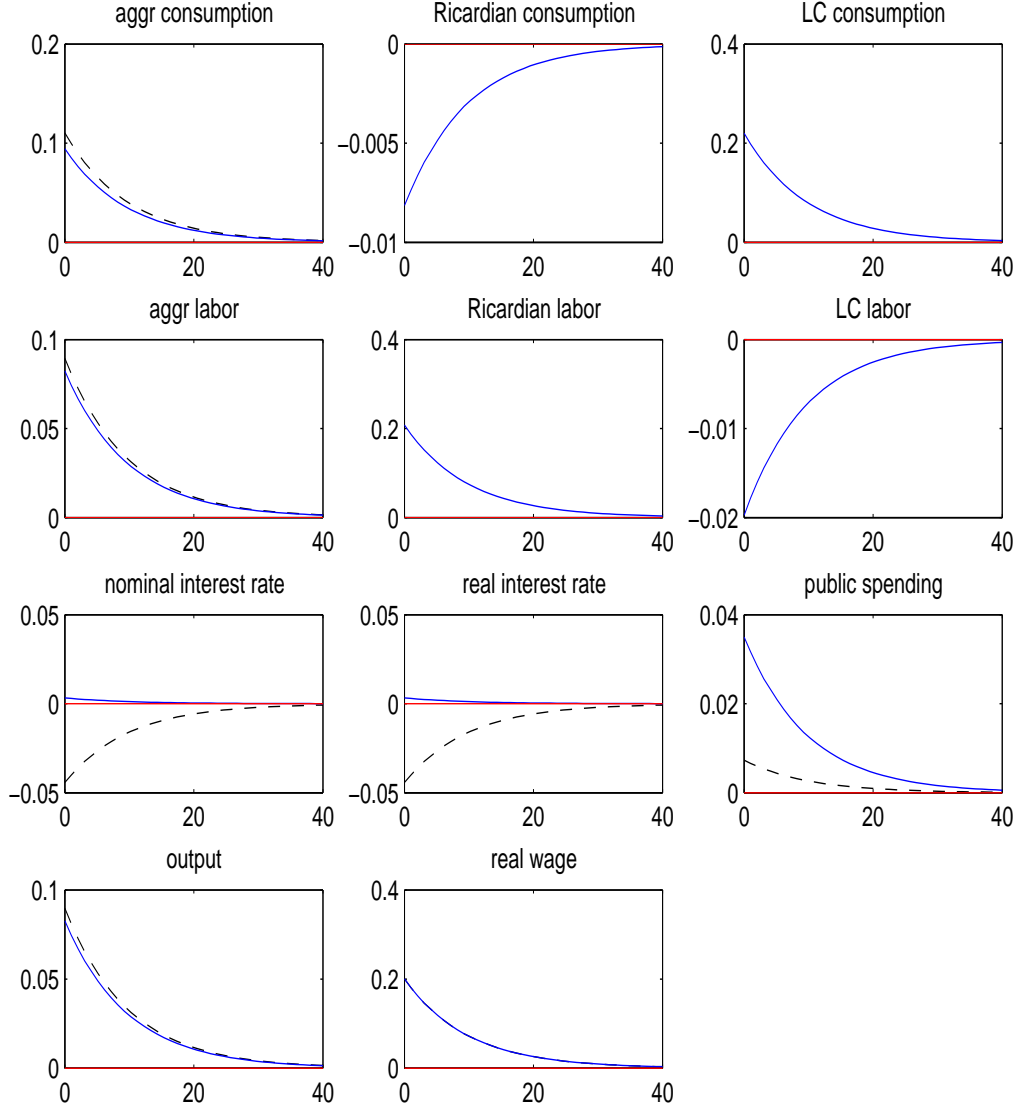


Figure 10: IRFs to a negative mark-up shock with fiscal leadership, conservative monetary policy and $\alpha = 1$

Notes: solid line for the model, dotted line for AB. Percentage variations from the steady state are shown. For inflation and interest rates annualized absolute variations are represented.