Federal Reserve Bank of Minneapolis Quarterly Review
Spring 1999, vol. 23, no. 2, pp. 13–29

Money and Debt in the Structure of Payments

Edward J. Green
Senior Research Officer
Research Department
Federal Reserve Bank of Minneapolis

Abstract

In Scott Freeman's (1996) model, payment system arrangements based on intermediated debt that is settled with money achieve higher welfare than does direct money payment. In a simplified version of Freeman's model, welfare can be further improved and efficiency achieved by a monetary authority participating in a secondary market for debt or by a private intermediary using a common clearinghouse device. The analysis clarifies that ordinary private agents can assume the role of central bank or clearinghouse; no artificial agent, posited solely to play that role and endowed with special capabilities for it, is necessary. The institutional features required for a central bank or a clearinghouse to achieve efficiency, particularly features related to central bank independence, are discussed informally.

This article is reprinted from *Monetary and Economic Studies* (May 1997, vol. 15, no. 1, pp. 63–87) with the permission of the Institute for Monetary and Economic Studies of the Bank of Japan.

An aphorism in economics is that money exchanges for goods, and goods for money, but goods do not exchange for goods. However, if one interprets *money* to mean base money or other outside money (such as balances held at a central bank), then the aphorism's simple pattern of money-for-goods exchange hardly captures the structure of actual transactions. The goals of this study are to understand the structure of transactions more closely and to address two major issues regarding it.

Notwithstanding the dissimilarities among various payment arrangements at a fine-grained level, most such arrangements have two main structural features in common. First, with few exceptions (such as cashiers checks and some wire-transfer networks based on real-time gross settlement), payment arrangements involve the creation of short-term debt of the payor to the payee that is settled through intermediaries. Second, although incurring short-term indebtedness is a substitute for using money for the purchase of a good, these debt-based arrangements do not wholly replace money, because money is used to settle the debt.[1]

Specifically, then, this study concerns payment arrangements based on intermediated debt that is settled using money. Such arrangements include checks, wire-transfer systems with netting arrangements, credit cards, and the like. The two features emphasized here lie at the root of current discussions regarding welfare and policy aspects of the payment system. Regarding large-value payments especially, there is controversy over whether or not the creation of debt is a desirable feature of a payment system. Given that there is a feasible way to make a cash transaction or to achieve gross settlement of an electronic transaction in real time, it is not obvious what the gain is from making payments in a way that involves creation of debt at an interim stage. In practice, the creation of debt carries at least a small risk of inability to settle, so one would not choose arrangements involving netting or other forms of debt creation if cash or gross-settlement arrangements were equally good in other respects. To the extent that the concentration of this debt in the possession of an intermediary should be cause for additional concern, this argument becomes even more persuasive. In order to make a good case for payment-system arrangements involving intermediated debt, therefore, some specific benefit must be found. Particularly in the case of electronic payments, where the real cost of making a transaction is extremely small, the mere fact that netting economizes on the number of transactions is unlikely to be a sufficient consideration. Thus, it is important to understand whether or not there is some additional benefit from using intermediated debt as a means of payment. The theoretical basis for such understanding is provided by Freeman (1996a, b), who shows that such a benefit does exist in some model environments.[2]

The use of cash settlement for the debt created in the payment system raises a further issue regarding the appropriate role of the public sector and especially of a central bank. Today, countries are taking various stands on this issue. In some countries, the government is solely a regulator of the payment system, while in others, the government is an active participant. In either case, there is a subordinate issue of how to apportion the responsibility for public-sector involvement among the treasury, the bank supervisory agency, and the central bank; and countries differ in their approach to this as well.[3]

Most current discussion of these issues considers the extent to which profit-maximizing operation of the payment system might potentially interfere with the conduct of monetary policy. There is consensus, although not unanimity, that this is not an urgent problem. However, there is another relevant issue that has not been much discussed: whether participation by the monetary authority can potentially enhance the economic efficiency of the payment system.[4] In this study, I address this efficiency issue in the context of Freeman's (1996b) model.[5]

Freeman shows that the potential of a central bank to enhance payment-system efficiency can only be evaluated by close study of the economy concerned. For some parametrizations of the model economy, a laissez-faire market in intermediated debt is efficient. For others, restrictions on private agents' market access entail that the monetary authority can improve welfare relative to some baseline by participating in a secondary market for debt that has not yet been settled.

The baseline to which I refer is the payment system that would be efficient if only a subset of the restrictions on market access were in force. Of course, to make a strong case for the need for the central bank to be a payment-system operator, one would have to show that its participation can improve welfare relative to the best payment system that a purely private system could implement in precisely the economy where its participation is being envisioned. Freeman's (1996b) model is not formulated at a sufficiently fundamental level to answer this question in a fully convincing way, but it comes close to doing so. I will show that efficiency requires an asset that is a perfect substitute for currency, in a sense that I will make precise. Following Freeman (1996a), I extend the model economy to permit a private-sector intermediary to trade its own debt obligation for the debt issued by the initial payor, thus providing such a perfect-substitute asset in the model environment.[6] Since the original debt claim is transferred from the payee to the intermediary, this trade of debt claims is tantamount to *novation and substitution*, a contractual device used by some clearinghouses.[7] Direct participation of the monetary authority is not essential to achieve efficiency in this model. This result can even hold in the extended version of a model environment that Freeman (1996a, b) studies, where intermediaries are unable to settle some of the debt that they issue.

Both the version of the model with central bank participation and the version with novation and substitution implicitly prejudge the issue of asset substitutability, since both versions abstract from aspects of the economy such as privacy of information and limited or costly enforceability of commitments, and these aspects might or might not give agents reason to regard a central bank as a more (or possibly less) trustworthy institution than a private clearinghouse. Although such issues related to credibility and institutional governance lie beyond the scope of the formal model, it is clear that they are inseparable from the market equilibrium issues which are formalized in the model. In particular, issues that determine the effectiveness of participation by a central bank in the payment sys-

tem appear closely related to those that arise with respect to political independence of a central bank.

Modeling Strategy

To address the welfare questions discussed in the introduction requires a model in which the following three means of payment, which are observed in actual economies, emerge endogenously in an equilibrium:

1. Money is used directly as a medium of payment for goods.
2. Some purchases of goods are also financed by the issuance of private debt, and money must be used to pay these purchases off. The use of money for settling debt is conceptually distinct from its direct use as a medium of exchange. In the equilibrium, one should be able to identify separate transactions where the two types of use occur.
3. Besides there being transactions in which money is exchanged for a good, there are also transactions in which money is exchanged for a debt that has not yet been settled.[8]

To formulate such a model, I follow the general strategy that was introduced by Sargent and Wallace (1982), who exhibit an equilibrium that has the first two attributes. The idea is to use an overlapping generations model, so that money can have value in equilibrium and its use can be essential for efficiency, and to posit some heterogeneity among agents within each generational cohort in order to provide an incentive and efficiency rationale for other types of transaction to occur. I proceed by first constructing two simpler model economies, in order to make clear how subsystems of the main model work. To begin, I specify the population and endowment structure that are common to all the models.

Before beginning the technical exposition, however, let me emphasize that the overlapping generations structure of the model is a technical convenience. The aim is to formulate the simplest possible model in which the various kinds of transaction observed in actual economies can all play a role and in which welfare questions regarding those transactions can be framed and analyzed. The spirit of the modeling exercise is that this model is exemplary of models with a lack of double coincidence of wants and with restrictions on agents' access to markets. These fundamental economic features of the model are what lead to the results; consequently, one would confidently expect parallel results from the analysis of more realistic models with the same features. From this perspective, the specific demographic structure of the model formulated here is a matter of convenience, although it might be of great significance in the case of other applications.

The Model

The Population

In each time period $t = 1, 2, 3, \ldots$, a set $A_t = C_t \cup D_t$ of agents is born. The populations C_t and D_t each consist of a continuum of agents, of measure 1. I will sometimes refer to the agents in C_t and D_t as *creditors* and *debtors*, respectively, since the debtors will borrow from the creditors in the equilibrium trading pattern of the model. Each agent lives for two periods (t and $t + 1$). Furthermore, there is a set C_0 of agents, the *initial old* (also a continuum of measure 1), who live only in period 1. Define $C = C_1 \cup C_2 \cup \ldots$ and $D = D_1 \cup D_2 \cup \ldots$.

Each agent in A_t is endowed with one unit of a perishable good in period t and with nothing in period $t + 1$. Agents in C and D are endowed with different goods.

Each agent in C_0 is endowed with one unit of fiat money, but with no consumption good.

Let x_{1t} (respectively, x_{2t}) be an agent's consumption of the endowment good of agents in C (respectively, D) in period t.

An agent must consume a nonnegative quantity of each good in each period. Let the utility function of an agent be

$$(1) \qquad w_i(x_{1t}, x_{2t}, x_{1(t+1)}, x_{2(t+1)}) = \begin{cases} u(x_{1t}) + v(x_{2(t+n)}) & \text{if agent } i \text{ is in } C \\ u_*(x_{1t}) + v_*(x_{2t}) & \text{if agent } i \text{ is in } D \\ v(x_{21}) & \text{if agent } i \text{ is in } C_0 \end{cases}.$$

Assume that all the functions on the right side are strictly increasing, are strictly concave, and satisfy the Inada condition that the limit of the derivative as the argument tends to zero from the right is infinite.

Given this specification of utility functions, and given the focus on stationary allocations in this study, the following notation that suppresses time subscripts will be convenient:

x_1 = consumption of x_{1t} by an agent in C_t

x_2' = consumption of $x_{2(t+1)}$ by an agent in C_t

x_1^* = consumption of x_{1t} by an agent in D_t

x_2^* = consumption of x_{2t} by an agent in D_t.

Note that agents in D wish to trade with members of their own age cohort in C, while agents in C wish to trade with members of the next age cohort in D. Thus, it will be seen that, as in the standard overlapping generations model of money (as well as in most other models in which fiat money is endogenously valued in equilibrium), there can be no mutually advantageous trades unless fiat money has value.

Efficiency

I concentrate on stationary allocations, that is, those in which corresponding agents in distinct age cohorts receive identical lifetime consumption bundles, except for the dating of their goods. (The consumption of an agent in C_0 is identical to the consumption of an agent in C_t at period $t + 1$.)

An *efficient stationary allocation* is a stationary allocation that solves the problem of maximizing a weighted sum of utilities of the members of C and D in each age cohort. That is, $(\hat{x}_1, \hat{x}_2, \hat{x}_1^*, \hat{x}_2^*)$ is efficient if, for some $\pi > 0$, it solves the problem

$$(2) \qquad \max\ [u(x_1) + v(x_2')] + \pi[u_*(x_1^*) + v_*(x_2^*)]$$

subject to the feasibility constraints that

$$(3) \qquad x_1 + x_1^* = 1$$

(4) $x_2' + x_2^* = 1.$

A necessary and sufficient condition for a feasible stationary allocation to be efficient is that

(5) $v'(x_2')/u'(x_1) = v_*'(x_2^*)/u_*'(x_1^*).$

I study this criterion of efficiency because of its technical simplicity and because it implies the standard Pareto-efficiency criterion. An efficient stationary allocation is Pareto efficient in the set of all feasible allocations of the infinite-horizon economy, as shown by Okuno and Zilcha (1980).

I am concerned with implementing a specific allocation under various constraints on market access. To define the allocation, consider a two-agent exchange economy. The first agent is endowed with one unit of good 1 and has the utility function $w(x) = u(x_1) + v(x_2')$. The second agent is endowed with one unit of good 2 and has the utility function $w(x^*) = u_*(x_1^*) + v_*(x_2^*)$. Define the stationary allocation $(\tilde{x}_1,\tilde{x}_2',\tilde{x}_1^*,\tilde{x}_2^*)$ by stipulating that $(\tilde{x}_1,\tilde{x}_2')$ and $(\tilde{x}_1^*,\tilde{x}_2^*)$ are the Walrasian consumption bundles of these two agents. Note that equation (5) is a necessary condition for a Walrasian equilibrium of the two-agent economy, so the corresponding stationary equilibrium of the infinite-horizon economy is efficient.

Clearly, the Walrasian price that supports this equilibrium is

(6) $\tilde{p} = (1/\tilde{x}_1^*, 1/\tilde{x}_2').$

Market Access, Securities, and Equilibrium

I complete the specification of the economy by imposing explicit constraints on agents' access to markets in each period.[9] In each period, there will be a sequence of subperiods. In each subperiod, only a subset of the agents currently alive will be able to trade or settle debts. In order for trade or debt settlement to be transacted between agents who do not have direct access to one another, money or another security must be accepted by a third agent or even by several intermediate agents.

Equilibrium is defined in terms of two features: agents are price takers who make optimal trading plans, given prices in the markets to which they have access (including correctly anticipated prices in markets to which they will have future access); and markets clear.[10]

For clarity, I consider three access-constraint specifications below. In the next section, I specify the constraints in such a way that only the use of fiat money is required to support an efficient equilibrium. Following that, I specify the constraints in such a way that debt, as well as fiat money, needs to be used. Finally, I specify the constraints in such a way that the debt must be intermediated in order to be settled. Also in this final specification, either the stock of money must fluctuate within each period, or the debt must be exchanged for debt issued by the intermediary (that is, novation must occur), in order for an efficient stationary equilibrium to be supported.

Modeling Money, Debt, and Intermediation

A Basic Overlapping Generations Structure: Valued Fiat Money

Suppose that, in each period $t = 1, 2, ...$, all of the agents currently alive are able to trade among themselves in the following pattern. First, the agents in C_{t-1} trade with those in D_t. Subsequently, the agents in D_t trade with those in C_t.

I show that, because each agent in C_0 holds a unit of money, there is a trading pattern for goods that can achieve efficiency in this market structure. Young D agents give some of their endowment to old C agents and subsequently receive some of the endowment of the young C agents. The entire money stock is passed in the opposite direction to goods at each stage, so that the old C agents continue to be the money holders at the beginning of each period. If prices are set appropriately, markets clear and all agents have incentive to make the efficient trades.

To formalize this idea, let each period t be divided into two subperiods, t.1 and t.2. Market participation is described in Table 1, which lists the agents who have access and the goods that are traded within each subperiod. Money is also traded in each subperiod, and it is the numeraire.[11] It will be represented as the last coordinate of a price vector.

That is, in the market in t.1, there is a price $p^1 = (p_2^1,1)$ that has only two coordinates, since good 2 (that is, the debtors' endowment good) is the only good available to be traded. In the market in t.2, there is a price vector $p^2 = (p_1^2,p_2^2,1)$, since both goods 1 and 2 are available in the market. (By the Inada condition on v_*, debtors will not trade away their entire endowments in t.1.)

I adopt the following notation to represent net trades. A net trade is always represented by the variable z, which can have the following superscripts and subscripts:

1. An asterisk superscript immediately following z indicates that the net trade belongs to an agent in D.
2. A numerical superscript indicates the subperiod in which the net trade is made.
3. A prime on the numerical superscript indicates that the net trade is made by an agent in the second period of life (that is, by an agent in A_{t-1} in period t).
4. A subscript indicates a coordinate of z. A numerical subscript 1 or 2 refers to one of the two goods available in the period of the market, and a letter subscript m (money), d (debt issued in the current period), d' (debt issued in the preceding period), or n (debt arising from novation, which will be introduced later) may also occur.

The letter p denotes a price vector. A numerical superscript on p indicates the subperiod of the market to which this price corresponds. A subscript on p, which can take the values just defined, indicates a coordinate.

An agent in C_t has access in t.2 (for $t > 0$) and in $(t+1)$.1. The agent makes a net trade $z^2 = (z_1^2,z_2^2,z_m^2)$ in t.2 and a net trade $z^{1'} = (z_2^{1'},z_m^{1'})$ in $(t+1)$.1. Thus, the market-constrained optimization problem of an agent in C_t is

(7) $\max\ u(x_1) + v(x_2')$

subject to

$$(x_1, x_2') \in \Re^2_+ \qquad z^2_m \geq 0$$
$$x_1 = 1 + z^2_1 \qquad z^{1'}_m \geq -z^2_m$$
$$x_2' = z^{1'}_2 \qquad p^2 z^2 \leq 0$$
$$z^2_2 \geq 0 \qquad p^1 z^{1'} \leq 0.$$

(Note that, by the specification of the agent's endowment and utility function, utility maximization will clearly imply that $z^2_2 = z^{1'}_1 = 0$ and $z^{1'}_m = -z^2_m$. An agent in C_0 only makes net trade z', and utility maximization clearly implies that $z^{1'}_m = -1$; that is, old creditors dispose of their entire money stock.)

An agent in D_t has access in $t.1$ and $t.2$ and makes net trades $z^{*1} = (z^{*1}_2, z^{*1}_m)$ and $z^{*2} = (z^{*2}_1, z^{*2}_2, z^{*2}_m)$ in these periods, respectively. This agent's market-constrained optimization problem is

$$\text{(8)} \qquad \max u_*(x^*_1) + v_*(x^*_2)$$

subject to

$$(x^*_1, x^*_2) \in \Re^2_+ \qquad z^{*1}_m \geq 0$$
$$x^*_1 = z^{*2}_1 \qquad z^{*2}_m \geq -z^{*1}_m$$
$$x^*_2 = 1 + z^{*1}_2 + z^{*2}_2 \qquad p^1 z^{*1} \leq 0$$
$$z^{*1}_2 \geq -1 \qquad p^2 z^{*2} \leq 0.$$

The structure of trading in this environment is indicated in Chart 1. Time is on the horizontal axis. An agent's life span is depicted by a thin horizontal line of two periods' length. Within each period, the subperiods in which an agent has market access are shown by a thickening of the line into a bar. (A thin vertical line connects these bars during each subperiod.) The top line (extending only through period 1) is C_0. After that, there are four generations having two lines each, representing D'_t and C'_t in descending order.

Since C_{t-1}, C_t, and D_t all have the same number of agents, the market-clearing conditions for this economy are that

$$\text{(9)} \qquad z^{1'} = -z^{*1}$$

$$\text{(10)} \qquad z^2 = -z^{*2}.$$

Now it is straightforward to verify, using equation (5), that the Walrasian stationary allocation $(\tilde{x}_1, \tilde{x}'_2, \tilde{x}^*_1, \tilde{x}^*_2)$ is an equilibrium allocation of this market structure. Equilibrium is supported by the following net trades and prices:

$$\text{(11)} \qquad p_1 = (1/\tilde{x}'_2, 1)$$

$$\text{(12)} \qquad z^{1'} = -z^{*1} = (\tilde{x}'_2, -1)$$

$$\text{(13)} \qquad p^2 = (1/\tilde{x}^*_1, 1/\tilde{x}'_2, 1)$$

$$\text{(14)} \qquad z^2 = -z^{*2} = (-\tilde{x}'_1, 0, 1).$$

Because the C agents closely resemble the agents in the standard overlapping generations model, and the D agents want only to trade their endowment good for a contemporaneous good, it is not surprising that the efficient equilibrium here bears very close resemblance to the efficient overlapping generations equilibrium. In particular, money has value, but there is no credit, and there is no role for a monetary authority.

Reversing the Order of Transactions Within a Period: Debt Securities

Now consider the opposite order of transactions. That is, suppose that first the agents in D_t trade with those in C_t and subsequently the agents in C_{t-1} trade with those in D_t.

For fiat money to be passed from the old C agents to the young ones, it would have to pass through the hands of the young D agents. But since those agents do not meet the old C agents until it is too late to deal with the young ones, that cannot happen.

If it is possible for young agents to issue debt securities that they pay in money when they are old, then there is a solution. The young D agents can use these securities to finance their consumption of goods purchased from young C agents, then give some of their endowments to old C agents in return for their money, and finally carry the money into the next period and use it to repay the holders (who will still be alive since they are young when the debt securities are issued). This repayment of debt requires an additional subperiod in each period, which I will assume to occur between the two subperiods where markets occur. Although from an ex post perspective, repayment of debt is a mandatory transfer, not a voluntary exchange, it will be treated formally as an exchange. That is, after a debt is repaid, the debtor's portfolio holds a zero quantity of debt.

The debt security traded in this economy is a commitment to pay one unit of money to the bearer, at some time during the period following the period in which the debt security is issued. The quantity of this security that an agent acquires is denoted d. That is, issuing a unit of debt corresponds to choosing $d = -1$.

Table 2 shows the order of transactions within each period t. The last column shows, for each subperiod, which goods (1 and 2) and assets (d and m) are traded. These are listed in the order they appear in the price vector. The numeraire is last.

The market-constrained optimization problem of an agent in C_t is to make net trades z^1, $z^{2'}$, and $z^{3'}$ that solve

$$\text{(15)} \qquad \max u(x_1) + v(x'_2)$$

subject to

$$(x_1, x'_2) \in \Re^2_+$$
$$x_1 = 1 + z^1_1 \qquad p^1 z^1 \leq 0$$
$$x'_2 = z^{3'}_2 \qquad p^3 z^{3'} \leq 0$$
$$z^1_2 \geq 0 \qquad z^{3'}_m \geq -z^{2'}_m$$
$$z^{2'}_m \geq 0 \qquad z^{2'}_m \geq z^1_d.$$

The market-constrained optimization problem of an agent in D_t is to make net trades z^{*1}, z^{*3}, and $z^{*2'}$ that solve

$$\text{(16)} \qquad \max u_*(x^*_1) + v_*(x^*_2)$$

subject to

$$(x_1^*, x_2^*) \in \Re_+^2$$

$$x_1^* = z_1^{*1} \qquad p^1 z^{*1} \le 0$$

$$x_2^* = 1 + z_2^{*1} + z_2^{*3} \qquad p^3 z^{*3} \le 0$$

$$z_2^{*1} \ge -1 \qquad z_m^{*2'} \ge -z_m^{*3}$$

$$z_m^{*3} \ge 0 \qquad z_m^{*2'} = z_d^1.$$

The structure of trading in this environment is indicated in Chart 2.

The market-clearing conditions for this economy are that

(17) $z^1 = -z^{*1}$

(18) $z^{2'} = -z^{*2'}$

(19) $z^{3'} = -z^{*3}$.

Again, it is straightforward to verify that the Walrasian stationary allocation $(\tilde{x}_1, \tilde{x}_2', \tilde{x}_1^*, \tilde{x}_2^*)$ is an equilibrium allocation of this market structure. Equilibrium is supported by the following prices and net trades:

(20) $p^1 = (1/\tilde{x}_1^*, 1/\tilde{x}_2', 1)$

(21) $z^1 = -z^{*1} = (-\tilde{x}_1^*, 0, 1)$

(22) $z^{2'} = -z^{*2'} = (-1, 1)$

(23) $p^3 = (1/\tilde{x}_2', 1)$

(24) $z^{3'} = -z^{*3} = (\tilde{x}_2', -1)$.

The efficient equilibrium in this transaction structure involves use of both valued fiat money and debt securities, but the debt securities are not intermediated, and there is no role for a monetary authority.

Separation Within a Cohort: Intermediated Debt Securities

Now I come to one of the two main market structures in this study. In this structure, not all agents of the same cohort can communicate directly with one another in the second period of their lives. Specifically, some debtors are not able to repay creditors to whom they have issued debt. Those creditors therefore need to sell their debt to other agents with whom the debtors can communicate. These purchasers of debt thus serve as intermediaries in the settlement of the original transactions.

To formalize this environment, define the partitions $C_t = C_t' \cup C_t''$ and $D_t = D_t' \cup D_t''$, for each $t \ge 1$. Define $C_0'' = C_0$. For each $t \ge 1$, let there be $\gamma \in (0,1)$ agents in C_t' and $\delta \in (0,1)$ agents in D_t'.

The market structure will be specified in such a way that creditors in C_t' cannot be repaid in $t + 1$ by debtors in D_t''. To make this specification, consider the following sequence of trading-opportunity subperiods within each period $t > 1$. (Only the first and last subperiods occur for $t = 1$.)

1. All agents in A_t trade with one another.
2. All agents in C_{t-1} enter the market. Agents in D_{t-1}' also enter the market and have the opportunity to pay the debt securities to their creditors.
3. All agents in C_{t-1} can trade money for outstanding debt securities that have not been settled. For now, assume that no new debt can be issued in this subperiod.
4. Agents in C_{t-1}' trade with agents in D_t and then leave the market.[12]
5. Agents in D_{t-1}'' enter and have the opportunity to pay their debt securities to anyone in C_{t-1}'' who is holding them.
6. Agents in C_{t-1}'' trade with agents in D_t.

This structure is represented in Table 3.

Subperiods t.2 and t.5 are distinct from subperiod t.3. In t.2 and t.5, debt is being settled at face value. In contrast, in t.3, debt is being purchased at market terms prior to settlement. Here, as in the other markets where voluntary exchange occurs, the price must be determined endogenously by agents' optimization together with market-clearing.

When an agent is young, the agent's incentive to trade with another member of the cohort is evidently affected by what the agent knows or believes about both its own subgroup and the trading partner's subgroup in the market structure when the two agents are old. I assume that no information about these matters is available until the second period of the agents' lives. Later, I discuss an implication of this assumption for welfare analysis.

Another question concerns the structure of debt security issuance. Is trade bilateral, so that each young D agent issues one debt security to a single young C agent? Or does each young D agent make small purchases from many young C agents, so that each C agent holds a diversified portfolio of small-denomination debt securities afterward? Risk-diversification considerations would seemingly lead the C agents to prefer the latter arrangement, if it is feasible.[13] The diversified, nonstrategic trading arrangement will be modeled here.

This arrangement implies an asymmetry in the interpretation of the quantity of debt securities held by an agent. If an agent holds a positive quantity of these, then that quantity represents a diversified portfolio of securities payable by all issuers in the economy, in proportion to those issuers' amounts of debt outstanding. If the quantity of debt is negative (that is, if the agent is an issuer of debt), then it represents debt issued by the agent.

Here, as in the market structures studied above, equilibrium is defined in terms of agents' optimization together with market-clearing. The objective function of an agent in C_t is slightly different here than above, since the agent's consumption can depend on whether the agent is in C_t' or C_t''. I assume that such an agent maximizes expected utility and assign probability γ to the event that the agent is in C_t' and consumes bundle x' and $1 - \gamma$ to the event that the agent is in C_t'' and consumes bundle x'', in period $t + 1$.

The optimization problem of an agent in C, then, is to choose net trades z^1, $z^{2'}$, $z^{2''}$, $z^{3'}$, $z^{3''}$, z^4, $z^{5''}$, and $z^{6''}$ to

(25) $\max u(x_1) + \gamma v(x_2') + (1-\gamma) v(x_2'')$

subject to constraints. The constraints and market conditions are conceptually straightforward, but they are numerous because the environment is so complex. They are presented in the Appendix.

The structure of trading in this environment is indicated in Chart 3.

Inefficiency of Equilibrium

The market structure just described permits trading of goods and three financial assets: money (m), new debt (d), and seasoned debt (d'). It is clear that there exists a pattern of trade—involving goods for new debt, goods for money, and seasoned debt for money market transactions, as well as settlement of seasoned debt—that achieves the stationary efficient allocation. That pattern of trade requires goods and assets to be exchanged in particular ratios. If those ratios are not the same as the price ratios in a competitive equilibrium, though, then the stationary efficient allocation will not be a competitive equilibrium allocation of the economy. Following Freeman (1996a, b), I show that equilibrium is inefficient in an economy where $\gamma > \delta$.

I begin the argument by supposing that, in subperiod 1 in period $t - 1$, each agent in C_{t-1} has acquired debt securities for one unit of money to be delivered in period t. (It is easy to see that, except in autarky equilibrium, the entire money stock of one unit must be passed from cohort to cohort in a stationary equilibrium.) Note that market-clearing in that subperiod implies that each agent in D_{t-1} owes one unit of money in period t. By diversification, in subperiod 2 in period t, each agent in C_{t-1} receives a total of δ units of fiat money from the agents in D'_{t-1} and is still owed $1 - \delta$ from the remaining agents in D_{t-1}. Agents in C'_{t-1} will not be able to collect their payments from those debtors in subperiod 4, though, so in subperiod 3 they will sell the debt securities still in their possession to other creditors who will participate in subperiod 4.

Agents in C'_{t-1} regard debt as worthless except in trade in subperiod 3. They will trade away their full inventories at any positive price.

Debt is certain to be paid by subperiod 5, and agents in C''_{t-1} do not need to use fiat money until subperiod 6, so these agents will be willing to pay up to the face value of debt to obtain fiat money in subperiod 3.

Thus, all money held by agents in C''_{t-1}, up to the face value of the debt held by agents in D''_{t-1}, will be exchanged for that debt. This determines the equilibrium price in the secondary market.

At the beginning of subperiod 3, the aggregate amount of money that will be provided in settlement of the debt in the possession of agents in C'_{t-1} is $\gamma(1-\delta)$. The total amount of money in the possession of agents in C''_{t-1} is $(1-\gamma)\delta$. Thus, the competitive price in subperiod 3 of a debt claim for one unit of money is

$$(26) \qquad p^3_{d'} = \min\,[1, (1-\gamma)\delta/\gamma(1-\delta)].$$

If $\delta < \gamma$, then $p < 1$.

Thus, if $\delta < \gamma$, then availability of money to intermediaries is a bottleneck in some sense. It remains to be shown that this bottleneck causes Pareto inefficiency. Freeman's (1996a, b) argument continues with a comparison of the amount of consumption enjoyed by an agent in C' with the amount enjoyed by an agent in C'' in equilibrium. The following allocation shows that the consumption of an agent in C' is lower, so the fact that too few debtors have market access in subperiod 2 induces consumption inequality among agents who are identical except for market access. This inequality is risk from an ex ante perspective, so from that perspective it is a Pareto-inefficient allocation among risk-averse agents.[14]

Specifically, an agent in C'_{t-1} receives δ units of money in settlement of debt in subperiod 2 and $p^3_{d'}(1-\delta)$ units of money from sale in subperiod 3 of debt not yet settled. Thus, an agent in C'_{t-1} holds less than one unit of fiat money to trade in subperiod 4.

In subperiod 3, an agent in C''_{t-1} spends all its money received in settlement of debt in subperiod 2 to purchase debt at price $p < 1$, which will be settled at par in subperiod 5. Thus, the agent will hold more fiat money in subperiod 6 than if the agent had not traded in the secondary market. That is, the agent will hold more than one unit of fiat money to trade in subperiod 6.

In equilibrium, agents in D_t must sell their endowment good for the same price in subperiod 4 as in subperiod 6. Therefore, an agent in C''_{t-1} consumes more of that good than does an agent in C'_{t-1}, since the agent in C''_{t-1} has more money to spend at the identical price for goods.

Chartering a Monetary Authority to Achieve Efficiency

Representing a Monetary Authority Within the Model

Before I present a result from Freeman (1996a, b) regarding the potential role of a monetary authority in achieving efficiency, it is worthwhile to reflect on what a monetary authority is and on how it ought to be modeled in this formal environment. First, consider what Freeman (1996b, pp. 1129, 1134) assumes about the monetary authority and how he characterizes its optimal policy:

> There exists . . . a monetary authority able to issue fiat money . . . . This authority issues an initial stock of . . . [money] to each initial old creditor . . . . Suppose that the . . . monetary authority (or "central bank") is now authorized to issue and lend fiat money equal to the nominal amount of debt presented by any of the late-leaving creditors . . . . This central-bank loan must be repaid with fiat money upon the arrival . . . of the late-arriving borrowers.

Because the monetary authority is described as dealing with the creditors in every cohort, superficially it might seem that the authority must be an infinite-lived agent. In that case, the monetary authority would be in a position to provide intermediation services that no private agent could provide.

There is a convincing argument that this is an inadvisable way to think about the role of a monetary authority or, in general, an agent that carries out public policy.[15] The criticism has to do with a dilemma regarding how to interpret the restrictions on market access in the model economy. These restrictions could be interpreted as reflecting technological restrictions, but then why the monetary authority is not bound by the same constraints that private agents face would be inexplicable. Alternatively, the restrictions could be interpreted as reflecting institutional or legal constraints from which the mone-

tary authority is exempt, but then the most natural welfare conclusion to draw from the inefficiency of competitive equilibrium would be that those constraints on private agents ought to be relaxed in general, not that there is a rationale for a distinguished agent to be granted a special exemption. These seem to be the only tenable interpretations of the market-access restrictions, and neither provides a good basis for understanding the role of a monetary authority.

On closer inspection, though, the monetary authority does not intermediate between agents who do not meet one another. In every period, it issues money in subperiod 3, which it uses to purchase seasoned debt. Then, in subperiod 5 in the same period, it absorbs the money it receives in settlement of this debt. Thus, rather than specifying that there is a special, infinite-lived agent in the model, one can equally well specify that, in subperiod 2 in each period t, one of the agents in C''_{t-1} is designated to be the monetary authority.

How a Monetary Authority Can Achieve Efficiency

Consider what can be accomplished by such a monetary authority, which consists of one agent in each cohort (specifically, in C''_{t-1}, in each period t) who is authorized to behave differently in one respect, and is constrained to behave differently in another respect, from the other agents. This distinguished agent is authorized to create money in subperiod 3 and is required to destroy in subperiod 5 an amount of money equal to that created. Specifically, the agent is authorized to create $\gamma(1-\delta) - (1-\gamma)\delta$ units of money for purchase of seasoned debt in subperiod 2, so that (by the argument leading to (26) in the laissez-faire case) $p^3_{d'} = 1$. This intervention eliminates the inequality of consumption between agents in C'_{t-1} and those in C''_{t-1}. Thus, it attains efficiency from the ex ante perspective.

Interpreting the monetary authority in this way, as being one of the private agents in the population who is selected to carry out a special responsibility, avoids making the suspect assumption that the monetary authority has a mysterious technological superiority over the private agents. This interpretation also has a clear implication regarding the nature of the contract to which the monetary authority is subject. That authority is exempt from the prohibition that other agents face against creating money (that is, against counterfeiting). However, it is expected to absorb the money received in settlement in subperiod 5 (with the exception of money received in settlement of debt in its private portfolio, as opposed to the debt initially purchased with newly created money), rather than to spend that money in subperiod 6 to finance consumption for itself. For such an expectation to be fulfilled, the monetary authority must be constrained in some way, or its incentives must be modified in some way, that is not represented explicitly in the model. This implicit assumption is analogous to the implicit assumption of some enforcement technology to compel repayment of debt. Subject to this assumption, the present analysis shows that the difference between a monetary authority and an ordinary private agent is simply one of incentives and not one of intrinsic opportunities or capabilities. (The one obvious advantage that a monetary authority typically enjoys with respect to private banks—a monopoly, or at least a competitive advantage, in note issuance—is an artifact of government policy rather than being intrinsic.)

Nothing in the formal model requires that this special incentive arrangement be offered only to a single agent. It could be supposed instead that all agents in C''_{t-1} would be subject to the arrangement. However, the implicitly assumed monitoring and enforcement functions are presumably costly to carry out. It would be inefficient to exercise them over all agents in C''_{t-1}, or even over several of them, if one agent can make all the transactions required for efficiency. This consideration suggests that the activity of central banking is probably a natural monopoly.

Relationship to Central Bank Independence

This agent-within-the-model interpretation of the nature of a monetary authority is different from the social-planner interpretation that economists often make. Nevertheless, the agent-within-the-model interpretation is consonant with the views expressed by distinguished scholars of central banking, such as Sayers (1967), Cairncross (1988), Goodhart (1988), and Cukierman (1995). Numerous central banks, including the Bank of England, were initially chartered as private joint-stock companies and continued to operate under that form of ownership long after their public policy roles were firmly established. In many countries today, including the United States, payment-system activities of the central bank continue to be conducted under a corporate charter, and the government is at most a minority owner. Thus, it is very appropriate to model the monetary authority as being identical to a private agent in most respects.

However, despite their corporate form, central banks are organized in a way that induces a markedly different outcome from the operation of an ordinary corporation. Ownership of a central bank is typically an entitlement to a fixed income stream (analogous to ownership of preferred stock, rather than common stock), with residual profits actually accruing to the government. From a perspective such as that taken by Jensen and Meckling (1976), the government is the true owner of the central bank (as the residual claimant of its profit stream), and thus control of it by the nominal owners is really a means of separating ownership and control in economic terms.

To the extent that the nominal owners of the central bank have the primary influence on the appointment and retention of its governor and other senior executives, it is foreseeable that the executives will have relatively small incentive to maximize profit. Other charter provisions, such as restrictions on the types of asset that can be held in the portfolio, complement the ownership structure by constraining the central bank from emulating the decisions that private agents would make to maximize profits.

The fact that central bank charters have such striking and idiosyncratic provisions, which are recognized to safeguard central bank independence from the residual claimant of the bank's profit, constitutes evidence in favor of the modeling approach taken here: to represent a monetary authority as an agent with the same intrinsic opportunities and capabilities as other agents, but with different induced incentives or legal constraints. Conversely, if the market structure specified above, which requires intermediaries to settle transactions, is the one that would exist under laissez-faire, then the fact that an

efficient allocation can be achieved by a departure from profit maximization on the part of the monetary authority provides a normative argument in favor of central bank independence.

Institutional and Contractual Alternatives to Central Bank Participation

A careful statement of the conclusion reached above is that, if the market structure requiring intermediaries were in effect and there were no participation by a monetary authority, then central bank participation in the form of open market operations, or equivalent intervention to support the secondary-market price of debt in subperiod 3, would support an efficient allocation that Pareto-dominates the laissez-faire equilibrium allocation from an ex ante perspective.

The applicability of this analysis to actual markets is an open question, because it is not certain that the market structure requiring intermediaries to settle transactions is the one that would emerge under laissez-faire. That market structure abstracts from private-sector agents that provide payment services, such as escrow agents and clearinghouses. It also abstracts from contractual features of payment, such as contract netting and novation.

In this section, I discuss one such private-sector arrangement that can achieve efficiency in the intermediaries environment. This arrangement resembles an intermediary that uses novation and substitution (that is, substitution of debt payable by itself for debt payable by the original issuer) to settle contracts. Although there are various types of intermediary in an actual economy that resemble this theoretical arrangement in some respects, the parallel with clearinghouses seems especially strong.[16]

A Market Structure With Novation Securities

An alternative to having a monetary authority is for agents in C''_{t-1} to issue debt securities—called *novation securities*—in subperiod t.3 in return for the debt securities of traders in C'_{t-1} that have not yet been settled. The agents in C'_{t-1} will exchange these novation securities for good 1 in subperiod t.4. The novation securities will be paid in subperiod t.6, when the agents in D_t who have acquired them will meet the agents in C''_{t-1} who issued them. In equilibrium, both the initial securities and the novation securities will trade at face value. Thus, again, the risk induced by trading-opportunity uncertainty will be fully insured, and efficiency will be achieved.

The asset structure of this economy is described by adding novation debt (denoted n) to the trading structure described in Table 4.

The budget constraints and market-clearing conditions for this market structure are straightforward modifications of those for the market structure that requires intermediaries to settle transactions.

With respect to the characteristics of securities that are represented explicitly in this model, there is hardly any difference between this novation security and the money issued and reabsorbed by the central bank above. Both money and the novation security are issued by agents in C''_{t-1} in subperiod t.3 to agents in C'_{t-1} in return for the debt held by those agents. The agents in C'_{t-1} trade the newly issued security (money or the novation security, depending on the payment arrangement) in subperiod t.4 to agents in D_t for those agents' consumption good. The security, or another security of the same type, is thereafter removed from circulation by the issuer. In the case of money, this happens in subperiod t.5 when the seasoned debt that was purchased with newly issued money is settled. In the case of novation debt, the money received in settlement of seasoned debt in subperiod t.5 is used to settle the novation debt in subperiod t.6. Only with respect to the specifics of how removal from circulation is accomplished does novation debt differ from money in more than name.

Implicitly, though, money and novation securities differ in much more than name. What differs between the two asset structures is the institutional framework of ownership, monitoring, and enforcement that must exist to support them. In contrast to the distinctive features of a central bank that have been mentioned above, a clearinghouse that operates by novation and substitution is subject to roughly the same framework of contract and enforcement as is a private debtor. Although it would be an exaggeration to claim that a central bank is totally unlike a clearinghouse (especially since a clearinghouse is typically chartered as a nonprofit corporation, is jointly owned by a group of the banks that it serves, and has restricted powers that prevent it from competing directly with them), in practice the distinction between them is substantial and easy to recognize.

Historically, clearinghouses preceded central banks in most industrialized countries, and central banks were chartered in part to address perceived inefficiencies in the payment systems where those clearinghouses were already operating. Despite the presence of central banks, which have tended to be advantaged relative to clearinghouses in point of their legal powers, clearinghouses continue to exist and to play a major role. These facts suggest that probably neither institutional form has an absolute advantage over the other. The basic model of intermediated debt and its extension in this section can perhaps provide a basis for thinking systematically about the relative advantages of each type of institution in various circumstances.

An Economic Definition of Novation and Substitution

A noteworthy feature of the extended model just discussed is that it permits an economic definition of novation and substitution. This operating procedure of a clearinghouse is typically described in institutional terms related to contract law, as in the following quotation from a report of a committee of the Bank for International Settlements (the Angell Report 1989, ¶2.5):

> One type of arrangement would establish a clearing house that would be substituted as the central counterparty in deals submitted for netting by participants in the arrangement, in order to effect a binding multilateral netting among those participants ("multilateral netting by novation and substitution").

Such substitution is exactly what takes place, in the equilibrium of the asset structure discussed above, when agents in C'_{t-1} swap debt issued by agents in D_{t-1} for novation securities (also debt) issued by agents in C''_{t-1}. Each agent in C_{t-1} has a different portfolio of specific debt securities after this swap than before. However, each of these agents has the same net credit position afterward

as before. Agents in C'_{t-1} hold debt securities both before and after the swap, and in equilibrium, the face value of the securities (as well as the market value) is the same. Agents in C''_{t-1} change from being only holders of debt before the swap to being both holders and issuers afterward, but again, there is no change in their net credit position. Thus, it is clear that *novation and substitution* can be defined in economic terms as an issuance and exchange of debt that leaves the net credit position of all agents unaffected.

The economic role of novation and substitution is to transfer debt from agents who do not have an opportunity to receive settlement of it to other agents who do have that opportunity, without affecting anyone's wealth position and in such a way that the initial debt holders have trading opportunities (that is, liquidity) equivalent to what they would have had if their initial debt holdings had been settled rather than traded.

Failure of a Clearinghouse to Settle

There is a consensus among payment-system experts that the failure of a clearinghouse to settle its obligations created by novation and substitution is an especially worrisome systemic risk. This view is clearly expressed by the Angell Report (1989, ¶6.27.iv):

> Multilateral netting by novation and substitution has the potential to reduce liquidity risks more than any other institutional form, but this depends critically on the financial condition of any central counterparty to the netting; if the liquidity of a central counterparty is weak, the liquidity risks of this institutional form may be greater than in the case of bilateral netting by novation; the credit risks of this institutional form are generally less than in other forms that have been considered, subject again to the identity and condition of any central counterparty.

Although Freeman (1996a, b) does not make such a claim, one tempting way to interpret his result is that the inability of agents in C'' to settle novation securities makes the involvement of a monetary authority indispensable in attaining efficiency in his model economy. Such an interpretation would be mistaken for two reasons.

Before I discuss these reasons, let me mention that Freeman's model has a feature that I have omitted from the efficient monetary authority version developed above. Freeman posits that, before the beginning of subperiod $t.6$, the agents in C_{t-1} and in D_t are exogenously and randomly dispersed among several "islands." (This sequestration lasts only for the duration of the subperiod, so the debtor agents are able to trade in period $t + 1$ exactly as specified in the model with a monetary authority or with novation securities, if they are traded.) If agent $\alpha \in D_t$ has accepted a novation security issued by agent $\alpha'' \in C''_{t-1}$, α is on island ι in subperiod $t.6$, and α'' is on island $\iota'' \neq \iota$ in subperiod $t.6$, then α'' cannot settle the novation security that α holds.

Despite this inability of prospective intermediaries in Freeman's (1996a, b) model economy to settle all (or even most) of the novation securities that they issue, the market structure involving those securities will still be efficient. To see this, suppose that there are I distinct islands. If the face value of novation securities issued by an agent in $\alpha'' \in C''_{t-1}$ is ϕ, and if those securities are traded to agents in D_t who are dispersed equally among the islands, then only a subset of the securities having value ϕ/I can be settled. In sharp contrast, agent α'' will receive settlement on all the seasoned debt d' for which the agent trades novation securities that it issues. Consequently, agents in C''_{t-1} will bid the novation security price of seasoned debt (that is, $p^3_{d'}/p^3_n$) up to I, rather than only up to par. Subsequently, in subperiod $t.4$, agents in D_t will recognize that only $1/I$ of the novation debt will be settled; so as a result of their optimization, the money price p^4_n of a unit of the novation security (specified to be settled in subperiod $t.6$ for one unit of money) in that subperiod will be only $1/I$. Thus, the amount of good 2 that an agent in C'_{t-1} can obtain by exchanging a unit of seasoned debt for novation securities and then exchanging those for consumption is $i(p^4_n/p^4_2) = 1/p^4_2$, which is the same amount that the agent could obtain in the model economy with novation securities. That is, equilibrium in a version of that model economy with islands would still be efficient, because agents with rational expectations would fully adjust in market equilibrium for the occurrence of settlement failure on the part of intermediaries.

The efficiency of this equilibrium is one reason it would be a mistake to suppose that participation of a monetary authority is necessary to attain efficiency in Freeman's (1996a, b) model. Of course, the argument in the preceding paragraph makes it clear that the intermediary's inability to settle in the model economy differs radically in its foreseeability from the type of settlement failure on the part of an actual intermediary that concerns policymakers so much. This is not to say that policymakers' concerns are necessarily warranted, but rather that models of settlement do not yet reflect some of the features of the actual economy that are crucial to reasoning conclusively about those concerns.

The other reason it would be a mistake to interpret Freeman's (1996a, b) model as justifying a necessary role for a monetary authority is directly related to the considerations discussed above regarding the constraints facing a central bank and their relationship to the constraints that face a clearinghouse. The import of my arguments is that a central bank cannot be regarded as an intrinsically better type of institution than a clearinghouse. Certainly, given the potential for the payment system to be abused for political ends, few people would be enthusiastic about transferring the main settlement responsibilities from a smoothly functioning clearinghouse to a central bank that lacked independence. However, as policymakers recognize, if the structure of a clearinghouse raises prudential concerns, one needs to examine whether the structure can be strengthened before concluding that the only solution is for the central bank to take over the clearinghouse's function.

Conclusion

This study has been concerned with the welfare analysis of central bank and clearinghouse intervention in payment arrangements. At a formal level, this analysis is done by extending a model of the use of intermediated debt for payment, so that private-sector intermediaries can issue debt that corresponds to the clearinghouse practice of novation and substitution. If such debt can be issued, then the resulting market equilibrium under laissez-faire is efficient, so there is no need for direct participation by a monetary authority. This result can

even hold in the extended version of a model environment (which is seen to be very special, however) where intermediaries are unable to settle some of the debt they issue.

Although issues of institutional governance lie beyond the scope of the formal model, the analysis makes it clear that they are inseparable from the market equilibrium issues that are treated explicitly. Whether or not efficiency might require a central bank to participate in the payment system depends on the degree to which a central bank can promise reliably and credibly to reabsorb money that it issues to facilitate payments and also on whether the commercial law framework governing the operation of a private-sector payment intermediary is sufficient to warrant agents' use of debt issued by the intermediary as a money-like medium of exchange.

The credibility of a central bank's promise about reabsorption evidently depends, in turn, on its governance structure. It is likely that the institutions of central bank governance necessary for credible participation in the payment system are essentially identical to those necessary for effective conduct of monetary policy in a narrow sense. Thus, to whatever extent there is a need for a central bank to participate directly in the payment system, this need reinforces the considerations in favor of chartering a politically independent central bank. Moreover, the need for political independence suggests that the central bank would typically be a more appropriate public-sector participant in the payment system than would the treasury or another agency under the immediate control of the government.

*This article is reprinted, with permission, from *Monetary and Economic Studies* (May 1997, vol. 15, no. 1, pp. 63–87). © 1997 by the Institute for Monetary and Economic Studies, Bank of Japan. The article was edited for publication in the *Federal Reserve Bank of Minneapolis Quarterly Review.*

The article is based on research conducted when the author was a visiting scholar at the Institute for Monetary and Economic Studies of the Bank of Japan. He would like to thank members of the institute's Research Division 1, especially Dr. Hiroshi Fujiki, for comments and suggestions. However, views expressed in the article are solely those of the author and do not necessarily represent those of the Bank of Japan, the Institute for Monetary and Economic Studies, the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of Chicago, or the Federal Reserve System.

[1]Throughout this study, the term *money* refers to outside money.

[2]Although I do not explicitly consider risk of the payor's inability to settle in this study, Freeman (1996a, b) does consider it. He finds that the benefit of using intermediated debt is robust to the existence of some level of settlement risk.

[3]An issue that is related, although beyond the scope of this study, concerns the scope of private-sector participation. By their regulatory policies, some governments are encouraging nonbank firms to enter the payment industry, while others are inclined to erect legal barriers to such entry.

[4]It is sometimes suggested that the central bank can enhance payment-system efficiency due to its ability to guarantee immediate and final payment, which a private-sector intermediary cannot do. This suggestion seems to reflect the view that a private intermediary would face potential liquidity crises analogous to bank runs, which the central bank would not face because of its ability to issue new fiat money. However, if a central bank is empowered to serve as a lender of last resort to a private payment-system intermediary, then this observation is not sufficient to establish that it must also participate in the payment system on a day-to-day basis, any more than the possibility of bank runs establishes that the central bank must conduct day-to-day business as a commercial bank.

[5]As a model of a central bank, Freeman's (1996b) model is clearly a partial-equilibrium model. An overall judgment about whether a central bank should participate in the payment system should take into account the opportunity cost of such participation with respect to the bank's other objectives. However, if the participation of the monetary authority can enhance the economic efficiency of the payment system, then there is at least a prima facie case for that participation.

[6]Freeman (1996a) posits an artificial agent, which must be endowed with a special capability (an infinite lifetime, in an environment where ordinary agents have two-period lives) to serve as the intermediary. In the model that I present here, ordinary agents are able to undertake the task of intermediation.

[7]The process of *novation and substitution* involves a contract between a pair of clearinghouse members *A* and *B* being replaced by two contracts: one between *A* and the clearinghouse and the other between the clearinghouse and *B*. In each contract, the clearinghouse is obligated to make the same net trade as was the party that it replaces.

[8]This secondary-market transaction can be structured in various ways. The debt can be in the form of a security payable to the bearer, or the debt can be assignable, or novation can occur.

[9]This access is called *market participation* elsewhere, but I have already used *participation* in a different sense in the introduction. In a formal sense, of course, the fact that each agent has access to markets in only two periods is already a constraint. The constraints to be introduced here will impede trade within an age cohort.

[10]That is, the definition of equilibrium is in the spirit of Radner (1972). A fully adequate equilibrium concept for this environment would allow for the endogenous introduction of securities, as do Allen and Gale (1988). Instead, for each market, I specify an exogenous set of securities to be traded. In principle, this is a shortcoming, but—particularly since the equilibria to be studied here support efficient stationary allocations—apparently there would be no scope for the introduction of further securities. That is, I believe that these equilibria would continue to be equilibria if a robustness-to-innovation requirement were explicitly imposed.

[11]The only equilibrium in which the price of money is zero is autarky.

[12]Alternatively, it could be specified that all agents in C_{t-1} trade with agents in D_t in this subperiod. In equilibrium, every agent in C''_{t-1} would make a zero net trade in this market.

[13]Moreover, if a bilateral arrangement is what one intends to have emerge as an equilibrium trading pattern, there must be some constraint on (or cost of) debt security issuance to induce it. In that case, the terms of trade would be negotiated by bargaining within each two-member trading coalition, rather than taken by agents as parametrically determined by an economywide price.

[14]The specification that all agents in *C* are identical ex ante is inessential to producing consumption inequality, although it simplifies the calculation of equilibrium by making all young creditors' decisions identical. Its significance is to make an allocation with consumption inequality, which would be Pareto incomparable to the equal-consumption allocation if agents were distinguishable ex ante, into a Pareto-inefficient allocation.

[15]A very clear development of this argument is by Wallace (1988), in a discussion of an analogous issue regarding the Diamond-Dybvig model of intermediation.

[16]The pricing below par of debt in subperiod *t*.3 seems to reflect one aspect of what occurred during bank panics under the U.S. National Banking System in the late 19th and early 20th centuries. The novation securities introduced below bear striking resemblance to the clearinghouse loan certificates that were issued during those episodes in the absence of a central bank. Those certificates and the central banking role played by U.S. clearinghouses at that time generally are described by Timberlake (1984).

Appendix
Optimization and Market-Clearing

Here I describe the constraints and market conditions facing agents in the first main market structure studied in the preceding paper: the model with separation within cohorts and a need for intermediated debt securities.

The optimization problem of an agent in *C* is to choose net trades z^1, $z^{2'}$, $z^{2''}$, $z^{3'}$, $z^{3''}$, $z^{4'}$, $z^{5''}$, and $z^{6''}$ to

$$\text{(A1)} \qquad \max\ u(x_1) + \gamma v(x_2') + (1-\gamma)v(x_2'')$$

subject to

$$(x_1, x_2', x_2'') \in \Re^3_+$$

$$x_1 = 1 + z_1^1 \qquad p^1 z^1 \le 0$$

$$x_2' = z_2^{4'} \qquad p^4 z^{4'} \le 0$$

$$x_2'' = z_2^{6''} \qquad p^6 z^{6''} \le 0$$

$$z_m^{2'} = \delta z_d^1 \qquad z_m^{2'} \ge 0$$

$$z_m^{2''} = \delta z_d^1 \qquad z_m^{2''} \ge 0$$

$$z_d^{3'} \ge (1-\delta) z_d^1 \qquad z_d^{3''} \ge (1-\delta) z_d^1$$

$$z_m^{3'} \ge z_m^{2'} \qquad p^3 z^{3'} \le 0$$

$$z_m^{3''} \ge -z_m^{2''} \qquad p^3 z^{3''} \le 0$$

$$z_m^{4'} \geq -(z_m^{2'}+z_m^{3'}) \qquad p^4 z^{4'} \leq 0$$

$$z_m^{5''} = (1-\delta)z_d^1 + z_d^{3''} \qquad z_m^{5''} \geq -(z_m^{2''}+z_m^{3''})$$

$$z_2^1 \geq 0 \qquad z_m^{6''} \geq -(z_m^{5''}+z_m^{2''}+z_m^{3''}).$$

The optimization problem of an agent in D is to choose net trades z^{*1}, z^{*4}, z^{*6}, $z^{*2'}$, and $z^{*5''}$ to

(A2) $\max u_*(x_1^*) + v_*(x_2^*)$

subject to

$$(x_1^*, x_2^*) \in \Re_+^2$$

$$x_1^* = z_1^{*1} \qquad x_2^* = 1 + z_2^{*1} + z_2^{*4} + z_2^{*6}$$

$$p^1 z^{*1} \leq 0$$

$$z_m^{*4} \geq 0 \qquad z_m^{*6} \geq -z_m^{*4}$$

$$z_m^{*2'} = z_d^1 \qquad z_m^{*2'} \geq -(z_m^{*4}+z_m^{*6})$$

$$z_m^{*5''} = z_d^1 \qquad z_m^{*5''} \geq -(z_m^{*4}+z_m^{*6}).$$

The market-clearing conditions in this model economy are

$$z^1 = -z^{*1} \qquad \gamma z^{2'} + (1-\gamma)z^{2''} = -\delta z^{*2'}$$

$$\gamma z^{3'} + -(1-\gamma)z^{3''} \qquad \gamma z^{4'} = -z^{*4}$$

$$(1-\gamma)z^{5''} = -(1-\delta)z^{*5''} \qquad (1-\gamma)z^{6''} = -z^{*6}.$$

References

Allen, Franklin, and Gale, Douglas. 1988. Optimal security design. *Review of Financial Studies* 1 (Fall): 229–63.

Angell Report. 1989. Report on netting schemes. Group of Experts on Payment Systems of the Central Banks of the Group of Ten Countries, Committee on Payment and Settlement Systems. Basle, Switzerland: Bank for International Settlements.

Cairncross, Alec. 1988. The Bank of England: Relationships with the Government, the Civil Service, and Parliament. In *Central banks' independence in historical perspective*, ed. Gianni Toniolo, pp. 39–72. Berlin: de Gruyter.

Cukierman, Alex. 1995. *Central bank strategy, credibility, and independence: Theory and evidence.* Cambridge, Mass.: MIT Press.

Freeman, Scott. 1996a. Clearinghouse banks and banknote over-issue. *Journal of Monetary Economics* 38 (August): 101–15.

_____. 1996b. The payments system, liquidity, and rediscounting. *American Economic Review* 86 (December): 1126–38.

Goodhart, Charles. 1988. *The evolution of central banks.* Cambridge, Mass.: MIT Press.

Jensen, Michael C., and Meckling, William H. 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* 3 (October): 305–60.

Okuno, Masahiro, and Zilcha, Itzhak. 1980. On the efficiency of a competitive equilibrium in infinite horizon monetary economies. *Review of Economic Studies* 47 (July): 797–807.

Radner, Roy. 1972. Existence of equilibrium of plans, prices, and price expectations in a sequence of markets. *Econometrica* 40 (March): 289–303.

Sargent, Thomas J., and Wallace, Neil. 1982. The real-bills doctrine versus the quantity theory: A reconsideration. *Journal of Political Economy* 90 (December): 1212–36.

Sayers, R. S. 1967. *Modern banking.* 7th ed. Oxford: Oxford University Press.

Timberlake, Richard H., Jr. 1984. The central banking role of clearinghouse associations. *Journal of Money, Credit, and Banking* 16 (February): 1–15.

Wallace, Neil. 1988. Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously. *Federal Reserve Bank of Minneapolis Quarterly Review* 12 (Fall): 3–16.

Communication Opportunities
Between Types of Agents in the Basic Structure . . .

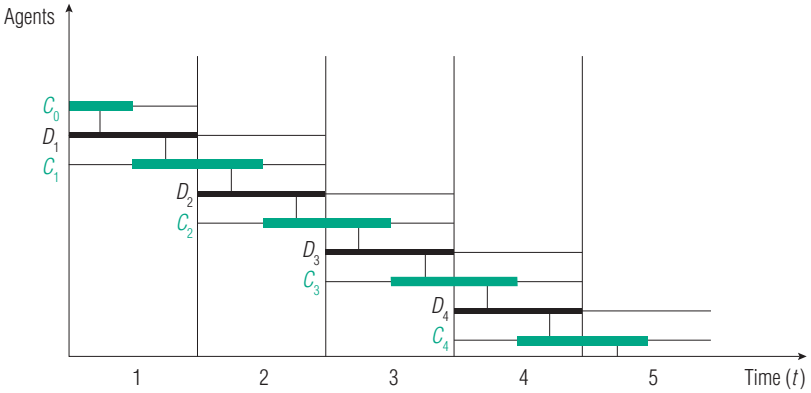
Table 1 **Trade**

Subperiod	Who Has Access	What Is Traded
$t.1$	C_{t-1}, D_t	2, m
$t.2$	C_t, D_t	1, 2, m

C = creditor, D = debtor
1, 2 = goods, m = money

Chart 1 **Structure**

When All Agents in Each Cohort Have Access to the Market
(Thin lines indicate life span; thicker lines indicate market access.)



. . . With Reversed Transaction Order . . .

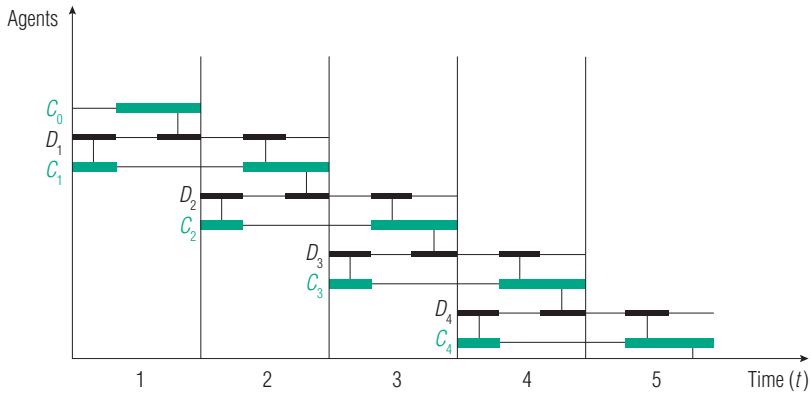
Table 2 **Trade**

Subperiod	Who Has Access	What Is Traded
$t.1$	C_t, D_t	1, 2, d
$t.2$	C_{t-1}, D_{t-1}	d, m
$t.3$	C_{t-1}, D_t	2, m

C = creditor, D = debtor
1, 2 = goods, m = money
d = debt security

Chart 2 **Structure**

When All Agents in Each Cohort Have Access to the Market
(Thin lines indicate life span; thicker lines indicate market access.)



. . . With Separation Within Cohorts . . .

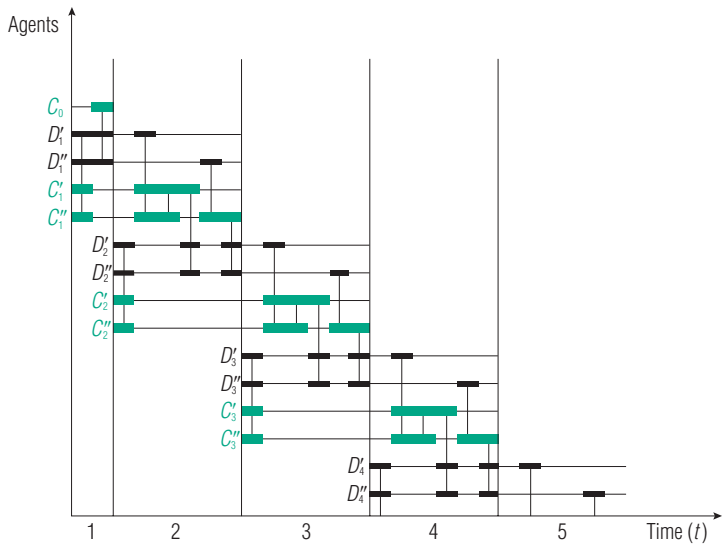
Table 3 **Trade**

Subperiod	Who Has Access	What Is Traded
$t.1$	C_t, D_t	1, 2, d
$t.2$	C_{t-1}, D'_{t-1}	d', m
$t.3$	C_{t-1}	d', m
$t.4$	C'_{t-1}, D_t	2, m
$t.5$	C''_{t-1}, D''_{t-1}	d', m
$t.6$	C''_{t-1}, D_t	2, m

C = creditor, D = debtor
1, 2 = goods, m = money
d = debt security

Chart 3 **Structure**

When All Agents in Each Cohort Have Access to the Market
(Thin lines indicate life span; thicker lines indicate market access.)



. . . And With Novation Debt

Table 4 Trade

Subperiod	Who Has Access	What Is Traded
$t.1$	C_t, D_t	1, 2, d
$t.2$	C_{t-1}, D'_{t-1}	d', m
$t.3$	C_{t-1}	d', n, m
$t.4$	C'_{t-1}, D_t	2, n, m
$t.5$	C''_{t-1}, D''_{t-1}	d', m
$t.6$	C''_{t-1}, D_t	2, n, m

C = creditor, D = debtor
1, 2 = goods, m = money
d = debt security, n = novation debt