Federal Reserve Bank of Minneapolis Quarterly Review
Vol. 24, No. 3, Summer 2000, pp. 2–10

Creating Business Cycles Through Credit Constraints

Narayana R. Kocherlakota
Adviser
Research Department
Federal Reserve Bank of Minneapolis
and Professor of Economics
University of Minnesota

Abstract

Business cycles appear to be large, persistent, and asymmetric relative to the shocks hitting the economy. This observation suggests the existence of an asymmetric amplification and propagation mechanism, which transforms the shocks into the observed movements in aggregate output. This article demonstrates, in a small open economy, how credit constraints can be such a mechanism. The article also shows, however, that the quantitative significance of the amplification which credit constraints can provide is sensitive to the quantitative specification of the underlying economy (especially factor shares).

Over the past two centuries, aggregate output in the United States has grown steadily, but not in a straight line; it has fluctuated around its upward trend. These output fluctuations have had three key properties:

- *Size.* Movements in aggregate output have been *large*. In an economy as big and varied as that of the United States, movements in various sectors might be expected to cancel each other, leaving only small movements in aggregate output. This is not what has happened.
- *Persistence.* The output movements have been highly *persistent*. Once output has fallen below its usual trend growth, for example, it has stayed below this trend for some time.
- *Asymmetry.* Output's movements have been *asymmetric:* downward movements have been sharper and quicker than upward movements (Falk 1986, Acemoglu and Scott 1997).

Economists have labeled the recurrent movements in aggregate output *business cycles*, but they have not yet satisfactorily explained why the movements have the particular properties they do. At least two types of explanations have been offered. One is simply that the economy has been frequently hit with aggregate shocks which have these properties. The problem with this explanation is that sufficiently large shocks like these are hard to find in the data (Summers 1986, Cochrane 1994). Candidates like sudden changes in government policy, the weather, and oil supplies have not been large enough to account for the large movements in aggregate output. The other potential explanation for the business cycle properties is that the economy has some as yet unidentified mechanism which transforms small, barely detectable, shocks to some or all parts of the economy into large, persistent, asymmetric movements in aggregate output. This economic mechanism propagates and amplifies shocks in a downwardly biased fashion. Here I argue that the mechanism might be *credit constraints*, or limits on how much economic agents can borrow.

Before turning to the formal specifics of my argument, let me explain the intuition behind it. Think of an entrepreneur who is the owner and manager of a firm which has two types of assets: savings in a bank account and computer equipment. Assume that the entrepreneur cannot borrow. However, note that the firm's scale of production is optimal; otherwise, the entrepreneur would use some of the firm's savings to buy more computer equipment.

Suppose this entrepreneur receives an unanticipated upward temporary shock to income. Will the entrepreneur use this income to buy more computers? No, the firm's scale of production is already optimal, so the extra income will simply be consumed or saved.

Now suppose instead that the entrepreneur receives a downward shock to income. If the shock is small, the entrepreneur will absorb it by reducing the firm's savings or consuming less or both. But if the shock is big enough to swamp the firm's savings, then the entrepreneur must lower the firm's scale of production by selling off some computer equipment. Note that in this instance, the entrepreneur would prefer to borrow, but cannot. If the marginal returns to computer equipment are diminishing, and the entrepreneur's marginal utility is diminishing, then returning to the firm's optimal scale of production will take many periods.

Thus, for an entrepreneur who faces credit constraints, upward shocks and small downward shocks to income have little or no effect on production. However, sufficiently large downward shocks can have persistent negative effects on production. Credit constraints (of virtually any form) are an asymmetric propagation mechanism.

Still, little in this intuition explains why credit constraints should amplify shocks. This is because only certain types of credit constraints do so. We just considered an entrepreneur who could not borrow at all. Now suppose instead that the entrepreneur also owns land, which is a complementary input with the computer equipment. We have seen that a downward income shock can lead a credit-constrained entrepreneur to reduce a firm's computer equipment. Suppose a large number of entrepreneurs are in this situation. Then, because computers and land are complementary inputs, land prices must fall. This fall will shrink the debt capacity of the entrepreneurs and lead to a further shrinkage in production. In this way, certain types of credit constraints can amplify the effects of income shocks.

I formalize this argument below. I construct a simple model in which productive agents use capital and land to produce output. The agents face a limit on how much they may borrow and an interest rate that is exogenously specified. I consider the effects of unanticipated increases or decreases in the agents' income. I show that unanticipated increases have no impact on output. However, sufficiently large unanticipated decreases in income reduce output, and after such reductions, output returns only slowly to its original level.

I then consider two types of credit constraints. First, I allow agents to borrow up to a fixed exogenous limit. I show that in this setting, the effects of income shocks are not amplified. Second, I allow agents to borrow up to the value of their land. In this setting, the effects of income shocks may be greatly amplified. However, the degree of amplification depends crucially on the shares of capital and land in the production function: if these shares sum to less than 40 percent (as is approximately true in the U.S. data), then the effects of income shocks are not amplified at all.

My work here is essentially a simpler presentation of ideas originally presented elsewhere, by me and by others. Many studies have pointed out that in economies with credit constraints, temporary income shocks can have persistent effects. (See, for example, Scheinkman and Weiss 1986, Kocherlakota 1996, and Cooley, Marimon, and Quadrini 2000.) Kiyotaki and Moore (1997) and Kiyotaki (1998) have emphasized the importance of borrowing limits that depend on asset values as an amplification mechanism.

A Simple Model

To start the analysis, I describe a simple model with a credit constraint. In technical terms, the model is essentially a small open economy version of a neoclassical growth model with complete depreciation.

I model a group of farmers. The farmers grow a special type of corn, which can be used equally well for food or for seed. In each period of time t, a farmer produces corn according to this production function:

(1) $Y_t = F(X_t, L_t)$

where X_t is the amount of corn planted last period and L_t is the amount of land used by the farmer. As is usual, I assume that F is concave and increasing and is as differentiable as I need it to be.

A farmer can split the produced corn Y_t into consumable sweet corn (C_t) and seed corn (X_{t+1}) for next period:

(2) $C_t + X_{t+1} = Y_t$.

The farmers have identical preferences over flows of sweet corn:

(3) $\sum_{t=1}^{\infty}\beta^{t-1}\ln(C_t)$

where $\beta^{-1} - 1$ represents the rate that farmers discount future utility. Each farmer begins life with X_1 units of planted seed corn and one unit of land.

The farmers can borrow and lend corn on the world market at an interest rate $R = \beta^{-1} - 1$. They can buy and sell land in an internal competitive market; the price of land in terms of corn in period t is denoted by Q_t. By *internal* market, I mean that, for legal or other reasons, nonfarmers cannot own land.[1]

The crucial element of this model is that farmers face credit constraints, or borrowing limits, in the world market. Let B_{t+1} denote the amount of seed corn borrowed by a typical farmer in period t. I consider two types of borrowing limits. Under the first, farmers cannot borrow more than a fixed amount of debt:

(4) $B_{t+1} \le B_*$.

I will refer to this borrowing limit as the *exogenous* credit constraint. Under the second type of borrowing limit, farmers must use their land as collateral when they borrow. In particular, farmers can disappear without repaying their debt, but if they do so, they must lose their land. Hence, the second type of borrowing limit says that farmers cannot borrow more than the current value of their land holdings:

(5) $B_{t+1} \le Q_t L_{t+1}$.

I will refer to this borrowing limit as the *endogenous* credit constraint.

In this economy, farmers take the sequence of land prices $(Q_t)_{t=1}^{\infty}$ as given and solve the following problem:

(6) $\max_{(C,X,L,B)} \sum_{t=1}^{\infty}\beta^{t-1}\ln(C_t)$

subject to

(7) $C_t + X_{t+1} + Q_t L_{t+1} + B_t(1+R) = F(X_t, L_t) + B_{t+1} + Q_t L_t$

(8) $B_{t+1} \le B_*$ or $B_{t+1} \le Q_t L_{t+1}$

(9) $C_t, X_t \ge 0$

(10) $L_1 = 1$

and $X_1 B_1$ are given. Note that B_1, the initial level of debt, may be nonzero, so that farmers may begin life in debt or with positive financial assets. A collection of sequences (Q, C, X, B) is an equilibrium if (C_t, X_t, B_t) and $L_t = 1$ solve a farmer's problem given Q.

Throughout, I define the economy's *equilibrium output* to be the farmers' total production of corn:

(11) $Y_t = F(X_t,1)$.

Let's assume that the economy has been running for a sufficiently long period of time, so that we can think of it beginning our analysis in a steady state. Formally, a *steady state* is a pair of initial debt and seed corn (B_{ss}, X_{ss}) such that if $B_1 = B_{ss}$ and $X_1 = X_{ss}$, then in equilibrium, for all t,

(12) $X_t = X_{ss}$,

(13) $B_t = B_{ss}$.

Solving for the steady state is easy. In a steady state, because the state variables are constant, a farmer's consumption of sweet corn must be too. The first-order condition for optimal planting of seed corn then implies that

(14) $\beta F_X(X_{ss},1) = 1$.

Suppose a farmer starts with X_{ss} units of seed corn. Over time, this farmer will use income to pay the interest on debt B_1 (but never the principal), will plant X_{ss} units of seed corn, and will consume a constant amount of sweet corn.

Thus, here, together with X_{ss}, any debt level at or below the borrowing limit is a steady-state level. With the exogenous constraint, that is, (B_{ss}, X_{ss}) is a steady state if and only if

(15) $B_{ss} \le B_*$

(16) $1 = \beta F_X(X_{ss},1)$.

Similarly, with the endogenous constraint, (B_{ss}, X_{ss}) is a steady state if and only if

(17) $B_{ss} \le Q_{ss}$

(18) $1 = \beta F_X(X_{ss},1)$.

Here Q_{ss} is the steady-state price of land, which is given by the present value of the rental payments of land:

(19) $Q_{ss} = \beta F_L(X_{ss},1)/(1-\beta)$.

In a steady-state equilibrium, consumption of sweet corn equals

(20) $C_{ss} = F(X_{ss},1) - X_{ss} - B_{ss}R$

and output equals

(21) $Y_{ss} = F(X_{ss},1)$.

Asymmetry and Persistence

Now I analyze purely unanticipated shocks. In particular, I assume that the economy begins in a steady state. In the first period, then, in addition to the structure described above, the farmers receive or lose an additional amount of income (Δ units of corn). They confront no additional positive or negative income shocks throughout their lifetime. I analyze the characteristics of the resulting equilibrium.

The interpretation of this exercise is straightforward. The farmers have acted in the past as if they would never face an income shock, and then they are unexpectedly faced with one. The model is sufficiently abstract so that we can think of this shock in many ways. Most straightforwardly, we can think of the farmers receiving unexpected revenue or paying an unexpected tax. Or as Kiyotaki and Moore (1997) argue, we can think of the shock as being a consequence of monetary policy. Suppose debt were purely nominal, and there were a sudden change in monetary policy. Then, if farmers' initial debt holdings were nonzero, there would be an income transfer to or from them.

In this section, I show that sufficiently large downward income shocks have persistent effects on the farmers' output of corn. However, upward shocks have no effect on that output. I conduct all of the analysis of this section in the context of the exogenous borrowing limit; the extension to the endogenous limit is straightforward.

Assume first that $X_1 = X_{ss}$ and that the shock to income Δ is positive. What is the nature of the implied equilibrium? Here, the economy is essentially equivalent to one in which farmers begin with a different initial level of debt:

(22) $$B_1' = B_1 - \Delta/(1+R).$$

But this new lower level of debt B_1' is still part of a steady state (given that the initial level of planted seed corn is X_{ss}). Hence, we see that the equilibrium level of output $Y_t = F(X_{ss},1)$ for all t. The upward temporary income shock has no impact on output. The farmers simply take the extra income and use it to pay off some of their debt.

Now assume that the shock to income Δ is negative. Here we must consider two separate cases.

Suppose first that (B_1, X_1) is a steady state and $B_1 < B_*$. Suppose that Δ is small enough that

(23) $$\Delta/(1+R) \le B_* - B_1.$$

Then, as above, this economy is equivalent to one in which $B_1' = B_1 + \Delta < B_*$. But this new economy is still in a steady state, so the resulting level of output is constant at $F(X_{ss},1)$. Again, the temporary income shock has no impact on output. Because the shock is temporary, the farmers borrow to keep their sweet corn consumption constant. But the requisite increase in debt is sufficiently small that they do not need to reduce their corn planting at all.

Now suppose that, instead, Δ is sufficiently large that

(24) $$\Delta/(1+R) > B_* - B_1.$$

Now the farmers cannot simply increase their debt level to smooth out the income shock; the shock is too large. So, instead, the farmers borrow as much as they can (with $B_t = B_*$ for all t). But this still leaves them with less corn to consume in the first period than they would have had in the steady state. Thus, they must reduce their seed corn planting in order to smooth their consumption adequately.

As a result, the economy moves back to its steady state along a transition path. The resulting equilibrium values (C, B, X) satisfy the following conditions:

(25) $$\beta F_X(X_{t+1},1)/C_{t+1} = 1/C_t$$

(26) $$B_t = B_*$$

for $t > 1$; and

(27) $$C_1 + X_2 = F(X_{ss},1) - B_1(1+R) + B_* - \Delta$$

(28) $$C_t + X_{t+1} = F(X_t,1) - B_*R$$

for $t > 1$. The standard arguments about transitions in the neoclassical growth model imply that for all $t > 1$

(29) $$Y_t < Y_{t+1} < F(X_{ss},1)$$

(30) $$\lim_{t\to\infty} Y_t = F(X_{ss},1).$$

The large downward temporary shock to income thus introduces an immediate shock to output. However, the economy recovers only slowly from this shock back to the steady-state level of output; the shock has persistent effects.

We can summarize this analysis simply. Temporary upward income shocks, no matter what their size, have no effects on output. Temporary downward shocks do have persistent effects on output, but only if the shocks are sufficiently large. Clearly, then, credit constraints are an asymmetric propagation mechanism.

Amplification

To be a mechanism capable of creating business cycles, however, credit constraints must also be able to amplify the effects of income shocks, to transform small income shocks into large movements in output. In this section, I consider how credit constraints might do that. Because of the preceding analysis, we know that we can ignore upward shocks. I assume that the initial debt holdings are sufficiently large that the borrowing limit holds with equality. This means that any downward shock (regardless of its size) triggers a persistent response in output.

Before tackling the main issue, though, we must decide how to measure *amplification* (versus persistence). I will define the amplification of a downward shock Δ to be how far output in the second period (Y_2) is from the steady-state output level, relative to the size of the original shock Δ. In other words, if the initial shock is of size Δ, then the amplification is given by

(31) $$|Y_2(\Delta) - Y_{ss}|/\Delta = (|Y_2 - Y_{ss}|/Y_{ss})/(\Delta/Y_{ss}).$$

Note that the assumption that the credit constraint initially holds with equality tends to magnify the degree of amplification. Otherwise, as we have seen, the farmers can offset some of the shock by borrowing more rather than changing the level of planting.

An Exogenous Constraint

Consider first what happens with an exogenous credit constraint.

As above, I assume here that $B_1 = B_*$, that $X_1 = X_{ss}$, and that there is a negative shock $-\Delta$ to first period income. To be concrete, I assume that $F(X, L) = X^{\alpha_1}L^{\alpha_2}$, where the capital and land shares $\alpha_1 + \alpha_2 \leq 1$. (As it turns out, the land share value α_2 is irrelevant with the exogenous credit constraint because land is inelastically supplied.)

In this world, the evolution of sweet corn consumption and planted seed corn (C_t, X_t) satisfies the equations

$$(32) \quad C_t^{-1} = \alpha_1\beta C_{t+1}^{-1}X_{t+1}^{\alpha_1-1}$$

$$(33) \quad C_t + X_{t+1} + RB_* = X_t^{\alpha_1} + \varepsilon_t$$

$$(34) \quad X_1 = X_{ss}$$

where the shock $\varepsilon_t = -\Delta$ if $t = 1$ and 0 otherwise. For small Δ, these equations are well-approximated by this system:

$$(35) \quad c_t = c_{t+1} + (1-\alpha_1)x_{t+1}$$

$$(36) \quad c_tC_{ss}/X_{ss} + x_{t+1} = \beta^{-1}\alpha_1x_t$$

for $t > 1$; and

$$(37) \quad c_1C_{ss}/X_{ss} + x_2 = -\Delta/X_{ss}$$

where $(c_t, x_t) \equiv \big(\ln(C_t/C_{ss}), \ln(X_t/X_{ss})\big)$. Substituting out for c_t gives a second-order difference equation in x_t, for $t > 1$:

$$(38) \quad x_3 + [-1 - \beta^{-1} - (1-\alpha_1)C_{ss}/X_{ss}]x_2 = \Delta/X_{ss}$$

$$(39) \quad x_{t+2} + [-1 - \beta^{-1} - (1-\alpha_1)C_{ss}/X_{ss}]x_{t+1} + \beta^{-1}x_t = 0.$$

The characteristic polynomial of the second-order difference equation is

$$(40) \quad z^2 + [-1 - \beta^{-1} - (1-\alpha_1)C_{ss}/X_{ss}]z + \beta^{-1}$$

and this polynomial has two roots. One of these roots is larger than β^{-1}, and the other lies in the set $[\alpha_1,1)$. The former root is irrelevant because it leads to a path for x_t that is explosive. (Technically, it violates a transversality condition.) Label the latter root γ. Then we know that, for $t > 1$,

$$(41) \quad x_{t+2} = \gamma x_{t+1}$$

$$(42) \quad x_3 + [-1 - \beta^{-1} - (1-\alpha_1)C_{ss}/X_{ss}]x_2 = \Delta/X_{ss}.$$

Substituting equation (41) into (42), we get that

$$(43) \quad \begin{aligned} y_2 &= \alpha_1x_2 \\ &= -\alpha_1\beta\gamma\Delta/X_{ss} \\ &= -\gamma\Delta/Y_{ss}. \end{aligned}$$

Thus, the amplification of Δ is given by γ.

How does γ depend on B_*? To understand this dependence, note from (16) and (20) that

$$(44) \quad C_{ss}/X_{ss} = \alpha_1^{-1}\beta^{-1} - (RB_*/X_{ss}) - 1.$$

When B_* is high, C_{ss}/X_{ss} is low, because the farmers are spending most of their income servicing their debt. Conversely, when B_* is low, C_{ss}/X_{ss} is high. In particular, if $B_* = 0$, then $C_{ss}/X_{ss} = \alpha_1^{-1}\beta^{-1} - 1$ and, by the quadratic formula, $\gamma = \alpha_1$. If B_* is so high that C_{ss}/X_{ss} is essentially zero, then (again by the quadratic formula) γ is well-approximated by 1.

Thus, the upper bound for amplification is 1, and the lower bound is α_1. With the exogenous credit constraint, the shock to farmers' income does not get amplified at all.

An Endogenous Constraint

I turn now to the endogenous credit constraint under which the farmers cannot borrow more than the value of their land. We naturally expect the amplification to be greater with such a constraint than with an exogenous constraint. Why? When farmers get a negative shock to their income, they lower their seed corn holdings X_t below X_{ss} for all t. Because seed corn and land are complementary inputs, this decrease must lead to a fall in the value of land. Hence, the farmers' borrowing constraint tightens. This, in turn, creates a need for a further decline in seed corn levels. Thus, the endogenous credit constraint creates an interaction between debt capacity and the income shock. This interaction multiplies the effect of the income shock.

While the qualitative impact of the endogenous credit constraint is clear, its quantitative impact is not. Here, as above, I assume that the initial level of debt is such that the borrowing limit holds with equality. Therefore,

$$(45) \quad 1 = \beta F_X(X_{ss},1)$$

$$(46) \quad B_1 = \beta F_L(X_{ss},1)/(1-\beta).$$

Again, I emphasize that this initial level of debt means that any downward income shock has a persistent effect on output. I parameterize $F(X, L) = X^{\alpha_1}L^{\alpha_2}$.

The equilibrium evolution of (C_t, X_t, Q_t) in this setting satisfies the following system of equations: The resource constraint (47), where $\varepsilon_t = \Delta$ if $t = 1$ and 0 otherwise,

$$(47) \quad C_t + X_{t+1} + (1+R)B_t = X_t^{\alpha_1} + B_{t+1} - \varepsilon_t;$$

the first-order conditions for seed corn (48) and land (49),

$$(48) \quad C_t^{-1} = \beta\alpha_1C_{t+1}^{-1}X_{t+1}^{\alpha_1-1}$$

$$(49) \quad \begin{aligned} C_t^{-1}Q_t = {} & \beta C_{t+1}^{-1}(\alpha_2X_{t+1}^{\alpha_1} + Q_t) \\ & + Q_t[C_t^{-1} - \beta(1+R)C_{t+1}^{-1}]; \end{aligned}$$

a borrowing limit (50) that binds throughout the transition,

$$(50) \quad B_{t+1} = Q_t;$$

and the initial conditions for debt (51) and seed corn (52),

$$(51) \quad B_1 = \beta\alpha_2X_{ss}^{\alpha_1}/(1-\beta)$$

$$(52) \quad X_1 = X_{ss}.$$

In the Appendix, I derive a log-linear approximation to this system of equations. I find that for small Δ,

$$(53) \quad Y_2 - Y_{ss} \approx -\alpha_1(1-\alpha_1\beta)\Delta/(1-\alpha_2-\alpha_1\beta)$$

$$(54) \quad Q_1 - Q_{ss} \approx -\alpha_1(1-\beta)\Delta/(1-\alpha_2-\alpha_1\beta).$$

Note that if $\alpha_2 = 0$, then the amplification of the effect of the income shock on output is α_1, which is the same as when farmers cannot borrow at all.[2]

These formulas show that the endogenous credit constraint can generate an arbitrarily high degree of amplification. Given the capital share α_1, output amplification is a strictly increasing function of the land share α_2. Amplification is bounded above by $(1-\beta)^{-1}$ and achieves this upper bound when $\alpha_1 = 0$ and $\alpha_2 = 1$. Hence, by setting α_2 and β sufficiently close to 1 and α_1 sufficiently close to 0, we can, theoretically, generate arbitrarily high degrees of amplification.

But this theoretical possibility is not robust. Specifically, suppose we parameterize $F(X, L) = X^{\alpha_1}L^{0.4-\alpha_1}$, so that the economy has another inelastically supplied input (for example, labor) with a share of 0.6. In the accompanying table, I display the results of setting $\beta = 0.97$ and calculating the degree of amplification (of land prices and output) for different values of capital share.[3] For these parameterizations, the price of land does not respond much to the income shock. The results are now similar to those with the exogenous constraint, when the degree of amplification is well-approximated by α_1.[4]

Thus, again, it is theoretically possible for small income shocks to lead to arbitrarily large output movements in a world with endogenous credit constraints. However, this possibility is not robust.

Anticipated Shocks

So far, I have assumed that farmers' income shocks are not anticipated. What happens if, instead, the farmers are faced with independent and identically distributed shocks to their income over time? (Assume that all farmers are hit with the same realizations of the shocks.) In this section, I provide an intuitive answer to that question (without going through analytical specifics).

The key to understanding the farmers' behavior is to realize that when they are deciding how much savings to maintain, farmers must balance two considerations: how impatient they are relative to the market interest rate and how likely they are to run into a borrowing constraint which would lead them to a suboptimal level of production.

Suppose first that $\beta(1+R) = 1$. Then the farmers are marginally indifferent about when they consume; their primary consideration is to avoid the borrowing constraint. They will accumulate savings in order to avoid the possibility of ever running into the borrowing constraint. In the limit, their savings will be infinite (as demonstrated in Sotomayor 1984), and no income shock will have any effect on their production levels.

If $\beta(1+R) < 1$, however, then the farmers' behavior is different. Their savings will bounce around stochastically; and with some positive probability, the farmers will end up constrained by their borrowing limit (after a sufficiently long run of bad shocks). In states of the world in which the farmers are unconstrained, the response to income shocks will be similar to that which we saw for unanticipated shocks. Upward shocks or small downward shocks will have no effect on the level of seed planting. However, large downward shocks will run farmers into their borrowing constraint and generate persistent effects on output.

If the farmers are constrained (as they are with positive probability), then their seed planting will be below X_{ss}. Upward income shocks will lead to an increase in the scale of production; downward income shocks, to a decrease in the scale of production. Both types of shocks will have persistent effects. However, because of curvature in marginal utility, the effect of downward shocks on output will be larger than the effect of upward shocks (Aiyagari 1994).

Even if shocks are anticipated, endogenous credit constraints lead to more amplification than exogenous credit constraints. How much more is an open question.

Conclusion

Macroeconomics is looking for an asymmetric amplification and propagation mechanism that can turn small shocks to the economy into the business cycle fluctuations we observe: large, persistent, downwardly biased movements in aggregate income. I have argued that credit constraints are potentially such a mechanism. However, I have shown that the degree of amplification provided by credit constraints seems to depend crucially on the parameters of the economy. This sets up a clear challenge for future work: to demonstrate, in a carefully calibrated model environment, that the amplification and propagation possible by credit constraints are quantitatively significant.[5]

A lot is at stake here. For if credit constraints can be shown to be significant in this way, then our understanding of macroeconomic policy must be modified in at least two fundamental ways.

One is that our view of the effects of fiscal and monetary policy must change; these effects may be much larger than our purely aggregate models predict. We have seen that in a world with credit constraints, the distribution of income is a key determinant of output. Especially if credit constraints are endogenous, changes in fiscal and monetary policy that trigger small changes in the income distribution can lead to big, persistent changes in aggregate output.

Related to that issue is our view of how the joint distribution of assets and productivity affects the impact of shocks (including policy shocks) on the economy. Consider, for a concrete example, a question often posed in policy circles: If the stock market were to drop drastically because a bubble burst, what would happen to aggregate output? According to my analysis here, the answer depends crucially on how close productive agents are to their borrowing limits. If agents have a lot of savings outside the stock market, then such a shock would lead to just a slight dip in output. But if productive agents are quite close to their borrowing limits, then this shock could depress output dramatically.

*The author thanks V. V. Chari, Harold Cole, and Patrick Kehoe for their comments, which greatly improved the article. The author also thanks Kathy Rolfe for excellent editorial assistance.

[1]This restriction simplifies the analysis (and exaggerates the effects of credit constraints) because it prevents farmers from using land, as well as debt, as a buffer against adverse shocks.

[2]The log-linearization also shows that the degree of persistence is α_1, in the sense that $(Y_t-Y_{ss})/Y_{ss} \approx \alpha_1(Y_{t-1}-Y_{ss})/Y_{ss}$ for $t > 3$.

[3]V. V. Chari has emphasized to me that it could well be interesting to explore specifications of the aggregate production function in which the elasticity of substitution between land and labor is less than 1. Such specifications could lead to bigger land price swings, even when the land share is relatively small.

[4]Some readers may be concerned that this table neglects nontrivial second-order dynamics. To check for this possibility, I used a shooting method to compute a more precise approximation. If I assume that $\Delta = Y_{ss}/100$ and that the system returns to steady state in 80 periods, then the amplification of output is within 0.001 of what is reported in the table.

[5]To this end, I have computed numerical solutions to versions of my model in which depreciation is less than full and in which the endogenous credit constraint is formulated in terms of capital rather than land. Making these changes seems to reduce the ability of the model to generate quantitatively significant amplification.

Appendix
Approximating the Amplification With an Endogenous Credit Constraint

In this Appendix, I derive an approximation for the amplification of the effects of the income shock on output when agents face an endogenous credit constraint. Here I use lowercase characters to refer to deviations of logged variables from steady states; thus, $x_t = \ln(X_t/X_{ss})$.

To start, recall that

(A1) $Q_{ss}/Y_{ss} = \alpha_2\beta/(1-\beta)$

(A2) $C_{ss}/X_{ss} = \alpha_1^{-1}\beta^{-1}(1-\alpha_2) - 1$

(A3) $X_{ss}/Y_{ss} = \alpha_1\beta$.

The log-linearized transition equations are

(A4) $c_t = c_{t+1} + (1-\alpha_1)x_{t+1}$

(A5) $q_t\beta^{-1}Q_{ss} = q_{t+1}Q_{ss} + \alpha_2\beta^{-1}x_{t+1}X_{ss}$

both for $t \geq 1$;

(A6) $c_tC_{ss} + x_{t+1}X_{ss} + \beta^{-1}q_{t-1}Q_{ss} = \beta^{-1}x_tX_{ss} + q_tQ_{ss}$

for $t > 1$; and

(A7) $c_1C_{ss} + x_2X_{ss} = q_1Q_{ss} - \Delta$.

By substituting (A5) into (A6), we get, for $t > 1$, that

(A8) $c_tC_{ss} + \alpha_2^{-1}\beta(\beta^{-1}q_tQ_{ss} - q_{t+1}Q_{ss}) + \beta^{-1}q_{t-1}Q_{ss}$
$= \beta^{-1}\alpha_2^{-1}q_{t-1}Q_{ss} - \alpha_2^{-1}q_tQ_{ss} + q_tQ_{ss}$.

By substituting (A5) into (A6), we get, for $t > 1$, that

(A9) $c_tC_{ss} - c_{t+1}C_{ss} = (1-\alpha_1)(C_{ss}/X_{ss})(\alpha_2^{-1}q_tQ_{ss} - \alpha_2^{-1}\beta q_{t+1}Q_{ss})$.

By combining (A7) and (A8), we get that

(A10) $\zeta(L^{-1})q_{t-1} = 0$

for $t > 1$, where $L^{-1}q_t \equiv q_{t+1}$ and

(A11) $\zeta(z) = \alpha_2^{-1}\beta z^3$
$+ [-2\alpha_2^{-1} - \alpha_2^{-1}\beta + 1 - \beta\alpha_2^{-1}(C_{ss}/X_{ss})(1-\alpha_1)]z^2$
$+ [2\alpha_2^{-1} - 1 + \alpha_2^{-1}\beta^{-1} - \beta^{-1}$
$+ (1-\alpha_1)\alpha_2^{-1}(C_{ss}/X_{ss})]z$
$+ (\beta^{-1} - \beta^{-1}\alpha_2^{-1})$.

The characteristic polynomial ζ has three real roots. One root is β^{-1}, a second is at least as large as β^{-1}, and the third is α_1. We can ignore the first two roots, because they lead q_t to violate the transversality condition. Hence, for $t \geq 1$,

(A12) $q_{t+1} = \alpha_1 q_t$.

By substituting this result into (A4)–(A7), we find that

(A13) $c_1C_{ss} = c_2C_{ss} + \alpha_2^{-1}\beta(\beta^{-1}-\alpha_1)q_1$

(A14) $c_1C_{ss} + \alpha_2^{-1}\beta(\beta^{-1}-\alpha_1)q_1Q_{ss} = -\Delta + q_1Q_{ss}$

(A15) $c_2C_{ss} + \alpha_2^{-1}\beta(\beta^{-1}-\alpha_1)q_1Q_{ss} + \beta^{-1}q_1Q_{ss}$
$= \alpha_2^{-1}(\beta^{-1}-\alpha_1)q_1 + \alpha_1q_1Q_{ss}$.

Combining terms, we get that

(A16) $[\zeta(\alpha_1) - \alpha_2^{-1}\alpha_1 - \beta^{-1}(1-\alpha_2^{-1})]\alpha_1^{-1}q_1Q_{ss} = -\Delta$

and, since $\zeta(\alpha_1) = 0$,

(A17) $q_1Q_{ss} = \Delta/[\alpha_2^{-1} + \beta^{-1}\alpha_1^{-1}(1-\alpha_2^{-1})]$.

This implies that

(A18) $x_2X_{ss}/Y_{ss} = \alpha_2^{-1}\beta(\beta^{-1}-\alpha_1)q_1Q_{ss}/Y_{ss}$
$= (\Delta/Y_{ss})\alpha_2^{-1}\alpha_1\beta(\beta^{-1}-\alpha_1)/[\alpha_2^{-1}\alpha_1 + \beta^{-1}(1-\alpha_2^{-1})]$.

Thus, since $Y_{ss}/X_{ss} = \alpha_1^{-1}\beta^{-1}$,

(A19) $y_2 = \alpha_1x_2$
$= (\Delta/Y_{ss})\alpha_2^{-1}\alpha_1(\beta^{-1}-\alpha_1)/(\alpha_2^{-1}\alpha_1+\beta^{-1}-\beta^{-1}\alpha_2^{-1})$
$= -\alpha_1(\Delta/Y_{ss})(1-\alpha_1\beta)/(1-\alpha_2-\alpha_1\beta)$

(A20) $q_1 = (\Delta/Y_{ss})(Y_{ss}/Q_{ss})\alpha_2\alpha_1/(\alpha_1+\beta^{-1}\alpha_2-\beta^{-1})$
$= (\Delta/Y_{ss})(1-\beta)\beta^{-1}\alpha_1/(\alpha_1+\beta^{-1}\alpha_2-\beta^{-1})$
$= -(\Delta/Y_{ss})(1-\beta)\alpha_1/(1-\alpha_1\beta-\alpha_2)$.

References

Acemoglu, Daron, and Scott, Andrew. 1997. Asymmetric business cycles: Theory and time-series evidence. *Journal of Monetary Economics* 40 (December): 501–33.

Aiyagari, S. Rao. 1994. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics* 109 (August): 659–84.

Cochrane, John H. 1994. Shocks. *Carnegie-Rochester Conference Series on Public Policy* 41 (December): 295–364.

Cooley, Thomas; Marimon, Ramon; and Quadrini, Vincenzo. 2000. Optimal financial contracts with limited enforceability and the business cycle. Manuscript. European University Institute, Universitat Pompeu Fabra.

Falk, Barry. 1986. Further evidence on the asymmetric behavior of economic time series over the business cycle. *Journal of Political Economy* 94 (October): 1096–109.

Kiyotaki, Nobuhiro. 1998. Credit and business cycles. *Japanese Economic Review* 49 (March): 18–35.

Kiyotaki, Nobuhiro, and Moore, John. 1997. Credit cycles. *Journal of Political Economy* 105 (April): 211–48.

Kocherlakota, Narayana R. 1996. Consumption, commitment, and cycles. *Journal of Monetary Economics* 37 (June): 461–74.

Scheinkman, Jose A., and Weiss, Laurence. 1986. Borrowing constraints and aggregate economic activity. *Econometrica* 54 (January): 23–45.

Sotomayor, Marilda A. de Oliveira. 1984. On income fluctuations and capital gains. *Journal of Economic Theory* 32 (February): 14–35.

Summers, Lawrence H. 1986. Some skeptical observations on real business cycle theory. *Federal Reserve Bank of Minneapolis Quarterly Review* 10 (Fall): 23–27.

Potential Amplification of an Income Shock With Various Capital Share Values

When $\beta = 0.97$ and $\alpha_2 = 0.4 - \alpha_1$

Value of Capital Share (α_1)	Amplification of Effect on	
	Land Price $(Q_1 - Q_{ss})\,\Delta^{-1}/Q_{ss}$	Output $(Y_2 - Y_{ss})\,\Delta^{-1}/Y_{ss}$
0.3	.008	.349
0.2	.006	.266
0.1	.004	.150