Veblen goods and neighbourhoods: endogenising consumption reference groups

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Abstract

One of the significant developments in the last four decades of economics is the growing empirical evidence that individual consumption preferences, as measured by self-reported life satisfaction, are neither fixed nor self-centred but are instead overwhelmingly dominated by externalities, partly in the form of reference levels set by others and by one's own experience. Welfare analysis recognising this fact is likely to indicate enormous revisions for macroeconomic policy and social objectives as well as for what is taught in economics at all levels. Yet the task of constructing general equilibrium models based on this microeconomic reality is still in its infancy. In this work I take the conventional stance that decision makers understand their own utility function. Therefore, they can choose the milieu in which they immerse themselves with the sophisticated understanding that it will affect their own consumption reference levels and therefore the degree of satisfaction they derive from their private consumption. At the same time, their private consumption will help to set the reference level for others in their chosen group. I treat theoretically the problem of such endogenous formation of consumption reference groups in the context of a simultaneous choice of neighbourhoods and home consumption amongst a heterogenous population. For both discrete and continuous distributions of types, I find general equilibrium outcomes in which differentiation of neighbourhoods occurs endogenously and I compare the welfare implications of growth in such economies.

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Our desires and pleasures spring from society; we measure them, therefore, by society and by the objects which serve for their satisfaction. Because they are of a social nature, they are of a relative nature. ... A house may be large or small; as long as the surrounding houses are equally small it satisfies all social demands for a dwelling. But let a palace arise beside the little house, and it shrinks from a little house to a hut ... the occupant of the relatively small house will feel more and more uncomfortable, dissatisfied and cramped within its four walls. [Marx and Engels, 1848, p. 163][1]

1 Introduction

A number of studies have shown large negative externalities in individual subjective well-being due to neighbours' income [Luttmer, 2005, Kingdon and Knight, 2007, Barrington-Leigh and Helliwell, 2007]. These externalities appear to reflect the role of nearby households as reference groups acting in individuals' reference-dependent preferences over income or consumption. At the same time, there are many reasons to expect positive spillovers from having prosperous neighbours. For instance, the quantity of tax-funded public goods and certain forms of social capital spillovers can be expected to be correlated with the incomes of nearby residents and thus to generate an apparent empathy effect. Alternatively, an idea pursued in this work is that neighbours' income may contribute to a local status level enjoyed by the entire neighbourhood, for instance through conspicuous displays of affluence.

An unresolved question is how such opposing positive and negative externalities of others' income relate to each other. It may, for instance, be that one effect is concentrated on a finer geographic scale than the other. In this work, I consider the possibility that individuals are fully aware of the structure of such returns. The motivating questions are then, firstly: when households properly anticipate the importance of reference groups and have some choice over where they live, can the simultaneous choice of whom to associate with and how much to consume lead to self-organisation of heterogeneous individuals into differentiated groups? Secondly, in a world in which such comparison effects are dominant, will a policy maker wish to curtail production of the status good or the freedom to sort? If relativities in preferences are to be acknowledged seriously in economics, general equilibrium outcomes including endogenous sorting must be understood.

In related empirical work, Barrington-Leigh and Helliwell [2007] combine high-resolution geographic data from three Canada-wide socio-economic surveys and the 2001 census to disentangle the spatial pattern of reference groups and to identify channels of positive and negative spillovers on life satisfaction. For instance, it appears that in Canadian urban regions the strongest reference group for the emulation of household income spans the entire metropolitan region. One might summarise this crudely with the finding that $\beta > 0$, $\beta = -\beta_R$ and $\beta_N = 0$ in the following linear estimate:

$$U = \beta_0 + \beta I + \beta_N \bar{I}_{\text{neighbourhood}} + \beta_R \bar{I}_{\text{region}} + \varepsilon \quad (1)$$

[1] Quoted in Kingdon and Knight [2007].

where I is own income and the other regressors are the mean incomes of one's neighbourhood and of one's region. This finding implies that, ignoring any additional national and international comparison effects, there is no social benefit to increasing the incomes of all households in a metropolitan region. On the other hand, there is a net benefit within a neighbourhood to increasing all its residents' incomes. With the aim to put the empirical work on geographic consumption reference groups in a more explicit framework, I develop a basic model of geographic organisation when such "Veblen" preferences are relevant. This represents an extension of previous work in two regards. In comparison to the symmetric Veblen equilibrium of Eaton and Eswaran [2006], I treat cases when (1) consumption is not homogeneous across individuals and (2) consumption reference groups are neither fixed nor common to all individuals. Thus, interdependent preferences drive both the segregation of types into dissimilar reference groups and the individual consumption choices given those reference groups. That is, reference groups are endogenised.[2] In the context pursued below, households choosing a home take into account the neighbourhood, judged in part by the look of other nearby houses. Simultaneously, within those neighbourhoods when building or maintaining their houses, yards, and even amenities like cars, consumers are influenced by the decisions of their neighbours and, in particular, tend to emulate local consumption norms.

I will not abstract from details of the functional dependence of utility on consumption of Veblen goods, since in investigating regional disparity one must depart from the symmetric consumption equilibria which provide elegant solutions in the analysis of Eaton and Eswaran [2006]. In addition, I depart from the representative agent formulation and assume exogenous heterogeneity. However, non-symmetric equilibria do not afford easy discussion of efficiency, since Veblen goods by their nature generate real utility benefits for some individuals at the expense of others.

Geographic proximity is only one of several plausible factors in delineating reference groups. Other natural reference groups include nuclear and extended family, work colleages, ethnic groups, and socioeconomic classes. Moreover, experience from one's own past and aspirations based on cognitive reasoning also provide reference levels which frame consumption evaluation. These contextual effects are all consistent with the evolutionary arguments of Rayo and Becker [2004][3]. Nevertheless, a focus on the interaction between interdependent preferences and settlement patterns that are spatially sorted according to income or consumption level is particularly important for its relevance to urban planning, real estate markets, and the empirical analysis of ge-

[2] The subjective well-being and social psychology literature indicates that there are likely systematic biases (generally in the direction of materialism) in individual choice, such that contemporary individuals are not acting to maximise their happiness [Dunn et al., 2003, Loewenstein et al., 2003]. However, there is no clear indication that people are confused more specifically about the competitive nature of consumption. In this work I do not assume any naiveté on the part of decision makers. The outcomes are driven by the collective action problem inherent in the consumption externality.

[3] They use a principal-agent framework to address the task of evolutionary forces in designing our internal reward circuitry, subject to the constraints that it has finite bounds. They argue that it therefore must have evolved with features that engineers would call automatic gain control and a (temporal) high-pass filter. That is, the comparison level and scale used for translating one's own consumption level into a psychological reward adapt to make best use of the available range of the reward experience.

ographic reference group effects[4]. The most obvious source of endogeneity for any spatial analysis, such as the empirical work motivating this study, is that people are mobile. Therefore, if reference effects are in play, households may have consciously chosen their reference group by moving to it.

The paper treats two general model formulations. Section 2 addresses the first, in which there are exactly two neighbourhood locations and two types of household. This simple case foreshadows most of the main results, but suffers from analytic intractability and assumes away the possibility of a (land) market being involved in the allocation of groups, or locations, to households. In Section 3 both the neighbourhood characteristics and the household types are continuously distributed and a land market regulates who lives where. Counterintuitively, this framework turns out to be more amenable to closed-form analysis than the discrete case. Section 4 provides some simulations of sample equilibria, and Section 5 concludes. A number of issues are addressed in more detail in the Appendix, which also contains proofs to propositions in the main text.

2 Discrete types and unpriced land

Consider a discrete set of household types, exogenously differentiated by their endowment of labour productivity $w \in [w_L, w_H]$. Each household chooses a consumption level of a pure Veblen good and also chooses which peer group to join. The sole industry may be taken to be the production of the pure Veblen good, housing, and the reference groups may be thought of as non-interacting neighbourhoods characterised by the average value of housing chosen by their residents. After choosing a residential neighbourhood, households compare their consumption of the Veblen good to average consumption in their own neighbourhood.[5] Nevertheless, agents are sophisticated rather than naïve in that prior to choosing a location, they are fully aware that their future consumption benefit will be framed by the neighbourhood that they have chosen. I will henceforth use the housing and neighbourhood context to describe model economies, although the relevance of the scenario extends to other Veblen goods with endogenous reference groups.

To elucidate the possibility of self-forming groups amongst Veblen consumers who make disaggregated decisons about their reference groups, I start by incorporating into the utility function a benefit of living in a wealthy neighbourhood, to act in tandem with the disutility imposed by having a higher consumption reference group.[6] Let preferences be defined[7] over leisure $x \geq 0$, the conspicuous extravagance $h \geq 0$ of

[4] Several empricial studies have, for reasons of empirical convenience and availability of data, assessed income reference groups on a geographic basis. See Barrington-Leigh and Helliwell [2007] and Clark et al. [2008].

[5] The simplifying assumption that neighbourhoods are non-interacting in this interpretation makes the model and those that follow non-spatial, strictly speaking. That is, there is no sense of physical proximity of one neighbourhood to another.

[6] Without any benefits to having wealthy neighbours, there cannot be any differentiation of types. See Appendix Section B.3 for a discussion of plausible positive consumption externalities in this geographic context.

[7] This form of utility is convenient in that it admits an equilibrium of the desired kind. See Section C for a discussion of the properties of the logarithm and exponential terms and how they relate to past literature exploring utility functions defined over differences — which may be positive or negative — and ratios of

one's house, the average value $\bar{h}$ of houses in one's choice of a neighbourhood, and the global average value of houses $\bar{\bar{h}}$. For convenience, utility is additively separable into a leisure term $F(\cdot)$, a Veblen term $H(\cdot)$ comparing own consumption with that of one's chosen peers, and a further Veblen term $N(\cdot)$ comparing one's neighbourhood to other neighbourhoods:[8]

$$U(x,h,\bar{h}) = \Phi \log(x) - \Lambda \exp\left(-\lambda\left[h-\bar{h}\right]\right) + N \log\left(1+\bar{h}/\bar{\bar{h}}\right) \tag{2}$$

Under these preferences, neighbourhood benefits accrue relative to a reference level $\bar{\bar{h}}$, which is the average consumption across neighbourhoods. The undesirable neighbourhood externality, on the other hand, comes about through a more local comparison between the neighbourhood standard $\bar{h}$ and the household's own consumption h. Using this form for $N(\cdot)$ is convenient in part because it allows the consideration below of a planner's policy which eliminates all production of the Veblen good[9] and also provides consistency with Section 3, to follow.

In choosing its optimal consumption, a household of type w is constrained by the budget

$$w[1-x] \geq h$$

Thus, given the optimality condition

$$x^{\star} = 1 - h/w \tag{3}$$

the household's decision problem may be reduced to a nested choice of an optimal housing purchase $h^{\star}(\bar{h})$ for each possible neighbourhood $\bar{h}$, followed by a choice of optimal neighbourhood $\bar{h}^{\star}$. In contrast to other superficially appealing forms for preferences, detailed in the Appendix, the utility function in equation (2) embodies bounded benefits to individual consumption of the Veblen good and a large penalty in utility for consuming much less than one's neighbours. Holding $\bar{h}$ fixed, $U(x^{\star}(h),h)$ is concave and its global optimum must be consistent with the first order condition

$$F'(1-\frac{h}{w}) = w\,H_h(h,\bar{h}) \quad \text{or} \quad h=0 \tag{4}$$

An explicit form for the optimal consumption choice $h^{\star}(w,\bar{h})$ for a household placed in a neighbourhood with average consumption $\bar{h}$ can be written in terms of the principal branch of the Lambert W function:[10]

quantities of goods.

[8] Also discussed in the Appendix are models incorporating an absolute utility benefit of wealthy neighbours, rather than the relative one posed here. This distinction is unlikely to be important except in as far as it affects analytic tractability and ease of welfare analysis.

[9] For this case, the limit of $1+\bar{h}/\bar{\bar{h}}$ is taken to be 2.

[10] The Lambert W function, also occasionally called the *omega* function or *product-log*, is the inverse function of $f(Z) = Z\exp(Z)$ [Corless et al., 1996]. Although less well known, it is very analogous to the logarithm. The real-valued principal branch is always implied in this work. $\text{LambertW}(x) > 0$ for $x > 0$. It is increasing, concave, and passes through the origin. Two identities used in this work are:

$$\log(\text{LambertW}(Z)) = \log(Z) - \text{LambertW}(Z)$$

$$h^{\star}(w,\bar{h}) = \max\left\{0, w - \frac{1}{\lambda}L(w,\bar{h})\right\} \tag{5}$$

where

$$L(w,\bar{h}) \equiv \text{LambertW}\left(\frac{\Phi}{\Lambda}e^{\lambda\left[w-\bar{h}\right]}\right)$$

Consumption $h^{\star}(w,\bar{h})$ is increasing (and leisure is decreasing) in $\bar{h}$: households will consume more when their neighbours do.[11] The corner solution, $h^{\star} = 0$, occurs where $\bar{h} < \frac{1}{\lambda}\log\left(\frac{\Phi}{\Lambda\lambda w}\right)$. The indirect utility $U(w,\bar{h})$ can then be expressed, as before, through substitution of $h^{\star}(w,\bar{h})$ into equation (2). Taking derivatives, this indirect utility is seen to be concave in both the interior and corner regions:

$$\frac{d^2U(w,\bar{h})}{d\bar{h}^2} = \begin{cases} -\frac{\lambda^2\Phi}{\left[L(w,\bar{h})+L(w,\bar{h})^2\right]} - \frac{N}{\left[\bar{h}+\bar{\bar{h}}\right]^2} < 0, & \text{for } \bar{h} > \frac{1}{\lambda}\log\left(\frac{\Phi}{\Lambda\lambda w}\right) \\ -\Lambda\lambda^2 e^{\lambda\bar{h}} - \frac{N}{\left[\bar{h}+\bar{\bar{h}}\right]^2} < 0, & \text{for } \bar{h} < \frac{1}{\lambda}\log\left(\frac{\Phi}{\Lambda\lambda w}\right) \end{cases}$$

Because the first derivative $\frac{dU(w,\bar{h})}{d\bar{h}}$ is continuous through $\bar{h} = \frac{1}{\lambda}\log\left(\frac{\Phi}{\Lambda\lambda w}\right)$, concavity ensures that there is a global maximum. Nevertheless, there is no general analytic form for the optimal $\bar{h}$, were a continuous choice available.

Moreover, households are not able to choose an arbitrary $\bar{h}$. Rather, they must choose between one of the two available neighbourhoods whose consumption levels $\bar{h}$ are equilibrium outcomes. For a separating equilibrium[12] in which $h = \bar{h}$ for each type, the equilibrium neighbourhoods lie at $\bar{h}_{\text{eq}} = \max\left\{0, w - \frac{\Phi}{\Lambda\lambda}\right\}$. Because for each type w there exists a global optimum $\bar{h} = \bar{h}_{\max U}$, it may be possible for certain fortuitous ranges of parameters to conspire to make $\bar{h}_{\text{eq}} \approx \bar{h}_{\max U}$ for each type. In this case, both types are content in their own neighbourhood and allocations form a separating equilibrium.

Figure 1a shows such a situation. By contrast, with different parameter values one or the other of the household types may prefer a deviation from $\bar{h}_{\text{eq}}$, as shown in Figure 1b where the high type prefers to move.Marked in the left hand panels of Figure 1 are the utility levels for each household type in the alternate, pooling equilibrium, as well as the homogeneous utility level for the case in which Veblen good production is prohibited and leisure is maximised. The pooling outcome is always an equilibrium and in cases such as that of Figure 1b it constitutes the unique equilibrium in pure strategies.[13]

For the case shown in Figure 1a, the high type is better off in the separating equilibrium, while the low type prefers the pooling equilibrium and could therefore be said

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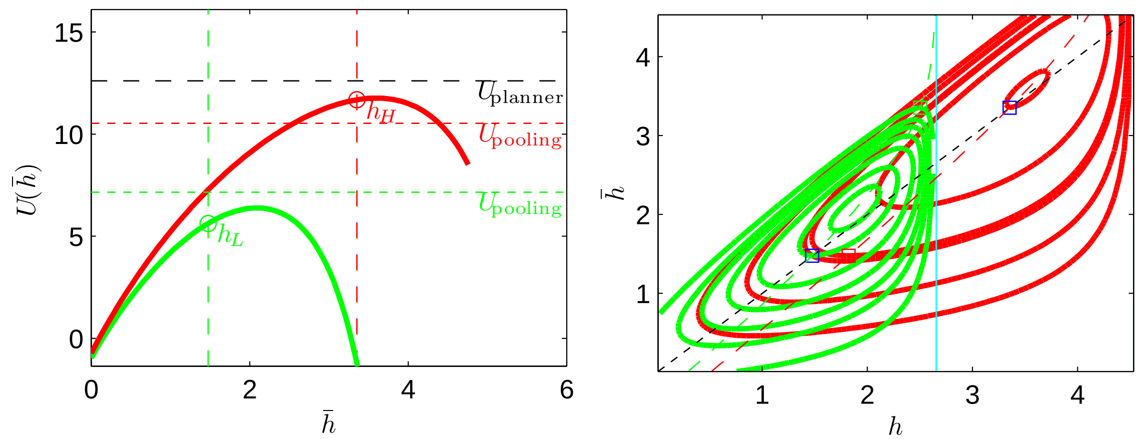
and

$$\frac{d}{dZ}\text{LambertW}(Z) = \frac{1}{Z}\frac{\text{LambertW}(Z)}{1+\text{LambertW}(Z)}$$

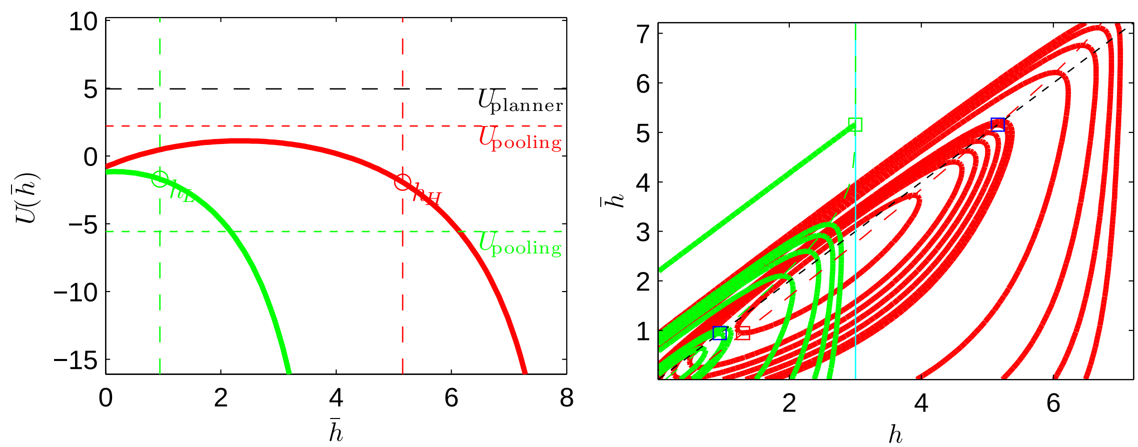
[11] See Equation (42) on page 40 for a contrasting case.

[12] A separating equilibrium is one in which neighbourhoods are differentiated according to household type. This equilibrium is more explicitly defined in the Appendix on page 40. An analogous equilibrium for the continuous case is also defined below in Section 3.

[13] See page 42 of the Appendix for a discussion of mixed strategies.



(a) Separating quilibrium for $\Phi \approx 3$, $\Lambda \approx 1$, $\lambda \approx 2$, $N \approx 20$, $w_L \approx 2.7$, and $w_H \approx 4.5$.



(b) No separating equilibrim exists for $\Phi \approx 8$, $\Lambda \approx 1$, $\lambda \approx 3$, $N \approx 9$, $w_L \approx 3$, and $w_H \approx 7$.

Figure 1: **Contingent existence of separating equilibrium.** Separating equilibrium (a) exists for "log-exp-log" preferences given by equation (2) but none exists (b) for other parameters in the same functional form. Also shown are utility levels in the pooling equilibrium for each type (U_{pooling}) and under the policy constraint of no Veblen good production (U_{planner}).

to favour policy designed to encourage neighbourhood integration across economic classes. Both types would prefer to have a planner remove the possibility of decentralised decision making about Veblen good production altogether, since the negative externality dominates the benefits even for the high type. This is reminiscent of the findings of Eaton and Eswaran [2006].

These qualitative features are not universal, however. In Figure 2, panel (a) shows a case when, conversely, the high type rather than the low type prefers an integrated neighbourhood, while in panel (c) both types prefer the pooling equilibrium. Numerous other orderings are possible. Figure 3 shows two cases in which the high type prefers to keep Veblen goods in production; that is, the planner's policy of eliminating Veblen goods would not be a Pareto improvement over either unregulated equilibrium. In the second case shown, the high type additionally prefers the integrated neighbourhood with Veblen goods to the one without.

Still other welfare orderings were found for different parameter values. Figure 4 shows that different regimes of exogenous parameters result in different welfare implications. Outside the region shown, separating equilibria were not found to exist. The distribution of points shows that endogenous group formation is not possible when within-group comparisons (Λ) receive considerably stronger weight in preferences than the between-group comparisons (N).

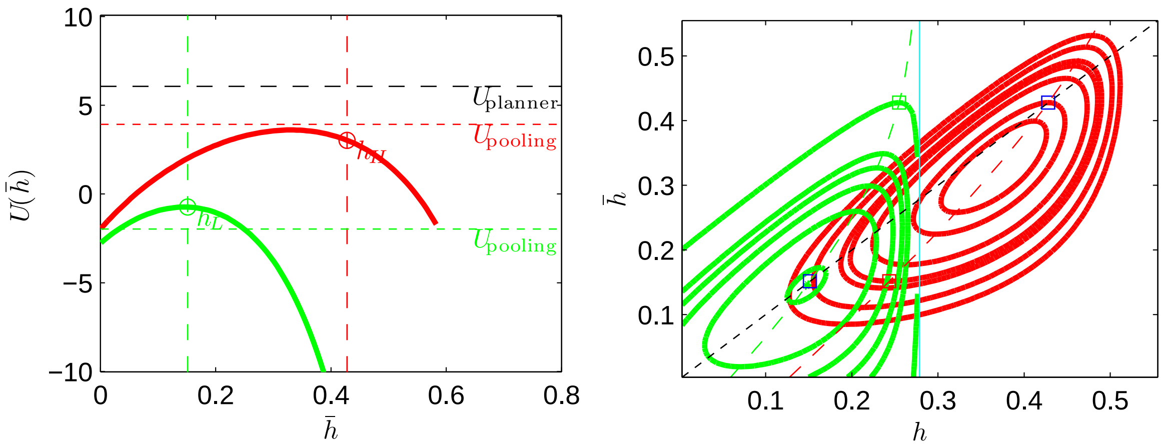
2.1 Summary

So far I have analysed the simplest case of a heterogeneous population choosing their own reference groups — the case of two types. Depending on the functional form of the utility, households may prefer to have higher or lower consumption of a Veblen good when they move to a higher consumption neighbourhood.[14] In all cases, there exists a pooling equilibrium conforming to the consistency condition that all households choose each neighbourhood with equal probability. Only for certain cases, on the other hand, does a pure strategy equilibrium exist in which different types prefer to remain segregated in neighbourhoods of internally homogeneous consumption levels. Nevertheless, the discrete nature of the choice amongst neighbourhoods makes it difficult to find closed form solutions or conditions on the existence of such equilibria.

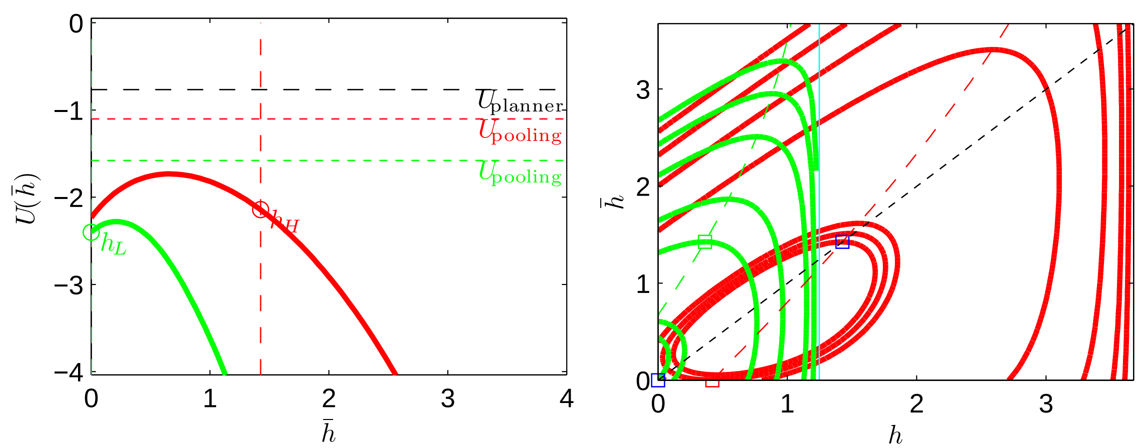
When both pooling and separating equilibria exist, numerical simulation indicates no simple universal welfare implications. Pure Veblen goods may be a desirable feature of the economy for wealthier households, and the freedom to relocate to form one's own reference groups may be desirable for one, both, or neither of the two types. These general features will be recaptured in the more analytical analysis to follow.

One reason for the awkwardness of the household problem and the condition for existence of a separating equilibrium is that there is no price to capture the benefit of a neighbourhood's consumption externalities. A natural way to do this is to allow a price for land, which heretofore has been costless. That is, for the case of a discrete set of neighbourhoods, separating equilibrian could more easily be supported if entry to a neighbourhood was competitive and exacted a cost to the household. However, two potential problems present themselves in this regard. First, prices relate to

[14] For the latter case, see, for example, Equation (42) on page 40.

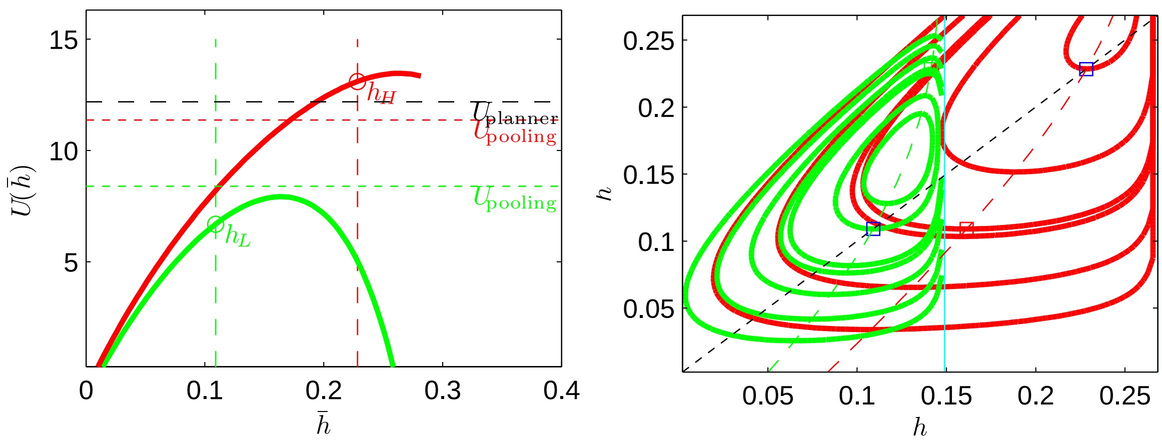


(a) Separating quilibrium for $\Phi \approx 4$, $\Lambda \approx 1$, $\lambda \approx 3$, $N \approx 13$, $w_L \approx 0.3$, and $w_H \approx 0.6$.

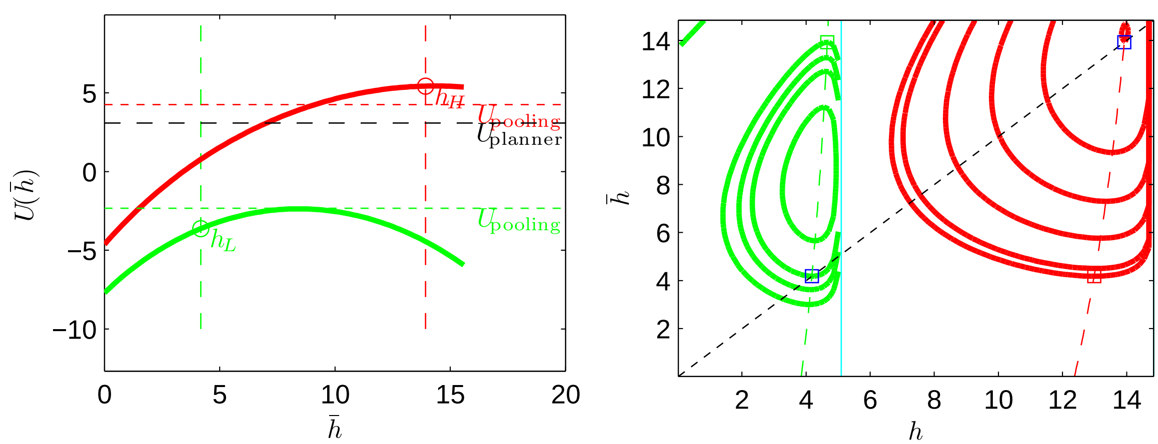


(b) Separating quilibrium for $\Phi \approx 2$, $\Lambda \approx 11$, $\lambda \approx 0.04$, $N \approx 3$, $w_L \approx 6$, and $w_H \approx 12$.

Figure 2: **Additional cases of equilibrium under "log-exp-log" preferences.**



(a) Separating quilibrium for $\Phi \approx 1$, $\Lambda \approx 2$, $\lambda \approx 19$, $N \approx 20$, $w_L \approx 0.15$, and $w_H \approx 0.27$.



(b) Separating quilibrium for $\Phi \approx 0.7$, $\Lambda \approx 9$, $\lambda \approx 0.1$, $N \approx 18$, $w_L \approx 5$, and $w_H \approx 15$.

Figure 3: **Further cases of equilibrium under "log-exp-log" preferences.**

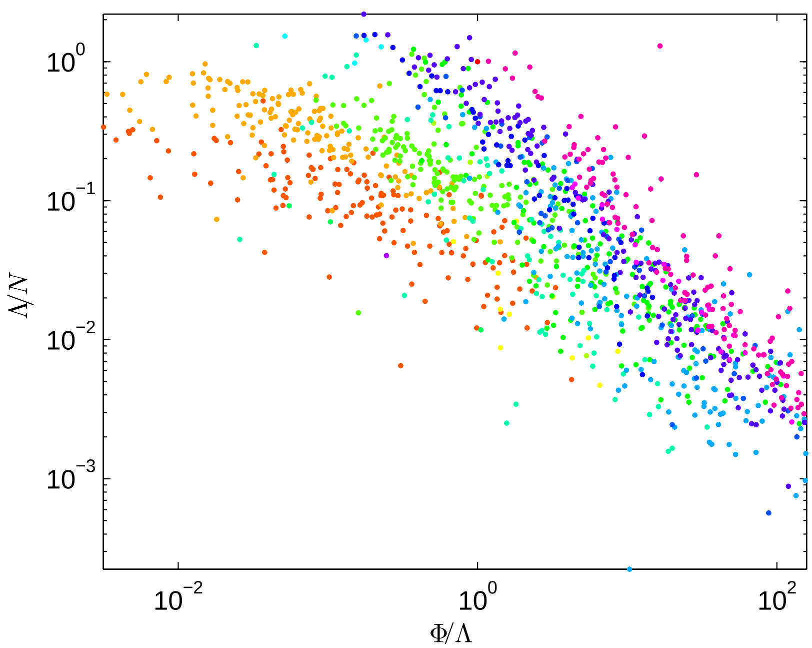


Figure 4: **Separating equilibrium parameter relationships.** Colours indicate different qualitative welfare orderings of pooling, planner, and separating outcomes.

marginal benefits in the real world and are therefore best incorporated into a model with a continuum of neighbourhood consumption levels $\bar{h}$. Secondly, in order to preserve a general equilibrium analysis, revenue from the sale or rental of land must be returned somehow to households.

These two issues are addressed in the following section by extending the endogenous reference group choice set to a continuum and by more realistically pricing land independently from housing.

3 A Continuum of types and a market for land

Consider then a framework in which, once again, static consumption reference-setting occurs both within a neighbourhood and between neighbourhoods. In choosing how much to spend on their own dwelling, household make a decision which is framed by the norm in their neighbourhood. In addition, households must choose a neighbourhood in which to position themselves. This affects not only the utility derived from their individual consumption choice but also provides a status payoff since they derive satisfaction from the relative standing of their neighbourhood.[15]

Therefore, as before, decisionmakers are faced with competing incentives to place themselves in a high or low affluence neighbourhood. In the analysis to follow, however, I introduce an additional direct cost associated with this choice. This comes about by relaxing the assumption of free land. When land is owned and rented, the marginal value to the renter of the reference level embodied by a particular location is captured in the price of land. This market can, as I show below, facilitate a disaggregated choice equilibrium of the kind already treated for discrete types.

In contrast to models such as that of Rothstein [2006] in which a small number of school districts confer peer effects to their residents,[16] a reasonable number of consumption reference group choices in the present context is large, since prospective homeowners can typically choose their neighbourhood from a nearly continuous set of affluence levels. Accordingly, I consider the case when there is a continuum of neighbourhoods rather than a discrete set. A crucial feature of the equilibrium to be defined below is that households have the option of moving to a neighbourhood with a marginally greater or lesser average consumption, just as they have the option of marginal changes to the size of their own house. Because households can relocate to their ideal reference group, there is no clustering of different types together in one neighbourhood.

[15] As mentioned previously, there are several possible reasons for neighbourhood status. For the sake of concreteness, I keep as the driver the same conspicuous consumption that drives house choice itself. That is, a neighbourhood's status value is determined by its average level of housing as compared with that of the greater region. This corresponds to the type 3 benefit on page 37. This specification is consistent with the findings of Barrington-Leigh and Helliwell [2007] and provides a coherent interpretation for welfare analysis of the consumption of neighbourhood quality. The drawback of this format is some superficial complexity: the household problem now represents two nested Veblen consumption choices. However, only one incorporates an endogenous choice of reference group, and it is the dynamics of this endogeneity that is the focus of the investigation.

[16] A different Tiebout equilibrium is defined in that case for each exogenously given integer number of discrete districts. In contrast, I consider continua of both household types and neighbourhoods and solve, below, for a unique equilibrium.

3.1 Agents' problem

As before, household agents are exogenously differentiated by their endowed labour productivity $w \in [w_L, w_H]$ in housebuilding, the sole industry. Now, however, types are continuously and uniformly distributed over this range. For each type w, there is a population of measure 1.

Agents maximise the following additively separable utility function through their choice of leisure $0 \leq x \leq 1$; the extravagance $h \geq 0$ of their house, which is the numeraire good; and their choice of a neighbourhood characterised by houses of average value $\bar{h}$:

$$U(x, h, \bar{h}) = F(x) + H(h, \bar{h}) + N(\bar{h}, \bar{\bar{h}})$$

The regional average level of housing consumption $\bar{\bar{h}}$ is perceived as identical by everyone. Each household is constrained by the budget

$$w[1-x] + r \geq h + p(\bar{h})$$

where r is any land dividend income received, and $p(\bar{h})$ is the competitive price of land in a neighbourhood with mean consumption $\bar{h}$. I will assume that land plots come in parcels that are independent of the size of the house that is built on them, and that this parcel size is uniform across neighbourhoods.

3.2 Firms' problem

Formally, there are two sectors of competitive firms.

Land management sector

Although the neighbourhood economy considered here is not explicitly spatial in that it abstracts from the arrangement and proximity of different neighbourhoods with respect to each other, the supply side of the land market must nevertheless be modeled in order for land price to be endogenous. Three scenerios present themselves as reasonable model assumptions:

free land: First of all, a simpler case is the one in which land is part of a commons. Then $r = p(\bar{h}) = 0$ and households choose their neighbourhood without any explicit cost, as in the discrete model of Section 2. Neighbourhoods are nevertheless mutually segregated.

absentee landowners: In this case, all plots of land are owned by absentee landlords who have no current use for them, and they rent individual plots to the highest bidder. Dividends r are zero for all households. For the purpose of welfare analysis, landowners are considered to be external to the economy.

uniform ownership: This case is similar to that of absentee landowners except that each plot of land is rented by an independent firm whose shares are equally owned by all households.[17] All rental income is profit and is distributed uniformly to shareholders. Thus each household, regardless of type, receives dividends r corresponding to the average price of rented land.

For reasons discussed below, land is assumed in much of what follows to be owned and rented by firms. Each plot of land is owned and managed by a separate price-taking firm whose equity is in turn owned in equal part by all households. Firms have no costs and simply receive rent p from the highest bidder for their land, subject only to the condition of nonnegative profit:

$$p \geq 0 \tag{6}$$

Firms then distribute all their profit to their shareholders.

Housebuilding sector

There is also a competitive housing production industry. Agents are endowed with an innate and universally visible productivity. Firms hire workers, pay them according to their productivity, and produce houses (or house maintenance, or conspicuous hosehold consumption goods more generally), making zero profit.

3.3 Definition of equilibrium

Given a continuous range of types $[w_L, w_H]$, a *separating neighbourhood equilibrium* consists of an average consumption $\bar{h}(n)$ for each neighbourhood n,[18] an overall regional average consumption $\bar{\bar{h}}$, market land prices $p(\bar{h})$ in each neighbourhood, rental dividends r, and allocations $\left\{x(w), h(w), \bar{h}(w)\right\}$, which

- satisfy consistency and aggregation requirements, in order that the perceived mean $\bar{h}$ is equal to the average consumption in each neighbourhood and that the global mean $\bar{\bar{h}}$ is the average over neighbourhoods,

$$\bar{h} = \int_{\left\{w \middle| \bar{h}(w)=\bar{h}\right\}} h(w)dw \quad \forall \bar{h} \tag{7}$$

$$\bar{\bar{h}} = \int \bar{h}(w)dw \tag{8}$$

- satisfy a non-profit condition on rental income (for the case when dividends are returned to households),

$$r = \int p\left(\bar{h}(w)\right) dw,$$

- satisfy the firms' incentive criterion,

$$p(\bar{h}) \geq 0 \tag{9}$$

[17] Unequal land ownership may be empirically more appealing and may represent a more acceptable middle ground between the two extremes, but it would constitute a complication at the moment.

[18] I will often refer to neighbourhoods, formally indexed by the continuous parameter n, by their equilibrium property, $\bar{h}$.

- satisfy each utility-maximising household who takes the allocations of others as given;
- and for which households are at least partly differentiated by type into different neighbourhood reference groups.

In addition, in order to eliminate degenerate solutions, I constrain the equilibrium to exclude allocations in which a disjoint set of types occupies a neighbourhood. For instance, this allows occupancy by the range $[w_1, w_2]$ but not by the discrete set $\{w_1, w_2\}$ for $w_1 \neq w_2$.

I restrict utility $U(\cdot)$ to be smoothly varying. For such functions, no continuous range of w will find the same (i.e., not varying with w) value of $\bar{h}$ to be optimal for interior allocations. Therefore, the above constraint against disjoint sets implies that equation (7) may be simplified to state that neighbourhoods are internally homogeneous:

$$h(w) = \bar{h}(w) \text{ for each } w \tag{7'}$$

The case when all occupied neighbourhoods exhibit identical average conspicuous consumption $\bar{h}$ is a *pooling neighbourhood equilibrium.*

3.4 Land markets are required for separating equilibria

Not all of the land ownership scenarios listed above admit separating equilibria. I first dispense with the *free land* possibility for a large set of functional forms and later, in Section 3.12, show that the absentee landlord case is also incompatible with separating equilibrium. As an additional refinement to Definition 3.3, let an *assortative separating neighbourhood equilibrium* be one in which the allocation of household types to neighbourhood types is one to one.

Proposition 3.1. (Requirement for land market) *If land is unpriced, there is no assortative separating equilibrium of continuous types. If land is unpriced, $N(\cdot)$ is concave or convex and $H(h,\bar{h})$ is a function of either $h-\bar{h}$ or $h/\bar{h}$, there is no pure strategy separating equilibrium of continuous types.*

Proof. Consider the choice of neighbourhood $\bar{h}$ by agents of type w when a continuum of neighbourhood types exist. The first order condition for the choice of $\bar{h}$, when an optimum exists, is

$$0 = \frac{\partial U(x,h,\bar{h},\bar{\bar{h}})}{\partial \bar{h}} = F_1(x)\frac{\partial x}{\partial \bar{h}} + H_2(h,\bar{h}) + N_1(\bar{h},\bar{\bar{h}}) \tag{10}$$

When $p(\bar{h}) = 0$, that is when the choice of neighbourhood has no direct bearing on a household's budget, $\partial x/\partial \bar{h} = 0$. Therefore, when (10) is evaluated at the equilibrium condition $h = \bar{h}$, it becomes

$$H_2(\bar{h},\bar{h}) + N_1(\bar{h},\bar{\bar{h}}) = 0 \tag{11}$$

which implicitly specifies the same choice(s) of $\bar{h}$ for all agents regardless of type, w. Therefore there is no unique sorting of types into neighbourhoods based on w — that is, no assortative separating neighbourhood equilibrium.

Furthermore, if $H(\cdot)$ takes the special forms $f\left(h-\bar{h}\right)$ or $f\left(\frac{h}{\bar{h}}\right)$, then $H_2(\bar{h},\bar{h})$ has value $-f'(0)$ or $-\frac{1}{\bar{h}}f'(1)$, respectively. In either case $\partial H_2(\bar{h},\bar{h})/\partial\bar{h} = 0$ and since $N_{11} \neq 0$, the left hand side of Equation equation (11) is monotonic and thus there is at most a unique solution for $\bar{h}$ and therefore no separating equilibrium.

□

This result may seem unintuitive in the context of the literature on discrete Tiebout equilibria, and it is difficult to find a good conceptual description to complement the proof. The impossibility of a separating equilibrium comes about because agents have two continuous choices to make but equilibrium requires that they align along a single dimension: the assortment of types into neighbourhoods. Without another price to clear the market in neighbourhood choice, the two sets of first order conditions cannot be simultaneously satisfied while meeting the equilibrium condition that $h = \bar{h}$.

3.5 Some general properties of equilibrium with a land market

Let $h^\star\left(\bar{h}\right)$ be the consumption level chosen optimally in a given neighbourhood with average consumption $\bar{h}$, and consider a utility function for which the indirect utility

$$U(w,\bar{h}) = U\left(w, h^\star\left(\bar{h}\right), \bar{h}\right) \tag{12}$$

is globally concave and in which x is essential, *i.e.*, $F(x) \to -\infty$ as $x \to 0$. Then the necessary optimality conditions for each household's choice of housing $h \geq 0$ and leisure $0 \leq x \leq 1$ take the following form:

$$F'(x) - wH_h(h,\bar{h}) - w\xi = 0 \quad \text{and} \tag{13}$$

$$\xi\left[r - h - p(\bar{h})\right] = 0 \tag{14}$$

where ξ is a Lagrange multiplier for the $x \leq 1$ constraint, which is equivalent to $h + p(\bar{h}) \geq r$. Since r and $p(\bar{h})$ are each nonnegative, this condition is stronger than $h \geq 0$, which therefore becomes redundant. For the choice of neighbourhood consumption $\bar{h} \geq 0$, necessary optimality conditions are:

$$\left[F'(x) - w\xi\right]p'(\bar{h}) + wH_{\bar{h}}(h,\bar{h}) - wN_{\bar{h}}(\bar{h},\bar{\bar{h}}) \geq 0 \quad \text{and} \tag{15}$$

$$\left[\left[F'(x) - w\xi\right]p'(\bar{h}) + wH_{\bar{h}}(h,\bar{h}) - wN_{\bar{h}}(\bar{h},\bar{\bar{h}})\right]\bar{h} = 0 \tag{16}$$

Considering interior values of h and $\bar{h}$, equation (13) can be used to eliminate $F'(x)$ in equation (16), providing a differential equation in $p'(\bar{h})$, h, $\bar{h}$, and $\bar{\bar{h}}$. Evaluating this at the equilibrium housing choice $h = \bar{h}$ gives:

$$p'(\bar{h}) = \frac{H_2(\bar{h},\bar{h}) + N_1(\bar{h},\bar{\bar{h}})}{H_1(\bar{h},\bar{h})} \tag{17}$$

Given a value for $p(0)$, equation equation (17) can be integrated to find the price of land for any neighbourhood. This property is used in the sections to follow.

3.6 Log-exp-log utility with equitable ownership

In order to find an explicit equilibrium solution for continuous types, I apply the equal ownership land model to the same functional form of utility used for discretely distributed types in Section 2:

$$U(x,h,\bar{h}) = \Phi \log(x) - \Lambda \exp\left(-\lambda\left[h-\bar{h}\right]\right) + N \log\left(1+\frac{\bar{h}}{\bar{\bar{h}}}\right) \tag{18}$$

For this specification, the house choice first order conditions (13) and (14), evaluated under the equilibrium condition $h=\bar{h}$, determine household leisure:

$$\begin{aligned} x(w) &= \min\left\{\frac{\Phi}{w\Lambda\lambda}, 1\right\} \\ &= \min\left\{\frac{w_0}{w}, 1\right\} \end{aligned} \tag{19}$$

where

$$w_0 \equiv \frac{\Phi}{\Lambda\lambda} \tag{20}$$

Households with productivity below w_0 choose not to expend any effort on building status symbols or buying into high-status neighbourhoods. Instead, they enjoy leisure $x=1$ and pool together in a low-status neighbourhood where spending is funded entirely by the universal dividend income, r. Because neighbourhoods in equilibrium are characterised by homogeneous consumption, the marginal value of housing consumption is uniformly equal to

$$\left.\frac{\partial H(h,\bar{h})}{\partial h}\right|_{h=\bar{h}} = \Lambda\lambda$$

As a result, the minimum wealth level for entry into the workforce is independent of the distribution of others' types. Moreover, it does not depend on the household's preference $N(\cdot)$ for neighbourhood status[19] but solely on the relative importance of leisure versus "keeping up with the Jones" in one's own neighbourhood.

Using 19 with the condition that no income is wasted generates an equation governing the neighbourhood allocations necessary for equilibrium:

$$\bar{h}(w) + p\left(\bar{h}\right) = r + \max\left\{0,\ w - w_0\right\} \tag{21}$$

Denote by $\bar{h}_{\min}$ the solution to $\bar{h} + p(\bar{h}) = r$; this neighbourhood is the lowest possible occupied neighbourhood.

[19]Nor would it depend on the intertemporal elasticity of substitution of leisure, which in the current formulation is fixed to 1.

The differential equation for price, equation (17), becomes[20]

$$p'(\bar{h}) = -1 + \frac{N}{\left[\bar{h} + \bar{\bar{h}}\right] \Lambda\lambda} \tag{22}$$

First, note that the sign of $p(\cdot)$ is indeterminate. In fact, while $p'(0)$ is positive if $N > \Lambda\lambda\bar{\bar{h}}$, it is negative otherwise; p has a maximum at $\bar{h} = \frac{N}{\Lambda\lambda} - \bar{\bar{h}}$. If $N < \Lambda\lambda\bar{\bar{h}}$, therefore, price is decreasing in neighbourhood affluence for all occupied neighbourhoods. This situation corresponds to preferences in which the neighbourhood status term N is relatively weak compared with consumption comparisons against immediate neighbours, and the average productivity is high.[21]

Land management firms would be willing to lend land for free but, in accordance with equation (9), are never willing to pay households to occupy land. Therefore, in equilibrium any continuous range of occupied neighbourhoods must bear a positive land price in order to meet condition equation (22).[22] It can be seen that for large $\bar{h}$, $p'(\bar{h}) \to -1$ and therefore p eventually crosses zero. Indeed, for $\bar{h}$ above some $\bar{h}_{\max}$, land price would need to be negative for neighbourhoods to be attractive to any household. The diminishing marginal returns to increasing status through neighbourhood choice are offset by the non-diminishing marginal cost to "keep up with the Jones" within a chosen neighbourhood.

In order to integrate equation (22) to find the price of land in any neighbourhood, a boundary condition on $p(\bar{h})$ is required. For the moment, let the integration constant remain unknown as p_0. Then equation (22) becomes

$$p(\bar{h}) = \max\left\{0,\, p_0 - \bar{h} + \frac{N}{\Lambda\lambda}\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right)\right\} \tag{23}$$

The price can now be eliminated from earlier expressions to find neighbourhood allocations as a function of r and $\bar{\bar{h}}$. Assuming that $p(\bar{h}(w)) > 0\ \forall w$, 21 and 23 can be combined to find

$$\begin{aligned}
\bar{h} &= r - p(\bar{h}) + \max\{0,\, w - w_0\} \\
&= r - p_0 + \bar{h} - \frac{N}{\Lambda\lambda}\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) + \max\{0,\, w - w_0\} \\
\to \log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) &= \frac{\Lambda\lambda}{N}\left[r - p_0 + \max\{0,\, w - w_0\}\right] \\
\to \bar{h}(w, r - p_0, \bar{\bar{h}}) &= \bar{\bar{h}}\exp\left(\Lambda\lambda\frac{r - p_0 + \max\{0,\, w - w_0\}}{N}\right) - \bar{\bar{h}} \qquad (24)
\end{aligned}$$

[20] A closed form of the indirect utility $U(\bar{h})$ based on results to follow shows that the second order condition for the choice of neighbourhood is satisfied for all values of p: $\frac{\partial^2 U\left(h^*(\bar{h}),\bar{h}\right)}{\partial \bar{h}^2} < 0$ for all parameter values. See Section 3.8.

[21] The endogenous value $\bar{\bar{h}}$ is expressed in terms of exogenous parameters below.

[22] This logic is the same reason that land pricing is necessary at all. See Proposition 3.1 on page 16.

This states that in equilibrium household consumption choice of the Veblen good increases convexly with productivity. Solving for $\bar{h}_{\min}$ gives

$$\bar{h}_{\min} = \bar{\bar{h}} \left[e^{\Lambda\lambda \frac{r-p_0}{N}} - 1 \right]$$

Let $\bar{h}_{\max}$ denote the upper root of $p(\bar{h}) = 0$ in equation (23). Below neighbourhood consumption level $\bar{h}_{\min}$, households cannot balance their budget in equilibrium without throwing income away. Above neighbourhood consumption level $\bar{h}_{\max}$, households would need to be compensated for occupying the land.[23]

If this upper limit on neighbourhood affluence is binding — that is, when $\bar{h}(w_H, r, \bar{\bar{h}}) > \bar{h}_{\max}$ — a separating equilibrium cannot exist. However, the next section shows that one can always find some price schedule which avoids this constraint.

3.7 General equilibrium averages

Denoting by $\langle \cdot \rangle$ an average over all types, the global average conspicuous consumption level is easily calculated from equation (21) as the total labour output in the production of housing:

$$\begin{aligned} \bar{\bar{h}} = \langle \bar{h} \rangle &= \big\langle r - p(\bar{h}(w)) + \max\{0,\ w - w_0\} \big\rangle \\ &= \langle \max\{w_0,\ w\} - w_0 \rangle \\ &= \begin{cases} \frac{w_H + w_L}{2} - w_0 & \text{if } w_L > w_0 \\ \frac{[w_H - w_0]^2}{2[w_H - w_L]} & \text{if } w_L \le w_0 \le w_H \end{cases} \end{aligned} \tag{25}$$

where I have used the fact that under uniform land ownership, $\langle r \rangle = \big\langle p(\bar{h}(w)) \big\rangle$.

Recalling that $w_0 = \frac{\Phi}{\Lambda\lambda}$, equation equation (25) states that when all households make interior choices, the average consumption of the Veblen good increases with the population average productivity in producing it, increases with the strength of the equilibrium *local* Veblen effect $\Lambda\lambda$ due to comparison with one's immediate neighbours, and decreases with the strength of preferences for leisure.

Defining $w_m \equiv \max\{w_L, w_0\}$ to be the lowest household type which chooses to work, a constraint on r follows from carrying out the integral over $p(w)$ explicitly.

[23] Equation equation (24) shows that, while pooling behaviour amongst the least endowed types is possible at $\bar{h} = \bar{h}_{\min}$, pooling of multiple types is not possible in any neighbourhood with a higher level of affluence. The implication of a downward-sloped price curve and a non-negative land price is that the market may unravel if a sufficiently wealthy type of household exists. For w high enough, the effective marginal cost of neighbourhood membership outweighs the status benefit, and demand for land at non-negative prices is zero in all more affluent neighbourhoods. In order to be induced to settle there, affluent types would need to be subsidised to compensate them for their contribution to the neighbourhood's status. However, once again the land holding firms are unwilling to subsidise (equation equation (23)). Households with w greater than some $w_{\max}$ will prefer a neighbourhood $\bar{h}$ in equation (24) which will exceed $\bar{h}_{\max}$. Above $\bar{h} = \bar{h}_{\max}$, the land price $p(\bar{h})$ sticks at 0 and there is no way to satisfy wealthy households with pure strategies. The most wealthy with $w > w_{\max}$ would, in the absence of any available neighbourhoods $\bar{h}(w)$, prefer to settle in a community with $\bar{h}_{\max}$, but doing so would raise the average consumption level there, making it unattractive for its original occupants if the rent remains at $p = 0$. Thus those original residents would prefer to move "down" to a less affluent neighbourhood, and so on; the separated neighbourhoods unravel.

Using equation (24),

$$
\begin{aligned}
\langle \bar{h} \rangle &= \bar{\bar{h}} \left\langle \exp\left(\Lambda\lambda \frac{r - p_0 + \max\{0,\, w - w_0\}}{N} \right) - 1 \right\rangle \\
1 + \frac{\langle \bar{h} \rangle}{\bar{\bar{h}}} &= \frac{1}{w_H - w_L} e^{\frac{\Lambda\lambda}{N}[r - p_0]} \int_{w_L}^{w_H} e^{\frac{\Lambda\lambda}{N} \max\{0,\, w - w_0\}} dw \\
\frac{\Lambda\lambda}{N}[r - p_0] &= \log\left(\left[1 + \frac{\langle \bar{h} \rangle}{\bar{\bar{h}}} \right] \frac{w_H - w_L}{\int_{w_L}^{w_H} e^{\frac{\Lambda\lambda}{N} \max\{0,\, w - w_0\}} dw} \right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
r - p_0 &= \frac{1}{\frac{\Lambda\lambda}{N}} \log\left(\frac{\left[1 + \langle \bar{h} \rangle / \bar{\bar{h}} \right] [w_H - w_L]}{\int_{w_L}^{w_m} 1 dw + \int_{w_m}^{w_H} e^{\frac{\Lambda\lambda}{N}[w - w_0]} dw} \right) \\
&= \frac{1}{\frac{\Lambda\lambda}{N}} \log\left(\frac{\left[1 + \langle \bar{h} \rangle / \bar{\bar{h}} \right] [w_H - w_L]}{w_m - w_L + \frac{1}{\frac{\Lambda\lambda}{N}} e^{-\frac{\Lambda\lambda}{N} w_0} \left[e^{\frac{\Lambda\lambda}{N} w_H} - e^{\frac{\Lambda\lambda}{N} w_m} \right]} \right) \\
&= \frac{1}{\frac{\Lambda\lambda}{N}} \log\left(\frac{\left[1 + \langle \bar{h} \rangle / \bar{\bar{h}} \right] [w_H - w_L]}{w_m - w_L + \frac{1}{\frac{\Lambda\lambda}{N}} e^{\frac{\Lambda\lambda}{N}[w_m - w_0]} \left[e^{\frac{\Lambda\lambda}{N}[w_H - w_m]} - 1 \right]} \right) \qquad (26)
\end{aligned}
$$

In equilibrium, $\langle \bar{h} \rangle / \bar{\bar{h}} = 1$. If $w_L > w_0$ (that is, for $w_m = w_L$), the above condition takes the form:

$$
\begin{aligned}
r - p_0 &= \frac{1}{\frac{\Lambda\lambda}{N}} \log\left(\frac{2 \frac{\Lambda\lambda}{N} [w_H - w_L]}{e^{\frac{\Lambda\lambda}{N}[w_L - w_0]} \left[e^{\frac{\Lambda\lambda}{N}[w_H - w_L]} - 1 \right]} \right) \\
&= w_0 - w_L + \frac{1}{\frac{\Lambda\lambda}{N}} \log\left(\frac{2 \frac{\Lambda\lambda}{N} [w_H - w_L]}{\left[e^{\frac{\Lambda\lambda}{N}[w_H - w_L]} - 1 \right]} \right) \qquad (27)
\end{aligned}
$$

According to equation (26) and equation (27), r has a fixed relationship to p_0 based on exogenous parameters. Because $\bar{h}(\cdot)$ in 24 depends only on the difference $r - p_0$, expressed above, any choice of base price p_0 results in the same consumption allocations amongst separating equilibria. On the other hand, according to equation (23) the value of $\bar{h}_{\max}$, where $p(\bar{h}) = 0$, is monotonically increasing in p_0. Therefore an equilibrium price schedule which accomodates the highest household type always exists. That is, for some p_0 high enough, $p(\bar{h}(w_H)) > 0$ and thus $w_{\max} > w_H$. A higher p_0 simply means higher dividends for all households and a higher base price for land. The insensitivity of equilibrium allocations and utility to the choice of p_0 simplifies welfare analysis somewhat but does not offset the redistributive effect of common land ownership as compared with an absentee land owner model. The slope of the price curve is unaffected by p_0 but is central to the equilibrium distribution of outcomes through the opposing effects of making high-income neighbourhoods exclusive and through more strongly redistributing wealth.

3.8 Concavity

As discussed in Section D.1 of the Appendix for the case of discrete types, it remains to ensure that the household's problem is characterised by a global maximum. A second order sufficiency condition is that the price schedule presents a concave objective function for the indirect utility $U(w,\bar{h}) = U(w,h^{\star}(\bar{h}),\bar{h})$. Given a neighbourhood choice $\bar{h}$, the optimal household consumption level is

$$h^{\star}(\bar{h}) \quad = \quad \max\left\{r - p(\bar{h}),\ \left[w + r - p(\bar{h})\right] - \frac{1}{\lambda}\mathscr{L}(w,\bar{h})\right\} \tag{28}$$

where

$$\mathscr{L}(w,\bar{h}) \equiv \text{LambertW}\left(\frac{\Phi}{\Lambda}e^{\lambda\left[w+r-p(\bar{h})-\bar{h}\right]}\right)$$

Therefore the indirect utility is

$$\begin{aligned} U(w,\bar{h}) \quad = \quad & \Phi \log\left(\min\left\{\frac{w_0}{w},\, 1\right\}\right) \\ & -\Lambda \exp\left(-\lambda\left[\max\left\{r - p(\bar{h}),\ \left[w + r - p(\bar{h})\right] - \frac{1}{\lambda}\mathscr{L}(w,\bar{h})\right\} - \bar{h}\right]\right) \\ & + N \log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) \end{aligned} \tag{29}$$

Consider the case of interior equilibria. Then $r - p(\bar{h})$ in the above expression can be eliminated in favour of the constant $[r - p_0]$ using equation (23):

$$\begin{aligned} r - p(\bar{h}) &= r - p_0 - [p - p_0] \\ &= [r - p_0] + \bar{h} - \frac{N}{\Lambda\lambda}\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) \end{aligned}$$

to find

$$\begin{aligned} U(w,\bar{h}) = {} & \Phi\log\left(\frac{w_0}{w}\right) + N\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) \\ & - \Lambda\exp\left(-\lambda\left[w + [r - p_0] - \frac{N}{\Lambda\lambda}\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right)\right.\right. \\ & \qquad \left.\left. - \frac{1}{\lambda}\text{LambertW}\left(\frac{\Phi}{\Lambda}e^{\lambda\left[w + [r-p_0] - \frac{N}{\Lambda\lambda}\log\left(1+\frac{\bar{h}}{\bar{\bar{h}}}\right)\right]}\right)\right]\right) \\ = {} & \Phi\log\left(\frac{w_0}{w}\right) + N\log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) \\ & - \Lambda\left[1 + \frac{\bar{h}}{\bar{\bar{h}}}\right]^{-\frac{N}{\Lambda}} e^{-\lambda[w + [r - p_0]]} \\ & \times \exp\left(\text{LambertW}\left(\frac{\Phi}{\Lambda}\left[1 + \frac{\bar{h}}{\bar{\bar{h}}}\right]^{-\frac{N}{\Lambda}} e^{\lambda[w + [r - p_0]]}\right)\right) \end{aligned}$$

which can be shown to have everywhere a negative second partial derivative with respect to $\bar{h}$.

3.9 Existence

The proof of the following existence claim is given in Section F on page 50 of the Appendix and follows by construction from the preceding discussion.

Proposition 3.2. (Existence of separating equilibrium) *For preferences of the "LEL" form and with a continuum of types and neighbourhood locations, there is a unique allocation of consumption $x(w)$, $h(w)$, and $\bar{h}(w)$ conforming to the equilibrium of Definition 3.3.*

3.10 Welfare analysis of interior equilibria

The equilibrium utility can now be written in terms of exogenous parameters,

$$\begin{aligned} U(w) \quad &= \quad \Phi \log\left(\min\left\{\frac{w_0}{w}, 1\right\}\right) - \Lambda + \Lambda\lambda \max\{0, w - w_0\} \\ &\quad + N \log\left(\frac{2[w_H - w_L]}{w_m - w_L + \frac{1}{\frac{\Lambda\lambda}{N}} e^{\frac{\Lambda\lambda}{N}[w_m - w_0]}\left[e^{\frac{\Lambda\lambda}{N}[w_H - w_m]} - 1\right]}\right) \end{aligned}$$

Note that the last term depends on the distribution of types but not on individual w. Also, the equilibrium welfare does not depend on the choice of base price p_0 in the land market. Using the notation $\Theta \equiv w_H / w_L$, the utility for the interior case, when $w_L > w_0$, takes the form

$$U(w) = \Phi \log\left(\frac{\Phi}{\Lambda\lambda w}\right) - \Lambda + \Lambda\lambda\,[w - w_L] + N \log\left(\frac{2\frac{\Lambda\lambda}{N}[\Theta - 1]w_L}{e^{\frac{\Lambda\lambda}{N}[\Theta - 1]w_L} - 1}\right)$$

For simplicity, the analysis to follow focuses on interior equilbria. Properties of this equlibrium can now be summarised as follows.

Intra-neighbourhood comparisons Welfare disparity is intensified not by the strength N of preferences over inter-neighbourhood comparisons, but by the strength of the local, intra-neighbourhood Veblen effect, $\Lambda\lambda$:

$$\frac{dU}{dw} = \Lambda\lambda - \frac{\Phi}{w} > 0$$

The negative term reflects the fact that to the extent that non-pecuniary pursuits are important to household utility, i.e. that Φ is large, endowment differences will not be reflected in welfare disparities.

Improvements to productivity As noted by Eaton and Eswaran [2006], improvements to productivity in the Veblen good industry can be harmful to welfare. Consider a multiplicative shift in the entire range of household productivities. This corresponds to raising or lowering w_L while holding Θ constant.

To assess the implication of an increase in productivity within a heterogeneous population, two marginal effects must be considered. A given household will experience individual productivity enhancement $dw = \frac{w}{w_L} dw_L$. The household's change in utility will be the sum of a component due to this individual shift within the distribution $U(w)$ and one due to the changing distribution. The latter effect is

$$\left.\frac{\partial U}{\partial w_L}\right|_{\Theta} = -\Lambda\lambda + \frac{N}{w_L} - \Lambda\lambda \frac{[\Theta-1] e^{\frac{\Lambda\lambda}{N}[\Theta-1] w_L}}{e^{\frac{\Lambda\lambda}{N}[\Theta-1] w_L} - 1} \tag{30}$$

$$= -\Lambda\lambda + \frac{N}{w_L} - \Lambda\lambda \frac{\Theta - 1}{1 - e^{-\frac{\Lambda\lambda}{N}[\Theta-1] w_L}} \tag{31}$$

which fits a form of the function $\Psi(\cdot)$ defined and characterised in Lemma E.3 on page 48 on page 48:

$$\left.\frac{\partial U}{\partial w_L}\right|_{\Theta} = -\Lambda\lambda + \Psi\left(-\Lambda\lambda[\Theta-1], \frac{w_L}{N}\right) < 0$$

The inequality follows from the property that $\Psi(-a, b) < 0$ for positive a and b. The overall marginal effect on a given household of rescaling productivity is the sum of the individual and distributional effects:

$$\begin{aligned}
dU &= \frac{\partial U}{\partial w} dw + \left.\frac{\partial U}{\partial w_L}\right|_{\Theta} dw_L \\
&= \frac{\partial U}{\partial w} \frac{w}{w_L} dw_L + \left.\frac{\partial U}{\partial w_L}\right|_{\Theta} dw_L \\
&= \left[\Lambda\lambda - \frac{\Phi}{w}\right] \frac{w}{w_L} dw_L + \left[-\Lambda\lambda + \Psi\left(-\Lambda\lambda[\Theta-1], \frac{w_L}{N}\right)\right] dw_L \\
&= \left[\Lambda\lambda \left[\frac{w}{w_L} - 1\right] - \frac{\Phi}{w_L} + \Psi\left(-\Lambda\lambda[\Theta-1], \frac{w_L}{N}\right)\right] dw_L
\end{aligned} \tag{32}$$

Numerical simulations of this function are explored below. The second and third terms are strictly negative for positive dw_L, and for large Φ in this pure Veblen labour economy every individual is worse off when productivities of each participant household are uniformly scaled up.

In general, growth in this context has negative welfare implications for the least wealthy, and may have positive benefits for the wealthiest.

The homogeneous population case from Eaton and Eswaran [2006] can be recovered by noting from Lemma E.3 on page 48 that

$$\lim_{\Theta \to 1} \left.\frac{dU}{dw_L}\right|_{\Theta} = -\frac{\Phi}{w}$$

That is, for homogeneous populations with sufficient productivity to merit production in the Veblen good industry, any increase in productivity is uniformly bad for welfare.

Helping the poor Indeed, even raising the productivity of only the poorest is bad for everyone else's welfare, a counterintuitive result when thinking is conditioned by non-Veblen goods models:[24]

$$\begin{aligned} \left.\frac{\partial U}{\partial w_L}\right|_{w_H} &= -\Lambda\lambda - \frac{N}{w_H - w_L} + \frac{\Lambda\lambda}{e^{\frac{\Lambda\lambda}{N}[w_H - w_L]} - 1} \\ &= -\Lambda\lambda - \Psi\left(\frac{w_H - w_L}{N}, \Lambda\lambda\right) < 0 \end{aligned}$$

Wealthy and Veblen good productivity Increasing productivity in this model is, however, not bad policy in all cases. Adding wealthy households to the economy is beneficial for everyone due to the redistributive effects outweighing the comparison externality:

$$\begin{aligned} \left.\frac{\partial U}{\partial \Theta}\right|_{w_L} = w_L \left.\frac{\partial U}{\partial w_H}\right|_{w_L} &= \frac{N}{\Theta - 1} - \frac{\Lambda\lambda w_L}{e^{\frac{\Lambda\lambda}{N}[\Theta - 1]w_L} - 1} \\ &= \Psi\left(\Lambda\lambda w_L, \frac{\Theta - 1}{N}\right) > 0 \end{aligned} \tag{33}$$

Disparity In order to investigate the effect of disparity, consider next a mean-preserving spread in the distribution of w. Rewriting $w_L = \langle w \rangle - \frac{1}{2}\Delta$ and $w_H = \langle w \rangle + \frac{1}{2}\Delta$, the effect of a change in the range Δ is:

$$\begin{aligned} \left.\frac{\partial U}{\partial \Delta}\right|_{\langle w \rangle} &= \left.\frac{\partial}{\partial \Delta}\right|_{\langle w \rangle} \left(\Phi \log\left(\frac{\Phi}{\Lambda\lambda w}\right) - \Lambda + \Lambda\lambda\left[w - \langle w \rangle + \frac{\Delta}{2}\right] + N \log\left(\frac{2\frac{\Lambda\lambda}{N}\Delta}{e^{\frac{\Lambda\lambda}{N}\Delta} - 1}\right)\right) \\ &= \frac{1}{2}\Lambda\lambda + \frac{N}{\Delta} - \frac{\Lambda\lambda}{1 - e^{-\frac{\Lambda\lambda}{N}\Delta}} \\ &= \frac{1}{2}\Lambda\lambda + \Psi\left(-\Lambda\lambda, \frac{\Delta}{N}\right) < 0 \end{aligned}$$

Increasing exogenous disparity at a constant mean productivity does not affect average consumption $\left(\frac{d\bar{\bar{h}}}{d\Delta} = 0\right)$ nor the price schedule $\left(\frac{dp(\bar{h})}{d\Delta} = 0\right)$ but is uniformly bad for welfare for all extant households. This comes about because when the spread Δ of household productivities increases, the average cost of housing of the new high types and new low types, combined, is less than the old average. In other words, the dividends r decrease:

$$\left.\frac{\partial r}{\partial \Delta}\right|_{\langle w \rangle} = \left.\frac{\partial}{\partial \Delta}\right|_{\langle w \rangle} \left(-\langle w \rangle - \frac{\Delta}{2} + \frac{\Phi}{\Lambda\lambda} + \frac{N}{\Lambda\lambda}\log\left(\frac{2\Lambda\lambda\Delta}{N\left[1 - e^{-\frac{\Lambda\lambda}{N}\Delta}\right]}\right)\right)$$

[24]This experiment consists of removing the least productive households from the economy. Therefore, the welfare of the removed households is not included. However, when the loss of dividends r by the removed households outweighs the extra income from their improved w, they too will prefer to remain within the economy. Thus, removing them represents a Pareto decline.

$$
\begin{aligned}
&= -\frac{1}{2}+\frac{N}{\Lambda\lambda\Delta}-\frac{e^{-\frac{\Lambda\lambda}{N}\Delta}}{1-e^{-\frac{\Lambda\lambda}{N}\Delta}} \\
&= -\frac{1}{2}+\frac{N}{\Lambda\lambda\Delta}-\frac{1}{e^{\frac{\Lambda\lambda}{N}\Delta}-1} \\
&= -\frac{1}{2}+\Psi\left(1,\frac{\Lambda\lambda}{N}\Delta\right)<0
\end{aligned}
$$

The inequality follows, once again, from Lemma E.3 which shows that

$$\lim_{b\to 0}\Psi(1,b)=\frac{1}{2}$$

and that $\frac{d}{db}\Psi(1,b)<0$.

3.11 Empirical interpretation

A central feature of the empirical results of Barrington-Leigh and Helliwell [2007] is that the well-being effect of a marginal change in the affluence of one's immediate neighbours is much smaller than the effect of a marginal change in broader consumption averages, which are strongly negative. Accordingly, the separating equilibrium modeled here has the feature that, after households have chosen their neighbourhood reference group, $dU/d\bar{h}=0$ but $dU/d\bar{\bar{h}}$ is significantly negative.

3.12 Log-exp-log utility with absentee landlords

In the previous section, feedback from the aggregate effects of the distribution over types onto the household decision problem comes through both r and $\bar{\bar{h}}$. When land rents are high, the equitable land ownership model significantly redistributes income by returning land rents uniformly to all households, thus narrowing the relative dispersion in wealth and consumption. Because the distribution of consumption is central to household choices and to welfare analysis, the details of how land equity is distributed matters in interpreting equilibrium outcomes.

An alternative extreme is for none of the land rents to be returned to households in the economy; this is the case of absentee landowners. Welfare analysis is also complicated, however, when rents are paid to absentee landowners unless the welfare of those landowners is somehow included in the accounting.

When $r=0$, households who choose not to work have no outside income with which to pay for land or housing. These households with $w<w_0$ prefer to pool together in a "slum" enjoying leisure $x=1$, no conspicuous housing consumption, and a reference neighbourhood with zero consumption. That is, $\bar{h}_{\min}$ becomes 0 and the price of land there, p_0, must also be zero. Thus, with $r-p_0=0$, equation 26 becomes a knife-edge constraint on parameters. Except for certain peculiar parameter sets, there are no separating equilibria when land is owned by absentees and rents leave the economy.

3.13 Pooling equilibria

When all neighbourhoods have an equivalent mix of types, $\bar{h} = \bar{\bar{h}}$ for all households. Household choice of consumption h and leisure $x = 1 - h/w$ maximises

$$U(x,h,\bar{h}) = \Phi \log(x) - \Lambda \exp\left(-\lambda \left[h - \bar{h}\right]\right) + N \log(2) \tag{34}$$

$$h(w) = \begin{cases} 0, & \text{for } w \leq w_0 e^{-\lambda \bar{h}} \\ w - \frac{1}{\lambda} \text{LambertW}\left(\frac{\Phi e^{\lambda w - \lambda \bar{h}}}{\Lambda}\right), & \text{otherwise} \end{cases}$$

The average consumption $\bar{\bar{h}} = \bar{h}$ can be expressed recursively by computing the average value of $h(w)$:

$$\begin{aligned} \bar{h} &= \left\langle \max\left\{0,\ w - \frac{1}{\lambda}\text{LambertW}\left(\frac{\Phi e^{\lambda w - \lambda \bar{h}}}{\Lambda}\right)\right\}\right\rangle \\ &= \frac{w_H^2 - w_m^2}{2\left[w_H - w_L\right]} - \frac{\text{LambertW}\left(\frac{\Phi e^{\lambda w_H - \lambda \bar{h}}}{\Lambda}\right)^2 - \text{LambertW}\left(\frac{\Phi e^{\lambda w_m - \lambda \bar{h}}}{\Lambda}\right)^2}{2\lambda^2\left[w_H - w_L\right]} \\ &\quad - \frac{\text{LambertW}\left(\frac{\Phi e^{\lambda w_H - \lambda \bar{h}}}{\Lambda}\right) - \text{LambertW}\left(\frac{\Phi e^{\lambda w_m - \lambda \bar{h}}}{\Lambda}\right)}{\lambda^2\left[w_H - w_L\right]} \end{aligned} \tag{35}$$

where $w_m = w_m(\bar{h}) \equiv \max\left\{w_L,\ w_0 e^{-\lambda \bar{h}}\right\}$. Equation 35 may be solved numerically for $\bar{h}$, from which values for $\bar{h}(w)$ and $U(w)$ follow. In this equilibrium, each household randomises its choice of neighbourhood since all neighbourhoods are alike and present the same environment $\bar{h} = \bar{\bar{h}}$. No deviation to another neighbourhood is beneficial and households choose only their individual consumption, h, given the global mean consumption level. This global consumption level is determined by the collective external effects of each household's choice of h. Properties of such pooling equilibria are demonstrated numerically, below.

3.14 Planner's problem

In an economy with a pure Veblen good such as the one modeled here, a reasonable policy for a planner is to prevent, for instance through prohibitive taxation,[25] any production of the Veblen good at all. Under this constraint, all households enjoy leisure $x = 1$ and inhabit identical neighbourhoods with $\bar{h} = 0$. The utility in this case is uniformly

$$U = -\Lambda + N \log(2)$$

[25] Here a relevant distinction is between a status good valued through a comparison of actual consumption and one which is valued by its cost to the buyer. The former is treated in this paper, while the latter is sometimes referred to as a "snob" good. In the snob good case, taxing the good may not affect net houshold expenditure on it, but it will still decrease its production and redistribute the revenue.

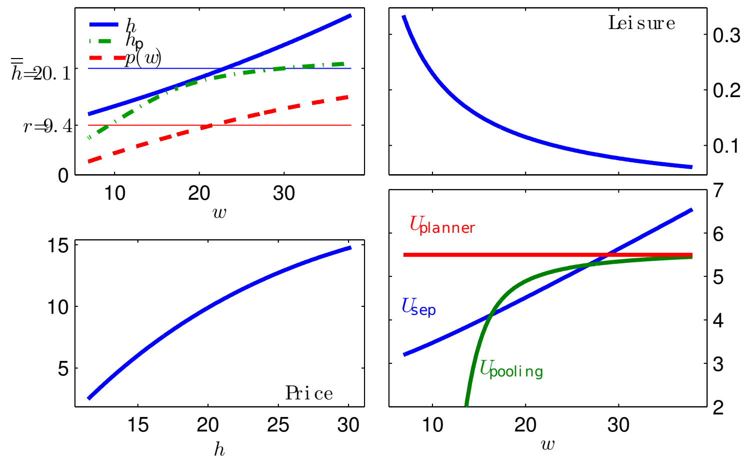


Figure 5: **An equilibrium with monotonically increasing price amongst occupied neighbourhoods.** Parameters: $w \in (6.9, 37.9), \Phi = 0.3, \Lambda = 0.2, \lambda = 0.6, N = 8.2$

Below I demonstrate numerically, echoing the earlier results using discrete neighbourhoods, that this outcome does not necessarily Pareto dominate the disaggregated decision equilibrium in which households consume the Veblen good and separate into reference neighbourhoods. This constitutes an important difference from the findings of Eaton and Eswaran [2006].

4 Numerical analysis

This section demonstrates through simulations some of the features that have been described analytically, and emphasises the diversity of possible outcomes given different choices of parameters. Appendix F outlines the method used here for numerically constructing separating equilibria for the problem described in Section 3.6. The pooling equilibria could not be characterised in closed form, so these have also been simulated numerically by solving equation 35.

Figure 5 depicts equilibria for one sample set of parameters. The top left panel shows the separating equilibrium distribution of household consumption $\bar{h}(w) = h(w)$ as well as its mean value $\bar{\bar{h}}$. Also shown is the rent $p(w)$ paid for land by each type w and its mean value r. For this economy, households all spend more on their housing than they do on buying their way into a neighbourhood. Also shown for comparison is the pooling equilibrium outcome $h_p(w)$; in this case all households spend less when mixed in identical heterogenous neighbourhoods than when they are sorted into their preferred reference groups.

The top right panel shows the dependence of leisure on type for the separating equilibrium. In all cases, it is weakly concave and decreasing. The lower left panel shows land price as a function of neighbourhood consumption. For the parameters used in this case, the price is an increasing function of neighbourhood affluence.

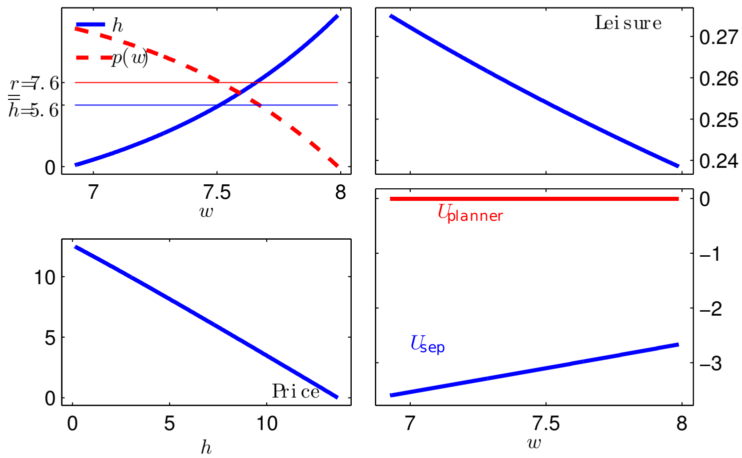


Figure 6: **An equilibrium for which $r > \bar{\bar{h}}$.** Parameters: $w \in (6.9, 8.0), \Phi = 2.3, \Lambda = 0.7, \lambda = 1.6, N = 1.0$

The lower right panel shows welfare distributions for three scenarios: the separating equilibrium (U_{sep}), the pooling equilibrium (U_{pooling}), and the planner's economy (U_{planner}, see Section Section 3.14), in which no one consumes the Veblen good and all households enjoy the maximum amount of leisure. The planner's economy Pareto dominates the pooling equilibrium but is preferred to the separating equilibrium only by the lower types.

Figure 6 shows a case with several qualitative differences. For these parameters, the land rent is a *decreasing* function of neighbourhood affluence, indicating that the marginal cost of a higher local reference group outweighs the marginal benefit of a higher-status neighbourhood.

The preferences in this example differ from those in Figure 5 in part by having a much higher relative weight on local comparisons as compared with neighbourhood comparisons; that is, $N/\Lambda\lambda$ is much smaller. In this case, the emphasis in preferences on consumption comparison at the local level as compared with leisure is also high, and the Veblen equilibrium is fully Pareto dominated by the planner's outcome with no Veblen good.[26] The significance of allowing for endogenous reference group choice is highlighted in this economy by the fact that households are spending more, on average, on reference group selection — *i.e.*, land — than on the underlying Veblen good itself.

Figure 7 shows a land rent schedule that is peaked at $\bar{h} = \frac{N}{\Lambda\lambda} - \bar{\bar{h}}$ (see Section 3.6). The equilibrium also includes pooling neighbourhoods in which households with low productivity choose not to work at all. In the case shown in Figure 8, every household type consumes more Veblen good and suffers lower utility in the pooling equilibrium than in the separating case. Both outcomes would be unanimously rejected in favour of the planner's allocation.

Figure 9 shows the effect on households of uniform growth in the economy. As

[26]No pooling equilibrium was found for this set of parameters.

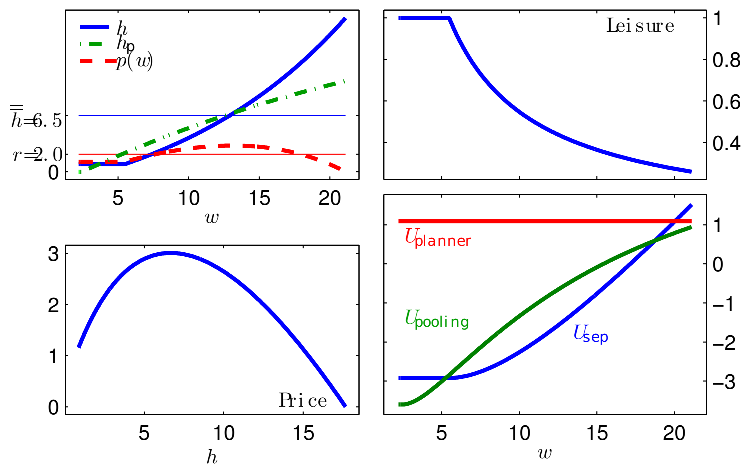


Figure 7: **An equilibrium with non-monotonic price.** Parameters: $w \in (2.2, 21), \Phi = 2.9, \Lambda = 3.8, \lambda = 0.1, N = 7.1$

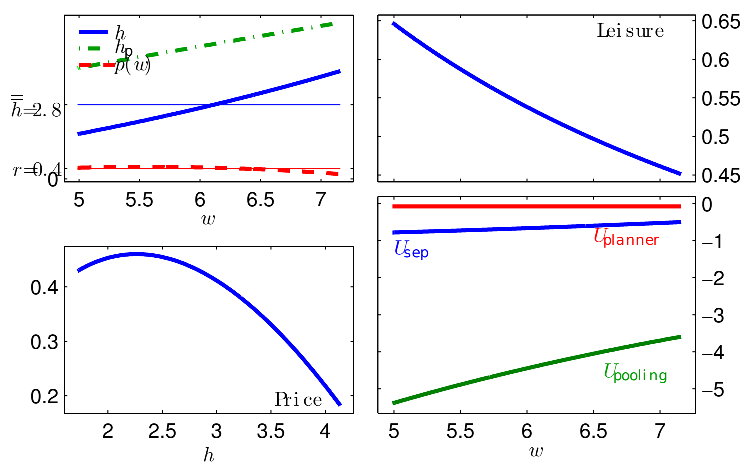


Figure 8: **An equilibrium in which all households prefer the separating equilibrium to the pooling one.** Parameters: $w \in (5.0, 7.2), \Phi = 0.9, \Lambda = 1.1, \lambda = 0.3, N = 1.4$

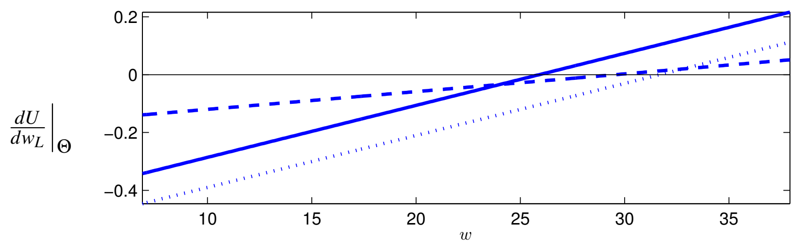


Figure 9: **Total marginal change to welfare in an economy subject to uniform growth.** The solid line represents the case of Figure 5 on page 28, the dashed line is the same case except that λ is decreased to 0.2, and the dotted line is the same case except that Φ is increased to 1.

represented in equation (32) on page 24, the overall benefits of growth for a given household may be positive or negative. The three cases shown in the figure illustrate that the strength of the dependence of growth effects on w is proportional to λ and that when households place a higher value on leisure, the effect of growth is worse for all.

5 Conclusion

Most of economic analysis is still predicated on the plausibility of fixed preferences over absolute consumption. In this paper I take seriously the idea that absolute consumption utility benefits are unlikely to be sensible to humans when modern consumption levels are orders of magnitude higher in real terms than during the vast majority of our evolutionary history. I take a modest step towards exploring some calculus of choice and macroeconomic equilibria when preferences are, indeed, purely relative to what we know and see and when, in addition, we are able to exert some choice over what it is that we know and see.

By considering utility functions with a "pure Veblen" component this work accounts for goods which are consumed conspicuously or "publicly" and therefore are likely preferentially to affect neighbours in close proximity to the consumer. The benefits of "privately" consumed goods are captured in the so-called "leisure" term, $\Phi(x)$, which may encompass not only activities involving social engagement but also other classes of relative preferences for which reference levels are set through means other than the observation of local contemporaries. For instance, expectations about lifestyle and consumption are influenced by advertising and by broad dissemination of cultural norms.

It should above all be kept in mind that the empirical work which motivated this investigation of relative consumption preferences indicates that market-oriented consumption (as proxied by income) benefits are not only relative to others' but are also relatively insignificant for well-being as compared with the contribution from other factors such as positive social engagement. Thus, the importance of social groups in this work might correspond to the lesser of two significant roles: in a broader view, pursuit of social groups is important for the direct social benefits they confer as well as for their influence on emulation behaviour through consumption externalities.

What can be learned from a purely theoretical investigation is limited. Nevertheless, even the extreme models presented here suggest some insights to add to those developed in past work. Firstly, the allowance for heterogeneity and reference group selection significantly modifies the characteristics of general equilibrium. When agents have the tendency to use their own social group rather than a global one as a reference, the ability to differentiate into like groups can lead to a more efficient outcome than that of a heterogeneous mix of types, as evidenced by Figure 8. A general interpretation is that the existence of regional diversity can mitigate the extreme Veblen problem described by Eaton and Eswaran [2006]. On the other hand, this mitigation is by no means certain. In Figures 5 and 7, only the highest and lowest types prefer the separating equilibrium to the pooling one.

The most significant findings from this paper and some key differences from those of Eaton and Eswaran [2006] are that (1) complete elimination of the Veblen good may not be in everyone's interest and (2) growth in productivity in the Veblen good industry may be beneficial to some households. Even though the economy takes the form of a "rat race" due to the existence of a pure Veblen good, the most wealthy and productive households may actually prefer to have the good permitted on the market and prefer to have policy geared towards increased productivity in producing it. These features are shown, for example, in Figures 7 and 9. If the high types which benefit from the Veblen economy also have a more than proportional share of influence over policy, they will find it in their interest to promote the production of a completely "useless" good with ever increasing efficiency.

The degree to which such preferences and consequent externalities form an important part of the real economy remains an active empirical question, but the conclusions from this exploration of heterogeneity and autonomous group selection suggest that one should take seriously warnings given by Eaton and Eswaran [2006] about the canonical assumptions and focus on material consumption growth by economists. If pursuit of Veblen good economies is not *pure* folly, it is only wise from the point of view of the wealthiest consumers.

References

C. P. Barrington-Leigh and John F. Helliwell. Empathy and emulation: life satisfaction and the urban geography of veblen effects. 2007.

A.E. Clark and A.J. Oswald. Comparison-concave utility and following behaviour in social and economic settings. *Journal of Public Economics*, 70(1):133–155, 1998.

Andrew E. Clark, Paul Frijters, and Michael A. Shields. Relative income, happiness, and utility: An explanation for the easterlin paradox and other puzzles. *The Journal of Economic Literature*, 46(1):95–144, 2008. URL http://www.aeaweb.org/articles.php?doi=10.1257/jel.46.1.95. doi:10.1257/jel.46.1.95.

R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffrey, and D.E. Knuth. On the LambertW function. *Advances in Computational Mathematics*, 5(1):329–359, 1996.

E.W. Dunn, T.D. Wilson, and D.T. Gilbert. Location, Location, Location: The Misprediction of Satisfaction in Housing Lotteries. *Personality and Social Psychology Bulletin*, 29(11):1421–1432, 2003.

B. Curtis Eaton and Mukesh Eswaran. Well-being and affluence in the presence of a veblen good. February 2006.

G.G. Kingdon and J. Knight. Community, Comparisons and Subjective Well-being in a Divided Society. *Journal of Economic Behaviour and Organization*, 2007.

G. Loewenstein, T. O'Donoghue, and M. Rabin. Projection bias in predicting future utility. *The Quarterly Journal of Economics*, 118(4):1209–1248, 2003.

Erzo F. P. Luttmer. Neighbors as negatives: Relative earnings and well-being. *Quarterly Journal of Economics*, 120(3):963–1002, August 2005.

K. Marx and F. Engels. Marx engels selected works. Progress Publishers (1986), 1848.

L. Rayo and G.S. Becker. Evolutionary Efficiency and Mean Reversion in Happiness. *mimeographed, University of Chicago*, 2004.

J.M. Rothstein. Good principals or good peers? Parental valuation of school characteristics, tiebout equilibrium, and the incentive effects of competition among jurisdictions. *The American economic review*, 96(4):1333–1350, 2006.

S. Scotchmer. Local Public Goods and Clubs. *Handbook of Public Economics*, 4: 1997–2042, 2002.

A. Tversky and D. Kahneman. Loss Aversion in Riskless Choice: A Reference-Dependent Model. *The Quarterly Journal of Economics*, 106(4):1039–1061, 1991.

Appendices

This appendix provides detail and more in-depth discussion of several issues raised in or tangential to the main body of the paper. Section A explains the relationship of this work to the literature on club economies. Section B discusses some very simple models of heterogeneity and Veblen preferences which give intuition for the more general case. Section C discusses the choice of functional forms used in the paper. Section E outlines proofs of propositions stated earlier, and Section F describes how to construct the separating equilibrium used in numerical examples.

A Endogenous reference groups are not club goods

There is a large existing literature on local public goods and agglomeration into clubs. The problem I address here is distinct in a couple of ways. Neighbourhoods are unlike clubs in that their membership does not explicitly choose an entry price, nor do they coordinate (typically through a voting scheme, or just through coordination in the core equilibrium) on the nature of the public good they provide. That is, I assume that neighbourhoods do not set standards for how lawns and gardens must be kept or how ornate new houses must be. Rather, in the models here the homogeneous behaviour within neighbourhoods is a result only of relativities in preferences and possibly of the individual ability to pay in a competitive market for land.

I also ignore for the moment congestion and the endogenous sizing of communities or of lots of land, although these are clearly relevant to spatial development patterns. A relevant observation is that in Canada the lowest density settlements are populated by the richest and poorest. High density areas are populated by more median incomes, presumably in urban high-rises.

Standard efficiency considerations for models of local public goods and "capitalisation" are not appropriate when the public goods are Veblen goods. Existing models tend to focus on tax and price systems which afford efficient allocations within jurisdictions and efficient location choice for individuals. Because I do not assume that differentiation of types occurs on the same geographic scale as tax taking institutions or those offering public services, I just ignore those policy instruments and look for equilibria without them.

Although the models I consider below do not include actions of coordinated neighbourhoods, insights from the work could inform local providers of public goods such as neighbourhood associations, clubs, or local governments. Along with public good problems, these groups face migration and changing distribution of local wealth, pressures on land price, and economic growth, which are the key concepts to follow.

B An introduction to neighbourhood segregation in the presence of Veblen goods

To introduce the ideas to come, consider a straightforward application of heterogeneity to the *Pure Veblen 1* formulation of Eaton and Eswaran [2006], as described below.

B.1 Exogenous segregation and Veblen consumption

Let there be two types of household, differentiated only by their endowed labour productivities, $w_H > w_L$. Preferences are generated by utility

$$U = F(x) + H(h - \bar{h})$$

where $0 < x < 1$ is chosen leisure and the numeraire good, h, is purchased according to the budget, $h \leq w[1-x]$. This good is a Veblen good in that its benefit is derived only through consumption relative to a reference consumption level, $\bar{h}$. We may imagine that h measures a form of intrinsically useless conspicuous consumption such as living in a grandiose house.[27] In this model economy and several of those to follow, building such houses is the only industry. The function $H(\cdot)$ then represents the status value of living with consumption level h amongst neighbours with average consumption level $\bar{h}$.

One can begin by considering the welfare implications of inequality. Is society better off with distinct types, w_H and w_L, segregated or integrated? If neighbourhoods characterised by their averge consumption $\bar{h}$ can be completely separated into homogeneous groups, then the utilities of the two types will be

$$\begin{aligned} U_L^s &= F(x_L^s) + H(0) \\ U_H^s &= F(x_H^s) + H(0) \end{aligned}$$

whereas in a homogeneously mixed community, outcomes are, for the case of equal populations in the two types,

$$\begin{aligned} U_L^m = &\quad F(x_L^m) + H(-\Delta) \quad < U_L^s \\ U_H^m = &\quad F(x_H^m) + H(\Delta) \quad > U_H^s \end{aligned}$$

where $\Delta \equiv \frac{1}{2}h_H^m - \frac{1}{2}h_L^m$. The given inequalities follow if $H(\cdot)$ is strictly concave. They indicate that the high types are better off in the integrated community while the low types are worse off. Furthermore, concavity of both $H(\cdot)$ and $F(\cdot)$ implies that $x_H^m > x_H^s$ and $x_L^m < x_L^s$. That is, the high productivity individuals will work less in the integrated community than in the segregated one, and conversely for the less productive type. In addition, $H(\Delta) + H(-\Delta) < 2H(0)$; that is, the summed benefits derived from housing alone are higher in the segregated case. However, without specifying functional forms, no definitive statement can be made about the relative efficiencies of the two cases on the basis of summed utilities which include both leisure and housing.[28]

[27] Such "pure Veblen" goods represent the case when any intrinsic value to increased consumption of the good suffers strongly from diminishing returns. Similar outcomes may be seen in the more general case when both absolute and relative benefits accrue from consumption. When others' consumption is roughly on par with one's own, the relative effects, or Veblen terms, remain while the absolute benefits saturate [Eaton and Eswaran, 2006]. In the current context, all houses are large enough to satisfy needs and provide most benefits that dwellings can provide their owners directly.

[28] Using the concept of "transferable utility" to justify the common practice of adding utilities and ordering outcomes on the basis of social (i.e., aggregate) welfare may seem a dubious method when utility has its normal modern interpretation as an abstract determinant of decision making. The widespread availability now of measured subjective well-being, however, gives the current exercise the slightly more empirical interpretation of comparing regional average life satisfaction levels under the two scenarios.

Similarly, the efficiency implications of growth are ambiguous. In the segregated case, growth in either w_H or w_L beyond some minimum level is unequivocally bad for welfare, as in Eaton and Eswaran [2006]. On the other hand and in contrast to their homogeneous case, the implications of growth for the mixed community is indeterminate without more assumptions.

Several of these ambiguities recur below when segregation is an endogenous outcome. Rather than pursue welfare analysis for specific functional forms at this stage, I consider next the implications of disaggregated choice in this simple economy in order to motivate necessary subsequent extensions to the model.

B.2 Endogenous segregation without neighbourhood benefits

To continue the introduction of heterogeneity to the competitive Veblen economy described in Eaton and Eswaran [2006], I now incorporate disaggregated decision making in the two-neighbourhood world of Section B.1. Consider the case where multiple neighbourhood locations exist and each household chooses where to live as well as how much to spend on its own consumption of housing. Assume that households are aware of the possibility of relocating, even though their consumption comparisons extend only to their own, chosen neighbours. This represents an endogenous choice of reference group for the Veblen good.

For any form of utility $U = F\left(x, v\left(h, \bar{h}\right)\right)$ with $\partial U / \partial \bar{h} < 0$, however, such neighbourhood differentiation is not possible. The uniform undesirability of high-consumption neighbours means that all decision makers will prefer the lowest available average neighbourhood consumption $\bar{h}$. That is, all types want to live with the poorest neighbours. Even if there is a free market for land in different neighbourhoods, the most able to buy are the most wealthy and therefore those with the least desirable externality to their neighbours. As a result, no differentiated neighbourhoods may exist in equilibrium.

B.3 Neighbourhood benefits

A segregated equilibrium where migration is a choice can thus only exist if there is coordination of some kind between members of a neighbourhood[29] or if there is another, countervailing externality acting in addition to the negative consumption externality considered above.

That is, in order to explain why wealthy types and poor types might each prefer to live amongst their own, there must be a neighbourhood benefit simultaneously with the local consumption comparison. There are several obvious reasons for higher productivity neighbourhoods to be desirable:

1. Productivity w could be an exogenous feature of location rather than of the individual; in cross-sectional or short-term studies this amounts to historical factors which determine opportunity or availability of resources.

[29]For instance, to assess taxes or membership fees. I do not consider this possibility, which as mentioned above is well covered by the literature on "club economies".

2. Neighbourhoods could be characterised by the intrinsic quality of their residents in a way which confers either pecuniary benefits to neighbours (through higher income opportunities due to networking or signaling) or social (non-wage) benefits through higher quality social interactions or more efficient child-rearing and home production.

3. Conspicuous consumption by neighbours (for instance and in particular, fancy houses) could determine a common status value enjoyed by all residents of a neighbourhood. In this case, the consumption comparison group is other neighbourhoods in the region.

In Section 2, I begin by considering benefits of type 2 and later focus on a self-consistent economy incorporating benefits of type 3. Either of these positive spillovers within a neighbourhood may exist simultaneously with the negative spillovers due to local, neighbour-to-neighbour consumption externalities.

One can imagine a model economy in which decision makers weigh these two effects against each other in order to choose a place to live. A household's choice of neighbourhood will be optimal if the benefit from that neighbourhood less the consumption externality suffered from living there is better than any other available option. This is the decision problem if entry into each neighbourhood is free to anyone who chooses it. I will show below that this scenario does not always lead to an economy with more than one neighbourhood.

Alternatively, entry into a neighbourhood might have a further cost due to the price of land. Whatever are the reasons behind the benefit from living in a particular location, that benefit may be captured in land prices that arise endogenously in the economy; this is known as *capitalisation* in the literature on club economies.[30] When no differentiated neighbourhoods are possible with free land, there may still be separated equilibria when a land market exists. I address both cases of priced and unpriced land in the discussion that follows.

Because the form of preferences under discussion, which incorporates relativities, is unusual in economics, it will prove useful to explore some qualitative features using simple functional forms which, based on experience with non-Veblen utility functions, one might expect to be tractable. It turns out that a prominent feature of even simple forms of preferences involving endogenous reference groups is that the utility function is not globally concave. As a result, in many cases a desirable equilibrium does not exist or exists only for special sets of parameters.

The next section introduces some functional forms used in subsequent analysis, before building simple equilibria in which endogenous choice of reference groups leads to differentiated neighbourhoods.

C Functional forms for Veblen preferences

Economists tend to use a narrow class of functional forms in parameterising utility. These functions are selected for their convenient macroeconomic properties and they

[30] See Scotchmer [2002] for a review.

typically have a domain restricted to positive values, which are appropriate for the study of preferences over absolute consumption levels. When describing preferences over relative consumption levels, these may be insufficient and new classes of functions which tend to be unfamiliar to consumption theory may be useful.

Clark and Oswald [1998] point out that utility which is concave in relative consumption leads to emulation, while comparison-convex utility leads to deviant behaviour. Tversky and Kahneman [1991]'s empirical findings on loss aversion might be rationale for expressing comparison utilities using a form of sigmoid curve, easily expressed using a hypertrigonometric form. The bounded extremes and slopes of such a function are also conducive to efficient numerical simulation. However, not being concave, sigmoid utility makes marginal analysis difficult.

Eaton and Eswaran [2006] consider two classes of comparison-concave utility. These are general concave increasing functions of either a difference or a ratio of own and average consumption levels. In the present work, I employ explicit forms for each of these two classes.

$$H(h,\bar{h}) = \Lambda \log\left(1 + h/\bar{h}\right)$$

and

$$H(h,\bar{h}) = -\Lambda \exp\left(-\lambda\left[h-\bar{h}\right]\right)$$

Both forms are increasing, comparison-concave, and continuous for any nonnegative h and positive $\bar{h}$. Both are relatively simple and likely to have analytic tractability.

D Nonexistence of separating equilibrium for discrete types model

In Section D.1, I introduce a slightly simplified utility form in which *benefits* from neighbours' consumption enters directly into the utility function, without comparison to other neighbourhoods. Section D.2 analyses one functional form in this class of utility functions, showing that there can be no equilibrium in which types separate. Sections D.3-D.5 consider several variations on the story which play a role in the treatment of a continuum of agent types in Section 3.

D.1 Direct neighbourhood benefits

This section formalises the endogenous reference group choice problem outlined above, in which households derive benefit from their relative consumption of housing but their absolute consumption of neighbourhood quality. Preferences are defined over leisure $x \geq 0$, the conspicuous extravagance $h \geq 0$ of one's house, and the average value $\bar{h}$ of houses in one's choice of a neighbourhood.

Utility is, as before, additively separable into a leisure term $F(\cdot)$, a pure Veblen term $H(\cdot)$ comparing own consumption with that of one's chosen peers, and a further absolute benefit $N(\cdot)$ derived from the consumption level of one's chosen peers:

$$U = F(x) + H(h,\bar{h}) + N(\bar{h}) \tag{36}$$

The benefits represented by $N(\cdot)$ are derived through one of the first two channels described in section Section B.3.

In maximising this utility function, an agent of type w is constrained by the budget

$$w[1-x] \geq h$$

Here $F(\cdot)$, $H(\cdot)$, and $N(\cdot)$ each obey standard convenient assumptions made concrete below. Given the optimality condition

$$x = 1 - h/w \tag{37}$$

the household's decision problem may be reduced to a nested choice of an optimal housing purchase $h^\star(\bar{h})$ for each possible neighbourhood $\bar{h}$, followed by a choice of optimal neighbourhood $\bar{h}^\star$. Holding $\bar{h}$ fixed, $U(h)$ is concave and its global optimum is consistent with the first order condition

$$F'(1 - \frac{h}{w}) = wH_h(h,\bar{h}) \quad \text{or} \quad h = 0 \tag{38}$$

The indirect utility $U(w,\bar{h})$ is then derived by substituting into the utility function (36) the housing choice $h^\star\left(\bar{h}\right)$ which would be selected in a given neighbourhood with average consumption $\bar{h}$:

$$U(w,\bar{h}) = U\left(w, h^\star\left(\bar{h}\right), \bar{h}\right) \tag{39}$$

If there is a discrete set of available neighbourhoods, each household must choose the one offering the highest utility in (39). However, in order to gain insight into the discrete choice optima, consider the case (treated further in Section 3) in which a continuum of neighbourhoods is available. Then (39) presents a continuous choice maximisation problem with no constraints on $\bar{h}$. Notice, however, that there is no guarantee that this optimisation over $\bar{h}$ is also characterised by a concave objective function. The slope $dU(w,\bar{h})/d\bar{h}$ may have a nonmonotonic dependence on $\bar{h}$, meaning that the global optimum may be difficult to find analytically. Moreover, a global maximum does not necessarily even exist, since $U(\cdot)$ may be unbounded even subject to the budget constraint equation (37).

One may understand this by noting that in the scenario described above there is no direct cost to choosing one neighbourhood over another. Without a price for entry to a neighbourhood, for instance in the form of a market for land that is independent from the cost of constructing a house, it is possible for the benefit from having wealthier neighbours to outweigh the penalty from having a relatively less desirable house compared with the one next door.

With this caveat about existence in mind, I now define an equilibrium of interest in which endogenously chosen reference groups are consistent with households being sorted by type. To be more precise, consider a world with, as before, two types of household differentiated only by their endowed labour productivities, $w_H > w_L$, and two neighbourhoods into which individuals may move and build a house.

Definition Then a *discrete separating Nash equilibrium* is a set of allocations $\{h_L \equiv h(w_L), \bar{h}_L \equiv \bar{h}(w_L), h_H \equiv h(w_H), \bar{h}_H \equiv \bar{h}(w_H)\}$ satisfying the necessary optimality conditions for each type w

$$\begin{aligned} \bar{h}(w) &= \bar{h}^\star(w) \\ h(w) &= h^\star(w, \bar{h}^\star(w)) \end{aligned}$$

and the consistency condition

$$\bar{h}(w) = h(w) \tag{40}$$

This last condition states that a neighbourhood's average consumption level $\bar{h}$ is equal to the consumption choice h of its residents.

It turns out that this equilibrium, in which types sort themselves into distinct neighbourhoods, is not possible for some preferences such as the one described next.

D.2 "Log-log-log" preferences with two types

Consider the following particular case of utility given in equation (36):

$$\begin{aligned} F(x) &= \Phi \log(x) \\ H(h, \bar{h}) &= \Lambda \log\left(1 + \frac{h}{\bar{h}}\right) \\ N(\bar{h}) &= N \log\left(\bar{h}\right) \end{aligned}$$

That is, let

$$U(x, h, \bar{h}) = \Phi \log(x) + \Lambda \log\left(1 + \frac{h}{\bar{h}}\right) + N \log\left(\bar{h}\right) \tag{41}$$

The optimal choice of housing within a given neighbourhood takes the simple form

$$h^\star(w, \bar{h}) = \max\left\{0, \frac{\Lambda w - \Phi \bar{h}}{\Phi + \Lambda}\right\} \tag{42}$$

with the corresponding leisure choices[31] given by equation (37):

$$x^\star(w, \bar{h}) = \Phi \frac{1 + \bar{h}/w}{\Phi + \Lambda} \tag{43}$$

Equation (42) states that households will choose to consume less (and enjoy more leisure) when their neighbours consume more. This substitution effect between neighbours' consumption and own consumption is a counterintuitive effect for a Veblen good.

Substituting these values into equation (41) generates the indirect utility $U(w, \bar{h})$ for each household type w. For interior solutions,

[31] Note that despite the superficial appearance of (41), the preferences do not conform to a Cobb-Douglas type, and the optimal allocation to leisure is not independent of others' allocations.

$$U(w,\bar{h}) = \log\left(\left[\frac{\Phi}{\Phi+\Lambda}\right]^{\Phi}\left[\frac{\Lambda}{\Phi+\Lambda}\right]^{\Lambda}\right) + \Phi\log\left(1+\frac{\bar{h}}{w}\right)+\Lambda\log\left(1+\frac{w}{\bar{h}}\right)+N\log\left(\bar{h}\right) \tag{44}$$

The household's problem involves finding the best choice amongst two alternative neighbourhoods $\bar{h}$ available in equilibrium. This goal, or finding a global optimum value $\bar{h}^{\star}(w)$ for this continuous equation, are both nontrivial tasks because $U(w,\bar{h})$ is not concave. Moreover, I next show that a separating equilibrium cannot exist.

Proposition D.1. *When group entry (land) is costless and preferences are given by equation (41), there is no discrete separating group Nash equilibrium with two types.*

A proof is given on page 46 in an Appendix. The only endogenous choice equilibrium is an unsorted one in which all households end up pooling in the same reference group, characterised by the average value of housing consumption. For instance, if productivities are high enough to avoid corner choices, there is a pooling equilibrium where $\bar{h}$ is given by equation (50) with w replaced by its population average.

In order to understand this result more intuitively, it is useful to consider the continuous properties of equation (44) in further detail. A significant feature of the indirect utility $U(w,\bar{h})$ is that it is in general neither monotonic nor concave in the choice of neighbourhood $\bar{h}$. The marginal utility of a shift in neighbourhood consumption is derived from equation (44):

$$\frac{dU}{d\bar{h}} = \frac{1}{w+\bar{h}}\left[\Phi+N+[N-\Lambda]\,\frac{w}{\bar{h}}\right] \tag{45}$$

When neighbourhood benefits are valued highly enough in comparison with local relative consumption, $N>\Lambda$ and $U(w,\bar{h})$ is strictly increasing in $\bar{h}$. In that case, the lower type will always prefer to move up to the higher type's neighbourhood when the two are separated.

If instead $\Lambda>N$, utility is initially decreasing but eventually increasing with $\bar{h}$. According to equation (45), utility is in this case unbounded as $\bar{h}\to 0$ and as $\bar{h}\to\infty$ and has a minimum value U_{min} at

$$\bar{h}_{\min U} = \frac{\Lambda-N}{\Phi+N}w \tag{46}$$

Because both $\bar{h}_{\text{eq}}$ and $\bar{h}_{\min U}$ scale directly with w, households occupying their separating equilibrium neighbourhoods will always either both prefer any higher neighbourhood to their own or both prefer any lower neighbourhood to their own. Thus it is impossible for both types to fulfill the equilibrium requirements.

Figure 10 on page 43 shows the possible cases for preferences conforming to equation (41). The left panels show the dependence of the indirect utility on the neighbourhood location $\bar{h}$ for the high type (red) and low type (green). The dependence is characterised by a minimum value which is proportional to the endowments w, in accordance with equation (46). Marked on each plot as h_L and h_H are the values $\bar{h}_{\text{eq}}$ for which a type's housing choice is consistent with that of its neighbours, *i.e.,* where $h^{\star}(\bar{h}) = \bar{h}$. The right panels show indifference contours for $U(h,\bar{h})$ for two values of w. The dashed lines indicate the optimum housing choice $h^{\star}(\bar{h})$ within each neighbourhood $\bar{h}$. The dotted line is the solution to $h = \bar{h}$, the blue squares show the values of $\bar{h}_{\text{eq}}$, and the red and green squares show each type's optimal choice of h in the *other* type's neighbourhood.

In (a), the left panel shows that $\bar{h}_{\text{eq}}$ is to the left of $\bar{h}_{\min U}$ for both types. Hence both types prefer to move to a less affluent neighbourhood and, in accordance with equation (42) and equation (43), to build a slightly smaller house and to consume less leisure. Because such a move is available to the high type, the $\bar{h}_{\text{eq}}$ values do not constitute an equilibrium. The right hand panel shows that the optimal housing consumption $h^{\star}(\bar{h})$ passes near a saddle point in the utility function $U(h,\bar{h})$.

Figure 10(b) shows the opposite case, when $\bar{h}_{\text{eq}}$ is greater than $\bar{h}_{\min U}$ and thus the low-type household prefers to move locations. Panels (c) are the same as (b) with the values of Λ and N reversed such that $N > \Lambda$. In this case, $U(w,\bar{h})$ is increasing in $\bar{h}$ and a move to a higher expenditure neighbourhood is always beneficial.

D.3 Mixed strategies

For simplicity, equation (40) describes a pure strategy equilibrium. A less restrictive definition of equilibrium in which mixed strategies are allowed would require only that for each neighbourhood j,

$$\bar{h}_n = \langle h \rangle_{\text{residents}(n)} \tag{47}$$

where the average $\langle \cdot \rangle$ is taken over all residents in the neighbourhood. This weaker condition will still not admit any separating outcome in which different types tend to live in different neighbourhoods. This is because for either type to be indifferent between two neighbourhoods, the neighbourhoods must have identical $\bar{h}$ and hence identical mixtures of the two types of household, resulting in a pooling equilibrium.

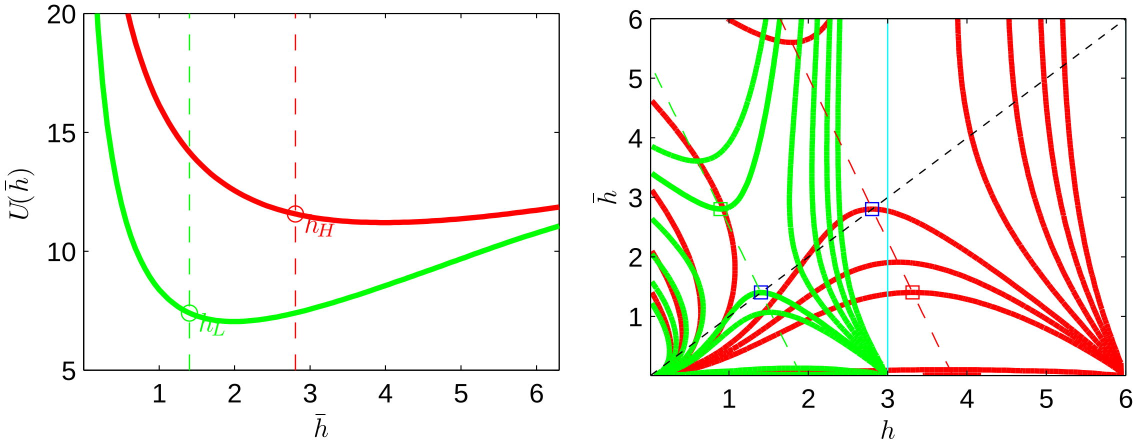
D.4 Neighbourhood benefits compared with other neighbourhoods

In equation (41), the functional form of $N(\cdot)$ provides unbounded benefits from consumption of the public good $\bar{h}$ while $H(\cdot)$ represents a bounded cost of Veblen comparison as $\bar{h}$ becomes large. As a result, households will always prefer moving to a sufficiently high-consumption neighbourhood rather than remain in their own.

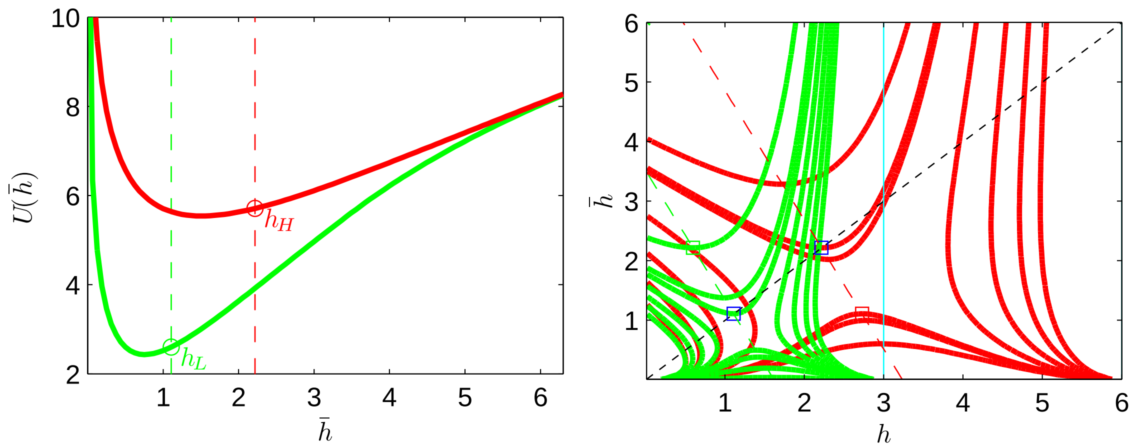
An alternate specification of preferences pertains to neighbourhood status benefits of type 3 on page 37 and is also more consistent with the empirical results outlined in Section 1. In this functional form, the neighbourhood consumption $\bar{h}$ confers utility only through comparison to a yet broader average consumption, $\bar{\bar{h}}$, which may be taken to be the average over all neighbourhoods. A new consistency condition states this additional relationship,

$$\bar{\bar{h}} = \langle \bar{h} \rangle$$

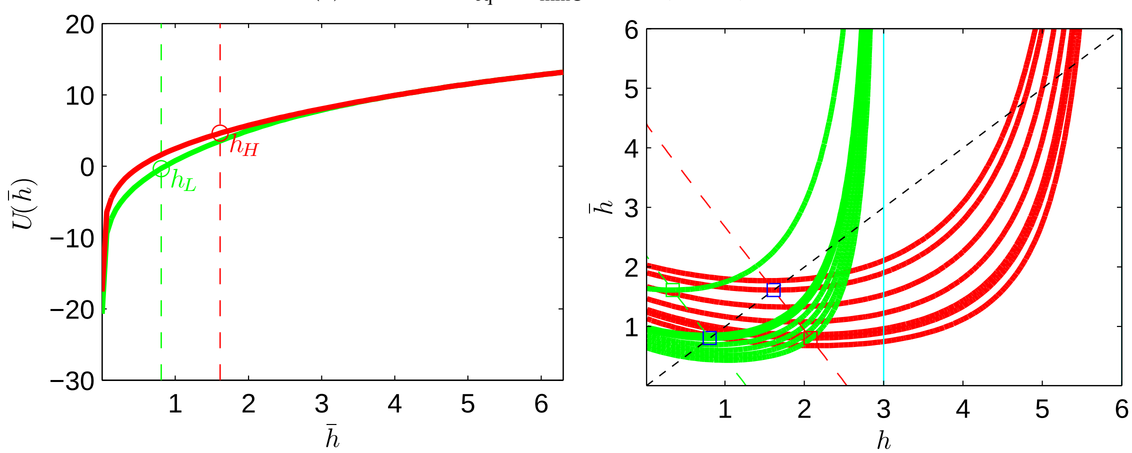
and the comparison between neighbourhoods is captured in the final term of the utility function, $N(\bar{h},\bar{\bar{h}})$. For instance, a form similar to that analysed in Section D.2 is



(a) Case with $\bar{h}_{\text{eq}} < \bar{h}_{\text{minU}}$: $\Phi = 9, \Lambda = 16$, $N = 6$



(b) Case with $\bar{h}_{\text{eq}} > \bar{h}_{\text{minU}}$: $\Phi = 6, \Lambda = 7$, $N = 4.5$



(c) Case with $N > \Lambda$: $\Phi = 6, \Lambda = 4.5$, $N = 7$

Figure 10: **Non-existence of separating equilibrium.** No separating equilibrium exists for "log-log-log" preferences given by equation (41). In all cases shown, $w_L = 3$ and $w_H = 6$.

$$U(x,h,\bar{h}) = \Phi \log(x) + \Lambda \log\left(1 + \frac{h}{\bar{\bar{h}}}\right) + N \log\left(1 + \frac{\bar{h}}{\bar{\bar{h}}}\right) \tag{48}$$

This utility function provides a more natural limit to the benefit obtained in equilibrium from neighbourhood consumption when the number of neighbourhoods is finite. Nevertheless, it is shown on page 47 in Appendix E that there is still no separating equilibrium for households with these preferences. The proof is similar to the case of absolute benefits, above.

D.5 "Log-log-exp" preferences with two types

In this section and the next, other convenient functional forms described in Section C are used to vary the qualitative assumptions on utility. Using the inverse exponential form for $N(\cdot)$ imposes a bound on the benefits from living in an affluent neighbourhood, which may be a more defensible assumption and circumvents one apparent problem with the specification give in equation (41). For simplicity, consider again the case in which $N(\cdot)$ depends only on absolute consumption of one's neighbours, but now with the following form:

$$U(x,h,\bar{h}) = \Phi \log(x) + \Lambda \log\left(1 + \frac{h}{\bar{\bar{h}}}\right) - N \exp\left(-\nu \bar{h}\right) \tag{49}$$

Topologically this form is richer than equation (41), with multiple inflection points in the indirect utility $U(w,\bar{h})$. Numerical analysis indicates that it also is incompatible with a separating equilibrium. Figure 11 on page 45 shows some parameter sets for which in (a) both types prefer to switch neighbourhoods if assigned to their $\bar{h}_{\text{eq}}$, and in (b) the high type prefers to switch and the low type prefers to stay.

E Proofs

Proposition E.1. (Decreasing leisure in economy with a continuum of types) *If $F(x)$ is concave and either (a) $H(h,\bar{h})$ takes the form $H = f\left(h - \bar{h}\right)$ or (b) $H(h,\bar{h})$ takes the form $H = f\left(\frac{h}{\bar{h}}\right)$ and $p = 0$, then leisure x is decreasing in w amongst interior equilibria.*

Proof. For interior equilibria, $F'(x) = wH_h(h,\bar{h})$ and $h = \bar{h} \forall w$. When $H = f\left(h - \bar{h}\right)$, taking a derivative gives

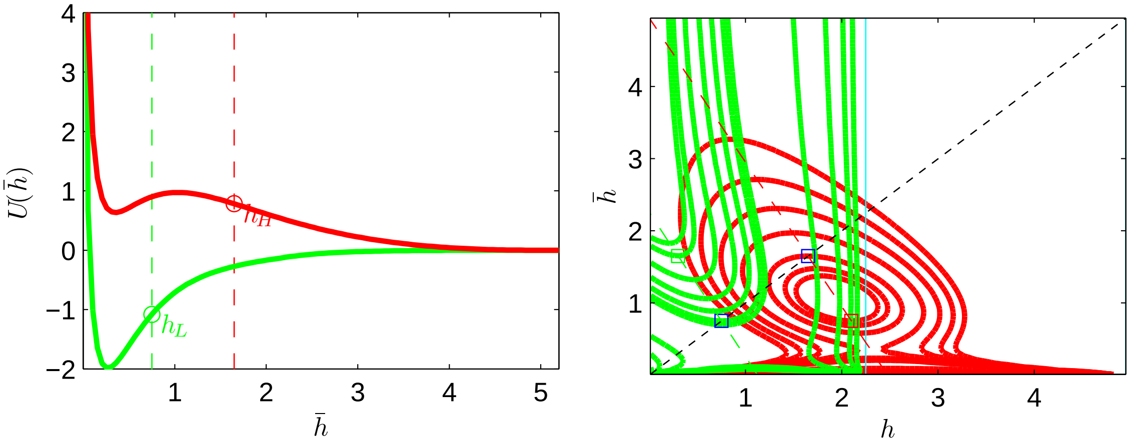
$$\frac{dx}{dw} = \frac{f_0'}{F''} < 0$$

For $H = f\left(\frac{h}{\bar{h}}\right)$,

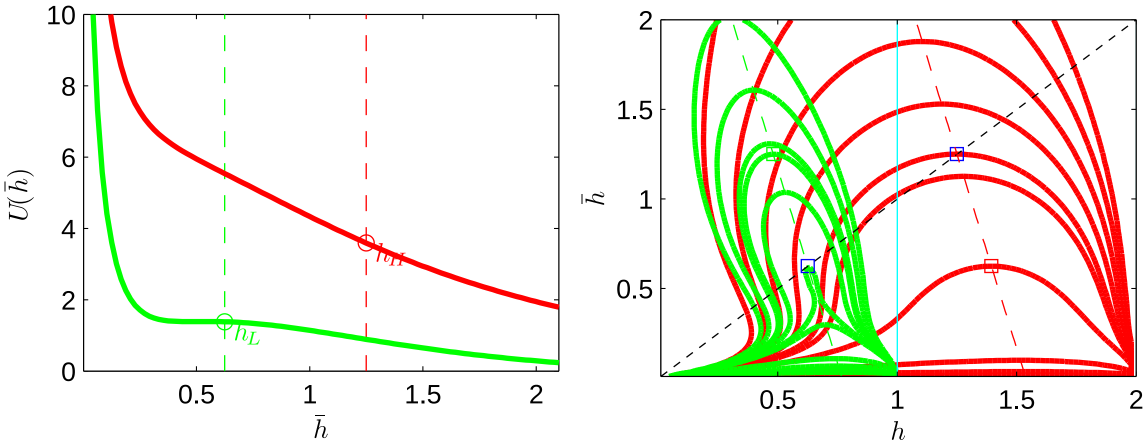
$$\frac{dx}{dw} = \frac{f_0'}{\bar{h}F''}\left[1 - \frac{d\bar{h}}{dw}\right]$$

and if $p(\bar{h}) = 0$,

$$\frac{d\bar{h}}{dw} = 1 - x - w\frac{dx}{dw}$$



(a) Case with $\Phi = 4$, $\Lambda = 4$, $N = 10$, $\nu = 2$, $w_L \approx 2.5$, and $w_H \approx 5$



(b) Case with $\Phi = 3$, $\Lambda = 10$, $N = 17$, $\nu = 3$, $w_L = 1$, and $w_H = 2$

Figure 11: **Non-existence of separating equilibrium.** No separating equilibrium exists for "log-log-exp" preferences given by equation (49).

Combining these expressions gives

$$\frac{dx}{dw} = \frac{1-x}{F'' - \frac{f_0'}{\bar{h}} w^2} < 0$$

□

Proposition D.1 on page 41.

Proof. According to equation (42) and equation (40), the interior equilibrium choices of type w can be written:

$$h_{\text{eq}} = \bar{h}_{\text{eq}} = \frac{\Lambda}{2\Phi + \Lambda} w \quad (50)$$

$$x_{\text{eq}} = \frac{2\Phi}{2\Phi + \Lambda} \quad (51)$$

This says that whenever a separating equilibrium exists, households of type w will always be seen to populate the same kind of neighbourhood, regardless of which other types also exist.

First, note that the equilibrium cannot include corner allocations. According to equation (42), the choice of h is interior whenever $\bar{h} \leq \frac{\Lambda}{\Phi} w$. Since $\bar{h}_{\text{eq}}$ given in equation (50) always satisfies $\bar{h}_{\text{eq}} \leq \frac{\Lambda}{\Phi} w$ and since, again according to equation (42), $h^\star > 0$ if $\bar{h} = 0$, allocations in the separating equilibrium must be interior.

Now, a sufficient condition for the existence of a separating equilibrium is that each type prefers to remain in its own neighbourhood. Formally, the net benefit ΔU from moving to the other available neighbourhood and choosing a new level of housing there must be negative for each type:

$$\Delta U_L \equiv U\left(w_L, \bar{h}_H\right) - U\left(w_L, \bar{h}_L\right) \leq 0 \text{ and} \quad (52)$$

$$\Delta U_H \equiv U\left(w_H, \bar{h}_L\right) - U\left(w_H, \bar{h}_H\right) \leq 0 \quad (53)$$

where $\bar{h}_L$ and $\bar{h}_H$ are the equilibrium neighbourhood housing choices in equation (50). These conditions can be evaluated using equation (44) with the convenient notation $\Theta \equiv \frac{w_H}{w_L} > 1$ and $B \equiv h_{\text{eq}}/w = \frac{\Lambda}{2\Phi+\Lambda} < 1$:

$$\begin{aligned} \Delta U_L &= \Phi \log\left(1 + \frac{h_H}{w_L}\right) + \Lambda \log\left(1 + \frac{w_L}{h_H}\right) + N \log\left(h_H\right) \\ &\quad - \Phi \log\left(1 + \frac{h_L}{w_L}\right) - \Lambda \log\left(1 + \frac{w_L}{h_L}\right) - N \log\left(h_L\right) \\ &= \Phi \log\left(\frac{1 + B\Theta}{1 + B}\right) + \Lambda \log\left(\frac{1 + 1/B\Theta}{1 + 1/B}\right) + N \log\left(\Theta\right) \\ &= \Phi \log\left(\frac{1 + B\Theta}{1 + B}\right) + \Lambda \log\left(\frac{1}{\Theta}\frac{1 + B\Theta}{1 + B}\right) + N \log\left(\Theta\right) \end{aligned}$$

$$= \quad [\Phi+\Lambda]\log\left(\frac{1+B\Theta}{1+B}\right)+[N-\Lambda]\log(\Theta) \tag{54}$$

Similarly,

$$\begin{aligned}
\Delta U_H &= \Phi\log\left(1+\frac{h_L}{w_H}\right)+\Lambda\log\left(1+\frac{w_H}{h_L}\right)+N\log(h_L) \\
&\quad -\Phi\log\left(1+\frac{h_H}{w_H}\right)-\Lambda\log\left(1+\frac{w_H}{h_H}\right)-N\log(h_H) \\
&= \Phi\log\left(\frac{1+B/\Theta}{1+B}\right)+\Lambda\log\left(\frac{1+\Theta/B}{1+1/B}\right)-N\log(\Theta) \\
&= \Phi\log\left(\frac{1}{\Theta}\frac{B+\Theta}{1+B}\right)+\Lambda\log\left(\frac{B+\Theta}{1+B}\right)-N\log(\Theta) \\
&= [\Phi+\Lambda]\log\left(\frac{B+\Theta}{1+B}\right)-[N+\Phi]\log(\Theta)
\end{aligned}$$

Note that the first term in equation (54) must be positive, since $\Theta > 1$. Whenever $N > \Lambda$ the second term is also positive and $\Delta U_L > 0$, which means that the low type will always prefer to move up to the high types's neighbourhood. This makes the separating equilibrium impossible when $N < \Lambda$, *i.e.* for agents who value neighbourhood-level benefits sufficiently more than they value their status within a neighbourhood.

The two terms of ΔU_H can also be unambiguously signed; the first is always positive and the second always negative. I now show that when the low types are content in their neighbourhood, the high types cannot be content in theirs.

A necessary equilibrium condition follows from combining the two inequalities in equation (52) and equation (53) into the weaker requirement that

$$\Delta U_H + \Delta U_L \leq 0$$

which becomes

$$\begin{aligned}
\Delta U_L+\Delta U_H &= [\Phi+\Lambda]\log\left(\frac{1+B\Theta}{1+B}\right)+[N-\Lambda]\log(\Theta) \\
&\quad +[\Phi+\Lambda]\log\left(\frac{B+\Theta}{1+B}\right)-[N+\Phi]\log(\Theta) \\
&= [\Phi+\Lambda]\log\left(\frac{1+B\Theta}{1+B}\,\frac{B+\Theta}{1+B}\,\frac{1}{\Theta}\right) \\
&= [\Phi+\Lambda]\log\left(\frac{\Theta\left[1+B^2\right]+\left[\Theta+\frac{1}{\Theta}\right]B\Theta}{\Theta\left[1+B^2\right]+2B\Theta}\right)
\end{aligned} \tag{55}$$

Because $\Theta + \frac{1}{\Theta} > 2$ for all $\Theta > 1$, the argument of log in equation (55) is always greater than 1; thus $\Delta U_L + \Delta U_H > 0$. Therefore, there is no separating equilibrium. □

Proposition E.2. *When group entry (land) is costless and preferences are given by equation* (48)*, there is no discrete separating group Nash equilibrium with two types.*

Proof. The proof closely resembles that of Proposition D.1; differences are noted here.

Let the equilibrium neighbourhoods be h_L and h_H according to equation (50). The global reference level can then be expressed

$$\bar{\bar{h}} = \Lambda \frac{w_L + w_H}{2\Lambda + 4\phi}$$

The conditions for an equilibrium then become:

$\Delta U_L = [\Phi + \Lambda] \log \left(\frac{\Phi + \Lambda \frac{1}{2}[1+\Theta]}{\Phi + \Lambda} \right) - \Lambda \log(\Theta) + N \log \left(\frac{1+3\Theta}{3+\Theta} \right) \leq 0$

$\Delta U_H = -[\Phi + \Lambda] \log \left(\frac{\Phi + \Lambda}{\Phi + \Lambda \frac{1}{2}\left[1+\frac{1}{\Theta}\right]} \right) + \Lambda \log(\Theta) - N \log \left(\frac{1+3\Theta}{3+\Theta} \right) \leq 0$

Again, a weaker necessary condition that follows from combining these two inequalities, $\Delta U_H \leq 0 \leq -\Delta U_L$, is that

$$\Delta U_H + \Delta U_L \leq 0$$

$$\begin{aligned} \Delta U_H + \Delta U_L &= [\Phi + \Lambda] \log \left(\frac{\Phi + \Lambda \frac{1}{2}\left[1 + \frac{1}{\Theta}\right]}{\Phi + \Lambda} \frac{\Phi + \Lambda \frac{1}{2}[1 + \Theta]}{\Phi + \Lambda} \right) \\ &= [\Phi + \Lambda] \log \left(\frac{\Phi^2 + \Phi\Lambda \frac{1}{2}\left[2 + \Theta + \frac{1}{\Theta}\right] + \Lambda^2 \frac{1}{4}\left[2 + \frac{1}{\Theta} + \Theta\right]}{\Phi^2 + 2\Phi\Lambda + \Lambda^2} \right) \leq 0 \end{aligned}$$

Because $\Theta + \frac{1}{\Theta} > 2$ for all $\Theta > 1$, the argument of log is always greater than 1 and therefore the above inequality is impossible. There is no separating equilibrium. □

Lemma E.3. (A useful exponential form) *Let* $\Psi(a,b) \equiv \frac{1}{b} - \frac{a}{e^{ab}-1}$*. Then for a and b positive,* $\lim_{b\to 0} \Psi(a,b) = \frac{1}{2}a$ *and* $\Psi(a,b)$ *is always positive. For* $\Psi(-a,b) = \frac{1}{b} - \frac{a}{1-e^{-ab}}$ *and a and b positive,* $\lim_{b\to 0} \Psi(-a,b) = -\frac{1}{2}a$ *and* $\Psi(-a,b)$ *is always negative. Furthermore,* $\frac{d\Psi(a,b)}{da} > 0$, $\frac{d\Psi(a,b)}{db} < 0$, $\frac{d\Psi(-a,b)}{da} < 0$, $\frac{d\Psi(-a,b)}{db} < 0$.

Proof. For $a > 0$, $b > 0$, the function $b \cdot \Psi(a,b)$ is nonnegative iff $ab \leq e^{ab} - 1$. For $ab = 0$, this is an equality. For $ab > 0$, the slope of the right hand side strictly dominates the slope of the left hand side. Therefore the inequality holds for all $ab \geq 0$. By similar reasoning, the inequality $ab \geq 1 - e^{-ab}$ holds for all positive ab.

Using a Taylor expansion to find the limits,

$$\begin{aligned} \Psi(a,b) &= \frac{1}{b} - \frac{a}{ab + \frac{1}{2}a^2b^2 + \frac{1}{3!}a^3b^3 + \dots} \\ &= \frac{b + \frac{1}{2}ab^2 + \frac{1}{3!}a^2b^3 + \dots - b}{b^2 + \frac{1}{2}ab^3 + \frac{1}{3!}a^2b^4 + \dots} \\ &= \frac{\frac{1}{2}a + \frac{1}{3!}a^2b + \dots}{1 + \frac{1}{2}ab + \frac{1}{3!}a^2b^2 + \dots} \end{aligned}$$

$$\rightarrow \quad \begin{cases} 0 & \text{as } a \rightarrow 0 \\ \frac{1}{2}a & \text{as } b \rightarrow 0 \end{cases}$$

A similar transformation finds the limits for $\Psi(-a,b)$. The above results can be used to sign the first derivatives as follows:

$$\begin{aligned} \frac{d}{da}\Psi(a,b) &= -\frac{1}{e^{ab}-1} + \frac{abe^{ab}}{\left[e^{ab}-1\right]^2} \\ &= -\frac{1}{e^{ab}-1}\left[1 - \frac{ab}{1-e^{-ab}}\right] \\ &= -\frac{1}{e^{ab}-1}\Psi(-ab,1) > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{d}{db}\Psi(a,b) &= -\frac{1}{b^2} + \frac{a^2 e^{ab}}{\left[e^{ab}-1\right]^2} \\ &= -\frac{1}{b^2} + \frac{a^2}{\left[e^{ab}-1\right]\left[1-e^{-ab}\right]} \\ &= -\frac{1}{b^2} + \frac{a^2}{e^{ab}+e^{-ab}-2} \\ &= -\frac{1}{b^2} + \frac{a^2}{\frac{2a^2b^2}{2!} + \frac{2a^4b^4}{4!} + \frac{2a^6b^6}{6!}\dots} \frac{a^2}{e^{ab}+e^{-ab}-2} \\ &= -\frac{1}{b^2}\left[1 - \frac{1}{1 + \frac{2a^2b^2}{4!} + \frac{2a^4b^4}{6!} + \dots}\right] < 0 \end{aligned}$$

For the modified function $\Psi(-a,b)$, both derivatives are negative:

$$\begin{aligned} \frac{d}{da}\Psi(-a,b) &= -\frac{1}{1-e^{-ab}} - \frac{abe^{-ab}}{[1-e^{-ab}]^2} \\ &= -\frac{1}{1-e^{-ab}}\left[1 + \frac{ab}{e^{ab}-1}\right] < 0 \end{aligned}$$

and

$$\frac{d}{db}\Psi(-a,b) = -\frac{1}{b^2}\left[1 - \frac{1}{1 + \frac{2a^2b^2}{4!} + \frac{2a^4b^4}{6!} + \dots}\right] < 0$$

It follows from these monotonic properties that $\Psi(a,b) \in (0, \frac{1}{2}a)$ and $\Psi(-a,b) \in (-\frac{1}{2}a, 0)$.

□

F Construction of equilibrium

This section outlines the steps taken to compute a separating equilibrium in the numerical examples to follow. Given exogenous parameters including the range of types w_L, w_H, values for $x(w)$, $w_m = \max\{w_L, w_0\}$, $\bar{\bar{h}}$, and $r - p_0$ can be directly computed from equations (19), (20), (25), and (26). From these, the remaining allocations $\bar{h}(w)$ follow using (24) . What remains is to calculate a price schedule $p(\bar{h})$.

In order to select a value of p_0 sufficiently high to clear the market both for the most affluent desired neighbourhoods, *i.e.,* $p\left(\bar{h}(w_H)\right) > 0$, and for the least affluent neighbourhood, *i.e.,* $p(\bar{h}_{\min}) \geq 0$, limiting values of p_0 for both conditions must be calculated and the higher of the two adopted. First, to ensure that there is a nonnegative price for the highest type, I impose $\bar{h}_{\max} = \bar{h}(w_H)$ and $p(\bar{h}_{\max}) = 0$ in (21), giving

$$\bar{h}_{\max} = r + w_H - w_0$$

Then (23) can be evaluated at this upper limit in order to solve for p_0:

$$\begin{aligned}
0 = p(\bar{h}_{\max}) &= p_0 - \bar{h}_{\max} + \frac{N}{\Lambda\lambda}\log\left(1 + \frac{\bar{h}_{\max}}{\bar{\bar{h}}}\right) \\
\frac{N}{\Lambda\lambda}\log\left(1 + \frac{r + w_H - w_0}{\bar{\bar{h}}}\right) &= [r - p_0 + w_H - w_0] \\
1 + \frac{r + w_H - w_0}{\bar{\bar{h}}} &= \exp\left(\frac{\Lambda\lambda}{N}[r - p_0 + w_H - w_0]\right) \\
r &= \bar{\bar{h}}\left[\exp\left(\frac{\Lambda\lambda}{N}[r - p_0 + w_H - w_0]\right) - 1\right] + w_0 - w_H
\end{aligned}$$

Hence,

$$\begin{aligned}
p_0 &= r - [r - p_0] \\
&= \bar{\bar{h}}\left[\exp\left(\frac{\Lambda\lambda}{N}[r - p_0 + w_H - w_0]\right) - 1\right] + w_0 - w_H - [r - p_0] \qquad (56)
\end{aligned}$$

Since the value of $[r - p_0]$ is already calculated in terms of exogenous parameters (56) provides a lower bound on the constant p_0. To calculate a second lower bound satisfying the condition $p(\bar{h}_{\min}) = 0$, note that the consumption level implied by this condition is $\bar{h}_{\min} = r$. Thus, the minimum p_0 can again be calculated in terms of $[r - p_0]$:

$$\begin{aligned}
0 &= p(r) \\
&= p_0 - r + \frac{N}{\Lambda\lambda}\log\left(1 + \frac{r}{\bar{\bar{h}}}\right) \\
\frac{N}{\Lambda\lambda}\log\left(1 + \frac{r}{\bar{\bar{h}}}\right) &= r - p_0 \\
1 + \frac{p_0 + [r - p_0]}{\bar{\bar{h}}} &= \exp\left([r - p_0]\frac{\Lambda\lambda}{N}\right)
\end{aligned}$$

$$
\begin{aligned}
&= p_0 + [r - p_0] - \frac{N}{\Lambda\lambda} \log\left(1 + \frac{p_0 + [r - p_0]}{\bar{\bar{h}}}\right) \\
\rightarrow 1 + \frac{p_0 + [r - p_0]}{\bar{\bar{h}}} &= \exp\left([r - p_0]\frac{\Lambda\lambda}{N}\right) \\
p_0 &= \bar{\bar{h}}\left[\exp\left([r - p_0]\frac{\Lambda\lambda}{N}\right) - 1\right] - [r - p_0] \qquad (57)
\end{aligned}
$$

For $r - p_0$ sufficiently high for a real solution $\bar{h}(w)$, above, a market-clearing price schedule can now be found by setting p_0 to the larger of the two values in (56) and (57). The price schedule follows from (23).