Federal Reserve Bank
of Minneapolis

Summer 1990

Quarterly Review

Deflating the Case for Zero Inflation (p. 2)

S. Rao Aiyagari

The Simple Analytics of Commodity Futures Markets: Do They Stabilize Prices? Do They Raise Welfare? (p. 12)

V. V. Chari
Ravi Jagannathan

Federal Reserve Bank of Minneapolis

Quarterly Review

Vol. 14, No. 3 ISSN 0271-5287

This publication primarily presents economic research aimed at improving policymaking by the Federal Reserve System and other governmental authorities.

Produced in the Research Department. Edited by Preston J. Miller, Kathleen S. Rolfe, Martha L. Starr, and Inga Velde. Graphic design by Barbara Birr, Public Affairs Department.

Address comments and questions to the Research Department, Federal Reserve Bank, Minneapolis, Minnesota 55480 (telephone 612-340-2341).

The Simple Analytics of Commodity Futures Markets: Do They Stabilize Prices? Do They Raise Welfare?*

V. V. Chari
Senior Economist
Research Department
Federal Reserve Bank of Minneapolis

Ravi Jagannathan
Visitor
Research Department
Federal Reserve Bank of Minneapolis
and Associate Professor of Finance
University of Minnesota

Modern futures trading—the organized trading of contracts to buy and sell things at a later date—began at the Chicago Board of Trade in the 1860s. Since then, the number of futures markets has grown exponentially. These markets strongly influence the prices and quantities of a vast array of foods, grains, livestock, metals, industrial materials, and financial assets. Almost since they began, futures markets have excited debate about whether they make prices more volatile. Organizations representing producers, especially farmers, have argued that they do. This public debate has generated a sizable academic literature.

The academic literature has studied futures markets from both empirical and theoretical perspectives. The empirical studies have compared how prices behave before and after the introduction of futures markets. (For a summary of the empirical research, see the references cited in Turnovsky 1983.) Although the empirical results are mixed, they seem to show that the volatility of prices tends to decrease when futures markets are introduced into an economy. The theoretical literature largely stems from Friedman's (1953) argument that speculation necessarily reduces the volatility of prices. The theoretical work suggests that under plausible assumptions about the sources of economic disturbances, futures markets reduce the volatility of prices. (See, for example, Turnovsky 1983 and Turnovsky and Campbell 1985.) It is difficult, however, to know how generally this result applies, since the literature uses very specific functional forms to model the objectives of market participants.

In this paper, we clarify the set of circumstances under which futures markets stabilize, or reduce the volatility of, prices. We show that, under a fairly stringent set of assumptions, the introduction of futures markets does stabilize prices. The assumptions we make are stringent, for we can easily construct examples in which they do not hold and in which prices become more volatile when futures markets are introduced.

We go beyond this issue, however. We also point out and question an implicit assumption in both the popular debate and the academic literature on the effect of futures markets—the assumption that lower price volatility is socially desirable. We show here that no direct or obvious link exists between price volatility and social welfare.

This perhaps more important finding suggests that price stability, in and of itself, should not be regarded as a policy goal. It clearly is a goal of U.S. policymakers now. For example, in the wake of the October 1987 stock market crash, a variety of regulations were proposed to limit the variability of stock prices. To the

*The models and most of the results presented in this paper were developed in collaboration with Larry Jones of Northwestern University. Parts of this paper have been published in a paper co-authored with Jones: "Price Stability and Futures Trading in Commodities," *Quarterly Journal of Economics* 105 (May): 527–34. The authors thank the MIT Press for permission to use extracts from that paper.

extent that policy proposals are based on the belief that lower price volatility is desirable, they start from the wrong premise. To ensure that any such proposed changes are beneficial, policymakers must examine their effects not on price stability, but rather on economic welfare.

A Model of Futures Markets

We now set about constructing a formal model of an economy with futures markets. So that the model can address the effect of futures markets on price stability and welfare, we must first take a stand on what they do. That is, exactly what role do these markets perform?

Defining Their Role

Again, futures markets are organized exchanges in which participants trade contracts to deliver or accept quantities of a specified commodity or asset at a specified later date at a currently agreed on price. (See Siegel and Siegel 1990 for an excellent, detailed discussion of futures markets; also see Hieronymus 1971, Gold 1975, and Carlton 1984 for detailed descriptions of the history and operations of these markets.) Since the price for future delivery is agreed on now, producers and owners of the commodity or asset, like farmers and grain elevator operators, clearly can use futures markets to protect themselves against the risk that prices will fall between today and the delivery date. Similarly, consumers, like grain mills and food companies, can protect themselves against the risk that prices will rise. By protecting themselves in this way, of course, producers and consumers forgo the profits they would make if, in the meantime, prices should move in the other direction. Still, by participating in futures markets, producers and consumers transfer the price risk to other market participants, typically called *speculators.* These are people who are more willing to accept the risks involved. A primary role of futures markets, therefore, is to allocate risks more efficiently than markets without futures trading. In this sense, futures markets perform an *insurance* role.

Making Assumptions

For our model economy, we make several natural simplifying assumptions. First, we assume that speculators are *risk neutral:* they care about only the average level of their profits, not how variable their profits are about this level. Second, we assume that producers are *risk averse:* they care about both the average level of profits and the variability about that level. Third, we assume that price variability occurs because consumer demand for the producers' good is variable. In the model, producers must decide how much of the good to produce before they know the demand for it. Once the good is produced, its demand is realized and its price is determined by the familiar requirement that supply equal demand. Fourth, for simplicity, we abstract away from inventory-holding decisions and assume that the good is not storable.

Describing the Environment

Our model has one industry. In it, a nonstorable good is produced each period. Production commitments must be made one period before output is realized. If a producer decides at time $t-1$ to produce q_t units of the good during time t, a cost of $c_t = C(q_t)$ is incurred in period t.

The inverse demand function for the industry's good at time t is given by $P(Q_t, \epsilon_t, \eta_{t-1})$, where Q_t is aggregate output at t and ϵ_t and η_{t-1} are shocks to demand. Aggregate output is the sum of the outputs of each producer. The demand shocks occur because of changes in the tastes or incomes of consumers. The shock ϵ is a temporary change in tastes or incomes, whereas the shock η is relatively permanent. The producer observes the shock η_{t-1} at $t-1$, before making the production decision. The shock ϵ_t is observed at t, after the production decision has been made. Basically, the production decisions respond to changes in the permanent demand component, but the decisions are made before the temporary movements in demand are known.

When this industry has no futures trading, the producer's profits are given by

$$(1) \qquad \pi_t = p_t q_t - c_t$$

where p_t denotes the price at time t. The producer's profits depend on both the price and the chosen quantity. Although able to control the quantity of the good produced, the producer is at the mercy of market forces, which determine the price of the good. Because of the shocks affecting demand, the price of the good varies, depending on the realization of the temporary shock ϵ. Consequently, profits vary, depending on the realization of this shock. So, how much the producer chooses to produce depends on his or her attitude toward risk. If extremely risk averse, for example, the producer might decide not to produce anything at all.

The traditional way to model decisions under uncertainty is to assume that the producer seeks to maximize the expected, or average, utility of profits. Typically, this formulation implies that the producer is willing to pay a premium to insure against risk. Formally, we assume that the producer maximizes the expected utility of profits $Eu(\pi)$, where u is a strictly concave function.

When futures trading is introduced into the environment, the producer, in addition to deciding how much to produce, must also make one more decision: the number of futures contracts to buy at $t-1$ for delivery of goods at t. Let m_{t-1} denote the number of futures contracts sold at $t-1$ for delivery of the good at t. Let f_{t-1} denote the price of each contract, which is a promise to deliver one unit of the good.

In the economy with futures trading, we use a superscript f to distinguish variables from their correspondents in the economy without futures trading. When futures trading is permitted, the producer's profits π_t^f at time t are

$$(2) \qquad \pi_t^f = p_t^f q_t^f + m_{t-1}(f_{t-1} - p_t^f) - C(q_t^f).$$

Equation (2) says that the producer must deliver m_{t-1} units of the good and does so by purchasing this good at the market (or *spot*) price p_t^f. In return, the producer receives f_{t-1} per unit delivered under the contract. From (2) it follows that a producer who sells contracts exactly equal to planned production can thereby eliminate all the risk in profits and guarantee profits equal to $f_{t-1} q_t^f - C(q_t^f)$.

As mentioned earlier, we assume that speculators are risk neutral. They buy and sell futures contracts to maximize expected profits, and they do not care about the risk in the variability of their profits. Consequently, when futures trading is permitted, speculators want to sell an infinite number of futures contracts if the average spot price they expect is higher than the futures price; if the expected spot price is lower than the futures price, then they want to buy an infinite number of contracts. Thus, the expected spot price must equal the futures price:

$$(3) \qquad f_{t-1} = E_{t-1}(p_t^f).$$

If this is true, then producers, by purchasing contracts equal to planned production, can guarantee their average profits. Since producers are risk averse, they will insure themselves completely, accept their guaranteed average profit levels, and let the speculators bear all the risk.

A Setup for Price Stability

Within the general model environment just presented, we now obtain a set of sufficient conditions for the introduction of futures trading to stabilize spot prices. We use a standard measure of how much prices move: the *unconditional variance* of spot prices. This measures the deviation of prices about their mean (or average) level over long periods and weights these deviations by both their magnitudes and their frequencies of occurrence. To analyze the effect of the introduction of futures markets on the variability of prices, we use a very intuitive graphical approach. We show that if the marginal (or incremental) costs of production are constant and if demand disturbances cause parallel shifts in the demand curve, then futures markets stabilize spot prices.

Let's suppose the cost function is given by $C(q)=cq$, where c is some positive constant. Here, the per-unit production costs do not depend on the amount or scale of production. Also suppose that the inverse demand function is given by

$$(4) \qquad P(Q_t, \epsilon_t, \eta_{t-1}) = G(Q_t) + \epsilon_t + \eta_{t-1}$$

where $E_{t-1}(\epsilon_t) = 0$ and $G(\cdot)$ is a decreasing function. Denote the conditional variance of ϵ by σ_ϵ^2 and the variance of η by σ_η^2. Note that the disturbances to demand, ϵ_t and η_{t-1}, are additive. For now, assume that the industry has a fixed number n of competitive producers, each of whom produces the same quantity of a good in equilibrium. Denote the expected, or average, price by $\bar{p}_t$. It follows that, in equilibrium,

$$(5) \qquad p_t = \bar{p}_t + \epsilon_t = G(nq_t) + \epsilon_t + \eta_{t-1}.$$

Our approach to the problem of determining the equilibrium price in this model is simple. First, we express the quantity produced as a function of the average price $\bar{p}_t$ to get an aggregate supply curve. Then, by combining the supply curve with the demand curve in (5), we can determine the market-clearing price. The supply curve is determined from the first-order condition to the producer's maximization problem without futures trading, given by

$$(6) \qquad E_{t-1}[u'(\pi_t)(p_t - c)] = 0$$

where the prime indicates the derivative of the utility function. From equation (5), a necessary condition for an equilibrium is that

$$(7) \qquad E_{t-1}\{u'[(\bar{p}_t + \epsilon_t - c)q_t](\bar{p}_t + \epsilon_t - c)\} = 0.$$

Equation (7) defines a unique solution for q_t as a func-

Figures 1 and 2

How Futures Markets Affect Price Volatility in the Original Setup*. . .

Figure 1 Average Supply and Demand Without Futures Markets

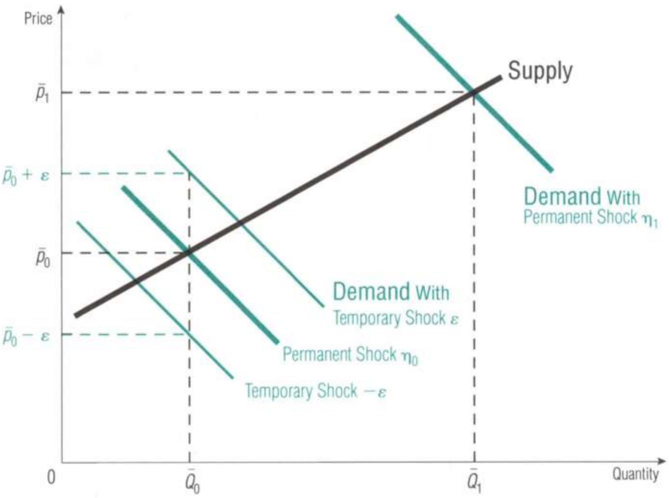
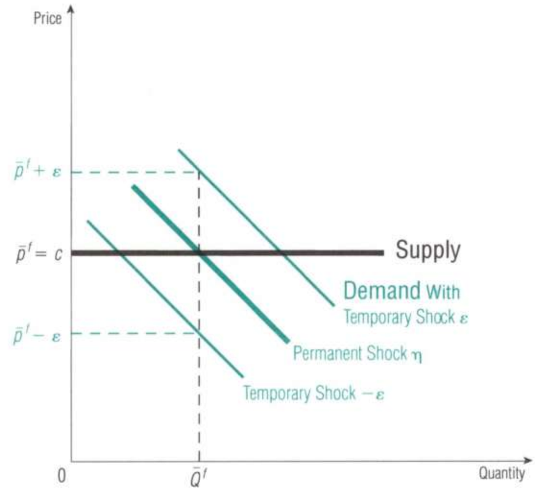


Figure 2 Average Supply and Demand With Futures Markets



*This setup assumes additive demand shocks, constant marginal costs, and a fixed number of producers.

tion of $\bar{p}_t$. Given the number of producers, the aggregate quantity produced is a function of the average price. This solution can be thought of as a supply curve. The intersection of this supply curve with the part of the demand curve given by $G(Q) + \eta$ yields the average price the producers expect to prevail.

Figure 1 shows how prices are determined for a given value of the persistent shock η. The intersection of the average supply and demand curves gives us the quantity produced Q. Demand curves are also drawn for a positive and a negative value of the temporary shock ϵ. When this shock is positive, the price is higher than average; when negative, the price is lower than average. So in each period, the variance of prices, given the quantity produced, is identical to the variance of the shock ϵ. Formally, we say that the conditional variance of the spot price equals the variance of ϵ.

However, prices also vary over time because of the persistent shock η which, in turn, makes producers' decisions vary over time. These effects cause the average spot price to vary over time as well. The nature of these variations can also be understood from Figure 1. An increase in the persistent shock η causes the average demand curve to make a parallel shift upward, thereby changing the quantity produced and the average price.

Price Volatility Decreases . . .

Now let's see what happens to price variability when futures markets are permitted to operate under this setup.

We have already argued that the futures price equals the average spot price in each period and that producers transfer all the risk to speculators. Suppose the futures price were to exceed the marginal cost of production (assumed to be constant). Then, since unit costs are constant, increasing production would increase profits indefinitely. But if the futures price were less than the cost of production, then producers would be better off not producing anything. It follows that the futures price, and thus the average spot price, always exactly equals the marginal cost of production.

This situation is shown in Figure 2. Here, the only source of variability in prices is the temporary shock ϵ. But the variability in prices due to ϵ is the same with and without futures markets because this shock affects demand additively. Without futures markets, the movements of output in response to the relatively permanent shock η are a source of price fluctuations. With futures markets, however, this source of variability is absent. Clearly, then, the variability in prices is lower with futures trading than without it.

... But So Does Producer Welfare

Does the lower volatility of prices imply that producers and consumers are better off with futures markets than without them? No. Futures markets make producers worse off and consumers better off.

To see that producers are made worse off, notice that when we allow futures trading, prices are always equal to the marginal costs of production. Therefore, producers make zero profits. In effect, producers are indifferent between producing and not producing at all. Without futures markets, however, producers are indifferent between producing and not producing one more unit at the margin. Consequently, producers are made worse off by the introduction of futures markets.

We can express this outcome formally. For any strictly concave function, it is well known that if profits $\pi \neq 0$, then

$$(8) \qquad u(\pi) - u(0) > u'(\pi)\pi.$$

By taking expectations on both sides and using (6), we can see that the right side of (8) is zero. Therefore, the left side is strictly positive.

Whereas producers are made worse off by the introduction of futures trading, consumers are made better off. This result follows from a fundamental result in welfare economics: complete, competitive markets yield Pareto-optimal allocations. That is, no one can be made better off without making someone else worse off. In the model with futures trading, markets are complete. Therefore, if shutting down futures markets makes producers better off, it must make someone else worse off—in this case, consumers. That means consumers are better off with futures markets than without them.

One caveat is required here. If a government can levy lump-sum taxes on consumers and make lump-sum transfers to producers, then a tax system could be devised to make everyone better off when futures markets are introduced. That such taxes and transfers are generally quite difficult to administer is well recognized.

A Change in Setup

Our results so far may well depend on the special assumptions we made about marginal cost being constant, demand being linear, and demand shocks being additive. The assumption of a constant marginal cost of production is particularly suspect in many applications. For instance, the scarcity of some resources, like high-quality land, tends to imply that the incremental costs of production increase with the quantity produced. We therefore change our earlier setup to cost functions with increasing, rather than constant, marginal cost.

To get sharp results about the effect of futures markets on price volatility, we need to impose restrictive assumptions on the demand curve and on the risk aversion of producers. Specifically, we impose three assumptions:

- The inverse demand function is given by $\eta_{t-1} - dq_t + \epsilon_t$.
- The cost function is given by $aq_t + bq_t^2$.
- The producer's utility function shows constant absolute aversion to risk.

The first assumption simply says that the demand curve is linear in output. The second implies that the marginal cost of production is also linear in output. The third, a special but widely used form of representing attitudes toward risk, says that the premium someone is willing to pay to eliminate a given risk is independent of the person's wealth. We adopt this third assumption because much of the literature uses it to model the risk preferences of producers (Kawai 1983, Turnovsky 1983).

Price Volatility Decreases Again ...

To analyze the effect of futures markets on the volatility of prices, we again use a simple graph of supply and demand. (See Figure 3.) And our results are similar to those in the earlier setup.

Let $\bar{p}_t$ denote the conditional expectation at time t of the spot price at $t-1$. With linear shocks to demand, the spot price is given by $\bar{p}_t + \epsilon_t$. A typical producer's profits are given by $(\bar{p}_t + \epsilon_t)q_t - a - bq_t^2$. The first-order condition to the producer's maximization problem without futures trading is given by

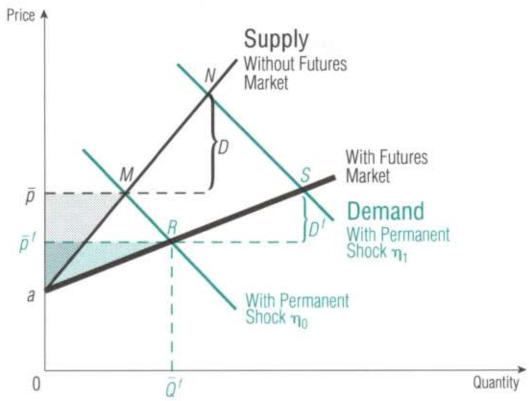
$$(9) \qquad E_{t-1}[u'(\pi_t)(\bar{p}_t + \epsilon_t - 2bq_t)] = 0.$$

This first-order condition gives the supply function of

Figure 3

. . . In the Basic Changed Setup*. . .

Average Supply and Demand



*The basic changes are to linear, increasing marginal costs and to constant absolute risk aversion for producers.

each producer as a function of the average price expected in the next period. When futures trading is permitted, the producer equates the futures price with the marginal cost of production. Therefore, the supply function of the producer is

$$(10) \quad \bar{p}_t^f = a + 2bq_t^f.$$

The supply curves, with and without futures trading, are shown in Figure 3. The supply curve without futures trading is the steeper one, both at every level of output and for every realization of the persistent shock η_{t-1}.

By totally differentiating equation (9) and collecting terms, we get

$$(11) \quad E[u''(\pi)(\bar{p}+\epsilon-a-2bq)\bar{p}] + Eu'(\pi) + E[u''(\pi)(\bar{p}+\epsilon-a-2bq)^2](dq/d\bar{p}) - 2bu'(\pi)(dq/d\bar{p}) = 0$$

where u'' denotes the second derivative of the utility function. (Hereon, where possible, we drop the time subscript for convenience.) A key property of constant absolute risk aversion preferences is that u'' is proportional to u'. Therefore, from (9) it follows that the first term in (11) is zero. The remaining terms can be rearranged to get the slope of the supply curve without futures trading:

$$(12) \quad dq_t/d\bar{p}_t = \left(2b - \{E_{t-1}[u''(\pi_t)(\bar{p}_t+\epsilon_t-a-2bq_t)^2] \div E_{t-1}[u'(\pi_t)]\}\right)^{-1}.$$

From equation (10), we have

$$(13) \quad dq_t^f/d\bar{p}_t^f = 1/2b.$$

Since $u''(\pi_t)$ is negative, the right side of equation (12) is less than the right side of (13). The slope of the supply curve is the inverse of the derivative of the quantity with respect to the price. Therefore, the supply curve without futures markets is steeper than the supply curve with futures markets.

Now consider any two distinct realizations of the persistent shock η_{t-1} and the corresponding expected values of the two inverse demand functions, based on information available at $t-1$. Let the supply curve without futures markets intersect the two demand functions at M and N. Let R and S be the points where the supply curve with futures trading intersects the demand functions.

To see what happens to the volatility of prices with futures markets, we observe that the vertical distance D between the points M and N is greater than the vertical distance D^f between the points R and S. This is readily seen to be true, since the slope of the supply curve with futures trading is a constant that is less than the slope of the supply curve without futures trading for every level of output, and the inverse demand curves are linear and downward sloping.

This geometry implies that as the persistent shock η varies, the spot price varies less with futures markets than without them. Here, as under the constant marginal cost setup, the variability of prices due to the temporary shock ϵ is the same with and without futures markets. Therefore, what remains is the variability caused by η, and we have shown that this variability decreases when futures markets are introduced.

Formally, our procedure involves the use of a standard decomposition theorem, which says that the unconditional variance of spot prices can be decomposed into the variance of the conditional mean and the mean of the conditional variance:

$$(14) \quad \text{var}(p_t) = \text{var}[E_{t-1}(\bar{p}_t+\epsilon_t)] + E[\text{var}_{t-1}(\bar{p}_t+\epsilon_t)].$$

The second term in equation (14) is the same with and without futures markets. Therefore, to show that the variance of the spot price decreases with the introduction of futures trading, we only need to show that the magnitude of the difference between the expected spot prices for any two different realizations of η_{t-1} decreases when futures trading is allowed.

The geometric argument also implies that the long-run average price without futures trading exceeds the long-run average price with futures trading. The supply is zero when the expected price in the next period equals a, the marginal cost at zero output. Hence, the two supply curves start from the same point on the vertical axis of Figure 3, and the steeper supply curve always stays above the flatter one.

. . . But What Happens to Welfare?

These changes in assumptions do not change the price volatility result, but they do change the welfare result. With these changes, the welfare effect of introducing futures markets becomes ambiguous.

One reason for this ambiguity is suggested by another look at Figure 3. For simplicity, suppose the industry has only one producer, so that this producer's output equals aggregate output. Then the area of the triangle $aR\bar{p}^f$ measures the producer's surplus or profit. This is because the producer's revenue is the product of price and quantity, given by the rectangle $0\bar{p}^fR\bar{Q}^f$, and the area under the marginal cost curve is the total production cost. The corresponding average surplus without futures markets is measured by the triangle $aM\bar{p}$. If the marginal cost curve is sufficiently flat or the demand curve sufficiently steep, then the average surplus without futures markets will be large relative to the surplus without them. Of course, in the environment without futures markets, the producer also bears some price risk. But Figure 3 suggests that the introduction of futures markets could very likely make producers worse off.

We formally demonstrate this possibility by considering the special case where the temporary shocks ϵ are drawn from a normal distribution with zero mean and variance σ_ϵ^2. Some tedious but straightforward algebra will show that a necessary and sufficient condition for increasing the producer's welfare as a result of the introduction of futures markets is

$$(15) \quad b\alpha\sigma_\epsilon^2 > d^2 + 2bd$$

where α is the coefficient of absolute risk aversion. If the slope of the marginal cost curve is relatively small and the slope of the inverse demand curve relatively large, then the producer can be made strictly worse off. In particular, as we have already shown, the producer is made worse off by the opening of futures markets when $b = 0$—that is, when the marginal cost of production is constant.

What about consumer welfare? In the case in which producers are made worse off by the introduction of futures markets, we can use the same arguments used earlier in the constant marginal cost setup to show that consumers are made better off. But if producers are made better off, then the effect on consumers' welfare is ambiguous.

Allowing Free Entry

So far, we have shown that under some assumptions, futures markets stabilize prices, but that even when they do, their effect on welfare is ambiguous. We now ask, Does the stability result hold for a large class of models? Until now, we have assumed that the number of producers in our model industry is fixed. This assumption is questionable. Moreover, the effects of futures markets on welfare as well as on price volatility clearly depend on it. For example, if some producers are made worse off by the introduction of futures markets, some of them might leave to pursue other activities, thereby attenuating the loss of welfare of other producers. This exit decision might also change how prices move. To consider these effects, we assume that there is a large number of risk-averse potential producers and that there is another input in production (say, land) that is in fixed supply. Aggregate output can be increased only by using land of lower productivity.

Suppose, then, that each producer can produce either one unit of the good or none. The cost of producing one unit is c_i for producer i for $i = 1, 2, 3, \ldots$. Assume that $c_i > c_j$ for $i > j$. Each producer acts as a price taker and chooses to produce one unit if

$$(16) \quad E\{u(p - c_i)\} \geq u(0)$$

where the producer's utility function for profits exhibits constant absolute risk aversion. Here $u(0)$ is interpreted as the utility available from other activities. The average demand curve, which gives the expected price as a function of the quantity consumed, is shown in Figure 4.

Now What Happens to Prices?

Recall that since producers face no risk with futures markets, the supply curve with futures trading is just the marginal cost curve. To consider the case without futures trading, we need some additional notation. Let $\bar{p}_i$ denote the expected price that would make producer i exactly indifferent to producing or not. Hence, $\bar{p}_i$ is implicitly defined by

$$(17) \quad E[u(\bar{p}_i + \epsilon - c_i)] = u(0).$$

It follows that $\bar{p}_i - c_i = \bar{p}_j - c_j$ for all i and j.

Define δ as $\bar{p}_i - c_i$. The term δ is the risk premium required to compensate producers when futures markets are absent. Hence, the supply curve with futures trading is the same as that without futures trading but shifted down by the amount of the risk premium δ, as shown in Figure 4. It follows that futures trading reduces the spot price on average. However, the effect on the variance of prices clearly depends on the shape of the supply curve. The variance could increase or decrease or, if marginal cost is linear, be unaffected.

Producer Welfare Increases

It can be seen that the number of producers with futures trading, n, is more than the number of producers without futures trading, N. The nth producer is clearly indifferent between producing and not producing and hence remains unaffected by the introduction of futures trading. In contrast, the Nth producer, who was indifferent to producing or not producing without futures trading, is clearly better off when futures trading is introduced.

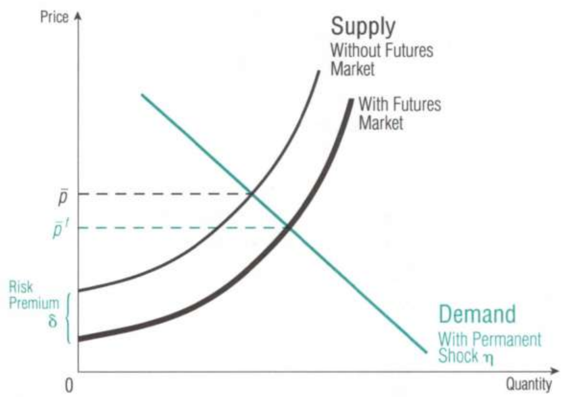
What about the other producers? Each producer is willing to give up δ units to get rid of the uncertainty associated with the price. But the average price declines by less than δ, since the demand curve slopes downward and the supply curve slopes upward. Therefore, producers are made better off. (The effect on consumers' welfare is ambiguous.)

The assumption that producers' preferences are identical and exhibit constant absolute risk aversion is crucial to the result that producers are made better off. For example, suppose absolute risk aversion decreases with wealth; that is, suppose poorer people are willing to pay a larger premium than richer ones to be insured against the same risk. Then the producer with the smallest cost of production (the one who earns the greatest profit) will be less averse to price uncertainty than the producer with a higher production cost. Even though the marginal producer is made better off, some low-cost producers may be made worse off if the supply curve is sufficiently steep.

Figure 4

. . . And in a Model With Free Entry*

Average Supply and Demand



*Here demand shocks are additive and the number of producers is changeable.

Relaxing Some Assumptions

We have shown that futures markets stabilize prices under three restrictive assumptions—when the producer's utility function displays constant absolute risk aversion, the marginal cost is linear, and shocks to demand are additive. We now show three examples of what happens when any one of these assumptions is relaxed: futures trading may increase the variance of spot prices. In each of the examples, the variability of the price due to the transitory shock ϵ is the same with and without futures markets. However, the variability of the price due to the persistent shock η is larger with futures markets than without them. (For a summary of all our results, see the accompanying table.)

Example 1. About Risk Aversion

First, let's relax the assumption of constant absolute risk aversion by the producer. Assume instead that $u(\pi) = \ln(\pi)$, $C(q) = q^2$, and $\epsilon_t = 1$ or -1 with equal probability. Assume also that the inverse demand curve is linear and downward sloping. We show that under these assumptions the supply curve, as a function of the expected price $\bar{p}_t$, is steeper with futures trading than without it. This means the magnitude of the difference between the expected spot prices that will result for any two different realizations of η_{t-1} increases when futures trading is allowed. Therefore, the variability of the spot

A Summary of Futures Markets Effects

↓ Decrease ↑ Increase ? Ambiguous Effect

Models and Assumptions	Effects on: Price Volatility	Welfare of: Producers	Welfare of: Consumers
INSURANCE MODELS			
Original Setup	↓	↓	↑
Additive Demand Shocks			
Constant Marginal Costs			
Fixed Number of Producers			
Basic Changed Setup	↓	?	?
Linear Demand			
Additive Demand Shocks			
Linear, Increasing Marginal Costs			
Constant Absolute Risk Aversion for Producers			
Fixed Number of Producers			
Other Changes			
Allowing Free Entry	?	↑	?
Relaxing Some Assumptions:			
Example 1 Constant Absolute Risk Aversion	↑	–	–
Example 2 Linear Marginal Costs	↑	–	–
Example 3 Additive Demand Shocks	↑	–	–
INFORMATION MODEL	?	↑	↑

price increases due to the introduction of futures markets.

Substituting for $u'(\cdot)$, π_t, ϵ_t, $a = 1$, and $b = 1$ in the first-order condition to the maximization problem given in (9) and simplifying, we get

$$(18) \quad \bar{p}_t^2 - 3\bar{p}_t q_t + 2q_t^2 - 1 = 0.$$

Solving equation (18) for q gives

$$(19) \quad q_t = [3\bar{p}_t - (\bar{p}_t^2 + 8)^{1/2}]/4.$$

We take the negative square root since the second-order conditions for a maximum are satisfied there. Totally differentiating (19) gives

$$(20) \quad d\bar{p}_t/dq_t = (3\bar{p}_t - 4q_t)/(2\bar{p}_t - 3q_t).$$

We can now compare this slope with the slope of the supply curve with futures trading. The slope with futures trading is simply the marginal cost of production, which is 2. Equation (20) easily implies that

$$(21) \quad d\bar{p}_t/dq_t < 2.$$

Thus, the supply curve is steeper with futures trading than without it.

Example 2. About Marginal Cost

Now let's see what happens when we relax the assumption that the marginal cost of production is linear. Instead, let the cost function $C(\cdot)$ be given by

$$(22) \quad C(q) = \begin{cases} q^2 & \text{if } q \leq 1 \\ -1 + 2q + 9.5(q-1)^2 & \text{if } q > 1 \end{cases}.$$

Let the expected utility function of the producer be given by

$$(23) \quad E_{t-1}[u(\pi_t)] = E_{t-1}(\pi_t) - \text{var}_{t-1}(\pi_t).$$

This utility function is consistent with constant absolute risk aversion if the disturbances are normally distributed. Let ϵ_t be normal with zero mean, the variance $\sigma_\epsilon^2 = 1$, and the inverse demand function be

$$(24) \quad p_t = \eta_{t-1} - 4q_t + \epsilon_t$$

where the persistent shock η_{t-1} is either 1 or 5 with equal probability.

It can be verified that in this economy, shocks of 1 and 5 lead to average prices of 0.80 and 4.00 without futures markets and 0.67 and 3.90 with them. The price variances without and with futures markets are 2.56 and 2.60. Thus, although futures trading decreases the spot price's average, it increases the spot price's variability. Futures trading reduces the average price significantly when demand is fairly low, but only slightly when demand is high. Physical constraints on production become more important at higher levels of production.

Although this example is rather artificial, it does capture the flavor of industries like agriculture, which have production factors that are fixed in the short run.

Example 3. About Demand Shocks

Finally, let's relax the assumption that shocks to demand are linear. Suppose the utility function of the producer is the same as (23) in Example 2. Let the inverse demand function be given by

$$(25) \quad p_t = 5 - dq_t + \epsilon_t$$

where d is either 0.1 or 10 with equal probability and ϵ_t is drawn from a standard normal distribution, again as in Example 2. Let the cost function of the producer be given by

$$(26) \quad C(q_t) = q_t^2.$$

It can be verified that the supply curves of this producer without and with futures markets are

$$(27) \quad E_{t-1}(p_t) = 4q_t$$

$$(28) \quad E_{t-1}(p_t^f) = 2q_t^f.$$

Hence, $E_{t-1}(p_t)$ will be either 4.88 or 1.43, depending on the realization of the shock to demand, while $E_{t-1}(p_t^f)$ will be either 4.76 or 0.83.

It is easy to see that the variance of the spot price increases with the introduction of futures trading. In this example, the supply curve becomes less steep with the introduction of futures trading. In addition, high demand periods are also periods with a flat demand curve. These effects cause the variability of the conditional expected spot price to increase when futures trading is introduced.

An Alternative Model

Do our largely negative results stem exclusively from our focus on the insurance role of futures markets? We think not. To make this point precise, we develop a model in which futures markets instead play an *informational* role; they aggregate and disseminate information about demand for goods. We show that in this model, too, futures trading can increase the variability of spot prices even though all participants are made strictly better off.

This model is closely related to one developed by Hart and Kreps (1986). In their model, inventory holders have superior information, but when inventory holding is prohibited, the variability of spot prices is reduced.

In our version of that model, again, futures markets serve as a channel for communicating information. Speculators have better information about the state of future demand than do producers. If futures markets are prohibited, speculators cannot transmit this information to producers. However, with futures markets, the futures price reveals it, and producers can better plan their production. As might be expected, both producers and consumers are made better off when producers get this information. Prices, however, might become more or less volatile.

Formally, we consider a linear-quadratic model so that the effect on variances is easily computed. We also assume that both the producer and the speculator are risk neutral.

At each time t, the producer decides on the quantity q_t to be produced. Note that here we do not require production commitments to be made one period in advance.

The cost incurred by the producer at t is given by

$$(29) \quad C(q_t) = c_t = \delta_0 q_t + (1/2)\delta_1 q_t^2 + (1/2)\delta_2(q_t + \gamma q_{t-1})^2.$$

For technical reasons, we assume that $0 < \gamma \leq 1$. Equation (29) says that the marginal cost of production at time t increases with the quantity q_{t-1} at $t-1$. Agriculture provides a simple example of this. If cereals are grown on the same land season after season, the productivity of the land falls. As a result, the farmer either has to leave the land idle for awhile or has to plant some other crop, such as legumes, that may be less profitable.

In this model, when there is no futures trading, the producer chooses a sequence $\{q_t\}_{t=0}^{\infty}$ so as to maximize the discounted value of profits

$$(30) \quad E_0\{\textstyle\sum_{t=0}\beta^t[p_t q_t - c]\}$$

where $E_0(\cdot)$ is the expectation operator conditioned on the information available to the producer at time 0 and β is a number between zero and one. The inverse demand function is given by

$$(31) \quad p_t = \eta_t - \alpha q_t$$

where the future shock to demand, η_{t+1}, is known to the speculator at t, but observed by the producer only at $t+1$. We assume that the demand shocks η_t are identically and independently distributed over time.

The key difference between this economy with and without futures markets is that with futures markets, the producer at time t knows, from the futures price, the value of the demand shock η_{t+1}. So, for example, if the current shock η_t is high and the future shock is low, then the producer can and will produce a large quantity today. Without futures markets, though, the producer may well be reluctant to produce a lot today because this decision would raise marginal costs of production tomorrow and so restrict the ability to produce a lot tomorrow.

This argument also suggests two effects that work at cross-purposes in determining what happens to the volatility of prices. With futures markets, the current output decision depends on both today's and tomorrow's demand shocks. Without futures markets, the output decision depends only on today's shock. Therefore, for any given value of today's shock, output and so today's price are more variable with futures markets. However, futures markets also make tomorrow's output more responsive to the demand disturbance; hence, they reduce the volatility of tomorrow's spot price.

Given these two effects, the introduction of futures markets may make price volatility increase or decrease, depending on the value of the parameters. In the Appendix, we establish sufficient conditions for an increase.

In this model, the effect of futures markets on welfare is unambiguous: Everyone is made better off.

Summary and Policy Implications

Here we have examined what happens to spot prices of nonstorable goods when trading in futures contracts is introduced into an economy. We have shown that there is no theoretical presumption that futures markets stabilize prices. We have also shown that lower volatility of prices is not necessarily associated with higher economic welfare.

We have used a simple, graphical approach to study the price effect. Futures trading turns out to stabilize prices when the supply curve becomes flatter, but not when the supply curve becomes steeper; then prices become more volatile. This graphical approach made it fairly straightforward to construct examples of both increases and decreases in price volatility resulting from the introduction of futures markets.

The previous academic literature finds that if fluctuations in prices primarily stem from disturbances in demand for goods, then the introduction of futures markets will stabilize spot prices. If, instead, inventory disturbances or production uncertainty are the preponderant shocks, then futures markets tend to destabilize prices. The literature assumes that producers, consumers, and speculators all have utility functions with constant absolute risk aversion, that marginal costs are linear, and that shocks are normally distributed. (See Kawai 1983 and Turnovsky 1983.)

Our assumptions are not the same as those in the literature. We have shown that even without inventory-holding decisions or production uncertainty, changing the assumptions can lead to increased volatility of prices with futures markets. Our results are more general along some dimensions since we do not make any assumptions regarding the nature of the probability distribution of prices. We do, however, assume that speculators are risk neutral. This assumption simplifies the analysis and lets us use more intuitive, geometric methods in proving the results. Since a primary function of futures markets is to provide an outlet for producers to purchase insurance, it seems natural to assume that speculators are less risk averse than producers. That speculators are risk neutral is just the extreme version of this assumption.

The fundamental policymaking issue here concerns the welfare implications of trading in futures markets. We have shown that the connection between spot price volatility and welfare is tenuous. Even when futures trading leads to a reduction in price volatility, some market participants can be made worse off. We need rather strong restrictions on the preferences of producers to ensure that everyone is made better off by the introduction of futures trading. Therefore, in judging whether or not policy changes are desirable, policymakers cannot simply argue that prices will become less volatile. Rather, for any proposed reforms, policymakers must weigh the benefits produced for those who will gain against the costs incurred by those who will lose.

Appendix
When Futures Markets Increase Price Volatility in the Information Model

Here we establish sufficient conditions for the introduction of futures markets to increase the variance of prices when futures markets are playing an informational role in an economy.

We start with the Euler equations for the producer's problem in equation (30) of the preceding paper:

$$\text{(A1)}\quad \beta^t\{p_t - \delta_0 - \delta_1 q_t - \delta_2(q_t + \gamma q_{t-1})\} - \beta^{t+1}\gamma\delta_2 E_{t-1}(q_{t+1} + \gamma q_t) = 0$$

for $t = 1, 2, \ldots$, with q_0 given and the transversality condition given by

$$\text{(A2)}\quad \lim_{T\to\infty} \beta^T\{p_T - \delta_0 - \delta_1 q_T - \delta_2(q_T + \gamma q_{T-1})\} = 0.$$

Note that p_t and q_t are in the information set of the producer at time $t-1$. Standard techniques (as in Sargent 1979) can be used to show that the solution to the Euler equations that satisfies the transversality condition is

$$\text{(A3)}\quad q_{t+1} = \lambda_1 q_t - (1/\lambda_2\beta\gamma\delta_2) \times \Sigma_{i=0}(1/\lambda_2)^i\{E_t(\eta_{t+1+i} - \delta_0)\}$$

where $\lambda_1\lambda_2 = 1/\beta$,

$$\text{(A4)}\quad -(\lambda_1 + \lambda_2) = (\alpha + \delta_1 + \delta_2 + \delta_2\gamma^2\beta)/\beta\gamma\delta_2$$

and λ_1 and λ_2 are the roots of the characteristic equation. Let $\lambda_2 < \lambda_1$. Then $\lambda_2 > -(\alpha + \delta_1 + \delta_2 + \delta_2\gamma^2\beta)/\beta\gamma\delta_2$. Solving (A3) and using the assumption that the demand shocks are identically and independently distributed, we have

$$\text{(A5)}\quad q_{t+1} = -(1/\lambda_2\beta\gamma\delta_2)\{\Sigma_{j=0}\lambda_1^j(\eta_{t+1-j} - \delta_0)\} + K_1$$

where K_1 is a constant. Using the demand function, we get the price without futures markets:

$$\text{(A6)}\quad p_{t+1} = \eta_{t+1} + (\alpha/\lambda_2\beta\gamma\delta_2)\{\Sigma_{j=0}\lambda_1^j(\eta_{t+1-j} - \delta_0)\} + K_2$$

where K_2 is a constant. Hence,

$$\text{(A7)}\quad \operatorname{var}(p_t) = [1 + (\alpha/\lambda_2\beta\gamma\delta_2)]^2 \operatorname{var}(\eta) + [\alpha^2\lambda_1^2/(\lambda_2\beta\gamma\delta_2)^2]\,[1/(1-\lambda_1^2)] \operatorname{var}(\eta).$$

In the economy with futures markets, both the producer and the speculator are risk neutral. Therefore, the only equilibrium is one in which the producer infers η_{t+1} (which is already known to the speculator at t) by observing the futures price. The equilibrium quantity of futures contracts is indeterminate.

Let p_t^f denote the spot price at t with futures trading, as before. It can then be shown that

$$\text{(A8)}\quad p_{t+1}^f = \eta_{t+1} + (\alpha/\lambda_2^2\beta\gamma\delta_2)(\eta_{t+2} - \delta_0) + [\alpha(1+\lambda)/\lambda_2\beta\gamma\delta_2]\,\Sigma_{j=0}\lambda_1^j(\eta_{t+1-j} - \delta_0)$$

where $\lambda = \lambda_1/\lambda_2$. The variance of the spot price with futures markets is then given by

$$\text{(A9)}\quad \operatorname{var}(p^f) = \{1 + [\alpha(1+\lambda)/\lambda_2\beta\gamma\delta_2]\}^2 + (\alpha^2/\lambda_2^4\beta^2\gamma^2\delta_2^2) + [\alpha^2(1+\lambda)^2\lambda_1^2/(\lambda_2\beta\gamma\delta_2)^2(1-\lambda_1^2)].$$

Since $\lambda > 0$, the last term in (A9) is strictly greater than the last term in (A7).

A sufficient condition for $\operatorname{var}(p^f)$ to be greater than $\operatorname{var}(p)$ is

$$\text{(A10)}\quad 1 + [2\alpha(1+\lambda)/\lambda_2\beta\gamma\delta_2] + [\alpha^2(1+\lambda)^2/(\lambda_2\beta\gamma\delta_2)^2] + [\alpha^2/\lambda_2^2(\lambda_2\beta\gamma\delta_2)^2] \geq 1 + (2\alpha/\lambda_2\beta\gamma\delta_2) + [\alpha^2/(\lambda_2\beta\gamma\delta_2)^2].$$

Simplifying (A10), we get the sufficient condition

$$\text{(A11)}\quad 2\lambda(\lambda_2\beta\gamma\delta_2 + \alpha) + (\alpha/\lambda_2^2) + \alpha\lambda^2 \geq 0.$$

Since $\lambda^2 > 0$, $\lambda_1\lambda_2 = 1/\beta$, and $\lambda_2\beta\gamma\delta_2 > -(\alpha + \delta_1 + \delta_2 + \beta\gamma^2\delta_2)$, (A11) holds if

$$\text{(A12)}\quad \alpha \geq 2(\delta_1 + \delta_2 + \beta\gamma^2\delta_2)/\beta.$$

Clearly, condition (A12) is satisfied if the slope of the demand curve α is sufficiently large. The expression in parentheses on the right side of (A12) is the slope of the supply curve when $\gamma = 1$. When that is true, (A12) is satisfied if the slope of the supply curve is sufficiently small.

References

Carlton, Dennis W. 1984. Futures markets: Their purpose, their history, their growth, their successes and failures. *Journal of Futures Markets* 4 (Fall): 237–71.

Friedman, Milton. 1953. The case for flexible exchange rates. In *Essays in positive economics,* pp. 157–203. Chicago: University of Chicago Press.

Gold, Gerald. 1975. *Modern commodity futures trading.* New York: Commodity Research Bureau.

Hart, Oliver D., and Kreps, David M. 1986. Price destabilizing speculation. *Journal of Political Economy* 94 (October): 927–52.

Hieronymus, Thomas A. 1971. *Economics of futures trading for commercial and personal profit.* New York: Commodity Research Bureau.

Kawai, Masahiro. 1983. Spot and futures prices of nonstorable commodities under rational expectations. *Quarterly Journal of Economics* 98 (May): 235–54.

Sargent, Thomas J. 1979. *Macroeconomic theory.* New York: Academic Press.

Siegel, Daniel R., and Siegel, Diane F. 1990. *Futures markets.* Hinsdale, Ill.: Dryden Press.

Turnovsky, Stephen J. 1983. The determination of spot and futures prices with storable commodities. *Econometrica* 51 (September): 1363–87.

Turnovsky, Stephen J., and Campbell, Robert B. 1985. The stabilizing and welfare properties of futures markets: A simulation approach. *International Economic Review* 26 (June): 277–303.